# Fiscal Policy and Government Debt: Theory and Empirical Evidence

## Dissertation

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**by**

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To my parents

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# Frequently Used Symbols

## General Notations

| | |
|---|---|
| $X_t$ | variable $X$ at time $t$ |
| $x$ | if not denoted otherwise, $x = \log X$ |
| $X^i$ | variable $X$ for country $i$ |
| $X_{i,t}$ | variable $X$ of the children-generation ($i = 1$), the parents-generation ($i = 2$), or the old generation ($i = 3$) in period $t$ (Chapter 2) |
| $X^{CA}$ | cyclically adjusted variable $X$ |
| $X_{ex}$ | variable $X$ for the model setup where debt is held by foreigners |
| $X_c$ | variable $X$ for the model setup where debt is held by own citizens |
| $\check{X}$ | variable $X$ in per capita terms |
| $\tilde{X}$ | variable $X$ expressed per labor |
| $\tilde{X}^*$ | steady state value of the per labor variable of $X$ |
| $\breve{X}$ | given path of variable $X$ |
| $\widehat{X}$ | deviation of variable $X$ from its steady state value $\bar{X}$ |
| $\bar{X}$ | steady state value of variable $X$ |
| $X^*$ | union-wide value of variable $X$ |

| | |
|---|---|
| $x^n$ | natural level of variable $x^i$, level that would occur in an economy without nominal rigidities |
| $x^{i,g}$ | gap variable of variable $x^i$, difference of $x^i$ and its natural level $x^n$ |
| $\ddot{x}$ | variable $X$ in log per capita, with $x = \log X$ |
| $f'$ | first derivative of function $f$ |
| $E_s$ | expectations operator at time $s$ |
| $\Delta$ | difference operator |
| var | variance term |
| $cov$ | covariance |
| $\vec{X}$ | matrices |
| $\vec{x}$ | vectors |

## Upper Case Latin Letters

| | |
|---|---|
| $A$ | total factor productivity |
| $A^i$ | technology level of country $i$ |
| $B$ | debt |
| $B^i$ | nominal debt level of country $i$ |
| $B^{i,r}$ | real debt level of country $i$ |
| $B_{cyc}$ | cyclical budget balance |
| $C$ | consumption |
| $C^i$ | consumption goods of country $i$ |
| $C^i_F$ | aggregate of consumption goods produced in a foreign country |
| $C^i_i$ | aggregate of consumption goods produced in the home country |
| $C^i_i(j)$ | consumption good $j$ produced and consumed in country $i$ |
| $C^i_f(j)$ | consumption good $j$ produced in country $f$ and consumed in country $i$ |
| $CFC$ | consumption of fixed capital in business sector |
| $CFC_{Gov}$ | consumption of fixed capital in government sector |
| $D^i_{t+1}$ | nominal payoff of the chosen portfolio held at the end of period $t$ |

| | |
|---|---|
| $\vec{E}$ | vector of structural shocks |
| $EX$ | exports |
| $F(K, L)$ | production function depending on capital and labor |
| $G$ | public good |
| $G^d$ | government spending excluding capital and interest spending |
| $GC$ | government consumption |
| $GFC$ | gross fixed capital formation |
| $I$ | investment, investment in business sector |
| $IG$ | investment in the government sector |
| $IH$ | investment in housing |
| $Im$ | overall imports of Germany |
| $IM$ | imports |
| $INV$ | investment |
| $K$ | capital, capital stock of the business sector |
| $L$ | labor |
| $L_{all}$ | overall working age population |
| $L_B$ | employment in the business sector |
| $L_B^{pot}$ | potential employment in the business sector |
| $L_{Gov}$ | employment in the government sector |
| $N$ | time spent working (Chapter 3) |
| $N_i$ | population size of generation $i$ (Chapter 2) |
| $N^i(j)$ | labor utilized to produce good $j$ |
| $NAWRU$ | smoothed non-accelerating wage rate of unemployment |
| $NAWRU_o$ | original series of the non-accelerating wage rate of unemployment |
| $NTIND$ | net indirect tax revenue |
| $P^i$ | price index of country $i$ |
| $\bar{P}^i$ | reset price in country $i$ |
| $P^i(j)$ | price of good $j$ in country $i$ |
| $P^*$ | union-wide price level |

| | |
|---|---|
| $P_c^i$ | consumer price index in country $i$ |
| $PART_{trend}$ | trend participation rate in the labor market |
| $Q_{t,t+1}$ | stochastic discount factor for one period ahead payoffs of period $t+1$ |
| $R_t^*$ | gross nominal return of a riskless one-period bond, which yields one unit in $t+1$ |
| $\log R_t^{*,n}$ | log natural level of the rate of return |
| $S$ | savings |
| $S^i$ | effective terms of trade for country $i$ |
| $S_f^i$ | bilateral terms of trade between country $i$ and country $f$ |
| $T^i$ | lump-sum tax introduced to finance a steady state subsidy that offsets the distortions caused by imperfect competition and the respective taxes in steady state |
| $T_i^d$ | tax revenue of the 5 tax categories personal income taxes, social security contributions, corporate income taxes, indirect taxes, residual term $ctrr$ |
| $TB_i$ | tax revenue of the respective tax category $i \in$ {personal income taxes, social security contributions} |
| $U$ | utility function |
| $\vec{U}$ | vector of estimation residuals |
| $UBG$ | average ratio of the unemployment benefit to total government expenditure |
| $ULF$ | unemployment rate |
| $W^i$ | nominal wage rate in country $i$ |
| $W_B$ | amount of real wages in the business sector |
| $W_{Gov}$ | compensation of employees |
| $WF^i$ | welfare function of country $i$ |
| $\vec{X}_t$ | vector of endogenous variables |
| $X^N$ | non tax revenues less interest on debt and net capital outlays |
| $Y$ | production, GDP |

| | |
|---|---|
| $Y^{pot}$ | overall potential output |
| $Y^i(j)$ | production of good $j$ in country $i$ |
| $Y_B$ | gross value-added in the business sector |
| $Y_{Gov}$ | value-added in the government sector |

## Lower Case Latin Letters

| | |
|---|---|
| $a^{tr}$ | trend factor productivity |
| $\widehat{b}^{i,r}$ | log deviation of real debt from steady state in country $i$ |
| $cir^j$ | annualized cumulative impulse response of variable $j$ |
| $\tilde{d}$ | donation per worker |
| $d_{ss}$ | steady state value of the debt per output ratio |
| $def$ | deficits |
| $e$ | real effective exchange rate |
| $e^j$ | structural shock of variable $j$ |
| $eeim$ | exchange rate elasticity of imports |
| $eelf$ | employment elasticity of the labor force |
| $eew$ | employment elasticity of wages |
| $f\left(\tilde{k}\right)$ | production function depending on the capital-labor ratio $\tilde{k}$ |
| $fb^*$ | union-wide variable in the government solvency constraint |
| $fy^*$ | union-wide variable in the change-in-output equation |
| $g$ | fiscal variable |
| $g_n$ | generation growth rate, equal across generations |
| $g_w$ | nominal money wage growth rate |
| $\ddot{g}$ | log government spending per capita |
| $i$ | nominal long-term interest rate |
| $\ddot{im}$ | imports from EMU member countries in logarithms per capita |
| $ir^j$ | impulse response of variable $j$ |
| $k$ | log business sector capital stock |

| | |
|---|---|
| $l_B^{pot}$ | log potential employment in the business sector |
| $mc^i$ | log real marginal costs in country $i$ |
| $oecit$ | output elasticity of corporate income taxes |
| $oee$ | output elasticity of employment |
| $oeit$ | output elasticity of indirect taxes |
| $oepit$ | output elasticity of personal income taxes |
| $oescc$ | elasticity of social security contributions with respect to output |
| $oet$ | output elasticity of transfers |
| $oetc_i$ | output elasticity of tax category $i$ |
| $q$ | fixed amount of income to be redistributed |
| $r$ | interest rate |
| $r^a$ | annual interest rate |
| $r^*$ | nominal interest rate |
| $rw^i$ | log real wages in country $i$ |
| $t$ | index for time, deterministic time trend |
| $\ddot{t}$ | log tax revenue per capita |
| $u$ | unemployment rate |
| $u^j$ | estimation residual of variable $j$ |
| $\vec{u}$ | vector of decision variables of the fiscal authority |
| $\tilde{w}$ | wage rate |
| $\tilde{w}^{net}$ | net wage rate |
| $\vec{x}$ | vector of predetermined and forward looking variables |
| $\vec{x}_{1,t}$ | vector of predetermined variables in time $t$ |
| $\vec{x}_{2,t}$ | vector of forward looking variables in time $t$ |
| $wepit$ | real wage elasticity of personal income taxes per worker |
| $wessc$ | real wage elasticity of social security contributions per worker |
| $wr$ | wage rate |
| $y_B^{pot}$ | log potential output in the business sector |

## Greek letters:

| | |
|---|---|
| $\alpha$ | capital share of income |
| $\alpha_{ge}$ | exchange rate elasticity of government spending |
| $\alpha_{gi}$ | interest rate elasticity of government spending |
| $\alpha_{gim}$ | import elasticity of government spending |
| $\alpha_{g\pi}$ | price elasticity of government spending |
| $\alpha_{gy}$ | output elasticity of government spending |
| $\alpha_{te}$ | exchange rate elasticity of net taxes |
| $\alpha_{ti}$ | interest rate elasticity of net taxes |
| $\alpha_{tim}$ | import elasticity of net taxes |
| $\alpha_{t\pi}$ | price elasticity of net taxes |
| $\alpha_{ty}$ | elasticity of net taxes with respect to output |
| $\alpha_{yg}$ | elasticity of output with respect to government spending |
| $\alpha_{yt}$ | elasticity of output with respect to net taxes |
| $\beta$ | discount factor of utility |
| $\beta_{gt}$ | elasticity of government spending with respect to net taxes |
| $\beta_{tg}$ | elasticity of net taxes with respect to government spending |
| $\chi^i$ | employment subsidy on the wage in country $i$ |
| $\gamma^i$ | steady state government expenditure per output ratio of country $i$ |
| $\gamma^{i,n}$ | natural level of government expenditure per output ratio of country $i$ |
| $\delta$ | depreciation rate |
| $\vec{\epsilon}$ | vector containing the shock residuals on the corresponding variables of vector $\vec{x}_1$ |
| $\varepsilon$ | elasticity of substitution between different varieties of public goods produced in country $i$ |
| $\varepsilon^{i,a}$ | white noise of the AR(1) process of log technology $a$ |
| $\lambda$ | Lagrange multiplier |
| $\lambda_{HP}$ | $\lambda$-parameter for the Hodrick-Prescott filter |
| $\lambda_{SP}$ | Lagrange multiplier of the social planner's maximization problem |

| | |
|---|---|
| $\log \mu^i$ | optimal price markup in the absence of sticky prices in country $i$ |
| $\nu$ | weight of the imported goods of country $i$ produced abroad, measure of openness |
| $\Pi^i$ | share of profits in the firms of country $i$ |
| $\pi$ | inflation rate |
| $\pi_c^i$ | CPI-inflation rate of country $i$ |
| $\pi^i$ | domestic inflation rate of country $i$ |
| $\pi^*$ | union-wide inflation rate |
| $\rho$ | rate of time preference |
| $\rho_a$ | coefficient of AR(1) process of log technology $a$ |
| $\sigma$ | coefficient of relative risk aversion |
| $\tau$ | lump-sum tax |
| $\tau^{i,l}$ | wage tax in country $i$ |
| $\tau_i^{i,s}$ | sales tax in country $i$ |
| $\tau^{f,s}$ | sales tax in country $f$ |
| $\vartheta_i$ | constant in the solution of a difference equation (Chapter 3) |
| $\theta$ | part of firms who are not able to reset prices to the reset price $\bar{P}^i$ |
| $\psi^i$ | relative weight of public goods $G^i$ to consumption goods $C^i$ in the utility function |
| $\xi_x^i$ | Lagrange multiplier of the union-wide welfare optimization problem concerning the constraint of variable $x$ |
| $\varphi$ | elasticity of labor supply |
| $\omega_{x^*}$ | Lagrange multiplier of the union-wide welfare optimization problem concerning the constraint of the union-wide variable $x^*$ |

# Chapter 1

# Introduction

## 1.1 Motivation

Over centuries, economists have been engaging in the question how fiscal policy affects the economy. Fiscal policy in this context is defined as using government expenditures and taxation to influence the level of aggregate demand in the economy.

The Keynesian theory advocates active fiscal policy to stabilize output over the business cycle. The main ideas of the Keynesian theory were developed during the Great Depression and were first presented by John Maynard Keynes in "The General Theory of Employment, Interest and Money" in 1936 [72]. Two types of policy are usually distinguished: expansionary and contractionary fiscal stimuli. A fiscal expansion is carried out in two ways. The government raises expenditures or it can cut net taxes, i.e., the difference between taxes and transfer payments. An expansion in spending raises the demand for goods and services directly. If the government cuts taxes or increases transfer payments, mainly the households' disposable income increases such that they can spend more on consumption. This indirectly raises the aggregate demand for goods and services. A contractionary fiscal stimulus is carried out by reducing government spending or raising net taxes.

Fiscal policy can be conducted pro- or countercyclically to the business cycle. Procyclicality means that fiscal actions intensify the booms and recessions. In contrast, a countercyclical policy, which is advocated by Keynesian economists, means that the policy acts against the tide of the business cycle: if an economy is in a recession, the government pursues expansionary fiscal policy to stabilize output. This can be successful if the fiscal expansion is well chosen in intensity and if there are production input factors such as the labor force or machinery that are idle. Output is raised and prices are rarely affected in that case. In contrast, if the unemployment rate is very low, an expansionary fiscal stimulus will affect inflation rather than output.[1] Moreover, fiscal policy is countercyclical if the government pursues a contractionary fiscal policy in a boom by raising taxes or cutting government expenditure to suppress inflation.

In the business cycle, the economic system is influenced by automatic stabilizers, which intensify expansionary fiscal policy in a recession and contractionary fiscal policy in a boom without taking an explicit policy decision. This coincides with countercyclical fiscal policy. Some examples are the unemployment benefits or the personal income taxes: the number of unemployed workers increase in a recession and thus, unemployment benefits as one component of government expenditure rise. Moreover, the income of the households declines due to the recession such that the personal income tax revenue as one component of the overall tax revenue declines. Thus, spending is raised and the tax revenue decreases in a recession, which corresponds to countercyclical fiscal policy.

Not only automatic stabilizers respond to the business cycle, but also the fiscal policy institutions directly react to changes in the economic situation. Two examples are the application of fiscal policy during the Great Depression (1929-end 1930s) and during the current economic crisis (2007/2008-?).

Figure 1.1 shows the world industrial output from June 1925 and from April 2008 onwards with monthly data.[2] The data are normalized to 100 for the starting period of the respective series, which is chosen as the last peak before the production begins to decline.

The run of the two series is more or less similar for the first year after the peak. World

[1] See for example Weil [126].

[2] The figure is taken from "A Tale of two Depressions", by B. Eichengreen and K. O'Rourke (2009/2010), [45].

industrial output declines severely in the Great Depression 1929 and in the financial crisis of 2008. But contrary to the Great Depression a lasting fall in production cannot be observed for the second and third year after the peak in the current crisis.

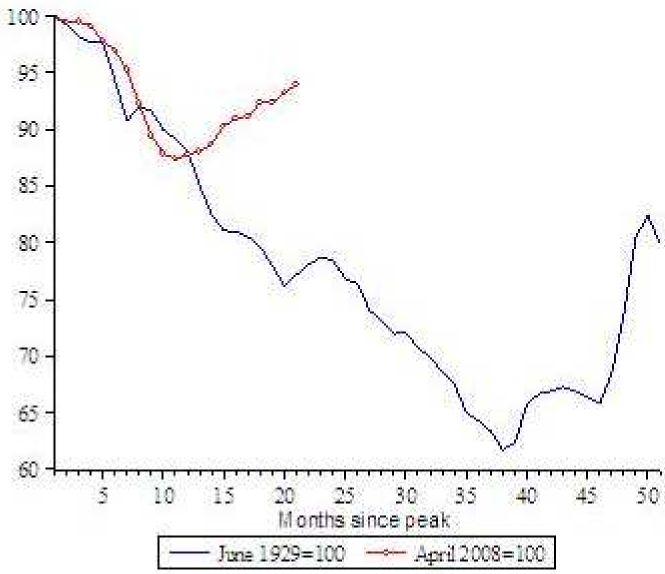


Figure 1.1: World Industrial Production

In the 1920s, the economic downturn began in the USA. Contrary to the traditional policy of laissez-faire in American depressions before 1929, US president Herbert Hoover decided to respond to the Great Depression shock with several bailout and spending programs. Farmers' subsidies were introduced and public works were intensified. In 1929, the government had a budget surplus and decided to cut taxes. But the higher government expenditures in combination with the decline in income caused a sharp government deficit—see Rothbard [115]. Since Hoover strictly believed in balanced budgets, the Congress enacted the Revenue Act of 1932. To stabilize the budget balance, tax rates were raised excessively.

The New Deal was engaged with the election of Franklin D. Roosevelt in 1932 during times of the lasting recession. This fiscal policy package was a series of economic programs containing amongst other things the Work Projects Administration, which employed unskilled workers in public works projects and the Social Security Act, which covered unemployment insurance and benefits for retirement, disability, survivorship, and death. Moreover, the Agricultural Adjustment Act, which should strengthen farmers and the National Recovery Administration, which cartelized the American industry, were part of the New Deal. The latter two Acts caused a further reduction in output. Overall, the USA did not recover from the depression until World War II.

Brown [26], Peppers [107], or Renaghan [111] amongst others examined fiscal policy for the period 1929-end 1930s and found that fiscal policy was at least a bit expansionary during this period. Overall, Roosevelt still believed in balanced budgets and did not set fiscal policy expansionary enough to help the economy to recover. “In brief, then, it took the massive expenditures forced on the nation by the second world war to realize the full potentialities of fiscal policy.”, see Brown [26].

The depression hit Europe at the beginning of the 1930s. Fiscal policy in Germany was first not very expansionary because the budget deficit as a percent of GDP did not increase very much. Cohn [37] states that German fiscal policy was even restrictive through 1932. Then it turned towards expansion. After 1934, the deficits rose, which was caused by massive spending on public works and rearmament—see Romer [114].

Figure 1.2[3] shows the fiscal surplus, i.e., the difference between government total revenue and government total spending, in percent of GDP as an approximation of fiscal net spending for the whole world from 1925 onwards, for advanced and emerging countries and the world from 2004 onwards. The data are plotted in years and start 4 years before the main crisis.

The fiscal surplus curve for the Great Depression time period is indeed positive or only very weakly negative for 1925-1929. It turned into a deficit in 1930, which grew with the contracting economy. But compared to the world fiscal surplus curve for 2004-2010, the deficits in the 1930s were very moderate as response to the economic crisis. To fight the 2008 crisis, a huge expansion in government spending took place, raising the deficit

[3] The figure is taken from “A Tale of two Depressions”, by B. Eichengreen and K. O’Rourke (2009/2010), [45].

per GDP ratio sharply.

It seems, the role of fiscal policy in economic crises is changing. Expenditures have been raised more excessively by now and the willingness to accept high deficits has increased.[4]

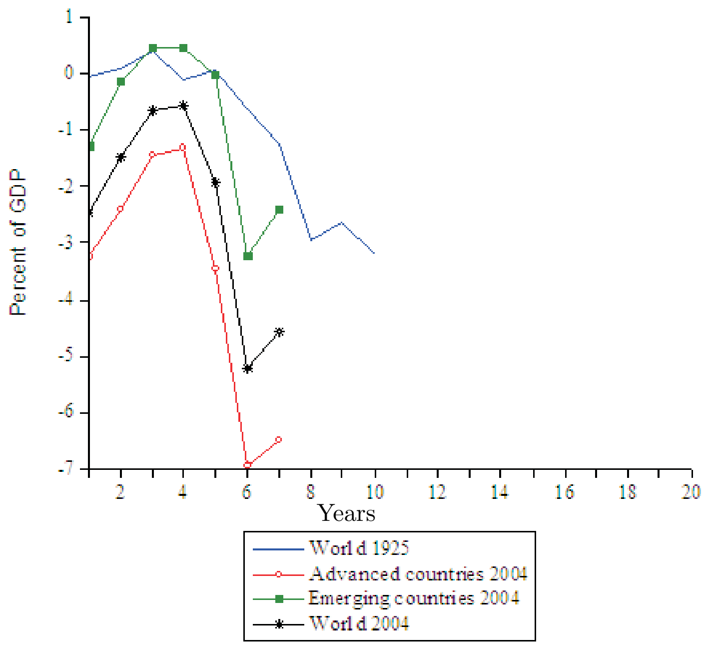


Figure 1.2: Fiscal Surplus in Percent of GDP

Examples for countercyclical fiscal policy in Germany are the German scrapping program for cars and the German fiscal stimulus packages in 2008 and 2009 as an answer to the 2008 recession. The stimulus packages focused on a child tax credit and on spending in transportation and education.

In theory, fiscal policy decisions can be made under two assumptions. On one hand, fiscal policy can be pursued under commitment. In this case, the government optimizes the present value of the objective function (e.g., a welfare function) and makes binding optimal policy choices. The future policy actions are specified in advance—see Barro [8]. In particular, Kydland and Prescott [77] look at binding rules for policy as a special form of commitment. On the other hand, fiscal policy decisions can be taken discretionary. In

[4]See B. Eichengreen and K. O'Rourke [45].

that case, the government reoptimizes its fiscal policy decision every period, i.e., it optimizes the objective function for each period. As a consequence, the economic business cycle can directly be taken into account. The optimal choice of fiscal policy today does not restrict future policy decisions—see Barro [8]. The main limitation of discretionary policy is its time-inconsistency problem due to the time lag emerging between the point of time when policy decisions are announced and the time when the fiscal decision is implemented (if it is indeed implemented).

Fiscal policy can be directly seen in the changes of the government's budget as already used in the comparison of the Great Depression and the 2008 crisis. The budget of a government can be described as the difference between net tax revenues and government expenditures within one period, e.g., a quarter or a year. Every rise in the budget corresponds to contractionary fiscal policy. In contrast, a reduction in the budget is said to be expansionary fiscal policy. If the budget is positive, the government is running a surplus. Contrary, if the budget is negative and thus if government spending exceeds government net tax revenues, the government is running a budget deficit. When the government raises its deficit by a fiscal expansion, it issues long-term bonds paying off interest and uses the proceeds to finance the deficit. By issuing bonds, it directly competes with private borrowers for money. If all other determinants of the process, in particular, the money supply, are held constant, the fiscal expansion raises the interest rates. As a consequence, private investment is reduced, affecting aggregate demand negatively.[5]

Whether a fiscal stimulus is effective or not depends on the magnitude of the crowding out effect; a question, which was heavily discussed between classical and Keynesian economists. Since an expansionary fiscal policy action raises aggregate demand on one hand though crowds out private investment on the other hand, the overall effect of the fiscal expansion depends on the relative changes in the directly and indirectly affected economic components. In an open economy, even more components of output are affected. Higher interest rates caused by the expansion of issuing bonds to finance an additional deficit attract foreign investors such that they acquire domestic assets. The

[5]See Weil [126].

increased demand of domestic currency causes an exchange rate appreciation in the short run. As a consequence, domestic goods are more expensive for foreigners and foreign goods are cheaper for residents. Imports rise and exports decline.[6]

The budget balance can be split into two parts, a cyclical and a structural component. The cyclical budget balance contains the difference between cyclical components of government revenue and expenditure. Moreover, the structural budget balance is defined as the difference of tax revenue and government spending if output would be equal to its potential level.

In most studies, the structural budget balance of a government household is used to evaluate the fiscal stance, i.e., the way fiscal policy is pursued. The structural budget balance can be calculated in several ways. The method used by the European System of Central Banks (ESCB) tries to capture changes in the structure of demand and income in more detail. Compared to other methods, it uses less aggregated data to calculate the budgetary elasticities, see, e.g., Bouthevillain et al. [23]. Furthermore, different components influencing the budget balance are chosen to take cyclical adjustment into account. A second method to calculate structural and cyclical budget balances is the OECD approach. Within this approach, output elasticities of government spending and revenues are calculated for different components of expenditure and tax categories. Then, potential output is derived. Finally, by combining the output elasticities and the potential output, the structural balance is calculated.

Economic institutions as the IMF or the OECD analyze the fiscal policy in several countries—see, e.g., the OECD Economic Outlook No. 74 [103], where fiscal policy in the three major countries Germany, France, and Italy for 1991-2002 is examined.

The efficacy of fiscal policy, in particular, the difference between shocks in government expenditure and net taxes and the existence of spill-overs caused by national fiscal policy to foreign countries are of major interest. The most frequently used working horse to analyze these questions is the structural Vector Auto-Regression (SVAR) approach. The main challenge of this approach is to identify uncorrelated structural shocks. In general,

[6] See Weil [126].

observations in time series data evolve from different influences. For example, the tax revenue is affected by changes in income. Moreover, if taxes are changed, the disposable income is affected and finally overall income changes. Thus, if fiscal variables and other economic variables (e.g., output, the inflation, or the interest rate) are examined in an SVAR framework, the resulting fiscal residuals are not independent of each other. The residuals have to be reestimated. This reestimation adjusts for automatic effects of unexpected movements in the other economic variables as output on the fiscal variables. Independent fiscal shocks are identified.

There are different methods to identify these fiscal shocks. A Choleski ordering of the variables is often used—see, e.g., Fatás and Mihov [47]. By imposing sign restrictions on the responses of economic variables to fiscal shocks, Mountford and Uhlig [100] identify uncorrelated shocks. A different approach is that of Blanchard and Perotti [21] who focus on the time lag in the decision making and implementation process of fiscal policy. Hence, they assume that a frequency of quarterly data is too short for a response of the fiscal variables to changes in output. This assumption is crucial in the identification process of shocks because cross-dependencies are excluded. Blanchard and Perotti [21] use institutional information, in particular, elasticities of the fiscal instruments (expenditures and net taxes) with respect to output and prices to identify the fiscal shocks.

The effect of fiscal policy in Germany including only national economic variables is examined, e.g., by Arcangelis and Lamartina [39], Höppner [64], Heppke-Falk et al. [61], Breuer and Buettner [25], or Bode et al. [22]. To analyze spill-over effects of German fiscal policy to foreign countries, Palacios-Salguero [105] or Giuliodori and Beetsma [57] introduce trade variables as the exchange rate or imports from member countries of the European Monetary Union (EMU).

A related topic is to analyze the dependence of national debt to fiscal policy. In this context, the national debt is defined as the aggregate total stock of government bonds and outstanding interest payments over time. When the government finances a rise in the deficit, e.g., for an expansionary fiscal stimulus by borrowing, the national debt is raised. If bonds are issued, debt can be held by own citizens or by foreigners (in the case of borrowing overseas). Greece for example is highly indebted (with 164% in 2011) and the debt is held mainly by foreigners or institutions that are not part of Greece. Japan is also highly indebted (about 232% in 2011), but debt is held primarily by domestic institutions and citizens.

Figure 1.3 shows the ratio of government debt held by non-residents to total government debt in 2009 for France, Germany, Greece, Italy, Japan, and the USA.[7]

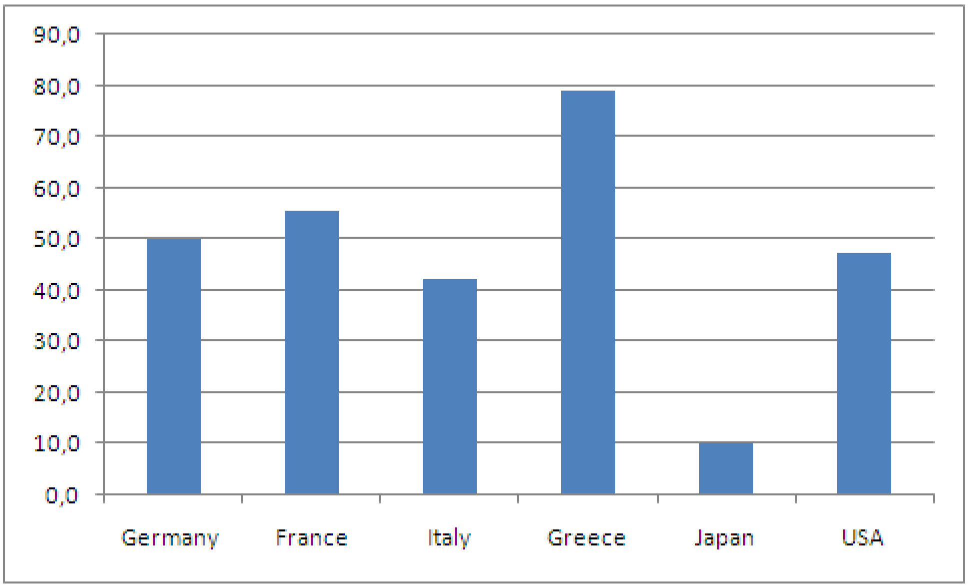


Figure 1.3: Ratio of Debt Held by Non-Residents to Total Government Debt (2009)

In this context, a quite interesting question emerges: does it make any difference to the level of the capital stock or to the level of utility of domestic citizens if the debt is internal compared to the one where it is external. Diamond [41] uses the standard two overlapping generations model implemented by Samuelson [116] to decide, which consequences arise. Barro [7] and Wickens [127] introduce a lump-sum tax payment where in one case debt is held by foreigners. In the other case, exclusively domestic citizens

[7]The data are taken from Eurostat for Italy, Germany, and France. The ratio for Greece is taken from Cabral [30]. The ratio for the USA is approximated by taking the marketable and non-marketable debt as total public debt. Moreover, the ratio of marketable and non-marketable debt held by non-residents taken from the OECD database to total debt is calculated. An alternative approach would be to derive the ratio with external public debt data taken from the U.S. Department of the Treasury, see [68]. The result is closely the same. The Japanese ratio is an imprecise estimate due to data unavailability.

hold the debt. They pay the interest due at the beginning of each period. Blanchard [19] analyzes the effect of debt on the level of steady state consumption and the interest rate in a finite horizon model. He uses explicit consumption aggregation but he needs special assumptions to do so, e.g., the assumption that the population does not grow.

Not only the distinction between the kind of debt, but also the effect of debt in special economic environments is of common interest. The establishment of the European Monetary Union (EMU) as an economic organization that has on one hand a unique currency and one monetary authority but on the other hand pursues national fiscal policies, which are uncoordinated and independent of other EMU member countries, raises new questions concerning national fiscal policy and debt levels. Possible problems of a monetary union, where fiscal authorities act independently, are analyzed by Uhlig [124]. In the current crisis, especially the consequences of high debt levels of member countries play a major role.

Figure 1.4 gives a broad overview of the public debt per GDP ratio of different countries. Each value is chosen as median value of the whole year.[8]

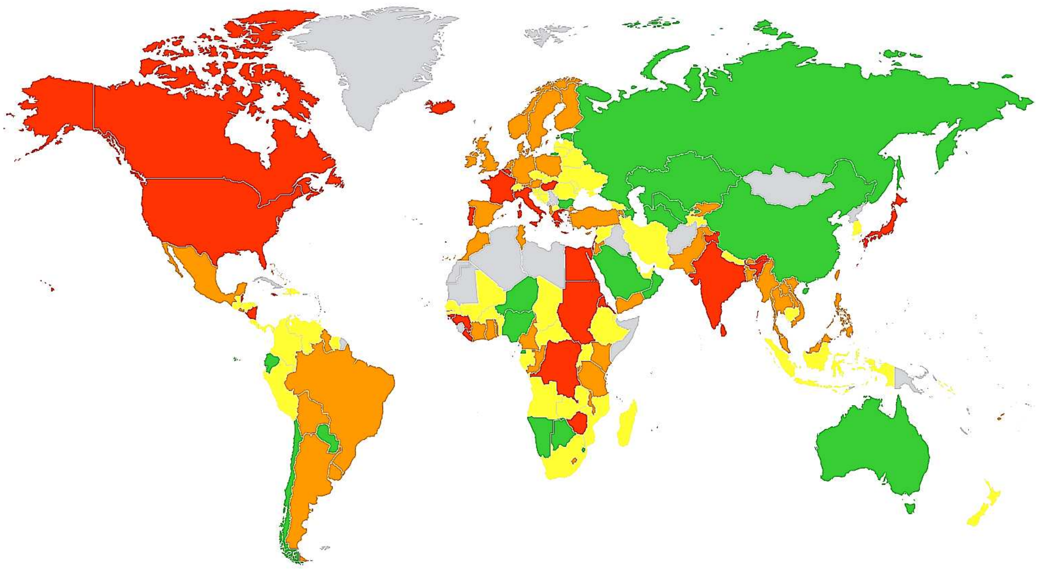

Figure 1.4: World Map of Public Debt in Percent of GDP

[8]The map is taken from the International Monetary Fund (IMF) website and visualizes the data from the Historical Public Debt Database (Fiscal Affairs Departmental Data) for 2009. See http://www.imf.org/external/datamapper/index.php.

The orange and deep red areas indicate medium to highly indebted countries, respectively. Highly indebted is defined as a debt per GDP ratio exceeding 75%. A country is colored green (yellow, orange) if the debt per GDP ratio is smaller than 20% (20-40%, 40-75%) and uncolored if no data is available for that country. Not only the US, but also Canada, Japan, India, some European, and some African countries are highly indebted according to the underlying classification scheme.[9]

Figure 1.5 shows the development of the public debt per GDP ratio for selected countries in 1980-2012.[10] The data for Germany begin in 1991 with the German reunification.

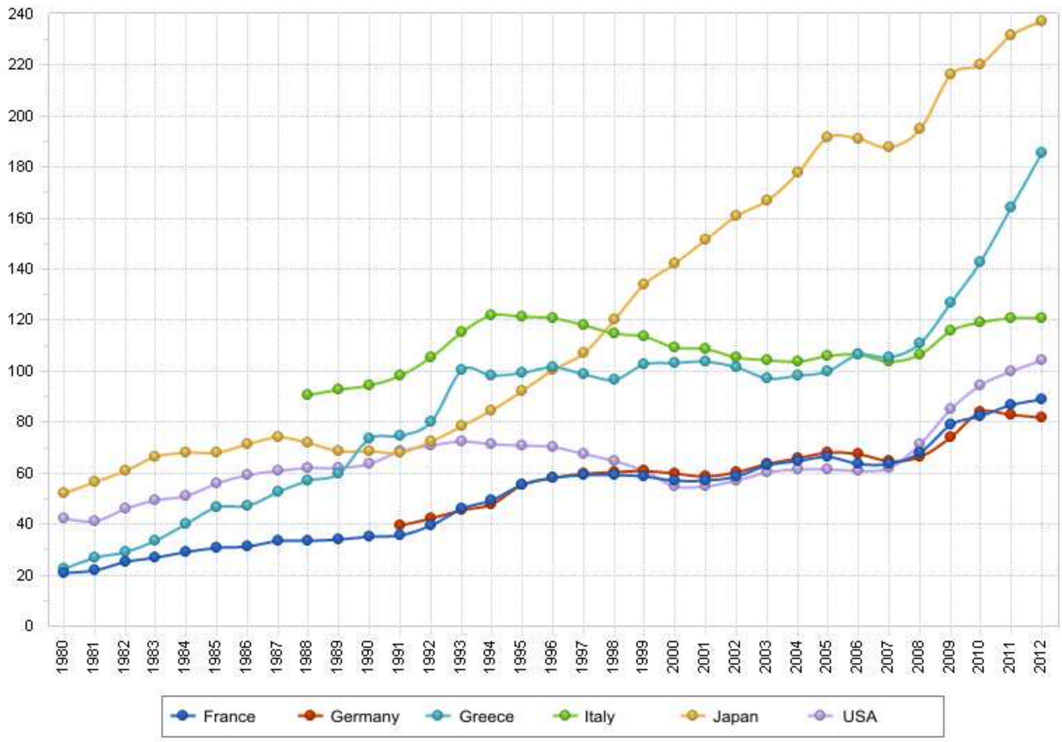


Figure 1.5: General Government Gross Debt in Percent of GDP

All countries (France, Germany, Greece, Italy, Japan, and the USA) have one thing in common: the government gross debt per GDP ratio increased over time. Starting with a

[9]Note, that the debt to GDP ratio crucially depends on the aggregation of public debt components. Every institution uses its own definition of debt such that the percentages are not equal for different databases.

[10]The graph is taken from the DSI Global Economic Statistics Database. All data are plotted in current prices.

20% debt per GDP ratio for France in 1980, the ratio grew up to 87% in 2011. The ratio for Japan increased from 52% in 1980 to 232% in 2011. Gross debt per GDP was 40% for Germany in 1991 and has risen to 83% in 2011. Germany and France show a comparable development of the debt per GDP ratio. The USA was indebted with a smaller ratio than Germany and France at the beginning of the 2000s. The ratio then rose from 2008 onwards and exceeded the respective ratios for Germany and France in 2011. Since 1992, the gross government debt per GDP in Italy amounts to more than 100% (121% in 2011). Greece's debt to GDP ratio has increased sharply from 2008 onwards. With 164% in 2011, it is the highest ratio measured for a European country. Japan has the highest debt per GDP ratio since 1998 (232% in 2011 and 237.5% in 2012).

In this context, an important question is how to reduce the debt levels without having too much loss in welfare.

Galí and Monacelli [54] develop an EMU-style model consisting of $n$ countries each with negligible size embedded in a monetary union environment. The country size restriction implies the absence of feedback effects from the other economies. The model contains national household and business sectors with imperfect competition and sticky prices, national fiscal authorities, and one union-wide monetary authority. The monetary authority maximizes union-wide welfare under the assumption that optimal fiscal policy is carried out. Fiscal authorities offer public goods efficiently and try to stabilize domestic inflation and the output gap. In contrast, Ferrero [49] includes a role for distortionary taxes and government debt (in a two country currency union). He assumes that the fiscal policy of one country affects the second country and thus derives a Phillips curve, a government solvency constraint, and a demand curve including the terms of trade. As a consequence, the terms of trade are part of the loss function and therefore, they are a target for fiscal policy. Alternative simple rules for fiscal and monetary policy are analyzed within this framework. Leith and Wren-Lewis [83] augment the Galí/Monacelli model by a government sector and examine fiscal and monetary policy if the policy makers optimize under discretion. They include income taxes as a tax instrument and analyze the consequences of debt shocks.

## 1.2 Outline and Preview of Results

This thesis analyzes the role of government debt and fiscal policy. The study is subdivided into 4 parts. The first two parts examine the role of government debt on a theoretical basis, whereas the last two parts study empirically the German fiscal stance, the effectiveness of fiscal policy, and national fiscal spill-over effects to foreign countries using time series data.[11] Chapter 6 draws conclusions and gives an outlook on new topics of research.

In particular, Chapter 2 analyzes the differences in the steady state capital-labor ratio and the steady state utility level of domestic citizens resulting if debt is held by foreigners or residents. Building on the standard two overlapping generations model by Samuelson [116] and Diamond [41], we introduce a third generation, which lives on redistributed income of another generation. This generation is called the children-generation and lives together with a parents-generation and an old generation in an overlapping generations model. Debt is included as external or internal debt and a lump-sum tax is introduced to ensure interest payments for the outstanding debt. We analyze the effect of debt on the steady state of the growth model, which can be dynamically efficient or dynamically inefficient. In the latter case, the individuals of the economy save too much and the capital stock is too high.

The following is shown: the more income is redistributed from the parents to the children, the higher is the steady state capital-labor ratio in the economy. At some critical point, the redistribution of income leads to a dynamically inefficient steady state. If the steady state is dynamically efficient, the steady state capital stock declines in both debt scenarios after the introduction of debt. Also the steady state utility level of individuals declines with the introduction of debt for both scenarios. If the steady state is dynamically inefficient, the steady state capital-labor ratio rises in the case of an external debt and falls with debt held by domestic citizens. For the external debt case, the effect on the utility level cannot be determined in general. However, if residents hold the debt,

[11] In the theoretical part as well as in the empirical part of this thesis, log or logarithm is used as a synonym for the natural logarithm.

they can substitute debt for physical capital and thus reduce the inefficiently high capital stock. This leads to a rise in utility. Overall, the consequences of debt highly depend on the debt holder structure.

The third chapter analyzes the effects of a national debt shock in a monetary union. Based on a European Monetary Union-style model with a single monetary institution and multiple fiscal authorities, we simulate a national debt shock and analyze different policy reactions to reduce the debt. Building on Galí/Monacelli [54] and Leith/Wren-Lewis [83], optimal monetary and optimal national fiscal policy under commitment with different fiscal instruments are examined.

Taking this context as given, the following is shown: optimal national fiscal policy under commitment and optimal monetary policy under commitment do not contradict each other. If debt suddenly increases in a single country, the national government should optimally lower government spending and raise taxes in order to reduce debt. Since changes in debt, government spending, and taxes influence the inflation rate, the intensity of a fiscal response is found to be very low to avoid high welfare losses. Thus, debt would be reduced very slowly. The higher the debt level in the occurrence of an additional positive shock in debt is, the stronger are the optimal fiscal policy responses and the higher is the loss in welfare. Comparing direct and indirect taxes, i.e., wage and sales taxes, as fiscal instruments, the optimal intensity of fiscal responses under commitment depends on the choice of the tax instrument: the fiscal authority has to raise sales taxes by a higher amount to obtain a comparable optimal debt reduction path. Hence, an optimal response of sales taxes to a debt shock causes higher national welfare losses. Overall, optimal fiscal policy including indirect taxes is less effective in reducing debt.

The fourth and fifth chapter present empirical findings about fiscal policy in Germany for 1991-2009. Chapter 4 examines the fiscal stance. In particular, we first estimate potential output and the elasticities of the two fiscal policy instruments, government spending and net tax revenues, with respect to output and prices using quarterly data. Combining the estimates of the elasticities and the estimated potential output, we obtain the structural budget balance. The derivation of the fiscal stance defined as changes in the structural budget balance concludes the analytical part of the situation in Germany for 1991-2009. Potential output increases within the analyzed period and actual output lies below its potential level, except for 2000-2002. Using the fiscal stance to assess fiscal policy, the following is found: the German government follows a pro-cyclical policy for most parts of 1991-2000. This behavior changes in 2001 such that fiscal policy is conducted countercyclically for most parts of 2001-2009. During challenging economic times fiscal policy responds clearly countercyclically to the economic development for the whole time horizon 1991-2009.

In the fifth chapter, the effect of fiscal shocks on real variables is examined. Fiscal shocks in this context are changes in government expenditure and revenues, i.e., fiscal policy actions. We follow a reduced form VAR approach and use the shock identification procedure introduced by Blanchard and Perotti [21] to study the effect of fiscal shocks on important economic variables like output.

First, we introduce a baseline model with the three endogenous variables real log government expenditure per capita, real log net tax revenue per capita, and real log GDP per capita. To analyze different cross-dependencies, this setting is augmented by two (the inflation and the long-term interest rate) and later four (moreover, the real log imports from EMU member countries per capita and the real effective exchange rate) additional endogenous variables. Finally, we do not allow the interest rate and the exchange rate to move by including the interest and the exchange rate as exogenous variables to decide which trade variables are affected and whether there exist spill-over effects from one EMU country to another in a perfectly functioning monetary union. The response of consumption and investment to fiscal shocks is discussed in detail for all model variations to find the economic theory matching best the empirical results for Germany. Since Germany's economy is export driven, special attention is paid to the response of exports to fiscal shocks. In this context, robust to all model variations, the following is shown: the overall deficit rises due to a positive shock in government expenditure and declines due to a positive net tax shock. Changing net taxes is a more effective instrument in changing deficits in the short run, whereas varying government expenditure influences deficits more strongly in the medium run. A positive shock in government expenditure first raises output but is followed by a decline in output after a while. Moreover, a positive shock in the net tax revenue affects output negatively. Overall, the estimated multipliers are low such that fiscal policy does not seem to be a strong instrument to stabilize output. Private consumption first rises and then declines due to a rise in government expenditure. The rise fits to Keynesian theory, whereas a decline would follow with Neoclassical theory. Thus, the observed response of consumption does not explicitly support one of the two theories. Investment declines due to a positive shock in government spending. Furthermore, imports from EMU member countries and exports are examined in detail. Both variables fall if government spending or net taxes are raised. Thus, we detect spill-over effects of German fiscal policy to foreign countries since the exports of EMU member countries and thus foreign GDPs are lowered by German fiscal shocks. A fiscal coordination for the EMU countries may be a valuable approach to stabilize not only the economic outcome of one single country but also the situation of the whole European Monetary Union.

# Chapter 2

# Debt in a Three Overlapping Generations Model

## 2.1 Introduction

In the current crisis, government debt of economies plays a major role. Within this context, there are different types of debt. Greece for example is highly indebted (with 164% in 2011) and the debt is held mainly by foreigners or institutions that are not part of Greece. Japan is also highly indebted (about 232% in 2011), but contrary to Greece, debt is held primarily by domestic institutions and citizens. Following Samuelson [116] and Diamond [41] who introduced an overlapping generations model type first, we examine an infinite horizon overlapping generations model including a household sector and a production sector in the baseline model. We augment the original household sector by a children-generation. The consumption decision of the additional generation is constrained by the transfer payments it receives from the parents-generation. The parents-generation works, saves, consumes, and supports the children without being altruistic. Thus, the chosen model structure differs from the model setting of Becker and Barro [9, 11], who implement altruism by intergenerational transfers into an overlapping generations model. They use a utility function that depends on the own as well as the utility of the descendant generation. No dynamically inefficient solution is possible in that case.[1]

Since we want to analyze the effect of different natures of debt for dynamically inefficient as well as efficient steady states, we choose the approach described above.

The old generation is the same as in the usual two overlapping generations model: it consumes the amount it saved when being a part of the parents-generation.

A three overlapping generations model is included in few other studies: three generations are implemented by Marchand et al. [93] and by Docquier et al. [42], who examine laissez faire and social optimal solutions in an endogenous growth model where human capital causes the growth of the economy. The three generations are connected by intergenerational transfers, i.e., parents finance the pensions of the old as well as the education of the young in these papers. Melindi Ghidi [97] uses a three overlapping generations model to analyze effects of demographic shocks on intertemporal consumption and saving decisions in different life cycle stages. Constantinides et al. [38] try to explain the equity premium puzzle in the context of a three overlapping generations model. They introduce debt as well as an equity in the model. The main focus lies on the accumulation of equity and the level of the interest rate.

So far, the role of debt has not been explicitly determined in a three overlapping generations framework. This is done within the underlying chapter.

To get an idea of what changes arise if we assume a model setting including debt, the new baseline model is augmented. Based on Barro [7] and Wickens [127], we introduce a lump-sum tax payment where in one case debt is held by foreigners. In the other case, domestic citizens, i.e., the parents-generation paying the interest due at the beginning of the period exclusively hold the debt. Blanchard [19] analyzes the effect of debt on the level of steady state consumption and the interest rate in a finite horizon model. He uses a consumption aggregation of different generations but he needs special assumptions to do so, e.g., the assumption that there is no population growth.

In this infinite horizon type study, the population grows with a constant rate $g_n > 0$.

[1] See Becker and Barro [9, 11] and Barro/Sala-i-Martin [10].

A visualization of the law of motion for the capital per labor ratio is undertaken using MATLAB [94] and a comparison of the model variations with different types of debt is discussed. Finally, the theoretic foundation of the effects of positive external or internal debt on the utility level of an individual is given. In this context, we differ between two cases: the model leads to a steady state that is either dynamically efficient or dynamically inefficient.[2] If consumption is too low and thus savings are too high in steady state, the steady state capital-labor ratio is too high. With too much capital, the steady state interest rate is too low. As a consequence, the steady state cannot be Pareto optimal and is dynamically inefficient. Similar to the original overlapping two generations model analyzed by Diamond [41], the underlying setting shows that debt lowers the utility level in a dynamically efficient long run equilibrium no matter who holds the debt. The steady state capital-labor ratio declines. If the steady state solution is dynamically inefficient, a rising debt raises utility for the internal debt case. The capital stock is inefficiently high such that with the introduction of debt, citizens substitute debt for physical capital in their portfolios. Hence, the dynamically inefficient steady state capital per labor ratio falls. If debt is held by foreigners, it is not part of the domestic citizens' portfolios. Capital cannot be substituted by debt. As a consequence, the introduction of debt and taxes being a subsidy in the dynamically inefficient case, raise disposable income. Savings are higher and the steady state capital-labor ratio increases. The debt effect on utility cannot be determined in general due to the existence of counteracting directions of effects.

The chapter is organized as follows: in the next three sections, a theoretical description of the new baseline model including three generations is given and the model is augmented by the existence of a debt that is held in the first case by foreigners and in the second case exclusively by residents. Section 5 calibrates the model using reasonable parameter values from the recent literature. A visualization as well as a comparison of the law of motion for the capital-labor ratio is given for the three model specifications.

[2]The transversality condition in the Ramsey Model ensures that dynamic inefficiency can never arise. In the underlying model type, it is possible to be in steady state where savings are too high and consumption is too low (i.e., the steady state is dynamically inefficient in that case).

To analyze the difference of a debt held by foreigners and an internal debt (i.e., debt is held by residents in this case), we follow Diamond [41] and derive the effect of a debt on the utility level of an individual in the sixth section. The last section gives some conclusions.

## 2.2 The Three Overlapping Generations Model

Suppose, there are three generations living at the same time: the children-generation, the parents-generation, and the old generation, which can be described as the grandparents of the economy. All variables in per capita terms relating to the population size of the respective generation are marked with a reversed hat, e.g., a variable $X$ changes to $\check{X}$. If a variable $X$ is expressed per labor, a tilde is used ($\tilde{X}$). Moreover, the model variables $X$ are indexed as follows. $X_{i,t}$ is the variable $X$ of the children-generation ($i = 1$), the parents-generation ($i = 2$), or the old generation ($i = 3$) in period $t$. Since no unemployment is introduced in the model, all members of the parents-generation work. The size of the parents-generation corresponds exactly to the number of workers, i.e., the magnitude of labor. As a consequence, a variable $X$ defined in per capita terms of the parents-generation is equal to variable $X$ in per labor terms. $\check{X}_{2,t} = \tilde{X}_t$.
In general, the optimization problem of households can be described as follows. Each member of one of the generations maximizes a standard constant relative risk aversion utility function depending only on consumption $U(\check{C}_{i,t}) = \frac{\check{C}_{i,t}^{1-\sigma}-1}{1-\sigma}$ for the corresponding lifetime horizon. $\sigma$ denotes the coefficient of relative risk aversion. $1/\sigma$ is the intertemporal substitution elasticity between consumption in any two periods, i.e., the willingness to substitute consumption in any two periods. The higher $\sigma$, the less willing is a household to substitute consumption over time. We assume that $\sigma = 1$ and thus choose as utility function $U(\check{C}_{i,t}) = \ln(\check{C}_{i,t})$. Moreover, the generation population size belonging to the initial parents-generation is normalized to $N_{2,2} = 1$. The initial first period to be analyzed is period 2 (it is the first period, where all three generations live together). The demographic structure is described as follows. Every generation grows with a fixed rate

$g_n \in [0, 1]$ in the following periods. It holds that $N_{i,t+1} = (1 + g_n) N_{i,t} \; \forall \; i \in \{1, \; 2, \; 3\}$ and $\forall \; t \geq 2$. If $g_n = 0.5$, the generation grows by 50%. Moreover, members of a generation do not die before they finish their lifetime as a member of the old generation. Hence, the generation size, e.g., of the children-generation in $t$ is equal to the member size of the parents-generation in $t + 1$: $N_{i,t} = N_{i+1,t+1} \; \forall \; i \in 1, \; 2$ and $\forall \; t \geq 2$. Since each individual lives for 3 periods, every period is about 25 years long.

The children-generation gets a transfer payment (a donation) from the parents-generation and decides what amount to consume and what amount to save for the time when the members of the children-generation are parents.

The parents-generation works and receives labor income. In addition, it gets the savings from the time when the members were children plus the interest payments net of depreciation. The expenditures of every member of the parents-generation can be described as follows: parents support the children in the economy and therefore transfer a donation, which is proportional to their wage. As in the original overlapping generations model the parents[3] decide what amount to save for the consumption when being old and what to consume now.

The members of the old generation consume the return on savings when they were parents.

## 2.3 The Baseline Model

The first part of the following section derives the optimal consumption-savings plans of an initial parents-, an initial old, and an arbitrary children-generation. Moreover, the law of motion for the capital-labor ratio in a three overlapping generations model is calculated. The second part deals with the assumptions, under which the steady state allocations are optimal. In this context, a condition is stated that can be checked to decide whether the equilibrium is dynamically efficient or not in the calibration exercise of Section 5.

[3]This generation corresponds to the young generation in the original two overlapping generations model.

### 2.3.1 Model Structure

The modeled economy consists of a household sector, which acts like an entrepreneur. It decides how much money to invest in the business sector by choosing an amount of saving. The depreciation appears therefore in the household sector. The second sector are the firms facing constant returns to scale. Hence, the profits of firms are zero. Labor and capital are used as input factors to produce an output good $Y$. Without loss of generality, we can assume that there is a single representative firm behaving competitively in the sense that it takes the rental prices for the inputs as well as the output price as given. Furthermore, in later sections, a government sector is introduced in the model.

Period 2 is the first period to be analyzed as it is the first period, where all three generations live together. The initial parents-generation, the initial old generation, and a common children-generation are part of this period. In the following, the optimal lifetime consumption-savings plans for the three generations are derived. We begin with the initial old generation living only for one period. Afterwards, the optimization problem for the initial parents-generation for their 2 periods of lifetime is solved. Finally, we analyze the maximization problem of an arbitrary children-generation. This includes the analysis of the parents- and the old generation's behavior for $t > 2$. We assume for all derivations shown in this and the next sections that $\sigma$, the coefficient of relative risk aversion, is equal to 1. The general solutions for the optimal consumption-savings plans of the analyzed generations assuming an arbitrary value for $\sigma$ can be found in Appendix A.1.

For the initial old generation, we observe the following: every member of the initial old generation consumes an amount of $\check{C}_{3,2}$ in period 2 and does not work, save, or financially support any other member in the economy. It maximizes the utility of consumption under the constraint that the whole initial old generation is endowed with the capital per labor ratio $\tilde{K}_2$ including the respective period interest payments with the interest rate $r_2$ net of depreciation. $(1 + r_2 - \delta)\tilde{K}_2$ can be unrestrictedly used for the initial old generation's consumption, where $\delta$ denotes the constant depreciation rate of capital. The endowment per member of the old generation is adjusted by the generation's population size and leads to an initial endowment of $\tilde{K}_2/(1+g_n)$ per old inhabitant. This is caused by the demographic structure $((1+g_n)N_{3,2} = N_{3,3} = N_{2,2})$.

Therefore, the maximization problem for every member of the initial old generation is given by

$$\max_{\check{C}_{3,2}} \quad U(\check{C}_{3,2}) \tag{2.1a}$$

$$\text{s.t.} \quad \check{C}_{3,2} \leq (1+r_2-\delta)\tfrac{\tilde{K}_2}{1+g_n}. \tag{2.1b}$$

The optimal consumption decision of the initial old generation is to consume all they own, i.e., $\check{C}_{3,2} = (1+r_2-\delta)\frac{\tilde{K}_2}{1+g_n}$. The optimal consumption choice of the initial old generation only depends on the initial endowment and the interest rate in period 2.

The initial parents-generation lives in period 2 and 3. It starts living as parent and becomes old in the next period. The initial parents-generation works and gets in return an amount of $\tilde{w}_2$ per worker. It supports the children-generation by paying $\tilde{d}_2$ per member to the children-generation and decides what amount to consume now $(\check{C}_{2,2})$ and what amount to save $(\check{S}_{2,2})$ for consumption when being old $(\check{C}_{3,3})$. Overall, the members of the initial parents-generation maximize the intertemporal utility function for their lifetime horizon:

$$\max_{\check{C}_{2,2},\ \check{S}_{2,2},\ \check{C}_{3,3}} \quad U(\check{C}_{2,2}) + \beta U(\check{C}_{3,3}) \tag{2.2a}$$

$$\text{s.t.} \quad \check{C}_{2,2} + \check{S}_{2,2} \leq \tilde{w}_2 - \tilde{d}_2 \tag{2.2b}$$

$$\check{C}_{3,3} \leq (1+r_3-\delta)\check{S}_{2,2}, \tag{2.2c}$$

where $\beta$ denotes the discount factor of utility. Moreover, $\beta = \frac{1}{1+\rho}$, where $\rho$ is the rate of time preference.

The optimal decisions concerning the consumption and saving in period 2 and the consumption in period 3 are given by

$$\check{C}_{2,2} = \frac{1}{1+\beta}\left(\tilde{w}_2 - \tilde{d}_2\right) \tag{2.2.1}$$

$$\check{S}_{2,2} = \frac{\beta}{1+\beta}\left(\tilde{w}_2 - \tilde{d}_2\right) \tag{2.2.2}$$

$$\check{C}_{3,3} = \frac{\beta}{1+\beta}\left(1 + r_3 - \delta\right)\left(\tilde{w}_2 - \tilde{d}_2\right). \tag{2.2.3}$$

Thus, the initial parents-generation splits its income $\tilde{w}_2 - \tilde{d}_2$ into consumption and savings in period 2. In period 3, the members consume all they receive for saving $\check{S}_{2,2}$ in period 2.

The initial parents-generation and the initial old generation living in period 2 have a shorter overall lifetime compared to an arbitrary parents- or old generation. Contrary, the children-generation in period 2 lives, as all other children-generations in time $t > 2$, for 3 periods. Thus, the maximization problem the children-generation faces in period 2 is the same for all $t \geq 2$.

The children decide what amount of their "income", (the donation $\tilde{d}_t$ they receive in period $t$ from the parents-generation) to consume and what amount to save. The amount of savings when being a child repays to the members of the next period's $(t+1)$ parents-generation with interest $r_{t+1}\check{S}_{1,t}$. Hence, the interest on investment (savings) in period $t$ is due in period $t+1$. Because the depreciation appears in this model in the household sector, the savings (the invested capital) depreciate by $\delta\check{S}_{1,t}$. The resulting component is a part of the period $t+1$ parents-generation's income.

In $t+1$, the former children, which are parents by now, work similar to the initial parents-generation in period 2. They get a wage, support the new children-generation in $t+1$ with a subsidy $\tilde{d}_{t+1}$ and decide what amount of income to consume now and what amount to save for consumption when being old. In $t+2$ the savings of period $t+1$ are paid back including the interest due in $t+2$ and reduced by the depreciation, which takes place within period $t+1$ and $t+2$. The remaining amount is completely used for the old generation's consumption. Therefore, the intertemporal maximization problem of the arbitrary children-generation in period $t \geq 2$ is

$$\max_{\check{C}_{1,t},\ \check{S}_{1,t},\ \check{C}_{2,t+1},\ \check{S}_{2,t+1},\ \check{C}_{3,t+2}} U(\check{C}_{1,t}) + \beta U(\check{C}_{2,t+1}) + \beta^2 U(\check{C}_{3,t+2}) \tag{2.3a}$$

$$\text{s.t. } \check{C}_{1,t} + \check{S}_{1,t} \leq \frac{\tilde{d}_t}{1+g_n} \tag{2.3b}$$

$$\check{C}_{2,t+1} + \check{S}_{2,t+1} \leq \tilde{w}_{t+1} - \tilde{d}_{t+1} + (1 + r_{t+1} - \delta)\, \check{S}_{1,t} \tag{2.3c}$$

$$\check{C}_{3,t+2} \leq (1 + r_{t+2} - \delta)\, \check{S}_{2,t+1}. \tag{2.3d}$$

Using the first order conditions, we obtain the following solutions for the optimal behavior of the children-generation

$$\check{C}_{1,t} = \frac{\tilde{w}_{t+1} - \tilde{d}_{t+1} + (1 + r_{t+1} - \delta)\frac{\tilde{d}_t}{1+g_n}}{(1 + r_{t+1} - \delta)\left[1 + \beta + \beta^2\right]} \tag{2.3.1}$$

$$\check{S}_{1,t} = \frac{\tilde{d}_t}{1+g_n} - \frac{\frac{\left(\tilde{w}_{t+1} - \tilde{d}_{t+1}\right)}{1 + r_{t+1} - \delta} + \frac{\tilde{d}_t}{1+g_n}}{\left[1 + \beta + \beta^2\right]} \tag{2.3.2}$$

$$\check{C}_{2,t+1} = \frac{\beta\left[\left(\tilde{w}_{t+1} - \tilde{d}_{t+1}\right) + \frac{\tilde{d}_t}{1+g_n}(1 + r_{t+1} - \delta)\right]}{\left[1 + \beta + \beta^2\right]} \tag{2.3.3}$$

$$\check{S}_{2,t+1} = \tilde{w}_{t+1} - \tilde{d}_{t+1} + (1 + r_{t+1} - \delta)\frac{\tilde{d}_t}{1+g_n} - \frac{\left[\tilde{w}_{t+1} - \tilde{d}_{t+1} + (1 + r_{t+1} - \delta)\frac{\tilde{d}_t}{1+g_n}\right](1+\beta)}{\left[1 + \beta + \beta^2\right]} \tag{2.3.4}$$

$$\check{C}_{3,t+2} = (1 + r_{t+2} - \delta)\left[\tilde{w}_{t+1} - \tilde{d}_{t+1} + (1 + r_{t+1} - \delta)\frac{\tilde{d}_t}{1+g_n} - \frac{\left[\tilde{w}_{t+1} - \tilde{d}_{t+1} + (1 + r_{t+1} - \delta)\frac{\tilde{d}_t}{1+g_n}\right](1+\beta)}{\left[1 + \beta + \beta^2\right]}\right]. \tag{2.3.5}$$

Because the arbitrary children-generation lives for three periods, the optimal solutions describe the lifetime consumption and savings plan. Hence, the optimal consumption and savings when being a child depend on the current income $\tilde{d}_t$ and the discounted future income $\tilde{w}_{t+1} - \tilde{d}_{t+1}$. The future income plays a role because each household optimizes for the whole lifetime (3 periods). The present value of income, i.e., income in $t$ and discounted future income, are split into consumption and savings in $t$. The optimal consumption and savings plan of the parents-generation (we now consider period $t+1$) depends on the current income $\tilde{w}_{t+1} - \tilde{d}_{t+1}$ in $t+1$ and the return for the savings made as a child. The disposable income is split into consumption and saving in $t+1$. Finally, the consumption when being old depends on the return obtained from saving in $t+1$. All optimal household decisions have been calculated so far. The economy is characterized by the goods, the asset, and the labor market. Thus, the market clearing conditions for the respective market are derived in the following.

1. For the goods market, consumption $C_t$ and investment $I_t$ have to equal production $Y_t$ $\forall t$

$$Y_t = C_t + I_t. \tag{2.4}$$

2. The asset market is cleared if the new capital stock for period $t+1$, $K_{t+1}$, is equal to the savings of the children- and the parents-generation in $t$. $N_{i,t}$ denotes the population size of generation $i$ in period $t$. The asset market clearing condition is derived in Appendix A.2.

$$\check{S}_{1,t}N_{1,t} + \check{S}_{2,t}N_{2,t} = K_{t+1}. \tag{2.5}$$

3. Finally, for the labor market it has to hold that the labor $L_t$ needed in $t$ is equal to the supplied work of the parents-generation $N_{2,t}$, i.e.,

$$N_{2,t} = L_t. \tag{2.6}$$

Firms maximize profits, given as the difference of the product $Y_t$ produced by labor $L_t$ and capital $K_t$ and the marginal costs of capital and labor. The amount of capital and labor is nonnegative because otherwise, nothing can be produced in the economy. The firms of the underlying economy are facing the maximization problem

$$\max_{K_t,\ L_t \geq 0} F\left(K_t,\ L_t\right) - \breve{r}_t K_t - \breve{w}_t L_t \tag{2.7}$$

under a given path of the interest rate $\breve{r}_t$ and the wage rate $\breve{w}_t$ (given variables are marked by a semicircle). The specific production function $F$ of the economy is given by

$$F\left(K_t,\ L_t\right) = Y_t = K_t^{\alpha} L_t^{1-\alpha},$$

where $\alpha$ denotes the capital share of income. We use the clearing condition for the labor market (equation (2.6)) and define the capital-labor ratio as $\tilde{K}_t = K_t/L_t = K_t/N_{2,t}$. It follows that the production per capita, i.e., $Y_t/N_t$ can be described as

$$\frac{Y_t}{N_t} = \frac{1+g_n}{3+3g_n+g_n^2}\tilde{K}_t^{\alpha}, \tag{2.8}$$

where $N_t = N_{1,t} + N_{2,t} + N_{3,t}$ is the overall population. A second equation for the output per capita ratio is obtained by using the clearing condition for the goods market (2.4) and the definition of capital accumulation $K_{t+1} = (1-\delta)K_t + I_t$:

$$\frac{Y_t}{N_t} = \frac{1+g_n}{3+3g_n+g_n^2}\left(\check{C}_{1,t}(1+g_n) + \check{C}_{2,t} + \frac{\check{C}_{3,t}}{1+g_n} + (1+g_n)\tilde{K}_{t+1} - (1-\delta)\tilde{K}_t\right). \tag{2.9}$$

Since the structure of the labor market does not change compared to the original two overlapping generations model, the production side of the underlying economy is the same as if only the two standard generations live at the same time. Thus, we obtain

$$Y_t = \tilde{w}_t N_{2,t} + r_t K_t. \tag{2.10}$$

The output per capita ratio is then

$$\frac{Y_t}{N_t} = \frac{1+g_n}{3+3g_n+g_n^2}\left(\tilde{w}_t + r_t\tilde{K}_t\right).$$

If we define the production function with the capital-labor ratio as argument as $f(\tilde{K}_t) = \tilde{K}_t^{\alpha}$, it results that

$$f\left(\tilde{K}_t\right) = \tilde{w}_t + r_t\tilde{K}_t.$$

Taking the derivative with respect to the capital-labor ratio $\tilde{K}_t$, we get

$$\frac{\partial f(\tilde{K}_t)}{\partial \tilde{K}_t} = f'(\tilde{K}_t) = \alpha\tilde{K}_t^{\alpha-1} = r_t.$$

The interest rate is therefore given as

$$r_t = \alpha\tilde{K}_t^{\alpha-1}. \tag{2.11}$$

With diminishing returns to labor and constant returns to scale, firms do not earn any profits and the wage rate for the parents-generation can be described as

$$\tilde{w}_t = \tilde{K}_t^{\alpha} - r_t\tilde{K}_t = f(\tilde{K}_t) - f'(\tilde{K}_t)\tilde{K}_t = (1-\alpha)\tilde{K}_t^{\alpha}. \tag{2.12}$$

Finally, the law of motion for the capital-labor ratio over time is derived. First, the law of motion for $\tilde{K}_{t+2}$ with $t \geq 2$ is calculated, whereas we need a special equation for $\tilde{K}_3$. Since the initial old and the initial parents-generation live in $t = 2$ and follow special optimal consumption-savings plans due to the restricted lifetime, the investment being a function of savings is different compared to all other periods. The accumulated capital changes in this period and thus, $\tilde{K}_3$ cannot be calculated with the law of motion derived for $t \geq 2$. We begin with the law of motion for the capital-labor ratio $\tilde{K}_{t+2}$ with $t \geq 2$. The asset market clearing condition (equation (2.5)) in per labor terms for period $t+2$ leads to

$$\tilde{K}_{t+2} = \check{S}_{1,t+1} + \frac{\check{S}_{2,t+1}}{1+g_n}. \tag{2.13}$$

Inserting the optimal solutions derived in equation (2.3.2) and (2.3.4), we obtain

$$\begin{aligned}\tilde{K}_{t+2} &= \frac{1}{(1+g_n)(1+\beta+\beta^2)}\Bigg[\left(\beta+\beta^2\right)\tilde{d}_{t+1} + \beta^2\left(\tilde{w}_{t+1} - \tilde{d}_{t+1}\right) \\ &\quad + \frac{(1+r_{t+1}-\delta)\,\beta^2\tilde{d}_t}{1+g_n} - \frac{1+g_n}{1+r_{t+2}-\delta}\left(\tilde{w}_{t+2} - \tilde{d}_{t+2}\right)\Bigg].\end{aligned}$$

We assume that the parents-generation supports the children-generation with a fixed amount $q$ of their income less the return on savings when being a child, i.e., in total $q\tilde{w}_t N_{2,t}$ in period $t$. The basis for subsidy payments to children does not include the income due to interest payments on savings when being a child. The intuition is that money earned by money only outweighs the effort of abstaining from it for one period and is not redistributed from the parents- to the children-generation. With this assumption, the law of motion for the capital-labor ratio changes to

$$\begin{aligned}\tilde{K}_{t+2} &= \frac{1}{(1+g_n)(1+\beta+\beta^2)}\Bigg[\left(\beta+\beta^2\right)q\tilde{w}_{t+1} + \beta^2(1-q)\,\tilde{w}_{t+1} \\ &\quad + \frac{(1+r_{t+1}-\delta)\,\beta^2 q\tilde{w}_t}{1+g_n} - \frac{1+g_n}{1+r_{t+2}-\delta}(1-q)\,\tilde{w}_{t+2}\Bigg].\end{aligned}$$

As we consider a three overlapping generations model where every generation lives for 3 periods, one period is about 25 years long. Therefore, we can assume that the average capital stock consisting of machinery, equipment, metal products, buildings etc. is depreciated completely in one period on average. Levy [85] estimates the average service lives of the aggregate capital stock in the US as 25 years in 1985. The average service lives decrease between 1950-1985. Schidlowski and Schmalwasser [117] estimate the average service lives for different assets in 2000 for Germany. Buildings and structures have an average service life of 66 years, whereas machinery and equipment have an average service life of 12 years. The Euro area estimates are similar: 68 years for housing construction and 14 years for metal products and machinery—see Derbyshire et al. [40]. If we assume that 25% of the capital stock consists of buildings and the remaining 75% are machinery and equipment, the overall average service life is close to 25 years. Thus, we set the depreciations rate $\delta$ equal to 1.

Substituting the interest rate and the wage rate corresponding to equation (2.11) and (2.12) and with $\delta = 1$, the law of motion of the capital-labor ratio in $t+2$ only depends on former capital-labor ratios (the ratios of the periods $t+1$ and $t$).

$$\tilde{K}_{t+2} = \frac{1}{(1+(1-q)(1-\alpha)/\left(\alpha\left(1+\beta+\beta^2\right)\right))\left(1+g_n\right)}\left[\frac{\beta(q+\beta)(1-\alpha)}{(1+\beta+\beta^2)}\tilde{K}_{t+1}^{\alpha} + \frac{(1-\alpha)\alpha q\beta^2}{(1+\beta+\beta^2)\left(1+g_n\right)}\tilde{K}_{t+1}^{\alpha-1}\tilde{K}_t^{\alpha}\right]. \tag{2.14}$$

As in a standard two overlapping generations model, we do not log-linearize the law of motion for the capital-labor ratio and work instead with the exact equation.

**Result 2.1.** $\tilde{K}_{t+2}$ for $t \geq 2$

- The time horizon comprises 3 periods with three overlapping generations.
- The law of motion for the capital-labor ratio comprises three periods, too.
- There is a component (last line of equation (2.14)) that combines two periods, namely $t$ and $t+1$ with each other. Therefore, there are dependencies exceeding one single period.

Our next step is to determine the capital-labor ratio for period 3, $\tilde{K}_3$. Since the initial parents-generation saves a different amount compared to an arbitrary parents-generation due to the restricted lifetime, the asset market clearing condition changes to $\tilde{K}_3 = \check{S}_{1,2} + \check{S}_{2,2}/\left(1+g_n\right)$. We insert the optimal choice for savings of the initial parents-generation and the children-generation (equation (2.2.2) and (2.3.2)). Furthermore, we use equation (2.11) and (2.12) as well as the assumption that capital is depreciated completely within one period, i.e., $\delta = 1$. Then, the law of motion for $\tilde{K}_3$ is

$$\tilde{K}_3 = \frac{1}{1 + \frac{(1-q)(1-\alpha)}{\alpha(1+\beta+\beta^2)}} \left( \frac{q\beta^2 + \left(1 + \beta + \beta^2\right) \beta}{\left(1 + g_n\right)\left(1 + \beta\right)\left(1 + \beta + \beta^2\right)} \right) (1 - \alpha)\, \tilde{K}_2^{\alpha}. \tag{2.15}$$

**Result 2.2.** $\tilde{K}_3$
The capital-labor ratio in period 3 is determined solely by the initial endowment $\tilde{K}_2$.

### 2.3.2 Optimality of Allocations

In a standard overlapping generations model, the stability of all solutions is not guaranteed—see for example Barro/Sala-i-Martin [10]. The same is true for the underlying setting. Not all steady state solutions of the model are stable. The necessary condition for stability of the solution is the same as in the standard overlapping two generations model (see for example Blanchard/Fischer [20]).

$$0 < \frac{dr_{t+1}}{dr_t} \leq 1.$$

A marginal increase in the interest rate $r_t$ today increases the interest rate in the next period.
Investment decreases today if the interest rate increases today. As a consequence, the capital stock of tomorrow decreases. With $f'(\tilde{K}_{t+1}) = r_{t+1}$ and since the slope of the production function $f$ decreases with an increasing capital-labor ratio, every decrease in the capital-labor ratio causes a rise in the interest rate. More intuitively, since the capital supply of tomorrow is lower due to the decrease in investment today, the interest rate of tomorrow increases.
As shown, e.g., by Barro and Sala-i-Martin [10], a solution in a standard overlapping generations model can be dynamically inefficient. The same is true for the underlying model specification. A dynamically inefficient steady state is a condition of the economy, where individuals save too much and do not consume enough. As a consequence, the capital stock is inefficiently high. Moreover, it is not Pareto-optimal. To characterize the steady states found in the model, it is necessary to define the conditions for dynamical

efficiency. We can simply use a modified traditional equation stated in Diamond [41] and do not need to worry about more general conditions as, e.g., developed by Abel et al. [1] due to the model structure.

In the following, this condition is derived. Moreover, it is shown why the steady state is indeed not Pareto-optimal if the condition holds. To focus on dynamic efficiency, we undertake the following steps. Starting with the goods market clearing condition (equation (2.4)), we transform the equation into $f\left(\tilde{K}_t\right) = \tilde{K}_t^{\alpha} = \check{C}_{1,t}(1+g_n) + \check{C}_{2,t} + \frac{\check{C}_{3,t}}{1+g_n} + (1+g_n)\tilde{K}_{t+1} - (1-\delta)\tilde{K}_t$. In steady state, it holds that $f\left(\tilde{K}^*\right) = \left(\check{C}_1^*(1+g_n) + \check{C}_2^* + \frac{\check{C}_3^*}{1+g_n}\right) + (\delta+g_n)\tilde{K}^*$, where $x^*$ is the steady state value for variable $x$. Let $\tilde{c}^* = \check{C}_1^*(1+g_n) + \check{C}_2^* + \frac{\check{C}_3^*}{1+g_n}$ denote the total consumption per worker in the steady state. We get $\tilde{c}^* = f\left(\tilde{K}^*\right) - (\delta+g_n)\tilde{K}^*$. Suppose that the steady state condition satisfies

$$f'\left(\tilde{K}^*\right) - \delta < g_n. \tag{2.16}$$

This steady state would be dynamically inefficient and not Pareto-optimal because $\frac{d\tilde{c}^*}{d\tilde{K}^*} = f'\left(\tilde{K}^*\right) - (\delta+g_n) < 0$: a positive change in the capital-labor ratio affects the steady state consumption negatively. Thus, a marginal decrease in capital would lead to higher available overall consumption. Moreover, the capital stock would be inefficiently high. Utility could be raised by substituting consumption for saving. Finally, this change in savings would cause a reduction in the capital stock.

Hence, equation (2.16) states the condition for dynamical inefficiency of a steady state solution. Because $f'\left(\tilde{K}^*\right) = r^*$ with equation (2.11), the steady state solution is dynamically inefficient if $r^* < g_n + \delta$ and dynamically efficient if $r^* > g_n + \delta$.

## 2.4 Governmental Debt and Taxes

To introduce government debt in the baseline model, we assume that there is an initial debt $B_3$, which is held by foreigners or by residents. For details, see Subsection 2.4.1 and 2.4.2, respectively. Debt is modeled by one-period bonds, which are sold in period $t$. The money received from the sale is used, e.g., for infrastructure spending, but an explicit government spending variable is not introduced in the model. The bonds plus an interest on the bonds have to be repaid after one period. So the initial debt $B_3$ is due in $t = 3$ and the government has to pay $(1 + r_3 - \delta) B_3$ to the debt holders. The rate of return for debt includes, similar to the rate of return for capital, the depreciation rate. This assumption is necessary to ensure that the residents, who hold the debt in the internal debt case are willing to hold debt as well as physical capital in their portfolios. If the rate of return for debt does not include the depreciation rate, debt pays off a higher amount and capital will not be part of the asset portfolio. For symmetry, we make the same assumption for the external debt case.[4]

In order to finance the required interest payments on debt $B_t$, the government taxes the parents-generation with a lump-sum tax[5] $\tilde{\tau}_t N_{2,t}$ and faces the budget constraint

$$B_t (1 + r_t - \delta) = B_{t+1} + \tilde{\tau}_t N_{2,t}. \tag{2.17}$$

The left hand side of equation (2.17) denotes the expenditures of the government and is equal to the repayment of bonds due in $t$. The right hand side shows the income of the government in period $t$. The government sells new bonds $B_{t+1}$, which are due in $t + 1$ and gets lump-sum taxes $\tilde{\tau}_t$ per member of the parents-generation, i.e., $\tilde{\tau}_t N_{2,t}$ in total. Since the economy grows with the population growth rate, debt has asymptotically no effect if the total amount of debt is assumed to be constant. The relative amount of debt to GDP shrinks in that case over time. Thus, we adjust for the growth rate and assume that the debt-labor ratio is defined as $\tilde{B}_t = \frac{B_t}{N_{2,t}}$ $\forall t$. $\tilde{B}_t$ is constant over time, i.e., $\tilde{B}_t = \tilde{B}$ $\forall t$. If $\tilde{B}_t$ is declining over time, the importance of debt in the model decreases for a falling amount of debt. If $\tilde{B}_t$ is increasing over time, the path for debt would be explosive.

With a constant debt-labor ratio, equation (2.17) becomes

$$\tilde{\tau}_t = \tilde{B} (1 + r_t - \delta) - \frac{B_{t+1} N_{2,t+1}}{N_{2,t+1} N_{2,t}}$$

[4] We follow Diamond [41].

[5] See also Barro [7] and Wickens [127].

and therefore

$$\begin{aligned} \tilde{\tau}_t &= \tilde{B}\,(1 + r_t - \delta) - \tilde{B}\,(1 + g_n) \\ &= \tilde{B}\,(r_t - g_n - \delta)\,. \end{aligned} \tag{2.18}$$

Lump-sum taxes $\tilde{\tau}_t$ are chosen according to the interest rate $r_t$ in time $t$ and the model specific constants $g_n$ (the population growth rate) and $\delta$ (the rate of depreciation). They can be positive as well as negative because the interest rate is positive and also values larger than one are reasonable: $(1 + r^a)^{25} - 1 = r_t$, where $r^a$ denotes the annual interest rate and $r_t$ is the overall interest rate for the whole period of 25 years. If we assume that $r^a = 3\%$, then $r_t$, the interest rate for one period, would be larger than 1.

If $r_t - \delta > g_n$, then $\tau_t > 0$ and the steady state allocation is dynamically efficient. We can summarize:

> **Result 2.3.** Taxes $\tilde{\tau}$
>
> The steady state tax is positive iff the steady state is dynamically efficient and negative iff the steady state is dynamically inefficient.

Since the interest rate depends on the capital supply, taxes can be formulated as a function of the capital-labor ratio $\tilde{K}$ if we use equation (2.11):

$$\tilde{\tau}_t = \tilde{B}\left(\alpha \tilde{K}_t^{\alpha-1} - (\delta + g_n)\right). \tag{2.19}$$

Hence, the higher the capital supply in an economy, the higher the lump-sum taxes to ensure a constant debt-labor ratio.

### 2.4.1 Debt held by Foreigners

Under the assumption that debt is held exclusively by foreigners, solely the constraint for the utility maximization problem of the parents-generation changes: every member is charged with a tax to guarantee the interest repayment on the government debt paid to the foreigners. If taxes are positive, parents have less disposable income. Negative taxes act as a subsidy and raise disposable income.

We substitute

$$\check{C}_{2,2} + \check{S}_{2,2} \leq \tilde{w}_2 - \tilde{\tau}_2 - \tilde{d}_2 \qquad \text{(2.DF-2b)}$$

for the inequality (2.2b) of the initial parents-generation. The optimal choices for consumption and saving $\check{C}_{2,2}$, $\check{S}_{2,2}$, and $\check{C}_{3,3}$ are:

$$\check{C}_{2,2} = \frac{1}{1+\beta}\left(\tilde{w}_2 - \tilde{\tau}_2 - \tilde{d}_2\right), \qquad \text{(2.DF-2.1)}$$

$$\check{S}_{2,2} = \frac{\beta}{1+\beta}\left(\tilde{w}_2 - \tilde{\tau}_2 - \tilde{d}_2\right), \qquad \text{(2.DF-2.2)}$$

$$\check{C}_{3,3} = \frac{\beta}{1+\beta}\left(1 + r_3 - \delta\right)\left(\tilde{w}_2 - \tilde{\tau}_2 - \tilde{d}_2\right). \qquad \text{(2.DF-2.3)}$$

As before, the coefficient of relative risk aversion $\sigma$ is assumed to be equal to one. The optimal consumption-savings plan for the initial parents-generation now depends on the disposable income, i.e., the baseline income net of a lump-sum tax. The arbitrary period-$t$ children-generation has to pay taxes in $t+1$ when being parents. Thus, also the budget constraint for the arbitrary children-generation, when being parents, changes. Similarly to above, we substitute

$$\check{C}_{2,t+1} + \check{S}_{2,t+1} \leq \tilde{w}_{t+1} - \tilde{\tau}_{t+1} - \tilde{d}_{t+1} + \left(1 + r_{t+1} - \delta\right)\check{S}_{1,t} \qquad \text{(2.DF-3c)}$$

for the constraint (2.3c) in the arbitrary children-generation's maximization problem. Since the optimal consumption and saving decisions of the children-generation depend on income today and discounted future income, also equations (2.3.1)-(2.3.5) change. The income component of the former equations is now net of the lump-sum tax. $\tilde{w}_s$ has to be substituted by $\tilde{w}_s - \tilde{\tau}_s$ for the corresponding point $s$ in time.[6]

We assume that the donation $\tilde{d}$ is once again a fixed part $q$ of the income less the return on savings when being a child, i.e., the wage reduced by the tax $\tilde{d}_t = q\left(\tilde{w}_t - \tilde{\tau}_t\right)$.

The new law of motion for the underlying model specification is

$$\begin{aligned}
&\tilde{K}_{t+2}\left(1 + \frac{(1-q)(1-\alpha)}{\alpha\left(1-\beta+\beta^2\right)}\right) + \frac{(1-q)(1+g_n)\tilde{B}}{\alpha\left(1+\beta+\beta^2\right)}\tilde{K}_{t+2}^{1-\alpha} = \frac{(1-q)\tilde{B}}{\left(1+\beta+\beta^2\right)} \\
&+\frac{1}{1+g_n}\left[\left((1-\alpha)\tilde{K}_{t+1}^{\alpha} - \alpha\tilde{K}_{t+1}^{\alpha-1}\tilde{B} + (1+g_n)\tilde{B}\right)\left(\frac{\beta(\beta+q)}{\left(1+\beta+\beta^2\right)}\right)\right. \\
&\left.+\left((1-\alpha)\tilde{K}_{t}^{\alpha} - \alpha\tilde{K}_{t}^{\alpha-1}\tilde{B} + (1+g_n)\tilde{B}\right)\frac{\alpha}{(1+g_n)}\left(q - \frac{(1+\beta)q}{\left(1+\beta+\beta^2\right)}\right)\tilde{K}_{t+1}^{\alpha-1}\right]
\end{aligned} \qquad \text{(2.DF-14)}$$

[6] For a detailed description see equation (A.DF-3.1)-(A.DF-3.5) of Appendix A.3.

for the arbitrary case $t \geq 2$ and

$$\tilde{K}_3\left(1+\frac{(1-q)(1-\alpha)}{\alpha(1-\beta+\beta^2)}\right)+\frac{(1-q)(1+g_n)\tilde{B}}{\alpha(1+\beta+\beta^2)}\tilde{K}_3^{1-\alpha}$$
$$=\frac{1}{1+g_n}\left[\frac{(1+g_n)\left(2\beta+\beta^2+\beta^3+q\beta^2-q\beta+1-q\right)\tilde{B}}{(1+\beta)(1+\beta+\beta^2)}\right.$$
$$\left.+\left((1-\alpha)\tilde{K}_2^{\alpha}-\alpha\tilde{K}_2^{\alpha-1}\tilde{B}\right)\left(\frac{\beta}{1+\beta}+\frac{\beta^2 q}{(1+\beta)(1+\beta+\beta^2)}\right)\right] \qquad \text{(2.DF-15)}$$

for $\tilde{K}_3$. If $\tilde{B}=0$, the law of motions (equation (2.DF-14) and equation (2.DF-15)) are the same as in the baseline model, namely equation (2.14) and (2.15). The necessary condition for stability of the solution is as before $0 < \frac{dr_{t+1}}{dr_t} \leq 1$.

### 2.4.2 Debt held by Residents

Assuming that debt is held exclusively by domestic citizens does not change the optimal consumption-savings plan for all generations. Thus, equation (2.DF-2.1)-(2.DF-2.3) of Subsection 2.4.1 as well as equation (A.DF-3.1)-(A.DF-3.5) of Appendix A.3 also hold in this model specification. No further constraints or conditions appear in the maximization problems of the three generations.

But the asset market equilibrium condition changes as the savings of the children as well as the parents do not only have to absorb the supply of the physical capital stock $K_{t+1}$ but also the supply of government bonds $B_{t+1}$ issued in every period. Therefore, the new asset market equilibrium condition is

$$\begin{aligned} K_{t+1}+B_{t+1} &= \check{S}_{1,t}N_{1,t}+\check{S}_{2,t}N_{2,t} \\ \Rightarrow \quad \tilde{K}_{t+1}+\tilde{B}_{t+1} &= \check{S}_{1,t}+\frac{\check{S}_{2,t}}{1+g_n}. \end{aligned} \qquad \text{(2.DC-5)}$$

The new law of motions for the underlying model specification are

$$\tilde{K}_{t+2}\left(1+\frac{(1-q)(1-\alpha)}{\alpha(1+\beta+\beta^2)}\right)+\frac{(1-q)(1+g_n)\tilde{B}}{\alpha(1+\beta+\beta^2)}\tilde{K}_{t+2}^{1-\alpha}=\frac{(1-q)\tilde{B}}{(1+\beta+\beta^2)}$$
$$+\frac{1}{1+g_n}\left[\left((1-\alpha)\tilde{K}_{t+1}^{\alpha}-\alpha\tilde{K}_{t+1}^{\alpha-1}\tilde{B}+(1+g_n)b\right)\left(\frac{\beta(\beta+q)}{(1+\beta+\beta^2)}\right)\right. \qquad \text{(2.DC-14)}$$
$$\left.+\left((1-\alpha)\tilde{K}_t^{\alpha}-\alpha\tilde{K}_t^{\alpha-1}\tilde{B}+(1+g_n)\tilde{B}\right)\frac{\alpha}{(1+g_n)}\left(q-\frac{(1+\beta)q}{(1+\beta+\beta^2)}\right)\tilde{K}_{t+1}^{\alpha-1}\right]-\tilde{B}$$

for the arbitrary case $t \geq 2$ and

$$\begin{aligned}
&\tilde{K}_3\left(1+\frac{(1-q)(1-\alpha)}{\alpha(1-\beta+\beta^2)}\right)+\frac{(1-q)(1+g_n)\tilde{B}}{\alpha(1+\beta+\beta^2)}\tilde{K}_3^{1-\alpha}\\
&=\frac{1}{1+g_n}\left[\frac{(1+g_n)\left(2\beta+\beta^2+\beta^3+q\beta^2-q\beta+1-q\right)\tilde{B}}{(1+\beta)(1+\beta+\beta^2)}\right.\\
&\left.+\left((1-\alpha)\tilde{K}_2^{\alpha}-\alpha\tilde{K}_2^{\alpha-1}\tilde{B}\right)\left(\frac{\beta}{1+\beta}+\frac{\beta^2 q}{(1+\beta)(1+\beta+\beta^2)}\right)\right]-\tilde{B}
\end{aligned} \tag{2.DC-15}$$

for $\tilde{K}_3$. As before, if we set $\tilde{B}=0$, the law of motions (equation (2.DC-14) and equation (2.DC-15)) are the same as in the baseline model, namely equation (2.14) and (2.15). The law of motions for the capital-labor ratio are very similar for internal and external debt. For internal debt, $\tilde{B}$ has to be subtracted on the right hand side of the equations due to the new asset market clearing condition. Also in this model specification, the necessary condition for stability of the steady state solution is $0 < \frac{dr_{t+1}}{dr_t} \leq 1$.
In the next section, we calibrate all three model specifications and compare the variations including government debt with the baseline model.

## 2.5 Calibration of the Models

This section contains a possible visualization of the law of motion for the capital-labor ratio over time with a reasonable choice of calibration parameters. As explained before, we assume that the whole capital stock is depreciated within one period ($\delta = 1$). Assuming that all generations are equally patient concerning the consumption today, we set the discount factor $\beta$ equal to 0.9. Suppose, the population of an economy grows with a rate of 1% per year. This leads to the following growth rate $g_n$ within one period (i.e., 25 years) in the given model: $(1+g_n) = 1.01^{25}$. Therefore, $g_n = 0.28$. Since $\alpha$ denotes the capital share of income, we set it equal to 1/3, which is commonly used. The average percentage $q$ of expenditure for children compared to pre tax income in a household with one child lies between 16% and 27% in 2009 for the US—see also Lino [87]. Based on data in 1998, the average percentage of expenditure for children to net income for a household with one child in Germany lies between 18.9% and 23.7%—see

Münnich and Krebs [101]. Therefore, we decide to set the average ratio of redistributed income $q$ equal to 0.25: the parents-generation transfers $q = 25\%$ of their labor income as a subsidy to the children-generation. To get a visualization of the law of motion for capital under consideration of equation (2.14), we choose different starting values $\tilde{K}_2$ for the capital-labor ratio and run for every starting value the law of motion for a chosen time horizon $t \in \{1, \ldots, T\}$. All paths include the steady state value, i.e., no matter what starting value is chosen, every capital per labor law of motion converges over time to $\tilde{K}^*$. The law of motion is three-dimensional, i.e., the dependence on two different points in time forces the law of motion to lie in a three and not in a two dimensional space.

We choose $\tilde{B}$ to be the same constant in all model simulations. This constant is independent of the respective starting value $\tilde{K}_2$ being the initial endowment of the economy. Since the Maastricht criterion postulates that government debt to GDP must not exceed 0.6, we set the debt-labor ratio $\tilde{B}$ to 60% of the smallest for simulation purposes chosen initial output produced with the initial capital per labor ratio $\tilde{K}_2$.

### 2.5.1 The Baseline Model

This subsection deals with the baseline model. The figures displayed in Appendix A.4 illustate the law of motion for the capital-labor ratio over time. The three-dimensional 45°-line shows all possible steady state allocations. It holds $\tilde{K}_t = \tilde{K}_{t+1} = \tilde{K}_{t+2}$ $\forall t$ on that line, which is equal to the steady state condition. Figure A.1 and Figure A.2, which projects the three-dimensional area of Figure A.1 to a two-dimensional view, show the following.

**Observation 2.1.**

- The law of motion for the capital-labor ratio is increasing for all starting values $\tilde{K}_2 < \tilde{K}^*$, where $\tilde{K}^*$ denotes the steady state capital-labor ratio value.
- The steady state value for the capital-labor ratio is unique for an appropriately chosen starting value $\tilde{K}_2$.

From a mathematical point of view, there is a unique positive and stable solution. For $\alpha = 1/3$, we obtain 2 real solutions unequal to zero because the equation of the steady state value is of order 4 and we have two complex solutions or a double root at zero for $q = 0$. Moreover, one real root is smaller than zero and one real root is larger than zero. The absolute value of the two real solutions is the same in all cases. For stability, the solution has to fulfill $0 < \frac{dr_{t+1}}{dr_t} = -\frac{\tilde{K} f''(\partial \tilde{S}/\partial \tilde{w})}{1-f''(\partial \tilde{S}/\partial r)} \leq 1$, where $\tilde{S}$ is an aggregate of the amount of savings from the children- and parents-generation, expressed in per-labor terms. $f''$ is the second derivative of the production function $f$, which has the capital-labor ratio as argument. $\frac{dr_{t+1}}{dr_t}$ is the change in the interest rate $r$ in $t+1$ due to a change in the interest rate at time $t$. With $A_c = \sqrt{(1-\alpha)}\sqrt{(\beta^2(1+4\alpha q-\alpha)+q(4+3\alpha q+2\alpha\beta+2\beta-3q))}$, the solution for the steady state capital per labor ratio is $\tilde{K}^* = \left[-\frac{1}{2(q(\alpha-1)\alpha\beta)}\left(\alpha\beta-\beta+\alpha q-q+A_c\right)(1+g_n)\right]^{\frac{1}{\alpha-1}}$ if $|\alpha\beta+\alpha q-(\beta+q)| < A_c \; \forall\alpha \in (0,1)$ (this is true for the underlying calibration) and $\tilde{K}^* = \left[-\frac{1}{2(q(\alpha-1)\alpha\beta)}\left(\alpha\beta-\beta+\alpha q-q-A_c\right)(1+g_n)\right]^{\frac{1}{\alpha-1}}$ otherwise, i.e., if $|\alpha\beta+\alpha q-(\beta+q)| > A_c \; \forall\alpha \in (0,1)$. The steady state capital-labor ratio in our calibration is $\tilde{K}^* = 0.0683$. To check if the steady state value is dynamically inefficient or not, we use equation (2.16), the condition for dynamical efficiency. Because $f'\left(\tilde{K}^*\right) = \frac{1}{3}\left(\tilde{K}^*\right)^{-2/3} = 1.9950 > 1.28 = g_n + \delta$, the solution is indeed dynamically efficient.

As the part $q$ of the parents' income transferred to the children-generation is set exogenously, based on average data, we vary $q$ and analyze the model implications. Let us assume that the parents support the children-generation by a higher amount of their income, then $q$ is set to a new value $q_1 = 0.3$. Parents possess less disposable income and save less. The children save more when they receive a higher income.

Figure A.3 shows that the steady state value for the capital-labor ratio is higher in that case. The blue area corresponds to the law of motion for the capital-labor ratio assuming that $q_1 = 0.3$, whereas the red area shows the ratio under the original parameter choice. We obtain as new steady state value $\tilde{K}_1^* = 0.0771$. The steady state with $q_1 = 0.3$ is still dynamically efficient.

In general, the higher the amount of redistributed income $q$, the more capital is accumulated in the economy. This is caused by the fact that all generations have the same time preference, but the children-generation lives for one more period. Hence, children save a higher amount of the additional unit of income compared to the decrease in the parents-generation's savings due to the redistribution of income.

At some critical point, the redistribution of income leads to a dynamically inefficient steady state capital-labor ratio. In the underlying calibration, the critical value $q^c$ is 0.58. If the amount of redistributed income $q$ exceeds the critical value the economy as a whole saves too much and consumes too little. The resulting steady state is dynamically inefficient. If we assume that the parents support their children with a very high amount, say 70% of their income, the steady state value is $\tilde{K}_{1a}^* = 0.1669$. $f'(k_{1a}^*) = 1.0993 < g_n + \delta$; the steady state is dynamically inefficient. Overall, the following holds.

**Observation 2.2.**

- The additional redistribution of income between two generations causes a higher steady state capital-labor ratio.
- The steady state becomes dynamically inefficient for a sufficiently high value of the average ratio of redistributed income $q$.

Finally, we vary the rate of substitution of capital to labor. With $\alpha_1 = 1/2$, the new steady state value of the capital-labor ratio (the intersection point of the 45°-line with the blue area in Figure A.4 of Appendix A.4), $\tilde{K}_2^* = 0.0287$, is lower and dynamically efficient. It holds also for the varied rate of substitution of capital to labor that a higher amount $q$ of income to be redistributed generates a higher steady state capital-labor ratio. The critical value of $q$, changing from a dynamically efficient to a dynamically inefficient steady state solution is higher, $q^{c2} = 0.87$. As a higher capital to labor rate of substitution leads to a lower steady state capital-labor ratio, the redistribution of income causing a higher amount of savings does not lead to a dynamically inefficient steady state capital-labor ratio for the old critical value $q^c = 0.58$. Thus, a higher amount can be redistributed before reaching a steady state solution in which too much capital is accumulated, i.e., too much is saved.

### 2.5.2 Models Including Government Debt

In the underlying subsection, we compare the modified models including government debt with the baseline model. All calibration parameters stay the same as in Subsection 2.5.1.

The law of motion for capital per labor in the debt model is harder to handle as the law of motion in the baseline model. With $\alpha = 1/3$, we get a polynomial equation of order 3 to be solved for the law of motion for the capital-labor ratio $\tilde{K}$. Two of the three solutions are complex. Hence, we obtain a unique real solution for equation (2.DC-14) and equation (2.DC-15) when solving for $\tilde{K}_{t+2}$.

There is no closed form solution for the steady state value. For $\alpha = 1/3$, the equation of the steady state value is of order 7 and we find four complex roots, one negative real solution and two positive real roots. The necessary condition for stability of the solution is to assume that $0 < \frac{dr_{t+1}}{dr_t} \leq 1$, where $r_t$ denotes the interest rate in period $t$—see Subsection 2.3.2. Therefore, the smaller real steady state value is unstable such that the law of motion for the capital stock does not lead to a convergence to the steady state, but the higher of the two different values is stable. Every law of motion of an arbitrary chosen initial capital per labor value converges to that higher value over time. Thus, we always choose the largest real steady state value as the unique solution. This procedure is also used in an arbitrary overlapping two generations model including government debt, see, e.g., Diamond [41].

As explained above, the debt-labor ratio $\tilde{B}$ is set to 60% of the smallest for simulation purposes chosen initial output produced with the initial capital per labor $\tilde{K}_2$. The steady state capital-labor ratio is labeled by a subscript $b_{ex}$ if debt is held exclusively by externals.

First, we analyze the model including debt held exclusively by foreigners, i.e, external debt. This is more or less close to the actual situation in Greece where about 80% of public debt is held by non-residents.

The red area in Figure A.5 of Appendix A.4 shows the simulated law of motion for the baseline model, whereas the blue area is the simulation result under the model setting assuming that there is an external government debt. For the standard calibration ($\alpha = 1/3$ and $q = 0.25$), the two areas in Figure A.5 converge closely to each other with proceeding time, but the red area always lies above the blue area. Therefore, the steady state value for $\tilde{K}$ is higher in the case of the baseline model. Indeed, the steady state value for the capital-labor ratio with external debt is smaller, namely $\tilde{K}^*_{b_{ex}} = 0.0669 < 0.0683 = \tilde{K}^*$. Because $f'(\tilde{K}^*_{b_{ex}}) = 2.0214 > g_n + \delta = 1.28$, the steady state value is dynamically efficient.

If debt held by foreigners is included in the model, the domestic citizens have to pay lump-sum taxes to finance the interest payments for debt due at the end of each period. Thus, the income of the parents-generation declines. Since the donation $\tilde{d}$ is a part of wage income less taxes, children receive less income leaving all parameters unchanged. As a consequence, the economy as a whole saves less because parents and children save less due to a lower amount of disposable income.

If $q$ is varied, the qualitative result is the same as for the baseline model. The redistribution of income lowers the parents' savings. The children's savings rise more than the savings of the parents-generation fall such that the steady state capital-labor ratio rises with increasing $q$.

If the amount $q$ of the parents' income redistributed to the children-generation is greater or equal to the critical value $q^{c_{ex}} = 0.58$, the economy saves too much and the capital stock is too high in the steady state. As a consequence, the economy does not consume enough. Choosing once again $q = 0.7$, we get a dynamically inefficient steady state value for the capital-labor ratio: $\tilde{K}^*_{b_{ex1}} = 0.1677$ and $f'(\tilde{K}^*_{b_{ex1}}) = 1.0961 < 1.28$.

The new steady state capital-labor ratio in the dynamically inefficient case is larger than the respective one in the baseline model. Since taxes are negative if the steady state is dynamically inefficient, the disposable income of the parents-generation increases. Hence, the members of the parents-generation consume and save more. Children obtain more income because the donation depends positively on the parents-generation's income and save more. As a consequence, overall savings increase and the capital-labor ratio in steady state is higher if debt and taxes are introduced.

**Observation 2.3.** External Debt

- There is a unique, stable, positive, and real steady state solution.
- The higher the amount of redistributed income, the higher the steady state capital-labor ratio.
- If the steady state is dynamically efficient, the introduction of debt held by foreigners lowers the steady state capital-labor ratio.
- If the steady state is dynamically inefficient, the introduction of external debt causes a higher steady state capital stock.

Finally, we consider the model variation where debt is held exclusively by residents. This corresponds more or less to a situation existent in Japan. The actual external public debt to total public debt ratio is very small such that the major part of public debt is held by own citizens. The steady state value is labeled with a subscript $b_c$ if debt is held exclusively by domestic citizens.

The red area in Figure A.6 of Appendix A.4 shows, as before, the simulation area under consideration of the baseline model, whereas the green area is the simulation result for the described model including government debt held by residents, all in the dynamically efficient calibration case. The green area in Figure A.6 runs clearly beneath the area of the baseline model.

By ensuring that $0 < \frac{dr_{t+1}}{dr_t} \leq 1$ (see Subsection 2.3.2 for a closer description of the stability condition), we get a unique stable real steady state value corresponding to the largest real value obtained when solving the differential equation. For the chosen model calibration, $\tilde{K}^*_{b_c} = 0.0594$, which is smaller than $\tilde{K}^*$ as well as $\tilde{K}^*_{b_{ex}}$. $\tilde{K}^*_{b_c}$ is dynamically efficient because $f'(\tilde{K}^*_{b_c}) = 2.1903 > 1.28$.

With the introduction of taxes, the overall disposable income decreases. Thus, the capital-labor ratio in steady state is smaller than that found for the baseline model. Moreover, capital can be substituted by debt in the asset portfolio of the parents-generation if debt is held by residents. Hence, the steady state capital-labor ratio is even smaller than the one found for the external-debt model.

The higher the amount of redistributed income $q$, the more capital is accumulated in steady state, i.e., the higher the steady state capital-labor ratio. The critical value $q^{c_c}$ for the amount of income redistributed to the children generation for the internal debt case turning the steady state capital-labor ratio dynamically inefficient is $q^{c_c} = 0.41$ and thus smaller than the value found for the other model settings. If parents transfer 41% of their income to the children, the resulting steady state is dynamically inefficient and the economy saves too much.

If we choose $q = 0.7$, $\tilde{K}^*_{b_{c1}} = 0.1586$ and $f'(\tilde{K}^*_{b_{c1}}) = 1.1375 < 1.28$. The steady state value for this model choice is dynamically inefficient. Since all generations have the same time preference, the children-generation would save a higher part than the parents save less of the redistributed income unit since children live one more period compared to the parents. Overall savings rise in that case.

**Observation 2.4.** Internal Debt

- There is a unique, stable, positive, and real steady state solution.
- The higher the amount of redistributed income, the higher the accumulated physical capital steady state value.
- The introduction of internal debt causes the steady state capital-labor ratio to decrease no matter if the steady state is dynamically efficient or not.

To compare the difference between a model including external and internal debt, we use Figure A.7. The green area symbolizing again the model with internal debt runs strictly beneath the blue area, the simulation under consideration of an external debt. The figure examines a calibration that leads to dynamically efficient steady state values. If debt is held by residents, the parents-generation can, contrary to the external debt setting, substitute debt for capital in their portfolio. The path for the capital-labor ratio runs below the corresponding external debt law of motion.

Overall, in the dynamically efficient case, the existence of taxes, which have to be paid to finance foreign debt, reduces disposable income and therefore savings. Thus, the capital stock decreases. Internal debt reduces the capital stock once more due to the substitution opportunity of government debt for capital in the portfolios of the individuals. In the external debt case, parents can only hold physical capital in their portfolio, whereas income is split in two possible assets, debt and physical capital if debt is held by residents. As a consequence, the overall capital stock is smaller if domestic citizens can substitute debt for capital.

> **Observation 2.5.**
> If the steady state is dynamically efficient, the steady state capital-labor ratio is always smaller for the internal debt case compared to a model where debt is held by foreigners.

In the dynamically inefficient case, $\tilde{K}^*_{b_{ex1}} > \tilde{K}^*_{b_{1a}} > \tilde{K}^*_{b_{c1}}$. Therefore, the existence of debt raises the capital-labor ratio in the steady state if the debt is held exclusively by foreigners and lowers $\tilde{K}^*$ for internal debt (debt held by residents). With external debt, the lump-sum tax needed to ensure a constant debt-labor ratio is negative and thus raises income. Savings increase and, as a consequence, the same is true for the capital stock. Contrary, if we compare an economy without debt to an economy where debt is held by own citizens, the residents reduce the inefficiently high capital stock by substituting capital by debt in their portfolio if the steady state is dynamically inefficient. The next section answers the question why the steady state capital-labor ratio is lowest in the internal debt case if the resulting steady states are dynamically inefficient. Moreover, it is analyzed why $\tilde{K}^*_{b_{ex}}$ has to be always higher than the respective steady state value for the baseline model in that case.

## 2.6 Theoretical Foundation of Debt Effects

We observed in the calibration section that the introduction of debt ($\tilde{B} > 0$) in the baseline model without debt ($\tilde{B} = 0$) causes a lower steady state capital-labor ratio if the steady state is dynamically efficient. It holds that $\tilde{K}^* > \tilde{K}^*_{b_{ex}} > \tilde{K}^*_{b_c}$. With the introduction of debt and taxes, the overall level of income is smaller, savings are lower, and the capital-labor ratio declines. For the case of a debt exclusively held by foreigners and a model calibration such that the old steady state (the steady state in the baseline model) is dynamically inefficient, an increase in government debt increases the steady state capital stock per worker. In this case, $0.1677 = \tilde{K}^*_{b_{ex1}} > \tilde{K}^*_{1a} = 0.1669$. Since taxes are negative in the dynamically inefficient steady state, overall income is higher and the amount of savings rises. The capital-labor ratio in steady state is thus higher.

For the internal debt model, the steady state capital-labor ratio falls compared to the baseline model ($0.1669 = \tilde{K}^*_{1a} > \tilde{K}^*_{bc1} = 0.160033$). On one hand, the negative tax raises the amount of income. On the other hand, there are substitution opportunities because the asset portfolios of residents contain physical capital and debt. Overall, the steady state capital-labor ratio declines.

We analyze the effect of debt on the utility of the individuals as it accounts for the substitution of debt for physical capital in the internal debt case.

Following Diamond [41], we compare the effect of external and internal government debt on the long run equilibrium capital-labor ratio as well as on the utility level ($\frac{dU}{d\tilde{B}}$) on a theoretical basis to verify and explain the results of the calibration section.

### 2.6.1 External Debt

First, we consider the model specification after the implementation of government debt held exclusively by foreigners. We use equations (A.DF-3.2) and (A.DF-3.4) as well as the fact that the donation is a fixed part $q$ of the net income $\tilde{w}^{net}$. Hence, the steady state total savings rate as a function of the net wage $\tilde{w}^{net} = \tilde{w} - \tilde{\tau}$ and the interest rate $r$ is

$$\begin{aligned} \tilde{S}\left(\tilde{w}^{net}, r\right) &= \check{S}_1 + \frac{\check{S}_2}{1+g_n} = \frac{\left(q\beta + \beta^2\right)}{\left(1+\beta+\beta^2\right)\left(1+g_n\right)}\tilde{w}^{net} \\ &\quad - \frac{(1-q)}{\left(1+\beta+\beta^2\right)\left(1+r-\delta\right)}\tilde{w}^{net} + \frac{q\left(1+r-\delta\right)}{\left(1+\beta+\beta^2\right)\left(1+g_n\right)^2}\tilde{w}^{net}. \end{aligned} \tag{2.20}$$

The stability condition can be stated as

$$0 < \frac{dr_{t+1}}{dr_t} = -\frac{f'' \frac{\partial \tilde{S}}{\partial \tilde{w}} \left(\tilde{K} + \tilde{B}\right)}{1 - f'' \frac{\partial \tilde{S}}{\partial r}} \leq 1.$$

To find a solution for the expression $\frac{dU}{d\tilde{B}}$ describing the change in the utility $U$ due to a change in debt, we use the fact that the following equations hold:

$$
\begin{aligned}
\frac{dr}{d\tilde{B}} &= \frac{d\tilde{w}^{net}}{d\tilde{B}}\frac{\partial f'}{\partial \tilde{w}} + \frac{dr}{d\tilde{B}}\frac{\partial f'}{\partial r} \\
&= \left(-\left(\tilde{K}+\tilde{B}\right)\frac{dr}{d\tilde{B}} - (r-g_n-\delta)\right) f''\frac{\partial \tilde{S}}{\partial \tilde{w}} + \frac{dr}{d\tilde{B}} f''\frac{\partial \tilde{S}}{\partial r} \\
\Leftrightarrow & \\
\frac{dr}{d\tilde{B}} &= \frac{(g_n+\delta-r)\, f''\frac{\partial \tilde{S}}{\partial \tilde{w}}}{1+\left(\tilde{K}+\tilde{B}\right) f''\frac{\partial \tilde{S}}{\partial \tilde{w}} - f''\frac{\partial \tilde{S}}{\partial r}}. \qquad (2.21)
\end{aligned}
$$

The expression $\frac{dr}{d\tilde{B}}$ is positive for $r > g_n + \delta$ because $f'' < 0$, $0 < \frac{\partial \tilde{S}}{\partial \tilde{w}} < 1$ and $0 < \frac{\partial \tilde{S}}{\partial r}$. This is the dynamically efficient case. An increase in debt causes the steady state interest rate to rise. $\frac{dr}{d\tilde{B}}$ is negative iff $r < g_n + \delta$. Hence, in the dynamically inefficient case, an increasing debt-labor ratio lowers the steady state interest rate.

**Result 2.4.** $\frac{dr}{d\tilde{B}} \gtrless 0$ iff $r \gtrless g_n + \delta$.
If debt is held exclusively by foreigners, the steady state interest rate rises with the introduction of debt for a dynamically efficient steady state. The change in the steady state interest rate due to a change in debt is negative if the steady state is dynamically inefficient.

The sign of the change in the interest rate due to changes in the debt-labor ratio is the same as the sign in the relation between the interest rate and the sum of the depreciation rate and the population growth rate. The argument of a rising steady state interest rate in a dynamically efficient steady state if debt is held by foreigners, can be explained by the capital market.

With the introduction of positive taxes in the external debt case, lifetime consumption of the respective taxpayer, i.e., the members of the parents-generation, is directly reduced. Taxes are always positive in the dynamically efficient case: $r - g_n - \delta > 0$ and $\tilde{\tau}_t = \tilde{B}(r - g_n - \delta) > 0$. The duty to pay taxes lowers the disposable income, which in return causes a reduction in savings of the parents-generation. Moreover, the decrease in disposable income leads to a smaller subsidy for the children-generation that is defined as a fixed amount of income. Thus, also children consume less and save less. Overall, the steady state capital-labor ratio declines. The capital supply in steady state is lower and therefore, the interest rate rises.

If the steady state is dynamically inefficient, $r - g_n - \delta < 0$. Hence, $\tilde{\tau}_t = \tilde{B}\left(r - g_n - \delta\right) < 0$; taxes are negative, i.e., a subsidy. The argument runs in the other direction in that case. The parents-generation as well as the children have more disposable income. The steady state capital-labor ratio increases. Moreover, the steady state interest rate falls ($\frac{dr}{d\tilde{B}} < 0$).

Overall, an increase in debt forces the steady state capital-labor ratio to fall in the dynamically efficient case ($0.0683 = \tilde{K}^* > \tilde{K}^*_{b_{ex}} = 0.0669$) and to rise in the dynamically inefficient case ($0.1669 = \tilde{K}^*_{1a} < \tilde{K}^*_{b_{ex1}} = 0.1677$). The corresponding steady state interest rate in the calibration exercise rises in the dynamically efficient case ($1.995 = f'(\tilde{K}^*) < f'(\tilde{K}^*_{b_{ex}}) = 2.0214$) and falls if the steady state is dynamically inefficient ($1.0993 = f'(\tilde{K}^*_{1a}) > f'(\tilde{K}^*_{b_{ex1}}) = 1.0961$).

The following expression determines the change in the net wage rate $\tilde{w}^{net}$ due to changes in the debt-labor ratio:

$$\begin{aligned} \frac{d\tilde{w}^{net}}{d\tilde{B}} &= \frac{d\tilde{w}}{dr}\frac{dr}{d\tilde{B}} - \frac{d\left[\left(r - g_n - \delta\right)\tilde{B}\right]}{d\tilde{B}} \\ &= -\tilde{K}\frac{dr}{d\tilde{B}} - \tilde{B}\frac{dr}{d\tilde{B}} - \left(r - g_n - \delta\right) \\ &= -\left(\tilde{K} + \tilde{B}\right)\frac{dr}{d\tilde{B}} + \left(g_n + \delta - r\right). \end{aligned} \tag{2.22}$$

For the dynamically efficient case, the change in the net wage rate is negative because $\frac{dr}{d\tilde{B}}$ is positive and $\frac{d\tilde{w}^{net}}{d\tilde{B}} > 0$ for $r < g_n + \delta$.

The total derivative shown in the following equation describes the effect of debt on utility:[7]

$$\frac{dU}{d\tilde{B}} = \frac{d\tilde{w}^{net}}{d\tilde{B}}\frac{\partial U}{\partial \tilde{w}} + \frac{dr}{d\tilde{B}}\frac{\partial U}{\partial r}. \tag{2.23}$$

The first missing component is $\frac{\partial U}{\partial \tilde{w}}$. We use the fact that $\frac{\partial U}{\partial \tilde{C}_1} = \frac{\partial U}{\partial \tilde{C}_2}\left(1 + r - \delta\right)$ and $\frac{\partial U}{\partial \tilde{C}_3} = \frac{\partial U}{\partial \tilde{C}_2}\left(\frac{1}{(1+r-\delta)}\right)$ and describe the derivative of utility with respect to the wage rate

[7] See Diamond [41].

as

$$\begin{aligned}\frac{\partial U}{\partial \tilde{w}} &= \frac{\partial U}{\partial \check{C}_1}\frac{\partial \check{C}_1}{\partial \tilde{w}} + \frac{\partial U}{\partial \check{C}_2}\frac{\partial \check{C}_2}{\partial \tilde{w}} + \frac{\partial U}{\partial \check{C}_3}\frac{\partial \check{C}_3}{\partial \tilde{w}} \\ &= \frac{\partial U}{\partial \check{C}_2}\left(\frac{\partial \check{C}_1}{\partial \tilde{w}}(1+r-\delta) + \frac{\partial \check{C}_2}{\partial \tilde{w}} + \frac{\partial \check{C}_3}{\partial \tilde{w}}\left(\frac{1}{1+r-\delta}\right)\right) \\ &= \frac{\partial U}{\partial \check{C}_2}\left(1-q+q\left(\frac{1+r-\delta}{1+g_n}\right)\right).\end{aligned} \tag{2.24}$$

Rearranging equations (2.3b)-(2.3d) such that the two savings rates are eliminated, we can write the steady state wage rate in dependence of the three different consumptions, where $\tilde{d} = q\tilde{w}$,

$$\tilde{w} = \frac{1}{1+r-\delta}\check{C}_3 + \tilde{d} - (1+r-\delta)\left(\frac{\tilde{d}}{1+g_n}\right) + (1+r-\delta)\,\check{C}_1 + \check{C}_2 \tag{2.25}$$

and obtain after differentiation with respect to $\tilde{w}$:

$$1 + \frac{(r-g_n-\delta)\,q}{1+g_n} = \frac{1}{1+r-\delta}\frac{\partial \check{C}_3}{\partial \tilde{w}} + (1+r-\delta)\frac{\partial \check{C}_1}{\partial \tilde{w}} + \frac{\partial \check{C}_2}{\partial \tilde{w}}. \tag{2.26}$$

Therefore, the equality of the second and last row of equation (2.24) holds. Next step is to find an expression for $\frac{\partial U}{\partial r}$, where

$$\frac{\partial U}{\partial r} = \frac{\partial U}{\partial \check{C}_1}\frac{\partial \check{C}_1}{\partial r} + \frac{\partial U}{\partial \check{C}_2}\frac{\partial \check{C}_2}{\partial r} + \frac{\partial U}{\partial \check{C}_3}\frac{\partial \check{C}_3}{\partial r}. \tag{2.27}$$

Once again using equation (2.25), we know that

$$\begin{aligned}\frac{\partial \tilde{w}}{\partial r} &= -\frac{1}{(1+r-\delta)^2}\check{C}_3 + \frac{1}{(1+r-\delta)}\frac{\partial \check{C}_3}{\partial r} - \frac{\tilde{d}}{1+g_n} + \check{C}_1 + (1+r-\delta)\frac{\partial \check{C}_1}{\partial r} + \frac{\partial \check{C}_2}{\partial r} \\ &= -\frac{1}{(1+r-\delta)^2}\check{C}_3 - \check{S}_1 + \frac{1}{(1+r-\delta)}\frac{\partial \check{C}_3}{\partial r} + (1+r-\delta)\frac{\partial \check{C}_1}{\partial r} + \frac{\partial \check{C}_2}{\partial r} \\ &= 0\end{aligned}$$

and thus

$$\begin{aligned}\frac{\partial U}{\partial r} &= \frac{\partial U}{\partial \check{C}_2}\left(\frac{1}{(1+r-\delta)}\frac{\partial \check{C}_3}{\partial r} + (1+r-\delta)\frac{\partial \check{C}_1}{\partial r} + \frac{\partial \check{C}_2}{\partial r}\right) \\ &= \frac{\partial U}{\partial \check{C}_2}\left(\check{S}_1 + \frac{1}{(1+r-\delta)^2}\check{C}_3\right) \\ &= \frac{\partial U}{\partial \check{C}_2}\left(\check{S}_1 + \frac{1}{(1+r-\delta)}\check{S}_2\right)\end{aligned} \tag{2.28}$$

such that equation (2.23) changes to

$$\begin{aligned}\frac{dU}{d\tilde{B}} &= -\frac{\partial U}{\partial \check{C}_2}\Bigg[\underbrace{(r-g_n-\delta)\left(1+\frac{(r-g_n-\delta)\,q}{1+g_n}\right)}_{I}+\underbrace{\left(1+\frac{(r-g_n-\delta)\,q}{1+g_n}\right)\tilde{B}\frac{dr}{d\tilde{B}}}_{II}\\ &\quad+\underbrace{\left(\tilde{K}-\check{S}_1-\frac{\check{S}_2}{1+r-\delta}\right)\frac{dr}{d\tilde{B}}}_{III}+\underbrace{\tilde{K}\frac{(r-g_n-\delta)\,q}{1+g_n}\frac{dr}{d\tilde{B}}}_{IV}\Bigg]. \qquad (2.29)\end{aligned}$$

The first term (I) denotes the change in utility arising from the tax needed to finance the amount that is additional to the outstanding debt. The respective term is multiplied by $\left(1+\frac{(r-g_n-\delta)q}{1+g_n}\right)$ due to the redistribution of income. It is positive if $r-g_n-\delta>0$.[8] The second term shows the change in the tax burden, once again multiplied by $\left(1+\frac{(r-g_n-\delta)q}{1+g_n}\right)$, which occurs because of a change in the interest rate and thus in the interest repayments due to the existing debt. Both terms are positive iff $r>g_n+\delta$, which corresponds to the condition for a dynamically efficient steady state. For $r<g_n+\delta$ (i.e., the dynamically inefficient case), the multiplier $\left(1+\frac{(r-g_n-\delta)q}{1+g_n}\right)$ has to be evaluated. We already know that $(r-g_n-\delta)<0$ and $\tilde{B}\frac{dr}{d\tilde{B}}<0$. With $0<r<g_n+\delta\le g_n+1$ and as a consequence $0<\frac{r}{1+g_n}<1$, it holds that $\frac{g_n+\delta}{1+g_n}\le 1$. Therefore, $-1<\frac{r}{1+g_n}-\frac{g_n+\delta}{1+g_n}<0$ and finally $0\le 1-q<1+q\left(\frac{r}{1+g_n}-\frac{g_n+\delta}{1+g_n}\right)<1$ with $0<q\le 1$. The chosen bound for $q$ is natural, because parents can at most give their whole wage income as a subsidy to their children. They would solely have the income from savings when being parents to decide what to consume and what to save in that case. We showed that the multiplier $\left(1+\frac{(r-g_n-\delta)q}{1+g_n}\right)$ is always positive. As a consequence, the first as well as the second term is negative iff $r<g_n+\delta$. The third expression in brackets can be rewritten as $\tilde{K}-\check{S}_1-\frac{\check{S}_2}{1+r-\delta}=\left(\tilde{K}-\check{S}_1\right)\frac{(r-g_n-\delta)}{(1+r-\delta)}=\check{S}_2\frac{(r-g_n-\delta)}{(1+g_n)(1+r-\delta)}$. Since $\frac{dr}{d\tilde{B}}$ has the same sign as $r-g_n-\delta$, term III is always positive: the movement of the interest rate causes the utility to fall for the third term in the dynamically efficient as well as in the dynamically inefficient case. Finally, as $r-g_n-\delta$ has the same sign as $\frac{dr}{d\tilde{B}}$, the last term (IV) is always positive.

[8]The tax is positive in that case, see equation (2.18).

Overall, we see that external debt causes the utility level of residents to fall in long-run equilibrium if the solution is dynamically efficient because all four terms are positive in that case. Thus, the negative sign in front of the bracket in equation (2.29) controls the direction of the effect. If the solution is dynamically inefficient ($r < g_n + \delta$), we do not know, which of the effects outweighs one or another. Term I and II are negative in that case and term III and IV are positive as before. A rise in debt causes counteracting effects such that the overall effect depends on the respective model parameters.

**Result 2.5.** $\frac{dU}{d\tilde{B}} < 0$ if $r > g_n + \delta$, $\frac{dU}{d\tilde{B}} =?$ if $r < g_n + \delta$.
If debt is held by foreigners, a rise in debt lowers the steady state utility level of the residents if the steady state is dynamically efficient. The effect of external debt on the utility level in a dynamically inefficient steady state cannot be determined in general.

### 2.6.2 Internal Debt

We now turn to the model extension assuming that debt is held exclusively by residents. The overall savings rate in steady state is $\tilde{S}\left(\tilde{w}^{net}, r\right) = \check{S}_1 + \frac{\check{S}_2}{1+g_n}$ and in equilibrium, we can describe the interest rate as

$$r = f'\left(\tilde{S}\left(\tilde{w}^{net}, r\right) - \tilde{B}\right). \tag{2.30}$$

If we are not in steady state, the interest rate is

$$r_{t+1} = f'\left(\tilde{S}\left(\tilde{w}^{net}_{t+2}, \tilde{w}^{net}_{t+1}, \tilde{w}^{net}_{t}, r_{t+2}, r_{t+1}\right) - \tilde{B}\right).$$

Therefore, we get as stability condition

$$0 < \frac{dr_{t+1}}{dr_t} = \frac{-\left(\tilde{K} + \tilde{B}\right) \overbrace{f''}^{<0} \overbrace{\frac{\partial \tilde{S}}{\partial \tilde{w}}}^{>0}}{1 - \underbrace{f''}_{<0} \underbrace{\frac{\partial \tilde{S}}{\partial r}}_{>0}} \leq 1. \tag{2.31}$$

We examine the effect arising on the future interest rate if the wage rate increases by a marginal unit. With

$$\begin{aligned}
\frac{dr_{t+1}}{d\tilde{w}_t} &= \frac{d\tilde{w}_t^{net}}{d\tilde{w}_t}\frac{\partial f'}{\partial \tilde{w}_t} + \frac{dr_{t+1}}{d\tilde{w}_t}\frac{\partial f'}{\partial r_{t+1}} \quad \text{and} \\
\frac{dr_{t+1}}{d\tilde{w}_t} &= \frac{\overbrace{f''}^{<0}\overbrace{\frac{\partial \tilde{S}}{\partial \tilde{w}}}^{>0}}{1-\underbrace{f''}_{<0}\underbrace{\frac{\partial \tilde{S}}{\partial r}}_{>0}} < 0, \qquad (2.32)
\end{aligned}$$

the interest rate in $t+1$ falls if the wage rate increases in $t$. This reaction is as expected: if the wage increases, then the savings increase and thus, with more capital supply, the interest rate decreases. Using the equations

$$\frac{dr}{d\tilde{B}} = \frac{d\tilde{w}^{net}}{d\tilde{B}}\frac{\partial f'}{\partial \tilde{w}} + \frac{dr}{d\tilde{B}}\frac{\partial f'}{\partial r} + \frac{d\tilde{B}}{d\tilde{B}}\frac{\partial f'}{\partial \tilde{B}}$$

and

$$\begin{aligned}
\frac{d\tilde{w}^{net}}{d\tilde{B}} &= \frac{d\tilde{w}}{dr}\frac{dr}{d\tilde{B}} - \frac{d\left[(r-g_n-\delta)\,\tilde{B}\right]}{d\tilde{B}} \\
&= \frac{d\tilde{w}}{dr}\frac{dr}{d\tilde{B}} - \tilde{B}\frac{dr}{d\tilde{B}} - (r-g_n-\delta) \\
&= -\left(\tilde{K}+\tilde{B}\right)\frac{dr}{d\tilde{B}} - (r-g_n-\delta)\,,
\end{aligned}$$

we get the effect of a change in debt on the interest rate. Because $\frac{d\tilde{B}}{d\tilde{B}}\frac{\partial f'}{\partial \tilde{B}} = f''(-1)$ and the inner derivative of $f'$ with respect to $\tilde{B}$ is -1 and $\frac{d\tilde{B}}{d\tilde{B}} = 1$, the change in the interest rate due to a change in debt is

$$\frac{dr}{d\tilde{B}} = \frac{-f''\left((r-g_n-\delta)\frac{\partial \tilde{S}}{\partial \tilde{w}}+1\right)}{1+\left(\tilde{K}+\tilde{B}\right)f''\frac{\partial \tilde{S}}{\partial \tilde{w}} - f''\frac{\partial \tilde{S}}{\partial r}}. \qquad (2.33)$$

The difference between equation (2.21) and equation (2.33) is the additional $-f''$ in the numerator of the fraction. Thus, the effect of debt held by foreigners has to be summed by a positive number in order to get the effect of the internal debt on the interest rate. To determine $\frac{dr}{d\tilde{B}}$ for the internal debt case, we have to analyze a dynamically efficient and a dynamically inefficient steady state. The change in the interest rate after an increase in the debt-labor ratio for the dynamically efficient case ($r > g_n + \delta$) is positive since the equation is positive. With the introduction of a positive tax to ensure a constant debt-labor ratio in steady state, disposable income is smaller and thus, consumption and savings of the parents- and the children-generation decline. Less capital in steady state is supplied. As a consequence, the steady state interest rate rises. Since we sum a positive number to the respective terms for $\frac{dr}{d\tilde{B}}$ in the external debt case, a rise in debt with the same intensity raises the steady state interest rate more if debt is held by residents. With the opportunity of domestic citizens to substitute debt for capital, less capital is held in that case. Due to the lower supply of capital, the interest rate rises and is thus higher than in the external debt case where a substitution of physical capital is not possible.

**Result 2.6.** $\frac{dr}{d\tilde{B}}$ with External Debt < $\frac{dr}{d\tilde{B}}$ with Internal Debt if $r > g_n + \delta$.
If internal debt is introduced, the steady state interest rate raises by a higher amount compared to the external debt scenario in the dynamically efficient case.

For the dynamically inefficient case, we have to examine the expression in brackets in the numerator. If $\frac{\partial \tilde{S}}{\partial \tilde{w}}(r - g_n - \delta) + 1 > 0$, the interest rate falls. Otherwise, the interest rate rises. The inequality $\frac{\partial \tilde{S}}{\partial \tilde{w}}(r - g_n - \delta) \gtrless -1$ is to be determined. In our model the following equation holds by assumption:[9]

$$0 < \frac{\partial \tilde{S}}{\partial \tilde{w}} < 1.$$

Therefore, we have to check if $g_n < r$ in the dynamically inefficient case for all plausible variable specifications. If the population growth rate $g_n$ is smaller than the steady state interest rate $r$, then $\delta + g_n - r < 1$ and therefore $r - g_n - \delta > -1$. With $0 < \frac{\partial \tilde{S}}{\partial \tilde{w}} < 1$, $\frac{\partial \tilde{S}}{\partial \tilde{w}}(r - g_n - \delta) > -1$ and thus $\frac{\partial \tilde{S}}{\partial \tilde{w}}(r - g_n - \delta) + 1 > 0$. Then $\frac{dr}{d\tilde{B}}$ is positive for the dynamically efficient as well as the dynamically inefficient case. As a consequence, the interest rate increases in the dynamically inefficient case.

[9] Following Diamond [41], p. 1131, we assume that consumption is always a normal good. Thus, the upper and lower bounds are true.

The problem to show that $g_n < r$ is that there is no closed form solution for the steady state value of $r$. If we consider equation (2.DC-14), we get for $\alpha = 1/3$ a polynomial with exponents $1,\ 1/3,\ 2/3,\ -1/3,\ -2/3,\ -4/3$. Multiplying this equation with $\tilde{K}^{4/3}$ delivers an equation with only positive exponents. If we finally substitute $\tilde{K}^{1/3}$ with a variable $z$, we get a polynomial of order 7 to be solved. For details see Appendix A.5. The nulls of the polynomial equation can only be found for given model parameter values. We always get 7 solutions and choose each time the largest positive real solution as steady state value just as in the traditional overlapping generations model with two generations.

We decide to test for the reaction of $r$ to a change in $q$, $\beta$, and $g_n$ in a plausible data domain, where $0 \leq q \leq 1$, $0.8 \leq \beta \leq 1$, and $0 \leq g_n \leq 1$. The upper bound for the population growth is chosen to be 100% within 25 years (more than a doubling of population in that time is per intuition not plausible). This corresponds to an annual population growth rate of about 2.8%. The data of the world bank [69] show that the industrial countries are always below this growth of population in average per year. Using a grid search with a step width of 0.0001 for each variable, we see that the steady state interest rate reacts in a linear, i.e., unique way to changes in each variable: $\beta \uparrow \Rightarrow\ r \downarrow$ & $q \uparrow \Rightarrow\ r \downarrow$ & $g_n \uparrow \Rightarrow r \uparrow$. Therefore, we have to compare the steady state interest rate in a setting with the largest discount factor $\beta$ possible ($\beta = 1$), the largest ratio of distributed income $q$ possible, this means, $q = 1$ and all possible population growth rates $g_n$.

Table 2.1 shows the results for 0.1 step changes in $g_n$. The steady state value of the interest rate $r$ in the dynamically inefficient case for the model including internal debt is larger than all reasonable population growth rates $g_n$ for 25 years.[10] Finally, we showed that $r > g_n$ holds for reasonable parameter choices. As a consequence, an increase in debt causes the steady state interest rate to rise also in the dynamically inefficient case. We once again argue with the capital market. On one hand, with a negative

[10] The interest rate for 25 years can be larger than 1 because even in the case of a 3% annual interest rate, we obtain $r = 1.09$.

tax being introduced in the dynamically inefficient case, disposable income rises and so do savings. The capital supply rises. On the other hand, if debt is held by domestic citizens, the inefficiently high capital stock is reduced by substituting debt for capital in the portfolio of the parents-generation. Overall, the substitution effect outweighs the rise in capital due to the additional disposable income such that the steady state capital-labor ratio decreases. Hence, the steady state capital-labor ratio declines for the dynamically efficient and the inefficient case.

| $g_n$ | r | $g_n$ | r |
|---|---|---|---|
| 0 | 0.5877 | 0.6 | 0.9502 |
| 0.1 | 0.6477 | 0.7 | 1.011 |
| 0.2 | 0.7077 | 0.8 | 1.073 |
| 0.3 | 0.7681 | 0.9 | 1.1339 |
| 0.4 | 0.8286 | 1 | 1.1955 |
| 0.5 | 0.8893 | | |

Table 2.1: Grid search to verify $r > g_n$ for the dynamically inefficient case.

As a consequence, if the supplied capital stock in steady state is lower, the steady state interest rate increases not only in the dynamically efficient but also in the dynamically inefficient case if the debt-labor ratio rises. This coincides with the calibration exercise: for the dynamically efficient case $0.0683 = \tilde{K}^* > \tilde{K}^*_{b_c} = 0.0594$, $1.995 = f'(\tilde{K}^*) < f'(\tilde{K}^*_{b_c}) = 2.1903$ and for the dynamically inefficient case $0.1669 = \tilde{K}^*_{1a} > \tilde{K}^*_{b_{c1}} = 0.1586$, $1.0993 = f'(\tilde{K}^*_{1a}) < f'(\tilde{K}^*_{b_{c1}}) = 1.1375$.

**Result 2.7.** $\frac{dr}{d\tilde{B}} > 0$
If debt is held by residents, the steady state interest rate rises no matter if the steady state is dynamically efficient or inefficient.

To answer the question, why own citizens substitute debt for capital such that the steady state capital-labor ratio decreases also for the dynamically inefficient case, we analyze the effect of a change in the debt-labor ratio to the utility level of individuals in long-run equilibrium. Similar to Subsection 2.6.1, we get[11]

$$\begin{aligned}
\frac{dU}{d\tilde{B}} &= \frac{d\tilde{w}^{net}}{d\tilde{B}}\frac{\partial U}{\partial \tilde{w}} + \frac{dr}{d\tilde{B}}\frac{\partial U}{\partial r} \\
&= \frac{\partial U}{\partial \check{C}_2}\left[\frac{d\tilde{w}^{net}}{d\tilde{B}}\left(1+q\left(\frac{r-g_n-\delta}{1+g_n}\right)\right) + \left(\check{S}_1 + \frac{1}{1+r-\delta}\check{S}_2\right)\frac{dr}{d\tilde{B}}\right] \\
&= -\frac{\partial U}{\partial \check{C}_2}\left[\left(1+q\left(\frac{r-g_n-\delta}{1+g_n}\right)\right)\left(\tilde{K}+\tilde{B}\right)\frac{dr}{d\tilde{B}}\right. \\
&\quad \left. + \left(1+q\left(\frac{r-g_n-\delta}{1+g_n}\right)\right)(r-g_n-\delta) - \left(\check{S}_1 + \frac{1}{1+r-\delta}\check{S}_2\right)\frac{dr}{d\tilde{B}}\right].
\end{aligned}$$

This is true because the constraint $\left(1+q\left(\frac{r-g_n-\delta}{1+g_n}\right)\right) = \frac{\partial \check{C}_1}{\partial \tilde{w}}(1+r-\delta) + \frac{\partial \check{C}_2}{\partial \tilde{w}} + \frac{1}{1+r-\delta}\frac{\partial \check{C}_3}{\partial \tilde{w}}$ holds as before and

$$\frac{\partial U}{\partial \tilde{w}} = \frac{\partial U}{\partial \check{C}_2}\left(\frac{\partial \check{C}_1}{\partial \tilde{w}}(1+r-\delta) + \frac{\partial \check{C}_2}{\partial \tilde{w}} + \frac{1}{1+r-\delta}\frac{\partial \check{C}_3}{\partial \tilde{w}}\right) = \frac{\partial U}{\partial \check{C}_2}\left(1+q\left(\frac{r-g_n-\delta}{1+g_n}\right)\right).$$

Moreover, we know that

$$\begin{aligned}
\frac{\partial U}{\partial r} &= \frac{\partial U}{\partial \check{C}_1}\frac{\partial \check{C}_1}{\partial r} + \frac{\partial U}{\partial \check{C}_2}\frac{\partial \check{C}_2}{\partial r} + \frac{\partial U}{\partial \check{C}_3}\frac{\partial \check{C}_3}{\partial r} \\
&= \frac{\partial U}{\partial \check{C}_2}\left(\frac{\partial \check{C}_1}{\partial r}(1+r-\delta) + \frac{\partial \check{C}_2}{\partial r} + \frac{1}{1+r-\delta}\frac{\partial \check{C}_3}{\partial r}\right) \\
&= \frac{\partial U}{\partial \check{C}_2}\left(\check{S}_1 + \frac{1}{1+r-\delta}\check{S}_2\right)
\end{aligned}$$

because

$$\frac{\partial \tilde{w}}{\partial r} = 0 = \frac{\partial \check{C}_1}{\partial r}(1+r-\delta) - \frac{\tilde{d}}{1+g_n} + \check{C}_1 + \frac{\partial \check{C}_2}{\partial r} + \frac{1}{1+r-\delta}\frac{\partial \check{C}_3}{\partial r} - \frac{1}{(1+r-\delta)^2}\check{C}_3.$$

Finally, the following expression for the change in utility due to a change in debt is found.

$$\begin{aligned}
\frac{dU}{d\tilde{B}} &= -\frac{\partial U}{\partial \check{C}_2}\left[\left(1+q\left(\frac{r-g_n-\delta}{1+g_n}\right)\right)(r-g_n-\delta) + \left(1+q\left(\frac{r-g_n-\delta}{1+g_n}\right)\right)\tilde{B}\frac{dr}{d\tilde{B}}\right. \\
&\quad \left. +q\left(\frac{r-g_n-\delta}{1+g_n}\right)\tilde{K}\frac{dr}{d\tilde{B}} + \left(\tilde{K} - \check{S}_1 - \frac{\check{S}_2}{1+r-\delta}\right)\frac{dr}{d\tilde{B}}\right] \qquad (2.34)
\end{aligned}$$

[11] See also Diamond [41].

$$\begin{aligned}
&= -\frac{\partial U}{\partial \check{C}_2}\left(r-g_n-\delta\right)\left[1+q\left(\frac{r-g_n-\delta}{1+g_n}\right)+\frac{q}{1+g_n}\left(\tilde{K}+\tilde{B}\right)\frac{dr}{d\tilde{B}}\right. \\
&\quad \left.+\frac{1}{r-g_n-\delta}\left(\tilde{B}+\tilde{K}-\check{S}_1-\frac{\check{S}_2}{(1+r-\delta)}\right)\frac{dr}{d\tilde{B}}\right] \\
&= -\frac{\partial U}{\partial \check{C}_2}\left(r-g_n-\delta\right)\left[1+q\left(\frac{r-g_n-\delta}{1+g_n}\right)+\frac{q}{1+g_n}\left(\tilde{K}+\tilde{B}\right)\frac{dr}{d\tilde{B}}\right. \\
&\quad \left.+\frac{1}{r-g_n-\delta}\left(-\frac{\check{S}_2}{1+r-\delta}+\frac{\check{S}_2}{1+g_n}\right)\right] \\
&= -\frac{\partial U}{\partial \check{C}_2}\left(r-g_n-\delta\right)\left[1+q\left(\frac{r-g_n-\delta}{1+g_n}\right)+\frac{q}{1+g_n}\left(\tilde{K}+\tilde{B}\right)\frac{dr}{d\tilde{B}}\right. \\
&\quad \left.+\frac{\check{S}_2}{(1+r-\delta)\,(1+g_n)}\frac{dr}{d\tilde{B}}\right] \\
&= -\frac{\partial U}{\partial \check{C}_2}\left(r-g_n-\delta\right)\left[1+q\left(\frac{r-g_n-\delta}{1+g_n}\right)+\frac{q}{1+g_n}\left(\tilde{K}+\tilde{B}\right)\frac{dr}{d\tilde{B}}\right. \\
&\quad \left.+\frac{\tilde{B}+\tilde{K}-\check{S}_1}{1+r-\delta}\frac{dr}{d\tilde{B}}\right]. \qquad (2.35)
\end{aligned}$$

Equation (2.34) can be interpreted as follows: the first term in brackets shows changes in the utility in terms of taxes needed to finance the increase in debt. The second term is characterized as the taxes needed to finance the increased interest payments on the existing debt and the third term is an additional term depending on capital and debt as well as the change in the interest rate due to a rise in debt. The last term is the changed value of factor payments.

As the signs of the first three terms only depend on $(r - g_n - \delta)$, the multiplier $(1+ q\left(\frac{r-g_n-\delta}{1+g_n}\right))$, which is always positive, and $\frac{dr}{d\tilde{B}}$, the effects on utility for these parts can be evaluated. But it is not obvious to assess the last term. Therefore, we rewrite the last term and derive equation (2.35). It can be seen that the last term only depends on the sign of $\frac{dr}{d\tilde{B}}$.

Overall, $r - g_n - \delta$ as well as all terms in brackets are greater than zero: utility decreases with an increase in the debt-labor ratio if the equilibrium is dynamically efficient. For the dynamically inefficient case, a positive debt-labor ratio raises the steady state interest rate and $\frac{dr}{d\tilde{B}} > 0$ holds. Moreover, $1 + q\frac{r-g_n-\delta}{1+g_n} > 0$. As a consequence, all terms in brackets are positive also for the dynamically inefficient case. With $r - g_n - \delta < 0$, the utility level increases with an increase in the debt-labor ratio if the equilibrium is dynamically inefficient. Hence, if domestic citizens hold a higher debt in a dynamically inefficient steady state, utility rises. If debt is introduced, individuals can substitute the two assets debt and physical capital in their portfolios. Since this substitution leads to a rise in utility, the amount of capital in the portfolios is indeed lowered. Thus, the inefficiently high capital stock is reduced.

**Result 2.8.** $\frac{dU}{d\bar{B}} \gtreqless 0$ iff $r \lesseqgtr g_n + \delta$.
If the steady state is dynamically efficient, the introduction of internal debt lowers utility. For a dynamically inefficient steady state, internal debt establishes substitution opportunities of debt for physical capital. Utility rises in that case and the inefficiently high capital stock is reduced.

To summarize, the steady state capital-labor ratio is smaller in a model including debt than the one found for the baseline model if the steady state is dynamically efficient. With the asset market clearing condition not only the steady state capital-labor ratio, but also the savings in the economy decline if debt is introduced. Therefore, the hypothesis that government spending or the existence of government debt lowers private savings is supported for a dynamically efficient steady state. Moreover, the introduction of external or internal debt causes the utility level of an individual in steady state to fall in that case. It is not possible to evaluate in general, which debt lowers utility more. On one hand, the steady state interest rate is more affected by an internal debt suggesting a higher reduction of utility in that case. On the other hand, the steady state capital-labor ratio is smaller if debt is held by residents. Since the steady state interest rate is higher, the steady state tax rate to ensure a constant debt-labor ratio is higher. Thus, income is smaller. Furthermore, citizens can substitute debt for physical capital in their asset portfolios. Overall, the steady state capital-labor ratio is smaller than in an external-debt setting. Thus, it lowers the negative influence of debt on the utility level if all other values would be the same for internal and external debt. As a consequence, the intensity of the reduction in utility caused by the internal debt cannot be determined in general in comparison to the respective intensity caused by the introduction of external debt.

In the dynamically inefficient case, utility in the long-run equilibrium with external debt is lower than the utility level without debt, because the already inefficiently high capital stock rises even more. Since the lump-sum taxes introduced together with debt are negative, the individuals have more disposable income. Thus, savings and the capital stock increase. If debt is held by residents, the parents-generation can substitute debt for capital in their portfolios and raise the utility level in long-run equilibrium. As a consequence, the inefficiently high capital supply is reduced.

## 2.7 Conclusions

In the last chapter, we have derived a variation of the known overlapping two generations model by implementing one additional generation, the children-generation. This generation interacts with the parents-generation corresponding to the known young generation of a general overlapping two generations model. Parents redistribute parts of their income to the children such that they can consume and save. The model setting of the baseline specification has been characterized by a household sector containing the three generations and a production sector. We have assumed that capital fully depreciates within 25 years and that government debt has a 25-year, this means, a one-period maturity. Government debt has been implemented by augmenting the baseline model. Two variations were analyzed: government debt held by foreigners and debt held by domestic citizens. In both cases, taxes have to be paid by the working generation to ensure interest repayments on debt. Since the economy grows due to the growing population, a fixed absolute amount of government debt would have asymptotically no effect. Hence, we adjusted for the population growth and chose the debt-labor ratio to be constant. The standard results for a general overlapping two generations model could be carried over to the underlying model extension. We found a unique stable, positive, real steady state solution for all model specifications.

A steady state can be dynamically efficient or dynamically inefficient. Dynamic inefficiency is characterized by a situation, where individuals consume too few and save too much. The capital stock is inefficiently high. If a higher amount of income in our model is redistributed from the parents- to the children-generation, the steady state capital-labor ratio is higher. For a critical amount of income redistribution, the accumulated capital stock becomes inefficiently high. The steady state changes from dynamic efficiency to dynamic inefficiency.

We analyzed the changes caused by the introduction of external or internal debt in dynamically efficient and inefficient steady states. The effect of debt in a dynamically efficient case is as expected. The introduction of government debt and a positive tax to ensure a constant debt-labor ratio reduces disposable income, no matter if debt is held by foreigners or residents. Savings decrease and the steady state capital-labor ratio is always lower in a debt scenario. As a consequence, the steady state interest rate is higher. The utility level of the individuals in long-run equilibrium declines. For the dynamically inefficient case, the effect of external and internal debt was analyzed separately. Since there are counteracting effects, it is not possible to determine the change in utility due to the introduction of external debt in general. On the one hand, utility increases because the tax needed to finance the additional debt burden is negative and thus raises disposable income. Overall savings are higher and more capital is accumulated. As a consequence, the steady state interest rate decreases and changes the tax burden, causing utility to rise. On the other hand, the utility level in steady state declines because the factor payments depend on taxes (which are negative) and the change of the interest rate due to the introduction of debt (which is negative, too). Thus, the product of these components is positive and has a negative influence on the utility level. The overall effect of external debt on utility depends on the relative intensity of the counteracting effects. If domestic citizens exclusively hold the government debt, utility increases in the dynamically inefficient case, because the individuals substitute government bonds for capital. Thus, the inefficiently high capital stock of the economy is reduced implying a gain in utility.

# Chapter 3

# Optimal Fiscal Policy Under Commitment in an EMU Model

## 3.1 Introduction

To analyze the impact of debt in the European Monetary Union (EMU) is still an important question. In this context, further questions occur: how does an optimal national fiscal policy under commitment in a monetary union framework look like? What are the effects of a policy as the response to a rise in debt? Do different tax rates induce different optimal fiscal policy decisions? What is the loss of welfare if a debt shock hits a country? Is there a difference if the country is already indebted at a higher level? Is the optimally chosen national fiscal policy compatible with an inflation stabilizing union-wide monetary policy?

We try to answer these questions by analyzing the effects of a national debt shock in a monetary union framework. Furthermore, we describe the optimal response of a single member country to a shock in debt and, finally, draw the connection to the policy of the union as a whole. Building on Galí/Monacelli [54] and Leith/Wren-Lewis [83], we analyze the optimal fiscal response to a debt shock in a situation where the government has access to different fiscal instruments. The basic model includes an IS-curve developed within the households sector and a New-Keynesian Phillips curve derived from profit maximizing firms in a Calvo-price setting framework.[1] There is monopolistic competition and price rigidities. The model is augmented by an active government sector, which sets taxes and the amount of government spending. This sector faces a solvency constraint, relating actual and former debt levels to government revenue (taxes) and expenditure (government spending). Two tax instruments are introduced separately, a sales tax or an income tax, in particular a wage tax.[2] We compare the effect of a debt shock on the optimal policy of the respective government as well as on the basic variables such as output or inflation for the two tax instruments. The sales tax causes an additional distortion in the goods market, whereas the wage tax distorts the labor market. A lump-sum tax and an employment subsidy is chosen to offset distortions in the markets. It is assumed that fiscal authorities as well as the union-wide monetary authority have access to a commitment technology.

Optimal monetary and Ramsey fiscal policy in a model including sticky prices in a one-country environment are analyzed by Schmitt-Grohé and Uribe [118]. Their main result is that price stickiness induces a deviation from the Friedman rule, i.e., an interest rate rule unequal to zero in equilibrium. Furthermore, the optimal inflation volatility is found to be close to zero such that a rise in inflation is avoided in any case. As a consequence, e.g., in the occurrence of a rise in government spending, debt and taxes are raised, both following a near random walk behavior. Fiscal as well as monetary policy have to be coordinated in this setting. Contrary to their model, we assume that optimal monetary and fiscal policy are conducted by different independent institutions.

Leith and Wren-Lewis [82] use a sticky-price New Neo-Classical Synthesis model to analyze optimal fiscal and monetary policy. On one hand, the government has access to a commitment technology, i.e., it optimizes the present value of an objective function (e.g., a welfare function) and obtains one optimal policy rule for all horizons. On the other hand, the government decides under discretion, i.e., the government reoptimizes

[1] See also Calvo [32].

[2] Since profits are not distributed, the income tax in the underlying model is levied only on wage income such that we call the income tax in the following a wage tax.

every period. The optimal choice of fiscal policy today does not restrict future policy decisions—see, e.g., Barro [8]. Leith and Wren-Lewis confirm that an optimal precommitment policy induces a random walk behavior of debt. For discretion, debt will always return to its initial steady state level even without having an explicit debt stabilizing target. Kirsanova and Wren-Lewis [75] analyze the impact of different degrees of fiscal feedback in a one country model with rigidities if monetary policy is optimal. Optimal fiscal feedback is small. If the intensity of fiscal feedback is raised, monetary policy becomes weaker and as fiscal policy is less effective, welfare declines. Leith and Wren-Lewis [78] find that fiscal policy has to be self-stabilizing if monetary policy seeks to respond to changes in inflation in a multi-good Blanchard-Yaari setup. This overlapping generations type model has a special structure. Agents face an exogenous probability to die and different generations exist at any point in time. The generations are not connected to previous generations—see also Yaari [132] and Blanchard [19]. Within this framework, Leith and Wren-Lewis show that monetary policy is forced to be passive if fiscal policy is not self-stabilizing.

Lombardo and Sutherland [89] analyze optimal fiscal and monetary policy interactions in a two country model, i.e., an open economy setting, under price stickiness. Two monetary institutions and two fiscal authorities are implemented. Assuming that monetary policy is set cooperatively, a cooperative fiscal policy leads to welfare gains.

Possible problems of a monetary union, where fiscal authorities act independently, are analyzed by Uhlig [124]. He examines, e.g., free-riding problems and crisis scenarios and concludes that bad national fiscal policies can lead to instability in the EMU even with an excellent monetary policy. He suggests a better structure of institutions to face crisis scenarios. Grimm and Ried [58] use a two-country model within an EMU framework and examine different static games where the monetary policy and the two fiscal authorities cooperate or do not cooperate, one leads, etc. They find that cooperation between all authorities and monetary leadership causes the lowest welfare losses. Moreover, a cooperation between the two fiscal policy makers is harmful not only for the single country welfare but also for the union-wide welfare. Leith and Wren-Lewis [81] show that the fiscal response for the stability of debt is higher if consumers do not live forever in a two country open economy EMU framework with overlapping generations. If the unique monetary policy tries to stabilize inflation, each country has to stabilize debt because one country can only compensate a weak fiscal policy of another country in a very limited way. In case of one fiscal authority who does not stabilize its debt, the other country cannot compensate without losing welfare. As a consequence, to stabilize debt in the whole union, the monetary authority has to abandon the inflation target. It conducts a passive policy such that a rise in inflation lowers debt in the respective country not stabilizing debt per se.

Optimal simple rules for fiscal policy in a two country model setting are examined by Kirsanova et al. [76]. They find welfare gains if governments respond on output gaps, national inflation, and national debt. The gains in welfare are negligible if the national government also focuses on the terms of trade and union-wide variables. Beetsma and Jensen [12] analyze the mechanisms of fiscal stabilization and commitment policy in a two-country monetary union. Fiscal policy rules stabilizing shifts in inflation as well as the terms of trade are examined. Leith and Wren-Lewis [79] investigate fiscal policy using different instruments for stabilization in open economies being no member or part of the EMU and compare effectiveness and welfare costs. Galí and Monacelli [54] develop a model consisting of $n$ countries each with negligible size in a monetary union environment. The small country size implies the absence of feedback effects from other economies. The monetary authority maximizes union-wide welfare. It assumes that optimal fiscal policy is carried out. Fiscal authorities want to offer public goods efficiently. Moreover, they try to stabilize domestic inflation and the output gap. In contrast, Ferrero [49] includes a role for distortionary taxes and government debt (in a two country currency union). She assumes that the fiscal policy of one country affects the second country and thus derives a Phillips curve, a government solvency constraint, and a demand curve including the terms of trade. In this case, the terms of trade are part of the loss function and are therefore a target for fiscal policy. In this context, alternative simple rules for fiscal and monetary policy are analyzed. Leith and Wren-Lewis

[83] augment the Galí/Monacelli model by a government sector and examine fiscal and monetary policy under the assumption that the policy makers optimize under discretion. They only consider income taxes as a fiscal instrument. In contrast to Leith and Wren-Lewis [83], we assume that the policy authorities optimally decide under commitment. Our equations derived from the wage tax scenario differ in some respect from those found by Leith and Wren-Lewis. Furthermore, we examine the effect of a sales tax as fiscal policy instrument. A theoretical foundation of optimal monetary policy is given for both scenarios. We simulate a positive national debt shock and analyze different policy reactions to reduce the debt. All optimal fiscal actions as a response to a debt shock and the union-wide optimal monetary policy do not contradict each other.

The optimal fiscal responses can be described as follows. If debt suddenly increases in a single country, the national government should optimally lower government spending and raise taxes in order to reduce debt. Since variations in debt, government spending, and taxes change the inflation rate, the intensity of the optimal fiscal response is found to be very low to avoid high welfare losses. As a consequence, debt is reduced very slowly.

The study shows that the higher the debt level is when an additional positive shock in debt occurs, the stronger are the optimal fiscal policy responses and the higher is the loss in welfare. Comparing wage with sales taxes as fiscal instruments, the fiscal authority has to raise sales taxes by a higher amount to obtain a comparable optimal debt reduction path. Hence, an optimal choice of sales taxes to reduce debt causes higher national welfare losses. Under commitment, the optimal intensity of fiscal responses depends on the choice of the tax instrument. Indirect taxes seem to be less effective in reducing debt.

The next section describes the main assumptions of the model and derives all major equations needed to simulate the model. The third section shows the social planner's optimization problem. In the fourth section, the union-wide as well as the national optimal policy problem are solved. The union-wide solution is derived from a theoretical basis, whereas for the national problem, the underlying model is calibrated. Optimal fiscal policy in response to a shock in debt is derived and different fiscal instruments are compared. The last section concludes.

## 3.2 The Model

The setup is based on the monetary union model of Galí and Monacelli [54]. Similar to Leith and Wren-Lewis [83], it consists of three sectors. The household sector optimizes utility subject to a budget constraint. The government sector maximizes the supply of public goods subject to a solvency constraint relating debt levels to expenditures and revenue of the government sector. Public goods are assumed to be equal to the consumed goods of the government. Due to an existing home bias, see e.g., Brulhart et al. [27], on domestic goods for the OECD countries, we assume that all goods consumed by the government sector are produced in the home country. This assumption is also made by Galí and Monacellli [54]. The firms sector optimizes profits subject to a Calvo-price setting rule. Since we analyze a multi country model, a section describing the terms of trade and the international risk sharing is added. Moreover, the equilibrium of the model is characterized. Finally, the model equations are rewritten with log deviation variables from the flex price equilibrium.

### 3.2.1 The Household Sector

In the underlying section, we derive the first order conditions for the utility maximization problem of the representative household in the underlying setup. Households maximize a utility function, where consumption and public goods raise while labor lowers the level of utility. Public goods are provided by the government sector. The goods consumed by the residents are partly produced abroad and partly in the domestic country. The households face a budget constraint. Expenditure, e.g., costs for consumption and assets, must not exceed the income, for example wages and payoffs of assets. There is no investment or capital in the setup.

The derivation is described in more detail in the following. In general, a variable $X_t^i$

denotes variable $X$ for country $i$ in period $t$. For simplification, we assume a perfect asset market.[3] Moreover, every member country of a monetary union has zero weight.[4] This negligible size of a single country implies that there are no feedback effects from the other economies, e.g., if fiscal policy in one country changes. Furthermore, every country is comparable in size. Comparability is assumed with respect to monetary policy. The central bank of a union as a whole should be independent of a single member country's movements. The same size means that the households of every country are facing the same budget constraints and thus pursue the same consumption plan.[5] The continuum of households is of size one and uniformly distributed on the interval $[0,1]$.

The representative household of a single member country $i$ maximizes the present value of utility depending on consumption $C_t^i$ at time $t$, a public good $G_t^i$ at time $t$, and on time spent working $N_t^i$ at time $t$.

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t^i, G_t^i, N_t^i\right), \tag{3.1}$$

where $\beta$ denotes the intertemporal discount factor. The period utility function $U\left(C_t^i,\right.$ $\left.G_t^i, N_t^i\right)$ is given by $\left(1-\psi^i\right)\log C_t^i + \psi^i \log G_t^i - \frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi}$. $\psi^i \in [0,1)$ denotes the relative weight of public goods $G_t^i$ to consumption goods $C_t^i$ in the utility function. The supply of public goods, which can be seen, e.g., as the level of infrastructure, raises the utility level of a household. It is defined as the aggregate of different goods $j$, all produced in the home country

$$G_t^i = \left(\int_0^1 G_t^i(j)^{(\varepsilon-1)/\varepsilon} dj\right)^{\varepsilon/(\varepsilon-1)}, \tag{3.2}$$

where $\varepsilon > 1$ is the elasticity of substitution between different varieties of public goods produced in country $i$. Since all countries are assumed to be homogenous with respect to public goods supply behavior, the elasticity is the same in every specific country $i$. The time spent working $N_t^i$ adds some disutility to the function at time $t$, where $\varphi$ is the elasticity of labor supply. Moreover, $C_t^i$ is the consumption aggregate defined as

[3] A consumer can trade claims to contracts with payoffs for all possible states.

[4] With this assumption, we follow Galí and Monacelli [54].

[5] The assumptions coincide with the ones found in the Galí/Monacelli paper [54].

a weighted product of consumption goods produced in the home country ($C^i_{i,t}$) and of consumption goods produced in a foreign country ($C^i_{F,t}$), i.e.,

$$C^i_t = \frac{\left(C^i_{i,t}\right)^{1-\nu}\left(C^i_{F,t}\right)^{\nu}}{(1-\nu)^{1-\nu}\,\nu^{\nu}}. \tag{3.3}$$

The subscript $i$ or $F$ shows the producing country of the good. Moreover, $\nu \in [0,1)$ is the weight of the imported goods of country $i$. With this approach, we follow Galí and Monacelli [54]. Since $\nu < 1$, every country also consumes domestic goods. The larger $\nu$, the more households are willing to consume goods produced in other countries. Thus, $\nu$ is a measure of openness. Since households in different countries may have different preferences concerning the willingness to import goods, the consumption baskets can be different. Even if the law of one price holds, the consumer price index (CPI) inflation can be different across countries—see Galí and Monacelli [54]. The domestic consumption goods $C^i_{i,t}$ are an aggregate of different goods $j$, which are all produced in country $i$

$$C^i_{i,t} = \left(\int_0^1 C^i_{i,t}(j)^{(\varepsilon-1)/\varepsilon}dj\right)^{\varepsilon/(\varepsilon-1)}, \tag{3.4}$$

where $\varepsilon > 1$ is the elasticity of substitution between different varieties of goods produced in country $i$. Every good is only produced in one country. $\varepsilon$ is the same for public goods as well as for consumption goods. We assume that households and the government are homogenous in that context. The amount of imported goods $C^i_{F,t}$ is defined as the aggregate quantity of goods produced in foreign countries $f$

$$C^i_{F,t} = \exp\int_0^1 c^i_{f,t}df, \tag{3.5}$$

where $c^i_{f,t} = \log C^i_{f,t}$ is the logarithm of an index containing the goods consumed by country $i$ and produced in country $f$

$$C^i_{f,t} = \left(\int_0^1 C^i_{f,t}(j)^{(\varepsilon-1)/\varepsilon}dj\right)^{\varepsilon/(\varepsilon-1)}. \tag{3.6}$$

Since we want to compare the effect of indirect and direct taxes, we differentiate between two scenarios. On one hand, we assume that the national government only levies wage taxes $\tau_t^{i,l}$. On the other hand, a sales tax with tax rate $\tau_{i,t}^{i,s}$ is raised. Some equations depend on the choice of the tax instrument such that they are displayed separately for the respective setup. The labels .W and .S in the equations stand for wage tax and sales tax, respectively. The budget constraint of the representative household of country $i$ in period $t$ can be written as follows.

$$\begin{aligned} \int_0^1 P_t^i(j)C_{i,t}^i(j)dj \quad &+ \quad \int_0^1 \int_0^1 P_t^f(j)C_{f,t}^i(j)djdf \\ +E_t\left[Q_{t,t+1}D_{t+1}^i\right] \quad &\leq \quad \Pi_t^i + D_t^i + W_t^i N_t^i \left(1-\tau_t^{i,l}\right) - T_t^i \end{aligned} \tag{3.7.W}$$

$$\begin{aligned} \left(1+\tau_{i,t}^{i,s}\right)\int_0^1 P_t^i(j)C_{i,t}^i(j)dj \quad &+ \quad \int_0^1 \left(1+\tau_t^{f,s}\right)\int_0^1 P_t^f(j)C_{f,t}^i(j)djdf \\ +E_t\left[Q_{t,t+1}D_{t+1}^i\right] \quad &\leq \quad \Pi_t^i + D_t^i + W_t^i N_t^i - T_t^i. \end{aligned} \tag{3.7.S}$$

The details are explained in the next paragraph. The first term denotes the home consumption of country $i$. This term is equal to the aggregate of the amount of domestically produced goods $C_{i,t}^i(j)$ consumed by residents, which are weighted with the respective prices $P_t^i(j)$. The second term is the consumption in country $i$ of products produced in other countries $f$, integrated over all foreign countries $f \in [0,1]$. The single goods $C_{f,t}^i(j)$ produced in country $f$ and consumed in country $i$ are evaluated with their respective prices $P_t^f(j)$. The last term on the left hand side ($E_t\left[Q_{t,t+1}D_{t+1}^i\right]$) is the expected discounted value of payoffs of a portfolio where $Q_{t,t+1}$ is the stochastic discount factor for one period ahead payoffs and $D_{t+1}^i$ is the nominal payoff of the chosen portfolio held at the end of period $t$. The discount factor $Q_{t,t+1}$ is assumed to be the same in all countries. The right hand side of the respective inequality is the "income" of the household: $\Pi_t^i$ denotes the profit share of the firms acting in a monopolistic competition framework[6], $W_t^i$ is the nominal wage rate, $N_t^i$ the time worked, and $T_t^i$ is a lump-sum tax introduced to finance a steady state subsidy, which offsets the distortions caused by imperfect competition and the respective taxes in the steady state. $\tau_t^{i,l}$ denotes the wage tax levied by the government and $\tau_{i,t}^{i,s}$, $\tau_t^{f,s}$ is a sales tax being a markup on the price $P_t^i$ for country $i$ or $P_t^f$ for country $f$. Every good within one country is subject to the same tax rate. There is no investment in the model. All products in the constraints are given

[6]This is due to the Calvo-pricing assumption.

in nominal terms. We assume that the law of one price holds in the monetary union. The representative household maximizes equation (3.1) subject to equation (3.7.W) and (3.7.S), respectively. It decides how to divide an available amount of expenditure between different available goods. To eliminate the integrals in the constraint inequalities, we use the demand function for a particular good $j$ produced in country $i, f$, country $i$'s domestic and imported goods price index, as well as the substitution rules for imported and domestic goods.

Since some equations depend on the chosen tax instrument, we derive the necessary equations for the wage and sales tax scenario separately. We begin with a setup assuming that national governments can only raise a wage tax. The demand function $C^i_{i,t}(j)$ for good $j$ produced in country $i$ is given by the aggregate domestic consumption goods weighted with the price of good $j$ relative to the domestic price index $P^i_t = \left(\int_0^1 P^i_t(j)^{1-\varepsilon}dj\right)^{1/(1-\varepsilon)}$, i.e.,

$$C^i_{i,t}(j) = \left(\frac{P^i_t(j)}{P^i_t}\right)^{-\varepsilon} C^i_{i,t}. \tag{3.8}$$

The demand function for imported goods is given by

$$C^i_{f,t}(j) = \left(\frac{P^f_t(j)}{P^f_t}\right)^{-\varepsilon} C^i_{f,t}, \tag{3.9}$$

where $P^f_t = \left(\int_0^1 P^f_t(j)^{1-\varepsilon}dj\right)^{1/(1-\varepsilon)}$ is the price index of the aggregate goods imported from country $f$ and, by the law of one price, is equal to country $f$'s domestic price level. With the former derivations, the first and the second integral in the budget constraint (equation (3.7.W)) is given by $\int_0^1 P^i_t(j)C^i_{i,t}(j)dj = \left(1/P^i_t\right)^{-\varepsilon} C^i_{i,t} \int_0^1 \left(P^i_t(j)\right)^{1-\varepsilon} dj = P^i_t C^i_{i,t}$ and $\int_0^1 P^f_t(j)C^i_{f,t}(j)dj = P^f_t C^i_{f,t}$. If we optimize across imported goods by a country, it holds that

$$P^f_t C^i_{f,t} = P^*_t C^i_{F,t}, \tag{3.10}$$

where $P^*_t = \exp \int_0^1 p^f_t df$ is the union-wide price index, i.e., an aggregate of the foreign countries' price indices and, from the perspective of a single country, the price index of imported goods. Finally, we need an expression for the last integral and can express the total expenditure on imported goods as

$$\int_0^1 P_t^f C_{f,t}^i df = P_t^* C_{F,t}^i. \tag{3.11.W}$$

As before, $C_{f,t}^i$ are the goods produced in country $f$ and consumed in country $i$, whereas $C_{F,t}^i$ are all imported goods consumed in country $i$. The consumer price index $P_{c,t}^i = \left(P_t^i\right)^{1-\nu} \left(P_t^*\right)^{\nu}$ is the weighted product of the domestic and the union-wide price index, weighted by $\nu$, the measure of openness. Since the consumed goods are weighted in the same manner, the representative basket being the basis for CPI calculation should have the same composition. Optimization between domestic and imported goods implies that $P_t^i C_{i,t}^i = (1-\nu) P_{c,t}^i C_t^i$ and $P_t^* C_{F,t}^i = \nu P_{c,t}^i C_t^i$. The overall consumption expenditures can be summarized as $P_t^i C_{i,t}^i + P_t^* C_{F,t}^i = P_{c,t}^i C_t^i$. Furthermore, the new budget constraint is given by

$$P_{c,t}^i C_t^i + E_t \left[Q_{t,t+1} D_{t+1}^i\right] \leq \Pi_t^i + D_t^i + W_t^i N_t^i \left(1 - \tau_t^{i,l}\right) - T_t^i. \tag{3.12.W}$$

The expenditure (for consumption and assets) are less or equal to the income, i.e., the sum of profits, assets, and net wage minus the lump-sum tax, which has to be payed by each household.

If we assume that governments only have access to sales taxes instead of a wage tax, the equations for the optimal choice of imported goods are different. Since the sales tax has no effect on the optimal choice within one country (every price of goods within one country includes the same tax rate) only the equations for the imported goods depend on the chosen tax instrument $\left(1 + \tau_t^{f,s}\right) P_t^f C_{f,t}^i = \left(1 + \tau_t^{*,s}\right) P_t^* C_{F,t}^i$, where $1 + \tau_t^{*,s} = \exp \int_0^1 \log \left(1 + \tau_t^{f,s}\right) df$. The total expenditure on imported goods is thus given by

$$\int_0^1 \left(1 + \tau_t^{f,s}\right) P_t^f C_{f,t}^i df = \left(1 + \tau_t^{*,s}\right) P_t^* C_{F,t}^i. \tag{3.11.S}$$

The measure of openness, $\nu$, weights the components for the index of sales taxes: $\left(1+\tau_t^{i,s}\right) = \left(1 + \tau_{i,t}^{i,s}\right)^{1-\nu} \left(1 + \tau_t^{*,s}\right)^{\nu}$. The optimal allocation of expenditures between domestic and foreign goods imported by country $i$ is given by $\left(1+\tau_{i,t}^{i,s}\right) P_t^i C_{i,t}^i = (1-\nu) \left(1+\tau_t^{i,s}\right) P_{c,t}^i C_t^i$ and $\left(1 + \tau_t^{*,s}\right) P_t^* C_{F,t}^i = \nu \left(1 + \tau_t^{i,s}\right) P_{c,t}^i C_t^i$. The overall consumption expenditure is $\left(1 + \tau_{i,t}^{i,s}\right) P_t^i C_{i,t}^i + \left(1 + \tau_t^{*,s}\right) P_t^* C_{F,t}^i = \left(1 + \tau_t^{i,s}\right) P_{c,t}^i C_t^i$.

Furthermore, the new budget constraint can be written as

$$\left(1+\tau_t^{i,s}\right) P_{c,t}^i C_t^i + E_t\left[Q_{t,t+1} D_{t+1}^i\right] \leq \Pi_t^i + D_t^i + W_t^i N_t^i - T_t^i. \tag{3.12.S}$$

The expenditure (for consumption and assets) on the left hand side has to be less or equal to the income, i.e., the sum of profits, assets, and wage minus the lump-sum tax, which is payed by each individual. Since the introduction of a sales tax raises the price for each good, the sales tax operates as a price premium.

As a next step, we derive the first order conditions of a household facing the optimization problem formulated in equation (3.1) subject to equation (3.12.W) or equation (3.12.S). Again, we start with the wage tax scenario. The derivative of the Lagrangian with respect to consumption, labor, and assets implies:

$$\begin{aligned}
\lambda_t &= \left(1-\psi^i\right) \frac{1}{C_t^i P_{c,t}^i}, \\
\lambda_t &= \frac{\left(N_t^i\right)^{\varphi}}{W_t^i\left(1-\tau_t^{i,l}\right)}, \text{ and} \\
\lambda_t &= E_t \lambda_{t+1} \beta R_t^*, \text{ respectively.}
\end{aligned}$$

$\lambda_t$ denotes the Lagrange multiplier at time $t$. $R_t^* = \frac{1}{E_t Q_{t,t+1}}$ is the gross nominal return of a riskless one-period bond, which yields one unit in $t+1$. Furthermore, $R_t^* = 1 + r_t^*$, where $r_t^*$ is the nominal interest rate and the monetary policy instrument of the unique monetary authority in the monetary union. Eliminating the Lagrange multiplier leads to the following two first order conditions

$$C_t^i \left(N_t^i\right)^{\varphi} = \frac{\left(1-\psi^i\right) W_t^i \left(1-\tau_t^{i,l}\right)}{P_{c,t}^i}, \tag{3.13.W}$$

$$E_t \frac{C_{t+1}^i P_{c,t+1}^i}{C_t^i P_{c,t}^i} = \beta R_t^*. \tag{3.14.W}$$

The last equation is the consumption Euler equation. Since $C^i$ denotes the consumption goods, $C^i P_c^i$ is the overall consumption value. The last equation links expected future consumption value with the current consumption value and the nominal rate of return.

To obtain a linear system of equations, the first order conditions are log-linearized

$$\begin{aligned}\widehat{rw}_t^i &= \varphi \widehat{n}_t^i + \widehat{c}_t^i - \widehat{\log 1 - \tau_t^{i,l}},\\ \widehat{c}_t^i &= E_t \widehat{c}_{t+1}^i - \left(\widehat{\log R_t^*} - E_t \pi_{c,t+1}^i\right),\end{aligned}$$

where small letters denote variables in logarithms. For the tax rate, the logarithms are explicitly labeled. $rw_t^i$ are real wages in logs, and $\widehat{x}$ is the deviation of a variable $x$ from its steady state value $\bar{x}$. The log deviation of real wages depends on the log deviation of labor, consumption, and taxes. Moreover, the log-linearized consumption Euler equation shows that the log deviation of consumption today depends on the expected log deviation of consumption tomorrow, the log deviation of the nominal rate of return, and the expected consumer price inflation rate tomorrow. If the equations are rewritten in log variables and not as log deviations from steady state, we obtain with $\widehat{\log 1 - \tau_t^{i,l}} = -\frac{\bar{\tau}^{i,l}}{1-\bar{\tau}^{i,l}} \widehat{\log \tau_t^{i,l}}$

$$\varphi n_t^i + c_t^i + \frac{\bar{\tau}^{i,l}}{1-\bar{\tau}^{i,l}} \log \tau_t^{i,l} - rw_t^i = \log\left(1-\psi^i\right) + \log\left(1-\bar{\tau}^{i,l}\right) + \frac{\bar{\tau}^{i,l}}{1-\bar{\tau}^{i,l}} \log \bar{\tau}^{i,l}, \quad (3.15.\text{W})$$

$$c_t^i = E_t c_{t+1}^i - \left(\log R_t^* - E_t \pi_{c,t+1}^i - \rho\right), \quad (3.16.\text{W})$$

where $\rho$ is the steady state value for the nominal gross rate of return. Using equation (3.14.W), we obtain $\log \bar{R} = -\log \beta = \rho$.

We follow the same steps for the scenario including a sales tax. We get the following first-order conditions

$$C_t^i \left(N_t^i\right)^{\varphi} = \frac{\left(1-\psi^i\right) W_t^i}{P_{c,t}^i \left(1+\tau_t^{i,s}\right)}, \quad (3.13.\text{S})$$

$$E_t \frac{C_{t+1}^i P_{c,t+1}^i \left(1+\tau_{t+1}^{i,s}\right)}{C_t^i P_{c,t}^i \left(1+\tau_t^{i,s}\right)} = \beta R_t^*. \quad (3.14.\text{S})$$

Contrary to the wage tax scenario, not the wage, but the prices are influenced by a tax. Thus, in both first order conditions, the price index is multiplied by the respective tax rate. The equations are log-linearized and finally stated in log level form.

With $\widehat{\log 1+\tau_t^{i,s}} = \frac{\bar{\tau}^{i,s}}{1+\bar{\tau}^{i,s}}\widehat{\log \tau_t^{i,s}}$, we obtain

$$\varphi n_t^i + c_t^i + \frac{\bar{\tau}^{i,s}}{1+\bar{\tau}^{i,s}} \log \tau_t^{i,s} - rw_t^i = \log\left(1-\psi^i\right) - \log\left(1+\bar{\tau}^{i,s}\right) + \frac{\bar{\tau}^{i,s}}{1+\bar{\tau}^{i,s}} \log \bar{\tau}^{i,s}, \quad (3.15.S)$$

$$c_t^i = E_t c_{t+1}^i - \left(\log R_t^* - E_t \pi_{c,t+1}^i - \rho\right) + \frac{\bar{\tau}^{i,s}}{1+\bar{\tau}^{i,s}} E_t \left(\log \tau_{t+1}^{i,s} - \log \tau_t^{i,s}\right). \quad (3.16.S)$$

The first equation shows that the log real wages depend on log labor, log consumption, log taxes, and some constants. Moreover, equation (3.16.S) denotes the corresponding consumption Euler equation. Log consumption today depends on the expected log consumption tomorrow, the difference of the log nominal rate of return and its steady state value, the expected CPI inflation tomorrow, and the difference of the expected log tax rate tomorrow and the log tax rate today.

### 3.2.2 The Terms of Trade

In the following subsection, we analyze the terms of trade that are defined by the relation of the price indices of two countries for the scenario including a wage tax and depend additionally on the relation between the tax rates in two countries if a sales tax is included in the model. Moreover, it is shown that the consumer price index (CPI) inflation rate is a linear combination of the log terms of trade and the domestic inflation rate. Union-wide CPI inflation is equal to union-wide inflation. The derivations are explained in detail in the following.

The bilateral terms of trade $S_{f,t}^i$ between a country $i$ and a country $f$ for the model including a wage tax is the ratio of the price index of country $f$ and $i$[7]

$$S_{f,t}^i = \frac{P_t^f}{P_t^i}, \quad (3.17.W)$$

whereas the effective terms of trade is defined as

$$S_t^i = \frac{P_t^*}{P_t^i} = \exp \int_0^1 \log S_{f,t}^i df. \quad (3.18.W)$$

Moreover, the CPI price index $P_{c,t}^i$ can be written as $P_{c,t}^i = P_t^i \left(S_t^i\right)^\nu = \left(P_t^i\right)^{1-\nu} \left(P_t^*\right)^\nu$. To obtain a system of linear equations, we log-linearize the equations above. The following

[7] See also Galí/Monacelli [54].

relationship holds between the CPI inflation $\pi_{c,t}^{i}$ and the domestic inflation $\pi_{t}^{i}$:

$$p_{c,t}^{i} = p_{t}^{i} + \nu s_{t}^{i} \quad \Leftrightarrow \quad \pi_{c,t}^{i} = \pi_{t}^{i} + \nu \Delta s_{t}^{i}. \tag{3.19.W}$$

As before, small letters denote variables in logs. If we take the equation on the left hand side (the log-linearized definition of the CPI price index) for two different points in time and subtract the equations, the CPI-inflation can be described as shown on the right hand side.

The overall log effective terms of trade over all countries are $\int_0^1 s_t^i di = 0$. This is due to the fact that the overall effective terms of trade sum up to 1. As a consequence, the log overall effective terms of trade are equal to zero. After integrating the right hand side equation of (3.19.W) over all countries $i$, it can be observed that the world inflation rate equals the CPI-inflation for the union as a whole: $\pi_{c,t}^{*} = \pi_{t}^{*}$, where $^*$ symbolizes a union-wide variable.

If the government chooses a sales tax as fiscal instrument, we obtain equation (3.17.S) for the terms of trade $S_{f,t}^{i}$ between two countries $i$ and $f$ and equation (3.18.S) for the effective terms of trade $S_t^i$ of country $i$.

$$S_{f,t}^{i} = \frac{P_t^f\left(1+\tau_{f,t}^{f,s}\right)}{P_t^i\left(1+\tau_{i,t}^{i,s}\right)}, \tag{3.17.S}$$

$$S_t^i = \frac{P_t^*\left(1+\tau_t^{*,s}\right)}{P_t^i\left(1+\tau_{i,t}^{i,s}\right)} = \exp\int_0^1 \log S_{f,t}^{i} df. \tag{3.18.S}$$

With the equations for the index of overall sales taxes and the CPI-price index, it holds that $P_{c,t}^{i}\left(1+\tau_t^{i,s}\right) = P_t^i\left(S_t^i\right)^{\nu}\left(1+\tau_{i,t}^{i,s}\right) = \left(P_t^i\left(1+\tau_{i,t}^{i,s}\right)\right)^{1-\nu}\left(P_t^*\left(1+\tau_t^{*,s}\right)\right)^{\nu}$ $\Leftrightarrow \quad P_{c,t}^{i} = P_t^i\left(S_t^i\right)^{\nu}\left(\frac{1+\tau_{i,t}^{i,s}}{1+\tau_t^{*,s}}\right)^{\nu} = P_t^i\left(\frac{P_t^*}{P_t^i}\right)^{\nu}$. The CPI price index depends on the sales tax rate. Since the CPI-inflation should be stated depending on domestic and world inflation, we take logarithms and obtain equation (3.19.S) for the CPI-inflation $\pi_{c,t}^{i}$ of country $i$

$$\pi_{c,t}^{i} = \pi_t^i + \nu\Delta s_t^i + \Delta\log\left(1+\tau_{i,t}^{i,s}\right) - \Delta\log\left(1+\tau_t^{i,s}\right) = (1-\nu)\,\pi_t^i + \nu\pi_t^*. \tag{3.19.S}$$

The two tax rate terms sum up to zero, which can also be seen from the fact that $P_{c,t}^{i} = P_t^i\left(\frac{P_t^*}{P_t^i}\right)^{\nu}$ and thus, in logs, $p_{c,t}^{i} = (1-\nu)\,p_t^i + \nu p_t^*$. If we then integrate over all countries $i$, we obtain $p^*_{c,t} = p^*_t$. Furthermore, $\int_0^1 \pi^i_t di = \pi^*_t$. As a consequence, $\pi^*_{c,t} = \pi^*_t$, i.e., the union-wide CPI-inflation is equal to the union-wide inflation rate.[8]

### 3.2.3 International Risk Sharing

In this subsection, we develop a relationship between log home consumption $c^i_t$ of a country $i$ and the log union-wide overall consumption $c^*_t$. The derivation is based on the consumption Euler equation obtained from the utility maximization problem of the households. Since the Euler equation depends on the choice of the fiscal tax instrument, both scenarios are analyzed separately.

If the government levies wage taxes, the corresponding consumption Euler equation (3.14.W), applied to two countries $i$ and $f$, implies

$$\beta \frac{C^f_t}{C^f_{t+1}} \frac{P^f_{c,t}}{P^f_{c,t+1}} = Q_{t,t+1} = \beta \frac{C^i_t}{C^i_{t+1}} \frac{P^i_{c,t}}{P^i_{c,t+1}}. \tag{3.20.W}$$

We assume that the stochastic discount factor $Q_{t,t+1}$ is the same for all countries. The solution for the difference equation (3.20.W) is $\vartheta_i C^f_t P^f_{c,t} = C^i_t P^i_{c,t}$.[9] Thus,

$$\vartheta_i C^f_t \frac{P^f_{c,t}}{P^i_{c,t}} = C^i_t = \vartheta_i C^f_t \left(S^i_{f,t}\right)^{1-\nu}. \tag{3.21.W}$$

If there are zero net foreign assets and if every country faces an identical initial environment, $\vartheta_i = \vartheta = 1 \quad \forall i \in [0,1]$. Equation (3.21.W) changes to $C^i_t = C^f_t \left(S^i_{f,t}\right)^{1-\nu}$. To obtain a linear system of equations, we take logarithms. Log domestic consumption $c^i_t$ depends on log foreign consumption $c^f_t$ in the following way: $c^i_t = c^f_t + (1-\nu)\, s^i_{f,t}$. After integrating over all countries $f$, we obtain an equation describing the relationship between log consumption $c^i_t$ of country $i$ to log world consumption $c^*_t$:

$$c^i_t = c^*_t + (1-\nu)\, s^i_t. \tag{3.22.W}$$

For the model including a sales tax, the consumption Euler equation (3.14.S) is different. Moreover, the effective price, i.e., the price including the tax markup determines the

[8] The result is the same as for a wage tax scenario.

[9] This is true, because $\beta(C^f_t P^f_{c,t})/(C^f_{t+1} P^f_{c,t+1}) = \beta(C^i_t P^i_{c,t})/(C^i_{t+1} P^i_{c,t+1}) \Leftrightarrow (C^f_t P^f_{c,t})/(C^f_{t+1} P^f_{c,t+1}) = (C^i_t P^i_{c,t})/(C^i_{t+1} P^i_{c,t+1})$. Inserting the solution leads to $(C^f_t P^f_{c,t})/(C^f_{t+1} P^f_{c,t+1}) = (\vartheta_i C^f_t P^f_{c,t})/(\vartheta_i C^f_{t+1} P^f_{c,t+1})$, which is true $\forall\ t$ and all constants $\vartheta_i \neq 0$.

exchange relation between the terms of trade of one and another country. With the Euler equation applied to two countries, we obtain a difference equation of the form:

$$\beta \frac{C_t^f}{C_{t+1}^f} \frac{P_{c,t}^f}{P_{c,t+1}^f} \frac{\left(1+\tau_t^{f,s}\right)}{\left(1+\tau_{t+1}^{f,s}\right)} = Q_{t,t+1} = \beta \frac{C_t^i}{C_{t+1}^i} \frac{P_{c,t}^i}{P_{c,t+1}^i} \frac{\left(1+\tau_t^{i,s}\right)}{\left(1+\tau_{t+1}^{i,s}\right)}, \tag{3.20.S}$$

which is solved by

$$\vartheta_i C_t^f \frac{P_{c,t}^f}{P_{c,t}^i} \frac{\left(1+\tau_t^{f,s}\right)}{\left(1+\tau_t^{i,s}\right)} = C_t^i = \vartheta_i C_t^f S_{f,t}^i \left(\frac{S_t^f}{S_t^i}\right)^{\nu} = \vartheta_i C_t^f \left(S_{f,t}^i\right)^{1-\nu}. \tag{3.21.S}$$

Assuming $\vartheta_i = 1 \ \forall i \in [0,1]$, we take logarithms and obtain $c_t^i = c_t^f + (1-\nu)\, s_{f,t}^i$. If we integrate over all countries $f$, the log consumption $c_t^i$ in country $i$ depends on the log world consumption and the log effective terms of trade, i.e.,[10]

$$c_t^i = c_t^* + (1-\nu)\, s_t^i. \tag{3.22.S}$$

### 3.2.4 The Government Sector

The government decides which amount of public goods optimally to provide. It faces a solvency constraint, which relates current debt to the present value of former debt, expenditure for public goods, and revenue in terms of taxes.

Public goods $G_t^i$, e.g., infrastructure, are produced in the home country $i$ and optimization between goods produced in country $i$ implies—see also equation (3.2)—that

$$G_t^i = \left(\int_0^1 \left(G_t^i(j)\right)^{(\varepsilon-1)/\varepsilon} dj\right)^{\varepsilon/(\varepsilon-1)},$$

where $\varepsilon$ denotes the elasticity of substitution between different goods produced in the same country. $\varepsilon > 1$ is the same for all countries and also enters the households' optimization of different goods produced in one country.

The government faces a solvency constraint relating present debt $B_t^i$ in country $i$ for period $t$ to income sources, expenditure, and former debt. We assume that seigniorage does not present any source of income for the national government sectors such that

[10] The equation is the same as for the wage tax scenario, equation (3.22.W).

taxes (wage and sales taxes, respectively) are the only component of government revenue. Expenditures are described as public goods $G_t^i$, which have to be payed by the government. Furthermore, a lump-sum tax $T_t^i$ as well as an employment subsidy $\chi_t^i$ on the wage is included. The steady state of the economy is only efficient if the distortions in both markets, the labor and the goods market, are offset. If there are taxes on income together with Calvo-pricing, both markets are distorted. The employment subsidy, which is financed by the lump-sum tax has to be chosen to offset the underlying distortions. With sales taxes and Calvo-pricing, only the goods market is distorted such that the employment subsidy is chosen to offset the corresponding distortion.

We follow Leith and Wren-Lewis [83] and define the intertemporal budget constraint of the government in country $i$ as a function of debt, government expenditure, and government revenue. Hence, the solvency constraint for the two scenarios is given by

$$B_t^i = R_{t-1}^* B_{t-1}^i + P_t^i G_t^i - W_t^i N_t^i \tau_t^{i,l} - T_t^i + \chi_t^i W_t^i N_t^i, \tag{3.23.W}$$
$$B_t^i = R_{t-1}^* B_{t-1}^i + P_t^i \left(1 + \tau_{i,t}^{i,s}\right) G_t^i - P_t^i Y_t^i \tau_{i,t}^{i,s} - T_t^i + \chi_t^i W_t^i N_t^i. \tag{3.23.S}$$

Debt today is equal to the present value of former debt. Moreover, government spending today is added and the tax revenue today is subtracted since actual expenditure augment debt and revenues reduce debt. $P_t^i G_t^i$ or $\left(1 + \tau_{i,t}^{i,s}\right) P_t^i G_t^i$ denotes the public goods valued with prices $P_t^i$ and $\left(1 + \tau_{i,t}^{i,s}\right) P_t^i$, respectively. An additional expenditure of the governmental authority is the employment subsidy, which is a part of the wage income. The tax revenue consists of the lump-sum tax and the amount of wage taxes and sales taxes, respectively.

If we define $B_t^{i,r}$ as the real debt (i.e., the nominal debt level $B_t^i$ divided by the price level $P_t^i$ of country $i$) multiplied by the rate of return for country $i$ at time $t$, $B_t^{i,r} = \frac{B_t^i R_t^*}{P_t^i}$, the respective budget constraint changes to

$$\frac{B_t^{i,r}}{R_t^*} = \frac{P_{t-1}^i}{P_t^i} B_{t-1}^{i,r} + G_t^i - \left(\tau_t^{i,l} - \chi_t^i\right) \frac{W_t^i}{P_t^i} N_t^i - \frac{T_t^i}{P_t^i},$$
$$\frac{B_t^{i,r}}{R_t^*} = \frac{P_{t-1}^i}{P_t^i} B_{t-1}^{i,r} + G_t^i - \tau_{i,t}^{i,s} \left(Y_t^i - G_t^i\right) + \chi_t^i \frac{W_t^i}{P_t^i} N_t^i - \frac{T_t^i}{P_t^i}.$$

Since the employment subsidy is chosen to offset distortions in the markets when being in steady state and is financed by the lump-sum tax, the last two terms sum up to zero.

Log-linearizing around the steady state delivers:

$$\beta\left(\widehat{b}_t^{i,r} - \widehat{\log R}_t^*\right) = \widehat{b}_{t-1}^{i,r} - \pi_t^i + \frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}}\log\left(\frac{G_t^i}{\bar{G}^i}\right) - \frac{\bar{\tau}^{i,l}\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}\left(\widehat{rw}_t^i + \widehat{n}_t^i + \nu\widehat{s}_t^i\right) - \frac{\bar{\tau}^{i,l}\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}\widehat{\log\tau_t^{i,l}}, \quad (3.24.W)$$

$$\beta\left(\widehat{b}_t^{i,r} - \widehat{\log R}_t^*\right) = \widehat{b}_{t-1}^{i,r} - \pi_t^i + \frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}}\left(1+\bar{\tau}_i^{i,s}\right)\log\left(\frac{G_t^i}{\bar{G}^i}\right) - \frac{\bar{\tau}_i^{i,s}\bar{P}^i\bar{Y}^i}{\bar{B}^i\bar{R}}\widehat{y}_t^i - \frac{\bar{\tau}_i^{i,s}\bar{P}^i}{\bar{B}^i\bar{R}}\left(\bar{Y}^i - \bar{G}^i\right)\widehat{\log\tau_{i,t}^{i,s}}. \quad (3.24.S)$$

### 3.2.5 The Business Sector

Within this section, the characteristics of firms acting in a market with monopolistic competition and price rigidities are analyzed. The optimal reset price under Calvo-price setting is calculated. Furthermore, we develop the marginal costs for the firms' production. Finally, the New-Keynesian Phillips curve describing the supply side of the model is derived.

We begin with the production function for a specific good $Y^i(j)$ produced in country $i$. The product $Y_t^i(j)$ depends on the technology level $A_t^i$ of country $i$ and the labor $N_t^i(j)$ utilized to produce good $j$, $Y_t^i(j) = A_t^i N_t^i(j)$. The log technology $a_t^i = \log A_t^i = \rho_a a_{t-1}^i + \varepsilon_t^{i,a}$ follows an AR(1) process with $\rho_a \in [0,1]$ and $\varepsilon_t^{i,a}$ is white noise. The demand function for a good $Y_t^i(j)$ depends on the price for good $j$, the overall price level of country $i$, the elasticity of substitution of different goods in one country, and the overall output produced in country $i$

$$Y_t^i(j) = \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon} Y_t^i. \quad (3.25)$$

The overall output produced in country $i$ is defined as the aggregate of the different goods $Y_t^i(j)$, $Y_t^i = \left(\int_0^1 \left(Y_t^i(j)\right)^{(\varepsilon-1)/\varepsilon} dj\right)^{\varepsilon/(\varepsilon-1)}$, and the overall labor used for production is equal to $N_t^i = \left(\int_0^1 \left(N_t^i(j)\right)^{(\varepsilon-1)/\varepsilon} dj\right)^{\varepsilon/(\varepsilon-1)}$. If we integrate $Y_t^i(j)^{(\varepsilon-1)/\varepsilon}$ over all firms $j$ and take it to the power of $\frac{\varepsilon}{\varepsilon-1}$, it holds that $Y_t^i = A_t^i N_t^i$. Taking logarithms leads to the following linear equation: $y_t^i = a_t^i + n_t^i$. Log output is equal to the sum of log technology and log labor.

Since only a specific part of firms can reset prices, the average price $P_t^i$ in country $i$ is given by a Calvo-price setting rule

$$P_t^i = \left[(1-\theta)\left(\bar{P}_t^i\right)^{1-\varepsilon} + \theta\left(P_{t-1}^i\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{3.26}$$

where $\theta$ denotes the part of firms not being able to reset prices to the reset price $\bar{P}_t^i$. The reset price is derived in the following. Firms maximize their profits, i.e., the difference of revenue and costs. If we optimize the objective function of a firm $j$ in real terms, we obtain the traditional optimal reset price in log-linearized form:

$$\bar{p}_t^i = (1-\beta\theta)\sum_{s=0}^{\infty}(\beta\theta)^s E_t\left(\log\mu_{t+s}^i + mc_{t+s}^i + p_{t+s}^i\right), \tag{3.27}$$

where $mc^i$ denotes the log real marginal costs weighted with the domestic price index and $\log\mu_t^i$ is the optimal price markup in the absence of sticky prices, i.e., in a flexible price equilibrium. Because the absence of sticky prices implies that there is zero inflation in steady state, $\log\mu_t^i$ is the markup apparent in a zero inflation steady state. In the flex price equilibrium, the marginal costs are exclusively driven by the optimal price markup. Thus, $mc_t^i = -\log\mu_t^i$.

To derive a New-Keynesian Phillips curve type equation, we take the following steps: using equation (3.27) we get for the reset price in $t+1$ that $\bar{p}_{t+1}^i = (1-\beta\theta)\sum_{s=0}^{\infty}(\beta\theta)^s$ $E_{t+1}\left(\log\mu_{t+1+s}^i + mc_{t+1+s}^i + p_{t+1+s}^i\right) \Leftrightarrow \frac{\bar{p}_t^i}{1-\beta\theta} = mc_t^i + p_t^i + \log\mu_t^i + \frac{\theta\beta}{1-\theta\beta}E_t\bar{p}_{t+1}^i$. If we log-linearize equation (3.26), we obtain $p_t^i = (1-\theta)\,\bar{p}_t^i + \theta p_{t-1}^i$ and therefore $\bar{p}_t^i = \frac{p_t^i - \theta p_{t-1}^i}{1-\theta}$. The optimal reset price in log-linearized form can be rewritten as

$$\begin{aligned}
\frac{1}{(1-\theta\beta)(1-\theta)}\left(p_t^i - \theta p_{t-1}^i\right) &= \frac{\theta\beta}{(1-\theta\beta)(1-\theta)}E_t\left(p_{t+1}^i - \theta p_t^i\right) \\
&\quad + w_t^i - a_t^i + \log\left(1-\chi_t^i\right) + \log\mu_t^i \\
\Leftrightarrow \frac{1}{(1-\theta\beta)(1-\theta)}\left(p_t^i - p_{t-1}^i\right) &= \frac{\theta\beta}{(1-\theta\beta)(1-\theta)}E_t\left(p_{t+1}^i - p_t^i\right) - \frac{1}{1-\theta\beta}p_{t-1}^i \\
&\quad + \frac{\theta\beta}{1-\theta\beta}p_t^i + w_t^i - a_t^i + \log\left(1-\chi_t^i\right) + \log\mu_t^i
\end{aligned}$$

and $\frac{\theta}{(1-\theta\beta)(1-\theta)}\left(\pi_t^i\right) = \frac{\theta\beta}{(1-\theta\beta)(1-\theta)} E_t\left(\pi_{t+1}^i\right) - p_t^i + w_t^i - a_t^i + \log\left(1-\chi_t^i\right) + \log\mu_t^i$. Thus, the New-Keynesian Phillips curve is

$$\begin{aligned} \pi_t^i &= \beta E_t \pi_{t+1}^i + \frac{(1-\theta\beta)(1-\theta)}{\theta}\left[w_t^i - a_t^i + \log\left(1-\chi_t^i\right) + \log\mu_t^i - p_t^i\right] \\ &= \beta E_t \pi_{t+1}^i + \frac{(1-\theta\beta)(1-\theta)}{\theta}\left[mc_t^i + \log\mu_t^i\right]. \end{aligned} \tag{3.28}$$

To obtain a formula for the log real marginal costs $mc_t^i$, we use the fact that real firms' costs are equal to $\frac{W_t^i N_t^i(j)\left(1-\chi_t^i\right)}{P_t^i} = \frac{W_t^i Y_t^i(j)\left(1-\chi_t^i\right)}{P_t^i A_t^i}$. Therefore, the real marginal costs are $MC_t^i = \frac{W_t^i\left(1-\chi_t^i\right)}{P_t^i A_t^i}$, and the log real marginal costs are

$$mc_t^i = w_t^i - p_t^i - a_t^i + \log\left(1-\chi_t^i\right). \tag{3.29}$$

If we use the first order condition of the household's optimization problem (equation (3.15.W) and (3.15.S)), we obtain the following equations for the two different model settings.

$$\begin{aligned} mc_t^i =& \varphi n_t^i + c_t^i + \frac{\bar{\tau}^{i,l}}{1-\bar{\tau}^{i,l}}\log\tau_t^{i,l} - \log\left(1-\psi^i\right) - \frac{\bar{\tau}^{i,l}}{1-\bar{\tau}^{i,l}}\log\bar{\tau}^{i,l} - \log\left(1-\bar{\tau}^{i,l}\right) \\ & -a_t^i + \log\left(1-\chi_t^i\right) + \nu s_t^i \\ =& \varphi y_t^i - (1+\varphi)\,a_t^i + c_t^i + \frac{\bar{\tau}^{i,l}}{1-\bar{\tau}^{i,l}}\log\tau_t^{i,l} - \log\left(1-\psi^i\right) - \frac{\bar{\tau}^{i,l}}{1-\bar{\tau}^{i,l}}\log\bar{\tau}^{i,l} \\ & -\log\left(1-\bar{\tau}^{i,l}\right) + \log\left(1-\chi_t^i\right) + \nu s_t^i, \end{aligned} \tag{3.30.W}$$

$$\begin{aligned} mc_t^i =& \varphi n_t^i + c_t^i + \frac{\bar{\tau}^{i,s}}{1+\bar{\tau}^{i,s}}\log\tau_t^{i,s} - \log\left(1-\psi^i\right) - \frac{\bar{\tau}^{i,s}}{1+\bar{\tau}^{i,s}}\log\bar{\tau}^{i,s} + \log\left(1+\bar{\tau}^{i,s}\right) \\ & -a_t^i + \log\left(1-\chi_t^i\right) + \nu s_t^i + \log\left(1+\tau_{i,t}^{i,s}\right) - \log\left(1+\tau_t^{i,s}\right) \\ =& \varphi y_t^i - (1+\varphi)\,a_t^i + c_t^i + \frac{\bar{\tau}^{i,s}}{1+\bar{\tau}^{i,s}}\log\tau_t^{i,s} - \log\left(1-\psi^i\right) - \frac{\bar{\tau}^{i,s}}{1+\bar{\tau}^{i,s}}\log\bar{\tau}^{i,s} \\ & +\log\left(1+\bar{\tau}^{i,s}\right) + \log\left(1-\chi_t^i\right) + \nu s_t^i + \log\left(1+\tau_{i,t}^{i,s}\right) - \log\left(1+\tau_t^{i,s}\right). \end{aligned} \tag{3.30.S}$$

Finally, the steady state tax rates are chosen such that they absorb all distortions caused by the markup shock. Moreover, the employment subsidy compensates the distortions caused by the tax. For the wage tax, it holds in steady state that $\bar{\tau}^{i,l} = 1 - 1/\bar{\mu}^i$ and with $1/\bar{\mu}^i = \bar{MC}^i = \frac{1-\bar{\chi}^i}{1-\bar{\tau}^{i,l}} = \left(1-\bar{\chi}^i\right)\bar{\mu}^i \Leftrightarrow \bar{\chi}^i = 1 - \left(1/\bar{\mu}^i\right)^2$. Furthermore, $\log\left(1-\bar{\tau}^{i,l}\right) = -\log\bar{\mu}^i$. The description of the steady state tax level or the natural level of taxes in Subsection 3.2.7 is comparable. The natural level of taxes is the tax rate in the flex price equilibrium.

For the steady state sales tax, it holds analogously that $\bar{\mu}^i - 1 = \bar{\tau}_i^{i,s}$ and $\bar{MC}^i = (1-\bar{\chi}^i)\left(1+\bar{\tau}_i^{i,s}\right) = 1/\bar{\mu}^i \Leftrightarrow \bar{\chi}^i = 1-\left(1/\bar{\mu}^i\right)^2$, $\log\left(1+\bar{\tau}_i^{i,s}\right) = \log \bar{\mu}^i$.

### 3.2.6 Equilibrium and the Fiscal Variable $g$

As a first step to analyze the equilibrium, the market clearing condition for a good $j$ produced in country $i$ can be stated as

$$\begin{aligned} Y_t^i(j) &= C_{i,t}^i(j) + \int_0^1 C_{i,t}^f(j)df + G_t^i(j) \\ &= \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon}\left[C_{i,t}^i + \int_0^1 C_{i,t}^f df + G_t^i\right], \end{aligned} \tag{3.31}$$

where the first term $C_{i,t}^i(j)$ in equation (3.31) denotes the consumption of domestic goods in country $i$, the second term symbolizes the overall exports $\int_0^1 C_{i,t}^f(j)df$ of domestic goods to foreign countries $f$, whereas the last term $G_t^i(j)$ are the public goods in country $i$. There is no investment in the model. For the second row, we use the optimality conditions obtained by optimizing between different goods produced in one country. Thus, the overall production of a good $j$ can be described as the sum of weighted consumption of overall domestic goods, overall public goods, and overall exports of domestically produced goods. With $P_t^i C_{i,t}^i = (1-\nu)P_{c,t}^i C_t^i$, $\left(1+\tau_{i,t}^{i,s}\right)P_t^i C_{i,t}^i = (1-\nu)\left(1+\tau_t^{i,s}\right)P_{c,t}^i C_t^i$, and equation (3.21.W), (3.21.S), the market clearing condition (equation (3.31)) changes to

$$Y_t^i(j) = \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon}\left[\left(S_t^i\right)^{\nu} C_t^i + G_t^i\right]. \tag{3.32}$$

Using $Y_t^i = \left(\int_0^1 \left(Y_t^i(j)\right)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$, we obtain for the overall production $Y_t^i$ in country $i$ the market clearing condition

$$Y_t^i = \left(S_t^i\right)^{\nu} C_t^i + G_t^i, \tag{3.33}$$

which holds for both model settings. We follow Leith and Wren-Lewis [83] and introduce the fiscal variable $g_t^i = -\log\left(1-\frac{G_t^i}{Y_t^i}\right)$ in the model setup. All model equations are rewritten with respect to the new fiscal variable. With $y_t^i - g_t^i = \log Y_t^i + \log\left(1-\frac{G_t^i}{Y_t^i}\right) = \log\left(Y_t^i - G_t^i\right)$ and after log-linearization, equation (3.33) can be described as

$$y_t^i - g_t^i = \nu s_t^i + c_t^i. \tag{3.34}$$

The log output in country $i$ depends on the fiscal variable $g_t^i$, log consumption $c_t^i$ and the log effective terms of trade $s_t^i$. As before, the overall log effective terms of trade sum up to zero $\int_0^1 s_t^i di = 0$ because the effective terms of trade for all countries are equal to one. We define the union-wide variables as follows: $y_t^* = \int_0^1 y_t^i di$, $c_t^* = \int_0^1 c_t^i di$, and $g_t^* = \int_0^1 g_t^i di$. After integrating over all countries $i$, the overall market clearing condition can be stated as

$$y_t^* = c_t^* + g_t^*. \tag{3.35}$$

Union-wide log output is equal to the sum of union-wide log consumption and the union-wide fiscal variable. If we use the overall market clearing condition and additionally the fact that $E_t\pi_{c,t+1}^i = E_t\left(p_{t+1}^i - p_t^i\right) + \nu E_t\left(s_{t+1}^i - s_t^i\right)$, $E_t\pi_{c,t+1}^i = E_t\pi_{t+1}^i + \nu E_t\Delta s_{t+1}^i + E_t\Delta\log\left(1+\tau_{i,t+1}^{i,s}\right) - E_t\Delta\log\left(1+\tau_{t+1}^{i,s}\right)$, respectively, the consumption Euler equations (3.16.W) and (3.16.S) can be rewritten as[11]

$$y_t^i = E_t y_{t+1}^i - \left(\log R_t^* - E_t\pi_{t+1}^i - \rho\right) - E_t\Delta g_{t+1}^i, \tag{3.36.W}$$

$$y_t^i = E_t y_{t+1}^i - \left(\log R_t^* - E_t\pi_{t+1}^i - \rho\right) - E_t\Delta g_{t+1}^i + \frac{\bar{\tau}_i^{i,s}}{1+\bar{\tau}_i^{i,s}} E_t\Delta\log\tau_{i,t+1}^{i,s}. \tag{3.36.S}$$

We want to substitute $g_t^i$ for $\log\frac{G_t^i}{\bar{G}^i}$ in the government solvency constraints. With $\log G_t^i = \log\left(\frac{G_t^i}{Y_t^i}Y_t^i\right) = \log\frac{G_t^i}{Y_t^i} + y_t^i = \log\left(1-e^{-g_t^i}\right) + y_t^i$, the term $\log\left(1-e^{-g_t^i}\right)$ can be linearized around steady state in the following way: $\log\left(1-e^{-g_t^i}\right) \approx \frac{1-\gamma^i}{\gamma^i}g_t^i + \log\gamma^i + \frac{1-\gamma^i}{\gamma^i}\log\left(1-\gamma^i\right)$. $\gamma^i = \frac{\bar{G}^i}{\bar{Y}^i}$ is defined as the steady state government expenditure per output ratio. Thus,

$$\log\frac{G_t^i}{\bar{G}^i} = \log G_t^i - \log\bar{G}^i = \frac{1-\gamma^i}{\gamma^i}\widehat{g}_t^i + \widehat{y}_t^i,$$

where $\widehat{g}_t^i$ and $\widehat{y}_t^i$ are the log deviations of the original variables from the respective steady state value. With equation (3.34), the two government solvency constraints (equation

[11] Note that $\frac{\bar{\tau}_i^{i,s}}{1+\bar{\tau}_i^{i,s}}E_t\Delta\log\tau_{i,t+1}^{i,s} = E_t\Delta\log\left(1+\tau_{i,t+1}^{i,s}\right)$.

(3.24.W) and (3.24.S)) become

$$\beta\left(\widehat{b}_t^{i,r} - \widehat{\log R_t^*}\right) = \widehat{b}_{t-1}^{i,r} - \pi_t^i + \left(\frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}}\left(\frac{1-\gamma^i}{\gamma^i}\right) + \bar{\tau}^{i,l}\frac{\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}\right)\widehat{g}_t^i + \left(\frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}} - \frac{\bar{\tau}^{i,l}\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}\right)\widehat{y}_t^i - \frac{\bar{\tau}^{i,l}\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}(1+\varphi)\left(\widehat{y}_t^i - a_t^i\right) - \frac{\bar{\tau}^{i,l}\bar{W}^i\bar{N}^i}{\left(1-\bar{\tau}_t^{i,l}\right)\bar{B}^i\bar{R}}\widehat{\log \tau_t^{i,l}}, \tag{3.37.W}$$

$$\beta\left(\widehat{b}_t^{i,r} - \widehat{\log R_t^*}\right) = \widehat{b}_{t-1}^{i,r} - \pi_t^i + \frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}}\left(\frac{1-\gamma^i}{\gamma^i}\right)\left(1+\bar{\tau}_i^{i,s}\right)\widehat{g}_t^i + \left(\frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}}\left(1+\bar{\tau}_i^{i,s}\right) - \frac{\bar{\tau}_i^{i,s}\bar{P}^i\bar{Y}^i}{\bar{B}^i\bar{R}}\right)\widehat{y}_t^i - \frac{\bar{\tau}_i^{i,s}\bar{P}^i}{\bar{B}^i\bar{R}}\left(\bar{Y}^i - \bar{G}^i\right)\widehat{\log \tau_{i,t}^{i,s}}. \tag{3.37.S}$$

Furthermore, equations (3.30.W) and (3.30.S) change with the introduction of the fiscal variable $g$ and with equation (3.34) to

$$mc_t^i = (1+\varphi)\left(y_t^i - a_t^i\right) - g_t^i + \log\left(1-\chi_t^i\right) - \log\left(1-\psi^i\right) - \log\left(1-\tau_t^{i,l}\right), \tag{3.38.W}$$

$$mc_t^i = (1+\varphi)\left(y_t^i - a_t^i\right) - g_t^i + \log\left(1-\chi_t^i\right) - \log\left(1-\psi^i\right) + \log\left(1+\tau_{i,t}^{i,s}\right). \tag{3.38.S}$$

Overall, all equations needed for the model setting have been derived. The demand side of the model is given by the Euler equation (3.36.W) and (3.36.S). The New-Keynesian Phillips curve is given by equation (3.28), where the log real marginal costs are defined in equation (3.38.W) and (3.38.S). To summarize, the following can be stated.

**Model Description 3.1.**

- The demand if wage taxes are used as fiscal instrument:
  $y_t^i = E_t y_{t+1}^i - \left(\log R_t^* - E_t\pi_{t+1}^i - \rho\right) - E_t\Delta g_{t+1}^i.$
- The demand if sales taxes are used as fiscal instrument:
  $y_t^i = E_t y_{t+1}^i - \left(\log R_t^* - E_t\pi_{t+1}^i - \rho\right) - E_t\Delta g_{t+1}^i + \frac{\bar{\tau}_i^{i,s}}{1+\bar{\tau}_i^{i,s}}E_t\Delta\log\tau_{i,t+1}^{i,s}.$
- The New-Keynesian Phillips curve:
  $\pi_t^i = \beta E_t\pi_{t+1}^i + \frac{(1-\theta\beta)(1-\theta)}{\theta}\left[mc_t^i + \log\mu_t^i\right].$

Finally, we develop an equation that sheds light on the relationship between domestic output, the terms of trade, and overall consumption. Equation (3.34) describes how log output depends on the fiscal variable, the log consumption, and the log terms of trade. Furthermore, equation (3.22.W) and (3.22.S) show that union-wide log consumption is a function of domestic log consumption and the log terms of trade. Combining the equations, we obtain

$$\Delta y_t^i = \Delta g_t^i + \Delta c_t^* + \Delta s_t^i.$$

If the government uses a wage tax as fiscal instrument, the log effective terms of trade depend on the domestic as well as the union-wide price level. For the sales tax scenario, the domestic and union-wide tax rate additionally influence the log effective terms of trade. If we substitute for the log effective terms of trade in the equation described above, the two following equations result, depending on the tax instrument.

$$\Delta y_t^i = \Delta g_t^i + \Delta c_t^* + \pi_t^* - \pi_t^i, \tag{3.39.W}$$

$$\Delta y_t^i = \Delta g_t^i + \Delta c_t^* + \pi_t^* - \pi_t^i + \Delta \log\left(1 + \tau_t^{*,s}\right) - \Delta \log\left(1 + \tau_{i,t}^{i,s}\right). \tag{3.39.S}$$

These additional equations show that the change in domestic log output depends on the change in the fiscal variable, the change in union-wide consumption, the difference in the union-wide and the domestic inflation rate, and, if sales taxes are levied, on the difference between the union-wide and the domestic log tax rate. Thus, the equations are a measure of competitiveness as they relate changes in domestic variables to changes arising between the domestic country and the union as a whole.

Finally, the model is closed by the policy maker's decision, who faces the government solvency constraint given by equation (3.37.W) and (3.37.S), respectively.

### 3.2.7 Gap Variables

The last step in the derivation of the setup is to rewrite the model equations in gap variables form, i.e., in deviations form. We define the natural level of a variable as the level that would occur if there are no nominal rigidities. Then, the gap of a variable $x$ is assumed to be the difference of the variable $x_t$ and its natural level $x_t^n$. The natural level of log output $y_t^n$ is thus the level that would occur in an economy without nominal rigidities. The gap variable $x_t^{i,g}$ of variable $x^i$ is given by $x_t^{i,g} = x_t^i - x_t^{i,n} = \widehat{x}_t^i - \widehat{x}_t^{i,n}$, where $\widehat{x}$ denotes the log-deviation of variable $x$ from steady state. Without nominal rigidities, the log of the real marginal costs is exclusively driven by the optimal markup on prices for a flex price equilibrium. Therefore, $mc_t^i = -\log \mu_t^i$ for both model settings. If we use equation (3.38.W) and (3.38.S), an expression for the natural output level of each model setup is obtained

$$\begin{aligned} y_t^{i,n} &= a_t^i + \frac{g_t^{i,n}}{1+\varphi} - \frac{\log\left(1-\chi_t^i\right)}{1+\varphi} + \frac{\left(1-\psi^i\right)}{1+\varphi} + \frac{\left(\log\left(1-\tau_t^{i,l}\right)\right)^n}{1+\varphi} - \frac{\log \mu_t^i}{1+\varphi} \\ y_t^{i,n} &= a_t^i + \frac{g_t^{i,n}}{1+\varphi} - \frac{\log\left(1-\chi_t^i\right)}{1+\varphi} + \frac{\left(1-\psi^i\right)}{1+\varphi} - \frac{\left(\log\left(1+\tau_{i,t}^{i,s}\right)\right)^n}{1+\varphi} - \frac{\log \mu_t^i}{1+\varphi}. \end{aligned}$$

The New-Keynesian Phillips curves can then be rewritten in gap form as

$$\pi_t^i = \beta E_t \pi_{t+1}^i + \frac{(1-\theta\beta)(1-\theta)}{\theta}\left((1+\varphi)\, y_t^{i,g} - g_t^{i,g} + \frac{\bar{\tau}^{i,l}}{1-\bar{\tau}^{i,l}}\left(\log \tau_t^{i,l}\right)^g\right), \quad (3.40.W)$$

$$\pi_t^i = \beta E_t \pi_{t+1}^i + \frac{(1-\theta\beta)(1-\theta)}{\theta}\left((1+\varphi)\, y_t^{i,g} - g_t^{i,g} + \frac{\bar{\tau}_i^{i,s}}{1+\bar{\tau}_i^{i,s}}\left(\log \tau_{i,t}^{i,s}\right)^g\right). \quad (3.40.S)$$

The Euler equations (3.36.W) and (3.36.S) change to

$$y_t^{i,g} = E_t y_{t+1}^{i,g} - \left(\log R_t^* - E_t \pi_{t+1}^i - \log R_t^{*,n}\right) - E_t \Delta g_{t+1}^{i,g}, \quad (3.41.W)$$

$$\begin{aligned} y_t^{i,g} = {} & E_t y_{t+1}^{i,g} - \left(\log R_t^* - E_t \pi_{t+1}^i - \log R_t^{*,n}\right) - E_t \Delta g_{t+1}^{i,g} \\ & + \frac{\bar{\tau}_i^{i,s}}{1+\bar{\tau}_i^{i,s}} E_t \Delta \left(\log \tau_{i,t+1}^{i,s}\right)^g, \end{aligned} \quad (3.41.S)$$

where the log natural level of the rate of return for the two settings is given by

$$\log R_t^{*,n} = E_t \Delta y_{t+1}^{i,n} + \rho - E_t \Delta g_{t+1}^{i,n}, \quad (3.42.W)$$

$$\log R_t^{*,n} = E_t \Delta y_{t+1}^{i,n} + \rho - E_t \Delta g_{t+1}^{i,n} + \frac{\bar{\tau}_i^{i,s}}{1+\bar{\tau}_i^{i,s}} E_t \Delta \left(\log \tau_{i,t+1}^{i,s}\right)^n. \quad (3.42.S)$$

The log natural level of taxes is chosen to offset the markup shock on inflation. Hence, it holds that

$$\begin{aligned} \left(\log\left(1-\tau_t^{i,l}\right)\right)^n &= -\log \mu_t^i = -\frac{\bar{\tau}^{i,l}}{1-\bar{\tau}^{i,l}}\left(\log \tau_t^{i,l}\right)^n, \\ \left(\log\left(1+\tau_{i,t}^{i,s}\right)\right)^n &= \log \mu_t^i = \frac{\bar{\tau}^{i,s}}{1+\bar{\tau}_i^{i,s}}\left(\log \tau_{i,t}^{i,s}\right)^n. \end{aligned}$$

Equations (3.39.W) and (3.39.S) change to

$$\Delta y_t^{i,g} = \Delta g_t^{i,g} + \Delta c_t^{*,g} + \pi_t^* - \pi_t^i - \Delta a_t^i + \Delta a_t^*, \tag{3.43.W}$$

$$\begin{aligned}\Delta y_t^{i,g} = {} & \Delta g_t^{i,g} + \Delta c_t^{*,g} + \pi_t^* - \pi_t^i + \Delta\left(\log\left(1+\tau_t^{*,s}\right)^g\right) - \Delta\left(\log\left(1+\tau_{i,t}^{i,s}\right)\right)^g \\ & -\Delta a_t^i + \Delta a_t^* + \underbrace{\Delta\left(\log\left(1+\tau_t^{*,s}\right)\right)^n - \Delta\left(\log\left(1+\tau_{i,t}^{i,s}\right)\right)^n}_{\Delta\widehat{\mu}_t^i},\end{aligned} \tag{3.43.S}$$

where $\widehat{\mu}_t^i = \frac{\log \mu_t^i}{\log \bar{\mu}}$ denotes the markup shock on prices.

Finally, the solvency constraints of the government are rewritten in gap form:

$$\begin{aligned}\beta\left(\widehat{b}_t^{i,r} - \widehat{\log R_t^*}\right) = {} & \widehat{b}_{t-1}^{i,r} - \pi_t^i + \left(\frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}}\left(\frac{1-\gamma^i}{\gamma^i}\right) + \bar{\tau}^{i,l}\frac{\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}\right)g_t^{i,g} \\ & + \left(\frac{1-\bar{R}}{\bar{R}}\right)y_t^{i,g} - \frac{\bar{\tau}^{i,l}\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}\left(\left(1+\varphi\right)y_t^{i,g} + \frac{\left(\log \tau_t^{i,l}\right)^g}{1-\bar{\tau}^{i,l}}\right) \\ & + \frac{1-\bar{R}}{\bar{R}}a_t^i - \frac{\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}\widehat{\mu}_t^i,\end{aligned} \tag{3.44.W}$$

$$\begin{aligned}\beta\left(\widehat{b}_t^{i,r} - \widehat{\log R_t^*}\right) = {} & \widehat{b}_{t-1}^{i,r} - \pi_t^i + \frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}}\left(\frac{1-\gamma^i}{\gamma^i}\right)\left(1+\bar{\tau}_i^{i,s}\right)g_t^{i,g} + \left(\frac{1-\bar{R}}{\bar{R}}\right)y_t^{i,g} \\ & - \left(\frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}} - \frac{1-\bar{R}}{\bar{R}}\right)\left(\log \tau_{i,t}^{i,s}\right)^g + \frac{1-\bar{R}}{\bar{R}}a_t^i \\ & + \left(\frac{1-\bar{R}}{\bar{R}} - \frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}}\right)\frac{1+\bar{\tau}_i^{i,s}}{\bar{\tau}_i^{i,s}}\widehat{\mu}_t^i.\end{aligned} \tag{3.44.S}$$

To summarize, the model consists of the following equations in gap form.

**Model Description 3.2.** The Wage-Tax Model

- The demand side of the model:

  $y_t^{i,g} = E_t y_{t+1}^{i,g} - \left(\log R_t^* - E_t\pi_{t+1}^i - \log R_t^{*,n}\right) - E_t\Delta g_{t+1}^{i,g}$

- The New-Keynesian Phillips curve:

  $\pi_t^i = \beta E_t\pi_{t+1}^i + \frac{(1-\theta\beta)(1-\theta)}{\theta}\left(\left(1+\varphi\right)y_t^{i,g} - g_t^{i,g} + \frac{\bar{\tau}^{i,l}}{1-\bar{\tau}^{i,l}}\left(\log \tau_t^{i,l}\right)^g\right)$

- The change in competitiveness:

  $\Delta y_t^{i,g} = \Delta g_t^{i,g} + \Delta c_t^{*,g} + \pi_t^* - \pi_t^i - \Delta a_t^i + \Delta a_t^*$

**Model Description 3.2.** The Wage-Tax Model—cont.

- The government solvency constraint:

$$\beta\left(\widehat{b}_t^{i,r}-\widehat{\log R_t^*}\right)=\widehat{b}_{t-1}^{i,r}-\pi_t^i+\left(\frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}}\left(\frac{1-\gamma^i}{\gamma^i}\right)+\bar{\tau}^{i,l}\frac{\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}\right)g_t^{i,g}+\left(\frac{1-\bar{R}}{\bar{R}}\right)y_t^{i,g}$$
$$-\frac{\bar{\tau}^{i,l}\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}\left((1+\varphi)\,y_t^{i,g}+\frac{\left(\log\tau_t^{i,l}\right)^g}{1-\bar{\tau}^{i,l}}\right)+\frac{1-\bar{R}}{\bar{R}}a_t^i-\frac{\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}\widehat{\mu}_t^i$$

**Model Description 3.3.** The Sales-Tax Model

- The demand side of the model:

$$y_t^{i,g}=E_ty_{t+1}^{i,g}-\left(\log R_t^*-E_t\pi_{t+1}^i-\log R_t^{*,n}\right)-E_t\Delta g_{t+1}^{i,g}+\frac{\bar{\tau}_i^{i,s}}{1+\bar{\tau}_i^{i,s}}E_t\Delta\left(\log\tau_{i,t+1}^{i,s}\right)^g$$

- The New-Keynesian Phillips curve:

$$\pi_t^i=\beta E_t\pi_{t+1}^i+\frac{(1-\theta\beta)(1-\theta)}{\theta}\left((1+\varphi)\,y_t^{i,g}-g_t^{i,g}+\frac{\bar{\tau}_i^{i,s}}{1+\bar{\tau}_i^{i,s}}\left(\log\tau_{i,t}^{i,s}\right)^g\right)$$

- The change in competitiveness:

$$\Delta y_t^{i,g}=\Delta g_t^{i,g}+\Delta c_t^{*,g}+\pi_t^*-\pi_t^i+\Delta\left(\log\left(1+\tau_t^{*,s}\right)^g\right)-\Delta\left(\log\left(1+\tau_{i,t}^{i,s}\right)\right)^g$$
$$-\Delta a_t^i+\Delta a_t^*+\Delta\left(\log\left(1+\tau_t^{*,s}\right)\right)^n-\Delta\widehat{\mu}_t^i$$

- The government solvency constraint:

$$\beta\left(\widehat{b}_t^{i,r}-\widehat{\log R_t^*}\right)=\widehat{b}_{t-1}^{i,r}-\pi_t^i+\frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}}\left(\frac{1-\gamma^i}{\gamma^i}\right)\left(1+\bar{\tau}_i^{i,s}\right)g_t^{i,g}+\left(\frac{1-\bar{R}}{\bar{R}}\right)y_t^{i,g}$$
$$-\left(\frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}}-\frac{1-\bar{R}}{\bar{R}}\right)\left(\log\tau_{i,t}^{i,s}\right)^g+\frac{1-\bar{R}}{\bar{R}}a_t^i+\left(\frac{1-\bar{R}}{\bar{R}}-\frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}}\right)\frac{1+\bar{\tau}_i^{i,s}}{\bar{\tau}_i^{i,s}}\widehat{\mu}_t^i$$

## 3.3 The National Problem

The following section derives the solution for the national social planner's problem as well as the national welfare function being the objective function to make government expenditure and tax decisions. First, the solution for the natural levels of output, labor, consumption, public goods, etc. in equilibrium is derived. Then, the national fiscal policy maker's optimization problem is analyzed in detail.

### 3.3.1 The Social Planner's Problem

The social planner maximizes the utility function $U\left(C_t^i, G_t^i, N_t^i\right) = \left(1-\psi^i\right)\log C_t^i + \psi^i \log G_t^i - \frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi}$ subject to the composition of output

$$\begin{aligned} \max \quad & U \\ \text{s.t.} \quad Y_t^i &= A_t^i N_t^i \quad \text{and} \\ Y_t^i &= \left(S_t^i\right)^{\nu} C_t^i + G_t^i = C_{i,t}^i + \underbrace{\int_0^1 C_{i,t}^f df}_{\text{exports}} + G_t^i, \end{aligned} \tag{3.45}$$

where $C_t^i = \frac{\left(C_{i,t}^i\right)^{1-\nu}\left(\exp\int_0^1 \log C_{f,t}^i df\right)^{\nu}}{(1-\nu)^{1-\nu}\nu^{\nu}}$ is the overall consumption of country $i$ and consists of the consumption of domestic and imported goods. Let $\lambda_{SP,t}$ denote the Lagrange multiplier of the maximization problem at time $t$. Taking the derivative of the Lagrangian resulting from the maximization problem (3.45) with respect to $C_{i,t}^i$, $C_{i,t}^j$, $N_t^i$, and $G_t^i$ gives the following first order conditions.

$$\begin{aligned} \lambda_{SP,t} &= \frac{\left(1-\psi^i\right)(1-\nu)}{C_{i,t}^i} \\ \lambda_{SP,t} &= \frac{\left(1-\psi^i\right)(\nu)}{\int_0^1 C_{i,t}^j dj} \\ \left(N_t^i\right)^{\varphi} &= \lambda_{SP,t} A_t^i \\ \lambda_{SP,t} &= \frac{\left(\psi^i\right)}{G_t^i}, \end{aligned}$$

such that $\left(N_t^i\right)^{\varphi} = A_t^i \frac{\left(1-\psi^i\right)(1-\nu)}{C_{i,t}^i} = A_t^i \frac{\left(1-\psi^i\right)(\nu)}{\int_0^1 C_{i,t}^j dj} = A_t^i \frac{\left(\psi^i\right)}{G_t^i}$.

$$\begin{aligned} \Leftrightarrow \qquad C_{i,t}^i &= \frac{A_t^i\left(1-\psi^i\right)(1-\nu)}{\left(N_t^i\right)^{\varphi}} \\ \Leftrightarrow \quad \int_0^1 C_{i,t}^f df &= \frac{A_t^i\left(1-\psi^i\right)\nu}{\left(N_t^i\right)^{\varphi}} \quad \text{and} \quad C_{i,t}^f = \frac{A_t^i\left(1-\psi^i\right)\nu}{\left(N_t^i\right)^{\varphi}} \\ \Leftrightarrow \qquad G_t^i &= A_t^i \frac{\left(\psi^i\right)}{\left(N_t^i\right)\varphi}. \end{aligned}$$

If we substitute the previous equations into $Y_t^i = A_t^i N_t^i = C_{i,t}^i + \int_0^1 C_{i,t}^f df + G_t^i$, we obtain $A_t^i N_t^i = \frac{A_t^i}{\left(N_t^i\right)^{\varphi}}$ and $\left(N_t^i\right)^{1+\varphi} = 1$ in the flex price equilibrium. As a consequence, the natural values of the respective variables are calculated as

$$
\begin{aligned}
N_t^{i,n} &= 1, \\
Y_t^{i,n} &= A_t^i, \\
C_{i,t}^{i,n} &= \left(1-\psi^i\right)(1-\nu) A_t^i = \left(1-\psi^i\right)(1-\nu) Y_t^{i,n}, \\
C_{i,t}^{f,n} &= \left(1-\psi^i\right)\nu A_t^i = \left(1-\psi^i\right)\nu Y_t^{i,n}, \quad \text{and} \\
G_t^{i,n} &= \psi^i A_t^i = \psi^i Y_t^{i,n}.
\end{aligned}
$$

With the last equation for the natural level of public goods, we obtain $1 - \frac{G_t^{i,n}}{Y_t^{i,n}} = 1 - \psi^i \Leftrightarrow \gamma^{i,n} = \psi^i$ and $g_t^{i,n} = -\log\left(1-\psi^i\right) \;\; \forall t$. Thus, the natural level of government expenditure per output is constant for all periods and the same is true for the natural level of the fiscal variable $g$. If we aggregate over all countries and assume that the weight of public goods to consumption goods in the utility function is the same for all countries, i.e., $\gamma^n = \gamma^{i,n} = \psi^i = \psi \;\; \forall i$, we obtain as union-wide natural levels

$$
\begin{aligned}
N_t^{*,n} &= 1, \\
Y_t^{*,n} &= A_t^*, \\
C_t^{*,n} &= \psi A_t^*, \\
G_t^{*,n} &= (1-\psi) A_t^*, \\
g_t^{*,n} &= -\log(1-\psi), \text{ and} \\
\frac{\bar{G}^i}{\bar{Y}^i} &= \frac{\bar{G}}{\bar{Y}} = \gamma^n = \psi,
\end{aligned}
$$

where $A_t^* = \exp \int_0^1 \log A_t^f df$ denotes the overall aggregated technology level of the entire monetary union. The natural level of the terms of trade is thus $S_t^{i,n} = \frac{A_t^i}{A_t^*}$. Taking logs, we obtain $s_t^{i,n} = a_t^i - a_t^*$. In the flex price equilibrium, i.e., in the absence of price rigidities, the terms of trade only depend on the difference of the domestic and the union-wide technology.

### 3.3.2 The National Fiscal Policy Maker's Problem

To solve the national fiscal optimization problem, the welfare function for the entire union as well as for a single country has to be calculated. The welfare function of one country is equal to the infinite sum of the discounted utility of the representative household. Moreover, the welfare for the union as a whole is the aggregate of the national welfare functions. As a consequence, the union welfare function is the integral over all national welfare functions. Following Woodford [130], we use a second order log linear approximation of the utility function to obtain a linear quadratic approximation of the welfare functions.

$(1-\psi)\log C_t^i + \psi \log G_t^i - \frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi}$ is the national utility function. Furthermore, the aggregate of the utility of households is given by $(1-\psi)\int_0^1 \log C_t^i di + \psi \int_0^1 \log G_t^i di - \int_0^1 \frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi} di$. For the first term, we know with equations (3.22.W), (3.22.S), and (3.33) that $c_t^i = y_t^i - g_t^i - \nu s_t^i$, $\nu s_t^i = \frac{\nu}{1-\nu}\left(c_t^i - c_t^*\right)$, $c_t^* = \int_0^1 c_t^f df$, $c_t^i = (1-\nu)\left(y_t^i - g_t^i\right) + \nu c_t^*$. Thus, the consumption term involves an exact first order log approximation such that a second order approximation is not possible. The last two terms are expanded to a second order. If the three approximated terms are written in gap variables form, we obtain

$$\begin{aligned}
c_t^{i,g} &= (1-\nu)\left(y_t^{i,g} - g_t^{i,g}\right) + \nu \int_0^1 c_t^{f,g} df + tip, \\
\frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi} &\approx y_t^{i,g} + \frac{\varepsilon}{2}\text{var}_j\{p_t^i(j)\} + \frac{1+\varphi}{2}\left(y_t^{i,g}\right)^2 + tip, \quad \text{and} \\
\psi \log G_t^i &\approx (1-\gamma^n)\, g_t^{i,g} - \frac{(1-\gamma^n)}{2\gamma^n}\left(g_t^{i,g}\right)^2 + \gamma^n y_t^{i,g} + tip,
\end{aligned}$$

where *tip* denotes the term independent of policy decisions and includes the residual term[12] absorbing the remaining part of the second order approximation. The second equation holds because $\frac{\left(N_t^i\right)^{1+\varphi}}{1+\varphi} \approx n_t^{i,g} + \frac{(1+\varphi)}{2}\left(n_t^{i,g}\right)^2 + tip$. With the demand function of a good $Y^i(j)$ (equation (3.25)) and $Y_t^i(j) = A_t^i N_t^i(j)$, it holds that $N_t^i = \frac{Y_t^i}{A_t^i}\int_0^1 \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon} dj \Leftrightarrow n_t^{i,g} = y_t^{i,g} + \log\left(\int_0^1 \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon} dj\right)$.

The second term $\log\left(\int_0^1 \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon} dj\right)$ changes with a second order approximation to the cross-sectional distribution of relative prices of second order, i.e., $\frac{\varepsilon}{2}\text{var}_j\{p_t^i(j)\}$—see Leith and Wren-Lewis [83]. The variance-term does not enter in the approximation of $n_t^{i,g}$ because it is of higher order than 2 and therefore absorbed by the *tip*-term. Finally, the last equation including government spending holds due to the definition of the fiscal

[12] I.e., $O(\|\xi\|^3)$-terms.

variable $g$ and with $\psi = \gamma^n$.

Collecting terms (i.e., the term for consumption, labor, and the public good), we can express the utility function as

$$\begin{aligned} U_t^i &= (1-\psi)(1-\nu)\left(y_t^{i,g} - g_t^{i,g}\right) + (1-\psi)\nu c_t^{*,g} + (1-\gamma^n) g_t^{i,g} \\ &\quad - \frac{(1-\gamma^n)}{2\gamma^n}\left(g_t^{i,g}\right)^2 + \gamma^n y_t^{i,g} - y_t^{i,g} - \frac{\varepsilon}{2}\mathrm{var}_j\{p_t^i(j)\} - \frac{(1+\varphi)}{2}\left(y_t^{i,g}\right)^2 + tip \\ &= -\frac{(1-\gamma^n)}{2\gamma^n}\left(g_t^{i,g}\right)^2 - \frac{\varepsilon}{2}\mathrm{var}_j\{p_t^i(j)\} - \frac{(1+\varphi)}{2}\left(y_t^{i,g}\right)^2 + tip. \end{aligned} \tag{3.46}$$

The remaining linear term $-(1-\gamma^n)\nu\left(s_t^{i,g}\right)$ is part of the *tip*-term because with the negligible size of one country, we can assume that there are no feedback effects from the rest of the countries in the EMU. Furthermore, by abandoning $s_t^{i,g}$, a conflict between the national and international welfare function is avoided. Since $\int_0^1 s_t^{i,g} di = 0$, the terms of trade component is not part of the union-wide welfare function. If we would assume that there is some kind of response from the other countries, the terms of trade could be an interesting part of the welfare function. One could try to include $s_t^i$ as linear term to obtain an approximation. A linear term in a linear quadratic scenario does not imply the correct first order welfare loss (see, e.g., Benigno and Woodford [16, 17]). Thus, it has to be eliminated, e.g., by a method first described in Benigno and Woodford [15] and utilized also by Lipinska [88] and Benigno and Benigno [14]. To be able to apply the method, a broader description of the terms of trade as well as the responses of other countries on national fiscal actions of one country is necessary. This is not part of the underlying setup.

With $\sum_{t=0}^{\infty} \beta^t \frac{\varepsilon}{2}\mathrm{var}_j\{p_t^i(j)\} = \frac{\varepsilon}{2}\frac{\theta}{(1-\theta\beta)(1-\theta)}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^i\right)^2$ (for the derivation of the equality see Woodford [130], Chapter 6.), the discounted sum of utilities, i.e., the welfare function for one country $i$, is given by

$$\begin{aligned} WF_t^i &= \sum_{t=0}^{\infty} \beta^t U\left(C_t^i, G_t^i, N_t^i\right) \\ &= -\sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}\left(\frac{(1-\gamma^n)}{\gamma^n}\left(g_t^{i,g}\right)^2 + \frac{\varepsilon}{\lambda}\left(\pi_t^i\right)^2 + (1+\varphi)\left(y_t^{i,g}\right)^2\right)\right] + tip, \end{aligned} \tag{3.47}$$

where $\lambda = \frac{(1-\theta\beta)(1-\theta)}{\theta}$. The equation states a loss function due to the negative sign in front of all terms.

Finally, we derive the welfare function for the union as a whole. Therefore, all welfare functions are aggregated. In the individual country's welfare function, the effective terms of trade are included in the *tip*-term. Taking the integral over all countries $i$, we obtain with $S_t^{i,n} = \frac{A_t^i}{A_t^*} \Leftrightarrow s_t^{i,n} = a_t^i - a_t^*$ that $-(1-\gamma^n)\,\nu \int_0^1 s_t^{i,g} di = (1-\gamma^n)\,\nu \left(\int_0^1 s_t^{i,n} di - \int_0^1 s_t^i di\right) = tip - 0$. As a consequence, the welfare function of the union as a whole is given by

$$\begin{aligned} WF_t &= \int_0^1 WF_t^i di \\ &= -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t \int_0^1 \left[\frac{(1-\gamma^n)}{\gamma^n}\left(g_t^{i,g}\right)^2 + \frac{\varepsilon}{\lambda}\left(\pi_t^i\right)^2 + (1+\varphi)\left(y_t^{i,g}\right)^2\right] di + tip. \end{aligned} \tag{3.48}$$

Since the union-wide welfare function consists of quadratic terms of the respective individual country's gap variables, the welfare function cannot be stated as a function of the union-wide variables ($\int \left(x_t^i\right)^2 di \neq \left(\int x_t^i di\right)^2$). As a consequence, we can only derive the optimal choice of the monetary union's optimal interest rate on a theoretical basis and we cannot simulate the optimization problem. A union-wide IS, a Phillips curve, and the overall consumption as a sum of the union-wide log output $y_t^*$ and the union-wide log fiscal variable $g_t^*$ can be calculated only depending on union-wide variables. But it is not possible to display the objective function (the welfare function) exclusively as a function of union-wide variables.

**Model Description 3.4.** The Welfare Functions

- The national welfare function:
  $WF_t^i = -\sum_{t=0}^{\infty}\beta^t \left[\frac{1}{2}\left(\frac{(1-\gamma^n)}{\gamma^n}\left(g_t^{i,g}\right)^2 + \frac{\varepsilon}{\lambda}\left(\pi_t^i\right)^2 + (1+\varphi)\left(y_t^{i,g}\right)^2\right)\right] + tip$
- The union-wide welfare function:
  $WF_t = -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t \int_0^1 \left[\frac{(1-\gamma^n)}{\gamma^n}\left(g_t^{i,g}\right)^2 + \frac{\varepsilon}{\lambda}\left(\pi_t^i\right)^2 + (1+\varphi)\left(y_t^{i,g}\right)^2\right] di + tip$

The first part of the next section derives the theoretical solution for the optimal choice of the interest rate. The second part of the section simulates the optimal policy choice under commitment of an individual country in the occurrence of a debt shock. It compares the model setting with direct or indirect taxes as fiscal instrument and analyzes the similarities and differences. It is shown that the optimal choices are consistent with the optimal union-wide monetary policy.

## 3.4 Solution for the Welfare Optimization Problems

The first subsection derives the solution for the union-wide problem by minimizing the loss of the union subject to the national constraints.

### 3.4.1 The Union-Wide Solution

The optimization problem for the union as a whole is summarized in the Lagrangian of the monetary authority, which is given by the union-wide welfare function (3.48) subject to the national constraints and the aggregation formulas. Because the national government cannot directly influence the union-wide variables, they are subsumed in one variable. Therefore, we rewrite the respective equations including union-wide terms as the rate of return or union-wide consumption, i.e., the equation on changes in output and the government solvency constraint. The union-wide variables $fy^*$ and $fb^*$ are introduced. If the government levies a wage tax, the changes in output (equation (3.43.W)) and the solvency constraint (equation (3.44.W)) of the national authorities become

$$\begin{aligned}
\Delta y_t^{i,g} &= \Delta g_t^{i,g} - \pi_t^i - \Delta a_t^i + fy_t^* && \text{(3.49.W)}\\
fy_t^* &= \Delta y_t^{*,g} - \Delta g_t^{*,g} + \pi_t^* + \Delta a_t^* \\
\beta \widehat{b}_t^{i,r} &= \widehat{b}_{t-1}^{i,r} - \pi_t^i + \left( \frac{\bar{G}^i \bar{P}^i}{\bar{B}^i \bar{R}} \left( \frac{1-\gamma^n}{\gamma^n} \right) + \bar{\tau}^{i,l} \frac{\bar{W}^i \bar{N}^i}{\bar{B}^i \bar{R}} \right) g_t^{i,g} \\
&\quad + \left( \frac{1-\bar{R}}{\bar{R}} \right) y_t^{i,g} - \frac{\bar{\tau}^{i,l} \bar{W}^i \bar{N}^i}{\bar{B}^i \bar{R}} \left( (1+\varphi) y_t^{i,g} + \frac{\left( \log \tau_t^{i,l} \right)^g}{1-\bar{\tau}^{i,l}} \right) \\
&\quad + \frac{1-\bar{R}}{\bar{R}} a_t^i - \frac{\bar{W}^i \bar{N}^i}{\bar{B}^i \bar{R}} \widehat{\mu}_t^i + fb_t^* && \text{(3.50.W)}\\
fb_t^* &= \beta \widehat{\log R_t}^* = \beta \left( -y_t^{*,g} + g_t^{*,g} + E_t y_{t+1}^{*,g} - E_t g_{t+1}^{*,g} + E_t \pi_{t+1}^* \right) + \beta \left( \rho_a - 1 \right) a_t^*.
\end{aligned}$$

For the sales tax model, we obtain new equations for the change in output (equation (3.43.S)) and for the government budget constraint (equation (3.44.S)):

$$\begin{aligned}
\Delta y_t^{i,g} &= \Delta g_t^{i,g} - \pi_t^i - \Delta\left(\log\left(1+\tau_{i,t}^{i,s}\right)\right)^g - \Delta a_t^i - \Delta\widehat{\mu}_t^i + f y_t^* && (3.49.S)\\
f y_t^* &= \Delta y_t^{*,g} - \Delta g_t^{*,g} + \pi_t^* + \Delta a_t^* + \Delta\left(\log\left(1+\tau_t^{*,s}\right)^g\right) + \Delta\left(\log\left(1+\tau_t^{*,s}\right)\right)^n\\
\beta\widehat{b}_t^{i,r} &= \widehat{b}_{t-1}^{i,r} - \pi_t^i + \frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}}\left(\frac{1-\gamma^n}{\gamma^n}\right)\left(1+\bar{\tau}_i^{i,s}\right)g_t^{i,g} + \left(\frac{1-\bar{R}}{\bar{R}}\right)y_t^{i,g}\\
&\quad - \left(\frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}} - \frac{1-\bar{R}}{\bar{R}}\right)\left(\log\tau_{i,t}^{i,s}\right)^g + \frac{1-\bar{R}}{\bar{R}}a_t^i\\
&\quad + \left(\frac{1-\bar{R}}{\bar{R}} - \frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}}\right)\frac{1+\bar{\tau}_i^{i,s}}{\bar{\tau}_i^{i,s}}\widehat{\mu}_t^i + f b_t^* && (3.50.S)\\
f b_t^* &= \beta\widehat{\log R}_t^* = \beta\left(-y_t^{*,g} + g_t^{*,g} + E_t y_{t+1}^{*,g} - E_t g_{t+1}^{*,g} + E_t\pi_{t+1}^*\right.\\
&\quad \left. + \frac{\bar{\tau}^{*,s}}{1+\bar{\tau}^{*,s}}E_t\Delta\left(\log\tau_{t+1}^{*,s}\right)^g\right) + \beta\left(\rho_a - 1\right)a_t^* - \beta\frac{\bar{\tau}^{*,s}}{1+\bar{\tau}^{*,s}}\widehat{\mu}_t^*,
\end{aligned}$$

where $\beta\left(E_t c_{t+1}^{*,n} - c_t^{*,n}\right) = \beta\left(\rho_a - 1\right)a_t^*$[13] and $c_t^{*,g} = y_t^{*,g} - g_t^{*,g}$ with equation (3.35). The Lagrangian for the wage tax model setting is equal to the infinite sum of the union-wide welfare function subject to the constraints given by the equations describing the setup. It can be expressed as

$$\begin{aligned}
\mathcal{L} &= \sum_{t=0}^{\infty}\beta^t\left[\int_0^1 -\frac{1}{2}\left(\frac{(1-\gamma^n)}{\gamma^n}\left(g_t^{i,g}\right)^2 + \frac{\varepsilon}{\lambda}\left(\pi_t^i\right)^2 + (1+\varphi)\left(y_t^{i,g}\right)^2\right)di\right.\\
&\quad + \xi_{y,t}^i\left(\Delta y_t^{i,g} - \Delta g_t^{i,g} + \pi_t^i + \Delta a_t^i - \Delta y_t^{*,g} + \Delta g_t^{*,g} - \pi_t^* - \Delta a_t^*\right)\\
&\quad + \xi_{b,t}^i\left(\beta\widehat{b}_t^{i,r} - \widehat{b}_{t-1}^{i,r} + \pi_t^i - \left(\frac{\bar{G}^i\bar{P}^i}{\bar{B}^i\bar{R}}\left(\frac{1-\gamma^n}{\gamma^n}\right) + \bar{\tau}^{i,l}\frac{\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}\right)g_t^{i,g} - \left(\frac{1-\bar{R}}{\bar{R}}\right)y_t^{i,g}\right.\\
&\qquad + \frac{\bar{\tau}^{i,l}\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}\left((1+\varphi)\,y_t^{i,g} + \frac{\left(\log\tau_t^{i,l}\right)^g}{1-\bar{\tau}^{i,l}}\right) - \frac{1-\bar{R}}{\bar{R}}a_t^i + \frac{\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}\widehat{\mu}_t^i\\
&\qquad \left. - \beta\left(-y_t^{*,g} + g_t^{*,g} + E_t y_{t+1}^{*,g} - E_t g_{t+1}^{*,g} + E_t\pi_{t+1}^*\right) - \beta\left(\rho_a - 1\right)a_t^*\right)\\
&\quad + \xi_{\pi,t}^i\left(\pi_t^i - \beta E_t\pi_{t+1}^i - \lambda\left((1+\varphi)\,y_t^{i,g} - g_t^{i,g} + \frac{\bar{\tau}^{i,l}}{1-\bar{\tau}^{i,l}}\left(\log\tau_t^{i,l}\right)^g\right)\right)
\end{aligned}$$

[13] See also Subsection 3.3.1.

$$+\omega_{y^*,t}\left(\int_0^1 y_t^{i,g}di - y_t^{*,g}\right)$$
$$+\omega_{g^*,t}\left(\int_0^1 g_t^{i,g}di - g_t^{*,g}\right)$$
$$+\omega_{\pi^*,t}\left(\int_0^1 \pi_t^{i,g}di - \pi_t^{*,g}\right)$$
$$+\omega_{\tau^*,t}\left(\int_0^1 \left(\log\left(1-\tau_t^{i,l}\right)\right)^g di - \left(\log\left(1-\tau_t^{*,l}\right)\right)^g\right)\Bigg].$$

Taking the derivative with respect to $y_t^{i,g}$, $\pi_t^{i,g}$, the fiscal variables $\left(\log\left(1-\tau_t^{i,l}\right)\right)^g$, $g_t^{i,g}$, $\widehat{b}_{t-1}^{i,r}$, and the union-wide variables $y_t^{*,g}$, $\pi_t^{*,g}$, $g_t^{*,g}$, and $\left(\log\left(1-\tau_t^{*,l}\right)\right)^g$, we rearrange the equations and obtain the following first order conditions:

$$\xi_{b,t} = \xi_{b,t-1} \tag{3.51.W}$$
$$\omega_{y^*,t} = -\omega_{g^*,t} \tag{3.52.W}$$
$$\omega_{\pi^*,t} = -\xi_{y,t}^i - \xi_{b,t-1}^i \tag{3.53.W}$$
$$\lambda\xi_{\pi,t}^i = \frac{\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}\xi_{b,t}^i \tag{3.54.W}$$
$$\begin{aligned} 0 &= (1+\varphi)\,y_t^{i,g} + \left(\frac{1-\gamma^n}{\gamma^n}\right)g_t^{i,g} - \xi_{\pi,t}^i\lambda\varphi \\ &\quad -\xi_{b,t}^i\left(\frac{1-\bar{R}}{\bar{R}} - \frac{\bar{\tau}^{i,l}\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}\varphi + \frac{\bar{Y}^i\bar{P}^i}{\bar{B}^i\bar{R}}\left(1-\gamma^n\right)\right) + \omega_{y^*,t} + \omega_{g^*,t} \end{aligned} \tag{3.55.W}$$
$$0 = \frac{\varepsilon}{\lambda}\pi_t^i + \xi_{y,t}^i + \xi_{b,t}^i + \xi_{\pi,t}^i + \omega_{\pi^*,t} - \xi_{\pi,t-1}^i. \tag{3.56.W}$$

The derivative with respect to $\widehat{b}_{t-1}^{i,r}$ (equation (3.51.W)) shows that the Lagrange multiplier is equal for all $t$. Thus, the budget constraints of governments affect the optimization problem of the authority always with the same intensity. Integrating equation (3.53.W) over all countries $i$ yields: $-\int_0^1 \xi_{y,t}^i di - \int_0^1 \xi_{g,t}^i di = \omega_{\pi^*,t}$. If we integrate equation (3.56.W) and combine it with the previous result, we obtain $\frac{\varepsilon}{\lambda}\pi_t^* + \int_0^1 \Delta\xi_{\pi,t}^i di = 0$. Restructuring and integrating the first order conditions with respect to $y_t^{i,g}$ and $g_t^{i,g}$, we can state that $\left(\frac{1-\gamma^n}{\gamma^n}\right)g_t^{*,g} = -\left(\frac{\bar{\tau}^{i,l}}{1-\bar{\tau}^{i,l}}\right)y_t^{*,g}$. Furthermore, we rewrite equation (3.55.W) as $0 = (1+\varphi)\,y_t^{i,g} + \left(\frac{1-\gamma^n}{\gamma^n}\right)g_t^{i,g} - \xi_{\pi,t}^i\left(\frac{1-\bar{R}}{\bar{R}} + \frac{\left(1-\bar{\tau}^{i,l}\right)\bar{W}^i\bar{N}^i}{\bar{B}^i\bar{R}}\varphi + \frac{\bar{Y}^i\bar{P}^i}{\bar{B}^i\bar{R}}\left(1-\gamma^n\right)\right)\frac{\lambda\bar{B}^i\bar{R}}{\bar{W}^i\bar{N}^i}$. If the modified equation is integrated and the previous results are used, we get $\varepsilon\pi_t^* const + \frac{1-\gamma^n}{\gamma^n}\Delta g_t^{*,g} + (1+\varphi)\,\Delta y_t^{*,g} = \varepsilon\pi_t^* const + \left(1+\varphi-\frac{\bar{\tau}^l}{1-\bar{\tau}^l}\right)\Delta y_t^{*,g} = 0$. $const = \frac{(1-\psi)\left(1-\bar{\tau}^l\right)}{1-\gamma^n} +$

$(1+\varphi)\left(\bar{\tau}^l-1\right)=\varphi\left(\bar{\tau}^l-1\right)$, with $\bar{N}^i=1\ \forall i$ (normalization) and $\bar{\tau}^l=\bar{\tau}^{i,l}=1-1/\bar{\mu}\ \forall i$ (the constant term *const* depends on the steady state tax rate, $\gamma^n$, $\psi$, and $\varphi$). The last equation reveals that the steady state tax rate is chosen to absorb distortions caused by the steady state price markup shock $\bar{\mu}$. Furthermore, we assume that the steady state value for the debt per output ratio is equal for all member countries ($\frac{\bar{B}^i}{\bar{Y}^i\bar{P}^i}=\frac{\bar{B}}{\bar{Y}\bar{P}}=d_{ss}\ \forall i$).

For $t=0$, it holds that $\varepsilon\pi_0^* const+\left(1+\varphi-\frac{\bar{\tau}^l}{1-\bar{\tau}^l}\right)y_0^{*,g}=0$. We follow the result of Schmitt-Grohé and Uribe [118], who show that the optimal inflation rate should be close to zero.

Hence, we assume that the monetary policy authority focuses on zero inflation. Then, $\pi_0^*=0$ such that $y_0^{*,g}=0$. Furthermore, $g_0^{*,g}=0$. With $E_0\pi_1^*=0$, it follows with the Phillips curve $\pi_t^*=\beta E_t\pi_{t+1}^*+\lambda\left((1+\varphi)y_t^{*,g}-g_t^{*,g}-\log\left(1-\tau_t^{*,l}\right)^g\right)$ that $\log\left(\tau_0^{*,l}\right)^g=0$. Because $\varepsilon E_0\pi_1^* const+\left(1+\varphi-\frac{\bar{\tau}^l}{1-\bar{\tau}^l}\right)\Delta E_0 y_1^{*,g}=0$ it holds that $E_0 y_1^{*,g}=0$.

The IS-curve for the union as a whole

$$\begin{aligned}
y_t^{*,g} &= E_t y_{t+1}^{*,g}-\left(\log R_t^*-E_t\pi_{t+1}^*-\log R_t^{*,n}\right)\\
\log R_t^{*,n} &= E_t\left(y_{t+1}^{*,n}-y_t^{*,n}\right)+\rho-E_t\left(g_{t+1}^{*,n}-g_t^{*,n}\right)=(\rho_a-1)\,a_t^*+\rho
\end{aligned}$$

implies that the difference between the log rate of return $R^*$ and the natural log rate of return is equal to zero. Thus, an optimal policy carried out by the single monetary policy authority for the union implies that the log rate of return is set equal to the log natural rate of return. Therefore, the interest rate is set equal to the natural rate of interest.

If the government raises a sales tax, the corresponding Lagrangian is given by

$$\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty}\beta^t\Bigg[&\int_0^1 -\frac{1}{2}\left(\frac{(1-\gamma^n)}{\gamma^n}\left(g_t^{i,g}\right)^2+\frac{\varepsilon}{\lambda}\left(\pi_t^i\right)^2+(1+\varphi)\left(y_t^{i,g}\right)^2\right)di\\
&+\xi_{y,t}^i\Bigg(\Delta y_t^{i,g}-\Delta g_t^{i,g}+\pi_t^i+\Delta\left(\log\left(1+\tau_{i,t}^{i,s}\right)\right)^g+\Delta a_t^i+\Delta\widehat{\mu}_t^i\\
&\qquad-\Delta y_t^{*,g}+\Delta g_t^{*,g}-\pi_t^*-\Delta a_t^*-\Delta\left(\log\left(1+\tau_t^{*,s}\right)\right)^g-\Delta\widehat{\mu}_t^*\Bigg)
\end{aligned}$$

$$
\begin{aligned}
&+\xi^i_{b,t}\left(\beta\widehat{b}^{i,r}_t-\widehat{b}^{i,r}_{t-1}+\pi^i_t-\frac{\bar{Y}^i\bar{P}^i}{\bar{B}^i\bar{R}}\left(1-\gamma^n\right)\left(1+\bar{\tau}^{i,s}_i\right)g^{i,g}_t-\left(\frac{1-\bar{R}}{\bar{R}}\right)y^{i,g}_t\right.\\
&\quad+\left(\frac{\bar{Y}^i\bar{P}^i\gamma^n}{\bar{B}^i\bar{R}}-\frac{1-\bar{R}}{\bar{R}}\right)\left(\left(\log\tau^{i,s}_{i,t}\right)^g+\frac{1+\bar{\tau}^{i,s}_i}{\bar{\tau}^{i,s}_i}\widehat{\mu}^i_t\right)-\frac{1-\bar{R}}{\bar{R}}a^i_t\\
&\quad-\beta\left(-y^{*,g}_t+g^{*,g}_t+E_ty^{*,g}_{t+1}-E_tg^{*,g}_{t+1}+E_t\pi^*_{t+1}\right)\\
&\quad\left.-\beta\frac{\bar{\tau}^{*,s}}{1+\bar{\tau}^{*,s}}E_t\Delta\left(\log\tau^{*,s}_{t+1}\right)^g-\beta\left(\rho_a-1\right)a^*_t+\beta\frac{\bar{\tau}^{*,s}}{1+\bar{\tau}^{*,s}}\widehat{\mu}^*_t\right)\\
&+\xi^i_{\pi,t}\left(\pi^i_t-\beta E_t\pi^i_{t+1}-\lambda\left(\left(1+\varphi\right)y^{i,g}_t-g^{i,g}_t+\frac{\bar{\tau}^{i,s}_i}{1+\bar{\tau}^{i,s}_i}\left(\log\tau^{i,s}_{i,t}\right)^g\right)\right)\\
&+\omega_{y^*,t}\left(\int_0^1 y^{i,g}_t di-y^{*,g}_t\right)\\
&+\omega_{g^*,t}\left(\int_0^1 g^{i,g}_t di-g^{*,g}_t\right)\\
&+\omega_{\pi^*,t}\left(\int_0^1 \pi^{i,g}_t di-\pi^{*,g}_t\right)\\
&+\omega_{\tau^*,t}\left(\int_0^1\left(\log\left(1+\tau^{i,s}_{i,t}\right)\right)^g di-\left(\log\left(1+\tau^{*,s}_t\right)\right)^g\right)\Bigg].
\end{aligned}
$$

Solving once again for the first order conditions by taking derivatives, similar to the above, it follows that $\frac{\varepsilon}{\lambda}\pi^*_t+\int_0^1\Delta\xi^i_{\pi,t}di=0$. As for the wage tax model, we integrate parts of the first order conditions to obtain a relationship between the inflation rate and the Lagrange multipliers of domestic output, of the domestic fiscal variable, and of the domestic and the union-wide inflation rate. Finally, integrating the first order conditions resulting from differentiation with respect to $y^{i,g}$ and $g^{i,g}$, the following relation between the output gap and the fiscal variable gap holds

$$
\frac{1-\gamma^n}{\gamma^n}g^{*,g}_t=-\left(\frac{\frac{1}{d_{ss}\bar{R}}\left(1+\bar{\tau}^{i,s}_i-\gamma^n\bar{\tau}^{i,s}_i+\gamma^n/\bar{\tau}^{i,s}_i\right)-\left(\beta-1\right)1/\bar{\tau}^{i,s}_i}{\frac{1+\bar{\tau}^{i,s}_i}{\bar{\tau}^{i,s}_i}\left(\frac{1}{d_{ss}\bar{R}}\gamma^n-\left(\beta-1\right)\right)}\right)y^{*,g}_t.
$$

A relationship between $\pi^*_t$, $\Delta y^{*,g}_t$, and $\Delta g^{*,g}_t$ can be obtained. With the Phillips curve and under optimal monetary policy, the tax gap variable $\left(\log\left(1+\tau^{*,s}_t\right)\right)^g$ is equal to zero. With $E_t\pi^*_{t+1}=0$, it follows that $E_ty^{*,g}_{t+1}=0$ and $E_tg^{*,g}_{t+1}=0$. With $E_{t+1}\pi^*_{t+2}=E_t\pi^*_{t+1}=0$, and with the Phillips curve for period $t+1$ and by taking expectations at time $t$, we obtain $E_t\log\left(1+\tau^{*,s}_{t+1}\right)^g=0$. In the respective IS-curve ($y^{*,g}_t=E_ty^{*,g}_{t+1}-\left(\log R^*_t-\log R^{*,n}_t\right)+E_t\pi^*_{t+1}-E_t\Delta g^{*,g}_{t+1}+E_t\Delta\left(\log\left(1+\tau^{*,s}_{t+1}\right)\right)^g$), the terms including output, inflation, and the fiscal variables are equal to zero. Therefore, $\left(\log R_t^* - \log R_t^{*,n}\right) = 0$. As a consequence, under optimal monetary policy, the central bank should follow $\log R_t^* = \log R_t^{*,n}$.

**Result 3.1.** Optimal Monetary Policy

For the union-wide monetary authority it is optimal under commitment to set the interest rate equal to its natural level. The result holds independent of the tax instrument used by the national government.

### 3.4.2 The National Solution

Finally, we analyze the national solution for the optimization problem under commitment. By doing so, the optimal rule for $g$ and the respective tax rate under commitment are found. To capture the dynamics of the model, the effect of a debt shock on the model variables is simulated.

The sources of a debt shock are quite different for the two settings due to the different solvency constraints, equations (3.50.W) and (3.50.S). Hence, we analyze different scenarios of shock occurrences.

Each setup includes a markup price/inflation shock as well as a shock in technology. Thus, we assume that the shock in national debt is a linear combination of the two shocks, which are already a part of the model.

The next part describes the optimal solution for the wage tax model and the second part carries out the whole analysis for the sales tax model.

For both model settings, the calibration parameters are chosen similar to the model parameters used by Galí and Monacelli [54]. Galí and Monacelli [54] choose a value of $\varphi = 3$, whereas Leith and Wren-Lewis [83] set the value to 1. As a consequence, we assume to set it to the mean value: $\varphi = 2$. The qualitative results are robust to the choice of $\varphi$. $\bar{\mu}^i$, the markup on prices is set to 1.2, such that $\varepsilon = 6$, where $\varepsilon$ is the elasticity of substitution between different goods all produced in one country. We set $\theta$, the part of firms who are not able to reset prices equal to 0.75. This corresponds to an annual price adjustment. Finally, the discount factor is set to $\beta = 0.99$, which implies an annual interest rate of 4% in steady state. The parameter of openness is set at $\nu = 0.4$.

The Euro Area average debt per output ratio lies since the foundation of the EMU between 0.68 and 0.85. The respective ratios for single member countries lie between 0.4 and 1.4. See also the ECB statistics website [67]. As a consequence, according to the Euro Area criterion, we choose a value of 0.6 for the steady state value $d_{ss}$, which corresponds to the debt per output ratio. This implies for the steady state government spending per output ratio $\gamma^n$ in the wage tax setting a value of about 20% and is equal to $\psi$, the weight of public goods in the utility function of households. The ratio of consumption to steady state GDP is therefore 80%, where 60% $(1 - \nu = 0.6)$ of the consumption goods are domestically produced.
The choice of the steady state debt per output ratio being 0.6 implies that the steady state government spending per output ratio $\gamma^n$ is 0.16 in the sales tax model. Hence, the ratio of overall consumption to steady state GDP is 84%. As before, 60% of the consumption goods are produced in the own country.
Smets and Wouters [122] estimated the parameters of a technology shock using Bayesian estimation techniques for the Euro Area. We use their results and assume that the AR(1) shock in technology, i.e., the productivity shock is modeled with a high degree of persistence ($\rho_a = 0.99$) and a standard deviation of 0.69. For the i.i.d. price markup shock, we assume a standard deviation of 0.19.

### 3.4.3 The National Solution in the Wage Tax Model

To obtain the optimal fiscal policy of a country $i$ under unconstrained commitment, we follow Söderlind [123]. As a first step, we state that

$$\vec{A}_0 \begin{pmatrix} \vec{x}_{1,t+1} \\ E_t \vec{x}_{2,t+1} \end{pmatrix} = \vec{A}_1 \begin{pmatrix} \vec{x}_{1,t} \\ \vec{x}_{2,t} \end{pmatrix} + \vec{B}_1 \vec{u}_t + \begin{pmatrix} \vec{\epsilon}_{t+1} \\ 0 \end{pmatrix},$$

where $\vec{x}_t = \begin{pmatrix} \vec{x}_{1,t} \\ \vec{x}_{2,t} \end{pmatrix}$ and $\vec{x}_{1,t} = \left(y_{t-1}^{i,g}, g_{t-1}^{i,g}, \widehat{b}_{t-1}^{i,r}, a_{t-1}^{i}, \pi_{t-1}^{i} - E_{t-2}\pi_{t-1}^{i}, \left(\log \tau_{t-1}^{i,l}\right)^{g}\right)'$ is the vector including all predetermined variables of the model, $\vec{x}_{2,t} = \left(fb_t^*, fy_t^*, a_t^i, \ \pi_t^i\right)'$ are all forward looking variables of the model, and $\vec{u}_t = \left(g_t^{i,g}, \left(\log \tau_t^{i,l}\right)^{g}\right)'$ consists of the decision variables of the fiscal authority. $\vec{\epsilon}_t$ is the vector containing the shock residuals on the corresponding variables of vector $\vec{x}_{1,t}$. A shock in technology is directly included because $a_t^i = \rho_a a_{t-1}^i + \epsilon_t^a$. The markup shock in prices and therefore in inflation can be implemented as $\pi_t^i = E_{t-1}\pi_t^i + \epsilon_t^\pi$, where $\epsilon_t^\pi = \widehat{\mu}_t$. The next step is to calculate $\vec{A}_0^{-1}$ such that $\vec{A} = \vec{A}_0^{-1}\vec{A}_1$ and $\vec{B} = \vec{A}_0^{-1}\vec{B}_1$. The shock vector is also multiplied by $\vec{A}_0^{-1}$. The influence of shocks on the respective variables of the model is captured in the structure of $\vec{A}_0^{-1}$.

Finally, the model can be formulated as

$$\begin{pmatrix} \vec{x}_{1,t+1} \\ E_t\vec{x}_{2,t+1} \end{pmatrix} = \vec{A} \begin{pmatrix} \vec{x}_{1,t} \\ \vec{x}_{2,t} \end{pmatrix} + \vec{B}\vec{u}_t + \vec{A}_0^{-1} \begin{pmatrix} \vec{\epsilon}_{t+1} \\ 0 \end{pmatrix}.$$

We derive the welfare function value which is given by $J_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\vec{x}_t'\vec{Q}\vec{x}_t + 2\vec{x}_t'\vec{U}\vec{u}_t + \vec{u}_t'\vec{R}\vec{u}_t\right]$.[14]

Then, we optimize under commitment: the fiscal authority maximizes the expected infinite welfare. The resulting optimal policy rule is constant over time. Hence, once an optimal rule is found, there are no changes in the policy for the following periods. The solution is time consistent.

Shocks, specifically a debt shock, are analyzed.[15] The wage tax model includes a government budget constraint, where a markup shock as well as a shock in productivity influences the debt of the respective country. Thus, the shock in debt is introduced as a linear combination of the two particular shocks. We calibrate the intensity of the shocks such that they are weighted with their standard deviations. Since the Phillips curve, the IS curve, and the government budget constraint cause a lagged shock processing in the system, the shocks are chosen in such a manner that debt rises by one unit 2 periods after the occurrence of the two shocks.

Because the debt variable $b$ depends on the rate of return, the steady state value for $b$ is a function of the natural rate of return. Furthermore, the monetary authority sets the interest rate equal to its natural rate, which is driven by the technology process.

[14] $\vec{Q}$ and $\vec{R}$ are symmetric, see Söderlind [123].

[15] All figures show the respective gap variables.

Because $\rho_a = 0.99$ (technology shows a close random walk behavior), also the steady state debt behaves close to a random walk. This corresponds to the results found by Schmitt-Grohé and Uribe [118].

If a shock in technology and inflation occurs that causes a positive one unit deviation from steady state debt, the impulse responses shown in Figure 3.1 are observed. One simulated period is equal to one quarter.

The rise in debt forces the fiscal authority to optimally raise taxes and to lower government spending if they want to stabilize debt (i.e., eliminate the deviation of debt from its steady state value) as soon as possible.[16] The simulated time horizon is long to illustrate that indeed all variables revert back to the zero line.

The optimal policy actions are very smooth such that the reversion process lasts for a long time. The low intensity of the optimal fiscal response to a debt shock can be explained as follows. On the one hand, debt should be lowered, on the other hand, domestic inflation should not be strongly distorted. A distortion in inflation would cause a loss of competitiveness and high welfare losses. To avoid an economic disadvantage, fiscal actions are pursued only very slightly.

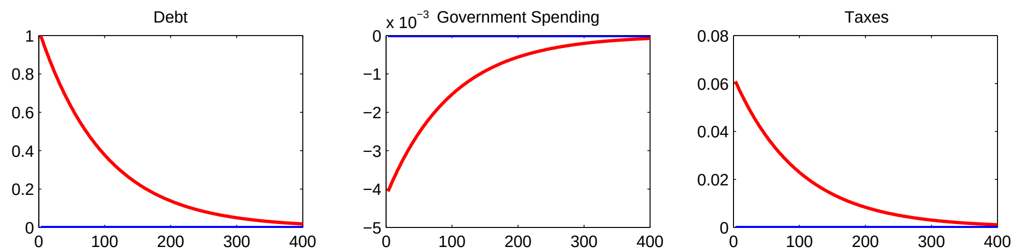


Figure 3.1: Impulse Responses to a 1 Unit Debt Shock (Wage Tax, $b, g, t$)

Figure 3.2 shows the impulse response of output and inflation to a one unit debt shock. The decrease in government spending forces output to decline. Taxes rise and lower output additionally.

Higher wage taxes do have a negative impact on the labor supply because workers earn

[16] Note, that log government spending is a linear combination of $y$ and $g$.

less disposable income. Furthermore, the marginal costs of firms are higher than before due to the positive influence of the log tax rate and thus lower output.

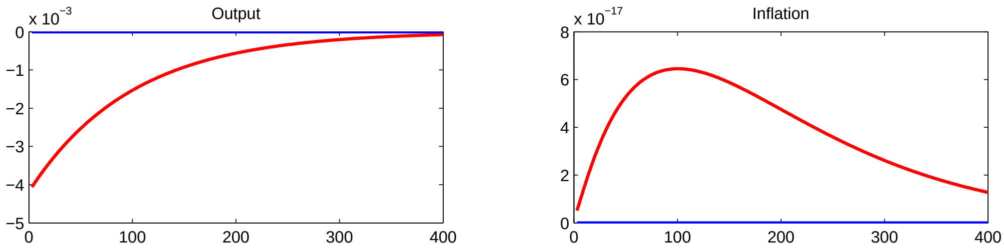


Figure 3.2: Impulse Responses to a 1 Unit Debt Shock (Wage Tax, $y, \pi$)

Inflation immediately rises before reverting to the zero line. The higher inflation lowers the competitiveness of the national country. This additionally reduces output. In the reversion process of all variables, country $i$ slowly regains competitiveness.

The half-time of the debt shock, i.e., the point in time after the occurrence of the shock, when debt diverges 0.5 units from steady state, is reached after 18 years. The optimal fiscal actions, lowering government spending and raising taxes, are indeed pursued with a very low intensity such that it lasts long until the debt deviation from the natural level drops by 50%.

The log rate of return directly captured in the foreign variable $fb_t^*$ converges from above to the log natural rate of return. The natural rate of return depends in general on constants and the technology process. As a consequence, it is not a constant but converges to a fixed value after the occurrence of a temporary shock because technology is an AR(1) process with $\rho_a < 1$.

Overall, the calculated optimal national fiscal policy does not contradict the optimal EMU monetary policy.

If only technology is shocked, all responses as well as the obtained welfare loss are closely the same. Thus, the shock in technology in the mixed shock scenario is the driving force of a shock in debt on the model variables.

**Result 3.2.** The Wage-Tax Model

- In the occurrence of a debt shock, under commitment, the government optimally lowers government spending and raises taxes to offset the shock.
- This lowers output.
- The inflation rate impulse response is hump-shaped and very weak.
- The optimal fiscal actions are very smooth.
- The half-time of debt under optimal commitment policy is simulated with 18 years.
- A shock in technology controls the shock in debt.
- Optimal monetary and fiscal policy under commitment coincide.

In the remainder of the subsection, we analyze the consequences of selective restrictions on the model structure and calculate the loss in welfare due to the debt shock. Moreover, robustness checks on the calibration parameters are performed.

First, we compare the optimally found fiscal policy under unconstrained commitment with one committing to a specific simple rule. Fiscal policy under unconstrained commitment means that the predetermined as well as the forward looking variables are included in the optimal policy description.

If the government commits to a simple rule, only the predetermined variables of the model are part of the rule. For a closer description, see Söderlind [123]. By setting the shadow prices of the forward looking variables in the model equal to zero, we automatically follow a commitment rule. Comparing the obtained optimal values for government spending and taxes when constraining the shadow prices to zero or without constraint, it shows that the difference for the optimally chosen tax rule is minimal and lies in the range of $2\text{x}10^{-13}$.[17] The differences for $g$ are even smaller ($2\text{x}10^{-17}$).

Moreover, the welfare function value is calculated if a one unit debt shock hits the system. We sum up all periodical values that are larger than $1\text{x}10^{-22}$ such that the approximated overall infinite welfare function value is sufficiently exact. The overall loss in welfare caused by the temporary debt shock is equal to $3.3564\text{x}10^{-8}$.

[17] This holds except for the very first optimal value. In this case, the difference is larger.

Robustness is tested by varying different calibration parameters. If we assume that the ratio of firms that are able to adjust their prices, is larger ($\theta_1 = 0.35 < \theta$): the price stickiness is small and all impulse responses are almost the same—see Figure 3.3 and 3.4. Only the impulse response of inflation changes quantitatively. Inflation reaches its maximum faster, already after 5 years, and reverts more quickly back to the zero line. Furthermore, the intensity of the response to a debt shock is higher if prices are more flexible (inflation is affected more). The response for the baseline calibration lies in a range of $10^{-17}$ and with $\theta = 0.35$ is $10^{-13}$. Overall, the response of inflation remains very small. But the change in the inflation response affects the welfare. The welfare loss rises to 9.488x$10^{-5}$, which is far more than 3.3564x$10^{-8}$ obtained with the basic calibration values.

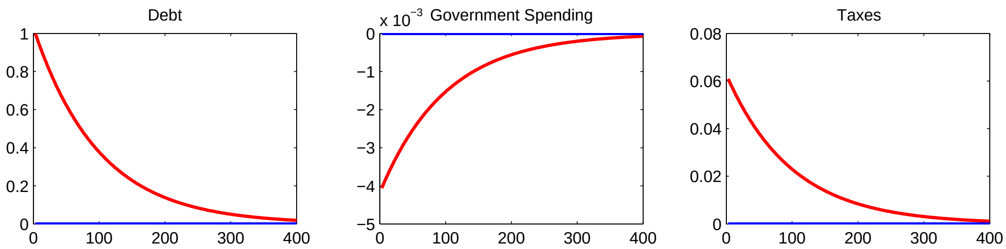


Figure 3.3: Impulse Responses to a 1 Unit Debt Shock with more Flexible Prices

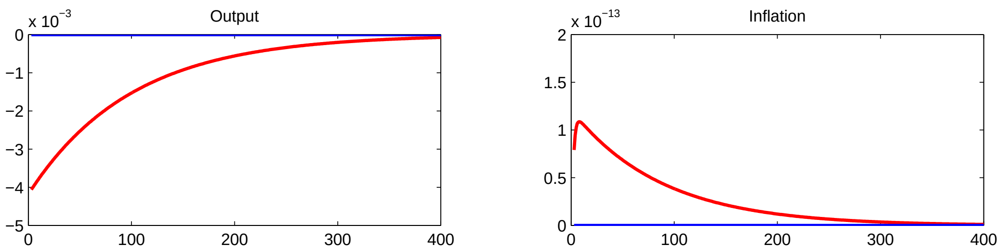


Figure 3.4: Impulse Responses to a 1 Unit Debt Shock with more Flexible Prices

As a second step on robustness, we vary the steady state debt per output ratio. The impulse responses of output, taxes, and government spending after the occurrence of a one unit debt shock are proportional to the amount the steady state debt per GDP ratio changes. If we assume that $d_{ss1} = 0.4 = 2/3 d_{ss}$ (see Figure 3.5 and 3.6), the output response e.g. for the 5th period, is 2/3 of the former response for the 5th period with the baseline calibration parameters.

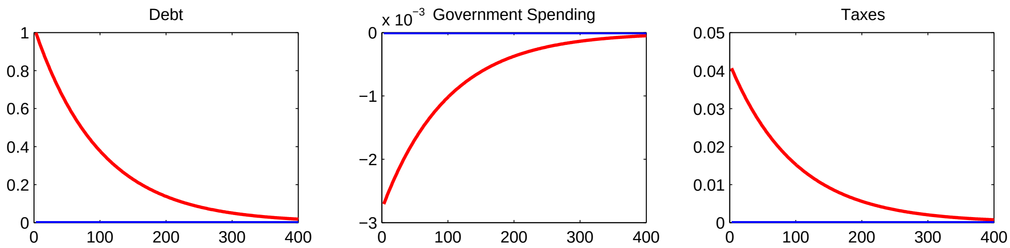


Figure 3.5: Impulse Responses to a 1 Unit Debt Shock with $d_{ss1} = 0.4$

Only inflation does not change proportionally. The response is more positive than multiplying the corresponding baseline calibration value with 2/3. Therefore, the change in inflation is less than proportional. The maximum response is reached faster and the reversion process starts after 6 years. This behavior is due to feedback effects of the other variables.

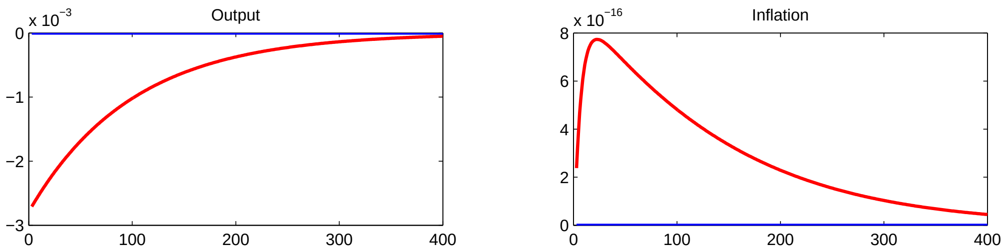


Figure 3.6: Impulse Responses to a 1 Unit Debt Shock with $d_{ss1} = 0.4$

If we raise the steady state value ($d_{ss2} = 0.85 = 17/12 d_{ss}$), the respective responses for government spending, output, and taxes are once again proportional to the standard impulse response when assuming that $d_{ss} = 0.6$—see Figures 3.1 and 3.2. The inflation rate responds less than proportional, just as for a decrease in the steady state debt output ratio. Moreover, the qualitative response changes.

First, inflation rises, lowering the competitiveness of the national country, but when regaining competitiveness, the inflation rate is even overcompensated by the positive tax rate such that it turns negative before reverting to the zero line.

3.7 and 3.8.

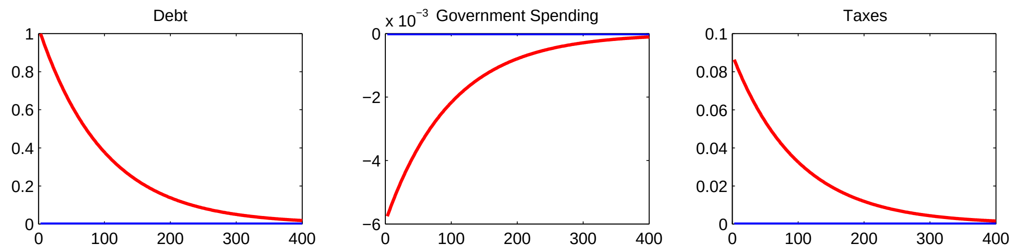


Figure 3.7: Impulse Responses to a 1 Unit Debt Shock with $d_{ss1} = 0.85$

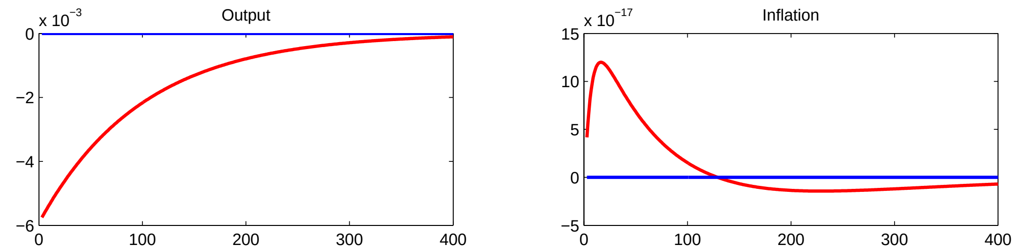


Figure 3.8: Impulse Responses to a 1 Unit Debt Shock with $d_{ss1} = 0.85$

Overall, the fiscal authority changes the optimally found government spending and tax rates with a higher intensity in the occurrence of a one unit debt shock if the debt ratio is already at a higher value. The loss for a debt level of $d_{ss1} = 0.4$ is lower ($J_1 = 1.4917\text{x}10^{-8}$), whereas the loss is higher for a higher debt level ($J_2 = 6.736\text{x}10^{-8}$ with $d_{ss2} = 0.85$.). The increase in the loss of welfare is more than proportional compared to the increase in steady state debt per output.

**Result 3.3.** The Wage-Tax Model

An additional unit of debt is more fatal if the economy is already indebted at a higher level.

### 3.4.4 The National Solution in the Sales Tax Model

For the optimization problem in the respective setting, we formulate the objective function as

$$\begin{pmatrix} \vec{x}_{1,t+1} \\ E_t\vec{x}_{2,t+1} \end{pmatrix} = \vec{A} \begin{pmatrix} \vec{x}_{1,t} \\ \vec{x}_{2,t} \end{pmatrix} + \vec{B}\vec{u}_t + \vec{A}_0^{-1} \begin{pmatrix} \vec{\epsilon}_{t+1} \\ 0 \end{pmatrix},$$

where the vector of predetermined variables $\vec{x}_{1,t} = \left(y^{i,g}_{t-1}, g^{i,g}_{t-1}, \widehat{b}^{i,r}_{t-1}, a^i_{t-1}, -E_{t-2}\pi^i_{t-1}, \pi^i_{t-1}, \left(\log \tau^{i,s}_{i,t-1}\right)^g\right)'$ contains 7 variables. The markup shock was included in the wage tax setup as one variable $\pi^i_{t-1} - E_{t-2}\pi^i_{t-1}$. Thus, $\vec{x}_{1t}$ consisted of 6 variables. Now, we split the shock variable in two separate parts ($-E_{t-2}\pi^i_{t-1}$, $\pi^i_{t-1}$). This is caused by the model structure: $\pi^i_{t-1}$ is needed separately for the welfare function implementation in the sales tax setting. The vector of forward looking variables $\vec{x}_{2,t} = \left(fb^*_t, fy^*_t, a^i_t, \pi^i_t\right)'$ is the same as before and the policy instrument vector is $\vec{u}_t = \left(g^{i,g}_t, \left(\log \tau^{i,s}_{i,t}\right)^g\right)'$. $\vec{\epsilon}$ contains all shocks. The welfare function is described in a comparable manner as the respective one for the wage tax scenario. We once again optimize under unrestricted commitment.

An analysis of the impulse responses in the occurrence of a one unit shock in debt is carried out. If the government raises an indirect tax (i.e., a sales tax in the underlying study), the shock processing is quite different compared to the one found for the wage tax model. Since the output gap and the technology process variable both enter the government budget constraint with exactly the same multiplier, a shock in productivity does not directly affect debt in the first period of the shock occurrence. After one period, the variables that are affected directly by a shock in technology and that are moreover a part of the government budget constraint, change the debt. In contrast, a price markup shock is directly included in the government solvency constraint, as for the wage tax setting, and influences debt immediately. To keep comparability, we include both shocks, once again weighted with the respective standard deviation and normalized such that the deviation of debt from its steady state value is one unit 2 periods after the shocks.

To find the driving force of a debt shock in the sales tax model, we compare a shock in technology with the linear combination of a technology and a price markup shock. Debt should respond to the respective shock(s) in the same way for both shock scenarios. If only a shock in technology takes place, a slightly stronger shock is needed to compensate for the missing shock in inflation.

Government spending, output, and inflation respond less strong. Taxes respond slightly stronger. Thus, the government optimally lowers government spending less strong and raises taxes more. As a consequence, output declines less strong because the smaller decrease in government spending outweighs the additional increase in taxes. The welfare loss is higher if additionally a price markup shock occurs. The result holds although the technology shock per se has a slightly higher intensity when being implemented uniquely. The price markup shock does not affect the optimal policy actions very strongly, but the loss in welfare raises. Overall, the shock in technology is the main driving force for the optimal policy responses.

We analyze the impulse responses of the model variables due to a shock in debt caused by the same linear combination of technology and price markup shock as for the wage tax model. Qualitatively, the impulse responses of the fiscal variables are the same as before—see Figure 3.9. If debt increases, the government decides to raise taxes and to lower government spending. The result is clear from intuition if we assume a debt stabilizing national fiscal policy. The debt smoothing path is similar for both scenarios. Moreover, the negative change in government spending is comparable in size: if the fiscal authority can only raise a sales tax, government spending decreases after 2 periods by 0.00406, whereas it declines to -0.004302 in the wage tax model. Thus, only a 5% change can be observed.

Since the government spending response and the debt path are closely the same for both models, the two tax instruments can be directly compared concerning their effectiveness. The increase in sales taxes as fiscal instrument is far more intense than before under the same model circumstances. The sales tax rate rises by 0.072 units, the wage tax rate was raised by 0.061 in the occurrence of a positive one unit debt shock. This corresponds to a rise of 18%. To compensate for the additional debt, sales taxes are raised more intensely. They are not that effective as a wage tax instrument to reduce debt again.

Thus, a higher sales tax rate is optimally needed under commitment to involve the same optimal debt reduction path.

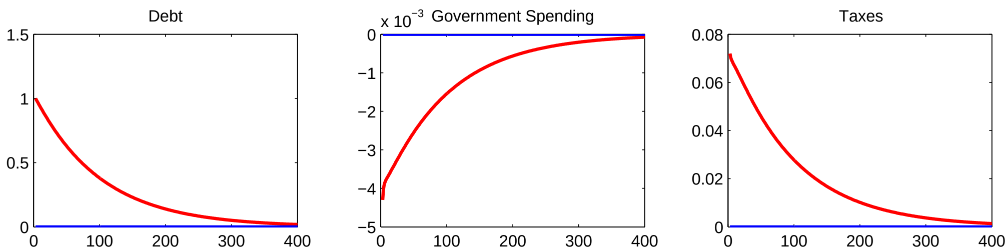


Figure 3.9: Impulse Responses to a 1 Unit Debt Shock (Sales Tax, $b, g, t$)

The response of output is comparable for both model settings. But the change in inflation, which is caused by the rise in debt differs qualitatively as well as quantitatively. In the very first period after the occurrence of the shock,[18] the change in inflation is positive. This lowers competitiveness. Then, it turns negative. Afterwards, it overshoots the zero line and reverts to the zero line from above—see Figure 3.10. Overall, the inflation rate responds far more intense to a debt shock in the sales tax model scenario.

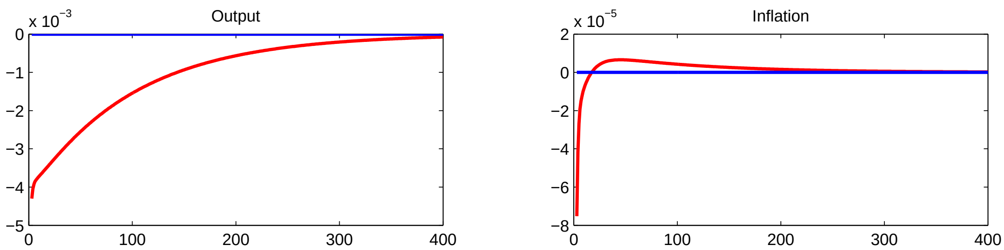


Figure 3.10: Impulse Responses to a 1 Unit Debt Shock (Sales Tax, $y, \pi$)

Debt diverges by 0.5 units from steady state after 18 years. The half-time of debt is closely the same as for the wage tax model. Moreover, the optimal debt reduction path is the same for both fiscal tax instrument setups. The rate of return and therefore, the interest rate, which is captured in variable $fb_t^*$ converges after the temporary shock to

[18]Note that the first period is not plotted.

its natural level. Once again, the natural level does not only depend on constants but also on the AR(1) process of technology and converges to a fixed value in the absence of a permanent shock. In this case, the log rate of return, which is set by the monetary authority converges from below to its natural level, whereas it converges from above for the wage tax rate case. Overall, the chosen optimal national fiscal policy does not conflict with the optimal EMU monetary policy.

Under commitment, the fiscal authority pursues a constant policy. We compare the optimal fiscal policy found under unrestricted commitment and a policy following a simple rule. In the first case, also the forward looking variables matter for the optimal decision, whereas policy with a simple rule means that the fiscal response only depends on the predetermined variables. The optimally chosen tax rate in the two cases differs significantly. The difference lies in the range of $8.8 \text{x} 10^{-3}$. The optimal tax rates would be chosen smaller when committing to a simple rule. The choice for $g$ would differ by only about $1.5 \text{x} 10^{-17}$. Committing to a simple policy rule would influence the optimal choice of taxes significantly.

**Result 3.4.** The Sales-Tax Model

- Under commitment, the government optimally lowers government spending and raises taxes in the occurrence of a positive debt shock.
- Output diminishes when a shock in debt takes place.
- The inflation rate impulse response is first positive, the turns negative before overshooting the zero line and finally reverting to zero.
- The optimal fiscal actions are very smooth.
- The half-time of debt under optimal commitment policy is 18 years.
- A shock in technology mainly influences the shock in debt.
- A price markup shock severely lowers welfare.
- Optimal monetary and fiscal policy under commitment coincide.

The loss function value in the occurrence of a one unit debt shock is $2.4921 \text{x} 10^{-6}$. Thus, a comparable debt shock causes a larger loss than in the wage tax model. If the government can only levy sales taxes, the inflation is affected more such that welfare losses are larger. Overall, if we compare the two settings, the following can be found.

**Result 3.5.** The Wage-Tax vs. the Sales-Tax Model

- Under optimal commitment policy, sales taxes are raised more compared to wage taxes to obtain the optimal debt reduction path.
- Thus, sales taxes are less effective in reducing debt.
- The inflation rate impulse response is different for the two scenarios.
- The half-time of debt under optimal commitment policy is closely the same for both tax instruments.
- Welfare losses are higher if the government can only raise sales taxes.

For robustness, we vary the price stickiness (i.e., the parameter $\theta$)—see Figure 3.11 and 3.12. If prices are more flexible (e.g., $\theta_1 = 0.35 < \theta$), all variables are affected less except of the inflation rate. The difference for the optimal choice of government spending is about 1x$10^{-3}$, whereas the optimal tax response changes by 5x$10^{-3}$. Overall, the shift in optimal government spending is smaller than in taxes if prices are more flexible. Thus, it suffices to raise taxes less and to lower government spending less to counteract the rise in debt with more flexible prices. The inflation rate responds more to a shock in debt when prices can adjust more quickly. Hence, inflation decreases more and then overshoots the zero line by a higher amount. The difference of the intensity lies in a range of 1x$10^{-4}$.

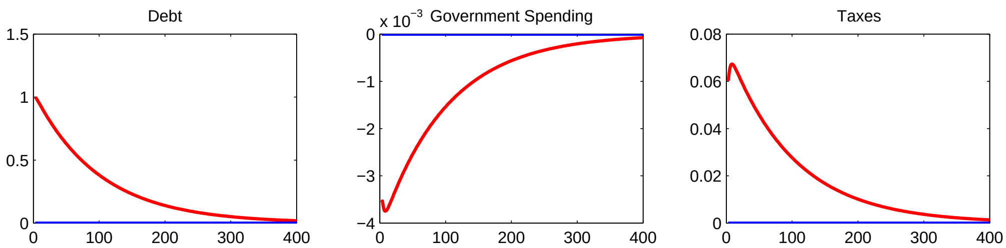


Figure 3.11: Impulse Responses to a 1 Unit Debt Shock with more Flexible Prices

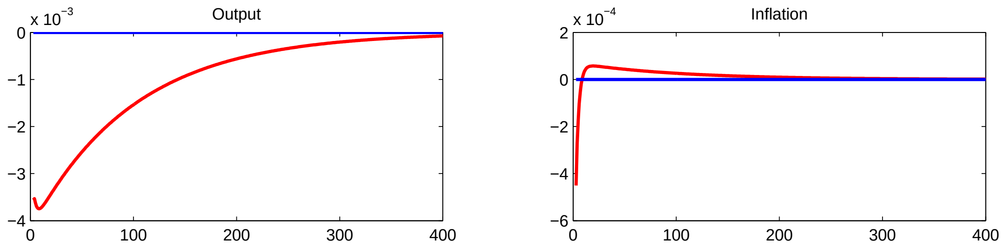


Figure 3.12: Impulse Responses to a 1 Unit Debt Shock with more Flexible Prices

The loss in welfare increases to 3.4413x$10^{-5}$. For the baseline calibration, it was 2.4921x$10^{-6}$. Thus, even with the weaker optimal fiscal policy actions and the less negative response of output, the loss in welfare strongly increases. The additional change in inflation due to more flexible prices severely affects the welfare in the economy.

As a second step for robustness checks, the steady state debt per output ratio is varied (see Figure 3.13-3.16).

As in the wage tax model, the impulse responses of output, government spending, and taxes change proportionally to a change in the steady state debt level. If $d_{ss1} = 0.4 = 2/3\ d_{ss}$, the impulse responses of the respective variables are less intense than before. They are equal to the original responses multiplied by a factor of 2/3.

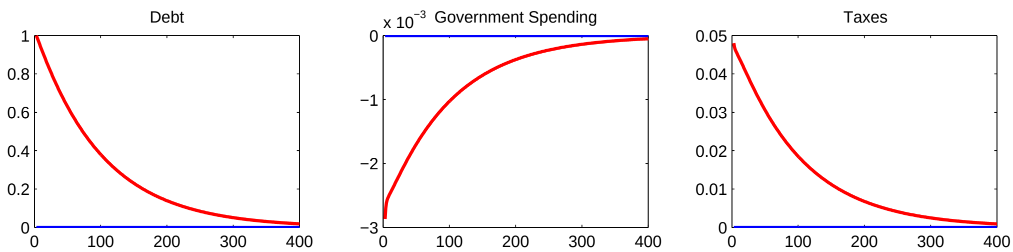


Figure 3.13: Impulse Responses to a 1 Unit Debt Shock with $d_{ss1} = 0.4$

Contrary to the wage tax model setting, not only the model variables responses described above change proportionally but also the respective one for inflation. The minimum and the maximum of the response curve are proportional to the change in the steady state debt.

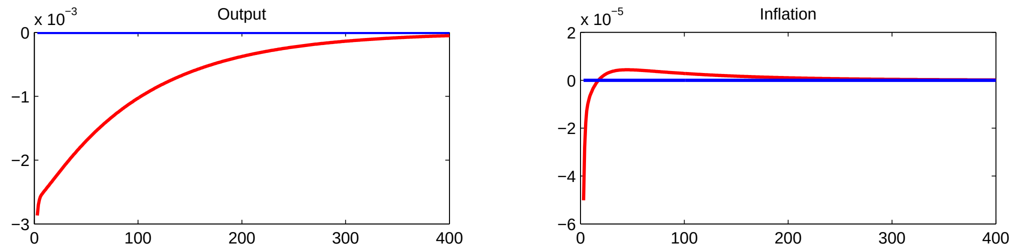


Figure 3.14: Impulse Responses to a 1 Unit Debt Shock with $d_{ss1} = 0.4$

Figure 3.15 and 3.16 show the impulse responses for a higher national steady state debt level. If $d_{ss}$ rises to $d_{ss2} = 0.85 = 17/12\ d_{ss}$, then government spending, taxes, and, as a consequence, output respond more strongly with a factor of 17/12. As in the wage tax model, due to an additional rise in debt, the policy actions in a more indebted economy are more intense to involve the optimal debt reduction path. Hence, also output is affected more.

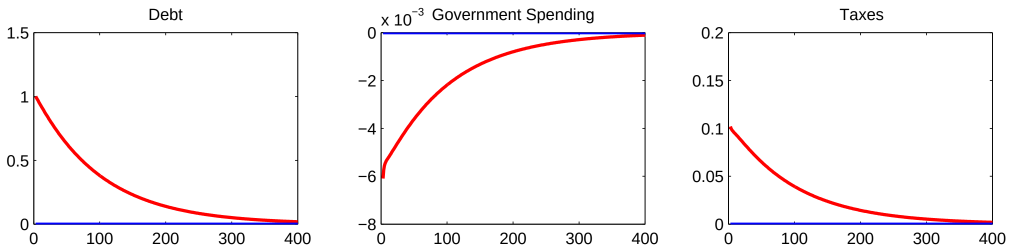


Figure 3.15: Impulse Responses to a 1 Unit Debt Shock with $d_{ss1} = 0.85$

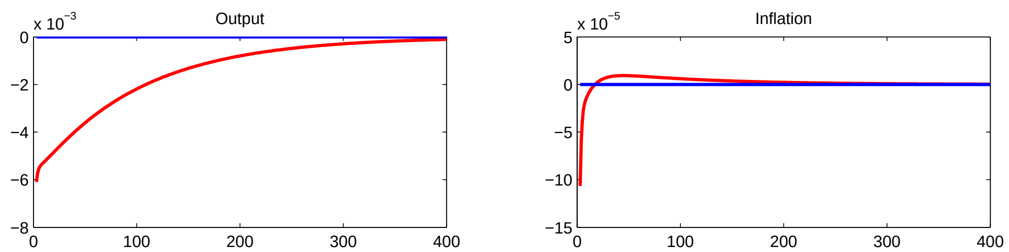


Figure 3.16: Impulse Responses to a 1 Unit Debt Shock with $d_{ss1} = 0.85$

The loss function value for $d_{ss1}$ is $J_1 = 1.1076\text{x}10^{-6}$, whereas it is calculated as $J_2 = 5.0015\text{x}10^{-6}$ for $d_{ss2} = 0.85$. The loss in welfare increases sharply in the occurrence of a positive debt shock if the economy is already indebted at a higher level.

**Result 3.6.** The Sales-Tax Model

If an economy is already highly indebted, an additional rise in debt causes a more than proportional loss in welfare when conducting optimal fiscal policy under commitment.

Overall, the real variables responses are fully robust to changes in the calibration values chosen according to recently found values for the EMU.

## 3.5 Conclusions

The underlying New-Keynesian model setting is an aggregate of different national countries and hence different national governments with one union-wide monetary authority. Following an inflation stabilizing monetary policy, the interest rate is set to preserve inflation stability. The optimal national fiscal policy under commitment does not conflict with the optimal monetary policy setting the interest rate equal to its natural level.

We have analyzed the effect of a positive debt shock on government expenditure, taxes, output, and inflation. Moreover, the loss in welfare has been calculated. Wage taxes or sales taxes as the only source of revenue for a national government have been included. The main difference arising in the two model settings is the shock processing. Debt is directly influenced by both shocks apparent in the model setting for the wage tax model. In contrast, for the sales tax setting, only the markup shock on prices affects debt directly.

With optimal fiscal policy, government spending is lowered whereas taxes are raised in the occurrence of a rise in debt. A strong response of inflation to the debt shock would cause a high amount of welfare loss. As a consequence, the fiscal variables are set under optimal commitment policy in a smooth way. Debt is only reduced very slowly. Moreover, the deviation of the remaining model variables from their steady state values lasts for long. The intensity of response depends proportionally on a change in the steady

state debt per output ratio. Assuming a higher debt output ratio, a positive debt shock causes a more than proportional loss in welfare. Thus, a one unit debt shock hits an economy indebted at a higher level harder and the fiscal response is stronger.
Overall, optimal fiscal policy under commitment with indirect taxes as fiscal instrument seems to harm welfare more than optimal policy with wage taxes. Furthermore, policy with indirect taxes is less effective in reducing debt.

# Chapter 4

# The Fiscal Stance in Germany for 1991-2009

## 4.1 Introduction

To create a meaningful environment for empirical studies as SVARs where fiscal shocks are examined, we analyze the de facto situation in Germany for the time after the reunification, i.e., for 1991-2009. The main objective is to evaluate the fiscal policy situation for which many steps have to be taken. To obtain a first insight into the situation of Germany, its potential output and the output gap is calculated.[1] Horn et al. [65] describe the importance of potential output and its calculation difficulties. There exists a variety of methods to obtain a measure for the unobservable potential output. Depending on the chosen method, the definition of potential output varies. The three most important approaches are the split time-trend method, the Hodrick-Prescott (HP) filter method (see also Hodrick and Prescott, [63]), and the production function based approach. The split time-trend method calculates for each business cycle the trend growth of GDP. One of the main disadvantages of this method is that it relies mainly on ad hoc judgements of the current technology level, capital stock, and labor force. Structural changes are not

[1] All estimations are carried out using MATLAB [94].

taken into account. The GDP smoothing method using a Hodrick-Prescott filter calculates a trend of the GDP data series for the whole horizon. Thus, structural breaks are smoothed over the whole period and cause measurement inaccuracies of the calculated detrended output series if suddenly, changes in the demand and supply of goods occur. Finally, the production function based approach calculates potential output as the magnitude that is consistent with the current technology, labor, and capital formation level. More details are given for example in Giorno et al. [56].

Contrary to methods like the split time-trend method the underlying study builds on the production function based approach used by the OECD: output is assumed to depend on the available input factors of an economy. Moreover, potential output is defined as the output produced with the capital stock and potential employment. Since we assume as an approximation that the government sector "produces" at its potential level,[2] i.e., the actual output in the government sector is equal to the potential output level, we apply the production function based approach only to the business sector of the economy. The business sector is defined as the entirety of economic actors excluding the government sector, see also Giorno et al. [56].

The output gap, which provides information on the business cycle, can be defined in several ways, e.g., as the deviation of output from its long-run stochastic trend. In some studies, the output gap is described as the deviation of output from the flexible-price output, which is obtained in an economy without nominal rigidities. A third possible definition of the output gap is the deviation of output from the level obtained by using the current technology level and the available capital and labor inputs to full capacity, i.e., the potential output. A broad overview on different definitions of the output gap and their implications is given by Kiley [73]. We choose the same definition as the IMF and the OECD and define the output gap as the difference between actual and potential output.

To assess the fiscal policy situation in Germany, one can use for example fiscal policy

[2]Implicitly, we assume that the government uses the entire available capital stock and the whole number of employees—see also Giorno et al. [56].

rules directly, correlation coefficients between the cyclical components of the real government expenditure and real GDP, the amplitude of the fiscal policy cycle, or the fiscal budget balance as a tool. Cimadomo et al. [35] examine five OECD countries and interpret fiscal policy behavior by estimating fiscal policy rules directly. They find that fiscal policy becomes more countercyclical in Germany over time. Kaminsky et al. [71] analyze fiscal policy for Germany by calculating the correlation coefficient between the cyclical components of real government spending and real GDP. Moreover, they calculate the amplitude of the fiscal policy cycle in 1960-2003. Fiscal policy is evaluated as pro-cyclical for the first indicator, whereas it is found to be countercyclical for the latter one. Thus, there is no clear tendency for Germany within the analyzed time horizon. Frankel et al. [51] examine the correlation coefficient between the cyclical components of real government spending and real GDP. They split their data sample in two parts, 1960-1999 and 2000-2009. For the first period, they find that Germany pursues a pro-cyclical fiscal policy, whereas it changes to a countercyclical policy for the second subperiod.

There are many studies that calculate fiscal balances and the associated elasticities, e.g., Mohr [99], Hermann et al. [62], Giorno et al. [56], or Van den Noord [125] with varying time context and data frequency. The budget balance measures the difference between revenues and expenditures of the government. A surplus (deficit) in the balance suggests a contractionary (expansionary) fiscal policy. Changes in the overall budget balance, the current fiscal balance, the primary balance, which excludes interest payments, or the structural balance, which is the cyclically-adjusted budget balance can be calculated to analyze the fiscal stance. We make use of the structural budget balance, i.e., the difference of tax revenue and government spending if output would be equal to its potential level to decide whether fiscal policy is performed pro-cyclically or not. If the structural budget balance is positive, the government receives more income than it spends and vice versa. To judge if fiscal policy is performed pro- or countercyclically, changes in the output gap are compared to changes in the structural budget balance of the economy. If the two changes have the same sign, fiscal policy is performed countercyclically, whereas if the changes have different signs, the fiscal policy responds pro-cyclically. In this context, the fiscal stance is defined as changes in the structural budget balance. For a closer description of the fiscal stance including other possible definitions see, e.g., Polito and Wickens [110], Clark and Dilnot [36], the OECD Economic Outlook No. 74 [103], or the IMF Pamphlet Series No. 49 [70]. Overall, the budget balance consists of two components, the structural and the cyclical budget balance. The former is the difference of government revenue and expenditure when output is equal to its potential level. If actual output is smaller than potential output, for instance additional transfer payments or unemployment benefits are caused by the fact that unemployment does not equal full employment. In particular, to derive the structural balance in that case, additional tax revenues that would occur if output was equal to potential output and thus full employment have to be added to and the additional transfer payments due to cyclical unemployment have to be subtracted from the actual budget balance. These additional budget components are captured in the cyclical budget balance.

The OECD Economic Outlook No. 74 [103] analyzes the fiscal policy in the three major countries Germany, France, and Italy for 1991-2002 using the structural balance. They use aggregated data and find that in most periods, fiscal policy is pro-cyclical.

In the underlying study, we calculate the budget balance and the fiscal stance defined above to decide if fiscal policy in Germany for 1991-2009 was pro- or countercyclical. There are several methods to obtain the structural and cyclical budget balance. The method used by the European System of Central Banks (ESCB) tries to capture changes in the structure of demand and income in more detail. It uses less aggregated data compared to other methods in order to calculate the budgetary elasticities, see, e.g., Bouthevillain et al. [23]. Moreover, different components influencing the budget balance are chosen to take cyclical adjustment more into account. In particular, average compensation of private employees, employment in the private sector, operating surplus of companies, private consumption, and unemployment are taken as variables for the macroeconomic environment instead of GDP. Budget items that are also part of the expenditure side and that are not cyclically adjusted as indirect taxes paid by the government to itself or direct taxes related to interest income are excluded from the analysis. A second method to calculate structural and cyclical budget balances is the OECD approach. According to this approach, output elasticities of government spending and revenues are calculated for different components of expenditure and tax categories. Then, potential output is derived using the production function approach. Finally, by combining the output elasticities and the potential output the structural and the cyclical balance is calculated.

We follow the OECD method. Giorno et al. [56] and Van den Noord [125] use annual data, Blanchard and Perotti [21] and Perotti [108] fit the methods to quarterly data to obtain the necessary output elasticities of government expenditure and tax categories. The results found in these studies are quite different because annual data smooth over one whole year, leaving quarterly changes unconsidered. Moreover, not every estimate is reasonable. If the estimated results are not statistically significant or unrealistic, the elasticities are set exogenously, see, e.g., Blanchard and Perotti [21] and Perotti [108]. We try to avoid exogenous choices and thus compare different methods to obtain meaningful estimates. The most realistic results are filtered and used for further calculations. Only a few exogenously given subelasticities are utilized. Combining the estimates of the elasticities and the estimated potential output, the structural budget balance is obtained. The derivation of the fiscal stance to examine fiscal policy concludes the analytical part of the actual situation in Germany for 1991-2009.

The labor share in the underlying study is found to be decreasing over time. Potential output increases for the examined time period and actual output lies below its potential level, except for 2000-2002. Moreover, after estimating the output elasticities of the fiscal instruments (government expenditure and revenue) and the budget balance, fiscal policy is assessed. The German government follows a pro-cyclical policy for most parts of 1991-2002. This behavior changes in 2003-2009. The largest part of this subperiod indicates a countercyclical fiscal policy and coincides with Keynesian economics. In challenging economic times as deep recessions fiscal policy responds clearly countercyclically to the economic development for the whole time horizon 1991-2009.

The remainder of the chapter is organized as follows. In the next section, we calculate the potential output and the output gaps using the methodology of the OECD. The third section derives the elasticities needed to calculate the budget balance. Moreover, the fiscal stance of Germany is characterized. The last section draws some conclusions.

## 4.2 Potential Output and the Output Gap for 1991-2009

In this section, we derive the potential output for Germany in the post-reunification period (1991-2009). Potential output is used to calculate the output gap. A decrease in the output gap suggests a cyclical downturn of the economy. We examine in later sections if downturns are accompanied by countercyclical fiscal policy.
Quarterly data are used for all calculations of the study. The data series, their sources, and their time horizons are described in detail in Appendix B.1.
As starting point, we assume that an economy is split into two sectors. The business sector includes all the self-employed, firms and service providers. Measurable value-added is produced in this sector. The second sector is the government sector, which produces public goods. The value-added of these goods is measured as the factor costs, i.e., the compensation of employees in the government sector. Additionally, the consumption of fixed capital is summed to obtain a measurable production value. We follow the definition of government production given in Nissen [102]. Moreover, we assume that the government sector produces at its potential level—see, e.g., Giorno [56]. The government sector uses the available capital stock and employs as many workers as possible. Thus, the corresponding actual government output value is used as one component of the overall potential output of an economy. For the business sector, the OECD Business Sector Database [104] is augmented by the period 2000-2009. The data base exists for 1960-1999 but is not supported by the OECD any more due to missing data series of many of the main countries. For Germany, most data series needed are available such that we add the necessary single data. To derive the potential output of the business sector, we apply the production function approach similar to the one used by the OECD. This approach takes a Cobb-Douglas production function as the basis to derive a data

series for the unobservable total factor productivity. In a first step, the labor share $(1-\alpha)$ defined as the fraction of real wages $(W_B)$ to gross value-added in the business sector $(Y_B)$ is calculated.

$$1-\alpha = \frac{W_B}{Y_B}.$$

The compensation of employees in the business sector $W_B$ is derived as the residual of the overall wages in Germany subtracting the wages in the government sector. All nominal wage series are deflated with the GDP-deflator with base year 2005. To obtain the actual gross value-added in the business sector $(Y_B)$, we take the difference between the overall gross value-added that is calculated as the difference between GDP $(Y)$ and the net indirect tax revenue $(NTIND)$ and the value-added in the government sector $(Y_{Gov})$. The government value-added is equal to the sum of the compensation of employees and the consumption of the fixed capital formation.

$$Y_B = Y - NTIND - Y_{Gov},$$

where $Y_{Gov} = W_{Gov} + CFC_{Gov}$. $W_{Gov}$ is the compensation of employees and $CFC_{Gov}$ denotes the consumption of fixed capital in the government sector.

The nominal GDP series is deflated with the GDP-deflator. The OECD Business Sector Database offers a deflator series for the investment and the wages in the government sector. Also a special deflator series for the net indirect taxes is given. See the description of the OECD Business Sector Database [104] for more details concerning the calculation of the business sector aggregates and the available data series.[3] Using the wages in the business sector, the labor share $1-\alpha$ is calculated and varies between 0.6125 in 1992Q3 and 0.5101 in 2008Q1.

Figure 4.1 shows the development of $1-\alpha$ for 1988-2009. To avoid endpoint problems, we augment all data series for 1988-1990. Before reunification, the data correspond only to West Germany.

We detect a statistically significant jump in 1991 due to the reunification. Overall, $1-\alpha$ diminishes (red solid line) over time and the average labor share for 1991-2009 is 0.5595

[3] All deflator series are normalized to 100 for the year 2005.

(black solid line). The blue solid line denotes the HP-filtered series to illustrate the negative slope of the series.[4]

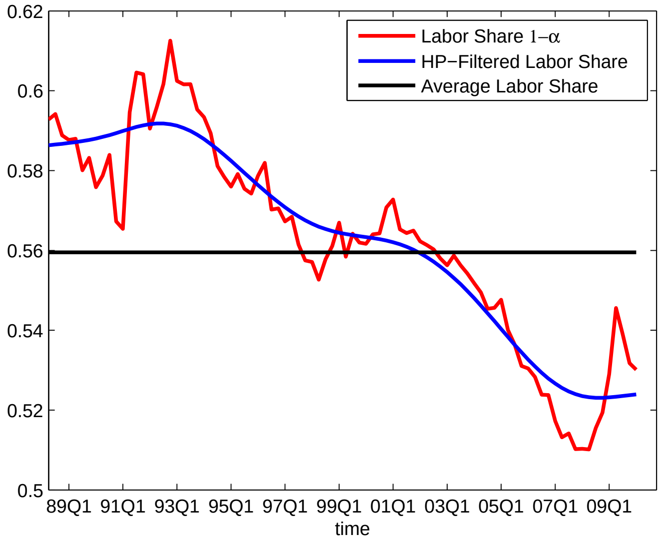


Figure 4.1: The Development of the Labor Share $1 - \alpha$ in 1988-2009

The next step in the calculation process of potential output in the business sector is to find an approximation for the unobservable total factor productivity (TFP) data-series $A$. We derive the TFP as the residual of the following Cobb-Douglas production function in logarithmized form.

$$Y_B = AL_B^{1-\alpha}K^{\alpha}, \tag{4.1}$$

where $Y_B$ denotes the gross value-added in the business sector, $A$ is the total factor productivity and $L_B$ is employment in the business sector. The employment in the business sector is calculated as the overall employment of the economy net of the general employment in the government sector. The self-employed of the economy are part of $L_B$. Finally, $K$ denotes the capital stock of the business sector. Since the data series of the capital stock $K$ in the OECD Business Sector Database ends in 1999Q4, it is approximated as follows. We assume that the capital stock tomorrow ($K_{t+1}$) is equal

[4]Following Hodrick and Prescott [63], the $\lambda_{HP}$-parameter of the filter is set to 1600 for all filtering procedures in the underlying study.

to the capital stock of today ($K_t$) net of depreciation ($\delta_t K_t$), where $\delta_t$ denotes the depreciation rate. Moreover, investment ($I_t$) today as the additional accumulated capital in period $t$ is added. We obtain the standard capital accumulation equation, applied to the business sector:

$$K_{t+1} = (1 - \delta_t) K_t + I_t. \tag{4.2}$$

The investment in the business sector is defined as the overall gross fixed capital formation ($GFC$) less investment in the government sector ($IG$) and investment in housing ($IH$):

$$I = GFC - IG - IH.$$

Since there is a special deflator series for all investment components, each component is deflated with the corresponding deflator series given by the OECD Database normalized for base year 2005.

The depreciation of capital in the business sector is modeled as follows. We take the scrapping rate data series of the OECD for the business sector as given. The data series ends in 1999Q4. Therefore, the last 5 years of available data are used to calculate the average growth rate of depreciation in the business sector. The missing 10 years of data are approximated by assuming that the growth rate of depreciation equals the calculated average growth.

To create a series for the business sector capital stock as consistent as possible, we take the 1991Q1 existing value and not the last existing value in 1999Q4 as the starting value. Thus, the 1991Q1 value $K_{1991Q1} = 1964935.608$ MN€ is chosen as starting point for the law of motion of the capital stock described in equation (4.2).

After log-linearization of the production function (equation (4.1)) and usage of the calculated data, we obtain a data series for the total factor productivity in logs. Finally, if the exponential function is applied to the log series, we arrive at the TFP data series.[5]

To avoid endpoint problems when applying the HP-filter, we augment all data series for 1988Q1-1990Q4. These data correspond only to West Germany.

[5] Since we apply the production function approach, it is not possible to assign a meaningful measure to the ordinate.

Figure 4.2 shows the total factor productivity for 1988-2009. The series comprises an upward trend for 1988-1990, which corresponds to expectations. The trend is disrupted by the reunification. The sharp decline in 1991Q1 is due to a disproportionately high increase in employment compared to the increase in gross value-added in the business sector due to the reunification of the two economic regions, the GDR and the FRG. The development in employment and the gross value-added in detail are shown in Figure C.1 and Figure C.2 of Appendix C.1, respectively. It seems that the average productivity per worker in the business sector for the former GDR is smaller than that of West Germany such that the reunification leads to a decrease of the total factor productivity. Thus, the reunification causes a jump of the TFP on a different development path. For 1991-2007, the total factor productivity increases but at a declining rate. In 2008, the TFP decreases due to a statistically significant decrease in the gross value-added of the business sector (Figure C.2 in Appendix C.1) because of the global crisis. The dashed line shows the HP-filtered TFP-series. As with the original data, the filtered series increases for a long time before declining in 2008.

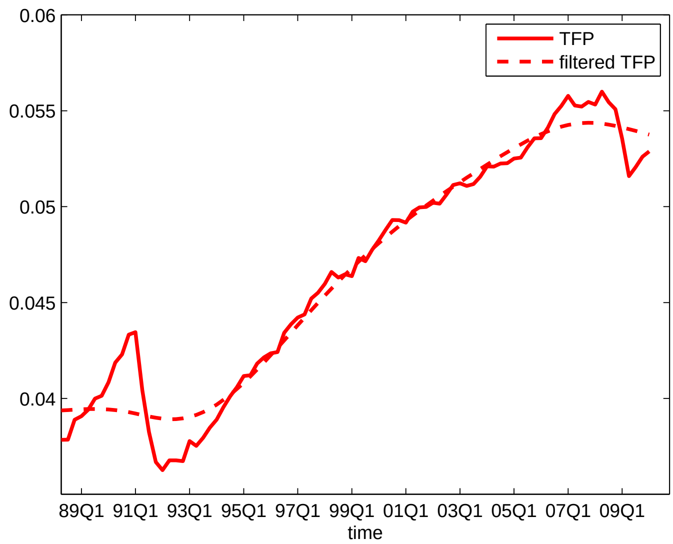


Figure 4.2: The Development of the Total Factor Productivity for 1988-2009

Finally, potential output in the business sector can be calculated using the log-linearized

Cobb-Douglas production function of equation (4.1)

$$y_B^{pot} = (1-\alpha)\, l_B^{pot} + \alpha k + a^{tr}, \tag{4.3}$$

where $y_B^{pot}$ denotes log potential output in the business sector and $l_B^{pot}$ is the log potential employment in the business sector. $k$ denotes the log business sector capital stock and $a^{tr}$ is the log trend factor productivity. A series for the capital stock $K$ already exists and the labor share $1-\alpha$ is known. The remaining series still have to be calculated. Following Giorno et al. [56], we use the HP-filtered log TFP series as a measure for the log trend factor productivity. Finally, potential employment in the business sector is calculated as

$$L_B^{pot} = L_{all} \cdot PART_{trend}\,(1 - NAWRU) - L_{Gov}, \tag{4.4}$$

where $L_B^{pot}$ is the potential employment in the business sector. $L_{all}$ denotes the overall working age population and $PART_{trend}$ is the trend participation rate in the labor market. The product of $L_{all}$ and the trend participation rate corresponds to the smoothed labor force. The $NAWRU$ denotes the smoothed non-accelerating wage rate of unemployment. The smoothing is proceeded using the HP-filter. Since potential output for the business sector is calculated, we subtract $L_{Gov}$, the employment in the government sector in equation (4.4) from the overall potential employment level of the economy. Consistent with the assumption that the government sector produces at its potential level, it also employs people at the potential level.

The potential overall employment level depends on the $NAWRU$ being the specific unemployment rate that would be associated with a constant wage inflation. Since we follow the concept of the OECD—see Giorno et al. [56]—to derive potential output, the $NAWRU$ is used. A different approach is to implement the $NAIRU$, the non-accelerating inflation rate of unemployment. For a closer discussion of alternative calculation methods of potential employment and output in Germany, see Horn et al. [65].

We decide to use the $NAWRU$-concept because we split the economic activity of a country into a government and a business sector. The $NAWRU$ is calculated for the whole economy, but it is reasonable to assume that the unemployment rate and the wages in the government sector follow a constant law of motion. Therefore, one can argue that the $NAWRU$ reflects the main reactions of the wage rate as well as the unemployment rate for the business sector. The inflation rate related $NAIRU$ cannot be split into different sectors, such that the usage of the $NAIRU$ in this two sector concept seems to be inappropriate.

Based on Elmeskov [46], we assume that there is a linear relationship between a change in wage inflation and the gap between the actual unemployment rate and the $NAWRU$. Thus, the $NAWRU$ is calculated as the filtered series of the original series $NAWRU_o$ defined as

$$NAWRU_o = u - \left(\Delta u / \Delta^2 g_w\right) \Delta g_w,$$

where $\Delta$ is the first difference operator, $u$ is the unemployment rate, and $g_w$ denotes the nominal money wage growth rate. Within this context, the money wage growth rate is defined as the difference between two successive log nominal wage rates.

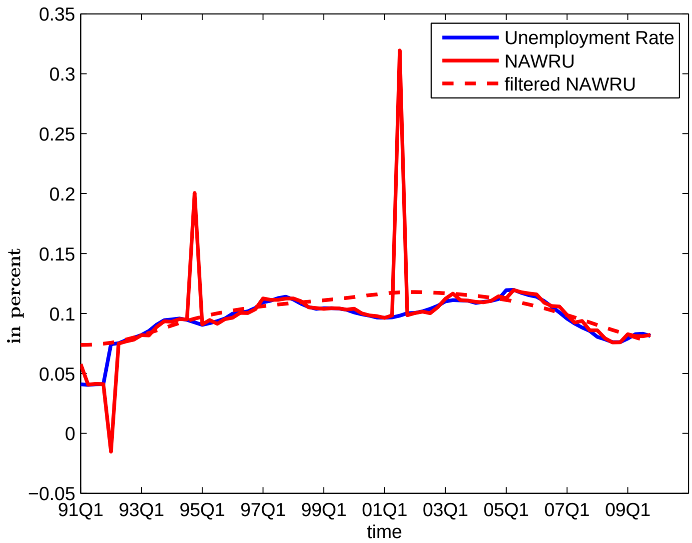


Figure 4.3: The Unemployment Rate and the $NAWRU$ for 1991-2009

Figure 4.3 shows the actual unemployment rate (blue solid line), the original $NAWRU$ (red solid line), and the smoothed $NAWRU$ series (red dashed line) for 1991-2009.

Once again, to avoid endpoint problems, we calculate the original $NAWRU$ for 1988-2009 and then filter the series. 1991-2009 is the relevant time period, therefore, we suppress the first 3 years. Since the original $NAWRU$ is very similar to the actual unemployment rate except of three outliers in 1992, 1994, and 2002, we filter the original $NAWRU$ series.[6]

As a last step to obtain potential employment in the business sector, the overall trend participation rate in the labor market has to be calculated that is defined as the smoothed ratio of overall employment to the labor force. We filter the calculated series of the participation rate with the HP-method. For more details, see Figure C.3 in Appendix C.1.

So far, the data series for the potential employment in the business sector, the trend total factor productivity, the labor share, and the capital formation have been derived. Hence, we calculate the log potential value-added in the business sector applying equation (4.3). The overall potential output $Y^{pot}$ for Germany is defined as the sum of potential output in the business and the government sector, net indirect tax revenue $NTIND$, and consumption of fixed capital in the business sector $CFC$. Since we assume that the government produces at its potential level, we take the actual output in the government sector $Y_{Gov}$ as a component of the overall potential output. It holds that

$$Y^{pot} = \exp\left(y_B^{pot}\right) + Y_{Gov} + NTIND + CFC. \tag{4.5}$$

To obtain a comparable measure to GDP, the value-added is augmented with the net indirect tax revenue and the depreciation of capital because both components are part of the GDP. The definition of government output includes the factor income, i.e., the wages and the depreciation of capital in the government sector.[7] Thus, we only have to add the depreciation of capital ($CFC$) for the business sector explicitly to get overall potential output.

The obtained estimate for potential output in Germany is larger than the one estimated

[6]In general, the $NAWRU$ is very volatile in many countries for most periods of time.

[7]If the original Deutsche Bundesbank series is not seasonally adjusted, as in the case of wages, the standard Census X12-ARIMA method is applied.

by Giorno et al. [56] even with the same conceptional methods.

On one hand, Giorno et al. use annual data, whereas we use quarterly data. Thus, more dynamics are caught in quarterly data because the period over which we average (three months) is shorter. On the other hand, the paper of Giorno et al. examines the time period 1986-1996. Since the potential output depends in most parts on unobservable variables as the total factor productivity calculated as a residual, the *NAWRU*, and the labor share—see Horn et al. [65]—it is reasonable that different estimates are obtained for different periods in time.

The estimates for the TFP and the *NAWRU* are not too different, but the labor share declines statistically significantly in the 1991-2009 period as shown in Figure 4.1. Calculating the average labor share for 1986-1996, we obtain a value of 0.5882. This is significantly higher than the value we obtained for 1991-2009 ($1 - \alpha = 0.5595$).

A closer analysis on the effect of different labor shares is found in Appendix C.1.3. The higher the labor share, the smaller the estimated potential output leaving all other data series unchanged. As a consequence, smaller values for the potential output are calculated in Giorno et al. [56].

The following figure shows the overall potential output (black solid line) and the actual GDP (blue solid line) for Germany in 1991-2009.

In the first third of the observed period, potential output is significantly higher than the actual output. This indicates a demand pull and a cost-push inflation. The first large difference between potential and actual output is caused by the fact that the input factor labor increases extraordinarily due to the reunification. Potential employment is calculated with very high values leading to a high potential output. The decline in factor productivity implies that the additional employees are not that productive such that potential output decreases. In the second third, the actual output converges to the potential output and for 2000-2002, actual output is even higher than potential output. Due to the turbulence of the world economy in the third part of the observed period, the actual output falls back below the potential output and especially for 2008 and 2009, the difference between actual and potential output becomes larger and larger.

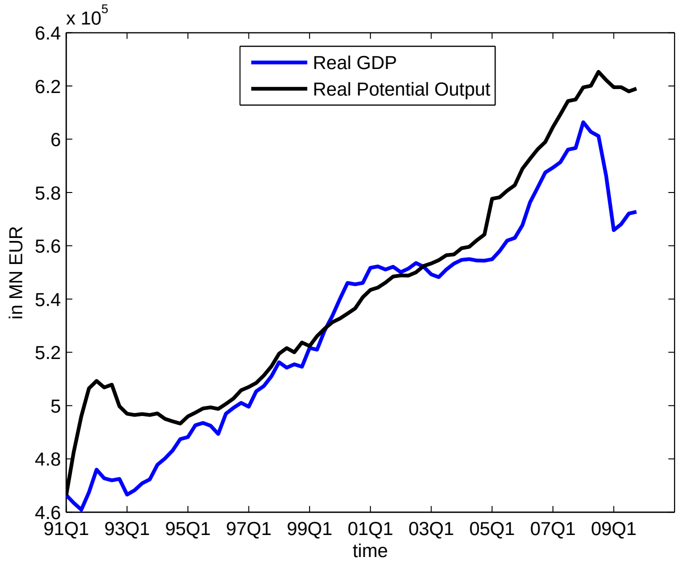


Figure 4.4: Potential and Actual Output for 1991-2009

As a last step in this section, we analyze the output gap. According to the recent literature, the output gap is defined as the deviation of actual from potential output as a ratio of the potential level.

Figure 4.5 shows the relationship between the output gap and the unemployment gap that is defined as the difference of the filtered $NAWRU$ series and the actual unemployment rate in percent ($NAWRU - u$).

A positive unemployment gap means that unemployment is below the non-accelerating wage rate level. Contrary, a positive output gap means that actual output is above the potential level. The output gap as a percentage of potential output is positive only for the period 1999Q4-2002Q4. In all other time periods the estimated potential output lies above the actual GDP. To check for consistency, we compare our results with the ones calculated by the IMF in spring 2006 for 1999 and 2001.[8] Since the estimates of potential output and the output gap depend on the calculation point in time—see Horn et al. [65]— the results cannot be the same. But the estimates we calculate, are comparable and consistent to the IMF values: the IMF finds the output gap for Germany to

[8] See the World Economic Outlook Database, 2006.

be +0.1% in 1999 and puts it at 1.5% in 2001. We find an annual output gap derived as the mean of the corresponding four quarterly data estimates to be -0.195% in 1999 and 1.135% in 2001.[9]

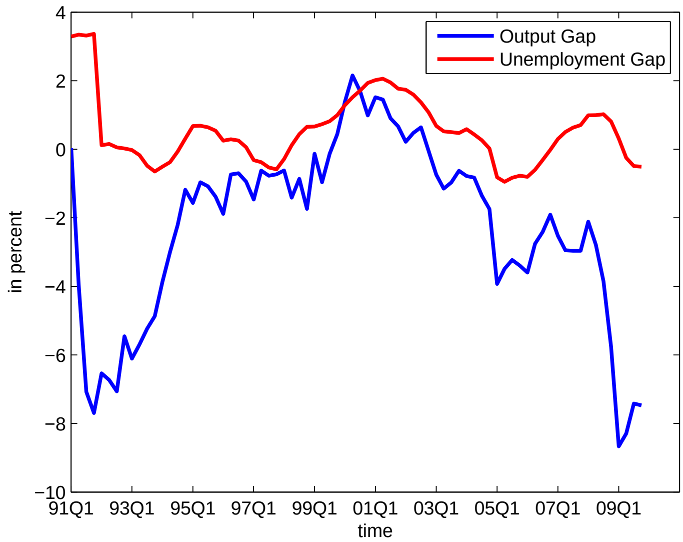


Figure 4.5: Output Gap and Unemployment Gap for 1991-2009

Thus, our estimates are always a bit lower than the IMF estimates. Since the smallest values for the labor share are calculated for winter 2006 to autumn 2008, our average value for $1 - \alpha$ is lower than the one used by the IMF at the beginning of 2006. As a consequence, the calculated output gap is lower. For a further discussion on the effect of different labor shares see Figure C.4 and C.5 in Appendix C.1.3.

## 4.3 Elasticities and the Budget Balance

To describe fiscal policy in Germany, the structural budget balance for 1991-2009 is used. Thus, the output elasticities for six different types of taxes and for the government expenditure have to be calculated. We approximate the government expenditure

[9]If we compare our estimates to the results of the World Economic Outlook Database in spring 2010, the IMF finds +0.175% for 1999 and 1.339% for 2001. Thus, also the application of the production function approach only to the business sectors seems to play a significant role concerning the value of the estimate.

similar to Perotti [108] as the sum of government purchases (government consumption) and capital purchases (government investment in the sense of the gross fixed capital formation in the government sector). The tax revenue is split into personal income taxes, contributions to social security, corporate taxes, indirect taxes, and a residual obtained by subtracting overall tax revenue by the four tax categories listed above. Moreover, government transfers in terms of subsidies, i.e., negative revenue, build the last category to be analyzed.

In the following subsection, we derive the elasticities of net taxes and government expenditure with respect to output and prices. Moreover, other elasticities as the elasticity of net taxes and government spending with respect to the interest or the exchange rate are calculated. These elasticities are needed for the SVAR estimation in Chapter 5. We compare our results to other recent studies and check for consistency of the results. Finally, the structural budget balance is derived to assess fiscal policy in Germany for 1991-2009.

### 4.3.1 Elasticities

Following Giorno et al. [56], we split the tax revenue of the government into five parts. Direct taxes are divided into three categories, the personal income tax, the contributions to social security, and the corporate tax. Furthermore, the government earns indirect taxes, which are divided into value-added tax (VAT) revenue and taxes on import duties. We consider an additional tax obtained as a residual of subtracting the available overall tax revenue with the direct and indirect tax types listed above. Finally, government transfers (subsidies) as a negative tax revenue are analyzed. For a closer description of the data sources see Appendix B.1.

The elasticity of net taxes or government spending with respect to output or, in other words, the output elasticity of net taxes or government spending describes the effect of varying output on taxes or government spending.

The elasticity of one of the five tax categories with respect to output is derived as the product of the elasticity describing the effect of a change in output on the tax base and the impact of varying the tax base on the tax revenue. Thus, it is equal to the product of the output elasticity of the tax base and the tax base elasticity of the tax rate. In some parts, we follow this concept (as in the case of indirect taxes), in other parts we use elasticity definitions suggested by Van den Noord [125]. In the next paragraphs, each output elasticity is explained in detail.

The output elasticity of personal income taxes ($oepit$) and the elasticity of social security contributions with respect to output ($oessc$) can be summarized as follows. $TB_i$ denotes the tax revenue of the respective tax category $i \in$ {personal income taxes, social security contributions}. $Y$, $L$, and $wr$ are the GDP, the employment, and the wage rate, respectively. Following Van den Noord [125], the output elasticity of personal income taxes and social security contributions $oe_i$ is defined as

$$\begin{aligned} oe_i &= \frac{\partial TB_i}{\partial Y}\frac{Y}{TB_i} = \frac{\partial\left[\left(TB_i/L\right)L\right]}{\partial Y}\frac{Y}{TB_i} \\ &= \frac{Y}{L}\left(\frac{\partial\left(TB_i/L\right)}{\partial Y}L + \frac{\partial L}{\partial Y}\frac{TB_i}{L}\right) \\ &= \left(\frac{\partial L}{\partial Y}\frac{Y}{L}\right)\left(1 + \left(\frac{\partial\left(TB_i/L\right)}{\partial wr}\frac{wr}{TB_i/L}\right)\left(\frac{\partial wr}{\partial L}\frac{L}{wr}\right)\right). \end{aligned}$$

The first term in the last row is the output elasticity of employment ($oee$), whereas $\left(\frac{\partial(TB_i/L)}{\partial wr}\frac{wr}{TB_i/L}\right)$ denotes the real wage elasticity of personal income taxes per worker ($wepit$) and the real wage elasticity of social security contributions per worker ($wessc$). $\left(\frac{\partial wr}{\partial L}\frac{L}{wr}\right)$ is the employment elasticity of wages ($eew$). The effect of a change in output on the personal income tax revenue is summarized as follows.

$$oepit = oee\left(1 + wepit \cdot eew\right), \tag{4.6}$$

where $oepit$ is the output elasticity of personal income taxes, $oee$ denotes the output elasticity of employment, $wepit$ is the real wage elasticity of personal income taxes per worker, and $eew$ denotes the employment elasticity of wages. The output elasticity of employment describes the effect of a change in output on the employment and indirectly captures the impact of changes in employment on taxes if a tax yield per worker is given. Since Okun's Law states that a change in output is partly absorbed by labor productivity changes, the output elasticity of employment has to be smaller than 1. The product in brackets of equation (4.6) shows the development of the tax yield per worker and is split into two parts: the real wage elasticity of personal income taxes and the employment elasticity of wages. The last term (the employment elasticity of wages, *eew*) can be seen as the Phillips curve effect on wages. It describes the impact of employment on wages. A sensitivity analysis due to the Phillips curve effect and the Okun's law effect is carried out below.

To obtain the output elasticity of employment (*oee*), we regress the log of total employment on the first lead, the actual value, and the first to fourth lag of the log gross value-added—see Perotti [108]. The choice of four lags is due to the usage of quarterly data. Additionally, we use a constant and a deterministic time trend as regressor. An LM-test on serial correlation is performed. Since the hypothesis of serial correlation is not rejected, we use Newey-West robust standard errors. Except of the first lag of the log gross value-added, all regressors are statistically significant at a 10% statistical significance level. The estimated output elasticity of employment defined as the coefficient of the log output today in the regression is statistically significant even at a 2% statistical significance level. $oee = 0.691092$ (2.459),[10] which is consistent with theory. An increase in output increases employment and the estimate is smaller than 1. Contrary to Perotti [108], we obtain a statistically significant, positive estimate such that we do not set the elasticity equal to zero. Van den Noord [125] uses annual data and obtains an output elasticity of 0.6 for the 1990th. The results are very similar for annual and quarterly data. Quarterly fluctuations do not seem to significantly influence the result. Giorno et al. [56] find an estimate of 0.45 for Germany in 1991, which is smaller than the estimate of Van den Noord [125]. Blanchard and Perotti [21] estimate the output elasticity of wages for the US as 0.42 in 1947-1997.

Similar to Perotti [108] and Blanchard and Perotti [21] who use the estimates of Giorno et al. [56] or Van den Noord [125] for their estimation with quarterly data, we adopt the estimates from Van den Noord and assume that $wepit = 1.5$. Giorno et al. [56] estimate that the wage elasticity of personal income taxes per worker (*wepit*) varies between 1.8

[10] The term in brackets denotes the t-statistic with Newey-West errors.

in 1978 and 2.0 in 1991, whereas Van den Noord [125] estimates it as 1.5 for the whole period 1991-1998. Thus, the elasticity seems to fall in the 1990th. As a consequence, we decide to use the lowest estimate available.

As a last step to obtain a value for the output elasticity of personal income taxes, we regress the log of real wages on one lead, the actual value, and four lags of the log employment to calculate the employment elasticity of wages ($eew$)—see Perotti [108]. Once again, a constant and a deterministic time trend are included as regressors.

The LM-test shows that we cannot reject the hypothesis of autocorrelation. Thus, we use Newey-West standard errors. All regressors are statistically significant at a 10% statistical significance level. The coefficient of the actual log employment, which is chosen as the measure for the quarterly employment elasticity of real wages, is equal to 0.675976 (1.96).[11] This qualitatively coincides with theory: an increase in employment causes an increase in real wages.

Contrary to Perotti [108] who calculates a negative estimate and sets the employment elasticity of real wages equal to zero, we obtain a statistically significant, positive estimate for the quarterly effect of a change in employment on real wages, $eew$. For annual data, Van den Noord [125] estimates the employment elasticity of real wages as 0.8 in 1985-1998 and Giorno et al. [56] estimate it as 0.57 in 1978-1991.

Blanchard and Perotti [21] use quarterly data for the US and obtain a value of 0.62 for the employment elasticity of real wages.

Finally, the output elasticity of personal income taxes is calculated as 1.3918 using equation (4.6). The calculated result is larger than that of Perotti [108] who sets the elasticity equal to 0. Van den Noord [125] obtains 1.3 and Giorno et al. [56] get 1.0. Both papers use annual data and a different time period. Additionally, Giorno et al. [56] use a different value for $wepit$. Thus, the estimates cannot be exactly the same.

Next, we derive the output elasticity of the second tax category in a similar manner. The output elasticity of social security contributions ($oessc$) is defined as

$$oessc = oee\left(1 + wessc \cdot eew\right), \tag{4.7}$$

[11] The term in brackets denotes the t-statistic with Newey-West standard errors.

where *wessc* denotes the wage elasticity of social security contributions per worker. Since social security contributions depend on personal income in a similar manner as the personal income tax, the definition of the *oessc* has the same structure as equation (4.6) defining the output elasticity of personal income taxes. We use the output elasticity of employment and the employment elasticity of wages calculated above. Giorno et al. [56] estimate a value of 0.9 for the wage elasticity of social security contributions in period 1978-1991, whereas Van den Noord [125] estimates a value of 0.8 for 1985-1998. Since the *wessc* seems to decline between 1978-1998 and the estimate of Van den Noord covers a big part of the analyzed time horizon of this study, we choose the smallest available estimate and assume that $wessc = 0.8$. Overall, the output elasticity of social security contributions is calculated as 1.0648 in this study. Perotti [108] sets the elasticity equal to 0 because his estimate is negative and statistically insignificant. Giorno et al. [56] get 0.7 and Van den Noord [125] calculates the output elasticity of social security contributions with 1.0. The estimates cannot be the same because the latter authors use annual data and different time periods. Moreover, Giorno et al. [56] utilize a different value for *wessc*.

The next step is to calculate the output elasticity of corporate income taxes (*oecit*). The estimation method of Van den Noord [125] assumes that the amount of corporate income tax revenue is strictly proportional to corporate income such that the output elasticity of corporate income taxes is equal to the output elasticity of corporate income (profits) and does not seem to be reasonable for quarterly data. Quarterly installments are paid on the liability of a firm that is assessed on the performance of the firm in the previous year. In more details, firms pay a specific tax. After publishing the tax balance sheet for the whole year they pay or receive a compensation related to the tax advance payments in the third quarter of the next year. This fact is not captured by the assumption of Van den Noord [125] that we can simply use the output elasticity of corporate income as a proxy. If the elasticity would be a good proxy, the elasticity of the tax base (the profits) to the corporate income taxes would be equal to one. This cannot be reasonable if the paid tax is geared not to the actual but to the previous year profit. The proxy does not seem to be a good one with quarterly data in reality due to the time lag described above. Thus, we decide to set the output elasticity of corporate income taxes analogous to Perotti [108] equal to 0.

We derive the output elasticity of the last important tax category, the indirect taxes, in the following. Perotti [108] assumes that the output elasticity of indirect taxes is equal to 1. He takes strict proportionality of the tax to the tax base as well as strict proportionality between the tax base and output as given. Van den Noord [125] presumes that the relevant tax base for the indirect taxes fluctuates proportionally to private consumption. He estimates the output elasticity of private consumption by 2SLS as an approximation for the output elasticity of indirect taxes and obtains 0.95 for the output elasticity of indirect taxes. We follow a different approach. As mentioned above, we split the indirect taxes into two categories: a VAT and an import duty tax. The overall output elasticity of indirect taxes ($oeit$) is derived as the weighted average of the estimated output elasticity of the VAT and the output elasticity of import duties. The weights are derived as the ratio of the VAT or the import duty taxes on the overall indirect taxes.

The VAT shows similar variations as private consumption. We assume that the amount of VAT reacts strictly proportional to consumption. Thus, the output elasticity of VAT is defined as the output elasticity of private consumption times the private consumption elasticity of VAT. The second term is equal to 1 due to the assumption of strict proportionality. We estimate the output elasticity of the VAT as $a_2$ of the equation $\log(C/Y^{pot})_t = const + a_1 t + a_2 \log(Y/Y^{pot})_t$, where $C$ denotes private consumption expenditures, $Y^{pot}$ is the potential output, $Y$ is GDP, and $t$ is a deterministic time trend. The subscript $t$ denotes the time period. We use the 2SLS method to correct for simultaneity and choose a constant, a deterministic trend, the log of gross fixed capital formation divided by potential output, and the log of government consumption divided by potential output as instruments. Since the LM-test result suggests that there is serial correlation, we use Newey-West robust standard errors. All regressors are highly statistically significant with a statistical significance level of 1%. We obtain 1.1690 with t-statistic 7.204 for the output elasticity of the VAT. The average ratio of the VAT to overall indirect taxes is 0.7657. The largest part of indirect tax revenue comes from the taxation of consumption goods.

The output elasticity of import duties is estimated in a similar manner as the output elasticity of VAT. We assume that the import duty taxes show the same variations as imports. The import elasticity of import duty taxes is set to 1. Thus, the output elasticity of the import duty tax is equal to the output elasticity of imports. The output elasticity of the import duty tax is calculated as $b_2$ of $\log(Im/Y^{pot})_t = const + b_1 t + b_2 \log(Y/Y^{pot})_t$, where $Im$ denotes the overall imports of Germany. The 2SLS method with a constant, a deterministic trend, the log of private consumption divided by potential output, the log of the gross fixed capital formation divided by potential output, and the log of government consumption divided by potential output as instruments is applied. Once again, the LM-test shows that there is serial correlation in the data such that we use Newey-West standard errors. All regressors are statistically significant at a 10% statistical significance level and the output elasticity of import duties is estimated as 1.2611 (1.773). The average ratio of import duty taxes to indirect taxes is 0.2343. As a consequence, we calculate the output elasticity of overall indirect taxes ($oeit$) as the weighted average of the output elasticities of VAT and import duty taxes with a value of 1.1905. Perotti [108] assumes an output elasticity of 1 for quarterly data in a different time period, whereas Van den Noord [125] estimates it as 0.95 for annual data, once again for a different time horizon.

The last component of the net tax revenues are the government transfers. Since only the unemployment benefits seem to react to a cycle, we derive the output elasticity of unemployment benefits as a proxy for the output elasticity of transfers ($oet$). The following formula is taken from Van den Noord [125] who uses the method described below. It implies that the labor force does not react to the cycle.

$$oet = -UBG \cdot oee(((1 - eelf)/ULF) - 1). \tag{4.8}$$

$UBG$ is the average ratio of the unemployment benefit to total government expenditure. $oee$ denotes the output elasticity of employment estimated above, $eelf$ is the employment elasticity of the labor force, and $ULF$ is the unemployment rate. We assume similar to Van den Noord [125] that the unemployment related expenditure is strictly proportional to unemployment. Furthermore, the unemployment benefit rates are postulated to be independent of the actual economic situation, i.e., the position in the business cycle. $-oee(((1 - eelf)/ULF) - 1)$ is the output elasticity of the unemployment related expenditure. Weighted with the ratio of unemployment benefit of total transfers ($UBG$), we obtain the output elasticity of government expenditures. Van den Noord [125] gets for $UBG$ 8.6%, whereas we get 2.82%. This is reasonable because the 2000-2009 time period is a period of enormous government transfers unequal to unemployment benefits in Germany. Thus, the ratio of the unemployment benefits is smaller than in 1985-1998. The unemployment rate ($ULF$) is 9.55%. Finally, we estimate the employment elasticity of the labor supply. The log of the labor force divided by potential labor is regressed on a constant and the log of total employment divided by potential labor. Due to serial correlation, we use Newey-West standard errors. All regressors are highly statistically significant and the employment elasticity of the labor force (the labor supply) is 0.5205 with t-statistic 11.23. Van den Noord [125] estimates it as 0.8 with annual data. Using equation (4.8), the output elasticity of government transfers ($oet$) is calculated as -0.0783. The negative sign of the estimate fits to theory: if the output declines, e.g., due to a crisis, unemployment rises and so do the unemployment transfer payments. The unemployment benefits act as automatic stabilizers in a countercyclical way.

For robustness reasons, we also look at an alternative method used by Van den Noord [125]. The unemployment rate in equation (4.8) is substituted by the average trend unemployment rate, the $NAWRU$. We obtain -0.0731 for the elasticity, which is very similar to the first result (-0.0783). Van den Noord [125] estimates the output elasticity of transfers as -0.0688 and rounds it to -0.1. To allow for additional effects of other transfer programs, which could be related to unemployment, we do not set the elasticity to the strict value of -0.0783 and round it, similar to Van den Noord, to -0.1.

As a last step, we assume that the residual calculated as the residual obtained as the difference of all tax categories examined above and the overall tax revenue has an output elasticity of 0, i.e., a change in output does not affect the tax revenue bundled in the residual.

Overall, the following output elasticities are obtained:

**Result 4.1.** Output Elasticities

| **Tax Category** | **Value** |
|---|---|
| Personal Income Tax (*oepit*) | 1.3918 |
| Social Security Contributions (*oessc*) | 1.0648 |
| Corporate Income Tax (*oecit*) | 0 (set exogenously) |
| Indirect Tax (*oeit*) | 1.1905 |
| Government Transfers (*oet*) | -0.1 (rounded) |
| Residual | 0 (set exogenously) |

For the sensitivity analysis mentioned above we first switch the Phillips curve effect off, i.e., we set the employment elasticity of wages equal to 0 such that tax progressivity does not play a role any longer. In this scenario, wages do not vary with employment over the cycle. The output elasticity of personal income taxes and social security contributions are affected: $oepit_s = oessc_s = 0.6911$. Before, $oepit = 1.3918$ and $oessc = 1.0648$. If the Phillips curve effect is switched off, the two elasticities are equal to the output elasticity of employment. The effect of a change in output on the tax revenue is smaller because changes in the employment do not affect wages and thus they do not supplementary change the tax revenue.

A second step of the sensitivity analysis is to additionally switch off Okun's Law. Wages are assumed to be constant, which implies that $oee = 1$. Output varies proportionately with employment and wages do not vary with employment in this scenario. According to equation (4.6) and (4.7), the output elasticities are equal to one in this case. I.e., a rise in output leads to an equal rise of the tax revenue because all other channels of reaction are switched off. If output changes affect employment strictly proportionally, the output elasticity of transfers changes. We obtain $oet_s = -0.1133$. For the alternative formula including the NAWRU, we get -0.1058. Hence, the absence of a channel where labor productivity absorbs changes in output causes the transfers to react more to a change in output: if a disturbance variable is switched off, the effect has to be stronger in general.

After the derivation of all tax category output elasticities, we obtain the overall elasticity of net taxes with respect to output $\alpha_{ty}$ as the weighted sum of all elasticities described above. The weights are chosen as the average share of the ith tax category to overall taxes. Because transfers are paid and not received by the government, we assign a negative weight to the output elasticity of transfers, see also Perotti [108] and Blanchard and Perotti [21]. The transfer payments decline due to a rise in output implying a rise in employment and a smaller amount of transfers has to be subtracted from overall taxes to obtain net taxes. Thus, net taxes rise due to the rise in output without taking changes in other tax categories into account. Overall, since transfer payments as unemployment benefits decline and the transfer component has a negative weight, the effect of a rise in output on the overall net tax due to a change in transfers is positive.

In particular, $\alpha_{ty} = 2.0857$, the output elasticity of net taxes for Germany in 1991-2009. Perotti [108] derives a value of 0.92 for West Germany in 1960-1989. An output elasticity larger than 1 is not unreasonable—Perotti calculates output elasticities of net taxes for the US and for Canada between 1.61 and 2.16. Thus, in the 1990th and 2000th, output seems to have a significantly higher impact on the net tax revenue. A 1% rise in output raises net tax revenue by 2.09%. Due to the dependency of the estimation method on quarterly data, e.g., by implementing four quarters of data in the regressions, we cannot simply include annual data and compare the results with those found by Van den Noord [125] and Giorno [56].

Heppke-Falk et al. [61] obtain a value of 0.95 for the period 1974-2008. They split the overall net taxes into different tax categories. As a consequence, the resulting elasticities are estimated with different values. Bode et al. [22] examine the period 1991-2005 and use aggregates of taxes that differ significantly from our choice. They obtain 0.46 as the output elasticity of net taxes. The main explanation for the deviation from our calculated value is indeed the usage of different tax categories.

The output elasticity of the government spending $\alpha_{gy}$ is assumed to be 0 because the government cannot react to changes in output within one quarter due to the rigidity of the government's decision making process. This assumption is only justified for quarterly data and is also made amongst others by Blanchard and Perotti [21], Heppke-Falk et al. [61], and Bode et al. [22].

> **Result 4.2.** Output Elasticities
>
> - $\alpha_{ty} = 2.0857$
> - $\alpha_{gy} = 0$ (set exogenously)

The price elasticity of net taxes $\alpha_{t\pi}$ is derived in a similar manner as the output elasticity. We decide for each tax category and the transfers if the corresponding taxes are affected by price changes. We follow Perotti [108] and assume that the price elasticity of real personal income taxes is equal to the wage elasticity of personal income taxes per worker (*wepit*) less 1. Implicitly, employment, output, and the real wage are held constant. Similarly, the price elasticity of social security contributions is set to be equal to the wage elasticity of social security contributions per worker (*wessc*) less 1. A change in the price level, i.e., inflation, affects corporate income taxes in both directions. Which direction overweighs is not clear. Thus, we set the price elasticity of corporate income taxes equal to 0. Following Perotti [108], the price elasticity of real indirect tax revenues is set equal to 0; the price elasticity of transfers is set to -1.

The price elasticity of the residual calculated by subtracting the tax revenue of the four tax categories from overall taxes is assumed to be 0.

As before the overall price elasticity of net taxes is derived as the sum of the weighted price elasticities of all tax categories. The weight is the average ratio of the ith category tax revenue to the overall tax revenue. We obtain $\alpha_{t\pi} = 1.0199$. In particular, the price elasticity of personal income taxes is positive as well as the impact of the negative price elasticity of transfers because transfers are assigned to a negative weight. Perotti [108] calculates a value of 0.87 for 1960-1989 where West Germany is examined. Heppke-Falk et al. [61] assume that the price elasticity of net taxes is equal to the output elasticity of net taxes minus one. They obtain a value of $-0.05$ as elasticity.[12]

Since some parts of government spending such as wages react on changes in the price level and some do not, we set the price elasticity of real government expenditure $\alpha_{g\pi}$

[12] The approach we follow is not comparable.

equal to -0.5. The price elasticity of the responding parts is -1. The other part such as spending for health does not react to changes in the price level at all. These parts have an elasticity of 0. We follow Perotti [108] with this approach. The government consumption of wages is about 40%-50% of the overall government expenditure such that we set the price elasticity of government spending on average equal to -0.5. Heppke-Falk et al. [61] set the price elasticity of government spending equal to the output elasticity of government spending minus 1, i.e., equal to -1.

**Result 4.3.** Price Elasticities

- $\alpha_{t\pi} = 1.0199$
- $\alpha_{g\pi} = -0.5$ (set exogenously)

To create a reasonable VAR model in the fifth chapter of the thesis, we need some more elasticities. The interest rate, the exchange rate, and imports are included in the SVAR model to analyze the effect of fiscal shocks on the exchange rate and trade variables. Thus, the interest rate elasticity, the exchange rate elasticity, and the import elasticity of net taxes and government spending are calculated in the following. First, we derive the amount of the interest rate elasticity of net taxes $\alpha_{ti}$ and government spending $\alpha_{gi}$. We use as approximation for government spending the sum of government consumption and government investment. Government consumption does not depend on the interest rate. On one hand, government investment depends on the interest rate. On the other hand, government investment is on average 8.2% of total government expenditure. Thus, the negative influence of interest rate changes on the government investment is negligible. Moreover, from intuition, we can rule out a de facto dependence of government expenditure on the interest rate in reality and set $\alpha_{gi} = 0$. Since the major part of the considered tax bases does not include property income, we assume that a change in the interest rate does not affect net taxes; $\alpha_{ti} = 0$. Also Heppke-Falk et al. [61] set the interest rate elasticity of net taxes and government spending equal to 0.

The next step is to calculate the import elasticity of net taxes. Taxes on imports of goods are part of the indirect taxes. We assumed above that the import duty tax revenue reacts strictly proportionally to imports. The import elasticity of import duty taxes is weighted with the average share of import duty taxes to overall net taxes. We obtain the import elasticity of net taxes $\alpha_{tim} = 0.0731$. The imports do not play any role in the decision process for government expenditure such that we set the import elasticity of government spending equal to 0: $\alpha_{gim} = 0$.

A rise in the exchange rate lowers imports. The import duty taxes and the net tax revenue decrease. Thus, we define the exchange rate elasticity of net taxes $\alpha_{te}$ as the import elasticity of net taxes $\alpha_{tim}$ multiplied by the exchange rate elasticity of imports ($eeim$). The $eeim$ is obtained by regressing the log real imports on a constant, the log real effective exchange rate, and the log real GDP, where the estimated coefficient of the log real effective exchange rate is the exchange rate elasticity of imports. We use Newey-West standard errors due to serial correlation. All coefficients are highly statistically significant on a 1% statistically significance level and $eeim = -0.8002$ (-2.895). Multiplying the exchange rate elasticity of imports with $\alpha_{tim}$, we obtain for the exchange rate elasticity of taxes $\alpha_{te} = -0.0585$. The small coefficient is due to the small weight of the import duty tax revenue vis-a-vis the overall net tax revenue. Finally, the exchange rate elasticity of government spending is assumed to be 0 because it is not reasonable that the policy decision process on government expenditure depends on the exchange rate. Thus, $\alpha_{ge} = 0$.

**Result 4.4.** Price Elasticities

- $\alpha_{ti} = 0$ (set exogenously)
- $\alpha_{tim} = 0.0731$
- $\alpha_{te} = -0.0585$
- $\alpha_{gi} = 0$ (set exogenously)
- $\alpha_{gim} = 0$ (set exogenously)
- $\alpha_{ge} = 0$ (set exogenously)

### 4.3.2 The Budget Balance

Before turning to the primary budget balance, the development of selected parameters as the net tax to GDP ratio or the government spending to GDP ratio for Germany is analyzed. The ratio of overall net taxes to GDP is on average 21.01%. Perotti [108] calculates it as 22.4% in West Germany for 1960-1989.

In the remaining part of the study (except of the derivation of the structural budget balance plotted in Figure 4.8), we follow the definition of Perotti [108] and Blanchard and Perotti [21] of government expenditures. The overall government spending is approximated by the sum of government consumption and the gross fixed capital formation of the government sector. The qualitative developments using the approximation are the same as for the original data used in the analysis of the cyclically-adjusted budget balance below. The average ratio of government spending to GDP is 20.99% where government consumption is 19.76% of GDP and government investment is 1.74%. Perotti [108] calculates these values as 21.4%, 17.9%, and 3.5% for West Germany in 1960-1989, respectively.

Figure 4.6 shows the net tax revenue (blue solid line) and the government spending (red solid line) as a ratio of GDP.

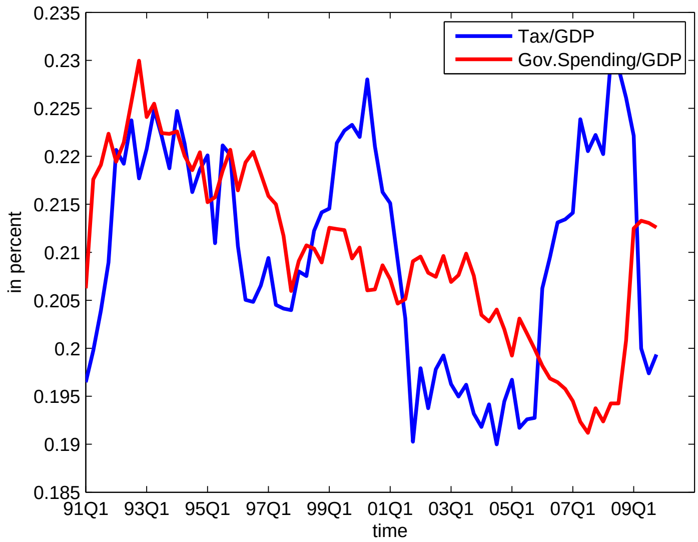


Figure 4.6: Net Tax Revenue and Government Spending as a Ratio of GDP

The ratio of government expenditure to GDP first increases for 1991-1992, where government spending rises more than GDP due to the reunification. In 1993-2008, the ratio decreases on average. The economic crisis in 2008/2009 causes output to decline and government spending to rise, e.g., due to the German fiscal stimulus package in late 2008 and 2009. Thus, the ratio sharply increases in 2009. The tax per GDP ratio first grows larger with the reunification of West Germany and the GDR. The tax revenue rises more than the GDP rises such that the ratio increases. For 1993-1998, the tax per GDP ratio declines and rises sharply till 2000. Afterwards it declines sharply till 2002 and rises once again in the period of 2002-2008. Finally, in 2009, the GDP declines due to the recession and as a consequence, the tax revenue decreases, too. The reduction in the tax revenue is more severe than the decline in GDP such that the ratio sharply falls. The fact that tax revenues are affected more coincides with the estimated elasticity of net taxes with respect to output exceeding 1.

Overall, the tax per GDP ratio is far more volatile than the government spending per GDP ratio. Both series clearly suggest a recession (an economic crisis) in 2009.

The differences of the log tax revenue to log government spending are plotted in Figure 4.7. The curve shows the overall effect of tax revenue and government spending in 1991-2009 and fluctuates around the zero line.

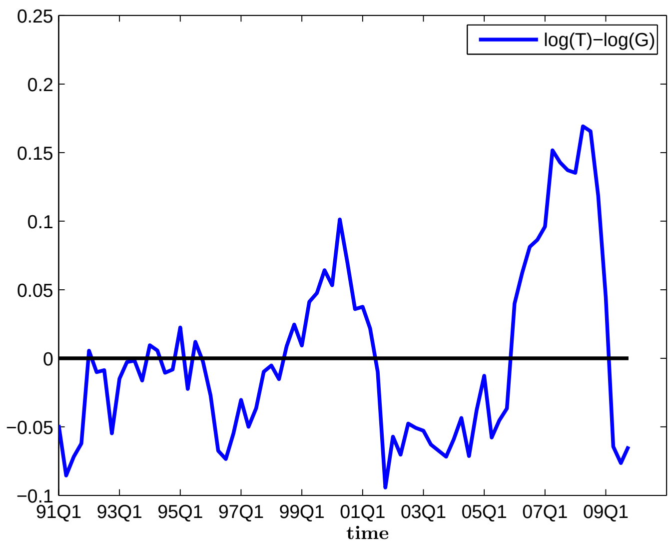


Figure 4.7: Difference of Log Net Tax Revenue and Log Government Spending

There are periods where government spending exceeds tax revenue and vice versa. The log tax revenue is larger than the log government spending in 1998-2001 and 2006-2008. The last period of positive values signal a high level of tax revenues indicating a boom. In 2009, the curve sharply declines, illustrating the economic crisis, which is accompanied by a high level of government spending.

The curve becomes more volatile over time. The fiscal actions are less balanced if we compare the 2000th with the first 10 years after the German reunification. Since the accession of Germany to the European Monetary Union (EMU) in 1999/2000, the difference of government expenditure and revenue becomes larger. Within 2000-2009, there are large changes in the economic performance of the EMU causing higher fluctuations in the fiscal aggregates of a single member country as Germany in the underlying case.

Finally, to decide whether fiscal policy responds pro- or countercyclically to the economic situation, we calculate the structural balance for Germany in 1991-2009.

The structural balance measures what the balance of tax revenue less government spending would be if the actual output is equal to the potential output.

We follow the definition given in Giorno et al. [56] and Van den Noord [125] and extend it by the residual tax category *ctrr*. The structural budget balance is the residual of the actual budget balance and the cyclical balance.

The actual balance is defined as the difference of government tax revenue and government spending. Moreover, the cyclical budget balance ($B_{cyc}$) is derived as a ratio of potential output specified in the following equation—see, e.g., Van den Noord [125].

$$\begin{aligned} B_{cyc} &= \frac{1}{Y}\left(\sum_{i=1}^{5} T_i^d \left(1 - \left(\frac{Y^{pot}}{Y}\right)^{oetc_i - 1}\right) + X^N \left(1 - \left(\frac{Y^{pot}}{Y}\right)^{-1}\right)\right. \\ &\quad \left. - G^d \left(1 - \left(\frac{Y^{pot}}{Y}\right)^{oet-1}\right)\right), \end{aligned} \tag{4.9}$$

where $Y^{pot}$ is potential output, $T_i^d$ is the tax revenue of the 5 tax categories personal income taxes, social security contributions, corporate income taxes, indirect taxes, and the residual term *ctrr*. $oetc_i$ denotes the output elasticity of tax category $i$ derived in Subsection 4.3.1. $G^d$ is government spending excluding capital and interest spending and *oet* is the output elasticity of transfers. $X^N$ denotes the non tax revenues less interest on debt and net capital outlays—see Giorno et al. [56] and Van den Noord [125]. As before, $Y$ is the GDP.

Figure 4.8 shows the actual, the calculated structural, and the cyclical budget balance as ratio of the actual GDP and the potential output.[13]

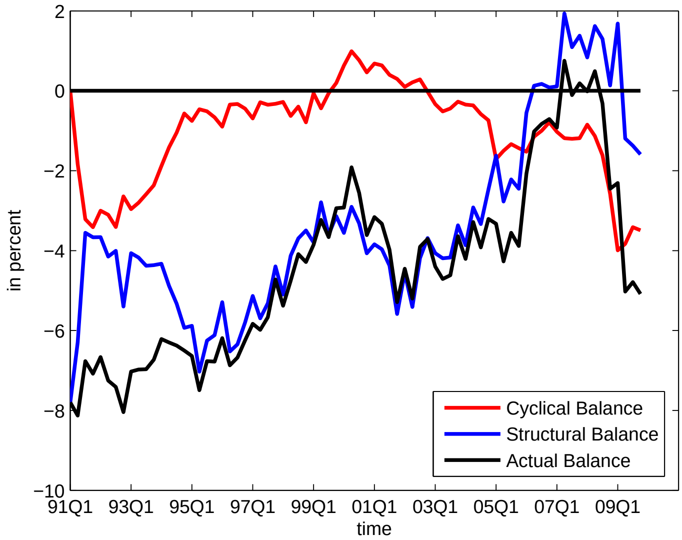


Figure 4.8: The Actual, Structural, and Cyclical Budget Balance

The cyclical balance fluctuates around the zero line. Starting with negative values for 1991-1999, it turns positive between 2000 and 2002 and becomes negative once again for the remaining plotted periods. There is a sharp decline in the cyclical balance from 2008 onwards due to the financial crisis.

The structural balance fluctuates around the actual balance. It lies above the actual balance for all periods with a negative cyclical balance. This holds per definition. After a decline in 1991-1995, the structural balance rises till 2001, falls again till 2003, and highly rises till 2008. The highest values are obtained between 2005-2008. In this period, the tax revenue rises due to a positive development of the German economic activities. In 2009, government expenditures sharply rise such that the structural balance sharply decreases and stays negative onwards. The dramatic rise in government spending and the sharp reduction of tax revenue at the end of 2008 has been already discussed with Figure 4.6.

The results for 1991-2002 are qualitatively comparable to those stated in the OECD

[13]There is an outlier in the data for the overall revenue of the government in 2003 quarter 3. We correct this value in the data series.

Economic Outlook No. 74 [103]. The OECD study examines the period 1980-2002 and offers only aggregated results for the large Euro-Area countries France, Germany, and Italy. Thus, the results cannot be quantitatively the same, e.g., because of the special role the reunification plays for Germany. But qualitatively, the cyclical balance in percent of potential output shows a comparable structure. The cyclical balance becomes negative in 1993 in the OECD study, whereas it is negative since 1991 in our study. The reunification causes this difference. Moreover, the cyclical balance is positive for 1999-2001 in both studies and negative again for the remaining year 2002. For most periods, the qualitative results are the same in both studies.

The highest value for the cyclical balance is 0.99% in our study, whereas it is about 1.2% for the OECD study. Thus, the results are not very different. One reason for discrepancies is the dependence of the results on the estimate for potential output, which has to be different.[14] The structural balance derived as the residual of the actual and the cyclical balance reaches the highest value in 1999/2000 if we compare the overlapping time horizon examined in both studies. We obtain a value of -2.905% of the potential output for the structural balance when the cyclical balance reaches its maximum, whereas the OECD estimates a value of about -1%.

Finally, we analyze the fiscal stance of Germany in 1991-2009. First, the development of the output gap is studied to assess the position in the business cycle. Then, we examine if the German government conducts a pro- or countercyclical fiscal policy. Defining cyclical downturns of the economy as the years in which changes in the output gap are non-positive relative to the previous year value, we identify 1992, 1998, 2001-2005 and 2007-2009 as cyclical downturns. These points in time are similar to the ones found in the OECD study [103]. To evaluate the fiscal stance of the German policy makers, we compare changes in the structural budget balance to changes in the output gap.[15] A fall in the structural budget balance means that government expenditures are raised more

[14] See the discussion above about the effect of different labor shares in the calculation of potential output.

[15] There is a variety of measures for the fiscal stance. For further information see the IMF Pamphlet Series No. 49 [70].

than tax revenues are lowered. Thus, a decrease or increase in the structural balance is equal to an expansionary or restrictive policy, respecively. Fiscal policy can be pro- or countercyclical. Countercyclicality is defined as a situation where fluctuations in the structural budget balance move in the same direction as the output gap, i.e., towards surplus in an upswing or towards deficit in a downturn. See also the OECD study [103].
In 1992, the output gap is smaller than in 1991. Furthermore, the structural budget balance falls. Thus, Germany acts countercyclically by following an expansionary fiscal policy. For 1993-1997, the output gap changes are positive; the structural budget balance falls till 1995 and rises afterwards. The fiscal policy responds pro-cyclically in 1993-1995 and countercyclically in 1995-1997: the upswing in 1993-1995 is accompanied by an expansionary policy, whereas it is followed by a restrictive policy for 1995-1997. Moreover, in 1998, the structural budget balance rises. As a consequence, the fiscal stance is pro-cyclical. For 1999-2000, the structural budget balance falls such that Germany follows a pro-cyclical policy by conducting an expansionary policy in an upswing. We identified a downturn in 2001-2005. In the first one to two years of this period a countercyclical policy takes place followed by a pro-cyclical policy for the last three to four years. In 2006-2008, fiscal policy responds more or less countercyclically to the business cycle: 2006 was found to be an economic upswing period, whereas 2007-2008 was identified as a downturn. Since the change in the structural balance is positive in 2006 and weakly negative for 2007-2008, an upswing is accompanied by a contractionary policy and the downturn in 2007-2008 by a weak expansionary policy. When the economic crisis becomes more and more severe in 2009, the structural budget balance strongly falls. The policy makers pursue an intense expansionary policy, see the huge economic stimulus packages in the end of 2008 (package I) and the beginning of 2009 (package II).
Overall, we find the German fiscal stance to be pro-cyclical for most parts of 1991-2000. This coincides with the results found for the three large Euro-Area countries in 1991-2002, see the OECD Economic Outlook No. 74 [103]. Frankel et al. [51] come to the same conclusion (for 1960-1999) using the correlation coefficient between the cyclical components of real government expenditure and real GDP as valuation basis. For 2001-

2009, fiscal policy is mainly countercyclical. Also in this case, Frankel et al. [51] find the same result. They examine the time horizon 2000-2009. During challenging economic situations (e.g., 2008-2009), the countercyclicality of fiscal policy in Germany is clearly detectable, also after the entry to the European Monetary Union.

Figure 4.9 summarizes this result:

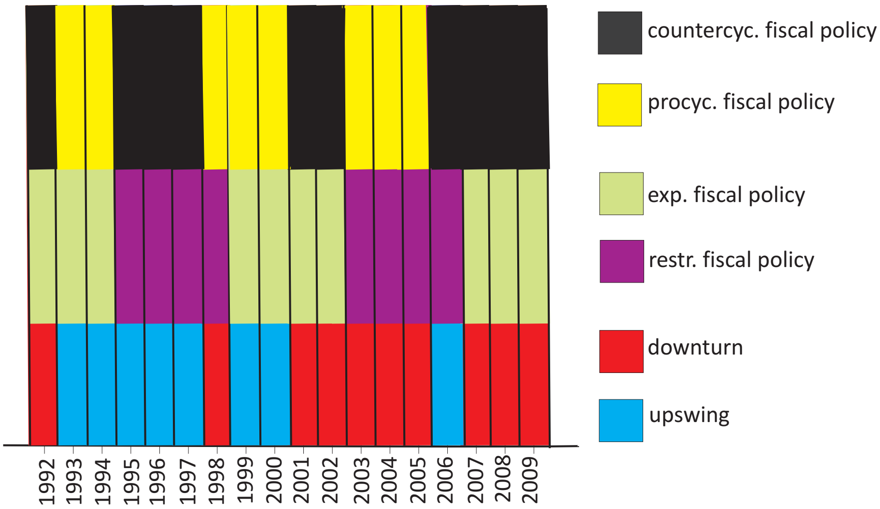


Figure 4.9: The Fiscal Stance in Germany for 1991-2009

Overall, the following is observed:

**Result 4.5.** Fiscal Policy in 1991-2009

The fiscal stance in 1991-2000 is mostly pro-cyclical. For 2001-2009, fiscal policy is conducted increasingly countercyclically. Moreover, during challenging economic situations as severe economic downturns, fiscal policy reacts strongly countercyclically for the whole analyzed period 1991-2009.

## 4.4 Conclusions

We have analyzed the fiscal policy situation in Germany between 1991-2009 in the underlying study. Potential output was calculated by applying the OECD production function approach to the business sector. Actual output lies below its potential level except for the periods 2000-2002. The study derived the elasticities of government expenditures and net taxes with respect to output and prices using quarterly data. The calculated output elasticity of net taxes is significantly larger than those found by former studies with a different time horizon. Thus, changes in output seem to have a stronger impact on net taxes for the last 20 years.

The output elasticities of different tax categories as personal income taxes, social security contributions, or indirect taxes do not differ strongly from elasticities derived with annual data for a comparable time horizon. Hence, the fluctuations caught in quarterly data do not seem to matter in the context of the reaction intensity. Finally, the fiscal stance of Germany has been analyzed and we found that fiscal policy in Germany is pro-cyclical for most parts of 1991-2000. This behavior changes in 2001-2009. The latter subperiod indicates mostly a countercyclical fiscal policy. The fiscal behavior coincides with Keynesian economics at least for severe economic downturns: during challenging economic situations as the financial crisis, the policy decision is found to be countercyclical for the whole analyzed time period 1991-2009.

# Chapter 5

# Fiscal Shocks in Germany - An SVAR Approach

## 5.1 Introduction

The question of how intensely fiscal policy affects an economy is more relevant than ever. In times of the financial crisis and large economic stimulus packages to support the business cycle, the first point of interest is the effectiveness of an increase in government expenditure. At the same time the ever lasting discussion of the impact due to a change in government expenditures or due to a change in government revenues gains more and more importance. In other words, which fiscal policy instrument is preferable?

With the very high debt in some European Monetary Union (EMU) member countries, an additional question emerges: are there spill-over effects of fiscal policy of one EMU member country to another? The underlying study gives answers to these questions. As a special issue for Germany, the impact of fiscal policy on exports as a main part of the GDP is examined in particular.

We analyze the effect of fiscal shocks on real variables in an economy. In this context, fiscal shocks are changes in government expenditure and revenues, i.e., fiscal policy actions. One main approach to study these effects is the analysis of special consolidation periods in an economy. The economic performance before, in, and after the special fiscal situation is compared (see, e.g., Alesina and Perotti [4] or Alesina and Ardagna [3]). Another approach examines the response of economic variables to fiscal shocks during a long period of time and not in the manner of a case study. The Vector Auto-Regression (VAR) or the Structural VAR (SVAR) are used as methodology.[1] The identification of independent fiscal shocks is the strongest challenge within the analysis. In general, fiscal shocks always depend on other variables. Thus, if fiscal variables and other real variables are estimated in a VAR framework, the resulting residuals are not independent of each other and have to be reestimated to identify independent fiscal shocks. There are different methods to identify these shocks. Fatás and Mihov [47] use a Choleski ordering of the variables in this context. Mountford and Uhlig [100] identify fiscal shocks by setting sign restrictions on the responses of economic variables to fiscal shocks. A different approach is that of Blanchard and Perotti [21] who assume that a frequency of quarterly data is too short for a response of the fiscal variables to changes in output. This assumption is crucial in the identification process of shocks because cross-dependencies are excluded. In detail, they calculate the cyclically adjusted residuals of a VAR estimation by using institutional information, i.e., output and price elasticities of the fiscal variables. Then, the authors regress the obtained cyclically adjusted residual of one fiscal variable on the VAR residual of the second fiscal variable to obtain independent residuals for both fiscal variables.

There are only few studies for Germany using an SVAR framework to evaluate the impact of fiscal policy on other economic variables. De Arcangelis and Lamartina [39] analyze West Germany. They compare the responses of economic variables to fiscal shocks under the assumption of different policy regimes, i.e., policy makers who first decide what to spend and then come to a tax decision and vice versa. An overidentified SVAR model is estimated in the study and the interest rate, the price level, the output level, and three fiscal variables, in particular, government expenditure for wages and transfers, other

[1] For a critical discussion of the VAR methodology, in particular, the inability of an SVAR model to account for the actual economic situation, see Parker [106].

government expenditure, and current revenue, are included as variables. They find that mostly government expenditure decisions are taken first but that expenditures do not seem to play a significant role for the dynamics of output because the estimates of the corresponding output responses are statistically insignificant with very low estimated coefficients. Overall, output and private consumption respond in a Keynesian way to expenditure and tax shocks but with low multipliers and if statistically significant then only weakly.

Höppner [64] examines fiscal shocks in a VAR framework. The endogenous variables of the model are government expenditure, government revenue, and GDP or government expenditures, revenues, private disposable income as well as private consumption. Höppner detects that the economic variables respond in a Keynesian way to fiscal shocks and finds that fiscal policy is not a useful tool for a stabilization policy due to the small and weakly statistically significant estimation results.

Heppke-Falk et al. [61] use real GDP, the inflation rate, the short-term interest rate, government expenditure, and government net revenue as variables for the SVAR model. They take quarterly data for 1974-2008 in Germany. The economic variables respond in a Keynesian way to fiscal shocks. Moreover, a government spending shock is found to affect output in a statistically significant way, whereas the estimates of the output response to a net tax shock are not statistically significant. Moreover, shocks in expenditure affect the economic variables only for a short time horizon after the occurrence of the shock. The authors find that the point estimate of output to a fiscal shock is low suggesting a weak effect of fiscal policy on output.

Breuer and Buettner [25] analyze fiscal shocks in an SVAR setting. They identify fiscal shocks by changes in tax laws for 1967-2008 and find statistically significant short-term effects of fiscal policy on economic variables. Finally, they forecast the response of output to the latest stimulus programme, which, based on their results, had a statistically significant short run effect on output.

The previous studies only examine economic variables within Germany. Trade or exchange rate variables are considered in the following field of research. Assuming some

policy coordination between the government and the central bank, Palacios-Salguero [105] examines the effect of monetary and fiscal shocks on the exchange rate for Germany in 1974-1998, i.e., before the European Monetary Union (EMU). He finds that an unexpected shock in government spending causes the German currency to appreciate.
Giuliodori and Beetsma [57] analyze the cross-border spill-over effects of fiscal policy shocks in an SVAR model framework for 1970-1998. They implement government expenditures and revenues, GDP, the inflation, the interest, and the exchange rate as well as imports from EMU member countries as endogenous variables to decide whether fiscal shocks of one country have international spill-overs or not. The imports connect the domestic country directly with foreign countries via the import-export channel: every imported good is exported by another country. The shocks are not identified with institutional information; all fiscal rule parameters are left unrestricted. Identification is based on a Choleski decomposition. In a second setting, the authors include the interest rate and the exchange rate as exogenous variables to simulate a situation with perfect EMU behavior. For the latter model framework, they find that a fiscal coordination in the EMU is of some help due to the obtained tendencies that there indeed exist spill-over effects of one country on the exports of another country.
We follow the SVAR approach. Since our aim is to analyze the effect of fiscal shocks on the economy without predictions for the direction of response after a shock, we do not use sign restrictions[2] but choose the identification method introduced by Blanchard and Perotti [21]. Quarterly data are used for all calculations of the study. All data are real and seasonally adjusted.[3] Methodologically, Structural VAR (SVAR) models are used. We analyze different model settings and start with the baseline model similar to that of Blanchard and Perotti [21] who only examine the US. Thus, taxes, government spending, and the GDP in logs and per capita are introduced as model variables. We augment the set of model variables in the course of the study. The first model variation

[2]The sign restriction approach solves the structural identification problem, but does not solve the model identification problem (there are many models with the same identified parameters fitting in the same way to the data). For more details, see Fry and Pagan [52].

[3]For more information on the data, their sources, and their time horizons see Appendix B.1.

additionally includes the inflation and the interest rate—see for example Perotti [108] or Heppke-Falk et al. [61]. In the second model variation, we add imports from European Monetary Union member countries and the real effective exchange rate to evaluate possible spill-over effects of the German fiscal policy. Within this setup, we examine two scenarios. First, all 7 variables are implemented as endogenous variables and second, we treat the exchange rate and the interest rate as exogenous variables. The last variation is based on the assumption that the EMU works perfectly. The interest rate and the exchange rate channel are switched off in that case. Implicitly, it is assumed that the European Central Bank sets one interest rate for the whole monetary union and that the monetary policy decisions are independent of the fiscal policy of single member countries. Moreover, there is only one common currency within the EMU such that the exchange rate is not affected by shocks in one national fiscal policy.

Overall, the effect of fiscal shocks on different model variables and the robustness of these effects are examined. Moreover, we want to decide, which economic theory matches best the empirical findings. Since for example consumption responds qualitatively different under New Classical or New Keynesian theory (positive to a positive government expenditure shock under consideration of New Keynesian theory and negative with New Classical theory), single components of the GDP as private consumption or investment are included and the respective impulse responses are analyzed. Special attention is paid to the response of exports due to fiscal shocks because exports play an important role for Germany and fiscal spill-over effects of national fiscal policy on other countries can be studied.

Our main findings are that the overall deficit rises due to a positive shock in government expenditure and declines due to a positive net tax shock. This fits to economic theory. Changing net taxes is a more effective instrument in affecting deficits in the short run, whereas varying government expenditure influences deficits more strongly in the medium run. A positive shock in government expenditure lowers output after a while. Moreover, a positive shock in the net tax revenue affects output negatively. Overall, the estimated multipliers are small such that fiscal policy does not seem to be a strong instrument in stabilizing output. Private consumption, investment, and exports diminish due to a rise in government expenditure. Furthermore, exports decline if net taxes are raised. Imports from EMU countries decrease due to the two fiscal shocks. Thus, we detect spill-overs of German fiscal policy to foreign countries. A fiscal coordination within the EMU may be helpful to stabilize the economic standing of the EMU.

The remainder of the chapter is organized as follows. In the next section, we shortly describe the VAR approach in general and the shock identification method used in the study. In the third to fifth section, we describe model setups with different sets of endogenous and exogenous variables. The estimation results are presented for each model setting separately. In the sixth section, the results, which are robust across all model variations are summarized and possible explanation approaches of the empirical findings are discussed. The last section draws conclusions.

## 5.2 The SVAR and its Identification Method

The German economic situation in 1991-2009 as a basis for the underlying study is of some interest to evaluate the quality of the VAR estimation. Thus, we look at the data. First, we perform the Augmented Dickey-Fuller (ADF) test to detect possible unit roots in the data. A selection of data plots can be found in Appendix C.2. All data plots except of the respective ones for the inflation rate and the real effective exchange rate clearly show an upward or downward trend in the data. Since the data plots indicate that there is a trend, we follow Heij, de Boer et al [60] and include additional to a constant a deterministic trend term in the test equation. If the trend term is omitted in the case of a trend in the data, the null hypothesis of a unit root, i.e., a stochastic trend, will not be rejected in nearly every scenario. In general, the null hypothesis of a unit root cannot be rejected if the test statistic exceeds the critical value.

For the net tax revenue and the government spending, we cannot reject the Null of a unit root in the data. Furthermore, a unit root is detected in the data series for the GDP, which corresponds to recent expectations. The data series of all components of

GDP, i.e., private and government consumption, investment, exports and imports, imports from member countries of the European Monetary Union, and the real effective exchange rate[4] contain a unit root.[5] Contrary to most other data series, the inflation rates stayed at a low level for the last 20 years—see Figure C.6 of Appendix C.2. The test statistic for the inflation rate leads to no rejection of the unit root hypothesis. Moreover, a unit root is found in the interest rate data series. To summarize, the unit root hypothesis is not rejected for all data series but the Null of a unit root is rejected for all first differences data series such that the data are I(1).

At least one cointegration relation is found by applying the Johansen trace and eigenvalue test for all model variations (combinations of model variables) examined in the underlying study. Following Sims et al. [120] and Lütkepohl and Reimers [92], the estimates of the (log) level VAR are consistent since the data series are nonstationary and cointegrated.

We now turn to the description of the general model setting. The reduced form Vector Auto-Regression model (VAR) can be written as

$$\vec{X}_t = A(L)\vec{X}_{t-1} + \vec{U}_t. \tag{5.1}$$

$\vec{X}_t$ denotes the vector including all endogenous variables as net tax revenue, government spending, and GDP in the baseline model. $A(L)$ is a polynomial in the lag operator $L$ and $\vec{U}_t$ is the vector of reduced form residuals. Additionally, we include several terms as a constant or a trend.

We estimate all model specifications with a deterministic, a log quadratic, and a stochastic trend. Due to the very similar results of a log quadratic trend to a linear trend, we suppress the results for this model variation. We follow Bhaskara [18] concerning the implementation of a stochastic trend. Seasonal dummies and, for the seven variables model, an exogenous variables component are added to the set of regression variables.

[4]Because the real effective exchange rate does not show a unique trend—see Figure C.9 in Appendix C.2—we cross-check our results by including a deterministic trend or not in the ADF-test. Both test results lead to no rejection of the unit root hypothesis.

[5]For robustness of results, we add regressors up to 4 lags. The null hypothesis of a unit root is not rejected for all scenarios.

These terms are omitted for simplicity in equation (5.1).

To determine the optimal lag length of the VAR model, we apply a likelihood-ratio test (LR), the Akaike (AIC), and the Schwarz information criterion (SIC). The maximum lag order is set to 10. The optimal lag order for all model variations found by the three methods lies between 1 and 6. For comparability, we choose the same lag order for all model specifications. Since a lag order of 4 seems to be a natural choice when using quarterly data, it is set to 4 for all model variations.

So far, we finish the pre-estimation tests and estimate the respective VAR model. After conducting a VAR-estimation, the obtained residuals for the two fiscal policy parameters are linear combinations of different influences: an automatic, a systematic, and a random response to changes in the other variables of the model—see also Blanchard and Perotti [21].

The random components of the fiscal variable residuals (government expenditure and net taxes) are not correlated with other structural shocks. Thus, they can be used to estimate the model variables' impulse responses to innovations in the fiscal variables. How the identification procedure of the random components continues is explained in detail for each model specification in the three following sections. In the following, every variable $x$ that is given as log per capita variable is labeled with $\ddot{x}$.[6] Besides the identification, the theoretical foundations of the estimation technique and the estimated results are described for each model. Furthermore, different model setups are compared. The last section checks for robustness in a general manner, discusses, and tries to explain the results, which are robust for all model specifications.

## 5.3 The Baseline Model

Following Blanchard and Perotti [21], the log government spending per capita, $\ddot{g}$, approximated by the sum of government consumption and the gross fixed capital formation in the government sector, the log tax revenue per capita, $\ddot{t}$, and the log GDP per capita,

[6] Small letters denote a variable in logarithms.

$\ddot{y}$, are chosen as endogenous variables of the model. Thus, $\vec{X}_t$, the vector of endogenous variables is equal to $\vec{X}_t = [\ddot{g}_t \;\; \ddot{t}_t \;\; \ddot{y}_t]'$.

The corresponding vector of residuals after the VAR-estimation is $\vec{U}_t = [u_t^g \;\; u_t^t \;\; u_t^y]'$. We rank government spending first, net taxes second, and output last. The underlying fiscal rules determining the cross-effects of one variable to the other variables are described in the following.[7]

Each residual of the three variables can be described as the linear combination of the other residuals and structural shocks for selected variables. Thus, w.l.o.g., the following equations hold:

$$\begin{aligned} u^g &= \alpha_{gy} u^y + \beta_{gt} e^t + e^g, \\ u^t &= \alpha_{ty} u^y + \beta_{tg} e^g + e^t, \quad \text{and} \\ u^y &= \alpha_{yt} u^t + \alpha_{yg} u^g + e^y, \end{aligned}$$

where $\alpha_{gy} = 0$ denotes the output elasticity of government expenditures and $\alpha_{ty} = 2.0857$ is the output elasticity of net taxes calculated in Chapter 4, Section 4.3.1. The output elasticities are derived independently of the model specification and reflect institutional information.

$\alpha_{yt}$ and $\alpha_{yg}$ clarify the influence of net taxes and government spending on output, respectively. $u^j$ is the residual for a variable $j \in \{g, t, y\}$ found in the VAR estimation and $e^j$ is the corresponding structural shock. The structural shocks are uncorrelated: $cov(e^i, e^j) = 0$ for $i \neq j$ and $i, j \in \{g, t, y\}$.

To identify the structural shocks of the model, we calculate the cyclically adjusted residual of government spending and the net tax revenue residual, which are linear combinations of the uncorrelated structural fiscal shocks:

$$\begin{aligned} u^{g,CA} &= u^g - \alpha_{gy} u^y = \beta_{gt} e^t + e^g, \\ u^{t,CA} &= u^t - \alpha_{ty} u^y = \beta_{tg} e^g + e^t. \end{aligned}$$

Following Perotti [108] and Blanchard and Perotti [21], we assume w.l.o.g. that $\beta_{tg} = 0$ holds. For robustness of results, we cross-check our results by assuming that $\beta_{gt} = 0$. The

[7] For a variation of the ordering of variables and its implications, see also Caldara and Kamps [31].

results were not far from the ones obtained with $\beta_{tg} = 0$. Thus, the chosen identification method does not seem to significantly matter. In particular, with $\beta_{tg} = 0$, it holds that $u^{t,CA} = e^t$ and $u^{g,CA} = \beta_{gt}e^t + e^g$. The structural shock $e^t$ for net taxes is equal to the cyclically adjusted residual $u^{t,CA} = u^t - \alpha_{ty}u^y$ calculated with the residuals obtained from the VAR estimation and the output elasticity of net taxes $\alpha_{ty}$. An OLS-regression of $e^t$ on $u^{g,CA}$ provides estimates for the coefficient $\beta_{gt}$ and the structural shock $e^g$ of government expenditure because $u^{g,CA} = \beta_{gt}e^t + e^g$. Hence, the two structural fiscal shocks are identified with this procedure.

Finally, the coefficients $\alpha_{yg}$ and $\alpha_{yt}$ as well as the structural shock $e^y$ in the original equation $u^y = \alpha_{yt}u^t + \alpha_{yg}u^g + e^y$ have to be identified. Since the two structural shocks of government expenditure and net taxes, $e^g$ and $e^t$, are independent, they can be used as instruments in an instrumental variable (IV) estimation for the correlated residuals $u^g$ and $u^t$ obtained from the VAR. We find estimates for $\alpha_{yt}$, $\alpha_{yg}$, and $e^y$ after performing the IV estimation.

The overall relationship between the residuals $u^j$ and the structural shocks $e^j$ for $j \in \{g, t, y\}$ can be written in matrix form as:

$$\begin{array}{cccc} \vec{C}_1 & \vec{U} & = & \vec{C}_2 \quad \vec{E} \\ \begin{pmatrix} 1 & 0 & -\alpha_{gy} \\ 0 & 1 & -\alpha_{ty} \\ -\alpha_{yg} & -\alpha_{yt} & 1 \end{pmatrix} & \begin{pmatrix} u^g \\ u^t \\ u^y \end{pmatrix} & = & \begin{pmatrix} 1 & \beta_{gt} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^g \\ e^t \\ e^y \end{pmatrix} \end{array}, \tag{5.2}$$

where $\vec{U} = [u^g, u^t, u^y]'$ and $\vec{E} = [e^g, e^t, e^y]'$. Thus, the underlying model is a so called AB-model described in Amisano and Giannini [5]. For further information see also Breitung, Brüggemann, and Lütkepohl [24].

After identifying the structural shocks of the model variables, we analyze the impulse responses of log government expenditure per capita, log net taxes per capita, and log output per capita to one unit positive shocks in the government expenditure and in the net tax revenue, respectively. For robust impulse responses, we perform a Monte Carlo simulation taking 500 draws from the distribution of the reduced form residuals. Residuals equally distributed as the original $u^i$ residuals for $i \in \{g, t, y\}$ are randomly chosen to create new time series. Next step is to carry out an impulse response analysis for the new data series. Finally, we save the impulse responses for all 500 replications and choose the 10% error band.[8]

The following subsections present the results for the baseline model setting including government expenditure, net taxes, and output as variables. Sensitivity is tested by varying the model setup, e.g., by implementing a deterministic and a stochastic trend (Section 5.3.1), or by splitting the period for 1991-2009 into two subsamples 1991-1998 and 1999-2009. The establishment of the European Monetary Union (EMU) plays a role in the 7 variables VAR model in Section 5.5. For better comparability of results, we split the sample into the two time periods before and after the integration of Germany into the EMU. The first sample only contains 32 data points such that we abstain from a discussion of the first subsample due to the small overall sample size and the associated problems.

We add components of the GDP (private and government consumption, investment, exports, and imports) to our model setting (Subsection 5.3.2) to discuss, which economic theory matches best the empirical results and what effects arise on the trade variables imports and especially exports in the occurrence of fiscal shocks.

### 5.3.1 Different Model Specifications

A Structural VAR including three endogenous variables, $\ddot{g}$, $\ddot{t}$, and $\ddot{y}$ is estimated, differing between two settings. First, we include a deterministic trend (DT) and second a stochastic trend (ST) in the set of model regressors. Moreover, a constant is added for both scenarios. We implement seasonal dummies for all model variations—see Perotti [108] and Blanchard and Perotti [21]. Since the implementation of a fourth seasonal dummy would make no sense because of the linear dependence of the seasonal dummies and the constant, we only add seasonal dummies for the first three quarters of a year—see also LeSage [84].

As described above, we find the coefficients $\beta_{gt}$, $\alpha_{yg}$, and $\alpha_{yt}$ via OLS and IV estimation

[8] For more information on the difficulties concerning error bands, see Sims and Zao [121].

after the VAR estimation. The results of a Portmanteau test on autocorrelation suggest that there is no autocorrelation in the VAR estimation.[9] Moreover, we do not find any autocorrelation in the OLS and the IV estimation.

The point estimates are shown in Table 5.1. The number in brackets is the value of the t-statistic.

Since $\beta_{gt}$ shows the relationship of the structural shock in net taxes to the cyclically adjusted residual of government spending, a positive change in net taxes lowers the cyclically adjusted government spending. The estimate is statistically insignificant and very small in size. $\alpha_{yg}$ and $\alpha_{yt}$ reflect the impact of government spending and net taxes on output, respectively. Note that $\alpha_{yg}$ and $\alpha_{yt}$ only show the effect of the correlated residuals of government spending and net taxes on output and not the effect of the respective structural shocks.

| **Variable** | **DT** | | **ST** | |
|---|---|---|---|---|
| $\beta_{gt}$ | -0.083 | (-1.5829) | -0.077 | (-1.477) |
| $\alpha_{yg}$ | 0.171** | (2.329) | 0.17** | (2.215) |
| $\alpha_{yt}$ | -0.201** | (-4.221) | -0.221** | (-4.349) |

* and ** denote statistical significance levels of 10% and 5%, respectively.

Table 5.1: Results for the Baseline Model

A government spending shock affects output positively, whereas a change in net taxes affects output negatively. The results for the regressors[10] do not vary strongly for the model variations including a stochastic and a deterministic trend.[11]

We perform a Granger-causality test between government expenditure, net taxes, and output. In general, a variable $x$ is Granger-causal for a variable $z$ if $z$ can be predicted more efficiently when adding the information on the $x$ process to the information set than without any information on $x$. For more information, see Lütkepohl [91]. Output does not Granger-cause government spending, but taxes have a Granger-causal impact

[9] For more information on the Portmanteau test see, e.g., Lütkepohl [91].

[10] We round all results off to three decimal places.

[11] In the VAR estimation, the stochastic and the deterministic time trend is statistically insignificant.

on government spending. Moreover, we cannot reject the Null of a Granger-causal relationship of government spending and output on net taxes for a 5% statistical significance level. Finally, no Granger-causality is found between net taxes and government expenditure on output.

Next, we analyze the impulse responses to fiscal shocks. After a Monte Carlo simulation with 500 draws, the following point estimates (the median values) for the responses to fiscal shocks in time period 0 (the ad hoc response) and the overall effect in period 1 are obtained. In general, the overall effect for period $t$ can be found by summing up the ad hoc response and the period response for all periods up to $t$.

| **Variable** | **G - 0** | **G - 1** | **T - 0** | **T - 1** | **T1 - 0** | **T1 - 1** |
|---|---|---|---|---|---|---|
| $ir^g$ | 1* | 1.429* | -0.084* | -0.135 | -0.122* | -0.198 |
| $ir^t$ | 0.253* | 0.531 | 0.685* | 0.926* | 1* | 1.351* |
| $ir^{def}$ | 0.747* | 0.898* | -0.769* | -1.062* | -1.122* | -1.55* |
| $ir^{t,CA}$ | 0* | 0.24* | 1* | 1.604* | 1.459* | 2.343* |
| $ir^y$ | 0.122* | 0.135 | -0.151* | -0.325* | -0.22* | -0.475* |
| $m$ | 0.122* | 0.095 | -0.229* | -0.351* | -0.22* | -0.352* |
| $m^{CA}$ | 0.122* | 0.095 | -0.151* | -0.203* | -0.151* | -0.203* |

* denotes statistical significance at 10% level.

Table 5.2: Influence of a Spending and a Tax Shock - Baseline Model

The left third of Table 5.2 shows the estimated effects of the model variables due to a positive one unit spending shock. The remaining part consists of the corresponding estimated effects due to a positive tax shock. The tax shock does not have 1 unit strength on taxes if a one unit change in net taxes is implemented because output immediately decreases and automatically lowers the influence of the positive tax shock on taxes. The Granger-causality test already suggested that there is an impact of output on net taxes. The columns marked with “T” are the results for a tax shock with the intensity that arises endogenously from the model. The columns marked with “T1” denote the results for a tax shock normalized to one unit intensity.

The impulse response coefficients of government expenditure, net taxes, and output are denoted as $ir^g$, $ir^t$, and $ir^y$, respectively. The deficit response $ir^{def}$ is defined as the difference of the impulse response of government spending and of net taxes to the particular fiscal shock ($ir^{def} = ir^g - ir^t$). The impulse response of the cyclically adjusted government spending $ir^{g,CA}$ is equal to the response of government spending, because we assume in Chapter 4, Section 4.3.1, that the output elasticity of government expenditure $\alpha_{gy}$ is zero and thus, $ir^{g,CA} = ir^g - \alpha_{gy} ir^y = ir^g$. As a consequence, it is suppressed in Table 5.2. We define the response of cyclically adjusted taxes $ir^{t,CA}$ as $ir^{t,CA} = ir^t - \alpha_{ty} ir^y$. $\alpha_{ty}$ is the output elasticity of net taxes—see Section 4.3.1 of Chapter 4. The last two rows of the table show the quarterly multipliers and the quarterly cyclically adjusted multipliers. In this context, the government spending and the tax shock multiplier are defined as the ratio of the impulse response of output to the impulse response of government spending $ir^y/ir^g$ and taxes $ir^y/ir^t$, respectively. Similarly, the cyclically adjusted multiplier is the ratio of the impulse response of output to the cyclically adjusted impulse response of government spending $ir^y/ir^{g,CA} = ir^y/ir^g$ and net taxes $ir^y/ir^{t,CA}$.

The statistically significant estimates are labeled with an asterisk. Statistical significance at a 10% statistical significance level is fulfilled if the zero line does not lie in the interval of the 10% and 90% error band of the 500 Monte Carlo simulations in a graph including the impulse response curves. Since the estimators in the two model variations with a deterministic or a stochastic time trend are quantitatively and qualitatively closely the same, we only list the results for the deterministic trend variation in detail.

Most of the estimates shown in Table 5.2 are statistically significant at a 10% statistical significance level. Per definition, the government spending shock affects government spending with 1 unit strength. Due to cross-effects, government expenditure rises even more after one period. Since a rise in government expenditure corresponds to expansionary fiscal policy, output increases and thus, also taxes increase. Both effects are strengthened after one period (the impulse responses are even larger one period after the occurrence of the shock in government expenditure). The rise in revenue due to a rise in output cannot compensate for the increased expenditure of the fiscal authority. As a consequence, the overall deficit reacts positively to a government spending shock. Debt is raised by the expansionary fiscal policy. The effect becomes even stronger one period after the change in expenditure. In the first period of simulation, the cyclically adjusted tax response to a government spending shock is 0,[12] whereas it becomes positive for the second period. A rise in government spending by one unit implies a rise in output by 0.122 units when the shock occurs. After one period, the multiplier decreases to 0.095.[13] Since the cyclically adjusted government spending response is equal to the original government spending impulse response, the cyclically adjusted expenditure multiplier is equal to the originally calculated multiplier: the last two rows of Table 5.2 are the same for a government spending shock.

If we consider a positive shock in net taxes, output decreases due to the contractionary fiscal policy. Since the effect on taxes strengthens one period after the shock, output decreases even more. Moreover, government spending decreases (with increasing intensity for the first two simulated periods). As a consequence, the overall deficit reacts negatively to a net tax shock. The rise in net taxes lowers the deficit and the decrease in government expenditure strengthens this effect additionally. Thus, debt is reduced by the contractionary policy. In the first simulation period, the cyclically adjusted tax response is 1 for a tax shock if we do not normalize the shock (columns marked with "T"). The interdependency with output is switched off by the cyclical adjustment. After one period, cyclically adjusted net taxes increase even more. The multipliers for the last four corresponding columns are the same as we only consider two different intensities of tax shocks. A rise in taxes by one unit lowers output by 0.229 units. The multiplier becomes larger for the second simulated period because all responses are stronger at this point in time. The cyclically adjusted multipliers show a smaller impact of the shock on output as the regularly calculated multiplier. This is caused by the fact that the cyclical

[12] This follows from the model specification, i.e., the ordering of variables.

[13] The quarterly multipliers only have a small size compared to Perotti [108], but we examine a different time horizon and use a different model setting by omitting the interest and the inflation rate in our baseline model.

adjustment already switches off cross-effects.

Figure 5.1 shows the impulse responses after a fiscal shock if a deterministic trend (DT) is implemented as regressor in the VAR model. Contrary, Figure 5.2 shows the respective impulse responses if a stochastic trend (ST) is added as regressor. The upper row of graphs consists of the responses of government expenditure, net taxes, and output to a positive one unit shock in government spending, whereas the lower row shows the estimated effect to a one unit net tax shock. The dash-dotted lines are the 10% and 90% statistical significance error bands. The solid blue line is the median value of the 500 draws obtained from the Monte Carlo simulation (the point estimate). If the zero line lies between the error bands, the point estimate of the impulse response is statistically insignificant.

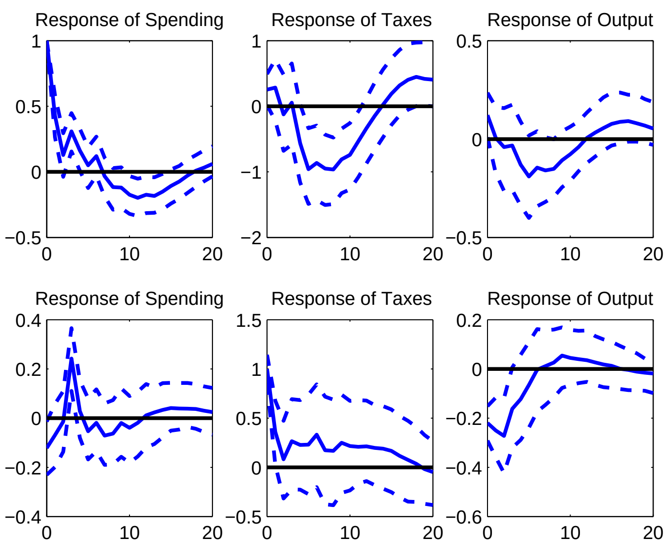


Figure 5.1: Impulse Responses to Fiscal Shocks (DT) - Baseline Model

As already indicated in Table 5.2, all responses of the model variables to a fiscal shock in period 0 except the point estimate of government spending to a net tax shock are statistically significant. The impulse responses to a government spending shock are very similar for both model variations. If a tax shock hits the system, taxes decline after a while due to the sharp decrease in output for the stochastic trend model. Contrary, output only responds smoothly to a positive tax shock in the deterministic trend model.

For robustness, we also cross-check the model with linear deterministic trend (DT) by implementing a log quadratic deterministic trend. The results are qualitatively, more or less quantitatively, and concerning the statistical significance closely the same.

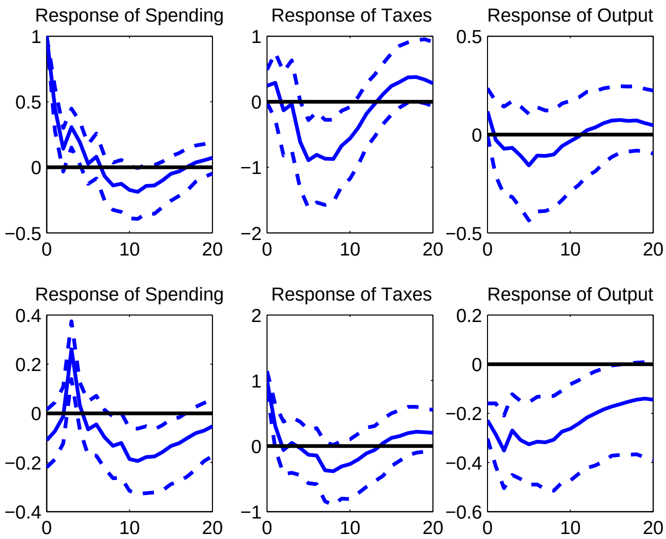


Figure 5.2: Impulse Responses to Fiscal Shocks (ST) - Baseline Model

Similar to Blanchard and Perotti [21], the positive ad hoc response of output to a government expenditure shock is followed by a sharp decline and a repeated increase after a while. Since the estimated ad hoc multiplier is larger in the Blanchard/Perotti study, our impulse response curve is shifted downwards such that the minimum of the curve lies in the negative quadrant. The negative estimates are not statistically significant. If taxes are raised, output responds statistically significantly negatively. The estimated multiplier is higher than for a comparable shock in government expenditure. For both model variations, the impulse response of government spending to a spending shock, the response of taxes to a tax shock and the response of output to a tax shock stay statistically significant for at least 3 periods after the occurrence of the shock.

The main difference of the two model specifications is the estimated effect of taxes to a positive net tax shock. For a stochastic trend scenario, taxes undershoot the zero line after a one unit tax shock. The responses to the tax shock are estimated with higher absolute values for all three variables including a stochastic trend. Moreover, output responds with a long lasting decline in the stochastic trend model. In contrast, output reacts similarly to a government spending shock for both model variations.

We cannot state long run implications due to the existence of unit roots when performing an SVAR in levels because the long run estimates tend to random variables and not to the true impulse response. See, e.g., Phillips [109]. As a consequence, we focus on the short- to medium-lived responses up to 12 periods after the occurrence of the shocks.

The cumulative impulse response of government expenditure, net taxes, and output are calculated by adding up the period impulse responses for the model variables and then choosing the median value. The sum of the impulse responses gives the entire impact of a shock on the respective variable because the VAR model is estimated in levels.

From a mathematical point of view, an accumulated impulse response is equal to the integral of the point estimate curve. Note that the accumulated median values of the point estimates do not have to be equal to the median value of the accumulated point estimates, i.e., the integral curve. To annualize, we divide the sum of the quarterly responses by 4—see Perotti [108]. The annualized cumulative responses of the model variables to fiscal shocks are listed in the following two tables.

Table 5.3 shows the results for a government spending shock if a deterministic (left half of the table) or a stochastic shock (right half of the table) is implemented. Moreover, Table 5.4 includes the respective results for a positive one unit net tax shock. The estimates for the 4th and the 8th time period (quarter) after the shock are shown.[14] The responses of government spending ($cir^g$), taxes ($cir^t$), output ($cir^y$) and cyclically adjusted taxes ($cir^{t,CA}$) are listed. Furthermore, we examine the cumulative response of the overall deficit ($cir^{def}$) defined as the difference between the cumulative response of spending and taxes to the corresponding fiscal shock.

The last row of Table 5.3 and Table 5.4 denotes the cyclically adjusted spending and the cyclically adjusted net tax multiplier, respectively. Similar to above, the multipliers are defined as the ratio of the cumulative response of GDP to the cyclically adjusted response of spending $cir^y/cir^{g,CA}$ or net taxes $cir^y/cir^{t,CA}$ for the same period.

[14] The shock takes place in period 0.

We compare the deterministic trend model (DT) with the stochastic trend model (ST) in Table 5.3 if a government spending shock occurs.

| **Variable** | **DT-4** | **DT-8** | **ST-4** | **ST-8** |
|---|---|---|---|---|
| $cir^g$ | 0.499* | 0.504* | 0.517* | 0.495* |
| $cir^t$ | -0.02 | -0.979 | -0.065 | -0.904 |
| $cir^{def}$ | 0.519* | 1.483* | 0.581* | 1.398* |
| $cir^{t,CA}$ | 0.001 | -0.62 | 0.007 | -0.574* |
| $cir^y$ | -0.01 | -0.172 | -0.034 | -0.158 |
| $sm^{CA}$ | -0.02 | -0.341 | -0.066 | -0.319 |

* denotes statistical significance at 10% level.

Table 5.3: Cumulative Responses to a Spending Shock for the Baseline Model

Since the positive response of output to a positive shock in government expenditure is weak and only short lasting before becoming negative, the accumulated response of output to a government spending shock is negative four and eight periods after the shock. The cumulative response is statistically insignificant at the 10% statistical significance level. Due to the negative output responses, tax revenue declines and the spending multipliers are negative, too.

After some periods, the expansion of spending causes a reduction in tax revenue. Hence, the overall deficit rises over time. The annualized multipliers suggest that a positive one unit spending shock lowers output by 0.02 units (DT) or 0.066 units (ST) after one year and by 0.341 (DT) or 0.319 (ST) units after 8 periods. The estimated decline in output after 8 quarters is smaller for the stochastic trend model. The results are not statistically significant.

Table 5.4 shows the estimates due to a one unit tax shock. All statistically significant annualized cumulative impulse responses, marked by an asterisk, have the same sign in the model with deterministic and with stochastic trend.

Deficits decline due to an increase in net taxes. The estimates are statistically significant at a 10% statistical significance level and have a greater absolute value for the deterministic trend model.

Output declines due to a net tax shock. The estimated negative response 4 and 8 periods after the positive tax shock is statistically significant for both model variations. Moreover, the tax multipliers are estimated to be statistically significantly negative with relatively small values, e.g., compared to the results found by Bode et al. [22], especially for a stochastic trend.

| **Variable** | **DT-4** | **DT-8** | **ST-4** | **ST-8** |
|---|---|---|---|---|
| $cir^g$ | 0.015 | -0.034 | 0.029 | -0.065 |
| $cir^t$ | 0.488* | 0.703* | 0.317 | 0.069 |
| $cir^{def}$ | -0.473* | -0.737* | -0.289 | -0.134 |
| $cir^{t,CA}$ | 1.026* | 1.26* | 1.077* | 1.49* |
| $cir^y$ | -0.258* | -0.267 | -0.361* | -0.681* |
| $tm^{CA}$ | -0.251* | -0.212 | -0.335* | -0.457* |

* denotes statistical significance at 10% level.

Table 5.4: Cumulative Responses to a Tax Shock for the Baseline Model

Overall, the deficit is affected positively by a positive government spending shock and negatively by a positive tax shock. Even after 8 periods (2 years), there is a statistically significant effect in the deterministic trend model. A rise in deficits is more intense after some years for a government expenditure shock.

In both model settings, output reacts as expected in the first periods after the occurrence of the respective shocks: it increases due to a positive shock in government expenditure. However, it becomes negative after a while. If a positive shock (an increase) in taxes occurs, output decreases. The tax multiplier is statistically significant and larger in size than the expenditure multiplier for both model variations.

Figure 5.3 and 5.4 show the accumulated impulse responses to a government spending and a net tax shock for the deterministic and the stochastic trend model. A positive slope in the figures means a positive value in the single variable accumulated response.

This does not mean that the variable responds still more and more positively in the respective period. The intensity of the estimated effect can diminish, i.e., the respective period impulse response can have a negative slope lying in the positive quadrant at that time.

As before, the first and the second row of each figure show the responses to a positive government expenditure shock and the responses to a positive net tax shock, respectively. Moreover, the dash-dotted lines are the 10% and the 90% error bands of the impulse responses found with a Monte Carlo simulation including 500 draws, whereas the solid lines denote the accumulated point estimates defined as the median values of the accumulated period impulse responses for 500 draws.

Most accumulated impulse responses are statistically significant at time 0. Only the response of spending to a spending shock, the response of taxes to a tax shock, and the response of GDP to a tax shock stay statistically significant in the deterministic trend case up to 12, 3, and 5 quarters, respectively.

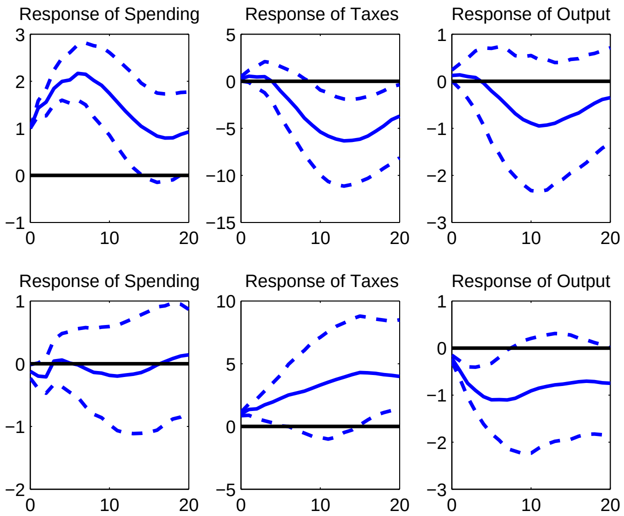


Figure 5.3: Accumulated Impulse Responses (DT) - Baseline Model

The overall effect of a government spending shock on output for the deterministic trend model is first positive, then becomes negative, but the estimate is not statistically significant. The estimated overall output response after a net tax shock is statistically significantly negative and lasting.

Figure 5.4 shows the impulse responses to fiscal shocks if a stochastic trend is included as regressor in the VAR model estimation. The responses to a net tax shock differ qualitatively for the two model variations. If a stochastic trend is implemented, the accumulated response of taxes become negative after a while, whereas they are positive for the whole simulated period with deterministic trend. Moreover, the estimated overall impact of a tax shock on output is stronger for the stochastic trend model and becomes more and more intense over time.

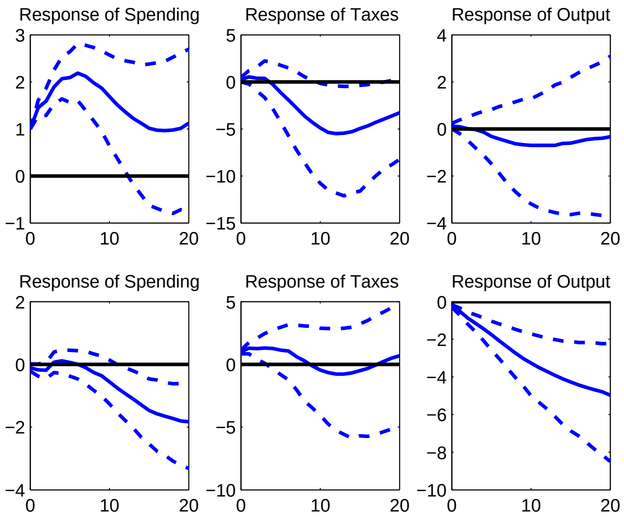


Figure 5.4: Accumulated Impulse Responses (ST) - Baseline Model

We summarize our results in the following.

**Result 5.1.** Expenditure vs. Net Tax Shock (Baseline Model)

- Output weakly rises due to a positive shock in government spending but declines some periods after the shock and becomes negative. The negative response is not statistically significant.
- Output declines statistically significantly due to a positive tax shock.
- All multipliers are estimated with small values.
- A shock in net taxes has an estimated higher impact on output as a comparable shock in government spending and seems to be a more effective instrument in stabilizing output in the short run.
- The overall deficit rises with an expansionary spending shock and declines if taxes are raised. A rise in taxes lowers deficits more in the first periods after the shock, but a reduction in government spending is more effective in the medium run.

For comparison, we split the whole time period for 1991-2009 into two parts; one part before (1991-1998) and one part after the foundation of the European Monetary Union (1999-2009). Due to small sample size problems and thus a lack of statistically significant results, we only compare the results of the second subsample with the results of the overall sample presented above. The institutional information needed (the output elasticities) are derived only for the subsample. The elasticity of net taxes with respect to output is 1.0094 and thus smaller than for the whole period. The impact of a change in output on net taxes seems to weaken after the German accession to the EMU.
Table 5.5 shows the estimated impulse responses at time zero. The first 2 columns are the estimation results for a government spending shock, whereas the remaining columns include the estimates for a net tax shock (the last tow columns assume a one unit tax shock).[15]

| **Variable** | **G - DT** | **G - ST** | **T - DT** | **T - ST** | **T1 - DT** | **T1 - ST** |
|---|---|---|---|---|---|---|
| $ir^g$ | 1* | 1* | -0.082 | -0.072 | -0.089 | -0.08 |
| $ir^t$ | -0.01 | -0.034 | 0.916* | 0.9* | 1* | 1* |
| $ir^{def}$ | 1.01* | 1.034* | -0.998* | -0.972* | -1.089* | -1.08* |
| $ir^{t,CA}$ | 0* | 0* | 1* | 1* | 1.092* | 1.111* |
| $ir^y$ | -0.01 | -0.034 | -0.083 | -0.099* | -0.091 | -0.11* |
| $m$ | -0.01 | -0.034 | -0.091 | -0.11* | -0.091 | -0.11* |
| $m^{CA}$ | -0.01 | -0.034 | -0.083 | -0.099* | -0.083 | -0.099* |

* denotes statistical significance at 10% level.

Table 5.5: Influence of a Spending and a Tax Shock for 1999-2009

The intensity of a government spending shock on government expenditure is per definition still 1. The impact of a tax shock on taxes is larger in the overall period for both model types. Since we cannot directly compare the effect of a tax shock on output because the magnitude of the tax shocks (the results displayed in the 3rd and fourth

[15] We only discuss the statistically significant results marked with an asterisk.

column of the table) are different for the two scenarios, we examine the last two columns of the respective tables where comparability is given.

In the subsample, output immediately responds negatively with a very small intensity to a government spending shock. The response is not statistically significant. Moreover, taxes diminish. Expenditure rises and revenues decline such that the overall deficit increases more by the government spending shock for 1999-2009. The impact factor of a government spending shock on deficits exceeds 1. Debt rises.

All multipliers are negative due to the decline in output, very small in size, and statistically insignificant.

If a shock in net taxes occurs, government spending declines weakly. Because the rise in taxes raise government revenues, the deficit is reduced. Compared to the whole sample 1991-2009, the reduction in the deficit is weaker due to a net tax shock for the subsample 1999-2009. Output declines due to the contractionary shock. The estimated response is statistically insignificant for the deterministic trend model and statistically significant for the stochastic trend model. The multipliers for a tax shock are negative. A one unit rise in taxes lowers output by 0.09-0.11 units, depending on the chosen model specification. Because all multipliers are smaller in absolute value compared to the whole sample results, a tax shock seems to have less impact on output after the accession to the EMU. Qualitatively, a one unit negative government expenditure shock seems to have closely the same impact on deficits as a one unit positive tax shock. For the overall period, a tax shock is found to be more effective. Thus, in the 2000s, the ability of a net tax shock to reduce overall deficits shrinks, whereas a government spending shock has a higher estimated impact factor on deficits as before.

**Result 5.2.** The Baseline Model - 1999-2009

- The impact of a rise in taxes on output weakens after the accession to the EMU.
- A reduction in government spending or a rise in taxes are equally effective in reducing the deficit for 1999-2009 in the short run.

The three variables model is facing statistical insignificance problems due to the small sample size for the subsamples. As a consequence, most of the impulse response estimates and the multipliers are statistically insignificant 4 or 8 periods after the shock. We refrain from discussing the results explicitly.

Overall, it can be noted that the response of output to a spending and to a tax shock is smaller than expected. One possible explanation is that fiscal policy is not that effective any longer in the 1990s and 2000s or the model is facing problems due to omitted variables. Hence, we augment the baseline model with several variables such that we can check for the robustness of results in the course of the study.

### 5.3.2 Components of GDP

Before augmenting the set of endogenous variables, several components of GDP as private and government consumption, investment, imports, and exports are added to the baseline model. On one hand, we try to answer the question, which economic theory (New Classical vs. New Keynesian) matches best the empirical results in the underlying model specification. On the other hand, the estimated impact of fiscal shocks on the trade variables imports and exports is analyzed.

Each additional component is ranked last in the vector $\vec{X}$ of endogenous variables. In particular, we estimate a four variables SVAR model. The residual of the GDP component is described in a similar manner as the residual of GDP: $u^{yc_i} = \alpha_{yc_i g} u^g + \alpha_{yc_i t} u^t + e^{yc_i}$, where $yc_i$ denotes one of the five GDP components listed above with $i \in \{1, ..., 5\}$. $yc_1$ is private consumption, $yc_2$ government consumption, $yc_3$ denotes investment, $yc_4$ are the exports, and $yc_5$ are the imports. All data are in logs and per capita.

Table 5.6 displays the ad hoc response and the annualized cumulative impulse response of the chosen component ($C$ is consumption, $GC$ government consumption, $INV$ investment, $IM$ imports, and $EX$ exports) in the occurrence of a positive spending shock or a positive tax shock for period 4 and 8 after the shock. Since we still assume that the respective shock occurs in period 0, the columns labeled with "0" show the estimated ad hoc coefficients of the variables. The upper half of the table corresponds to the spending shock, the lower half to the tax shock, respectively. As before, the deterministic trend model is displayed on the left (labeled with "DT"), whereas the stochastic trend model is shown on the right (labeled with "ST"). The first row of each block gives the estimated impulse response of GDP from the baseline Structural VAR—see Subsection 5.3.1.

We describe the estimated annualized overall impact of the fiscal shocks on the GDP components in the following. Since we want to analyze whether national fiscal policy within the EMU also affects other EMU member countries via the trade channel and Germany is highly export driven, the response of exports is analyzed in detail.

| Variable | DT - 0 | DT - 4 | DT - 8 | ST - 0 | ST - 4 | ST - 8 |
|---|---|---|---|---|---|---|
| $cir^{y}$ | 0.122* | -0.01 | -0.172 | 0.117 | -0.034 | -0.158 |
| $cir^{C}$ | 0.038 | -0.008 | -0.1335 | 0.035 | -0.018 | -0.203 |
| $cir^{GC}$ | 0.798* | 0.405* | 0.396* | 0.796* | 0.42* | 0.451* |
| $cir^{INV}$ | 0.313 | -0.387 | -1.225 | 0.276 | -0.337 | -1.2 |
| $cir^{EX}$ | -0.304 | -0.123 | -0.414 | -0.333 | -0.261 | -0.572 |
| $cir^{IM}$ | 0.268 | 0.343 | 0.107 | 0.275 | 0.308 | 0.312 |
| $cir^{y}$ | -0.22* | -0.258* | -0.267 | -0.229* | -0.361* | -0.681* |
| $cir^{C}$ | -0.109* | 0.066 | 0.212 | -0.111* | 0.037 | 0.118 |
| $cir^{GC}$ | -0.028 | 0.009 | 0.0075 | -0.053 | -0.035 | -0.052 |
| $cir^{INV}$ | -0.248 | -0.285 | 0.057 | -0.225 | -0.294 | -0.014 |
| $cir^{EX}$ | -0.39* | -0.863* | -0.959 | -0.447* | -1.363* | -2.673* |
| $cir^{IM}$ | -0.447* | -0.775* | -0.759 | -0.486* | -0.943* | -1.32 |

* denotes statistical significance at 10% level.

Table 5.6: Influence of Fiscal Shocks on GDP Components - Baseline Model

The government spending shock first raises private consumption. The positive estimated ad hoc coefficient is very small. After a while, the impulse response of consumption becomes negative. Reasonable values concerning the size of response are all negative.

The estimated overall impact of a spending shock on government consumption is positive and statistically significant for both model variations as government consumption is de-

fined as a part of government spending. Investment responds to a government spending shock first positively and after some periods negatively. The overall response is negative for the main part of the forecasted periods.
The impact on imports is positive for the first years after the shock. Exports decline for almost 10 quarters. The cumulative impulse responses show that an increase in government spending lowers exports and raises imports at least for the analyzed 12 quarters after the occurrence of the expenditure shock in the baseline model.
A tax shock lowers all components of GDP when the shock occurs. The direction of the response changes for consumption, government consumption, and investment such that the estimated responses are positive after some periods. Contrary, a positive tax shock (an increase in taxes) persistently lowers imports and exports for at least two years.
The responses of the GDP components are qualitatively the same for the deterministic trend and the stochastic trend model, but the intensity varies.
The sum of the estimated GDP component responses significantly differs from the GDP response for an expenditure shock. In the occurrence of a tax shock, the sum of the GDP component responses are close to the response of GDP. Most of the single GDP components react intensely to a fiscal shock compared to the impact factor on output. The longer the forecast horizon, the larger is the difference of the summed responses of GDP components and the directly estimated impulse response of GDP.
The impulse responses concerning their fit to economic theory is analyzed in the following. With neoclassical theory, consumption decreases if government spending is raised, whereas it responds with a positive sign according to the Keynesian theory. The spending shock affects private consumption first positively then negatively. The positive ad hoc point estimate is statistically insignificant. It cannot be shown which theory matches best the estimated impulse response of private consumption.
The estimated coefficients of investment have in most parts converse signs for the two analyzed fiscal shocks. An expansionary (government spending) shock and a contractionary (net tax) shock affect investment in opposite directions. This fits to standard Keynesian theory.

Finally, we examine the impulse response of exports to fiscal shocks in detail.


Figure 5.5: Impulse Responses of Exports - Baseline Model

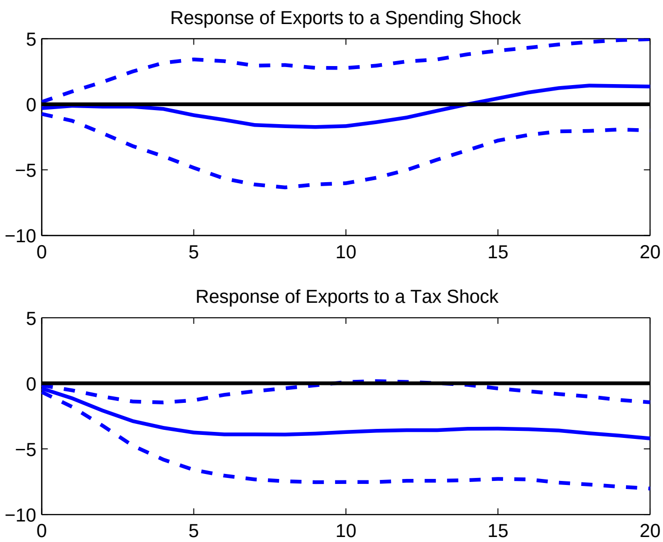


Figure 5.6: Accumulated Impulse Responses of Exports - Baseline Model

The period responses are shown in Figure 5.5. Exports respond negatively to both positive fiscal shocks at the occurrence of the respective shock. In the course of time, they begin to increase, fluctuating around the zero line. Overall, the estimated response to an expenditure shock is more volatile.

Figure 5.6 shows the annualized accumulated effect of a fiscal shock on exports. The estimated overall impact of a positive shock in government spending is negative for all 3 analyzed years after the shock. The response is statistically insignificant. Moreover, exports decline if taxes are raised. The impulse response of exports to a net tax shock is statistically significant for most periods.

Both accumulated impulse responses are negative for the whole analyzed simulation period of 12 quarters. A net tax shock has a lasting negative overall effect on exports since it does not revert to the zero line, whereas the estimated impact on a positive government spending shock weakens over time.

Within this subsection, we find the following.

**Result 5.3.**

- Keynesian theory matches the impulse responses of investment to fiscal shocks.
- Analyzing the impact of a government spending shock on consumption, we cannot decide, which theory is supported by the empirical result.
- Exports decline if a fiscal shock hits the economy. The estimated cumulative response is statistically significant if taxes are raised.

Due to the limited model framework, we cannot explain the responses of the trade variables in this setting. Thus, the model is augmented by new endogenous model variables to argue about interdependencies. As a first step, the interest rate and the inflation rate are included in the following.

## 5.4 The SVAR Model Including the Interest and Inflation Rate

We augment the baseline model with the nominal interest rate $i$ and the GDP-deflator inflation rate $\pi$. The two interest rates to be considered are the short-term interest rate and the long-term interest rate.

There are different arguments to include the one or the other.
On one hand, the interest rate play a role thinking of monetary policy, on the other hand, it has some influence on the demand variables of an economy.
The nominal short-term interest rate plays the major role in the context of monetary policy because it is the main tool to influence the economy. The importance of long-term interest rates for monetary policy is discussed widely. Some studies analyze if the long-term interest rate should be used as monetary policy instrument—see amongst others McGough, Rudebusch, and Williams [96] or Woodford [131]. McCallum [95] assumes a positive relation between short-term and long-term interest rates and thus includes the long-term interest rate with a positive sign in the monetary policy decision rule. Contrary, Casellina and Uberti [33] figure out that the optimal response of the short-term interest rate to a positive shock in the long-term interest rate is negative. In this case, the model setting is an IS-LM-Phillips curve model including rational expectations and the long-term interest rate with an extended Taylor rule to analyze optimal monetary policy. Woodford [130] states that real output depends more on the long-term interest rate than on the short-term interest rate. Thus, the efficacy of monetary policy crucially relies on the ability to influence the term structure of an economy and thus the long-term interest rate.
Standard macro models as the IS-LM model state a relationship between macro variables as the GDP, consumption, saving, or investment and the interest rate. In this case, the long-term interest rate plays the major role because the actors in an economy decide what amount to save, consume, or invest depending on the value of the real long-term interest rate although it is not observable. As a consequence, the expected real long-term interest rates are relevant in the decision making process. The Fisher-Hypothesis claims that the real interest rate is equal to the difference of the nominal interest rate and the expected rate of inflation—see Fisher [50]. Hence, consumption, saving, and investment decisions depend implicitly on the nominal long-term interest rate.
Since the demand response on changes in fiscal policy is one of the basic points of interest in this study and the analyzed structural VAR model includes the inflation rate,

we implement the nominal long-term interest rate rather than the short-term interest rate as regressor following Perotti [108].[16] Contrary, most other studies as, e.g., Heppke-Falk et al. [61] or Giuliodori and Beetsma [57] include the short-term interest rate.

The new endogenous variables vector is defined as $\vec{X}_t = [\ddot{g}_t, \ddot{t}_t, \ddot{y}_t, \pi_t, i_t]'$, whereas the corresponding vector of residuals after the VAR-estimation is $\vec{U}_t = [u_t^g, u_t^t, u_t^y, u_t^\pi, u_t^i]'$. We rank the inflation rate past to the endogenous variables of the baseline model to allow for effects of output, government spending, and taxes on the price level. Finally, we include the long-term interest rate as the variable ordered last. After estimation of the VAR model, we obtain the estimation residuals $u_t^j \; \forall j \in \{g, t, y, \pi, i\}$. The cyclically adjusted residuals for the two fiscal variables are defined as

$$\begin{aligned} u^{g,CA} &= u^g - \left(\alpha_{gy} u^y + \alpha_{g\pi} u^\pi + \alpha_{gi} u^i\right) = \beta_{gt} e^t + e^g \\ u^{t,CA} &= u^t - \left(\alpha_{ty} u^y + \alpha_{t\pi} u^\pi + \alpha_{ti} u^i\right) = \beta_{tg} e^g + e^t. \end{aligned}$$

The output elasticity of government spending $\alpha_{gy}$ is set to zero and the output elasticity of net taxes is calculated as $\alpha_{ty} = 2.0857$. The price elasticity of government spending $\alpha_{g\pi}$ is set to -0.5 and the price elasticity of net taxes is estimated as $\alpha_{t\pi} = 1.0199$. As before, we assume that neither government spending nor net taxes react to changes in the interest rate such that $\alpha_{gi} = \alpha_{ti} = 0$ holds. For more information on the derivation of the respective elasticities see Section 4.3.1 of Chapter 4.

We follow the same identification method as for the baseline model. The overall relationship of the estimation residuals $\vec{U}$ to the structural shock variables $\vec{E}$ is characterized by an equation similar to equation (5.2):

$$\underbrace{\begin{pmatrix} 1 & 0 & -\alpha_{gy} & -\alpha_{g\pi} & -\alpha_{gi} \\ 0 & 1 & -\alpha_{ty} & -\alpha_{t\pi} & -\alpha_{ti} \\ -\alpha_{yg} & -\alpha_{yt} & 1 & 0 & 0 \\ -\alpha_{\pi g} & -\alpha_{\pi t} & -\alpha_{\pi y} & 1 & 0 \\ -\alpha_{ig} & -\alpha_{it} & -\alpha_{iy} & -\alpha_{i\pi} & 1 \end{pmatrix}}_{\vec{C}_1} \underbrace{\begin{pmatrix} u^g \\ u^t \\ u^y \\ u^\pi \\ u^i \end{pmatrix}}_{\vec{U}} = \underbrace{\begin{pmatrix} 1 & \beta_{gt} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}}_{\vec{C}_2} \underbrace{\begin{pmatrix} e^g \\ e^t \\ e^y \\ e^\pi \\ e^i \end{pmatrix}}_{\vec{E}},$$

[16] We cross-checked our results by implementing the short-term interest rate rather than the long-term interest rate. All impulse responses stay qualitatively the same in that case.

where $\vec{U} = [u^g, u^t, u^y, u^\pi, u^i]'$ and $\vec{E} = [e^g, e^t, e^y, e^\pi, e^i]'$. The missing values for $\alpha_{jk}$ are found, as before, by stepwise IV estimation using the former identified structural shock variables as instruments for the correlated residuals. The following subsection describes the results for the first augmented model variation.

### 5.4.1 Results for the Five Variables Model

The SVAR model including the inflation and the nominal long-term interest rate is estimated. After obtaining the residuals, we perform an instrumental variable estimation of the correlated residuals found with the VAR model estimation and the identified structural parameters. Selected point estimates are shown in Table 5.7. For comparability, we list the same estimates as in Subsection 5.3.1. As before, the number in brackets denotes the t-statistic.

| **Variable** | **DT** | **ST** |
|---|---|---|
| $\beta_{gt}$ | -0.095* (-1.79) | -0.093* (-1.727) |
| $\alpha_{yg}$ | 0.118* (1.734) | 0.114* (1.68) |
| $\alpha_{yt}$ | -0.177** (-4.007) | -0.18** (-4.03) |

* and ** denote statistical significance levels of 10% and 5%, respectively.

Table 5.7: Results for the 5 Variables Model

No autocorrelation is detected in the estimation residuals of the VAR if we perform a Portmanteau test. The estimates are statistically significant (at 5% or 10% statistical significance level) and are estimated with similar values as these found for the baseline model—see Table 5.1.

The Granger-causality test performed after the VAR suggests that we cannot reject a Granger-causal impact of output on net taxes on a 1% statistical significance level. Furthermore, the long-term interest rate Granger-causes output. The Null of a Granger-causal relation of taxes and the inflation rate to the interest rate is not rejected. The results are robust to the choice of a stochastic or deterministic trend as regressor. Since the results including one of the two trend types are similar, we only discuss the model

setups with deterministic trend in detail in the remainder of the study.
The qualitative change in one fiscal variable due to a change in the other fiscal variable in the underlying model specification is comparable to the respective change found for the baseline model: if government spending is affected by a positive one unit shock, net taxes immediately rise. Expenditure decreases due to a positive one unit tax shock.
Table 5.8 shows the point estimates for the deficits and output. Moreover, the cyclically adjusted responses of government spending and net taxes as well as the multipliers are given.

| **Variable** | **G - 0** | **G - 4** | **G - 8** | **T1 - 0** | **T1 - 4** | **T1 - 8** |
|---|---|---|---|---|---|---|
| $cir^{def}$ | 0.785* | 0.547* | 1.391* | -1.113* | -0.527* | -0.825* |
| $cir^{y}$ | 0.093 | -0.052 | -0.265 | -0.179* | -0.168* | -0.162 |
| $cir^{g,CA}$ | 1.01* | 0.494* | 0.446* | -0.145* | 0.011 | -0.036 |
| $cir^{t,CA}$ | 0.001 | 0.033 | -0.421* | 1.438* | 0.938* | 1.18* |
| $m$ | 0.093 | -0.104 | -0.615 | -0.179* | -0.307* | -0.201 |
| $m^{CA}$ | 0.092 | -0.105 | -0.594 | -0.124* | -0.179* | -0.137 |

* denotes statistical significance at 10% level.

Table 5.8: Influence of a Spending and a Tax Shock - 5 Variables Model

The deficit is more affected in almost all cases in the underlying augmented model specification. Qualitatively, the result found for the baseline model is still true: deficits decline most in the first periods after a net tax shock, but the reduction in deficits due to a negative shock in government expenditure is lasting longer and is estimated stronger in intensity for the overall analyzed horizon of 12 quarters.
Output responds positively to a government spending shock, but the intensity is smaller compared to that of the three variables baseline model. In particular, the ad hoc response of output to a government spending shock is 0.093 (before 0.122), 1 period after the shock, the output response decreases to 0.04 (before 0.135), whereas the annualized cumulative responses for the 4th and 8th period are $-0.052$ (before -0.01) and $-0.265$ (before -0.172), respectively. Output is affected less at the time the shock occurs, but

the estimated negative response following after some periods is stronger. Overall, the impulse response of output due to a rise in government spending is shifted downwards if we include the inflation and the interest rate. For comparison, see also Table 5.2 and 5.3. All estimates are not statistically significant at the 10% statistical significance level. The impulse response of output after a positive government spending shock is qualitatively not too different compared to the results found by Heppke-Falk et al. [61]. The ad hoc increase in output due to a spending shock is followed by a decrease in output after some periods. Only the time periods are different. Furthermore, the intensity of the estimated effect is very similar as a one unit expenditure shock causes an ad hoc rise in output of around 0.12 in Figure 6 of Heppke-Falk et al. [61], whereas our response is 0.093. If taxes rise due to a shock, the ad hoc response of output as well as the impact 4 and 8 quarters after the occurrence of the tax shock on output are weaker. The response is negative also for this model specification and thus, the qualitative response is similar in both setups. Heppke-Falk et al. [61] estimate a smaller response of output to a net tax shock. The ad hoc response is calculated as -0.019, whereas our response is -0.179. As before, the estimated ad hoc response of output is larger in size for a net tax shock such that taxes are the more effective instrument in stimulating the economy for a short period of time.

Due to the smaller absolute value of the estimated cyclically adjusted ad hoc multipliers, the immediate influence of the fiscal shocks to output seems to be smaller in the five variables model setting compared to the baseline model.

The negative multipliers for a shock in government spending, which are all statistically insignificant, are larger for the underlying model specification after a while such that a rise in expenditure affects output more negatively in the course of time. The multipliers for a net tax shock are all calculated with smaller absolute values independent of the time elapsed after the shock. The impact of a rise in net taxes on output is detected to be smaller if the model additionally includes the interest and the inflation rate as regressors.

Figure 5.7 shows the period responses of output to fiscal shocks, whereas the accumulated responses, i.e., the overall effect of a shock in one of the two fiscal variables government expenditure or net taxes on output are given in Figure 5.8.

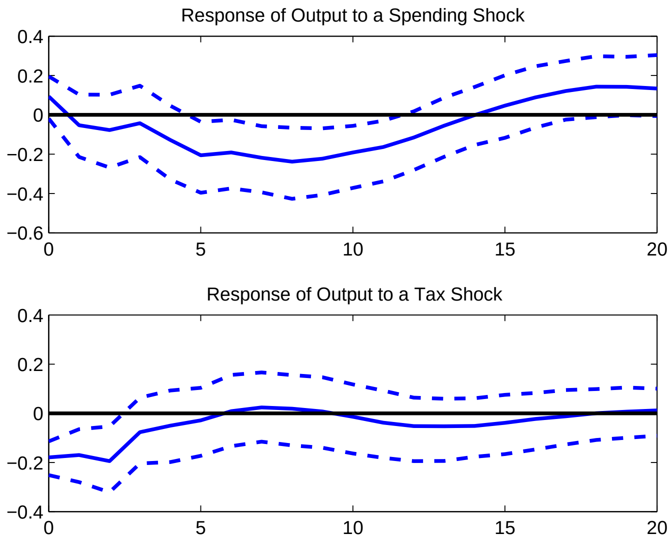


Figure 5.7: Impulse Responses of Output - 5 Variables Model

Output first rises due to a positive shock in government spending. But the estimated period response is negative from the second period onwards. If net taxes rise, output declines almost for all simulated periods.

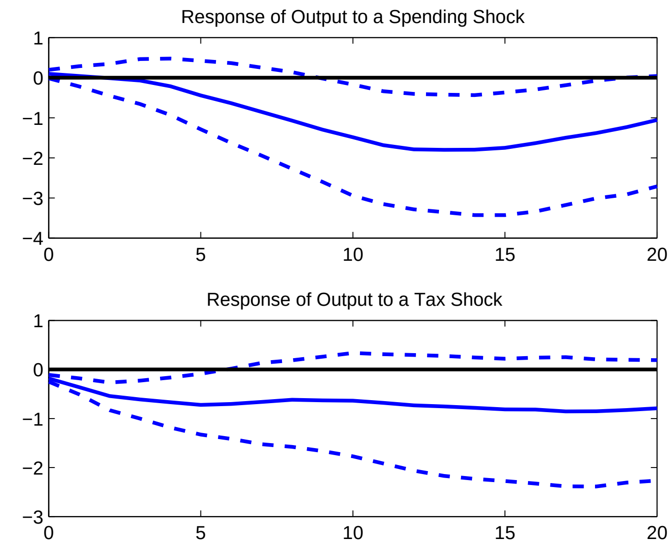


Figure 5.8: Accumulated Impulse Responses of Output - 5 Variables Model

As suggested in Table 5.8, the accumulated response of output to a positive government expenditure shock shown in Figure 5.8 becomes negative 3 periods after the occurrence of the shock. The estimated negative response is statistically significant 10 periods after the shock. To summarize, the expansionary government expenditure shock first raises output, but the multiplier is only very small. Output declines after a while and the response is estimated with statistically significant negative values 2.5 years after the shock.

The estimated cumulative impulse response of output to a net tax shock is negative for all simulated periods and statistically significant for the first 1.5 years after the occurrence of the shock. Hence, a rise in taxes lowers output which corresponds to standard macroeconomic theory.

The period responses of the inflation rate and the long-term interest rate are shown in Figure 5.9. As before, the upper row shows the responses to an expenditure shock, whereas the lower row consists of the estimated effects on a tax shock. The results for the stochastic trend implementation are omitted because the results are similar to those found for the deterministic trend model.

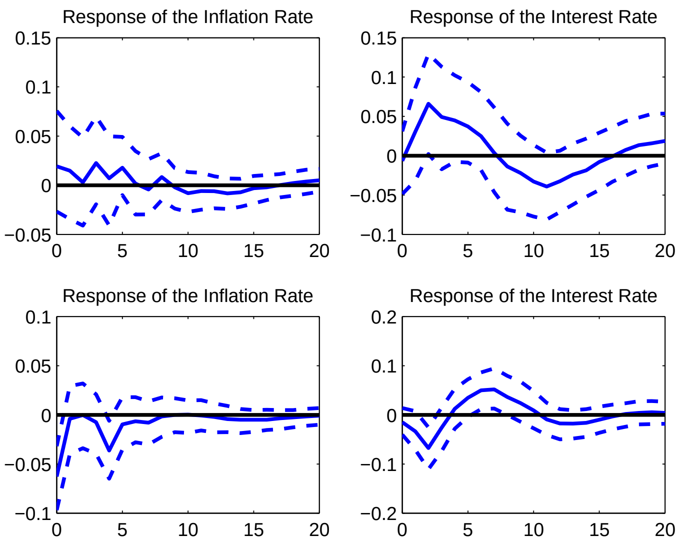


Figure 5.9: Impulse Responses of the Inflation and the Interest Rate - 5 Var.

The inflation rate increases due to a positive government spending shock with a very low intensity and fluctuated after some periods around the zero line. The ad hoc response of the inflation rate (0.019) is very small; the annualized cumulative response for the 4th period after the shock is 0.017 and 0.023 for the 8th period and not statistically significant. After 12-14 periods, the inflation rate approximates to the zero line.

Due to a positive net tax shock, the inflation rate decreases statistically significantly. The ad hoc response is -0.063. The annualized cumulative response after 4 and 8 periods are -0.028 and -0.034, respectively. Moreover, the inflation rate approximates to the zero line after some periods.

The interest rate is affected negatively but with a very small value (the ad hoc response is $-0.007$) due to the expenditure shock. It then increases, overshoots the zero line, and afterwards decreases again fluctuating around the zero line. The annualized cumulative responses after 4 and 8 periods amount to 0.046 and 0.059, respectively. Overall, the response is small in intensity. The increase after some periods and the subsequent decline in the period responses of the interest rate are statistically significant.

In the occurrence of a one unit tax shock, the interest rate is first negative, increases and then overshoots the zero line. The ad hoc response is -0.015, whereas the annualized cumulative response after 4 (8) periods is -0.033 (0.011). The decline in the interest rate and the following increase are statistically significant.

Figure 5.10 shows the estimated overall effect of fiscal shocks on the inflation and the interest rate. The inflation rate and the long-term interest rate are affected positively by a positive government spending shock, where both cumulative impulse responses are not statistically significant. One may argue that the rise in the interest rate after a positive government expenditure shock is reasonable because overall deficits rise–see Table 5.8. As a consequence, the government issues additional bonds. If the money supply is not adjusted by the same amount, the interest rate rises.

If net taxes rise temporarily, the estimated overall impact on the inflation rate is negative and long lasting. The interest rate is affected first negatively and overshoots the zero line 1.5 years after the occurrence of the shock. Both responses to a net tax shock are statistically significant at least for 4 quarters.

Overall, the estimated intensity of the fiscal shock impact factors is significantly higher for the response of output than the ones of the inflation or the interest rate.

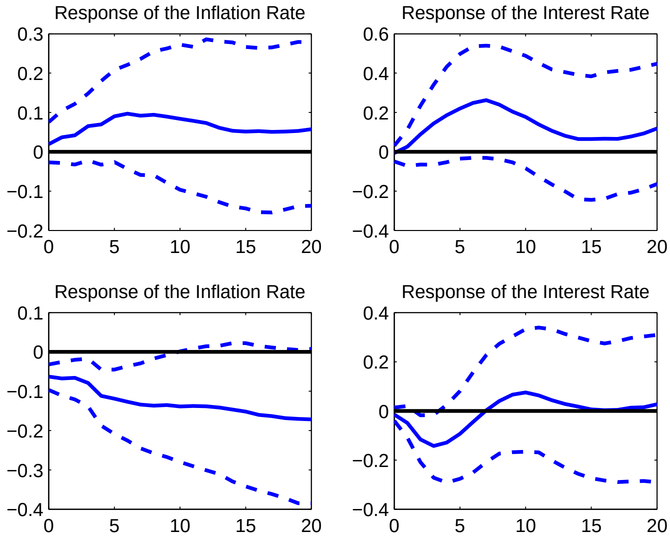


Figure 5.10: Acc. Impulse Responses of the Inflation and the Interest Rate - 5 Vars.

**Result 5.4.** Expenditure vs. Net Tax Shock (5 Variables Model)

- Output weakly rises due to a positive shock in government spending but declines some periods after the shock and becomes negative. The negative response is statistically significant 10 quarters after the shock.
- Output statistically significantly declines due to a positive tax shock.
- All multipliers are estimated with small values.
- If the model includes the inflation and the interest rate, changes in taxes affect output less strongly.
- As for the baseline model, a shock in net taxes has a higher estimated impact on output and is a more effective instrument in stabilizing output in the short run.
- The overall deficit rises with an expansionary spending shock and declines if taxes are raised. A rise in taxes lowers deficits more in the short run, but a reduction in government spending is more effective in the medium run.
- If the inflation and the interest rate is included, deficits can be influenced more with one unit shocks in government expenditure and net taxes.

For a closer examination of the near past, the model is applied to 1999-2009, i.e., the period after Germany's entry to the European Monetary Union. We calculate the elasticities needed to identify the fiscal shocks for the subsample period. The elasticity of net taxes with respect to output is 1.0094 (for the whole sample 2.0857), whereas the price elasticity of net taxes is 1.0279 (for the whole sample 1.0199). For the subsample, the impact of a change in output on net taxes seems to weaken and the effect of a change in prices on the net taxes seems to strengthen in comparison to the whole sample (1991-2009). As before, all other institutional information is set exogenously and independently of the time horizon.

The qualitative tendency of the estimated output response to a government spending shock is similar to the one found for the whole period, but the intensity of the estimated effects is different. The ad hoc response of output to a government spending shock is $0.166 > 0.093$ where the value on the right hand side is the respective estimate for the whole sample and $0.701 > 0.04$ for the first period after the shock. Moreover, the overall output response stays positive for a longer simulation period. Even the 4th quarter estimate is positive (0.005) in that case.

In the occurrence of a net tax shock, output declines first for the subsample but becomes positive afterwards. The ad hoc response is $-0.019 > -0.179$ and we estimate the response of the first period as $0.026 > -0.349$. The positive response is statistically insignificant.

Raising government expenditures becomes more effective. Moreover, the estimated impact of a net tax shock on output declines in the short run. Thus, the results are comparable to those found for the subsample in the baseline model setting. The inflation rate and the interest rate are found to respond more volatile in the subperiod than in the whole period to fiscal shocks.[17]

**Result 5.5.** The 5 Variables Model - 1999-2009

- Net tax shocks seem to affect output less strongly after the accession of Germany to the EMU. Furthermore, output seems to respond more strongly to a government expenditure shock in that case.
- The response of the inflation and the interest rate are more volatile.

[17]The corresponding figures are available on request.

### 5.4.2 Results for the Five Variables Model - Components of GDP

We implement one of the five components of GDP as a sixth variable in the SVAR model setting in this subsection. Since we try to decide, which theory matches best the empirical results, private consumption and investment are analyzed in detail. If New Keynesian theory is presumed, consumption increases due to a positive government spending shock, whereas the opposite is true for New Classical theory. If investment responds conversely to an expansionary government spending and a positive net tax shock, this behavior matches with New Keynesian theory. Apart from that, the effect of German fiscal shocks on the trade variables is examined and the responses of German exports are explicitly shown.

The GDP components are included in $\vec{X}$, the vector of endogenous variables, and are ordered past output and prior to the inflation rate. The responses of all five variables after the implementation of the additional variables are qualitatively the same as for the standard five variables model.

The following table shows the responses of the components in the deterministic trend model for three different points in time, similar to Table 5.6. The first row reports the response of output found in the standard five variables model.

| **Variable** | **G - 0** | **G - 4** | **G - 8** | **T1 - 0** | **T1 - 4** | **T1 - 8** |
|---|---|---|---|---|---|---|
| $cir^y$ | 0.093 | -0.052 | -0.265 | -0.179* | -0.168* | -0.162 |
| $cir^C$ | 0.086 | 0.025 | -0.134 | -0.082 | 0.066 | 0.113 |
| $cir^{GC}$ | 0.773* | 0.378* | 0.293 | -0.047 | 0.005 | -0.017 |
| $cir^{INV}$ | 0.138 | -0.739 | -1.788* | -0.031 | 0.206 | 0.595 |
| $cir^{EX}$ | -0.601* | -0.472 | -0.777 | -0.296* | -0.492 | -0.346 |
| $cir^{IM}$ | 0.076 | -0.048 | -0.575 | -0.332* | -0.273 | -0.031 |

* denotes statistical significance at 10% level.

Table 5.9: Influence of Fiscal Shocks on GDP Components - 5 Variables Model

Consumption first responds positively and then becomes negative for a positive government spending shock. Most parts of the response are statistically insignificant. Due to a rise in net taxes, consumption first declines because also output and thus income declines. After a while, consumption rises. This positive response is statistically insignificant, whereas some parts of the negative response are statistically significant at a 10% statistical significance level.

Overall, the response cannot be used to find a match with a specific economic theory.

The estimated impact of an expenditure shock on government consumption is (as expected) high because government consumption is one aggregation component of government expenditure, whereas a net tax shock affects government consumption only weakly.

Investment responds qualitatively similar to a fiscal shock as in the baseline model. The response due to an expansionary government expenditure shock is first positive and then becomes negative. Since the interest rate rises if government expenditure is increased, it is observed that investment is crowded out by the rising government spending. If a net tax shock occurs, investment is first negative and becomes positive after some periods. Overall, the impulse responses of investment show a converse behavior in the occurrence of a shock in government expenditure and a shock in net taxes. Standard Keynesian theory matches these results.


Figure 5.11: Impulse Responses of Exports - 5 Variables Model

Figure 5.11 shows the period impulse responses of exports to fiscal shocks. Exports are

estimated to decline due to both fiscal shocks. Some period responses are statistically significant at a 10% statistical significance level. The two shocks seem to have a long lasting overall impact on exports. After a negatively estimated response, exports rise due to the expenditure shock. The response in the first periods after the occurrence of the shock is statistically significant. If a positive net tax shock takes place, exports fluctuate around the zero line. Only parts of the negative response are statistically significant.
The accumulated impulse responses are shown in Figure 5.12. Since both curves run below the zero line for all forecasted periods, the graphs indicate a decline in exports due to the fiscal shocks: the estimated impact is negative and long lasting.


Figure 5.12: Accumulated Impulse Responses of Exports - 5 Variables Model

The response of exports to a rise in government expenditure or net taxes can be explained by the response of the inflation rate. If the deficit is extended by raising government spending, the inflation rate increases. Prices rise and thus exports decline.
If taxes are raised, the inflation rate is smaller than before, but all private customers from abroad have to pay higher taxes on German products. Hence, exports decline.[18]

[18] Firms are unaffected by higher taxes because they do not have to pay German taxes on German products when buying from abroad. Only private customers from abroad have to pay VAT, just like the domestic consumers.

Imports first rise due to a positive shock in expenditure but decline after some periods. All values are statistically insignificant at a 10% statistical significance level. Similar to exports also imports decline in the occurrence of a positive net tax shock: import taxes probably increase or are even expected to be higher if overall net taxes increase. The estimated intensity of the accumulated impulse response for imports is smaller than that for exports except for the ad hoc response to a net tax shock.

As a consequence, the ad hoc response of net exports to a tax shock is very small and positive (0.036) but becomes immediately negative after one period. The impact of a positive shock in government expenditures on net exports is negative for all analyzed simulation periods. Net exports are indeed affected negatively by fiscal shocks.

The sum of the annualized cumulative responses for the GDP components differs from the response of GDP for the ad hoc response. The longer the forecast, the higher the deviation of the sum of single responses to the overall response of the aggregated GDP. Overall, we find the following.

**Result 5.6.**

- Keynesian theory matches the impulse responses of investment to fiscal shocks.
- The impulse response of private consumption to a government spending shock cannot be used to decide about economic theory.
- Exports are statistically significantly reduced if taxes are raised. Moreover, exports decline due to an increase in government expenditure.
- Net exports are reduced due to the fiscal shocks.

## 5.5 The Model Including EMU-Imports and the REER

In the following, the five variables model is augmented by imports from member countries of the European Monetary Union and the real effective exchange rate. In the first part of the section, all variables are implemented as endogenous variables, whereas the second part shows the results when assuming that the exchange rate channel and the interest rate channel are switched off in a monetary union with one currency and one central bank conducting monetary policy. In the latter case, the interest rate and the exchange rate are implemented as exogenous variables.

The augmented seven variables model is estimated as before. The imports from EMU member countries are ordered past to the output and prior to the inflation rate, whereas the real effective exchange rate is ordered last in the vector of endogenous variables, i.e., $\vec{X} = [\ddot{g} \quad \ddot{t} \quad \ddot{y} \quad \ddot{im} \quad \pi \quad i \quad e]'$, where $\ddot{im}$ denotes imports from EMU member countries and $e$ is the real effective exchange rate (REER). The corresponding vector of residuals is $\vec{U} = [u^g \quad u^t \quad u^y \quad u^{im} \quad u^\pi \quad u^i \quad u^e]'$. After performing a VAR, the cyclically adjusted residuals for the two fiscal variables are calculated as

$$\begin{aligned} u^{g,CA} &= u^g - \left(\alpha_{gy} u^y + \alpha_{gim} u^{im} + \alpha_{g\pi} u^\pi + \alpha_{gi} u^i + \alpha_{ge} u^e\right) = \beta_{gt} e^t + e^g, \\ u^{t,CA} &= u^t - \left(\alpha_{ty} u^y + \alpha_{tim} u^{im} + \alpha_{t\pi} u^\pi + \alpha_{ti} u^i + \alpha_{te} u^e\right) = \beta_{tg} e^g + e^t. \end{aligned}$$

As before, we use as value for the elasticity of net taxes with respect to output $\alpha_{ty} = 2.0857$ and for the price elasticity of net taxes $\alpha_{t\pi} = 1.0199$—see Section 4.3.1 of Chapter 4. The price elasticity of government spending $\alpha_{g\pi}$ is set to -0.5. We assume that neither government spending nor net taxes respond to changes in the interest rate such that $\alpha_{gi} = \alpha_{ti} = 0$ holds (Section 4.3.1 of Chapter 4).

The elasticities additionally needed are stated in the last part of Section 4.3.1. The decision process of government expenditure is not influenced by the amount of imports or the exchange rate such that we set $\alpha_{gim} = \alpha_{ge} = 0$. The elasticity of imports and the real effective exchange rate to taxes was estimated as $\alpha_{tim} = 0.0731$ and $\alpha_{te} = -0.0585$, respectively. The relatively small value for the import elasticity of net taxes is due to the small import tax revenue vis-a-vis the overall tax revenue.

The shock identification method is the same as before. An OLS estimation is performed to identify the uncorrelated two fiscal shocks. Furthermore, the identified residuals are used as instruments in a stepwise instrumental variable estimation to obtain the $\vec{C}_1$ and $\vec{C}_2$ matrices. In this case, we get 7x7 matrices with a similar structure as the 5x5 matrices described in detail in Section 5.4.

For the second part of the section, the interest rate and the exchange rate are included as exogenous variables and are thus no longer part of the endogenous variables vector $\vec{X}$. The VAR is performed with a vector of five endogenous variables and a vector of exogenous variables, including the constant and the trend as before and additionally the interest and the exchange rate. With this procedure, it is assumed that the monetary union is perfectly working: monetary policy is conducted by one independent, single institution and there is one currency for the whole union.

### 5.5.1 Results for the Seven Variables Model

First, the VAR model is estimated. A Portmanteau test is performed and no autocorrelation is detected in all estimations. The point estimates of the three main coefficients obtained from the IV estimations are shown in the following table. Compared to the former results, the estimated coefficients are small and in most parts statistically insignificant, but the signs are the same.

| **Variable** | **DT** | | **ST** | |
|---|---|---|---|---|
| $\beta_{gt}$ | -0.064 | (-1.171) | -0.067 | (-1.272) |
| $\alpha_{yg}$ | 0.051 | (0.875) | 0.057 | (0.957) |
| $\alpha_{yt}$ | -0.09** | (-2.782) | -0.102** | (-2.964) |

* and ** denote statistical significance levels of 10% and 5%, respectively.

Table 5.10: Results for the 7 Variables Model

The Granger-causality test shows that output is influenced by the inflation and the interest rate. Also net taxes Granger-cause output. Furthermore, the interest rate and imports from EMU member countries are Granger-caused by the inflation rate.
The cumulated impact of the fiscal shocks on output, the deficit, the cyclically adjusted government expenditures and net taxes, and the multipliers are listed in Table 5.11.
The deficit response to a positive government expenditure shock is estimated with higher values in comparison to the five variables model. The ad hoc response of deficits to a net tax shock is estimated with smaller values, whereas the 4th and 8th period response has a higher estimate compared to the 5 variables model. The estimates indicate that

a positive net tax shock first lowers deficits less if we also take the union-variables into account, whereas the effect is estimated to be stronger after a while in that case. As before, a net tax shock is the more effective instrument in reducing the deficit for a short time and a government expenditure has a stronger medium run effect on deficits.

| **Variable** | **G - 0** | **G - 4** | **G - 8** | **T1 - 0** | **T1 - 4** | **T1 - 8** |
|---|---|---|---|---|---|---|
| $cir^{def}$ | 0.919* | 0.825* | 1.609* | -1.06* | -0.549* | -0.884* |
| $cir^{y}$ | 0.041 | -0.152 | -0.292 | -0.096* | -0.074 | -0.031 |
| $cir^{g,CA}$ | 0.998* | 0.447* | 0.303* | -0.072 | 0.064 | 0.07 |
| $cir^{t,CA}$ | -0.0002 | -0.036 | -0.732* | 1.262* | 0.81* | 1.035* |
| $m$ | 0.041 | -0.335 | -0.994 | -0.096* | -0.12 | -0.033 |
| $m^{CA}$ | 0.041 | -0.34 | -0.734 | -0.076* | -0.091 | -0.03 |
| $cir^{im_{EMU}}$ | -0.13 | -0.546 | -0.784 | -0.441* | -0.286 | -0.304 |

* denotes statistical significance at 10% level.

Table 5.11: Influence of a Spending and a Tax Shock - 7 Variables Model

The cyclically adjusted ad hoc multiplier for a government spending shock is very small such that a rise of one unit in government expenditure raises output only by 0.041 units. The decline of output after a short increase due to the government spending shock, which was also found for the other model specifications, takes place one period after the occurrence of the shock. Thus, the multiplier for the first period after the shock has a negative sign. Moreover, the cyclically adjusted multipliers for the 4th and 8th period after the occurrence of the shock are estimated with higher absolute values: the negative impact of a rise in government spending on output seems to strengthen over time. All spending multipliers shown in Table 5.11 are statistically insignificant.
The results of the underlying model specification lead to the conclusion that an increase in government expenditure has a negative impact on output, which is stronger in intensity than the effect found for the two former models setups. The estimated tax multipliers are all smaller in absolute value compared to the ones found in the five variables model. Thus, if the trade variables imports from EMU member countries and the exchange rate

are implemented in the model, the estimated impact of a net tax shock is smaller. If we compare the spending and the tax multiplier, output is affected stronger by a net tax shock, but both ad hoc multipliers are very small such that fiscal policy does not seem to be an effective instrument in stabilizing variations in output. The imports from EMU member countries are affected negatively for both fiscal shocks and the ad hoc response to a net tax shock is statistically significant.

In the following, we analyze the period impulse responses on fiscal shocks in the augmented 7 variables model. Special attention is paid to the response of output.

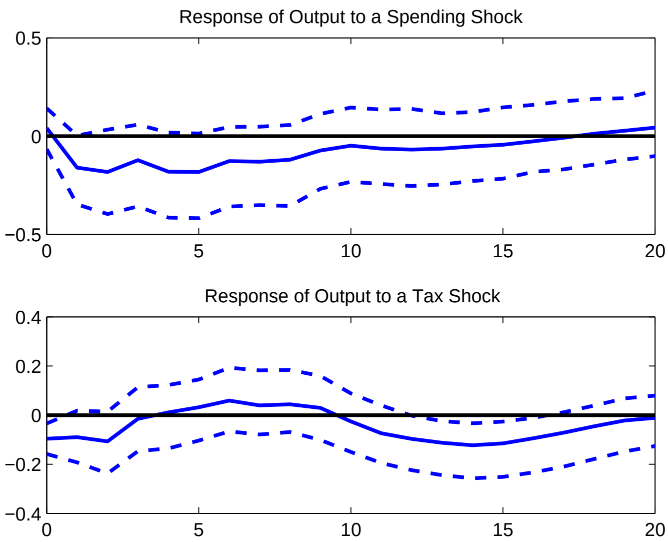


Figure 5.13: Impulse Responses of Output - 7 Variables Model

First, a government spending shock has a positive impact on output as in the other model specifications. But output declines faster compared to the 5 variables model. After reaching a minimum in the negative quadrant,[19] output increases again, similar to the three and five variables model after the occurrence of the shock. The impulse response is statistically insignificant at a 10% statistical significance level for almost all simulated periods. The impact of a net tax shock on output is qualitatively the same compared to the five variables model but shows a higher volatility in the underlying

[19] The negative values are also found by Giuliodori and Beetsma [57] who use a similar model setting.

model specification—see also Figure 5.7 in Section 5.4. Only the negative responses for the first periods after the shock are statistically significant.

To analyze the overall effect of a fiscal shock on output, the following figure shows the accumulated responses of output to a positive government spending or net tax shock.

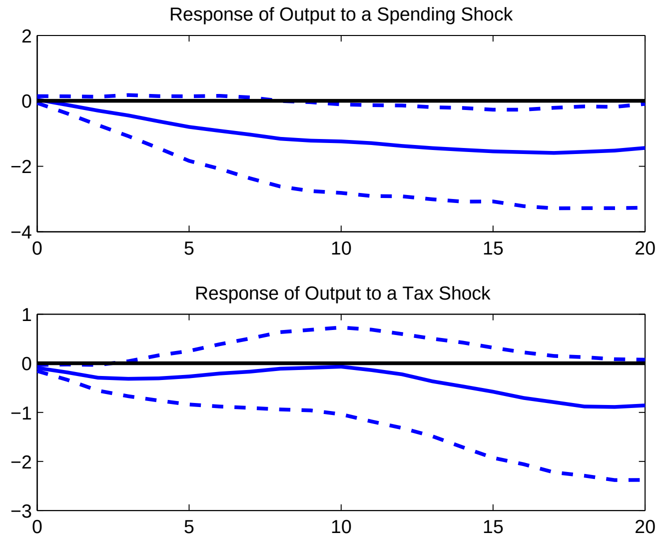


Figure 5.14: Accumulated Impulse Response of Output - 7 Variables Model

The estimated output responses indicate that a government spending shock affects output negatively. The estimated response is statistically significant 9 periods after the occurrence of the shock. In the five variables model, the qualitative response is closely the same. The first statistically significant negative response is obtained in the 5 variables setting after 2.5 years, i.e., 10 periods after the shock.

A net tax shock has a long lasting negative impact on output. The response is statistically significant at a 10% statistical significance level only for 3 periods after the shock, whereas it is estimated to be statistically significant for 5 periods in the 5 variables model setting.

The inflation and the interest rate responses to a fiscal shock are shown in Figure 5.15 (period impulse responses) and 5.16 (cumulative impulse responses).

Inflation declines due to a shock in government expenditure in the underlying model setting. After a while, the overall impact becomes positive. The estimated cumulative response is statistically significant after 6 quarters with positive values.

The overall response of the interest rate to a government expenditure shock is positive and statistically significant for the first 5 quarters after the occurrence of the shock. The government issued bonds to finance the additional spending. Since Germany does not print money to adjust the money supply, the interest rate rises.

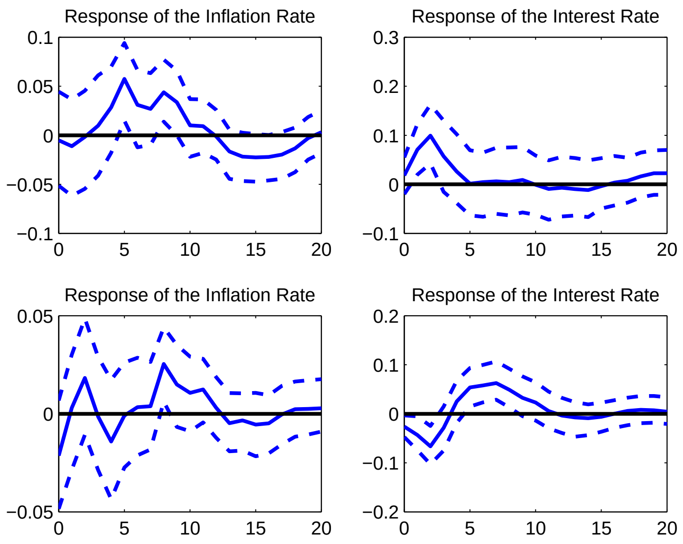


Figure 5.15: Impulse Responses of the Inflation and the Interest Rate - 7 Vars.

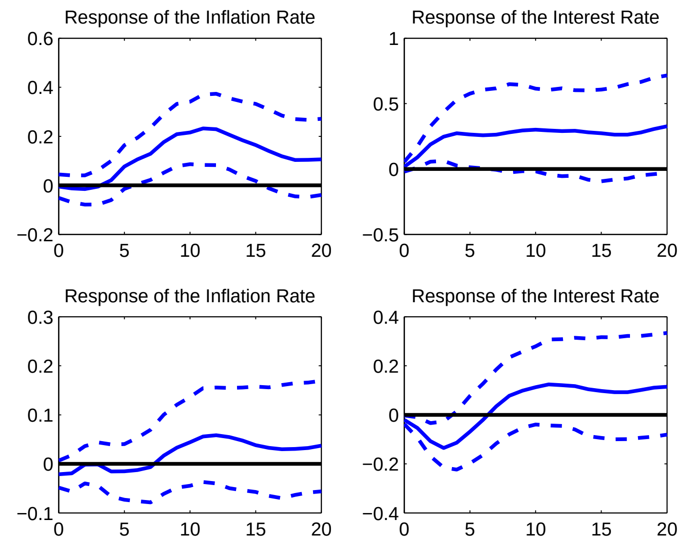


Figure 5.16: Acc. Impulse Responses of the Inflation and the Interest Rate - 7 Vars.

The accumulated response of the inflation rate to a net tax shock first fluctuates around the zero line and is positive from 8 periods onwards after the shock. All estimates are statistically insignificant at a 10% statistical significance level.
As in the 5 variables model, the interest rate first responds statistically significantly negative to a rise in net taxes. The estimated overall impact of the net tax shock becomes positive after 6 periods.
The effect of the fiscal shocks on the imports from EMU member countries and the exchange rate, i.e., the newly implemented variables, are shown in Figure 5.17 and 5.18.

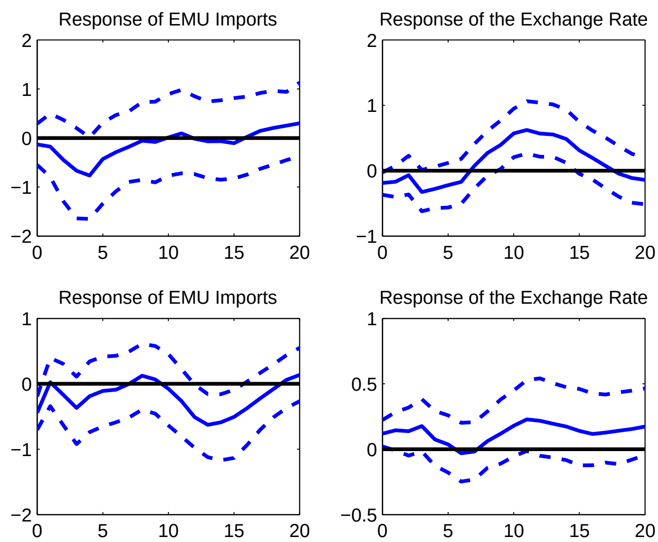


Figure 5.17: Responses of EMU-Imports and the Exchange Rate - 7 Vars.

Imports from EMU member countries respond negatively to a positive government spending shock. Within one year, imports sharply decrease before increasing again. A positive net tax shock affects imports negatively. The ad hoc response is statistically significant. In the remaining simulated period, imports fluctuate around the zero line staying in most parts in the negative quadrant. The exchange rate reacts to a positive spending shock with an immediate statistically significant decrease and a statistically significant increase 9-12 periods after the shock. The exchange rate response to a rise in net taxes lies above the zero line for almost all periods, indicating a lasting positive influence. The response is statistically significant only in period 0.

The overall impacts of the fiscal shocks are shown in Figure 5.18. Imports from EMU member countries diminish due to fiscal shocks, whereas the effect on the exchange rate is overall positive for a net tax shock. The currency depreciates in that case.

The exchange rate is affected negatively for the first 10 periods after a government expenditure shock, which corresponds to an appreciation of currency and becomes positive afterwards. Since the increased interest rate due to the shock attracts foreign investors, the demand for home currency rises such that it appreciates.

Especially, a net tax shock seems to have long lasting effects on both variables.

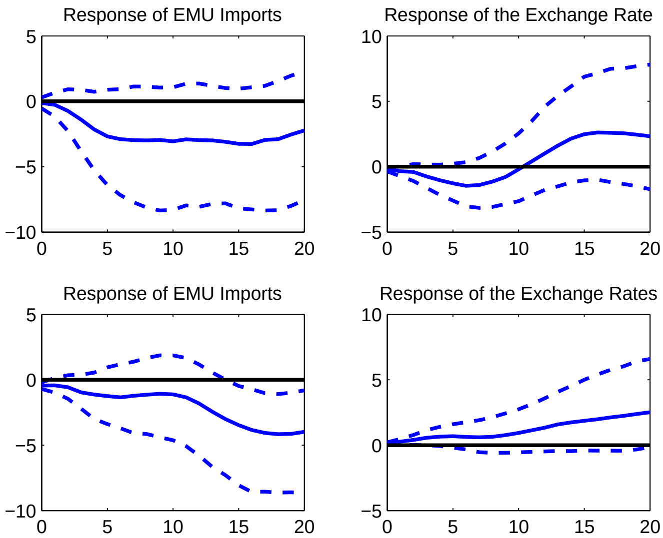


Figure 5.18: Acc. Impulse Responses of EMU-Imports and the Exchange Rate - 7 Vars.

The negative response of imports after an expenditure shock is due to the rise in the inflation rate (the rise in the price level). The statistically significant decline in imports due to a net tax shock is caused by higher taxes on imports or the expectation that import taxes are raised if overall net taxes are higher than before. The response of the inflation rate in that case is statistically insignificant, very small in size and seems to have closely no effect.

We cross-check the results by implementing the short-term interest rate instead of the long-term interest rate. The accumulated impulse responses of the inflation rate to a positive government spending shock is at least weakly positive for all analyzed periods. The response to a net tax shock is negative. Furthermore, the accumulated impulse response of the exchange rate to a net tax shock is close to the zero line. All remaining responses are qualitatively more or less the same in comparison to the results found for the long-term interest rate and the explanations for the impulse responses are still reasonable.

**Result 5.7.** Expenditure vs. Net Tax Shock (7 Variables Model)

- Output rises very weakly due to a positive shock in government spending but declines one period after the shock and becomes negative. The negative response is statistically significant 9 quarters after the shock.
- Output statistically significantly declines due to a positive tax shock.
- All multipliers are estimated with very small values such that an important role of fiscal policy in the stabilization process of output is not detected.
- If the model includes the exchange rate and imports from EMU member countries, the estimated responses of output to changes in net taxes is smaller.
- As for the two other model variations, a shock in net taxes has a higher ad hoc impact on output and is a more effective instrument in stabilizing output in the short run.
- The overall deficit rises with an expansionary spending shock and declines if taxes are raised. A rise in taxes lowers deficits more in the short run, but a reduction in government spending is more effective in the medium run.
- German fiscal policy indirectly lowers the exports of EMU member countries and thus the corresponding GDP.
- The home currency is directly affected: the currency appreciates with a positive government expenditure shock and depreciates with a positive net tax shock.

The period after the foundation of the EMU is not examined separately. Due to the high number of endogenous variables and the low number of observations, a meaningful discussion is not possible. The ad hoc responses are qualitatively the same for all variables but the 10% error bands become very large within a short period after the occurrence of a fiscal shock such that the uncertainty of response increases extraordinarily.

Finally, the model is augmented by the GDP components. The respective component is ordered prior to the imports from EMU member countries and past to output in the vector of endogenous variables. Similar to Table 5.9, the annualized cumulative responses are shown in Table 5.12.

As for all other model variations, the ad hoc response of consumption to a government expenditure shock is positive and very small. The response becomes negative after a while and is in most parts statistically insignificant. The estimated overall impact of government expenditure on consumption is negative and long lasting. If a positive shock in net taxes occur, we estimate first a negative response of consumption, which becomes positive and finally negative again. The overall impact is negative only for the first periods after the occurrence of the shock and statistically insignificant at a 10% statistical significance level.

| **Variable** | **G - 0** | **G - 4** | **G - 8** | **T1 - 0** | **T1 - 4** | **T1 - 8** |
|---|---|---|---|---|---|---|
| $cir^y$ | 0.041 | -0.152 | -0.292 | -0.096* | -0.074 | -0.031 |
| $cir^C$ | 0.01 | -0.026 | -0.173 | -0.023 | 0.069 | 0.099 |
| $cir^{GC}$ | 0.743* | 0.37* | 0.194 | -0.014 | 0.039 | 0.021 |
| $cir^{INV}$ | 0.201 | -0.749 | -1.805 | 0.264 | 0.383 | 0.673 |
| $cir^{EX}$ | -0.769* | -0.702 | -0.496 | -0.265 | -0.441 | -0.31 |
| $cir^{IM}$ | 0.098 | -0.153 | -0.66 | -0.468* | -0.414 | -0.511 |

* denotes statistical significance at 10% level.

Table 5.12: Influence of Fiscal Shocks on GDP Components - 7 Variables Model

Government consumption reacts as expected positively to an increase in government expenditures and shows a similar pattern as the response of government expenditure to a net tax shock. Overall, the responses do not differ in a qualitative way from the responses found in the other model variations.

Investment immediately increases due to a positive government spending shock. After some periods, it declines. The estimate is negative and statistically significant in that case. This is caused by the increase in the interest rate due to a positive government expenditure shock. Investment is crowded out by the increase in government expenditure. The impact of a net tax shock on investment is first positive and becomes negative after 10 periods. The intensity of response is high compared to the components of GDP discussed so far. The estimated overall effect of a shock in government expenditure on investment is negative and statistically significant from 5 periods after the shock onwards. Moreover, the estimated overall impact of a net tax shock on investment is positive for all 12 periods but statistically insignificant. Except of the ad hoc response, investment always responds to a spending shock in the opposite direction as if a shock in net taxes occurs. Thus, we can, as in the other model setups, conclude that Keynesian theory matches the empirical results.

Exports also respond with a high intensity: the period response is negative first and then fluctuates around the zero line for both fiscal shocks. The estimated overall responses are negative for both fiscal shocks. The same was true for the three and five variables model for the 12 analyzed periods. Imports are affected positively by a government spending shock only at the occurrence of the shock. The remaining period responses are estimated with negative values. The response to a net tax shock runs below the zero line for all 12 analyzed periods. When examining the overall impact, the government spending shock affects imports negatively except for the period of the shock occurrence. Moreover, the accumulated impulse response of imports to a net tax shock is negative for all periods and statistically significant for the first 3 quarters after the shock. The impact factors are high. The impacts of fiscal policy on overall imports and on imports from EMU member countries do not qualitatively differ from each other.

Next, we analyze the impact of the fiscal shocks on exports. Figure 5.19 shows the period impulse responses of exports. As described above, both period responses fluctuate around the zero line. The cumulative impulse responses of exports to fiscal shocks are given in Figure 5.20.

All curves indicate that fiscal policy has a long lasting negative effect on exports: the government spending shock causes the inflation rate to rise such that the price level rises. As a consequence, exports decline. In general, the home currency would depreciate with a higher inflation rate leading to a rise in exports. But the positive response of the interest rate caused by the fiscal expansion finally leads to an appreciation of the home currency such that home goods are more expensive for foreigners. Hence, exports lower. A net tax shock has a negative impact on exports because the private consumers from abroad expect higher prices due to an increase in taxes. As in the other model variations, the estimated cumulative impact of a fiscal shock on exports is negative.


Figure 5.19: Impulse Responses of Exports - 7 Variables Model

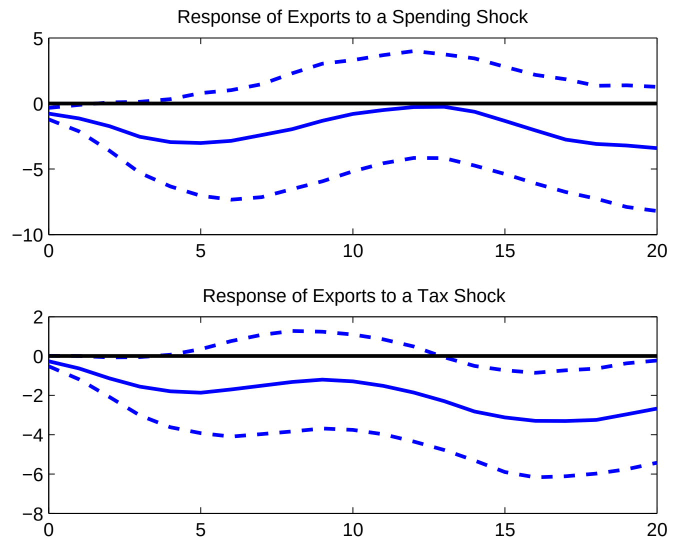


Figure 5.20: Accumulated Impulse Responses of Exports - 7 Variables Model

Overall, summing up the effect on all components of GDP delivers a significantly larger value than the overall response of output to the fiscal shock, but the estimated GDP component multipliers are smaller than in the other two model settings.

**Result 5.8.**

- Keynesian theory matches the impulse responses of investment to fiscal shocks.
- The response of consumption to an expenditure shock is not supported by classical or Keynesian theory.
- Consistently with the former models examined in Section 5.3 and 5.4, exports are affected negatively by increasing government spending or net taxes.

In the next subsection, we compare the seven variables model with a version of the model where the interest rate and the exchange rate channel is switched off such that fiscal shocks of one country do not influence the long-term interest rate or the currency (in terms of a perfect monetary union with one monetary authority). The model is simulated for 1991-2009. We assume that the two channels do not work since 1991, i.e., 9 years before the foundation of the EMU as approximation.

### 5.5.2 Results for the Reduced Form SVAR Model

After performing a VAR model estimation,[20] we get the point estimates obtained from the IV estimations of the VAR residuals shown in Table 5.13. Contrary to the 7 variables model, the found values are statistically significant.

| **Variable** | **DT** | | **ST** | |
|---|---|---|---|---|
| $\beta_{gt}$ | -0.091* | (-1.738) | -0.095* | (-1.857) |
| $\alpha_{yg}$ | 0.136* | (1.957) | 0.147** | (2.074) |
| $\alpha_{yt}$ | -0.173** | (-3.783) | -0.203** | (-4.254) |

* and ** denote statistical significance levels of 10% and 5%, respectively.

Table 5.13: Results for the Reduced Form Model

[20] The hypothesis of no autocorrelation is not rejected after a Portmanteau test.

The statistically insignificant small estimated coefficients of the seven variables model seem to be caused by the high number of endogenous variables relative to the low number of observations.

The Granger-causality test indicates that the inflation rate Granger-causes imports from EMU member countries. The same was found for the 7 variables model.

Table 5.14 shows the response of the overall deficit, output, the imports from EMU member countries, and the multipliers for 0, 4, and 8 periods after the occurrence of a positive government expenditure or a net tax shock.

| **Variable** | **G - 0** | **G - 4** | **G - 8** | **T1 - 0** | **T1 - 4** | **T1 - 8** |
|---|---|---|---|---|---|---|
| $cir^{def}$ | 0.789* | 0.634* | 1.154* | -1.116* | -0.528* | -0.565 |
| $cir^{y}$ | 0.092 | -0.17 | -0.298* | -0.189* | -0.109 | -0.085 |
| $cir^{g,CA}$ | 1.013* | 0.431* | 0.316* | -0.15 | 0.043 | 0.01 |
| $cir^{t,CA}$ | -0.003 | 0.174* | -0.157 | 1.513* | 0.865* | 1.039* |
| $m$ | 0.092 | -0.399 | -0.975 | -0.189* | -0.191 | -0.1 |
| $m^{CA}$ | 0.091 | -0.394 | -0.943* | -0.125* | -0.126 | -0.082 |
| $cir^{im_{EMU}}$ | -0.064 | -0.751* | -1.437* | -0.692* | -0.327 | -0.34 |

* denotes statistical significance at 10% level.

Table 5.14: Influence of a Spending and a Tax Shock - Reduced Form

As for the other model variation, overall deficits rise due to an increase in government expenditure and lower with an increase in net taxes. The ad hoc response of overall deficits and the impact for the 4th and the 8th period after the respective shock are statistically significant in almost all cases. As in the other model variations, a net tax shock affects overall deficits more for a short period of time compared to a government spending shock. Thus, increasing taxes by one unit is more effective in reducing deficits than lowering government expenditures by one unit in the short run. On the other hand, the reduction in the deficit is stronger in the medium run if government expenditure is reduced.

The response of output to a government expenditure shock is first estimated to be weakly positive but becomes negative after one period. The cumulative negative response 8 periods after the shock is even statistically significant. Overall, the cyclically adjusted government spending multiplier is small and becomes negative after one period.[21] Output declines, as in all other model setups, for a positive shock in net taxes. The net tax multiplier is negative and estimated with comparable values as for the 5 variables model. As for all other setups except of the seven variables model the estimated impact of a net tax shock on output is stronger than the impact found for a shock in spending. The imports from EMU member countries are affected negatively by both fiscal shocks. The estimates are almost all larger in absolute values compared to the ones found for the seven variables model. If we allow for a response of the interest rate and the exchange rate on a fiscal shock, the interdependencies between these variables and the imports from EMU member countries seem to weaken the effect of the fiscal shock.

The impulse responses of output and imports from EMU member countries are discussed in the following—see Figure 5.21 and 5.22 for the period and the cumulative impulse responses, respectively.

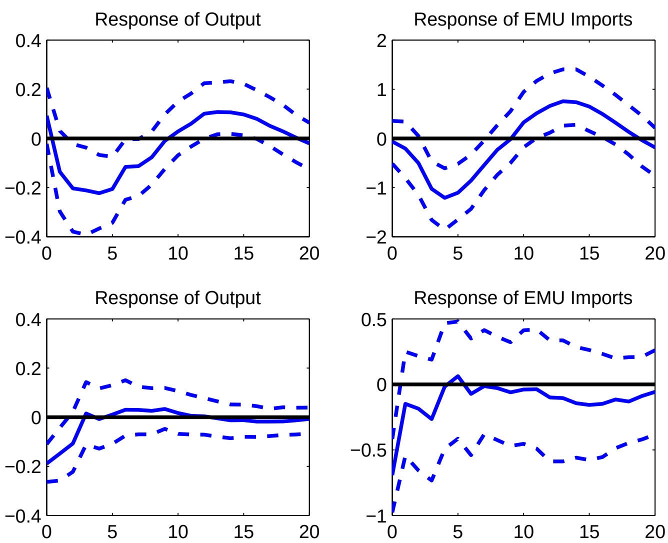


Figure 5.21: Impulse Responses of Output and EMU-Imports - Reduced Form

[21]This is due to the sharp decline in output after a first increase. See also Figure 5.21.

The impact of the examined fiscal shocks on output and imports from EMU member countries are closely the same. Similar to the other model variations, output increases immediately after the occurrence of the shock, then declines before increasing again. The estimated negative response is statistically significant after one year. Almost the same is true for imports from EMU member countries. The ad hoc response to a government spending shock is positive and the impact becomes negative after a while.

Also in the occurrence of a net tax shock, the response pattern of the two examined real variables is comparable. Output and imports from EMU member countries respond statistically significantly negative to a positive one unit net tax shock. Afterwards, both variables rise, staying close to the zero line for the remaining periods.

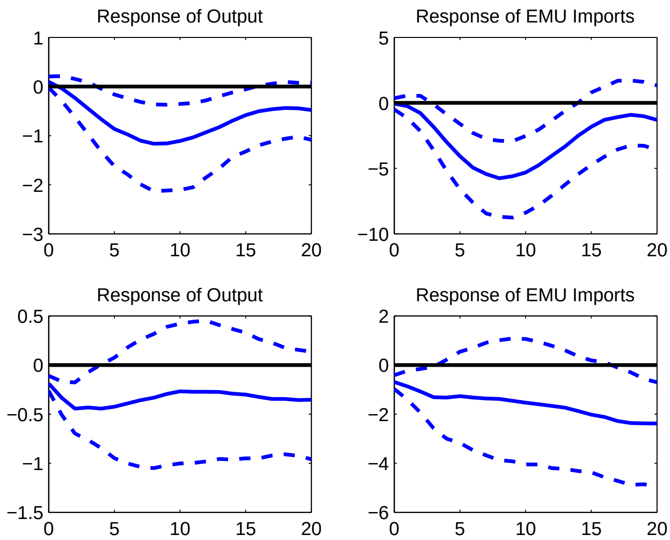


Figure 5.22: Accumulated Responses of Output and EMU-Imports - Reduced Form

The overall impact of fiscal shocks on the two variables is negative—see Figure 5.22. The pattern of the curves for output and imports from EMU member countries is closely the same. Raising government spending affects both variables first positively, but is afterwards followed by a long lasting decline. The negative response is statistically significant at a 10% statistical significance level 4-12 and 5-12 periods after the occurrence of the shock for EMU imports and output, respectively. A positive net tax shock affects both variables negatively for the simulated periods but the results are only statistically significant for a short simulation period.

Figure 5.23 and 5.24 show the period and the cumulative impulse responses of the inflation rate to fiscal shocks.

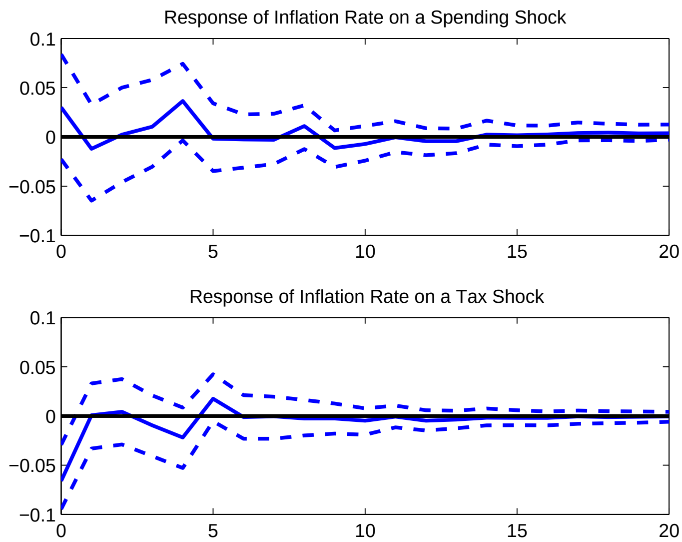


Figure 5.23: Impulse Responses of the Inflation Rate - Reduced Form

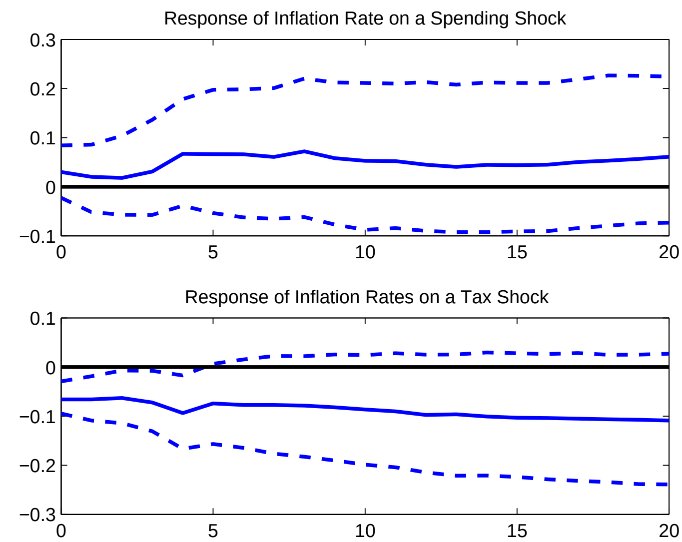


Figure 5.24: Accumulated Responses of the Inflation Rate - Reduced Form

The inflation rate rises after a government spending shock and declines after a net tax shock. Both curves fluctuate around the zero line and the estimated impact of the fiscal shocks weakens over time.

The accumulated responses of the inflation rate shown in Figure 5.24 are positive for a government expenditure shock and negative for a net tax shock. Both effects are weak and long lasting. The response to a government expenditure shock is statistically insignificant whereas the response to a net tax shock is statistically significant for the first 4 simulation periods. Overall, they are qualitatively the same as the estimated responses for the 5 variables model.

We can summarize the following.

**Result 5.9.** Expenditure vs. Net Tax Shock (Reduced Form Model)

- Output rises very weakly due to a positive shock in government spending but declines one period after the shock and becomes negative. The negative response is statistically significant 5 quarters after the shock.
- Output declines after a positive shock in net taxes.
- All multipliers are estimated with small values.
- As for all other model variations, a shock in net taxes has a higher ad hoc impact on output and seems to be a more effective instrument in stabilizing output in the short run.
- The overall deficit rises due to an expansionary spending shock and declines if taxes are raised. This result is robust to all model variations. A rise in taxes lowers deficits more in the short run, but a reduction in government spending is more effective in the medium run.
- German fiscal policy indirectly lowers the exports of EMU member countries and thus the corresponding GDP. The impact of the fiscal shocks on imports from EMU member countries is high.

We closer analyze the period after Germany's entry to the EMU (1999-2009) in the following. The elasticities needed to identify the fiscal shocks are estimated for the subsample period. The elasticity of net taxes with respect to output is 1.0094 (for the whole sample 2.0857). Hence, the output elasticity of net taxes differs significantly for the whole and the subsample period. In 1991-1998, there seems to be a strong influence of output on net taxes. In 1999-2009, the German tax law changes such that the tax revenue in percent of GDP declines. Hence, a one unit change in output changes net tax revenue by a lower number of units for 1999-2009. For more information on changes in tax laws see, e.g., Hayo and Uhl [59].

The price elasticity of net taxes is 1.0279 (for the whole sample 1.0199). Moreover, the import elasticity of net taxes is 0.0747 (for the whole sample 0.0731). As before, all other institutional information is set exogenously and is therefore independent of the time horizon.

The qualitative output response to a government spending shock is similar to the one found for the whole period, but the intensity of the estimated effect is different. The ad hoc response of output to a government spending shock is $0.075 < 0.092$ where the value on the right hand side denotes the respective estimate for the whole sample. It holds that $0.174 > -0.04$ for the first period after the shock.[22] Thus, the estimated response is positive for a longer time period, but the decline in output after a while is more severe for the subsample. I.e., a shock in government expenditure affects output stronger if the estimated response is negative.

In the occurrence of a net tax shock, the ad hoc response of output is estimated as $-0.248 < -0.189$ and the response of the first period is $-0.644 < -0.337$.[23] Raising net taxes lowers output more in the subsample than for the whole period.

Contrary to the impulse response of the inflation rate to a government expenditure shock for the whole sample, the estimated response for the subsample is negative for most parts of the simulation.

The inflation rate response to a net tax shock is qualitatively the same for both analyzed periods.

The imports from EMU member countries respond qualitatively similar to both fiscal shocks for the two analyzed periods. The estimated impact of fiscal policy on the EMU-imports is higher after the German accession to the EMU.

[22] The responses of the 4th and the 8th quarter are $-0.172 < -0.17$ and $-0.692 < -0.298$, respectively.

[23] The responses for the 4th and the 8th period are $-0.167 < -0.109$ and $-0.338 < -0.085$, respectively.

**Result 5.10.** The Reduced Form Model - 1999-2009

- Net tax shocks affect output more intensely after the entry of Germany to the EMU. Furthermore, output responds more strongly to a government expenditure shock in this period.
- The response of the inflation rate is mostly negative for a government spending shock.
- The imports from EMU member countries decline more due to fiscal shocks in the subsample period.

We finally discuss the impulse responses of the respective GDP components private consumption, government consumption, investment, exports, and imports to the fiscal shocks. As before, the additional variables are ranked past to the log output per capita. Almost all annualized ad hoc responses of the GDP components and output shown in Table 5.15 are more intense after the occurrence of a fiscal shock if we cancel out the interdependencies of the interest rate and the exchange rate to the other variables, i.e., if we implement the two variables as exogenous regressors. Also most other cumulative responses are estimated with higher values in the reduced form model. The additional variables in the seven variables model cause that the estimated impact of fiscal shocks to the GDP components is smaller.

| **Variable** | **G - 0** | **G - 4** | **G - 8** | **T1 - 0** | **T1 - 4** | **T1 - 8** |
|---|---|---|---|---|---|---|
| $cir^y$ | 0.092 | -0.17 | -0.298* | -0.189* | -0.109 | -0.085 |
| $cir^C$ | 0.094 | 0.048 | -0.051 | -0.121* | 0.061 | 0.174 |
| $cir^{GC}$ | 0.743* | 0.36* | 0.213 | -0.074 | 0.016 | 0 |
| $cir^{INV}$ | 0.351 | -0.767 | -1.696* | 0.051 | 0.437 | 0.846 |
| $cir^{EX}$ | -0.734* | -1.041* | -1.097* | -0.388* | -0.371 | -0.486 |
| $cir^{IM}$ | 0.066 | -0.625 | -1.012 | -0.511* | -0.363 | -0.48 |

* denotes statistical significance at 10% level.

Table 5.15: Influence of Fiscal Shocks on GDP Components - Reduced Form

The responses of consumption correspond qualitatively to the ones found in the former

model settings: the overall impact of a government expenditure shock is in most parts negative and statistically insignificant. Consumption responds positively from the 2nd period onwards after a transitory rise in net taxes. The cumulative response is statistically significant from 2 years after the shock onwards.
As in all other setups, government consumption responds positively and strongly to a government expenditure shock and only weakly to a net tax shock.
Investment responds similar as in the former model variations. The single responses and the overall responses of the two respective fiscal shocks are almost reflections of each other such that empirical results fit to Keynesian theory.
Exports and imports mainly respond negatively to fiscal shocks. The results are qualitatively equal to the ones found for the 5 and 7 variables model. Imports respond similarly for the seven and the reduced form model. The estimated overall effects of the two fiscal shocks are negative, long lasting, and more intense for a government spending shock if the interest rate and the exchange rate channel are working.[24]
We finally closer examine the response on exports in Figure 5.25 and 5.26.

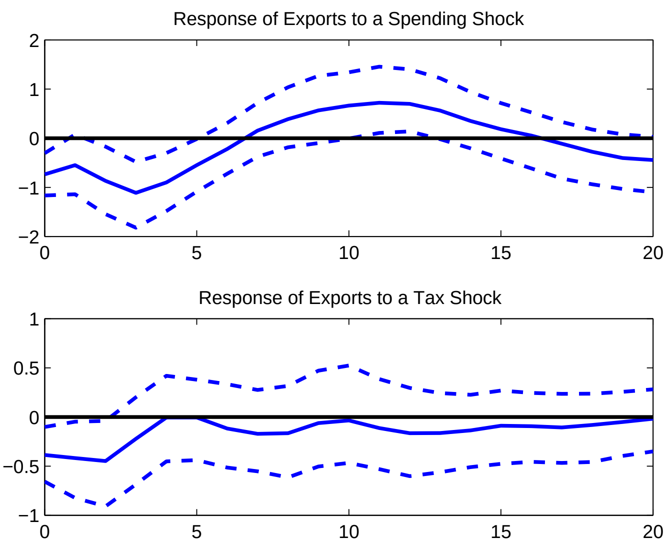


Figure 5.25: Impulse Responses of Exports - Reduced Form

[24]We found a comparable result for the EMU imports.

Similar to the other model variations, exports decline due to a rise in government spending for the first periods after the shock and then overshoot the zero line—see for comparison the seven variables model. The intensity of the estimated effect is stronger for a government spending shock. If a positive net tax shock occurs, exports respond negatively for all simulated periods.

The estimated accumulated responses shown in Figure 5.26 indicate that exports decline due to the two fiscal shocks. The negative cumulative responses for both shocks are statistically significant for at least 2.5 years. Switching off the exchange rate and interest rate channel intensifies the medium run impact of a government expenditure shock on exports.[25] Moreover, exports are affected more by a net tax shock in the first periods after the shock. Since the inflation rate responds statistically significantly negative, the Euro appreciates with respect to other currencies. Exports are more expensive and thus decline. This effect accompanies the reduction of exports due to the expectation of foreigners that prices will rise in the future if net taxes rise.


Figure 5.26: Accumulated Responses of Exports - Reduced Form

[25]Since the inflation rate response is statistically insignificant, it cannot be used as explanation.

The estimated overall impacts of a net tax shock for the seven variables and the reduced form variables model are not too different. Thus, the interest and the exchange rate do not seem to play a significant role, if net tax shocks change exports. This is compatible with the explanation that exports mainly decrease due to a change in taxes and thus the expectation on the future price development, see also the discussion in Subsection 5.5.1.

**Result 5.11.**

- Keynesian theory matches the impulse responses of investment to fiscal shocks.
- The response of consumption cannot be used to decide, which theory is supported by the empirical results.
- Consistently with all former setups, exports are affected negatively when increasing government spending or net taxes. The responses are statistically significant for at least 2.5 years.

## 5.6 Summary of Results

In this section, we summarize the results that are robust for all model settings in the given time period 1991-2009.[26] Possible explanation approaches are discussed.
The first two results describe the response of the overall deficits to fiscal policy in terms of an expansion in government spending or net taxes, respectively. We try to give an answer on the everlasting question which instrument is preferable to influence debt in the short and the medium run:

1. Deficits rise if government expenditures are raised and decline if the net tax revenues are reduced.

If the government expenditures increase, the short run rise in net tax revenue caused by the expansive effect of the shock on output cannot compensate for the expansion in spending. The response is too weak compared to the intensity of the rise in government expenditures. Thus, overall deficits rise in the short run. After some periods, the net

[26] We abstain from augmenting the time horizon and leave this for future research.

tax response becomes negative due to the estimated decline in output as a consequence of the expenditure shock and even strengthens the increase in deficits. Moreover, if the net tax revenue is raised, the estimated government expenditure response is negative. Thus, overall deficits decline.

2. Changes in net tax revenue are more effective in changing deficits in the short run. Moreover, deficits are affected more by a change in government expenditure in the medium run.

The negative ad hoc response of government expenditure to a positive net tax shock intensifies the reduction of the overall deficit in the short run.
The positive response of net taxes weakens the positive response of overall deficits to a rise in government spending. Output declines due to a positive shock in government expenditure after a while such that the net tax revenue is strongly reduced.
Overall, the expansion in expenditure is accompanied by a contraction in net tax revenues and leads to a strong rise of deficits in the medium run. As a consequence, if Germany conducts a policy that aims to reduce debt in a short period of time, taxes should be increased. Contrary, if Germany aims to effectively reduce debt in the medium run, accepting a weaker effect of policy in the short run, it should reduce government spending.

3. Output rises only weakly due to a positive shock in government expenditure. After some periods, the impulse response becomes negative.

In the very first periods after the occurrence of the expansionary fiscal policy action of raising government expenditure, output increases. But despite recent theory, output declines after some periods. This result is robust to all model specifications.
If we analyze the impulse responses of the GDP components, the estimated effect of investment and output are qualitatively the same. One hypothesis is that the response of output is driven by the investment response. Since the interest rate rises due to a

government expenditure shock, private investment might be strongly crowded out.[27]
A different explanation approach for the decline in output due to a positive government expenditure shock, which does not depend on the endogenous model variables is the following: if a government is already indebted at a high level as it is true for Germany, an additional increase in expenditure and thus an extension of overall deficits being a contribution to the existing debt, raises the scepticism of the economic entities to fiscal policy. Thus, the credibility channel of fiscal policy is disturbed and the economic agents expect an economic crisis such that they reduce their economic activities. As a consequence, output declines.

Moreover, the structure of the German government expenditure aggregate could give some hints on the output response pattern: on one hand, spending is used to stabilize output, on the other hand, it is used to stabilize the labor market. The reduction of unemployment as well as the stabilization of the labor market in the economic crisis has been successful in the last years. But in particular in Germany, a rise in labor does not necessarily lead to a rise in output.[28] To decide whether this hypothesis fits to the data, the government expenditure aggregate and its effect on the labor market has to be analyzed in greater detail. A closer analysis is left for future research.

A last explanation approach discussed in detail would be the methodological weakness of the SVAR model. The identification process of shocks and thus the impulse responses depend crucially on the institutional information, in detail, on the estimated elasticities. Thus, we cross-check our results by implementing elasticities that are estimated in comparable studies as in Heppke-Falk et al. [61] or Bode et al. [22]. The elasticities of net taxes with respect to output were calculated with smaller values. This was due to the chosen aggregation methods and time horizons. For a closer discussion, see also Section 4.3.1 of Chapter 4. If we use the smaller elasticities of net taxes with respect to output,

[27] The reduction in investment could also be caused by the actual situation in 2007-2009. Thus, we analyze the correlation of investment to the IFO climate, the IFO situation, or the IFO expectation index to decide whether the changes in investment could be explained by the actual economic situation. No statistically significant correlation is found.

[28] Amongst others, Christensen et al. [34] estimate a negative contribution of labor to growth for Germany in the context of growth accounting.

the cumulative impulse response of output to a government expenditure shock stays positive for a longer simulation period. This result is robust to all model variations. If we especially compare the results of the five variables model including the output elasticity of net taxes used by Heppke-Falk et al. [61] to our empirical results the output responses are qualitatively almost the same and output increases due to the shock. Moreover, the decline of output due to a rise in government expenditure takes place at the same time period as in the original study. As a consequence, if we implement different institutional information, the estimated decline in output is statistically significant in an advanced simulation period.
But the elasticities we derive in Chapter 4 are reasonable and consistent with the used data set and methodology.
Robustness of the shock identification could be tested by identifying the fiscal shocks using different methods as, e.g., a Choleski decomposition.

4. Output declines due to a positive shock in net taxes. The estimated response is statistically significant at least for some quarters after the shock.

Since net taxes are raised, fiscal policy is contractionary and affects output negatively. This fits to economic theory. Contrary to the results found in this study, Heppke-Falk et al. [61] estimate very small and statistically insignificant tax multipliers such that they conclude that varying taxes does not change output statistically significantly.

5. In general, the spending and tax multipliers are estimated with weak values. But the ad hoc response of output to a tax shock is stronger vis-a-vis the spending multiplier.

We estimate that output is more affected by a change in net taxes in the short run for all model variations examined in the study. This is mainly due to the fact that the estimated spending multiplier in the occurrence of a spending shock is very small.
All multipliers indicate that fiscal policy does not seem to be a very effective instrument in stabilizing output in Germany. The impact factors are estimated only with weak val-

ues. The same was found in other comparable studies with German data—see, amongst others, Heppke-Falk et al. [61].

6. Except of the reduced form model, the impact of a rise in net taxes on output weakens for 1999-2009.

We find for almost all model specifications that net taxes affect output with a lower intensity since Germany's accession to the European Monetary Union. The results for the reduced form model suggest that fiscal tax decisions are even more effective after the entry to the EMU. Thus, overall, we cannot draw a definite conclusion.

7. The inflation rate rises for most analyzed simulation periods due to a positive government expenditure shock and declines if net taxes are raised.

Thus, an expansionary fiscal shock (a rise in government spending) raises the inflation rate and the price level increases. The opposite is true for a restrictive fiscal policy action in terms of a rise in net taxes: it lowers the inflation rate. The responses are not very strong and often of weak statistical significance.

8. The impulse response of consumption to a government expenditure shock cannot be used to decide which theory fits the empirical results best. However, Keynesian theory matches the investment responses to fiscal shocks.

For the first simulation period, the consumption response to a government expenditure shock is positive. But contrary to Heppke-Falk et al. [61], the estimated cumulative impulse response of consumption after 4 quarters is negative and statistically insignificant. Since consumption depends on income and output declines due to a positive government expenditure shock, consumption decreases. Thus, we cannot decide with the consumption response which theory matches the empirical results. Moreover, contrary to Heppke-Falk et al. [61], investment responds statistically significantly to fiscal shocks. The responses suggest that Keynesian theory matches the empirical results since they respond conversely to a positive government expenditure and a positive net tax shock.

9. Exports decline statistically significantly due to fiscal shocks with high impact factors.

There are spill-over effects of German fiscal policy to other countries. An explanation was partly given in the sections including the interest, the inflation, and the exchange rate as endogenous variables: exports decrease due to a rise in government expenditure (to an expansionary fiscal stimulus) because the expansion causes the inflation rate to rise such that the price level rises. Goods are more expensive if we assume that the exchange rate does not adjust such that the foreign demand lowers. The interest rate increases due to the additional deficit, which is partly financed by issuing bonds. The higher interest rate attracts foreign investors such that they acquire domestic assets. As a consequence, the home currency appreciates such that domestically produced goods are more expensive for foreigners. This lowers exports additionally. If the interest rate does not rise leading to a currency appreciation, the higher inflation rate would cause the currency to depreciate and exports would be cheaper for foreigners such that they augment. Thus, the interest rate response prevents a rise in exports due to expansionary fiscal policy in terms of an increase in government expenditure.
A net tax shock has a negative impact on exports because the private consumers from abroad expect higher prices due to an increase in taxes.

10. Imports from EMU member countries are affected negatively and strongly in intensity due to fiscal shocks.

The result is robust to the reduced form model, where the interest rate and the exchange rate channel are switched off. In an economic environment being a monetary union or not, national fiscal policy lowers the imports from EMU member countries. The inflation rate rises if government spending is raised and causes the imports to decline, whereas the expectation of higher prices due to a rise in net taxes leads to a decline in imports, too. As a consequence, exports and the GDP of EMU member countries are affected negatively by a rise in German government expenditures or a rise in German net taxes. Overall, since the responses of exports and EMU-imports to fiscal shocks are statistically significant at least for some periods, we detect spill-overs of German fiscal policy to foreign countries.

## 5.7 Conclusions

We have studied the effect of fiscal shocks in an SVAR framework in the underlying chapter and have tried to answer the question whether fiscal policy is effective in stabilizing output and how economic variables are affected by fiscal shocks. Different model settings have been used to analyze the robustness of results. We have found the following: a positive shock in government expenditure leads to an increase in overall deficits, whereas a rise in the net tax revenue lowers the deficit. Changing government spending is more effective in the medium run, whereas tax changes seem to be more effective in the short run.

When comparing a positive government spending shock with a positive shock in net taxes, output first responds as expected, it rises due to an expansion in government expenditure and declines due to the restrictive tax shock. But after some periods, output sharply declines due to the expansionary spending shock for all model variations. There were several approaches to explain this response. We observed a strong decline in investment, which could cause the decline in output: if the crowding out effect of investment is strong enough, output could even decline due to the expansionary shock. Moreover, the negative response of output to a positive government spending shock could be due to a loss in credibility of the fiscal policy institution. By raising government spending, debt rises and individuals lower their economic activity. A different approach was that the additional spending in the case of a positive expenditure shock is mainly used to stabilize the labor market such that output is not extended. Moreover, one can argue that the methodology of an SVAR model depends on institutional information and is thus sensitive to the estimated output and price elasticities lowering the robustness of results. A shock in net taxes has a long lasting negative overall impact on output. The restrictive fiscal policy action affects output as expected.

Overall, the estimated impact of fiscal shocks is very low. Thus, fiscal policy does not seem to be a strong instrument in stabilizing output.

Contrary to the weak influence of fiscal shocks on output, the estimated impact on other components of GDP is stronger. Investment, exports, and imports respond with a high intensity to fiscal shocks. The inflation rate responds in most parts positively to a government spending shock and negatively to a net tax shock. The impact on inflation is not very strong and in some cases statistically insignificant.

Exports are affected negatively by both fiscal shocks. The estimated impact factor is high. Thus, German fiscal policy affects the import behavior of foreign countries. Moreover, the last two model variations (the 7 variables and the reduced form model) have analyzed if there are spill-over effects of German fiscal policy specifically on other EMU member countries. EMU-imports decline due to a fiscal shock with high estimated impact factors. As a consequence, the exports and thus the GDP of the respective EMU member countries are affected.

Fiscal shocks lower exports and the imports from EMU member countries statistically significantly if we assume that the exchange rate channel and the interest rate channel does not work, which would be true in a perfectly functioning European Monetary Union with one currency and one monetary policy authority.

We have found that fiscal policy seems to be not very effective in stabilizing the GDP but that the negative effect on exports and imports is significant. Overall, the empirical results indicate that the EMU is sensitive to German fiscal shocks. We detect severe spill-over effects.

As a consequence, single EMU member countries should pursue fiscal policy very carefully such that they do not jeopardize the stability of the whole monetary union as economic entity.

A fiscal coordination might be a valuable tool to strengthen the economic standing of the EMU.

# Chapter 6

# Conclusions and Outlook

In times of the late 2000s economic crisis, lasting debt problems in member countries of the European Monetary Union as Greece and Italy, and necessary fiscal policy decisions as the stepwise enhancement of the European Financial Stability Facility (EFSF), the analysis of fiscal policy is more important than ever.

The underlying thesis highlights different aspects of this complex topic as, e.g., the influence of the public debt structure on the economic performance of a country. The role of external debt or, in particular, external debt defaults are studied in the literature—see amongst others Eichengreen [43], Eichengreen and Lindert [44], or Lindert and Morton [86].

However, domestic public debt is analyzed only rarely so far. In most theoretical models, it is assumed that government debt is always honored such that there is no need of policy decisions or such that a debt reduction policy is irrelevant because Ricardian equivalence holds as in Barro [7]. Furthermore, domestic public debt is mostly included in the former type of models as the driving force for the level of prices operating through the budget constraint of the government—see Woodford [128] or with overlapping generations—see Diamond [41].[1]

There are only few available data for domestic public debt[2] and almost no empirical

[1]See also Reinhart and Rogoff [112].

[2]This is true particularly for data prior to 1980.

studies. The analysis of domestic debt defaults, which took place in the past, for example in Argentina in 1989, is neglected in most parts. Reinhart and Rogoff [112] suggest that the public domestic debt should be studied in more detail to fully understand the public sector. They give a starting point by describing domestic debt crises. Moreover, they develop a historical database for internal public debt (Reinhart and Rogoff [113]).
As shown in Chapter 2 of the underlying thesis, where an economy with different types of debt has been compared, the relation between internal and external public debt, that means its composition, is crucial for the performance of the economy if debt rises. Particularly, the analysis of domestic public debt in fiscal policy decisions might be one important topic for future academic and policy research.
Overall, it may be worthwhile that fiscal policy institutions take the debt structure into account to ensure a precise efficacy of fiscal stimuli.
The European Monetary Union (EMU) as a special economic entity of national countries with independent fiscal authorities and one union-wide central bank (the European Central Bank, ECB) is subject of many studies. The compatibility of fiscal and monetary policy as well as the consequences of this special economic structure are widely discussed—see, e.g., the chapters published in Butti [29]. Moreover, the fiscal and monetary behavior before and after the establishment of the EMU are compared. With the new economic structure, the ECB conducts monetary policy with the main aim to preserve price stability within the union. The national fiscal policy behavior is constrained by the Stability and Growth Pact requiring that national deficits never exceed 3% of GDP. It seems reasonable to suppose that the policy behavior is different for the pre-EMU and past-EMU period. Ballabriga and Martinez-Mongay [6] show that policy changed less drastic than anticipated before the establishment of the EMU.
Since some member countries of the European Monetary Union are indebted with very high rates, the analysis of debt shocks becomes more important than ever. For example the debt to GDP ratio in Greece grew rapidly from 110.7% in 2008 (being 127.1% in 2009 and 142.8% in 2010) to about 165% in 2011. This increase was not gradual over many years but rather resembled a shock.

We examined in Chapter 3 of this thesis the national effect of a rise in debt and compared different debt levels as a starting point for a debt shock analysis. Direct and indirect taxes were used as national fiscal instruments to reduce debt without causing high welfare losses. In this context, the impact of fiscal spill-overs and the role of fiscal coordination across member countries of the EMU can be seen as one important topic for future academic research. With the European Financial Stability Facility (EFSF), a first move of fiscal coordination has been made such that time series data will be available soon to cross-check theoretical findings in this field of research.

Chapter 4 and 5 of the underlying thesis have provided an empirical analysis of German fiscal policy. In particular, we have stated in Chapter 4 that there are periods of contractionary as well as expansionary fiscal policy as shown in the following figure:

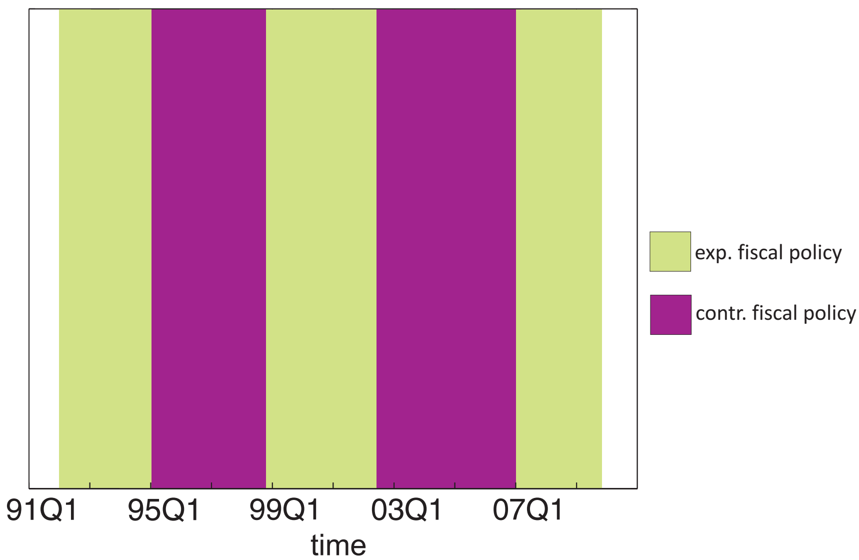


Figure 6.1: Periods of Restrictive and Expansionary Fiscal Policy in Germany

According to economic theory, the respective chosen manner of fiscal policy influences the GDP. If the government pursues an expansionary fiscal policy, the GDP should increase whereas it should decrease if restrictive fiscal policy is conducted. Figure 6.2 shows the development of real GDP in times of expansionary or contractionary fiscal policy in Germany for 1991-2009. Two factors of influence are reflected in the run of the curve: the real GDP data show fluctuations, which are partly due to business cycles and partly due to fiscal decision responses.

In the first period of expansionary fiscal policy (1991-1995), GDP first decreases but rises in the course of time. For the first purple area, i.e., restrictive fiscal policy in 1995-1999, it cannot be observed that GDP decreases or that the increase in GDP weakens. The GDP data series is interfered with an opposing business cycle fluctuation or fiscal policy does not work as expected. 1999-2000 is a period with high growth rates as well as expansionary fiscal policy. This fits to our expectation. Also for 2000-2002, expansionary policy is conducted, but the observed high increase of GDP weakens in this period. For 2002-2005, restrictive fiscal policy is conducted and the increase in GDP seems to be effectively damped. In the second half of this last period of restrictive fiscal policy (2005-2007), GDP strongly increases although restrictive policy is pursued. Finally, in 2009, the sharp decline in GDP indicates that the economic crisis takes place in a period of expansionary fiscal policy.

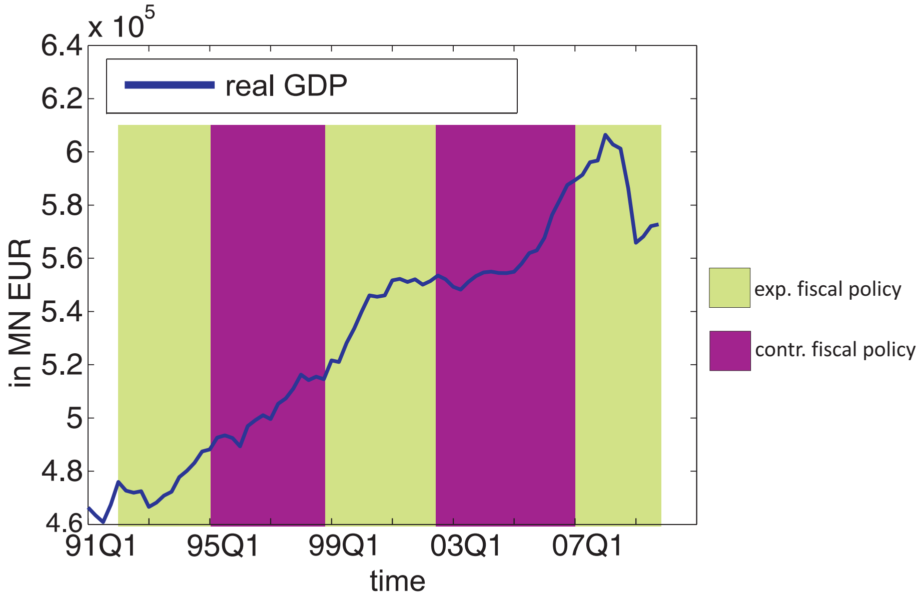


Figure 6.2: The GDP in Periods of Contractionary and Expansionary Fiscal Policy

Thus, the effectiveness of fiscal policy has to be discussed. For some periods, the fiscal stimulus seems to be strong enough to affect GDP in the expected way. But there are also periods, where the fiscal actions are too weak to influence GDP sufficiently. In other periods, fiscal policy does not seem to affect GDP at all in the expected way as, e.g., for restrictive fiscal policy in 2006 when cyclical fluctuations entirely overlay the fiscal effect.

Not only the GDP but also the actual budget balance responds to fiscal policy according to theory. The actual budget balance in the underlying context is defined as the difference of government tax revenue and government spending. Since contractionary fiscal policy is characterized as a reduction in government spending or a raise in net taxes, the budget balance increases in that case. Conversely, if expansionary fiscal policy is conducted (i.e., government spending is raised or net taxes lowered), the budget balance decreases. Figure 6.3 shows the actual budget balance in percent of GDP.

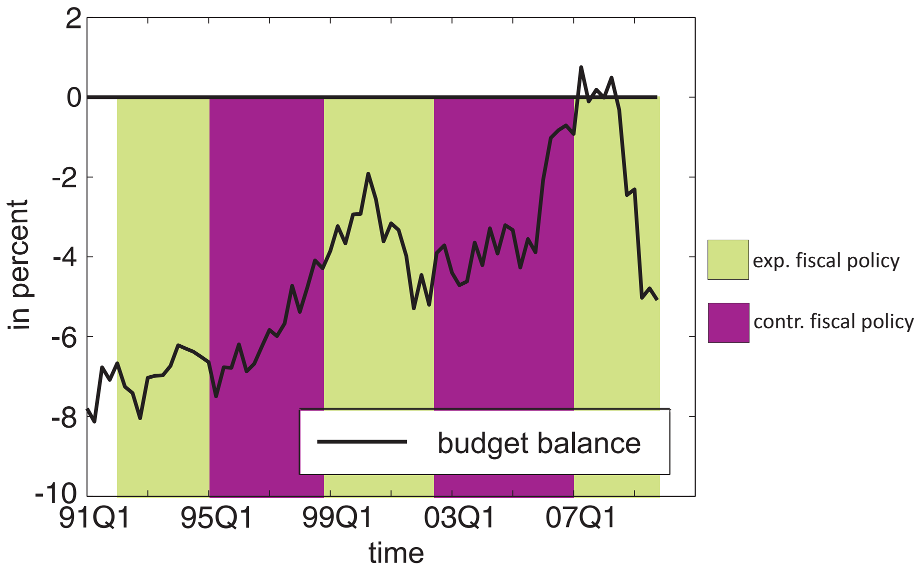


Figure 6.3: The Budget Balance with Restrictive and Expansionary Fiscal Policy

If restrictive and expansionary fiscal policy would be effective in a way perfectly compatible with theory, the budget balance in percent of GDP would show a clear pattern: with restrictive fiscal policy, the budget balance should increase and GDP should decrease such that the ratio of these two components overall would increase. The converse is true for expansionary fiscal policy: in that case, the budget balance should decrease and GDP should increase such that the ratio decreases. But GDP does not respond in the expected way, probably occasionally influenced by the business cycle. Nonetheless, the actual budget balance in percent of GDP is affected in the theoretically expected way. In periods of contractionary fiscal policy (purple area), the actual budget balance in percent of the actual GDP increases whereas it decreases within a period of expansionary fiscal policy (green area). Especially the strong fall in 2000-2003 and in 2007-2009 as well as the strong increase in 2006-2007 illustrates the direct influence of the type of fiscal policy on the budget balance, which seems to be so strong that it outweighs the unexpected response of output on the respective type of fiscal policy.

Additional to the results discussed so far, we have found in Chapter 4 that, taking the business cycle into account, there are periods, where fiscal policy is conducted pro-cyclically as well as periods, where fiscal policy is countercyclical. From a theoretical point of view, countercyclical fiscal policy is expected to support the economic performance of an economy. The following figure summarizes this result:

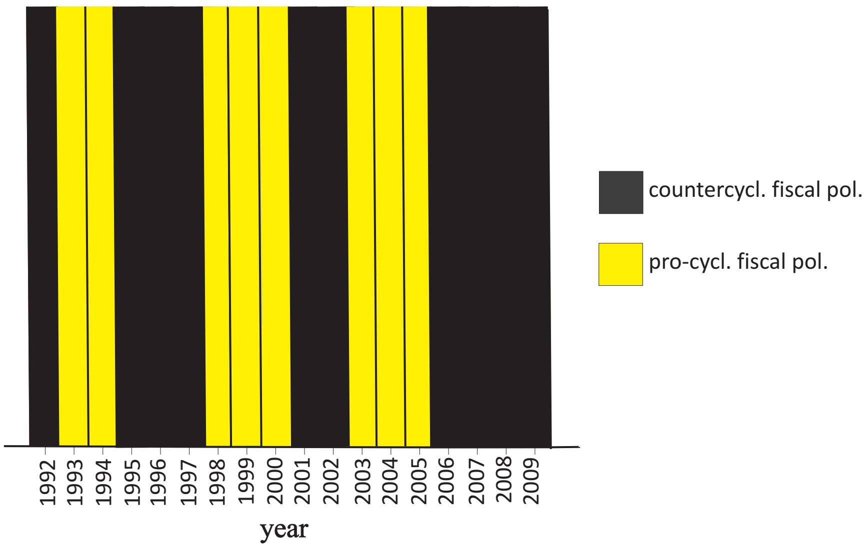


Figure 6.4: Pro- and Countercyclical Fiscal Policy in Germany

Since there are also long lasting periods of pro-cyclical fiscal policy as in 1998-2000 or in 2003-2005, the German government does not seem to make fiscal policy decisions solely due to the economic situation and thus does not pursue countercyclical fiscal policy as suggested by Keynesian theory. The periods of pro-cyclical fiscal policy are mostly periods of expansionary fiscal decisions–compare also Figure 6.1. Hence, the German government seems to raise expenditures often independent of the economic situation, which is rather strange and needs further investigations.

In Chapter 5, the effect of fiscal policy on economic variables as output has been analyzed using SVAR techniques. In this context, the response of output to a positive government spending shock differs from the standard reaction of output to an expansionary fiscal shock based on economic, in particular, on Keynesian theory. Output declines some periods after the shock. One important topic for future research would be to verify the suggested explanations in Chapter 5. In particular, German government expenditures have a special structure since they are used not only to stabilize output, but also in major parts to stabilize the labor market. This labor market stabilization seems to be successful so far and could perhaps be used to explain the decline in output. More work is needed to test the formulated hypotheses.

The empirical findings in Chapter 5 have indicated that there are spill-over effects of German fiscal policy on other EMU member countries or even on the rest of the world. As a consequence, fiscal coordination may be a valuable tool in stabilizing the European Monetary Union as one of the largest existing economic entities.

# Appendix A

## A.1 Optimal Choices for the Baseline Model

The following section shows the optimal choices of consumption and saving for the initial parents- and the children-generation without the restrictive assumption that $\sigma = 1$. The optimal consumption choice of the initial old generation only depends on the initial endowment and the interest rate in period 2. Hence, it is independent of $\sigma$, the coefficient of relative risk aversion.

The optimal decisions concerning the consumption and saving in period 2 and the consumption in period 3 for the initial parents-generation is given by

$$\begin{aligned}
\check{C}_{2,2} &= \left(\tilde{w}_2 - \tilde{d}_2\right)\left(1 - \frac{1}{1+\beta^{-1/\sigma}\left(1+r_3-\delta\right)^{1-1/\sigma}}\right), \\
\check{S}_{2,2} &= \frac{\left(\tilde{w}_2 - \tilde{d}_2\right)}{1+\beta^{-1/\sigma}\left(1+r_3-\delta\right)^{1-1/\sigma}}, \text{ and} \\
\check{C}_{3,3} &= \left(1+r_3-\delta\right)\check{S}_{2,2} = \frac{\left(1+r_3-\delta\right)\left(\tilde{w}_2 - \tilde{d}_2\right)}{1+\beta^{-1/\sigma}\left(1+r_3-\delta\right)^{1-1/\sigma}}.
\end{aligned}$$

Finally, using the first order conditions of the maximization problem of the children-generation, we obtain as optimal choices for consumption and saving

$$\check{C}_{1,t} = \frac{\left(\tilde{w}_{t+1} - \tilde{d}_{t+1}\right)/\left(1+r_{t+1}-\delta\right) + \tilde{d}_t/\left(1+g_n\right)}{\left[1+\left(1+\beta^{1/\sigma}\left(1+r_{t+2}-\delta\right)^{1/\sigma-1}\right)\beta^{1/\sigma}\left(1+r_{t+1}-\delta\right)^{1/\sigma-1}\right]},$$

$$\begin{aligned}
\check{S}_{1,t} &= -\frac{\left(\tilde{w}_{t+1}-\tilde{d}_{t+1}\right)/\left(1+r_{t+1}-\delta\right)+\tilde{d}_t/\left(1+g_n\right)}{\left[1+\left(1+\beta^{1/\sigma}\left(1+r_{t+2}-\delta\right)^{1/\sigma-1}\right)\beta^{1/\sigma}\left(1+r_{t+1}-\delta\right)^{1/\sigma-1}\right]} \\
&\quad +\frac{\tilde{d}_t}{1+g_n}, \\
\check{C}_{2,t+1} &= \frac{\beta^{1/\sigma}\left(1+r_{t+1}-\delta\right)^{1/\sigma-1}\left[\left(\tilde{w}_{t+1}-\tilde{d}_{t+1}\right)+\frac{\tilde{d}_t}{1+g_n}\left(1+r_{t+1}-\delta\right)\right]}{\left[1+\left(1+\beta^{1/\sigma}\left(1+r_{t+2}-\delta\right)^{1/\sigma-1}\right)\beta^{1/\sigma}\left(1+r_{t+1}-\delta\right)^{1/\sigma-1}\right]}, \\
\check{S}_{2,t+1} &= -\frac{\left[\tilde{w}_{t+1}-\tilde{d}_{t+1}+\left(1+r_{t+1}-\delta\right)\frac{\tilde{d}_t}{1+g_n}\right]\left(1+\beta^{1/\sigma}\left(1+r_{t+1}-\delta\right)^{1/\sigma-1}\right)}{\left[1+\left(1+\beta^{1/\sigma}\left(1+r_{t+2}-\delta\right)^{1/\sigma-1}\right)\beta^{1/\sigma}\left(1+r_{t+1}-\delta\right)^{1/\sigma-1}\right]} \\
&\quad +\tilde{w}_{t+1}-\tilde{d}_{t+1}+\left(1+r_{t+1}-\delta\right)\frac{\tilde{d}_t}{1+g_n}, \text{ and} \\
\check{C}_{3,t+2} &= \left[-\frac{\left[\tilde{w}_{t+1}-\tilde{d}_{t+1}+\left(1+r_{t+1}-\delta\right)\frac{\tilde{d}_t}{1+g_n}\right]\left(1+\beta^{1/\sigma}\left(1+r_{t+1}-\delta\right)^{1/\sigma-1}\right)}{\left[1+\left(1+\beta^{1/\sigma}\left(1+r_{t+2}-\delta\right)^{1/\sigma-1}\right)\beta^{1/\sigma}\left(1+r_{t+1}-\delta\right)^{1/\sigma-1}\right]}\right. \\
&\quad \left.+\tilde{w}_{t+1}-\tilde{d}_{t+1}+\left(1+r_{t+1}-\delta\right)\frac{\tilde{d}_t}{1+g_n}\right]\left(1+r_{t+2}-\delta\right).
\end{aligned}$$

## A.2 Derivation of the Asset Market Clearing Condition

Equating the two equations (2.8) and (2.9), we get

$$\tilde{K}_{t+1}=\frac{1}{1+g_n}\left(\tilde{K}_t^{\alpha}+(1-\delta)\tilde{K}_t-\check{C}_{1,t}(1+g_n)-\check{C}_{2,t}-\frac{\check{C}_{3,t}}{1+g_n}\right). \tag{A.1}$$

From the constraints of the optimization problem for the arbitrary children-generation, we can describe $\check{C}_{1,t}$, $\check{C}_{2,t}$, and $\check{C}_{3,t}$ with equation (2.3.1), (2.3.3), and (2.3.5). Inserting this into equation (A.1), we get:

$$\begin{aligned}
\tilde{K}_{t+1} &= \frac{1}{1+g_n}\left(\tilde{K}_t^{\alpha}+(1-\delta)\tilde{K}_t+(1+g_n)\check{s}_{1,t}-\tilde{w}_t-(1+r_t-\delta)\check{S}_{1,t-1}+\check{S}_{2,t}\right. \\
&\quad \left.-\frac{1+r_t-\delta}{1+g_n}\check{S}_{2,t-1}\right).
\end{aligned}$$

With equation (2.12), $\tilde{K}_{t+1}$ can be described as:

$$\begin{aligned}
\tilde{K}_{t+1} &= \frac{1}{1+g_n}\left((1+r_t-\delta)\tilde{K}_t+(1+g_n)\check{S}_{1,t}-(1+r_t-\delta)\check{S}_{1,t-1}+\check{S}_{2,t}\right. \\
&\quad \left.-\frac{1+r_t-\delta}{1+g_n}\check{S}_{2,t-1}\right).
\end{aligned} \tag{A.2}$$

If we rearrange equation (A.2) in absolute terms and not in per capita or per labor terms, the following difference equation results:

$$K_{t+1} - \check{S}_{1,t}N_{1,t} - \check{S}_{2,t}N_{2,t} = (1 + r_t - \delta)\left[K_t - \check{S}_{1,t-1}N_{1,t-1} - \check{S}_{2,t-1}N_{2,t-1}\right]. \qquad \text{(A.3)}$$

The only chance to solve equation (A.3) is to set $K_{t+1} = \check{S}_{1,t}N_{1,t} + \check{S}_{2,t}N_{2,t}$ $\forall t$, which corresponds to the asset market clearing condition (2.5).

## A.3 Optimal Choices for the Modified Model

The optimal choices for the arbitrary children-generation assuming that debt is held exclusively by foreigners are:

$$\check{C}_{1,t} = \frac{\tilde{w}_{t+1} - \tilde{\tau}_{t+1} - \tilde{d}_{t+1} + (1 + r_{t+1} - \delta)\frac{\tilde{d}_t}{1+g_n}}{(1 + r_{t+1} - \delta)\left[1 + \beta + \beta^2\right]}, \qquad \text{(A.DF-3.1)}$$

$$\check{S}_{1,t} = \frac{\tilde{d}_t}{1+g_n} - \frac{\frac{\left(\tilde{w}_{t+1} - \tilde{\tau}_{t+1} - \tilde{d}_{t+1}\right)}{1+r_{t+1}-\delta} + \frac{\tilde{d}_t}{1+g_n}}{\left[1 + \beta + \beta^2\right]}, \qquad \text{(A.DF-3.2)}$$

$$\check{C}_{2,t+1} = \frac{\beta\left[\left(\tilde{w}_{t+1} - \tilde{\tau}_{t+1} - \tilde{d}_{t+1}\right) + \frac{\tilde{d}_t}{1+g_n}(1 + r_{t+1} - \delta)\right]}{\left[1 + \beta + \beta^2\right]}, \qquad \text{(A.DF-3.3)}$$

$$\begin{aligned}\check{S}_{2,t+1} &= \tilde{w}_{t+1} - \tilde{\tau}_{t+1} - \tilde{d}_{t+1} + (1 + r_{t+1} - \delta)\frac{\tilde{d}_t}{1+g_n} \\ &\quad - \frac{\left[\tilde{w}_{t+1} - \tilde{\tau}_{t+1} - \tilde{d}_{t+1} + (1 + r_{t+1} - \delta)\frac{\tilde{d}_t}{1+g_n}\right](1+\beta)}{\left[1 + \beta + \beta^2\right]}, \qquad \text{(A.DF-3.4)}\end{aligned}$$

$$\begin{aligned}\check{C}_{3,t+2} &= (1 + r_{t+2} - \delta)\Bigg[\tilde{w}_{t+1} - \tilde{\tau}_{t+1} - \tilde{d}_{t+1} + (1 + r_{t+1} - \delta)\frac{\tilde{d}_t}{1+g_n} \\ &\quad - \frac{\left[\tilde{w}_{t+1} - \tilde{\tau}_{t+1} - \tilde{d}_{t+1} + (1 + r_{t+1} - \delta)\frac{\tilde{d}_t}{1+g_n}\right](1+\beta)}{\left[1 + \beta + \beta^2\right]}\Bigg]. \qquad \text{(A.DF-3.5)}\end{aligned}$$

## A.4 Figures

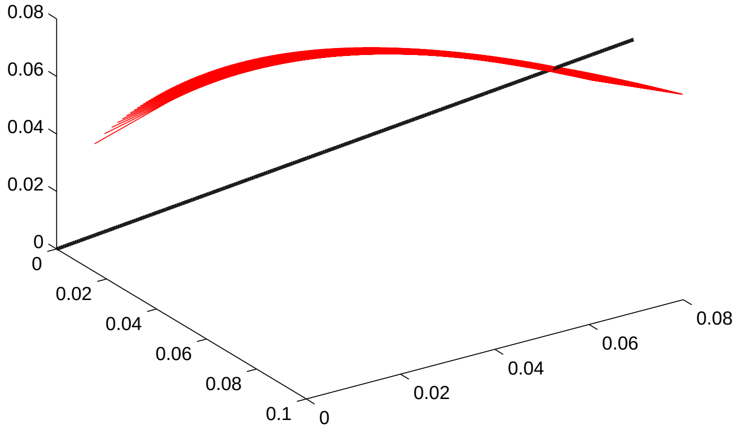


Figure A.1: Law of Motion for the Capital-Labor Ratio with $\alpha = 1/3$ and $q = 0.25$

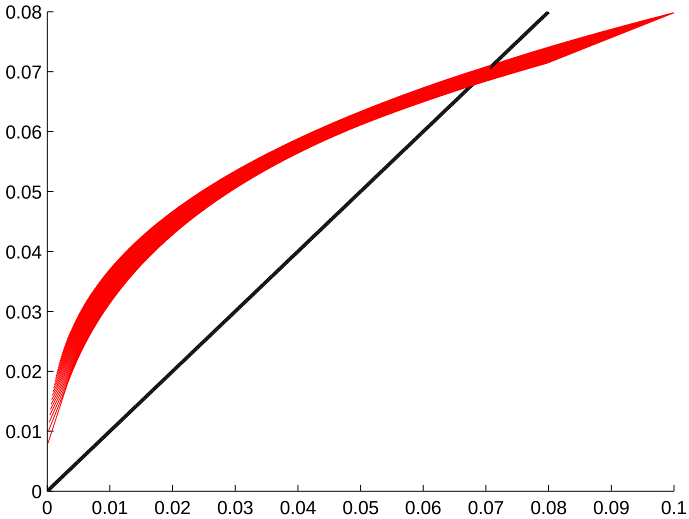


Figure A.2: Law of Motion for the Capital-Labor Ratio with $\alpha = 1/3$ and $q = 0.25$ in a Two Dimensional View

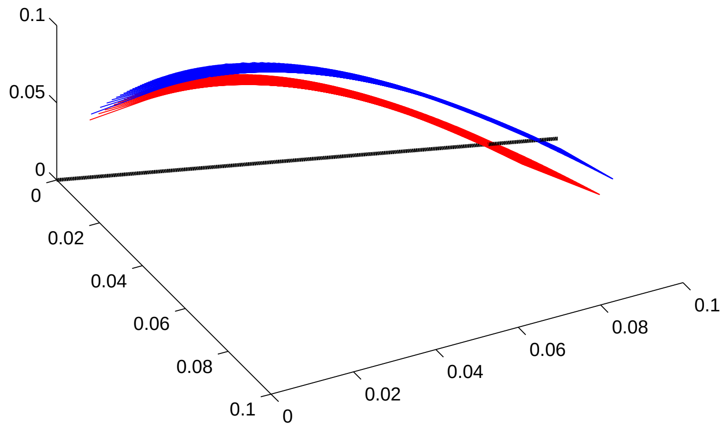


Figure A.3: Law of Motion for the Capital-Labor Ratio with $\alpha = 1/3$ and $q = 0.25$ (red area), $q_1 = 0.3$ (blue area)

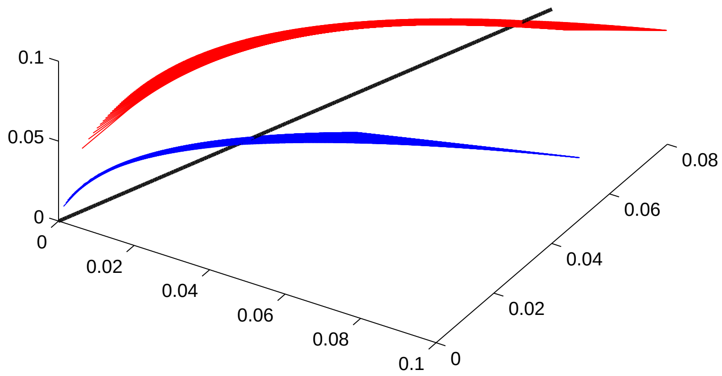


Figure A.4: Law of Motion for the Capital-Labor Ratio with $\alpha = 1/3$ and $q = 0.25$ (red area), $\alpha = 1/2$ (blue area)

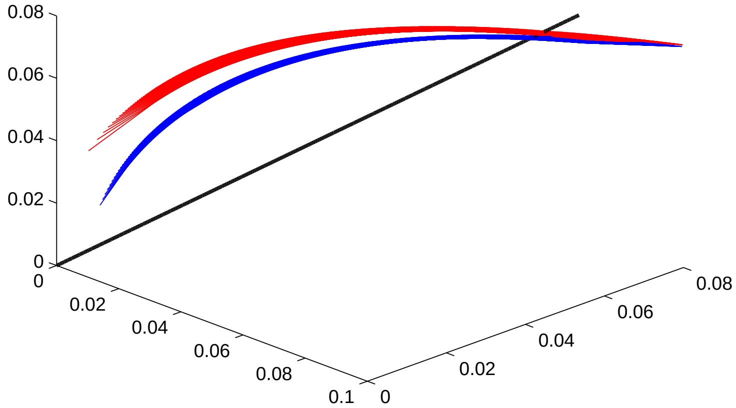


Figure A.5: Law of Motion for the Capital-Labor Ratio for the Baseline Model (red area) and the Model Specification Including Debt Held by Foreigners (blue area)

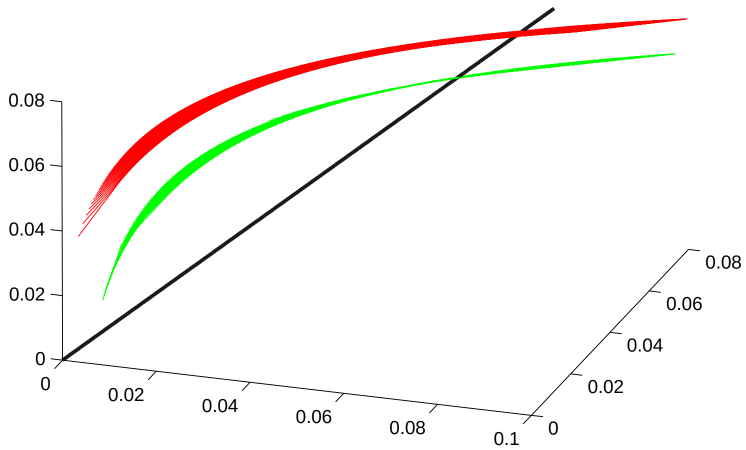


Figure A.6: Law of Motion for the Capital-Labor Ratio for the Baseline Model (red area) and the Model Specification Including Debt (green area) Held by Domestic Citizens

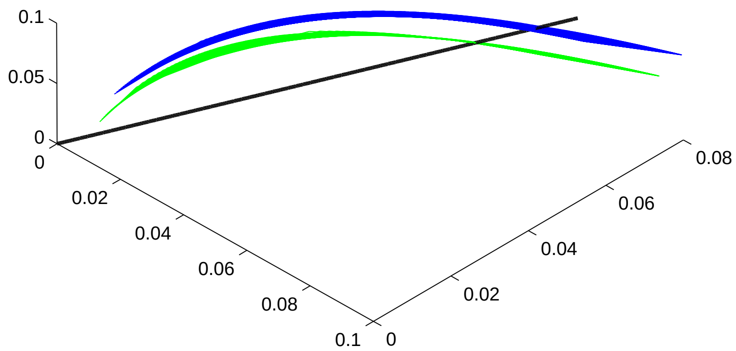


Figure A.7: Law of Motion for the Capital-Labor Ratio for the Model Specification Including External (blue area) and Internal (green area) Debt

## A.5 Derivation of the Polynomial for $\tilde{K}^*_{b_c}$

Starting with equation (2.DC-14), we get for the steady state value of the capital-labor ratio ($\tilde{K}_t = \tilde{K}_{t+1} = \tilde{K}_{t+2} = \tilde{K}^*$) the following equation:

$$\tilde{K}^* \left(1 + \frac{(1-q)(1-\alpha)}{\alpha(1+\beta+\beta^2)}\right) + \frac{(1-q)(1+g_n)\tilde{B}}{\alpha(1+\beta+\beta^2)} \left(\tilde{K}^*\right)^{1-\alpha} = \frac{(1-q)\tilde{B}}{(1+\beta+\beta^2)}$$
$$+\frac{1}{1+g_n}\left[\left((1-\alpha)\left(\tilde{K}^*\right)^{\alpha} - \alpha\left(\tilde{K}^*\right)^{\alpha-1}\tilde{B} + (1+g_n)\tilde{B}\right)\left(\frac{\beta(\beta+q)}{(1+\beta+\beta^2)}\right)\right.$$
$$\left.+\frac{\left((1-\alpha)\left(\tilde{K}^*\right)^{\alpha} - \alpha\left(\tilde{K}^*\right)^{\alpha-1}\tilde{B} + (1+g_n)\tilde{B}\right)\alpha}{(1+g_n)}\left(q - \frac{(1+\beta)q}{(1+\beta+\beta^2)}\right)\left(\tilde{K}^*\right)^{\alpha-1}\right]$$
$$-\tilde{B}.$$

For an arbitrary $0 < \alpha = 1/u < 1$, we get

$$\begin{aligned}
&\tilde{K}^* \left(1 + \frac{(1-q)(1-\alpha)}{\alpha(1+\beta+\beta^2)}\right) + \frac{(1-q)(1+g_n)\tilde{B}}{\alpha(1+\beta+\beta^2)} \left(\tilde{K}^*\right)^{\frac{u-1}{u}} = \\
&\left(\frac{(1-q)}{(1+\beta+\beta^2)} - 1 + \left(\frac{\beta(\beta+q)}{(1+\beta+\beta^2)}\right)\right)\tilde{B} + \left(\frac{\beta(\beta+q)}{(1+\beta+\beta^2)}\right)\frac{(1-\alpha)}{1+g_n}\left(\tilde{K}^*\right)^{\frac{1}{u}} \\
&-\left(\left(\frac{\beta(\beta+q)}{(1+\beta+\beta^2)}\right) - \left(q - \frac{(1+\beta)q}{(1+\beta+\beta^2)}\right)\right)\frac{\alpha\tilde{B}}{1+g_n}\left(\tilde{K}^*\right)^{\frac{1-u}{u}} \\
&+\frac{(1-\alpha)\alpha}{(1+g_n)^2}\left(q - \frac{(1+\beta)q}{(1+\beta+\beta^2)}\right)\left(\tilde{K}^*\right)^{\frac{2-u}{u}} \\
&-\frac{\alpha^2\tilde{B}}{(1+g_n)^2}\left(q - \frac{(1+\beta)q}{(1+\beta+\beta^2)}\right)\left(\tilde{K}^*\right)^{\frac{2-2u}{u}}
\end{aligned}$$

and therefore with $u = 3$:

$$\begin{aligned}
&\tilde{K}^* \left(1 + \frac{(1-q)(1-\alpha)}{\alpha(1+\beta+\beta^2)}\right) + \frac{(1-q)(1+g_n)\tilde{B}}{\alpha(1+\beta+\beta^2)} \left(\tilde{K}^*\right)^{\frac{2}{3}} = \\
&\left(\frac{(1-q)}{(1+\beta+\beta^2)} - 1 + \left(\frac{\beta(\beta+q)}{(1+\beta+\beta^2)}\right)\right)\tilde{B} + \left(\frac{\beta(\beta+q)}{(1+\beta+\beta^2)}\right)\frac{(1-\alpha)}{1+g_n}\left(\tilde{K}^*\right)^{\frac{1}{3}} \\
&-\left(\left(\frac{\beta(\beta+q)}{(1+\beta+\beta^2)}\right) - \left(q - \frac{(1+\beta)q}{(1+\beta+\beta^2)}\right)\right)\frac{\alpha\tilde{B}}{1+g_n}\left(\tilde{K}^*\right)^{-\frac{2}{3}} \\
&+\frac{(1-\alpha)\alpha}{(1+g_n)^2}\left(q - \frac{(1+\beta)q}{(1+\beta+\beta^2)}\right)\left(\tilde{K}^*\right)^{-\frac{1}{3}} \\
&-\frac{\alpha^2\tilde{B}}{(1+g_n)^2}\left(q - \frac{(1+\beta)q}{(1+\beta+\beta^2)}\right)\left(\tilde{K}^*\right)^{-\frac{4}{3}}.
\end{aligned}$$

We see that we have a polynomial with positive as well as negative exponents. To obtain an equation representation containing only positive exponents, w.l.o.g., we multiply the equation with $\tilde{K}^{\frac{4}{3}}$. Using this procedure, we add additional zero roots. Since we are only interested in the largest positive real root, the procedure is valid. The following is obtained

$$\begin{aligned}
&\left(\tilde{K}^*\right)^{\frac{7}{3}} \left(1 + \frac{(1-q)(1-\alpha)}{\alpha(1+\beta+\beta^2)}\right) + \frac{(1-q)(1+g_n)\tilde{B}}{\alpha(1+\beta+\beta^2)} \left(\tilde{K}^*\right)^{\frac{6}{3}} = \\
&\left(\frac{(1-q)}{(1+\beta+\beta^2)} - 1 + \left(\frac{\beta(\beta+q)}{(1+\beta+\beta^2)}\right)\right)\tilde{B}\left(\tilde{K}^*\right)^{\frac{4}{3}} \\
&+\left(\frac{\beta(\beta+q)}{(1+\beta+\beta^2)}\right)\frac{(1-\alpha)}{1+g_n}\left(\tilde{K}^*\right)^{\frac{5}{3}}
\end{aligned}$$

$$-\left(\left(\frac{\beta\left(\beta+q\right)}{\left(1+\beta+\beta^2\right)}\right)-\left(q-\frac{\left(1+\beta\right)q}{\left(1+\beta+\beta^2\right)}\right)\right)\frac{\alpha\tilde{B}}{1+g_n}\left(\tilde{K}^*\right)^{\frac{2}{3}}$$
$$+\frac{\left(1-\alpha\right)\alpha}{\left(1+g_n\right)^2}\left(q-\frac{\left(1+\beta\right)q}{\left(1+\beta+\beta^2\right)}\right)\tilde{K}^*-\frac{\alpha^2\tilde{B}}{\left(1+g_n\right)^2}\left(q-\frac{\left(1+\beta\right)q}{\left(1+\beta+\beta^2\right)}\right).$$

Finally, we substitute $\tilde{K}^{\frac{1}{3}}=z$ and get a polynomial of order 7:

$$z^7\left(1+\frac{\left(1-q\right)\left(1-\alpha\right)}{\alpha\left(1+\beta+\beta^2\right)}\right)+\frac{\left(1-q\right)\left(1+g_n\right)\tilde{B}}{\alpha\left(1+\beta+\beta^2\right)}z^6=$$
$$\left(\frac{\left(1-q\right)}{\left(1+\beta+\beta^2\right)}-1+\left(\frac{\beta\left(\beta+q\right)}{\left(1+\beta+\beta^2\right)}\right)\right)\tilde{B}z^4+\left(\frac{\beta\left(\beta+q\right)}{\left(1+\beta+\beta^2\right)}\right)\frac{\left(1-\alpha\right)}{1+g_n}z^5$$
$$-\left(\left(\frac{\beta\left(\beta+q\right)}{\left(1+\beta+\beta^2\right)}\right)-\left(q-\frac{\left(1+\beta\right)q}{\left(1+\beta+\beta^2\right)}\right)\right)\frac{\alpha\tilde{B}}{1+g_n}z^2$$
$$+\frac{\left(1-\alpha\right)\alpha}{\left(1+g_n\right)^2}\left(q-\frac{\left(1+\beta\right)q}{\left(1+\beta+\beta^2\right)}\right)z^3-\frac{\alpha^2\tilde{B}}{\left(1+g_n\right)^2}\left(q-\frac{\left(1+\beta\right)q}{\left(1+\beta+\beta^2\right)}\right).$$

It is not possible to state a closed form solution for an arbitrary polynomial of order 7. Using the Newton-procedure to receive a numerical approximation for the roots of the given equation, MATLAB [94] finds the 7 solutions. The solution set consists of 4 complex, one negative, and two positive real roots.

# Appendix B

## B.1 The Data

The data used in this study are taken from three main sources: the IMF Database, the OECD Database, and the database supplied by the Deutsche Bundesbank (BBK)/Genesis. Only the number of employees in the government sector are taken from the Eurostat Database. All data are seasonally adjusted with the standard Census X12-ARIMA procedure for seasonal adjustment and quarterly.[1] We convert all series such that they are given in MN euro. If the original series is given in DM, we use the euro reference quotation in 1999: 1€ = 1.95583DM. Moreover, the data are transformed in a number e.g. for population, employment, etc. Most series taken from BBK/Genesis start in 1991Q1, therefore if we additionally need data for 1988-1990, we use for this pre-period OECD data. If not labeled otherwise, all series are in current prices. The following tables show, which series of the corresponding sources are used:

| **Description** | **Data Source** |
|---|---|
| GDP | OECD |

Table B.1: Data Sources: GDP

[1] In the OECD Database all quarterly data are annualized so that we divide with four to get the quarterly flows.

| Description | Data Source |
|---|---|
| Income on Investments (Earnings) | BBK/Genesis |
| Income on Investment (Expenditure) | BBK/Genesis |
| Gross Operating Surplus | Genesis |
| Profits | BBK/Genesis |
| Private Consumption | Genesis |
| Imports | Genesis |
| Exports | Genesis |
| Imports from EMU Countries | BBK/Genesis |

Table B.2: Data Sources: GDP Components

| Description | Data Source |
|---|---|
| Total Gross Fixed Capital Formation | Genesis |
| Scrapping Rate of Capital in the Business Sector | OECD |
| Investment in Housing | OECD |
| Real Investment in Housing | OECD |
| Gross Fixed Investment (Capital Formation) in Government Sector | OECD |
| Total Consumption of Fixed Capital | IMF |
| Consumption of Fixed Capital in Government Sector | OECD(1988-1998)& IMF(1999-2009) |

Table B.3: Data Sources: Capital

| Description | Data Source |
|---|---|
| Real Effective Exchange Rate | OECD |
| Nominal Effective Exchange Rate | IMF |
| Long-Term Interest Rate | OECD |
| Short-Term Interest Rate | OECD |

Table B.4: Data Sources: Interest and Exchange Rates

| Description | Data Source |
|---|---|
| GDP Deflator with Base Year 2005 | IMF |
| Government Final Wage Consumption Expenditure Deflator | OECD |
| Government Fixed Capital Formation Deflator | OECD |
| Gross Total Fixed Capital Formation Deflator | OECD |
| CPI Index with Base Year 2005 | OECD |

Table B.5: Data Sources: Deflators and CPI

| Description | Data Source |
|---|---|
| Net Indirect Tax Income (Indirect Taxes - Subsidies) | OECD |
| Real Net Indirect Taxes | OECD |
| Personal Income Tax | BBK/Genesis |
| Social Security Contributions | BBK/Genesis |
| Corporate Tax | BBK/Genesis |
| VAT | BBK/Genesis |
| Import Duty Tax | BBK/Genesis |
| Total Tax Income | BBK/Genesis |
| Government Tax Income Excluding Social Security Contributions | BBK/Genesis |
| Interest on Public Debt | BBK/Genesis |
| Capital Outlay of the Government Sector | BBK/Genesis |
| Government Transfers to Households | Genesis |
| Subsidies | DIW/IMF |
| Government Final Consumption Expenditure | Genesis |
| Total Expenditure of the Government Sector | BBK/Genesis |
| Unemployment Benefits for East and West Germany | BBK/Genesis |

Table B.6: Data Sources: Government Sector Variables

| **Description** | **Data Source** |
|---|---|
| Population | Genesis |
| Total Labor Force | OECD |
| Total Employment | OECD |
| Total Number of Unemployed | OECD |
| Total Self-Employed | OECD |
| Employment in the Government Sector | Eurostat |
| Wages for All Sectors | OECD |
| Compensation Per Employee in Private Sector | OECD |
| Wages in the Government Sector | OECD(1988-1990)& BBK/Genesis(1991-2009) |

Table B.7: Data Sources: Population, Labor, and Wages

# Appendix C

## C.1 Figures for Germany

The first subsection of Appendix C.1 examines the calculated employment and gross value-added in the business sector to give an overview of the actual development in Germany. The second subsection analyzes the participation ratio for the whole labor market pointing out the problems of data collection in the reunification period of Germany. The importance of different labor shares is shown by comparing potential output and the output gap derived under different labor shares $1 - \alpha$ in the third subsection.

### C.1.1 The Business Sector

Figure C.1 shows the time dependent behavior of the employment in the business sector, which consists of employees and self-employed. The graph is plotted for 1988-2009 where the data for 1988-1990 correspond only to West Germany. We see a sharp increase in the number of employment due to the reunification of two different economic entities. The gross value-added in the business sector does not increase that sharply—see Figure C.2.[1] Despite the sharp increase in employment, the additional workers can not produce a proportionately high additional outcome. They seem to be less productive on average—see also Figure 4.2 on the total factor productivity in Section 4.2.

[1] The data for 1988-1990 correspond to West Germany.

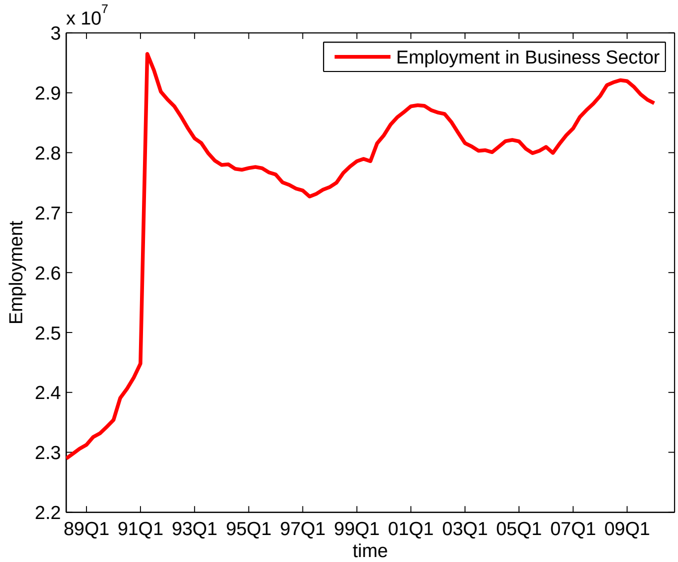


Figure C.1: The Development of Employment in the Business Sector for 1988-2009

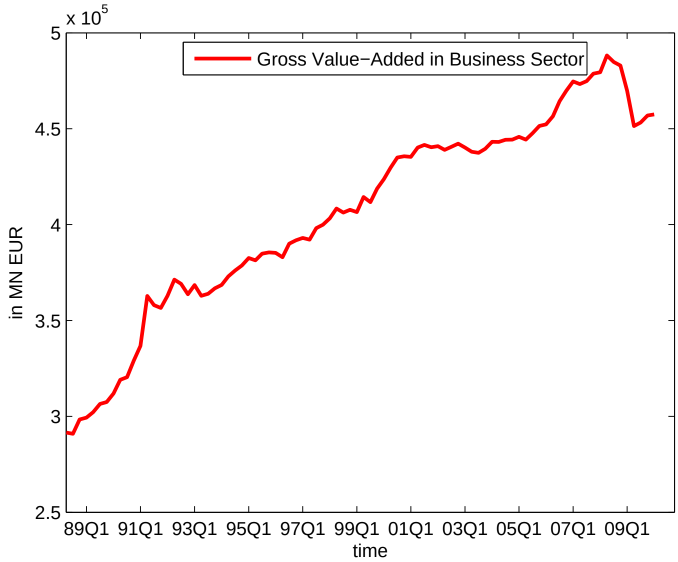


Figure C.2: Gross Value-Added in the Business Sector for 1988-2009

### C.1.2 The Participation Ratio of The Labor Market

The graph of the participation ratio in the labor market between 1991-2009 is shown in Figure C.3. Due to the reunification one faces some problems when capturing the data such that the first two plotted observations in 1991Q1 and 1991Q2 can be seen as outliers because the two data points are unexpectedly high. To avoid endpoint problems, the series is augmented with data for 1988-1990 and then smoothed using a HP-filter with $\lambda_{HP} = 1600$.

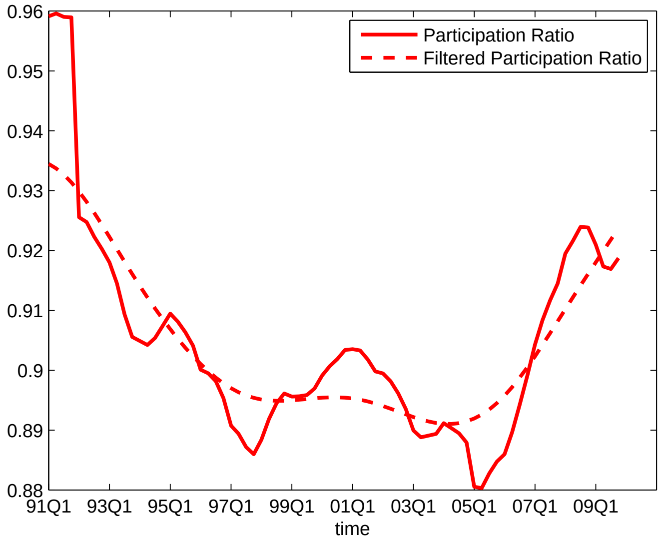

Figure C.3: The Labor Market Participation Ratio for 1991-2009

The filtered series (dashed line) lies far below the two outliers[2], whereas in the remaining run of the curve it is closer to the very volatile participation ratio (solid line). Participation in labor declines within 1991-2004 and increases afterwards.

### C.1.3 The Effects of Varying the Labor Share $1 - \alpha$

Figure C.4 shows the potential output for two different choices of the labor share. The black line takes the originally calculated average labor share ($1 - \alpha = 0.5595$) into account, whereas the blue line assumes that $1 - \alpha_{new} = 0.65$. We see that all calculated

[2] Our presumption of an outlier in the data series is confirmed.

values for the potential output are significantly lower, the higher $1 - \alpha$.

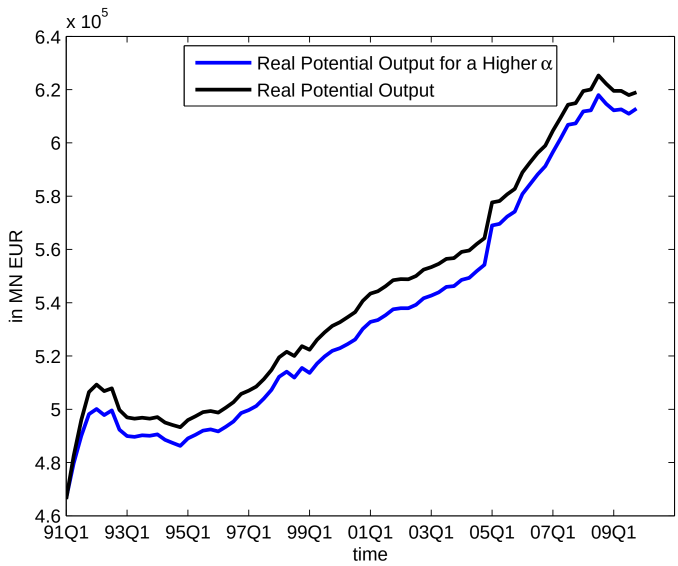


Figure C.4: Potential Output for Two Different Labor Shares

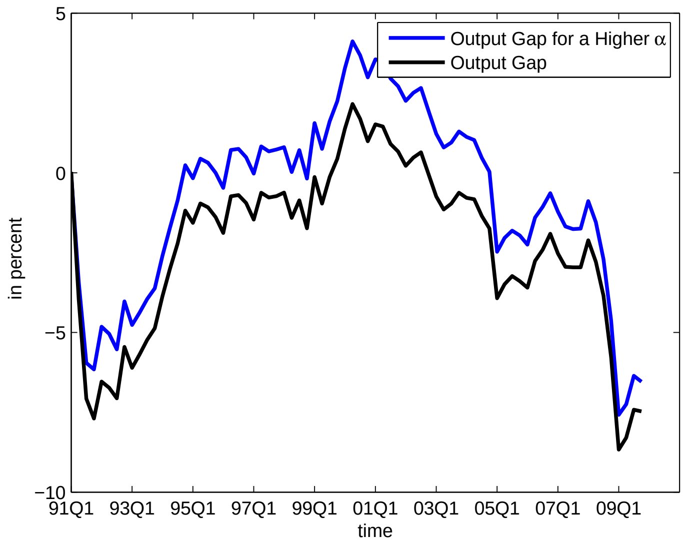


Figure C.5: The Output Gap for Two Different Labor Shares

Since potential output is estimated with lower values for a higher average labor share, the corresponding output gap being the deviation of actual from potential output as a ratio of the potential level is more often positive when $1-\alpha$ is higher. The black line for the output gap with $1-\alpha = 0.5595$ in Figure C.5 lies below the blue line with $1-\alpha_{new} = 0.65$ for all periods. Since the average labor share is higher without taking 2006-2009 into account, it is reasonable that the IMF calculates a higher output gap for 1999 and 2001.

## C.2 Selected Data Plots for Germany

The following section offers a closer description of the inflation rate, the long term interest rate, overall imports and imports from EMU member countries, and the real effective exchange rate for Germany in 1991-2009.

The discussed period 1991-2009 is influenced by a low inflation rate. The quarterly GDP-deflator inflation rate was high at the beginning of the examined period but fluctuated for most parts (for 1995-2009) around 0.2% on average.

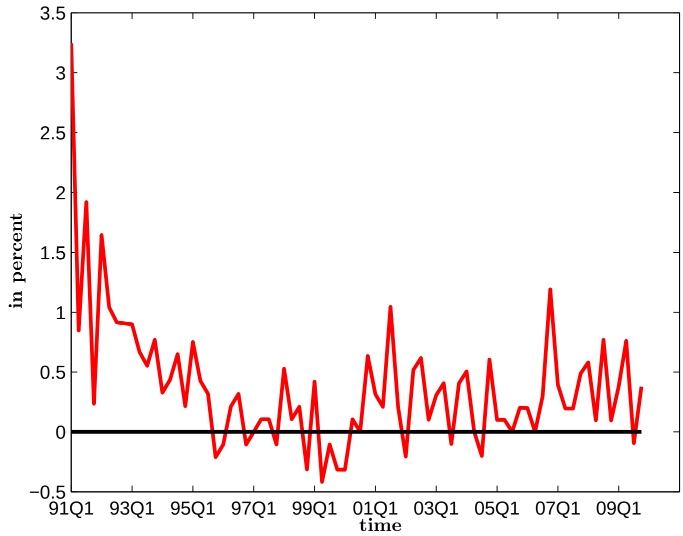


Figure C.6: The Inflation Rate in Germany

Figure C.7 shows the German long-term interest rate. In 1991-2009, the long-term interest rate in percent was volatile and is characterized by a negative slope. Thus, it was highest in 1991 and lowest in 2009.

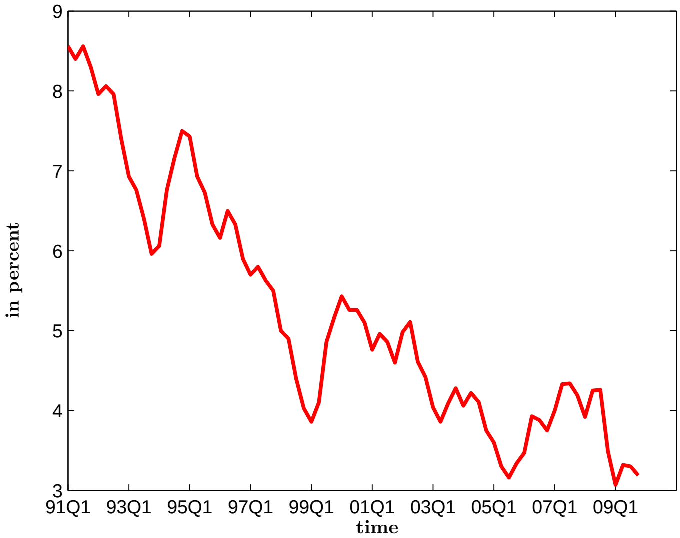


Figure C.7: The Interest Rate in Germany

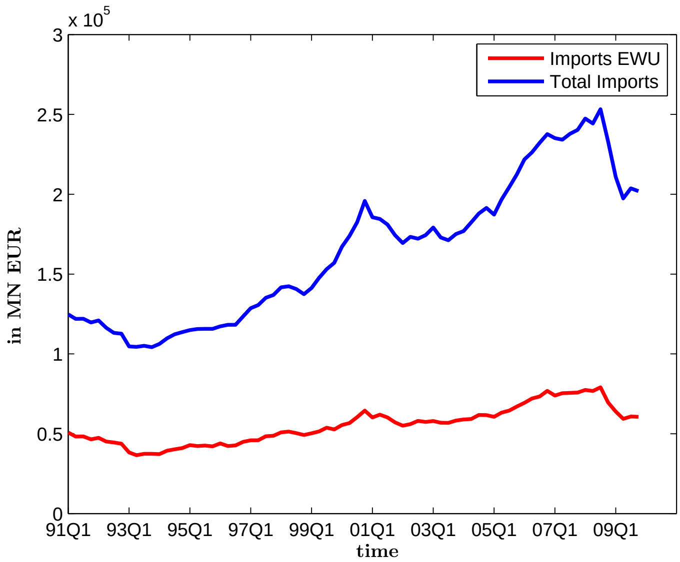


Figure C.8: Imports in Germany

Figure C.8 shows the German imports for 1991-2009. The blue line displays the overall imports of Germany, whereas the red line are the imports exclusively from EMU member countries. Imports from EMU countries play a special role because within the EMU, there is one unique central bank interest rate and the exchange rate reaction channel is not working any longer after the entry of Germany to the EMU in 1999.

Germany imported a high amount of goods and services from member countries of the European Monetary Union. The overall imports showed a positive trend, which was stronger than the trend found for imports solely purchased from the EMU member countries. Thus, imports from the rest of the world increased more for 1999-2009.

The real effective exchange rate does not show a clear trend—see Figure C.9. It fluctuated for the first two third of the plotted period (1991-2003) more than for 2003-2009. In the latter subperiod, the real effective exchange rate varied only within a small range.

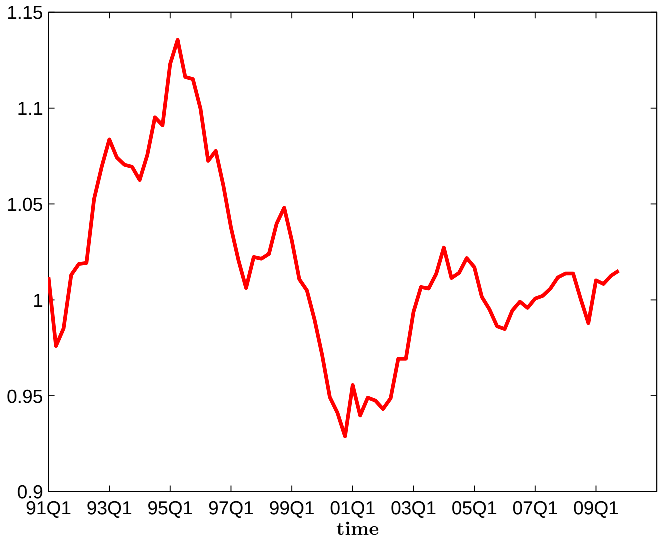


Figure C.9: The Real Effective Exchange Rate

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I hereby declare that I have written the present dissertation by my own and have not used other than the acknowledged resources and aids. Furthermore, I confirm that the dissertation has neither been accepted nor graded 'failed' in a previous doctoral procedure.