Impact of economic inventory and payment policies on working capital optimization in purchase-to-pay processes

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Zusammenfassung

Die vorliegende Dissertation umfasst acht Kapitel und ist wie folgt strukturiert: Nach einer kurzen Einführung in die generelle Thematik in *Kapitel 1* folgt in *Kapitel 2* im Rahmen eines strukturierten Literaturüberblickes eine Übersicht über den aktuellen Stand der Forschung zu Bestands- bzw. Losgrößenmodelle. Dies dient neben der Einordnung des Beitrages auch der Verdeutlichung der unterschiedlichen Strömungen innerhalb des Forschungsgebietes sowie deren Relevanz für Forschung und Praxis. Zunächst werden in diesem Abschnitt die methodischen Grundlagen der strukturierten Literaturanalysen diskutiert gefolgt von einer deskriptiven Auswertung des Literatursamples. Anschließend wird ein inhaltsbezogenes Klassifikationsschema für Bestands- bzw. Losgrößenmodelle entwickelt und die im finalen Literatursample identifizierten Beiträge werden im Kontext dieses Klassifikationsschemas diskutiert. Wie die Analyse zeigt, sind im Laufe der Jahre verschiedene Erweiterungen des grundlegenden Losgrößenmodells von Harris vorgenommen worden. Dies beinhaltet beispielsweise die Koordination in mehrstufigen Bestandssystemen sowie die Berücksichtigung von möglichen Anreizsystemen. Beide Themenkomplexe werden nachfolgend wieder aufgegriffen.

Zunächst fokussiert sich der Beitrag auf die Identifikation optimaler Bestell- und Zahlungspolitiken bei gegebenen Lieferantenkrediten. Da im Rahmen von progressiv ausgestalteten Zinsvereinbarungen in Lieferantenkrediten der innerhalb des Zahlungsziels zu berücksichtigende Zinssatz von Periode zu Periode ansteigt, bieten sich dem Debitor in dieser Situation unterschiedliche Optionen, die offenen Rechnungen zu begleichen, wobei die finanziellen Auswirkungen jeder Option von der aktuellen Zinsstruktur und den alternativen Investitionsbedingungen abhängen. *Kapitel 3* greift diese Problematik auf, indem ein Bestandsmodell unter Berücksichtigung von Lieferantenkrediten mit progressiven Zinsschema erweitert wird um a) den Fall, dass der Kreditzins des Käufers den vom Lieferanten in Rechnung gestellten Zinssatz überschreiten kann, b) die Möglichkeit des Käufers, den ausstehenden Saldo innerhalb der Kreditlaufzeiten fortlaufend zu tilgen, c) die Berücksichtigung von Zinseszinsen und d) die potenzielle Substitution von Lieferantenkrediten durch alternative Bankkredite.

Anschließend werden in *Kapitel 4* verschiedene Lösungsalgorithmen zur Ableitung der optimalen Bestell- und Zahlungspolitik des Käufers untersucht und erweitert. Basierend auf der Erkenntnis, dass die abschnittsweise definierte Gesamtkostenfunktion konvex, aber nicht notwendigerweise kontinuierlich ist, wird zunächst ein modifizierter Lösungsalgorithmus entwickelt und anschließend im Verlauf eines Simulationsexperiments mit in der Literatur diskutierten Algorithmen verglichen. Die Ergebnisse zeigen, dass der modifizierte Algorithmus alle globalen Optima lokalisieren und damit die vorhandenen Ansätze in Bezug auf die Lösungsqualität verbessern kann.

Die *Kapitel 5* und *6* erweitern anschließend den Umfang der Analyse um die Entwicklung von optimalen Bestell- und Zahlungspolitiken unter Berücksichtigung einer bestandsabhängigen

Endkundennachfrage. Eine solche Problematik lässt sich häufig im Einzelhandel beobachten, wo die Nachfrage normalerweise durch die in den Regalen ausliegenden Warenbestände beeinflusst wird. Die Ergebnisse verdeutlichen neben der durch die eingeführten Erweiterungen gestiegene Praxistauglichkeit auch die enge Verknüpfung zwischen operativen und finanziellen Aspekten des Supply Chain Managements, die durch den Einsatz integrierter Planungsansätze berücksichtigt werden kann.

Nachfolgend wird die Betrachtung zudem um die Verwendung des Barwertkalküls erweitert. Da Entscheidungen über die Working-Capital-Struktur eines Unternehmens, im vorliegenden Fall definiert durch die optimale Bestands- und Zahlungspolitik, die zukünftigen Cashflows und damit die zeitliche Allokation von Zahlungen maßgeblich beeinflussen, sollten sie auch hinsichtlich ihrer langfristigen Rentabilität unter Berücksichtigung des Kapitalwerts bewertet werden. Insbesondere in Situationen, in denen Lieferantenkredite über einen langen Zeitraum hinweg mit variierenden Zinssätzen verwendet werden, hilft die explizite Berücksichtigung des Kapitalwertes, die Planung realistischer zu gestalten. Dieser Aspekt wird in *Kapitel 7* eingehender behandelt, welches die optimalen Bestell- und Zahlungspolitiken eines Käufers unter Minimierung des Barwertes aller entscheidungsrelevanten Kosten untersucht.

Schlussendlich wird in *Kapitel 8* ein weiterer Aspekt aufgegriffen und ein integriertes Bestandsmodell für den Fall eines multi-sourcing Szenarios unter stochastischer Nachfrage entwickelt. Der Käufer verwendet dabei eine (Q,s) Lagerhaltungspolitik zur Bestimmung der optimalen Bestellmengen und -zeitpunkte. Etwaige Lieferzeiten werden als deterministisch, aber von der Bestellmenge abhängig angenommen, weshalb die effektive Lieferzeit und das damit verbundene Ausfallrisiko sowohl durch die Variation der Losgröße als auch der Anzahl der Vertragslieferanten beeinflusst werden kann. Nach der Entwicklung entsprechender Entscheidungsmodelle für dieses sogenannte Multi-Vendor-Single-Buyer-Problem mit stochastischer Nachfrage und variabler Durchlaufzeit wird die Auswirkung verschiedener Lieferstrukturen auf das Lagerunterdeckungsrisik, die erforderlichen Lagerbestände und die damit verbundenen Lager- und Bestellkosten untersucht.

Abstract

The thesis at hand includes eight chapter and is structured as follows: Following a brief introduction of the topic in *Chapter 1*, *Chapter 2* provides a survey of literature reviews in the area of lot sizing. Its intention is to show which streams of research emerged from Harris' seminal lot size model, and which major achievements have been accomplished in the respective areas. It first develops the methodology and then descriptively analyzes the sample. Subsequently, a content-related classification scheme for lot sizing models is developed, and the reviews contained in the sample are discussed in light of this classification scheme. The analysis reveals that various extensions of Harris' lot size model have been developed over the years, such as lot sizing models that include multi-stage inventory systems, incentives, or productivity issues. The aims of such a tertiary study are the following: firstly, it helps primary researchers to position their own work in the literature, to reproduce the development of different types of lot sizing problems, and to find starting points if they intend to work in a new research direction. Secondly, the study identifies several topics that offer opportunities for future secondary research apart from the ones covered in this thesis.

In the presence of a progressive payment scheme, the supplier offers a sequence of credit periods, where the interest rate that is charged on the outstanding balance usually increases from period to period. If a buyer faces a progressive trade credit scheme, various options for settling the unpaid balance exist, where the financial impact of each option depends on the current credit interest structure and the alternative investment conditions. *Chapter 3* takes up this issue by generalizing the trade credit inventory model with progressive interest scheme by considering a) the case where the credit interest rate of the buyer may (but not necessarily has to) exceed the interest rate charged by the supplier, b) where the buyer has the option to settle the outstanding balance continuously within the credit periods, c) where compound interest accrues at the retailer, and d) bank loans are available as a substitute for the trade credit. In addition, some inaccuracies in earlier formulations of the effective interest cost are corrected.

Subsequently, *Chapter 4* studies and extends solution algorithms for deriving the optimal ordering and payment policies of a retailer on the condition that the supplier provides a progressive interest scheme. Based on the finding that the piecewise total cost functions are convex but not necessarily continuous, a modified solution algorithm is developed and collated with existing ones in the course of a simulation experiment. The results indicate that the modified algorithm can locate all optimal solutions and outperforms existing approaches.

Chapters 5 and *6* further extend the scope of the analysis by considering models aimed at finding ordering and payment policies for a buyer with stock-dependent demand and a supplier that offers a progressive payment scheme. Such a setting can frequently be observed in retail stores where the demand rate is usually influenced by the amount of inventories displayed on the shelves. These chapters correct some errors in the formulation of previously published approaches and extend those works by assuming that the credit interest rate of the retailer may exceed the interest rate charged by the supplier. Several numerical examples illustrate the

benefits of the suggested modifications. The results also illustrate the close linkage between operational and financial aspects in supply chain management, which should be considered by employing more integrated planning approaches.

As decisions on the working capital structure of the company defined by an appropriate inventory and payment policy significantly influence future cash-flows and thus the temporal allocation of payments, they should also be evaluated in terms of long-term profitability by considering their net present value or equivalent measures. Especially in situations where trade credit agreements are used over a long period of time and where discount rates are varying, explicitly considering the time-value of money in inventory models helps to make them more realistic. This aspect is considered in *Chapter 7* that studies the optimal ordering and payment policies of a buyer assuming that the supplier offers a progressive interest scheme. The models proposed enable decision makers to improve decision making and the results reveal that taking into account the temporal allocation of payments, the prevailing interest relation influences replenishment policies significantly.

Finally, *Chapter 8* studies a buyer sourcing a product from multiple suppliers under stochastic demand. The buyer uses a (Q,s) continuous review, reorder point, order quantity inventory control system to determine the size and timing of orders. Lead time is assumed to be deterministic and to vary linearly with the lot size, wherefore lead time and the associated stock-out risk may be influenced both by varying the lot size and the number of contracted suppliers. After presenting several mathematical models for a multiple supplier single buyer integrated inventory problem with stochastic demand and variable lead time, the impact of different delivery structures on the risk of incurring a stock-out during lead time and the required inventories is analyzed.

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I. Introduction

Abstract: This introductory chapter provides an overview of the general contextual framework as well as of the scope of the thesis facilitating its positioning and the critical assessment of the presented findings. After a brief introduction to the emergence of global and fragmented supply chains, the pillars of supply chain finance as an attempt towards the integration and optimization of financing at an inter-organizational level are discussed. This is followed by a discussion of the role of trade credits in such a globalized and fragmented environment as well as its opportunities and challenges at the company level. This section concludes with an overview of the different chapters outlining the considered research approaches and most important findings.

1. Evolution of supply chain finance

In past decades, intensified off-shoring and outsourcing have fundamentally altered businesses and the way they operate (Brennan et al., 2015). Whereas at first predominantly large multinationals began to set up facilities abroad, in the early years of this millennium this trend also spread to small and medium sized companies. In 2002-2003, approximately every third European manufacturing company had off-shored parts of its operations seeking for cost advantages, improved flexibility, proximity to key customers or the opening of new markets (Dachs et al., 2006). Even in the presence of gradually deteriorating cost advantages in low-cost countries caused by increasing factor cost and the emergence of new technologies, this trend remains unbroken (Dachs and Zanker, 2014) although the rationales have shifted from factor-cost arbitrage to proximity to demand and innovation. As a consequence, the share of global manufacturing value added of the G7 nations dropped between 1990 and 2010 from 64 to 46 per cent, a decrease by 18 percentage points fueled by intensified off-shoring and taken up by emerging economies in Asia (mainly China, but also Korea, Indonesia, Thailand and India) and (South-) Eastern Europe (Baldwin and Lopez-Gonzalez, 2014). Simultaneously, companies that previously sought for power, control and rationalization by conglomeration, vertical and horizontal integration realized that their evolution from push-based material processing and assembly units to demand-driven systems converting ideas or needs into marketable product and service bundles required the reconfiguration of the organizational boundaries (McCarthy and Anagnostou, 2004). Based on the premise that superior performance is realized by strategically contracting external suppliers to carry out processes more effectively and efficiently than it had previously been done in-house (Kroes and Gosh, 2010), outsourcing initiatives gained momentum and more and more companies began to focus on their core business by externalizing a wide range of activities ranging from support functions to core manufacturing-related functions. In the European automotive industry, for example, large OEM (original equipment manufacturers), such as BMW, Fiat, Mercedes or Porsche outsource design as well as large parts or even entire manufacturing and assembly processes to suppliers (Ciravegna et al., 2013). As a result, the degree of manufacturers' vertical integration averages around 28% but even can go down to 10% (Wannenwetsch, 2010). Evidently, outsourcing provides important financial and strategic benefits, however, apart from rising coordination requirements and increasing dependence on suppliers, this is often also associated with the transfer of knowledge to the external partners (McCarthy and Anagnostou, 2004) which may induce further risks in the long-term. In conclusion, both, increasing vertical specialization driven by several waves of outsourcing as well as global dispersion of manufacturing in the expectation of lowered factor-cost or proximity to customers and innovation have turned operations into a predominantly inter-firm activity capable of managing highly complex and global networks (Brennan et al., 2015).

These new challenges in their consequence also led to the emergence of supply chain management, coined in the early 1980's (cf. Oliver and Webber, 1982), as a concept of intra- and inter-organizational integration and coordination across activities, functions and organizations in order to create customer value and competitive advantages (Cooper et al. 1997, Ballou, 2007). Supply chain management represents a fundamental paradigmatic shift in modern management by recognizing that businesses no longer operate as autonomous entities, but rather as supply chains competing against supply chains (Lambert and Cooper, 2000). In its broadest sense, this embraces all value-adding activities involved in the creation and delivery of product and service bundles which are frequently dispersed over a network of vertically specialized organizations. Thus, supply chains exhibit rather complex structures and range from the initial source to the ultimate customer (cf. Cooper et al. 1997 and Mentzer et al., 2001). As companies are rarely part of only one supply chain, the scope of the network that should actually be proactively managed depends on various factors such as complexity, uncertainty and vulnerability (Lambert and Cooper, 2000, Manuj and Mentzer, 2008). Although it was stated early that the management of supply chains should apart from bi-directional material, component and product flows also take into account the corresponding information and financial flows (cf. Mentzer et al., 2001), the management of financial aspects within supply chains has only recently made its way onto the agenda of supply chain researchers and professional (see, Pfohl and Gomm, 2009 or Kouvelis and Zhao, 2012).

Following the economic downturn in 2008/2009, many industries faced a considerable reduction in the availability of loans combined with an increase in the cost of corporate borrowing (Ivashina and Scharfstein, 2010). In the consequence, access to finance became, apart from attracting customers, the most pressing concern especially for small- and medium-sized companies (ECB, 2013). To overcome this 'credit crunch', that increased financial asymmetries among the supply chain partners and threatened the competitiveness of entire supply chains (Hale and Arteta, 2009), companies made increasingly use of alternative ways of financing such as trade credit schemes, while organizations less affected took the role of liquidity providers (Garcia-Appendini and Montoriol-Garriga, 2013). This contributed considerably to the development of supply chain finance as an attempt to integrate and optimize financing at an inter-organizational level often supported by financial intermediaries and/or technology service providers (Pfohl and Gomm, 2009, Gomm, 2010). In practice, many supply chain finance initiatives aim at improving cash flow by extending a supplier's payment terms, while at the same time enabling suppliers to receive early payments. However, analogous to the general integration across the supply chain, the scope of supply chain finance in terms of the companies as well as assets and liabilities involved may vary and ranges from rather short-term solutions primarily focused on accounts payable and/or receivable such as reverse factoring (cf. Wuttke et al., 2013b) to collaborative working capital management (cf. Randall and Farris, 2009, Wuttke et al., 2013a) up to shared fixed asset financing within the supply chain (see Pfohl and Gomm, 2009 and note that the extended concept is often also referred to as financial supply chain management in order to distinguish the scope of the potential solutions, Wuttke et al., 2013a). Especially in such a scenario of an economic downturn, supply chain finance provides an indispensable opportunity to improve the performance of the individual companies involved and the supply chain as a whole by taking advantage of the varying financial capabilities in order to grant access to loans, to lower corporate debt cost or to improve working capital management at the supply chain level (Pfohl and Gomm, 2009, Randall and Farris, 2009, Wuttke et al., 2013b). The value-effect of these initiatives can be captured by considering its impact on the volume of assets affected, the duration of financing required and the effective cost of capital rate in order to assess the cost of capital within the supply chain (Pfohl and Gomm, 2009).

2. Trade credits and purchase-to-pay solutions

An application of supply chain finance that has received considerable attention in recent years is trade credits (see Seifert et al., 2013, for a recent review of the literature). Trade credits are short-term debt financing instruments that enable buyers of intermediate goods or services to delay the payment to their suppliers for a predefined credit period, either free of cost or in exchange for a contracted interest rate. The main advantage of delayed payments is that suppliers grant access to loans and thus enable their customers to increase order sizes without approaching a liquidity bottleneck. Besides diminishing credit rationing, customers may also benefit from reduction of cost by pooling transactions, and increased financial flexibility as compared to bank loans in the case of financial distress (Garcia-Teruel and Martinez-Solano, 2010). From a supplier’s point of view, payment delays can improve the competitive position, as they can be used instead of price discounts to promote sales and to develop their product market position (cf. Summers and Wilson, 2002). Other enablers facilitating the supply of trade credits are differences in the price elasticity between suppliers and buyers, collateral values of goods sold, credit intermediation between buyers and banks as well as the protection of non-salvageable investments in buyers (cf. Seifert et al., 2013). For many businesses, trade credits have become the most important source of short-term funding. In the UK, for example, it is stated that more than 80% of business-to-business transactions include trade credit agreements (cf. Summers and Wilson, 2002) and also internationally, it is assumed that trade credits exceed short-term bank financing by far (cf. De Blasio, 2005) with an upward trend. Especially in the case of a reduction of credit flows, companies try to extract liquidity, as much as possible, through better and more efficient management of their operations (see Camerinelli, 2009). During the economic downturn 2008/2009, Anheuser-Busch InBev, for example, increased its DPO (days payables outstanding) by the factor four within less than a month, freeing up nearly $1 billion in working capital. Similarly, other large companies used the financial crises to renegotiate payment terms (cf. Strom, 2015). However, squeezing suppliers by forcing them to accept prolonged payment terms does not necessarily prove to be a winning strategy if this leads to increasing unit cost in the long term (Camerinelli, 2009).

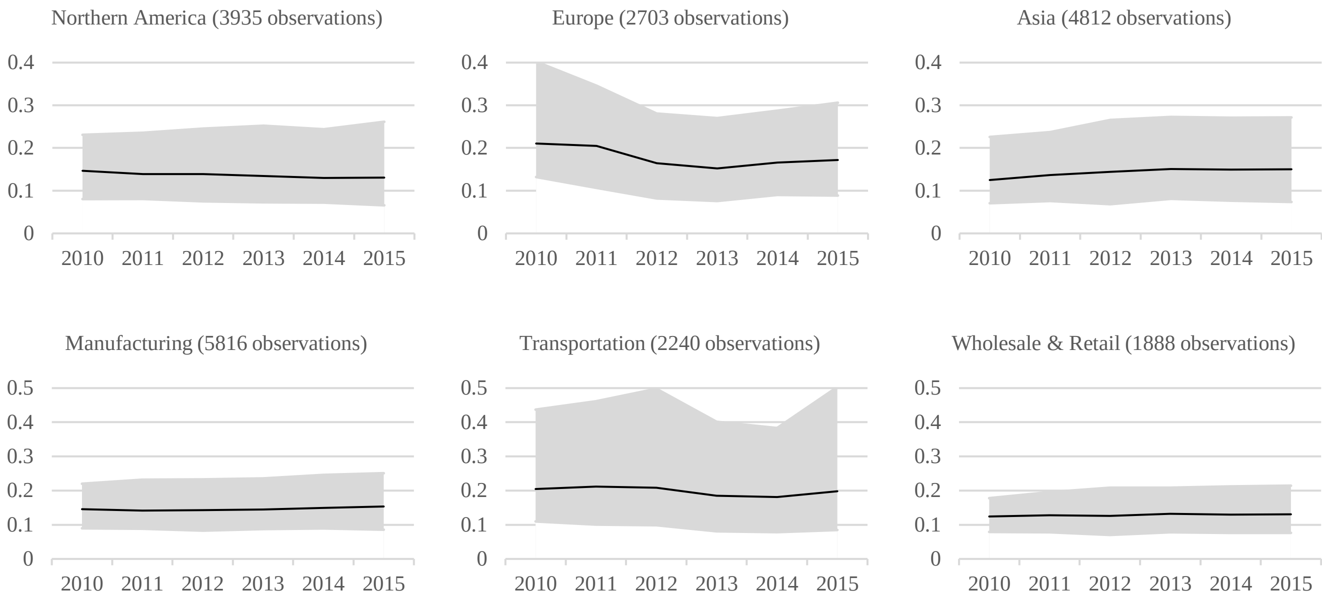

Figure 1: Accounts payable as percentage of COGS (Source: Bureau van Dijk – Orbis)

Although the general importance of trade credits is supported by numerous studies, the amount of trade credit financing varies significantly between different regions and industries (see Ng et al., 1999 and Figure 1 for a more recent overview of trade credit volumes in selected regions and industries). A comparison of accounting data from companies in three different regions and in three different industries illustrates that median accounts payable range between 12% and 21% of COGS (Cost of Goods Sold) and are comparatively stable over time (cf. Figure 1). Moreover, actual payment delays usually exhibit a high degree of variation which cannot be captured at the aggregated level. In some industries, companies even seem to vary credit terms and actual delays from customer to customer (cf. Summers and Wilson, 2002), creating a plethora of short-term liabilities with different maturities, interest rates and obligations. In combination with fragmented data storage across multiple ERP systems and local adoptions of company standards, the management of cash flows can become quite inefficient. Consequently, based on the ongoing implementation of electronic invoicing, the full integration and digitization of the purchase-to-pay processes and the entire financial supply chain is expected to leverage significant benefits for working capital optimization (see Camerinelli, 2009 and Caluwaertz, 2010). Beginning with the migration to e-invoicing and the automation of invoicing processes, efficiency can be improved by the elimination of a considerable amount of low value-added activities such as data entry from paper documents, manual settlement of documents or correction of non-conformities due to typing errors (Caluwaertz, 2010). When fully integrated with the internal systems, intelligent purchase-to-pay solutions accessing all contract- and billing-related information can also facilitate improved cash management by taking advantage of individual payment conditions to coordinate and optimize ordering and payment decisions. Apart from the alignment of end-to-end processes and the improved coordination among the finance and the procurement function, this does also require the development of appropriate prescriptive models considering general financial conditions and diversity of existing trade credit terms in order to automate decision processes. This issue will be taken up in the present thesis aimed at the development of inventory models supporting optimal working capital management. This includes the generalization of the trade credit inventory models with progressive interest scheme by considering the case where a) the credit interest rate of the buyer may exceed the interest rate charged by the supplier, b) the buyer has the option to settle the outstanding balance continuously during the credit periods by the help of electronic payment systems, c) compound interest accrues at the retailer, and d) bank loans are available as a substitute for the trade credit as well as the introduction of a stochastic multi-supplier integrated inventory model facilitating the reduction of inventories.

3. Structure of the conducted analyses

The remainder of this thesis is structured as follows: *Chapter 2* provides a survey of literature reviews in the area of lot sizing. Its intention is to show which streams of research emerged from Harris' seminal lot size model, and which major achievements have been accomplished in the respective areas. It first develops the methodology and then descriptively analyzes the sample. Subsequently, a content-related classification scheme for lot sizing models is developed, and the reviews contained in the sample are discussed in light of this classification scheme. The analysis reveals that various extensions of Harris' lot size model have been developed over the years, such as lot sizing models that include multi-stage inventory systems, incentives, or productivity issues. The aims of such a tertiary study are the following: firstly, it helps primary researchers to position their own work in the literature, to reproduce the development of different types of lot sizing problems, and to find starting points if they intend to work in a new research direction. Secondly, the study identifies several topics that offer opportunities for future secondary research apart from the ones covered in this thesis.

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II. The Lot Sizing Problem: A Tertiary Study[1]

Abstract: This paper provides a survey of literature reviews in the area of lot sizing. Its intention is to show which streams of research emerged from Harris' seminal lot size model, and which major achievements have been accomplished in the respective areas. We first develop the methodology of this review and then descriptively analyze the sample. Subsequently, a content-related classification scheme for lot sizing models is developed, and the reviews contained in our sample are discussed in light of this classification scheme. Our analysis shows that various extensions of Harris' lot size model were developed over the years, such as lot sizing models that include multi-stage inventory systems, incentives, or productivity issues. The aims of our tertiary study are the following: firstly, it helps primary researchers to position their own work in the literature, to reproduce the development of different types of lot sizing problems, and to find starting points if they intend to work in a new research direction. Secondly, the study identifies several topics that offer opportunities for future secondary research.

[1] This chapter has been published as: Glock, C.H, Grosse, E.H, Ries, J.M., 2014. The Lot Sizing Problem: A Tertiary Study. International Journal of Production Economics, Vol. 155, pp. 39-51. https://www.sciencedirect.com/science/article/pii/S0925527313005689

III. The influence of financial conditions on optimal ordering and payment policies under progressive interest schemes[2]

Abstract: In many business-to-business transactions, the buyer is not required to pay immediately after the receipt of an order, but is instead allowed to postpone the payment to its suppliers for a certain period. In such a situation, the buyer can either settle the account at the end of the credit period or authorize the payment later, usually at the expense of interest that is charged by the supplier on the outstanding balance. Some payment terms, which are often referred to as trade credit contracts, contain progressive interest charges. In such cases, the supplier offers a sequence of credit periods, where the interest rate that is charged on the outstanding balance usually increases from period to period. If a buyer faces a progressive trade credit scheme, various options for settling the unpaid balance exist, where the financial impact of each option depends on the current credit interest structure and the alternative investment conditions. This paper studies the influence of different financial conditions in terms of alternative investment opportunities and credit interest structure on the optimal ordering and payment policies of a buyer on the condition that the supplier provides a progressive interest scheme. For this purpose, mathematical models are developed and analyzed.

[2] This chapter has been published as: Ries, J.M., Glock, C.H, Schwindl, K, 2017. The influence of financial conditions on optimal ordering and payment policies under progressive interest schemes. Omega, Vol. 70, pp. 15-30.
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IV. Solution algorithms for optimal ordering policies under progressive payment schemes[3]

Abstract: This paper studies and extends solution algorithms for deriving the optimal ordering and payment policies of a retailer on the condition that the supplier provides a progressive interest scheme. Based on the finding that the piecewise total cost functions are convex but not necessarily continuous, a modified solution algorithm is developed and collated with existing ones in the course of a simulation experiment. The results indicate that the modified algorithm can locate all optimal solutions and outperforms existing approaches.

1. Introduction

Trade credits have received considerable attention as a means of short-term debt financing in recent years and became the most important source of funding for businesses suffering from the credit crunch. In the retail industry, for example, that is heavily reliant on trade credits (cf. Klapper et al., 2012), accounts payable can reach on average one fifth of the firm's total assets and one third of the firm's total liabilities (see Ries et al., 2017). Nevertheless, the amount of trade credit funding and also credit terms vary significantly across industries (cf. Ng. et al., 1999, Seifert et al., 2013).

In their simplest appearance, trade credits contain a specified period of time in which the retailer is allowed to defer the payment to its supplier without incurring any additional cost. However, if the retailer fails to settle payables within that predefined period, interest is charged on the outstanding balance. This type of trade credit was initially analyzed in the context of economic order quantity (EOQ) policies by Goyal (1985), who showed that the order quantity increases if predefined payment delays are permitted, as compared to the classical EOQ model that assumes immediate settlements. Subsequently, Dave (1985) modified the model by considering different purchasing and selling prices, and Chung (1998) presented a simplified solution procedure. Teng (2002) further extended the model of Goyal (1985) and demonstrated that for certain cases, it is beneficial for the retailer to reduce its order quantity if trade credits are offered in order to benefit from the permissible delay in payments more frequently. To counterbalance such effects, Huang (2007) introduced trade credit terms with threshold order quantities, in which the full trade credit is only granted if the retailer's order quantity exceeds the minimum quantity. Similarly, Chung et al. (2005) and Yang et al. (2013) assumed that if the order quantity does not exceed the threshold, the supplier does not offer a trade credit at all. Another variation of credit terms was introduced by Goyal et al. (2007), who were among the first to consider progressive payment schemes with more than a single credit period in an EOQ setting. The general idea of such a progressive payment scheme is that similar to the simple trade credits no interest is charged in the first credit period, and that the interest rate then increases from credit period to credit period. This paper has been revised by Chung (2009) who

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[3] This chapter contains material from a currently unpublished manuscript.

improved the solution procedure and was, more recently, extended by considering a stock-dependent demand rate inducing higher customer demand early in a replenishment cycle and lower customer demand at the end of a replenishment cycle (see, Soni and Shah, 2008; 2009 or Glock et al., 2015). In addition, it was shown that under different financial conditions, the buyer may benefit from postponing the payment to its supplier due to temporary arbitrage gains (cf. Glock et al., 2014 or Ries et al., 2017). Further extensions include the consideration of product deterioration (Teng et al., 2011), the time value of money (e.g. Ries et al., 2016), or limited storage space (e.g., Teng et al., 2011). A review of the trade credit literature including different modeling perspectives is provided in Seifert et al. (2013).

When analyzing the literature, it becomes apparent that most research frequently aimed at relaxing limiting assumption of earlier works in order to develop extended models covering a wide range of practical scenarios. However, with the notable exception of Soni and Shah (2008) and (2009), most studies either neglect the development of appropriate solution algorithms or refer to the approach outlined in Goyal et al. (2007) that leads to good but, not in all cases to optimal solutions. This is insufficient as most trade credit inventory models are based on piecewise total cost functions that are convex but not necessarily continuous. Finding the global optimum in such a case can results in unnecessarily large memory and computational requirements or can even become computationally prohibitive in case of a large number of credit periods (Patrinos and Sarimveis, 2011). Therefore, the paper at hand takes up this issue by a) developing a modified solution procedure for deriving the optimal ordering and payment policies of a retailer on the condition that the supplier provides a progressive interest scheme and by b) conducting a simulation study for analyzing the performance of the different algorithms and its determinants. All analyses are exemplified in the case of a linear demand function as studied in Goyal et al. (2007) and Ries et al. (2017) but do equally apply to other demand characteristics as in Soni and Shah (2008) and Glock et al. (2015) or other interest conditions as in Ries et al. (2017) given a piecewise convex total cost function. The remainder of the paper is structured as follows: The next section outlines assumptions and notations used throughout the paper and introduces formal models. Section 3 then illustrates relevant model characteristics and develops a novel solution algorithm. Sections 4 presents the results of a simulation study while Section 5 finally concludes the article.

2. Model formulation

2.1. Assumptions and notation

This paper is concerned with finding the optimal ordering and payment policies of a retailer on the condition that the supplier provides a progressive interest scheme. In such cases, the supplier offers a sequence of credit periods, where the interest rate that is charged on the outstanding balance usually increases from period to period. If a retailer faces such a progressive trade credit scheme, various options for ordering and settling the unpaid balance exist, where the financial impact of each option heavily depends on the current trade credit terms (e.g. the interest rates charged and credit periods allowed) as well as the alternative investment conditions. The outlined problem will subsequently be analyzed under the conditions that:

1. The inventory system involves a single item and has an infinite planning horizon.
2. Shortages are not allowed and the demand rate is constant and deterministic.
3. Lead time is zero and replenishments are made instantaneously.
4. The supplier provides a trade credit with progressive interest rates to the retailer. If the retailer pays before time M, the supplier does not charge any interest, whereas in case the retailer pays between times M and N with $M < N$, the supplier charges interest at the rate of Ic_1. In case the retailer pays after time N, the supplier charges interest at the rate of Ic_2, with $Ic_1 \leq Ic_2$.
5. The retailer has the option to deposit money in an interest bearing account with a fixed interest rate of Ie. Thus, s/he may use sales revenues to earn interest until the account is completely settled. However, to exclude arbitrage activity, it is assumed that $Ie \leq Ic_1 \leq Ic_2$

The following terminology is used throughout the paper:

Parameters:

A	cost of placing an order
C	unit purchasing cost with $C < P$
D	demand rate per unit of time
ϵ	infinitesimal number
h	physical unit holding cost per unit and unit of time
Ic_1	interest rate per unit of time charged between times M and N
Ic_2	interest rate per unit of time charged after time N
Ie	interest rate earned on deposits per unit of time
M	permissible delay in payments without any interest charges
N	permissible delay in payments at which the borrowing rate increases
P	selling price per unit

Decision variables

Q	order quantity of the retailer (can implicitly be derived from T)
T	replenishment interval

2.2. Piecewise total cost function

The total relevant costs of the retailer are given as the sum of ordering, inventory carrying and interest costs, reduced by interest earnings. Whereas ordering and inventory holding cost are determined by the length of the replenishment cycle, incurred interest costs and/or realized interest earnings are also dependent on the ratio of the interest rates (i.e. the ratio of Ie, Ic_1 and Ic_2) and the lengths of the credit periods, M and N. Based on Goyal et al. (2007) and as extended in Ries et al. (2017), the total cost function for the retailer $Z(T)$, given the generally assumed the interest structure $Ie \leq Ic_1 \leq Ic_2$, can be expressed as follows:

$$Z(T) = \begin{cases} Z_1(T), \; T \le M & (a) \\ Z_{2.1}(T), \; M < T \le N \wedge U_1 = 0 & (b) \\ Z_{2.2}(T), \; M < T \le N \wedge U_1 > 0 & (c) \\ Z_{3.1}(T), \; N < T \wedge U_1 = 0 & (d) \\ Z_{3.2}(T), \; N < T \wedge U_1 > 0 \wedge U_2 = 0 & (e) \\ Z_{3.3}(T), \; N < T \wedge U_1 > 0 \wedge U_2 > 0 & (f) \end{cases} \quad (1)$$

where $U_1 = CDT - PDM(1 + IeM/2)$ denotes the unpaid balance of the retailer at time M and $U_2 = \left(CDT - PDM(1 + IeM/2)\right)\left(1 + Ic_1(N - M)\right) - PD(N - M)(1 + Ic_1(N - M)/2)$ the unpaid balance at time N, respectively. The total relevant cost for all potential subcases are given as:

$$Z_1(T) = \frac{A}{T} + \frac{hDT}{2} - IePD\left(M - \frac{T}{2}\right) \quad (2)$$

$$Z_{2.1}(T) = \frac{A}{T} + \frac{hDT}{2} - \frac{IePDM^2}{2T} \quad (3)$$

$$Z_{2.2}(T) = \frac{A}{T} + \frac{hDT}{2} + \frac{Ic_1}{2PDT}\left(CDT - PDM(1 + IeM/2)\right)^2 - \frac{IePDM^2}{2T} \quad (4)$$

$$Z_{3.1}(T) = \frac{A}{T} + \frac{hDT}{2} - \frac{IePDM^2}{2T} \quad (5)$$

$$Z_{3.2}(T) = \frac{A}{T} + \frac{hDT}{2} + \frac{Ic_1}{2PDT}\left(CDT - PDM(1 + IeM/2)\right)^2 - \frac{IePDM^2}{2T} \quad (6)$$

$$Z_{3.3}(T) = \frac{A}{T} + \frac{hDT}{2} + \frac{Ic_1(N-M)}{T}\left(CDT - PDM\left(1 + \frac{IeM}{2}\right) - \frac{PD(N-M)}{2}\right) + \frac{Ic_2}{2PDT}\left(\left(CDT - PDM\left(1 + \frac{IeM}{2}\right)\right)\left(1 + Ic_1(N - M)\right) - PD(N - M)\left(1 + \frac{Ic_1(N-M)}{2}\right)\right)^2 - \frac{IePDM^2}{2T} \quad (7)$$

As Eqs. (3) and (5) reveal that $Z_{2.1}(T) = Z_{3.1}(T)$, just as Eqs. (4) and (6) reveal that $Z_{2.2}(T) = Z_{3.2}(T)$, Eq. $(1a - 1f)$ can be simplified (see also Chung, 2009). Integrating the boundary conditions and rearranging, leads to the following total cost function $Z(T)$ for the retailer:

$$Z(T) = \begin{cases} Z_1(T), \; T \le M & (a) \\ Z_{2.1}(T), \; M < T \le M\left(\frac{P}{C}\right)\left(1 + \frac{IeM}{2}\right) & (b) \\ Z_{3.2}(T), \; M\left(\frac{P}{C}\right)\left(1 + \frac{IeM}{2}\right) < T \le M\left(\frac{P}{C}\right)\left(1 + \frac{IeM}{2}\right) + \left(\frac{P\left(2(N-M) + Ic_1(N-M)^2\right)}{2C\left(1 + Ic_1(N-M)\right)}\right) & (c) \\ Z_{3.3}(T), \; M\left(\frac{P}{C}\right)\left(1 + \frac{IeM}{2}\right) + \left(\frac{P\left(2(N-M) + Ic_1(N-M)^2\right)}{2C\left(1 + Ic_1(N-M)\right)}\right) < T & (d) \end{cases} \quad (8)$$

3. Soution development

For convenience, we assume that all $Z_i(T)$ with i representing the respective case ($i \in \{1,2.1,3.2,3.3\}$) are defined on $T > 0$. The first-order conditions $dZ_i(T)/dT$ and second-order conditions $d^2Z_i(T)/dT^2$ for the relevant subcases in Eqs. (2)-(7) result in:

$$Z_1'(T) = -\frac{A}{T^2} + \frac{hD}{2} + \frac{IePD}{2} \quad (9)$$

$$Z_{2.1}'(T) = -\frac{A}{T^2} + \frac{hD}{2} + \frac{IePDM^2}{2T^2} \quad (10)$$

$$Z_{3.2}'(T) = -\frac{A}{T^2} + \frac{hD}{2} + \frac{Ic_1DC^2}{2P} - \frac{PDM^2}{2T^2}(Ic_1(1 + IeM/2)^2 - Ie) \quad (11)$$

$$Z_{3.3}'(T) = -\frac{A}{T^2} + \frac{hD}{2} + \frac{Ic_1PD(N-M)}{T^2}\left(M\left(1 + \frac{IeM}{2}\right) + \frac{(N-M)}{2}\right) + \frac{Ic_2DC^2}{2P}\left(1 + Ic_1(N-M)\right)^2 - \frac{Ic_2PDM^2}{2T^2}\left(\left(1 + \frac{IeM}{2}\right)\left(1 + Ic_1(N-M)\right) + \frac{(N-M)}{M}\left(1 + \frac{Ic_1(N-M)}{2}\right)\right)^2 + \frac{IePDM^2}{2T^2} \quad (12)$$

$$Z_1''(T) = \frac{2A}{T^3} \quad (13)$$

$$Z_{2.1}''(T) = \frac{1}{T^3}(2A - IePDM^2) \quad (14)$$

$$Z_{3.2}''(T) = \frac{1}{T^3}\left(2A + PDM^2(Ic_1(1 + IeM/2)^2 - Ie)\right) \quad (15)$$

$$Z_{3.3}''(T) = \frac{1}{T^3}\left(2A + Ic_2PDM^2\left(\left(1 + \frac{IeM}{2}\right)\left(1 + Ic_1(N-M)\right) + \frac{(N-M)}{M}\left(1 + \frac{Ic_1(N-M)}{2}\right)\right)^2 - 2Ic_1PD(N-M)\left(M\left(1 + \frac{IeM}{2}\right) + \frac{(N-M)}{2}\right) - IePDM^2\right) \quad (16)$$

Lemma 1. For $T > 0$, $Z_i(T)$ is convex for $i \in \{1,2.1,3.2,3.3\}$ and $T_i^* = \underset{T\in\mathbb{R}^+}{\arg\min}\, Z_i(T)$ exists.

Proof:

(a) As $2A/T^3 > 0$ for all $T > 0$, $Z_1(T)$ is convex. By rearranging (9), we get $F(T) = A/T$ and $G(T) = T/2\,(hD + IePD)$. Since $F'(T) = -A/T^2 < 0$, $F(T)$ is a strictly decreasing function in T. In contrast, since $G'(T) = 1/2\,(hD + IePD) > 0$, $G(T)$ is a strictly increasing function in T. In addition, $F(0) > G(0)$, whereas $F(\infty) < G(\infty)$. Consequently, there is a unique $T > 0$ such that $F(T) = G(T)$, which implies that T_1^* exists.

(b) As $(2A - IePDM^2)/T^3 > 0$ for all $T > 0$ given the presence of this subcase (not that by setting the first-order condition equal to zero, solving this equation for T and replacing T in the lower bound condition $M < T$, we obtain the stricter condition $2A - IePDM^2 + hDM^2$ required for this subcase), $Z_{2.1}(T)$ is convex. By rearranging (10), we get $F(T) =$

$(2A - IePDM^2)/T$ and $G(T) = hDT$. Since $F'(T) = -(2A - IePDM^2)/T^2 < 0$, $F(T)$ is a strictly decreasing function in T. In contrast, since $G'(T) = hD > 0$, $G(T)$ is a strictly increasing function in T. In addition, $F(0) > G(0)$, whereas $F(\infty) < G(\infty)$. Consequently, there is a unique $T > 0$ such that $F(T) = G(T)$, which implies that $T_{2.1}^*$ exists.

(c) As $\left(2A + PDM^2(Ic_1(1 + IeM/2)^2 - Ie)\right)/T^3 > 0$ for all $T > 0$ given that $Ic_1 > Ie$, $Z_{3.2}(T)$ is convex. By rearranging (11), we get $F(T) = \left(2A + PDM^2(Ic_1(1 + IeM/2)^2 - Ie)\right)/T$ and $G(T) = TD(hP + Ic_1C^2)/P$. Since $F'(T) = -\left(2A + PDM^2(Ic_1(1 + IeM/2)^2 - Ie)\right)/T^2 < 0$, $F(T)$ is a strictly decreasing function in T. In contrast, since $G'(T) = D(hP + Ic_1C^2)/P > 0$, $G(T)$ is a strictly increasing function in T. In addition, $F(0) > G(0)$ whereas $F(\infty) < G(\infty)$. Consequently, there is a unique $T > 0$ such that $F(T) = G(T)$, which implies that $T_{3.2}^*$ exists.

(d) As $\left(2A + Ic_2PDM^2\left(\left(1 + \frac{IeM}{2}\right)\left(1 + Ic_1(N - M)\right) + \frac{(N-M)}{M}\left(1 + \frac{Ic_1(N-M)}{2}\right)\right)^2 - 2Ic_1PD(N - M)\left(M\left(1 + \frac{IeM}{2}\right) + \frac{(N-M)}{2}\right) - IePDM^2\right)/T^3 > 0$ for all $T > 0$ given that $Ic_1 > Ie$, $Z_{3.3}(T)$ is convex. By rearranging (12), we get $F(T) = \left(2A + Ic_2PD\left(M(1 + IeM/2)\left(1 + Ic_1(N - M)\right) + (N - M)(1 + Ic_1(N - M)/2)\right)^2 - Ic_1PD(N - M)\left(M(2 + IeM) + (N - M)\right) - IePDM^2\right)/T$ and $G(T) = TD\left(hP + Ic_2\left(C + CIc_1(N - M)\right)^2\right)/P$. Since $F'(T) = -\left(2A + Ic_2PD\left(M(1 + IeM/2)\left(1 + Ic_1(N - M)\right) + (N - M)(1 + Ic_1(N - M)/2)\right)^2 - Ic_1PD(N - M)\left(M(2 + IeM) + (N - M)\right) - IePDM^2\right)/T^2 < 0$, $F(T)$ is a strictly decreasing function in T. In contrast, since $G'(T) = D\left(hP + Ic_2\left(C + CIc_1(N - M)\right)^2\right)/P > 0$, $G(T)$ is a strictly increasing function in T. In addition, $F(0) > G(0)$, whereas $F(\infty) < G(\infty)$. Consequently, there is a unique $T > 0$ such that $F(T) = G(T)$, which implies that $T_{3.3}^*$ exists. □

Consequently, all $Z_i(T)$ with $i \in \{1,2.1,3.2,3.3\}$ are convex on $T > 0$ and Z: $\mathbb{R} \to \mathbb{R}$ is a piecewise convex function that can be decomposed into $Z(T) = \min\{Z_i(T) \mid i \in K\}$, where $Z_i : \mathbb{R} \to \mathbb{R}$ is convex for all $i \in K = \{1,2.1,3.2,3.3\}$. For finding the global optimum, one has to compare for each value of the parameter vector the corresponding value functions for all the pieces of the function to identify the indices of the pieces corresponding to the minimum value. This procedure can obviously become computationally prohibitive if the number of pieces of the function is large. Furthermore, this approach results in unnecessarily large memory and computational requirements (cf. Patrinos and Sarimveis, 2011). Therefore, the proposed algorithm aims at providing a more efficient procedure based on the structural characteristics of $Z(T)$. To examine the characteristics of Eq. (8) in more detail, let:

$$\Delta_1 = \frac{PM}{C} + \frac{IePM^2}{2C} = \frac{PM}{C}\left(1 + \frac{IeM}{2}\right) \quad (17)$$

$$\Delta_2 = \frac{PM}{C}\left(1 + \frac{IeM}{2}\right) + \frac{P}{C}(N - M)\frac{\left(2+Ic_1(N-M)\right)}{2\left(1+Ic_1(N-M)\right)} \quad (18)$$

From Eqs. (2)-(7) it can be inferred that:

$$Z_1(M) = \lim_{T\to M} Z_{2.1}(T) \quad (19)$$

$$Z_{2.1}(\Delta_1) = \lim_{T\to \Delta_1} Z_{3.2}(T) \quad (20)$$

$$Z_{3.2}(\Delta_2) \neq \lim_{T\to \Delta_2} Z_{3.3}(T) \quad (21)$$

Lemma 2. For $T \to \Delta_2$, the total cost of subcase $Z_{3.2}(T)$ exceed the total cost of subcase $Z_{3.3}(T)$.

Proof: Rearranging $\lim_{T\to \Delta_2} Z_{3.3}(T) - Z_{3.2}(\Delta_2)$ leads to $-PDI{c_1}^3(N-M)^4/8\Delta_2\left(1+Ic_1(N-M)\right)^2$ with $\Delta_2 > 0$. Hence, it can be concluded that $\lim_{T\to \Delta_2} Z_{3.3}(T) - Z_{3.2}(\Delta_2) < 0$ or $\lim_{T\to \Delta_2} Z_{3.3}(T) \neq Z_{3.2}(\Delta_2)$. □

In addition from Eqs. (9)-(12) it follows that:

$$Z_1'(M) = -\frac{A}{M^2} + \frac{hD}{2} + \frac{IePD}{2} \quad (22)$$

$$\lim_{T\to M} Z_{2.1}'(T) = -\frac{A}{M^2} + \frac{hD}{2} + \frac{IePD}{2} \quad (23)$$

$$Z_{2.1}'(\Delta_1) = -\frac{A}{{\Delta_1}^2} + \frac{hD}{2} + \frac{IePDM^2}{2{\Delta_1}^2} \quad (24)$$

$$\lim_{T\to \Delta_1} Z_{3.2}'(T) = -\frac{A}{{\Delta_1}^2} + \frac{hD}{2} + \frac{IePDM^2}{2{\Delta_1}^2} \quad (25)$$

$$Z_{3.2}'(\Delta_2) = -\frac{A}{{\Delta_2}^2} + \frac{hD}{2} + \frac{IePDM^2}{2{\Delta_2}^2} - \frac{Ic_1DC^2}{2P}\left(\frac{{\Delta_1}^2}{{\Delta_2}^2} - 1\right) \quad (26)$$

$$\lim_{T\to \Delta_2} Z_{3.3}'(T) = -\frac{A}{{\Delta_2}^2} + \frac{hD}{2} + \frac{IePDM^2}{2{\Delta_2}^2} + \frac{Ic_1DC^2}{2P}\left(2\frac{\Delta_1}{{\Delta_2}^2}\frac{P}{C}(N-M) + \frac{1}{{\Delta_2}^2}\frac{P^2}{C^2}(N-M)^2\right) \quad (27)$$

Lemma 3. The first-order conditions $dZ_i(T)/dT$ at the interval boundaries for $T \in \{M, \Delta_1, \Delta_2\}$ have the following structure $Z_1'(M) < Z_{2,1}'(\Delta_1) < Z_{3.2}'(\Delta_2) < \lim_{T\to \Delta_2} Z_{3.3}'(T)$.

Proof: Given that $M < \Delta_1 < \Delta_2$, Eqs. (22) to (26) reveal that $Z_1'(M) < Z_{2,1}'(\Delta_1) < Z_{3.2}'(\Delta_2)$. Rearranging $Z_{3.2}'(\Delta_2) < \lim_{T \to \Delta_2} Z_{3.3}'(T)$, it can be shown that the condition is satisfied for all $\left(2 + Ic_1(N - M)\right)/2\left(1 + Ic_1(N - M)\right) < 1$. As this is a strictly monotonically decreasing function of $N - M$ with a maximum of 1 at $N = M$ (note that $N > M$ was a prerequisite for the existence of this case), the condition generally holds true. Hence, it follows that $Z_1'(M) < Z_{2,1}'(\Delta_1) < Z_{3.2}'(\Delta_2) < \lim_{T \to \Delta_2} Z_{3.3}'(T)$. □

Theorem 1. For $T > 0$, there exists a unique $T^* = \arg\min_{T \in \mathbb{R}^+} Z(T)$.

(a) If $Z_1'(M) \geq 0$, then $T^* = T_1^*$ or $T^* = \Delta_2 + \epsilon$ associated with the least cost.
(b) If $Z_1'(M) < 0 \leq Z_{2.1}'(\Delta_1)$, then $T^* = T_{2.1}^*$ or $T^* = \Delta_2 + \epsilon$ associated with the least cost.
(c) If $Z_{2.1}'(\Delta_1) < 0 \leq Z_{3.2}'(\Delta_2)$, then $T^* = T_{3.2}^*$ or $T^* = \Delta_2 + \epsilon$ associated with the least cost.
(d) If $Z_{3.2}'(\Delta_2) < 0 \leq \lim_{T \to \Delta_2} Z_{3.3}'(T)$, then $T^* = \Delta_2 + \epsilon$.
(e) If $\lim_{T \to \Delta_2} Z_{3.3}'(T) <$, then $T^* = T_{3.3}^*$.

Proof:

(a) If $Z_1'(M) \geq 0$, Lemma 3 implies $0 < Z_1'(M) < Z_{2,1}'(\Delta_1) < Z_{3.2}'(\Delta_2) < \lim_{T \to \Delta_2} Z_{3.3}'(T)$ and from Lemma 1 it follows that
  (i) $Z_1(T)$ is decreasing on $(0, T_1^*]$ and increasing on $[T_1^*, M]$.
  (ii) $Z_{2.1}(T)$ is increasing on $(M, \Delta_1]$.
  (iii) $Z_{3.2}(T)$ is increasing on $(\Delta_1, \Delta_2]$.
  (iv) $\lim_{T \to \Delta_2} Z_{3.3}'(T)$ is increasing on (Δ_2, ∞).
  Combining (i)-(iv), Lemma 2 and Eqs. (22)-(27), we can conclude that $T^* = T_1^*$ or $T^* = \Delta_2 + \epsilon$ associated with the least cost.
(b) If $Z_1'(M) < 0 \leq Z_{2.1}'(\Delta_1)$, Lemma 3 implies $Z_1'(M) < 0 < Z_{2,1}'(\Delta_1) < Z_{3.2}'(\Delta_2) < \lim_{T \to \Delta_2} Z_{3.3}'(T)$ and from Lemma 1 it follows that
  (i) $Z_1(T)$ is decreasing on $(0, M]$.
  (ii) $Z_{2.1}(T)$ is decreasing on $(M, T_{2.1}^*]$ and increasing on $[T_{2.1}^*, \Delta_1]$.
  (iii) $Z_{3.2}(T)$ is increasing on $(\Delta_1, \Delta_2]$.
  (iv) $\lim_{T \to \Delta_2} Z_{3.3}'(T)$ is increasing on (Δ_2, ∞).
  Combining (i)-(iv), Lemma 2 and Eqs. (22)-(27), we can conclude that $T^* = T_{2.1}^*$ or $T^* = \Delta_2 + \epsilon$ associated with the least cost.
(c) If $Z_{2.1}'(\Delta_1) < 0 \leq Z_{3.2}'(\Delta_2)$, Lemma 3 implies $Z_1'(M) < Z_{2,1}'(\Delta_1) < 0 < Z_{3.2}'(\Delta_2) < \lim_{T \to \Delta_2} Z_{3.3}'(T)$ and from Lemma 1 it follows that
  (i) $Z_1(T)$ is decreasing on $(0, M]$.
  (ii) $Z_{2.1}(T)$ is decreasing on $(M, \Delta_1]$.
  (iii) $Z_{3.2}(T)$ is decreasing on $(\Delta_1, T_{3.2}^*]$ and increasing on $[T_{3.2}^*, \Delta_2]$.

(iv) $\lim_{T\to\Delta_2} Z'_{3.3}(T)$ is increasing on (Δ_2, ∞).

Combining (i)-(iv), Lemma 2 and Eqs. (22)-(27), we can conclude that $T^* = T^*_{3.2}$ or $T^* = \Delta_2 + \epsilon$ associated with the least cost.

(d) If $Z'_{3.2}(\Delta_2) < 0 \leq Z'_{3.3}(\Delta_2)$, Lemma 3 implies $Z'_1(M) < Z'_{2,1}(\Delta_1) < Z'_{3.2}(\Delta_2) < 0 < \lim_{T\to\Delta_2} Z'_{3.3}(T)$ and from Lemma 1 it follows that

(i) $Z_1(T)$ is decreasing on $(0, M]$.

(ii) $Z_{2.1}(T)$ is decreasing on $(M, \Delta_1]$.

(iii) $Z_{3.2}(T)$ is decreasing on $(\Delta_1, \Delta_2]$.

(iv) $\lim_{T\to\Delta_2} Z'_{3.3}(T)$ is increasing on (Δ_2, ∞).

Combining (i)-(iv), Lemma 2 and Eqs. (22)-(27), we can conclude that $T^* = \Delta_2 + \epsilon$.

(e) If $Z'_{3.3}(\Delta_2) < 0$, Lemma 3 implies $Z'_1(M) < Z'_{2,1}(\Delta_1) < Z'_{3.2}(\Delta_2) < \lim_{T\to\Delta_2} Z'_{3.3}(T) < 0$ and from Lemma 1 it follows that

(i) $Z_1(T)$ is decreasing on $(0, M]$.

(ii) $Z_{2.1}(T)$ is decreasing on $(M, \Delta_1]$.

(iii) $Z_{3.2}(T)$ is decreasing on $(\Delta_1, \Delta_2]$.

(iv) $\lim_{T\to\Delta_2} Z'_{3.3}(T)$ is decreasing on $(\Delta_2, T^*_{3.3}]$ and increasing on $[T^*_{3.3}, \infty)$.

Combining (i)-(iv) and Eqs. (22)-(27), we can conclude that $T^* = T^*_{3.3}$.

Incorporating all arguments, we have completed the proof of Theorem 1. □

Given that the total cost function $Z(T)$ is a piecewise convex function that can be decomposed into $Z(T) = \min\{Z_i(T) \mid i \in K\}$, where $Z_i : \mathbb{R} \to \mathbb{R}$ is convex for all $i \in K = \{1, \dots, k\}$ and Lemmas 1-3 hold true, we can easily derive T^* by the iterative procedure outlined in Theorem 1 and summarized below:

Step1: Compute $Z'_i(T_i^{max})$ and $\lim_{T\to T_i^{\max}} Z'_{i+1}(T)$ for all $i \in K = \{1, \dots, k-1\}$ and sort all $Z'_i(T_i^{max})$ and $\lim_{T\to T_i^{\max}} Z'_{i+1}(T)$ in ascending order beginning with $i = 1$.

Step2: If $0 < Z'_i(T_i^{max}) = \lim_{T\to T_i^{\max}} Z'_{i+1}(T)$, then $T_i^* = \arg\min_{T\in\mathbb{R}^+} Z_i(T)$ and go to next step, otherwise $i = i + 1$ and repeat step until $i = k$ and $T_i^* = \arg\min_{T\in\mathbb{R}^+} Z_k(T)$.

Step3: If $Z_i(T_i^*) \leq \lim_{T\to T_{k-1}^{\max}} Z_k(T)$, then $T^* = T_i^*$, otherwise $T^* = T_{k-1}^{\max} + \epsilon$.

4. Numerical studies

To analyze the performance of the developed approach aimed at identifying the optimal ordering and payment policy in the presence of a progressive interest scheme, this section outlines the results of a simulation experiment. According to Kritchanchai and MacCarthy (2002), simulation experiments provide valuable insights into distinctive behavior of real-life systems and enable the exploration and comparison of different control policies. Therefore, in the present case, the policy as proposed in Goyal et al. (2007) will serve as a benchmark to test the performance of the developed algorithm for different sets of input parameters. In the absence of representative data, the reference state of the simulation model was deduced by the help of data from existing studies (e.g. Goyal et al., 2007, Chung, 2009 and Ries et al., 2017). In addition, the debugging technique was used to validate the simulation model. Errors that occurred during the simulation were analyzed and corrected. For various parameter combinations the system outcome was tested against the predicted results to confirm that the system behaves in a consistent manner (cf. Kritchanchai and MacCarthy, 2002). After defining the parameter ranges (see Table 1), 1,000 instances were generated using random sampling (note that these instances were also tested for their space-filling properties, cf. Kleijnen et al. 2005) and analyzed in the following.

D	∈	[500,1500]	demand in units per year
A	∈	[15,600]	ordering cost per order
C	∈	[10,40]	unit purchase cost
P	∈	[max{C,30},50]	unit selling price
h	∈	[2,8]	inventory holding cost per unit and year
Ic_1	∈	[0.005,0.08]	interest rate per year for the first credit period
Ic_2	∈	[0.08,0.16]	interest rate per year for the second credit period
Ie	∈	[0.005,min{Ic_1, 0.08}]	interest rate on deposits per year
M	∈	[10;60]	first permissible credit period
N	∈	[max{M,40},90]	second permissible credit period

Table 1: Parameter ranges for simulation data sets

The analysis of the simulation results reveals that the developed algorithm outperforms the benchmark approach in 55% of all instances while realizing equal results in all other instances. This leads to an average reduction of the replenishment intervals and order quantities by 1.99% whereas the on average the total cost decrease by 0.09% only which can be explained by the robustness property of the economic order quantity (cf. Stadtler, 2007). However, for some instances the average replenishment intervals and order quantities decrease by 17.43% which also induces a reduction of total cost by 2.37%. It’s worth mentioning that these cost savings come at no cost and are solely induced by the improvements of the solution algorithm. Consequently, the proposed solution procedure can lead to a significant modification of the retailer’s ordering behavior associated with lower total cost (note that the algorithm as proposed by Soni and Shah, 2008, under certain conditions leads to equivalent results, but at the expense of higher computational efforts). In addition to the descriptive results, we also analyze the impact of the different model parameters on the relative performance of the developed algorithm by the help several multivariate regressions. The results of the regression analyses with the problem parameters as independent variables and the ratio of the replenishment intervals ($T^{New}/T^{\mathrm{Goyal}}$) or the ratio of expected total costs ($TC^{New}/TC^{\mathrm{Goyal}}$) as the dependent variables are shown in Table 2. As can be seen, a statistically significant relationship (with Sig. < 0.05) is found between all problem parameters and the ratio of the replenishment intervals, with the exception of *D* and Ic_2. An increase in the time span between *M* and *N* as well as an increase in Ic_1 or a decrease in *Ie* leads to comparatively lower replenishment intervals and order quantities. Similarly, for the ratio of the total costs a statistically significant relationship (with Sig. < 0.05) can be found for all problem parameters, with the exception of *P* and Ic_2. Even though the effects are less pronounced, an increase in the time span between *M* and *N* as well as an increase in Ic_1 or a decrease in *Ie* also induces a lower ratio of total cost and consequently increasing cost advantages for the developed algorithm. Thus, especially in the case of less generous payment terms offered by the supplier and a low deposit rate of the retailer, lowered inventories and cost savings can be realized by applying more accurate solution approaches.

Model parameter	Coefficient	t-value	Sig.
D	0.000	-0.540	0.589
A	0.000	-9.461	0.000
C	-0.001	-15.417	0.000
P	0.000	3.310	0.001
h	0.004	8.615	0.000
Ic_1	-0.491	-10.508	0.000
Ic_2	-0.009	-0.285	0.776
Ie	0.129	2.114	0.035
M	0.165	8.639	0.000
N	-0.074	-3.700	0.000

Model parameter	Coefficient	t-value	Sig.
D	0.000	-3.270	0.001
A	0.000	-2.420	0.016
C	0.000	-8.990	0.000
P	0.000	1.447	0.148
h	0.000	7.202	0.000
Ic_1	-0.038	-8.964	0.000
Ic_2	-0.001	-0.267	0.790
Ie	0.011	2.000	0.046
M	0.011	6.450	0.000
N	-0.007	-3.934	0.000

Table 2: Results of the regression analysis for the ratio of payment intervals (left part of Table 2; adjusted $R^2 = 0.379$) and the ratio of total costs (right part of Table 2; adjusted $R^2 = 0.227$)

5. Conclusion

The purpose of this paper was to study and extend solution algorithms for deriving the optimal ordering and payment policy of a retailer on the condition that the supplier provides a progressive interest scheme by a) developing a modified solution procedure to identify the optimal length of the replenishment interval and by b) analyzing realized performance improvements in comparison to earlier approaches by the help of a simulation experiment. The results provide evidence that in case of a progressive interest scheme inventory models are based on piecewise total cost functions that are convex but not necessarily continuous which require efficient solution procedures based on the structural characteristics of the function. In addition, the simulation experiment reveals that the proposed approach outperforms the traditional solution approach in 55% of all generated instances which leads to significant reductions of inventories and additional cost savings. Those savings in inventories and total cost can especially be realized in cases of less generous payments intervals (e.g. a higher time span between M and N), higher interest rates charged by the supplier and low deposit rates earned by the retailer.

The results of this paper have several implications for research and practice. As has been shown, in the case of progressive trade credits with two credit periods, inaccurate solution algorithms may lead to significantly higher inventories and unnecessary additional cost. However, it is easy to imagine, that the supplier may extend its credit policy by introducing contracts with *k* credit periods. In this case, finding an appropriate replenishment policy will become even more challenging and require advanced solution procedures as outlined before.

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V. A note on: Optimal ordering policy for stock-dependent demand under progressive payment scheme[4]

Abstract: In a recent paper, Soni and Shah [Soni, H., Shah, N. H., 2008. Optimal ordering policy for stock-dependent demand under progressive payment scheme. European Journal of Operational Research 184, 91-100] developed a model to find the optimal ordering policy for a retailer with stock-dependent demand and a supplier that offers a progressive payment scheme to the retailer. This note corrects some errors in the formulation of the model of Soni and Shah. It also extends their work by assuming that the credit interest rate of the retailer may exceed the interest rate charged by the supplier. Numerical examples illustrate the benefits of these modifications.

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https://www.sciencedirect.com/science/article/abs/pii/S0377221713006127

VI. Ordering policy for stock-dependent demand rate under progressive payment scheme: A comment[5]

Abstract: In a recent paper, Soni and Shah [2009. Ordering policy for stock-dependent demand rate under progressive payment scheme. International Journal of Systems Science 40, 81-89] developed a model for finding the optimal ordering policy for a retailer facing stock-dependent demand and a supplier offering a progressive payment scheme. In this note, we correct several errors in the formulation of the models of Soni and Shah and modify some assumptions to increase the model's applicability. Numerical examples illustrate the benefits of our modifications.

[5] This chapter has been published as: Glock, C.H, Ries, J.M., Schwindl, K., 2015. Ordering policy for stock-dependent demand rate under progressive payment scheme: A comment. International Journal of Systems Science, Vol. 46, No. 5, pp. 872-877. https://www.tandfonline.com/10.1080/00207721.2013.798446

VII. Economic ordering and payment policies under progressive payment schemes and time-value of money[6]

Abstract: Trade credits have received considerable attention in recent years and have become one of the most important sources of short-term funding for many companies. The paper at hand studies the optimal ordering and payment policies of a buyer assuming that the supplier offers a progressive interest scheme. The contribution to the literature is twofold. First, the different financial conditions of the companies involved are taken into account by assuming that the credit interest rate of the buyer may, but not necessarily has to, exceed the interest rate charged by the supplier. In addition, the time-value of money is considered in this scenario which is relevant when trade credit terms are valid for a long period of time and payment flows need to be evaluated by their net present value to ensure long-term profitability. The models proposed enable decision makers to improve ordering and payment decisions and the results reveal that taking into account the temporal allocation of payments, the prevailing interest relation influences replenishment policies significantly.

[6] This chapter has been published as: Ries, J.M., Glock, C.H., Schwindl, K., 2016. Economic ordering and payment policies under progressive payment schemes and time-value of money. International Journal of Operations and Quantitative Management, Vol. 22, No. 3, pp. 101-121. http://www.ijoqm.org/v22no3.asp#

VIII. Reducing lead time risk through multiple sourcing: The case of stochastic demand and variable lead time[7]

Abstract: This paper studies a buyer sourcing a product from multiple suppliers under stochastic demand. The buyer uses a (Q, s) continuous review, reorder point, order quantity inventory control system to determine the size and timing of orders. Lead time is assumed to be deterministic and to vary linearly with the lot size, wherefore lead time and the associated stockout risk may be influenced by varying the lot size and the number of contracted suppliers. This paper presents mathematical models for a multiple supplier single buyer integrated inventory problem with stochastic demand and variable lead time and studies the impact of the delivery structure on the risk of incurring a stockout during lead time.

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https://www.tandfonline.com/10.1080/00207543.2011.644817