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PHYSICS

Nonabelian generalization of Topological Dipole Field Theory

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ABSTRACT

The Standard model of particle physics is based on nonabelian gauge theories. Since there are observed phenomena which cannot be explained with ordinary Standard model, this theory can be further generalized. This paper treats an extension of the Standard model by introducing a generalization of nonabelian gauge theories.

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INTRODUCTION

The most general model which is experimentally verified is the Standard model of particle Physics. A few years ago, the Higgs boson was observed at the Large Hadron Collider (Chatrchyan et al. 2012). Standard model of particle physics describes the electroweak and the strong interaction which are fundamental forces of nature. Both interactions are described by a nonabelian gauge theory based on the gauge group $U(1) \times SU(2) \times SU(3)$. The electromagnetic interaction which relies on the Lie group $U(1)$ is obtained by symmetry breaking of the electroweak interaction. Gravitational interactions are not included in the Standard model.

However, the Standard model of particle physics cannot describe some experiments with sufficient accuracy. An example of a phenomenon where the Standard model fails is the asymmetry of matter and antimatter in the universe (Canetti et al. 2012). This asymmetry is also called “Baryon asymmetry” and is still an unsolved problem in physics. A model which explains the baryon asymmetry is an intrinsic electric dipole moment in elementary particles (The ACME Collaboration 2014). Such a dipole moment would lead to a difference in the decay rates of matter and antimatter. The validity of the electric dipole argument for explanation of baryon asymmetry is still an open question in physics.

Recently, an intrinsic dipole moment in elementary particles with electric charge is proposed as an additional degree of freedom in Topological Dipole Field Theory (Linker 2015). The intrinsic dipole moment has a topological nature, i.e. the physical system does not depend explicitly on the magnitudes of this dipole moment. Due to this fact it is constructed a topological quantum field theory for this intrinsic dipole moment. By adding a 2-form dipole field B to the ordinary electromagnetic field strength tensor an extension of Quantum electrodynamics is obtained. This implies a modified dynamics of the force carriers of electromagnetism. Another generalization of electrodynamics is the Born-Infeld model (Goenner 2014). It is a nonlinear generalization of Maxwell’s field equations.

In this research paper it is showed how nonabelian gauge theories can also be further generalized. Generalizations of Yang-Mills theory were proposed in supersymmetric theories. There are existing several examinations about supersymmetric Yang-Mills theories (Beisert 2012). May be X a quantum field which can be expressed as a $n \times n$ matrix with dimension n. In nonabelian field theory the field can be decomposed as

$$X = X_a T^a. \qquad (1)$$

Here, T^a is the constant Lie group generator and a is the generator index which runs from 1 to n^2-1. For generator indices and spacetime indices Einstein's summation convention is used. May be A the 1-form gauge connection. Then the nonabelian field strength tensor F is given by

$$F = dA + gA \wedge A. \qquad (2)$$

With a coupling constant g. This research paper shows how the field strength tensor (2) can be generalized. The generalization of the gauge field strength tensor is very similar to the generalization procedure performed in the original paper of Topological Dipole Field Theory (TDFT). It is respected the principle of gauge invariance during derivation of equations. After that, a simple computation with TDFT is shown.

THEORY

A further generalization of TDFT can be obtained in similar manner to the derivation of TDFT in the original research paper. After the derivation of nonabelian TDFT, a calculation to extended Quantum chromodynamics is performed.

FORMULATION OF NONABELIAN TDFT

A plausible generalization of the field strength tensor (2) has the following form:

$$F = dA + gA \wedge A + B'. \qquad (3)$$

Here, B' is the intrinsic dipole moment corresponding to the gauge interaction. This intrinsic dipole moment satisfies $\Delta B' = 0$ with the general Čech coboundary map Δ and is an observable of the theory. This map is a gauge covariant map that satisfies $\Delta Y \mapsto \sigma \Delta Y \sigma^{-1}$ for an arbitrary 2-form field Y. Since the field strength tensor transforms under a gauge group σ as $F \mapsto \sigma F \sigma^{-1}$ it must hold the transformation rule

$$B' \mapsto \sigma B' \sigma^{-1}. \qquad (4)$$

Also the general dipole field B transforms by the rule (4) under gauge transformations. Because σ is a local group, the ordinary Čech coboundary map δ has to be modified. Due to the transformation property $A \mapsto \sigma A\, \sigma^{-1} + \sigma(d\sigma^{-1})$ one can construct a gauge connection Q that satisfies the gauge transformation condition $Q \mapsto \sigma Q\, \sigma^{-1} + \sigma(\delta\sigma^{-1})$. It holds the relation

$$\delta X(x) = \sum_{i=1}^{m} (-1)^i X(x+a_i) \approx \sum_{i=1}^{m} (-1)^i (X(x) + < \nabla X(x), a_i >) \qquad (5)$$

with the number of intersecting topological bases m which surround a certain spacetime point, the spacetime point position vector x pointing at the intersection of all topological bases and the vector a_i which points from x to the intersection of $m-1$ bases with the i-th base excluded. The vector a_i is an infinitesimal quantity. From (5) it follows that it must be:

$$Q = \sum_{i=1}^{m} (-1)^i * (A \wedge * a_i). \qquad (6)$$

It is easy to show that the following quantity is a differential operator:

$$\Delta X = \delta X + QX - XQ = \delta X + [Q, X]. \qquad (7)$$

Moreover, equation (7) is a proper gauge covariant derivative since the ordinary covariant derivative $DX = dX + [A, X]$ has the property $D(\sigma X \sigma^{-1}) = \sigma D X \sigma^{-1}$. With above considerations, a gauge-invariant topological action can be constructed. For abelian gauge fields, the topological term of the TDFT has the form

$$S_{top} = \int_M (B \wedge \delta B + \lambda \delta B \wedge \delta B) \qquad (8)$$

with the Minkowski spacetime manifold M and the Lagrange multiplier λ. If B is a nonabelian field and after replacing the operator δ with Δ, the Lagrangian density 4-form L transforms as $L \mapsto \sigma L \sigma^{-1}$. Hence, the gauge-invariant topological term of TDFT in the nonabelian case reads:

$$S_{top} = \int_M tr(B \wedge \Delta B + \lambda \Delta B \wedge \Delta B). \qquad (9)$$

Due to linearity of any differential operators and the trace, it can be shown in a similar way as in the original research paper of TDFT that (9) represents a Witten-type topological quantum field theory in the intrinsic curvature B. Moreover, it holds the exactness condition $\Delta^2 = 0$ since the operator Δ evaluates the Čech coboundary map only in the base space M of the fiber bundle that represents the nonabelian gauge theory. Equation (9) yields equation (8) if the dipole fields B are abelian.

PERTURBATIVE CALCULATION WITH TDFT

To evaluate the integral

$$s = \int d[A] d[B] d[\lambda] \exp\left(i \int_M \frac{1}{4} tr(F \wedge * F) + i S_{top} \right) \qquad (10)$$

the factor $\exp(i \int_M F \wedge * F)$ is expanded into Taylor series. After the Taylor expansion the general 2-form field B can be decomposed into an exact term B' and into a non-exact term W, i.e.

$$B = B' + bW \qquad (11)$$

with a coupling constant b such that $d[B] = d[B'] d[W]$. Without loss of generality, the topological bases which generate the Čech coboundary are chosen such that they are absorbing local gauge transformations, i.e. it can be set $\delta = \Delta$. Since B runs over the fields $B(x + a_i)$ for all $i \in \{1, \ldots, m\}$ one can pick an arbitrary field $B(x + a_i)$ for arbitrary i that is set equal to W. For matching the spacetime point where B is defined, the field W lies on the intersection of all topological bases. All other fields $B(x + a_j)$ with $i \neq j$ can be obtained by considering all possible generalized Čech cocycles. From (11) it follows $\Delta B = b\, \Delta W$, hence:

$$S_{top} = b \int_M tr(B' \wedge \Delta W) + b^2 \int_M tr((W + \lambda \Delta W) \wedge \Delta W). \quad (12)$$

The evaluation of the generalized coboundary map on W yields also a term $\pm W$ and by choosing a positive sign it follows from (12) the topological action:

$$S_{top} = b \int_M tr(B' \wedge W) + b^2 \int_M tr((\lambda + 1) W \wedge W). \quad (13)$$

May be the incoming gauge boson fields A fixed and $A \in SU(3)$, i.e. it must not be integrated over all possible gauge connection states. Additionally it is set $tr(T_a T_b) = \frac{1}{2}\delta_{ab}$. It is easy to show that all nonzero powers less than the fourth power which can be formed with $B'_{\mu\nu}$ are vanishing when weighted with the factor $\exp(iS_{top})$. Since it can be set $\lambda + 1 \mapsto \lambda$ due to translational invariance of the integration measure one obtains:

$$S_{top} = \frac{b}{2} \int_M d^4x \epsilon^{\mu\nu\alpha\beta} B'^{a}_{\mu\nu} W_{a\,\alpha\beta} + \frac{b^2}{2} \int_M d^4x \lambda \epsilon^{\mu\nu\alpha\beta} W^a_{\mu\nu} W_{a\,\alpha\beta}. \quad (14)$$

When integrating over multinomials in $B'_{\mu\nu}$ one can use the basic property $\frac{d^n}{dx^n}\delta(x) = \frac{i^n}{2\pi}\int_{-\infty}^{\infty} dk k^n e^{ikx}$. After this integration, the integration over W can be performed. Finally, the averaging of multinomials in $B'_{\mu\nu}$ with weight factor $\exp(iS_{top})$ yields a number which is independent on the choice of the topological bases which generates the Čech complex. Perturbative evaluations of (10) show that 5-boson-scattering or higher order scattering can occur. However, quantum chromodynamics is a field theory which has to be treated non-perturbative in many cases. The additional coupling constant b is unique for every kind of gauge boson and has to be determined by experiments with particle colliders.

CONCLUSIONS

Topological Dipole Field Theory offers a higher complexity in calculations than the ordinary Standard model does. Phenomena where the ordinary Standard model fails like the baryon asymmetry can be predicted more precisely by TDFT. The main advantage of TDFT is that except the topological dipole moments no supersymmetric partners of every Standard model particle or other hypothetical concepts which require a lot of rigorous experimental verifications is introduced. More insights in phenomena in particle physics and cosmology that are still undiscovered are possible by TDFT.

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