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An advanced quadratic displacement approach for the analytical stress analysis of adhesive bonded joints

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In this work an advanced analytical model for the efficient prediction of stress distributions in the adhesive layer of bonded joints is presented. In order to better account for local deformation effects occurring in thicker adhesive layers, a quadratic-extended displacement approach is introduced. The use of a general sandwich-type model for the overlap region enables a wide applicability to the analysis of different joint shapes and load cases.

1 Introduction

In the field of composite structures and lightweight engineering, adhesive bonding is one of the most commonly used joining techniques. However, the comparably soft adhesive layer is a component that is prone to failure. To avoid this, a series of analytical models have been developed to predict stress distributions in the adhesive layer to ensure safe applications. Volkersen [1] as well as Goland and Reissner [2] presented simple linear-elastic models for stress calculations, where the adherends were modeled as beams and the adhesive as a layer of smeared springs. This so-called weak-interface approach in combination with the general sandwich-type model, which was first introduced by Bigwood and Crocombe [3], enables a convincingly accurate calculation of the stresses in thin adhesive layers of different joint configurations under different load cases. For thick adhesives, however, the accuracy decreases, which is why we have developed an improved model as follows.

2 Stress derivation using the variational principle

The current model is based on the concept of the general sandwich-type model [3] as it is shown in Fig. 1, where only the overlap region is considered and different joint geometries and external loads are introduced by sectional forces and moments at the left and right edges of the adherends. Using a displacement based approach [4] as can be seen in equation (1), we apply the first order shear deformation theory to the adherends (indexed with i=1,2) and an advanced approach to the adhesive (index a). In order to improve the accuracy for thick adhesive layers, we modify an existing linear displacement approach from Ojalvo and Eidinoff [5] by adding a new quadratic term. This new term is intended to more accurately account for transverse contraction effects that influence the adhesive deformations. The displacements in the adhesive layer are described by those of the adherends, evaluated at their interface to the adhesive $\hat{u}^{(i)}(x)$ and $\hat{w}^{(i)}(x)$ as well as the newly introduced adhesive variable $\phi(x)$, which result in

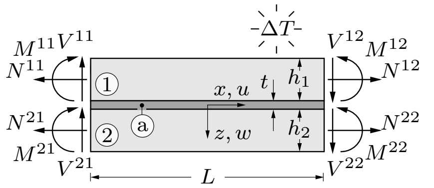

Fig. 1: General sandwich-type model applied on the overlap region

$$u^{(i)}(x,z) = u_0^{(i)} + z\psi^{(i)}, \qquad u^{(a)}(x,z) = \frac{\hat{u}^{(1)}+\hat{u}^{(2)}}{2} + \frac{z}{t}\left(\hat{u}^{(2)} - \hat{u}^{(1)}\right) + \phi\left(1 - \frac{4}{t^2}z^2\right),$$
$$w^{(i)}(x,z) = w_0^{(i)}, \qquad w^{(a)}(x,z) = \frac{\hat{w}^{(1)}+\hat{w}^{(2)}}{2} + \frac{z}{t}\left(\hat{w}^{(2)} - \hat{w}^{(1)}\right). \tag{1}$$

Once the displacements are defined, the strains are calculated by $\varepsilon_{xx} = \frac{\partial u}{\partial x}$, $\varepsilon_{zz} = \frac{\partial w}{\partial z}$ and $\gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$. Assuming linear elastic Hooke's law, plane strain condition and applying beam theory to the adherends yields the constitutive equations

$$\begin{pmatrix}\sigma_{xx}\\ \tau_{xz}\end{pmatrix}^{(i)} = \begin{pmatrix}\frac{E}{1-\nu^2}\left[\varepsilon_{xx} - (1+\nu)\alpha_T\Delta T\right]\\ kG\gamma_{xz}\end{pmatrix}^{(i)}, \qquad \begin{pmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \tau_{xz}\end{pmatrix}^{(a)} = E^{(a)^*}\begin{pmatrix}1-\nu & \nu & \nu & 0\\ \nu & 1-\nu & \nu & 0\\ \nu & \nu & 1-\nu & 0\\ 0 & 0 & 0 & \frac{1-2\nu}{2}\end{pmatrix}^{(a)}\begin{pmatrix}\varepsilon_{xx} - \alpha_T\Delta T\\ -\alpha_T\Delta T\\ \varepsilon_{zz} - \alpha_T\Delta T\\ \gamma_{xz}\end{pmatrix}^{(a)}, \tag{2}$$

with $E^{(a)^*} = \frac{E^{(a)}}{(1+\nu^{(a)})(1-2\nu^{(a)})}$. Contrary to previous studies, we also consider the influence of the adhesive normal stresses $\sigma_{xx}^{(a)}$ on the overall stress distribution, which was neglected before. Thermal behavior of the joints is taken into account as well. With the governing equations we then set up the potential energy Π for the entire overlap region, consisting of the strain energy in the

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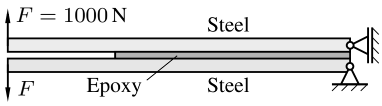

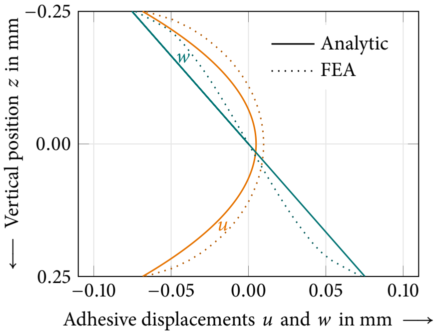

Fig. 2: Adhesive displacements in through-thickness direction at the left edge of the overlap region of a double cantilever beam.

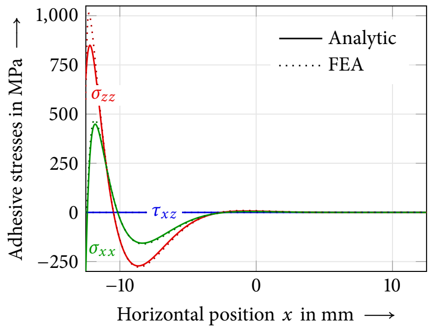

Fig. 3: Stress distributions of the double cantilever beam evaluated at the midplane of the adhesive and verified by FEA.

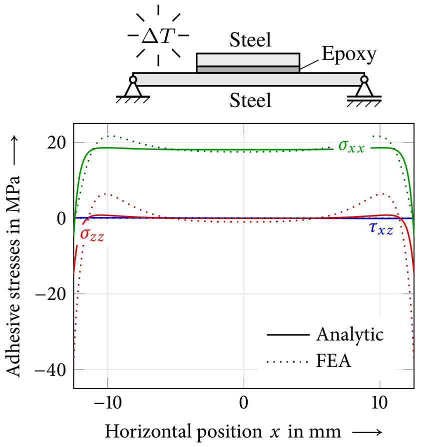

Fig. 4: Detected shear and normal stresses in the midplane of a reinforcing steel-epoxy-steel patch under pure thermal load.

adherends (V_i) and the adhesive (V_a), containing the stresses $\boldsymbol{\sigma}$ and strains $\boldsymbol{\varepsilon}$ and the potential energy resulting from external surface stresses $\mathbf{t}$ acting distributed at the edges of the adherends (S_i) and the corresponding displacement variables $\mathbf{u}$, as

$$\Pi = \frac{1}{2}\int_{V_i} \boldsymbol{\sigma}^{(i)}\boldsymbol{\varepsilon}^{(i)}\mathrm{d}V_i + \frac{1}{2}\int_{V_a} \boldsymbol{\sigma}^{(a)}\boldsymbol{\varepsilon}^{(a)}\mathrm{d}V_a - \int_{S_i} \mathbf{t}^{(i)}\mathbf{u}^{(i)}\mathrm{d}S_i. \tag{3}$$

Applying the minimum total potential energy principle by setting $\delta\Pi = 0$ leads to a homogeneous second-order system of ordinary differential equations with constant coefficients and a corresponding set of boundary conditions. Reducing the order of the equation system to a first-order differential equation system yields the form

$$\Psi' = \mathbf{A}\Psi, \qquad \text{with} \qquad \Psi = \left(u_0^{(1)},\ u_0'^{(1)},\ u_0^{(2)},\ u_0'^{(2)},\ w_0^{(1)},\ w_0'^{(1)},\ w_0^{(2)},\ w_0'^{(2)},\ \psi^{(1)},\ \psi'^{(1)},\ \psi^{(2)},\ \psi'^{(2)},\ \phi,\ \phi'\right)^T, \tag{4}$$

depending on 14 undetermined displacement variables and their derivatives with regard to the coordinate x. With the exponential standard solution approach an eigenvalue problem with real, complex, and zero eigenvalues is obtained, where the overall solution is derived from a linear combination of the different eigenvalue solutions also including generalized eigenvectors. The unknown constants are determined from the boundary conditions.

3 Results and discussion

The described calculation procedure has been validated for different lap joint configurations and verified with finite element analyses (FEA). In Fig. 2 the adhesive displacements of a symmetrical double cantilever beam under mechanical load have been displayed. The results have been evaluated at the left edge of the overlap region in through-thickness direction. According to FEA, the transverse contraction of the adhesive yields an approximately quadratic shaped deformation. Due to the additionally applied quadratic term, the new analytical model succeeds convincingly in detecting this deformation effect. Looking at the stress predictions for the same joint configuration in Fig. 3, evaluated at the midplane of the adhesive, the peeling stresses $\sigma_{zz}^{(a)}$ are dominating the stress distributions as the adherends are vertically pulled apart. In Fig. 4, we see the stress results of a purely thermally loaded reinforcing patch configuration. The detected normal stresses $\sigma_{xx}^{(a)}$, resulting from the different thermal expansion coefficients of adherends and adhesive, provoke the occurrence of peeling stresses that are confirmed by FEA. With the normal stresses $\sigma_{xx}^{(a)}$ being neglected in previous research, these peeling stresses could not be detected at all.

4 Conclusion

The present model provides improved accuracy on the prediction of stresses in adhesive joints resulting from a more realistic consideration of the adhesive displacements. Despite the advanced approach it is very simple and efficient in its performance and allows for various lap joint geometries and different load cases.

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