Chapter 1 Introduction and Motivation

For a long period of time, repeatedly observed point positions have been a source of information as to the deformation of the Earth's crust. Perhaps the main reason resides with the facts that this method (that compares repeated positions) is seemingly the most straightforward approach and that, a huge source of data has been available (Vanicek and Krakiwsky, 1986). Traditionally, horizontal and vertical control points have been treated separately. Today, not only the rate of available repeated point positions has considerably increased but also the vertical and horizontal positions of points can be simultaneously determined. This is due to space-geodetic techniques like the Global Positioning System (GPS) (Leick, 1995; Hoffmann et al., 1992). In the light of these new possibilities, we are now capable to analyze the interdependence of the vertical and horizontal components of point positions on deformation parameters. Deformation of the Earth's crust is a three-dimensional phenomenon in nature. Therefore, its study in horizontal dimensions by reducing the observations onto the surface of a horizontal reference datum or, considering it as a pure vertical process can lead to a systematic error in the result obtained.

Several authors have tried to study the deformation in three-dimensions. For example: Brunner (1979) proposed a method for analyzing the *overall deformation* of the Earth's crust based on the idea that the deformation can be taken as an affine transformation (Sokolnikoff, 1956), which maps the unstrained state of a body to its strained state. An affine transformation is literally a linear functional relation, therefore theoretically, this mathematical representation of the problem cannot accommodate inhomogeneous deformations. Grafarend (1986) formulates global scale deformations, like tidal induced deformations, in terms of spherical harmonics. He has also formulated local deformations in terms of irregularly shaped finite elements. Lichtenegger and Sünkel (1989) reviewed the theoretical aspects of both three and two-dimensional analysis as well as the kinematics and dynamics of deformation. They also discussed the plate kinematics in the theory of plate tectonics. Wittenburg (2003) argued that due to the deficit of required information a three-dimensional description of deformation, based on geodetic surveys is not possible. This argument is not generally correct and depends on the methodology to be used for the analysis of deformation. For example: the theory of analytical surface deformation analysis (Altiner, 1999) takes into account the three-dimensional nature of deformation through the computation of the so-called external measures of deformation. However, it is not possible to establish a functional relation between the external measures of deformation (as the parameters that characterize the deformation) and stress, constitutive equations, for all types of rehologies. In addition to that, the interpretation of the external measures of deformation is also difficult. Therefore, development of a new approach that can consider the three dimensional nature of deformation and produces immediately applicable and easily interpretable results is evidently still of great interest.

In this work, two new approaches have been implemented and worked out for analyzing the 3D-kinematics of deformations. One is based on the Lagrangian representation of the problem and the other is based on its Isoparametric form (Flügge, 1972).

The new approaches implemented in this research benefit from the simplicity of the extension of the well-established two-dimensional methods to three-dimensions. In the light of the new approaches, the cross-correlation effect of the vertical and horizontal deformations on the deformation parameters can be analyzed. The new approaches propose a set of new constraints on the deformations of the Earth's crust. The key point resides in the treatment of the problem in a geocentric Cartesian coordinate system. To see if the infinitesimal approximation of deformation is self-sufficient or the non-linearity of deformation should also be taken into account, the finite element approach (Pope, 1972; Dermains and Grafarend, 1993) has been avoided. Once the 3D-deformation tensor of one station is computed in a geocentric Cartesian coordinate system, it can be transformed to any desired system of curvilinear coordinates. This is done using the known rotations between the two coordinate systems and the deformation quadratic. This process greatly simplifies the computations and adds versatility to the new approaches compared to the traditional two-dimensional ones. This is because, the improvement of the computational accuracy in traditional two-dimensional methods, for example from spherical to ellipsoidal approximation, requires reformulation of the both deterministic and stochastic mathematical models of the problem.

The extension of the well-known two-dimensional methods to three-dimensions is not as straightforward as it may seem. Characteristics like the network size and the network configuration challenge this straightforward process. To analyze the upcoming problems, sensitivity analysis of the least-squares estimates in relation to the unknowns (Higham, 2002) is considered first. Based on simulation studies it is shown that the 3D-Lagrangian representation of deformation is not sensitive to the perturbations of input parameters. For real measurements, the instability of the least-squares solution to the 3D-Isoparametric and Lagrangian representations of deformation is assured through the analysis of discrete Picard condition. Singular value decomposition and principal component analysis have aided to formulate a capable method for the detailed diagnosis of the problems (sensitivity analysis of deformation tensor).

Regularization and optimization techniques are used to reduce the sensitivity of the solution when the stability of the least-squares solution is not assured. This is at the cost of loosing the resolution. The analysis of regularization error in the solution of a regularized solution is normally based on some a-priori information on the solution of the problem. However, it is shown that in three-dimensional analysis of deformation, two-dimensional results of deformation analysis can be used for analyzing regularization errors of estimated 3D-parameters of deformation. Nevertheless, physical evidences as well as independent results obtained from previous works on the analysis of deformation in the study area, using data sources other than GPS, also support the three-dimensional regularized solutions of this study.

The statistical inference of deformation parameters has been used as a mathematical tool for analyzing the change of deformations. The development and implementation of statistical methods have been systematically restricted to displacement fields (e.g. Caspary et al., 1999; Brunner, 1979). The statistical inference of the deformation tensor can profit from the invariance property of the deformation parameters as compared to at least some of the traditional inference techniques. Since the signal to noise ratio for the change in deformation is normally small both in space and time, the assessment and interpretation of such small variations requires sophisticated mathematical techniques. A new two-step method has been devised that can fulfill the requirements mentioned above. The method is capable of identifying the significance of the change in deformations at various stations of a network (the change in deformations in space) and similar stations in the course of time (the change in deformations in time). Instead of the standard multivariate test statistics such as the Hotteling's T^2 test, the method is based on 1) the Procrustes analysis (Mosier, 1939; Green, 1952; Cliff, 1966;

Schönemann, 1966; Schönemann and Carroll, 1970; Gower, 1975; Lissitz and Schönemann, 1976; Ten Berg, 1977; Goodall, 1991; Dryden and Mardia, 2002) of deformation tensors and 2) screening of the corresponding procrustean residual tensors, hence it is here referred as *the Procrustean Statistical Inference of Deformation.*

Systematic errors are a potential source for falsifying the estimate of the deformation parameters and their subsequent interpretations. This is because observational and model source systematic errors can propagate to the unknowns, an effect that depends on the adopted methodology for formulating and solving the problem. The 3D-analysis of the Earth's surface deformations accounts for the model source systematic errors. For the sake of completeness, an attempt has been made to formulate the effect of observational systematic errors on deformation parameters. Since this study makes use of the GPS technique, the impacts of the systematic errors in GPS measurements are of the most interest.

The cross-correlation effect of the height and horizontal coordinates is expected to be larger where the deformation rate in the vertical dimension is higher or at least comparable to the rate of deformation in horizontal ones. Therefore, an area where the deformation in vertical direction is considerable will be of special interest. One good example would be the Kenai Peninsula in south-central Alaska, which serves as the test area in this research. It is well known that Southern Alaska (together with Aleutian Island chain, extending from Fairbanks on the north to the Gulf of Alaska on the south) is one of the world's most active seismic zones. About seven percent of the Earth's annual seismic energy comes from this area (Stacy, 1977). The subduction of the Pacific plate beneath the North American plate along the Aleutian subduction zone and the 1964 Prince William Sound (PWS) earthquake (the biggest event in the instrumentally recorded earthquake history in North America) are two major sources for crustal deformation in this area. The area is now undergoing pronounced post-seismic uplift with a maximum rate of $30\ mm/yr$ in response to the 1964 PWS event (see for example: Cohen and Freymueller, 1997). Therefore, the area seems to fulfill the requirements of this research.

The roadmap of this study is as follows. The test area and its tectonic setting will be firstly introduced in Chapter two. This chapter includes a literature survey on other researchers' results that can both help better interpret and validate the newly obtained results, especially in the third dimension. In Chapter three, the mathematical frameworks of the two new approaches that have been implemented in this study for the 3D-analysis of deformation are introduced. The upcoming problems that challenge the straightforward process of the extension of two-dimensional methods are briefly discussed in this chapter. In the fourth chapter, theoretical considerations of the previous chapter are worked out in further detail and numerically analyzed using simulated fields of deformation. This process deepens our understanding of the phenomenon and provides the valuable chance of checking computer codes that have been developed for practical applications in this research. Chapter five introduces the mathematical framework of the procrustean statistical inference of deformations. There, the efficiency of the method will be shown by applying it to synthetic deformations. The GPS systematic errors are briefly discussed in the sixth chapter. The impacts of these systematic errors are also analytically worked out for Isoparametric representation of deformations. Chapter seven considers the 3D-analysis of the crustal deformations within the Kenai Peninsula. In the first place, GPS measurements and their analysis process are discussed. Then, 2D- and 3D-numerical results of deformations analysis in the test area are given. Regularization errors of the 3D-deformation tensors of this study have been assessed by comparing the 2D- and 3D- numerical results. Physical evidence and the results of other studies that support the 3D-results obtained here are also given and discussed there. Finally, The research is closed in the concluding chapter by reviewing the new contributions and proposing the possibilities of further research.