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A simple closed-form analytical model for the column buckling of omega-stringer-stiffened panels with periodic boundary conditions

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A R T I C L E I N F O

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A B S T R A C T

During the preliminary design of stiffened panels, the stability behaviour is critical and both buckling modes, global and local, have to be considered in order to avoid panel configurations in which the minimum stiffness of the stringers is not achieved. In the present work, a new simple computational model is presented that computes the critical column buckling load of an omega-stringer-stiffened panel with periodic boundary conditions. This is achieved by obtaining the effective stiffness properties for an equivalent composite column. The model is able to predict column buckling conservatively as numerical studies show and can easily be used, e.g. as a constraint for optimization studies.

1. Introduction

In a previous study by the authors, a new closed-form analytical method has been developed that predicts the linear local stability behaviour of omega-stringer-stiffened composite panels [1]. This computational method is intended for preliminary design, e.g. optimization of the stringer geometry. However, since only the linear local stability behaviour is covered, a computational highly efficient approach for the column buckling behaviour is necessary, i.e. for the case the bending stiffness of the stringers is too small and no local buckling occurs. Since periodic boundary conditions are assumed along longitudinal edges, as also presented by Mittelstedt [2] in a previous study for open-cross-sectional stringer geometries, the case of a panel simply-supported along all edges is not applicable. The deformation of the current problem is sketched in Fig. 1 where column buckling is shown. Thus, the present work aims to introduce an approach for stringer-stiffened composite panels that are modelled with periodic boundary conditions which is exemplarily derived and discussed for omega stringers $((\cdot)_{st})$. For this case no previous work is known to the authors that covers the global buckling behaviour in a closed-form analytical fashion.

2. Computational method

The idea of the present approach is to idealize the column buckling of omega-stringer-stiffened panels as a column that is simply supported at both ends, as shown in Fig. 1. Euler's corresponding column buckling formula is presented in Equation (1) (see Timoshenko and Gere [3]).

$$F_{cr} = \frac{\pi^2 EI}{a^2} \tag{1}$$

Consequently, two steps are needed in order to obtain an expression for the critical column buckling load N_{cr}. First, the critical buckling force F_{cr} is remodelled as line load related to the skin $((\cdot)_{sk})$ by requiring that the elongation due to the force is equal to the elongation due to the line load, as shown in Equation (2).

$$\varepsilon = \frac{F}{\widehat{EA}} = \frac{Nb_1}{E_{sk} b_1 t_{sk}} \quad \text{with} \quad E_{sk} = \frac{1}{t_{sk} a_{11,\ sk}} \tag{2}$$

By combining Equation (2) with Equation (1) and solving for N_{cr}, the following expression is obtained.

$$N_{cr} = \frac{\pi^2\, E_{sk}\, b_1\, t_{sk}\, \widehat{EI}}{\widehat{EA}\, b_1\, a^2} \tag{3}$$

In the second step, expressions for the effective stiffnesses $\widehat{EI}$ and $\widehat{EA}$ for the panel assembled from composite plates are obtained according to the method described by Kollár and Springer [4]. The panel cross-section is divided into sub-plates of which their respective contribution to the effective bending stiffness is summed. For the present case, this leads to Equations (4) and (5).

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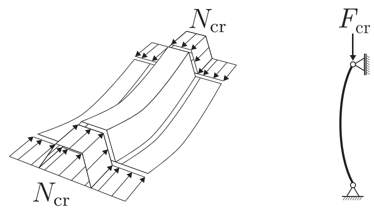

Fig. 1. Column buckling of an omega-stringer-stiffened panel under uniaxial compressive load (left) and an idealized model of Euler case II (right).

$$\widehat{EI} = \sum_{k=1}\left[\left(\frac{b_k\, z_k^2}{a_{11,\,k}} + \frac{b_k^3 \sin^2\alpha_k}{12\, a_{11,\,k}}\right) + \frac{b_k \cos^2\alpha_k}{d_{11,\,k}}\right] \tag{4}$$

$$\widehat{EA} = \sum_{k=1}\frac{b_k}{a_{11,\,k}} \qquad z_k = \overline{z}_k - z_c = \overline{z}_k - \frac{\sum_{k=1}^{K} \overline{z}_k \frac{b_k}{a_{11,\,k}}}{\sum_{k=1}^{K} \frac{b_k}{a_{11,\,k}}} \tag{5}$$

In Fig. 2 the $\overline{z}$-coordinates in reference to the arbitrary coordinate system are given for each of the plates. The index k denotes the number of the sub-plate and K the overall number of sub-plates.

This finally results in the following expression for the critical global buckling load (Eq 6).

$$N_{cr} = \frac{\pi^2}{a^2\, a_{11,sk}\left(\frac{2\,b_2+2\,b_3+b_4}{a_{11,st}} + \frac{b_1}{a_{11,sk}}\right)}\left(\frac{b_1}{d_{11,sk}} + \frac{b_1\, z_1^2}{a_{11,sk}} + \frac{\frac{1}{6}\, b_3^3\cos(\alpha)^2 + 2\, b_3\, z_3^2 + 2\, b_2\, z_2^2 + b_4\, z_4^2}{a_{11,st}} + \frac{2\, b_2 + b_4 - 2\, b_3\left(\cos(\alpha)^2 - 1\right)}{d_{11,st}} + \frac{b_1 z_1^2}{d_{11,sk}}\right) \tag{6}$$

In this formula, the geometric parameters can be taken from Fig. 2 and a_{11} and d_{11} are elements of the compliance matrices a_{ij} and d_{ij} obtained from the constitutive relations for cross-ply laminates based on the classical laminated plate theory ($a_{ij} = (A_{ij})^{-1}$, $d_{ij} = (D_{ij})^{-1}$) (See e.g. Jones [5] and Kollár and Springer [4]).

3. Results and discussion

The new closed-form analytical solution is evaluated in combination with the previously mentioned work covering the local buckling behaviour [1]. Exemplary results are shown in Fig. 3 for different widths b_1 and

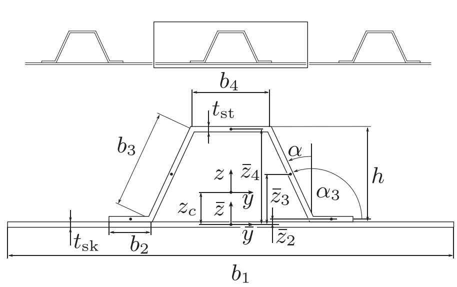

Fig. 2. Geometry of omega-stringer and skin in chosen unit cell with $\overline{z}_1 = 0$, $\overline{z}_2 = \frac{1}{2}(t_{st} + t_{sk})$, $\overline{z}_3 = \frac{h}{2}$ and $\overline{z}_4 = h$.

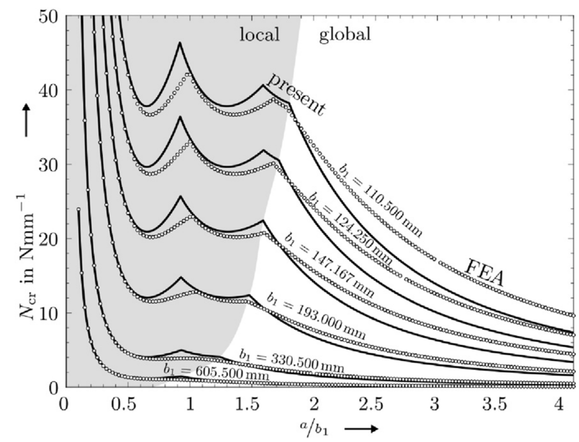

Fig. 3. Buckling curves for different stringer spacings b_1 of a stringer with $b_2 = 0.1(b_1 - 10.5$ mm); $b_3 = 6.263$ mm; $b_4 = 5.206$ mm; $h = 5.676$ mm; $\alpha = 25°$; stringer lay-up [0° 90° 0° 90° 0° 90° 0°]; skin lay-up $[0°\ 90°\ 0°\ 90°]_s$.

a cross-ply laminate with a different number of layers for skin and stringer. It becomes clear that the local buckling mode is modelled with good agreement and the global buckling load prediction of the present work is able to capture the stability behaviour for high aspect rations $\frac{a}{b_1}$. It is notable that the new method gives a conservative result, because the edges are modelled free and are consequently not stiffened by the adjoining panels, as periodic boundary conditions imply.

4. Conclusion

In combination with the approximative model for the local buckling mode [1] the present approach to the global column buckling behaviour is able to solve the very complex problem of the linear stability behaviour of an omega-stringer-stiffened composite panel in an approximate closed-form fashion. Therefore, a set of computationally highly efficient methods is available for use in preliminary design for, e.g. optimization of aircraft fuselage panels.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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