Darmstadt Discussion Papers in Economics

Nominal rigidities and the dynamic effects of a monetary shock

Rafael Gerke

Nr. 107

Arbeitspapiere
des Instituts für Volkswirtschaftslehre
Technische Universität Darmstadt

Economic **T**heory

Nominal rigidities and the dynamic effects of a monetary shock

Rafael Gerke*)

Darmstadt University of Technology

August 2001

Abstract

Two dynamic sticky price models with monopolistic competition in the goods market are presented. In the first model, each intermediate goods producer faces quadratic costs of adjusting its nominal price as introduced by Rotemberg (1982); the second model incorporates staggered price setting as proposed by Taylor (1980) and recently discussed by Chari/Kehoe/McGrattan (2000). Using the approximation method and the toolkit of Uhlig (1999) these models are used to derive theoretical impulse response functions. One aim is to check whether these two different forms of nominal price rigidities imply quantitatively and qualitatively different impulse response functions. Interestingly, both models do not seem to imply as much persistence as empirical impulse response functions typically indicate. However, qualitative differences do exist.

*) Rafael Gerke
Technische Universität Darmstadt
Institut für VWL
gerke@vwl.tu-darmstadt.de

Nominal Rigidities and the Dynamic Effects of a Monetary Shock[1]

1 Introduction

Of crucial importance for both the analysis of monetary policy and the study of the business cycle is the concept of the monetary transmission process. Recent econometric work based on VAR models suggests that monetary policy shocks do have real effects that last for many quarters (see the summaries in e.g. Christiano/Eichenbaum/Evans, 1999 or Favero, 2001). One important feature of the evidence is the persistent movement in output (and other variables) after a monetary shock, that is, aggregate output seems to display an inverse j-shaped response ("hump-shaped") and a zero long-run effect after a expansionary monetary shock.

Still a matter of debate are the mechanisms through which monetary shocks affect real economic activity. However, one popular explanation for non-neutrality is to emphasize nominal rigidities, i.e. sluggish adjustment of goods prices or money wages (or both). Nominal rigidities are of potential interest because they imply that nominal shocks can be transmitted by the propagation mechanisms of the model economy. An unattractive way of generating persistence is to simply assume that prices or wages are fixed for a long period of time. A more appealing way is to consider small frictions that lead to endogenous price or wage rigidities and therefore to persistent movement of output.

A number of recent papers have incorporated nominal rigidities in dynamic general equilibrium models. The specific source of the nominal rigidities range from a setting in which prices or wages are set in advance for one or more periods (e.g. Cho, 1993) to models where the adjustement prices incur some costs (e.g. Hairault/Portier, 1993) or to models where only a fraction of firms have the possibility to change their prices. (e.g. Yun, 1996). These models generate real effects of monetary shocks, but they rarely explain the persistence. It still seems, that the real effects in the data tend to have a longer life than is reasonable to assume for the types of rigidities imposed in the models (Bergin/Feenstra, 2000).

Taylor (1980) already referred to the problem of persistence in models with nominal rigidities.[2] However, he showed that a rational expectational model, in which wage contracts

---

[1] I am grateful to Ingo Barens for many helpful comments.

were the only source of rigidity, was capable of endogenously generating persistence, significantly outlasting the duration of the contract period. Two assumptions underlie this result: (1) wage contracts are staggered, that is, not all wage decisions in the economy are made at the same time; (2) when making wage decisions, firms (and unions) take into account the wage rates which are set by other firms. In effect, because of the staggering, each contract is written relative to other contracts, and this causes shocks to be passed on from one contract to another. Blanchard (1983) applied the idea of staggering to firms setting their prices in an asynchronized fashion and showed that the results also hold when firms set prices in a staggered fashion. Recently however, Chari/Kehoe/McGrattan (2000) have questioned the potential of the staggered price setting to generate endogenous persistence. They demonstrate that staggering of price changes alone does not generate endogenous persistence in a dynamic general equilibrium model.

As the considered nominal rigidities do not produce persistent real effects, one might be inclined to conclude that the specific sources of rigidities imply similar dynamics. This seems to be the perception in the literature (for instance Roberts, 1995, Jeanne, 1998, Mankiw, 2001). However, this view has recently been questioned. Recent work shows that nominal price rigidity and nominal wage rigidity do differ in their potential of producing persistence (see e.g. Huang/Liu (1999). Using an analytical approach, Kiley (1998) concludes that different forms of nominal price inertia imply different dynamics, at least for typical parameteriziations of dynamic general equlibrium models.

The present paper is in the spirit of Kiley (1998). It illustrates that although two types of nominal price rigidities do not generate persistence of the aggregate output they imply different dynamics. To do so, a quantitative dynamic general equilibrium model with capital accumulation and monopolistic competition in the goods market is formulated. Two model variations are discussed by introducing two specific sources of nominal price inertia. In the first variant each intermediate goods producer faces quadratic costs of adjusting its nominal price as introduced by Rotemberg (1982); the second variant incorporates staggered price setting. The following can be concluded. Simulated impulse response functions show that adjustment costs do not generate persistent output movements, even if the adjustment costs are increased to an unrealistic magnitude. Furthermore, staggered price setting as an alternative source of nominal price inertia can lead to impulse response functions that

---

[2] Another early paper is Fischer (1979a).

oscillate. This confirms the analytical results of Chari/Kehoe/McGrattan (2000). Obviously, the simulation results are at odds with the empirical evidence However, the results indicate that different forms of nominal price inertia can imply qualitatively different dynamics. It might be concluded that the formulation of a standard quantitative general equilibrium model is unsufficient because of the lack of a powerful transmission mechanism.

2 Monopolistic competition and adjustment costs

The economic environment

The specification of the model is based on the business cycle literature. The model formulation takes its principle features from Ireland (1997) who builds on earlier work by Blanchard/Kiyotaki (1987) and Hairault/Portier (1993). The specification of nominal price inertia is based on Rotemberg (1982). The economy is populated by a representative household, a representative firm, which produces the finished goods, a continuum of intermediate goods producing firms and a monetary authority. The representative household has preferences over consumption, leisure and real money balances. The household purchases consumption and investment goods from the finished goods producing firm and receives income from its labour and from ist capital supply to the intermediate goods producing firms in competitive markets. The final goods producers behave competitively. In each period *t* they choose inputs produced by the intermediate goods producers and produce output to maximize profits. The intermediate goods producing firms, indexed by $i \in [0, 1]$, each produce a distinct intermediate good with labor and capital supplied by the representative household. Since intermediate goods substitute imperfectly for one another, the intermediate goods producing firms sell their output in a monopolistically competitive market. Each intermediate good producer faces a quadratic cost of adjusting its nominal output price. The nominal money supply follows an exogenous stochastic process.

Description of the representative household

The representative household has preferences defined over consumption of the finished good, leisure, and real cash balance. It chooses an optimal quatriple of consumption, leisure, real balances and capital subject to a budget constraint. The preferences are described by the expected utility function

$$U = E_t\left[\sum_{t=0}^{\infty}\beta^t u(c_t, l_t, m_t)\right], \tag{1.}$$

where $u(c_t, m_t, l_t)$ is given by (see Fischer, 1979b, Walsh, 1998)

$$u(c_t, m_t, l_t) = \frac{\left(c_t m_t^b\right)^{1-\Phi}}{1-\Phi} + \Psi\frac{l_t^{1-\eta}}{1-\eta}. \tag{2.}$$

with β as a constant discount factor, $0 < \beta < 1$. c_t is consumption of the final good, l_t is leisure and $m_t \equiv M_t / P_t$ is real cash balances, P_t is the price level and M_t is money. The household carries M_{t-1} units of money and k_{t-1}^H units of capital into period *t* (where "^{H}" denotes the household variables). During period *t*, it supplies $n_t^H(i)$ units of labour at the real wage w_t and consumes l_t units of leisure. Total available time is normalized to one, i.e. $n_t^H + l_t = 1$. The supply of labour must satisfy $n_t = \int_0^1 n_t^H(i)di$. The household accumulates the physical capital of the economy which is supplied at the real rental rate r_t to each intermediate goods producing firm. The household choices must satisfy $k_{t-1} = \int_0^1 k_{t-1}^H(i)di$. The capital accumulation constraint is standard and given by

$$k_t^H = (1-\delta)k_{t-1}^H + I_t,$$

where I_t denotes investment and δ denotes the capital depreciation rate, $0 \le \delta \le 1$.

In addition to the factor payment the household receives a lump-sum transfer τ_t and the dividends Π_t from the intermediate goods producers, where $\Pi_t = \int_0^1 \Pi_t(i)di$.

The budget constraint can be written as

$$P_t\left(1 + r_t^K - \delta\right)k_{t-1}^H + P_t w_t n_t + M_{t-1} + \tau_t + \Pi_t = P_t c_t + P_t k_t + M_t, \tag{3.}$$

Description of the firms

The representative finished goods producing firm uses $y_t(i)$ units of each intermediate good *i* during period *t* to produce y_t units of the finished goods. The technology of the finished goods producer is described by the following constant returns to scale production technology

$$y_t = \left[\int_0^1 y_t(i)^{(\theta-1)/\theta} \, di \right]^{\theta/(\theta-1)} \tag{4.}$$

where $\theta > 1$. The elasticity of substitution is equal to $-\theta$.

In each period *t*, the intermediate goods producing firm *i* hires $n_t^U(i)$ units of labour and $k_{t-1}^U(i)$ units of capital from the representative household to produce $y_t(i)$ units of output according to a constant returns to scale technology $y_t^s(i) = f\left(k_{t-1}^U(i), n_t^U(i), z_t\right)$ (where "^{U}" denotes the firm variables). The analysis will be based on a Cobb-Douglas production function of the form

$$y_t^s(i) = z_t \left[k_{t-1}^U(i)\right]^{\alpha} \left[n_t^U(i)\right]^{1-\alpha} . \tag{5.}$$

The technology shock z_t is described by a stationary process

$$\log z_t = (1-\psi_z)\log \bar{z} + \psi_z \log z_{t-1} + \varepsilon_t^z . \tag{6.}$$

where $\varepsilon_t \sim i.i.d.N\left(0, \sigma^2\right)$ and $0 < \psi_z < 1$.

Deriving the optimal plans for the representative houshold and the firms

The optimization problem of the representative household is to maximimize

$$\max_{\{c_t,\, l_t,\, k_t^H,\, m_t^H\}_{t=0}^{\infty}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t u\left(c_t, m_t, 1-n_t^H\right) \right\} \tag{7.}$$

subject to

$$(1 + r_t - \delta) k_{t-1}^H + w_t n_t^H + \frac{M_{t-1}}{P_t} + \frac{\tau_t}{P_t} + \frac{\Pi_t}{P_t} = c_t + k_t^H + \frac{M_t}{P_t} . \tag{8.}$$

The first order conditions for this problem can be denoted as

$$u_c\left(c_t, m_t, 1-n_t^H\right) = \lambda_t , \tag{9.}$$

$$u_l\left(c_t, m_t, 1-n_t^H\right) = \lambda_t w_t , \tag{10.}$$

$$\lambda_t = \beta E_t \lambda_{t+1} (1 + r_{t+1} - \delta). \tag{11.}$$

$$u_m\left(c_t, m_t 1-n_t^H\right) + \beta E_t \frac{\lambda_{t+1}}{1+\pi_{t+1}} = \lambda_t , \tag{12.}$$

where $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ and $\lambda_t = \Lambda_t / \beta^t$. Λ_t is the Lagrange-multiplier of the budget constraint.

Every period *t*, the finished goods producer chooses $y_t(i)$ units to maximize its profits

$$\max_{y_t(i)} \; P_t y_t - \int_0^1 P_t(i) y_t(i) di , \tag{13.}$$

where P_t is the nominal price of the finished good and $P_t(i)$ is the nominal price of the intermediate good *i*. The first order condition results in the demand function for intermediate good *i* as a function of its output y_t and the relative price $P_t(i)/P_t$:

$$y_t^d(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} y_t . \tag{14.}$$

As the finished goods producer operates under perfect competition, it earns zero profits in equilibrium. The zero-profit condition can be used to determine the price-level as

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di\right]^{1/1-\theta} . \tag{15.}$$

Each intermediate goods producing firm sells its output in a monopolistically competitive market and faces a quadratic cost function when adjusting its nominal price. The functional form of the cost function is expressed as

$$\frac{\phi_P}{2}\left[\frac{P_t(i)}{P_{t-1}(i)} - 1\right]^2 y_t . \tag{16.}$$

Unlike some menu-costs which are unchanged for each price change, equation (16.) highlights the notion that price changes might have negative effects on customer-firm relationships. These negative effects increase with the magnitude of the price change and the level of economic activity.

The optimization problem of the intermediate goods producer is to maximize

$$\max \; E_t \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t \Pi_t(i)}{P_t} , \tag{17.}$$

where $\beta^t \lambda_t / P_t$ is the marginal utility value to the representative household of an additional dollar of profits during period *t*. The nominal profits $\Pi_t(i)$ are defined as

$$\Pi_t(i) = P_t(i)y_t(i) - P_t w_t n_t^U(i) - P_t r_t^K k_{t-1}^U(i) - P_t \frac{\phi_P}{2}\left[\frac{P_t(i)}{P_{t-1}(i)} - 1\right]^2 y_t . \tag{18.}$$

When maximizing the nominal profits intermediate goods producing firm has to take into consideration the following constraint

$$y_t^s(i) = z_t\left[k_{t-1}^U(i)\right]^{\alpha}\left[n_t^U(i)\right]^{1-\alpha} = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} y_t = y_t^d(i). \tag{19.}$$

The first order conditions for this problem are:

$$\xi_t f_k\left(k_{t-1}^U(i), n_t^U(i), z_t\right) = \lambda_t r_t , \tag{20.}$$

$$\xi_t f_n\left(k_{t-1}^U(i), n_t^U(i), z_t\right) = \lambda_t w_t , \tag{21.}$$

$$\begin{aligned} &\lambda_t(1-\theta)\left[\frac{P_t(i)}{P_t}\right]^{-\theta}\frac{y_t}{P_t} - \lambda_t\phi_P\left[\frac{P_t(i)}{P_{t-1}(i)} - 1\right]\frac{y_t}{P_{t-1}(i)} + \xi_t\theta\left[\frac{P_t(i)}{P_t}\right]^{-\theta-1}\frac{y_t}{P_t} \\ &+ \beta E_t\left\{\lambda_{t+1}\phi_P\left[\frac{P_{t+1}(i)}{P_t(i)} - 1\right]y_{t+1}\frac{P_{t+1}(i)}{\left[P_t(i)\right]^2}\right\} = 0 \end{aligned} \tag{22.}$$

where ξ_t is the Lagrange-multiplier of the constraint. Equation (20.) and (21.) equate the marginal rate of substitution between labour and capital in production to the relative factor price r_t/w_t. Equation (22.) shows the optimal setting of the nominal price. In a symmetric equilibrium, where $P_t(i) = P_t$ for all $i \in [0, 1]$, equation (20.) and (21.) indicate that $\lambda_t/\xi_t \equiv \mu_t$ denotes an gross markup of price over marginal cost. Equation (22.) then shows that in the absence of the adjustment costs, when $\phi_P = 0$, the markup will be equal to $\theta/(\theta - 1)$.

The monetary authority

The monetary authority supplies the economy with money. In every period *t*, the nominal money supply grows with an exogenous rate g_t, i.e. $M_t = (1 + g_t)M_{t-1}$. The newly created money is paid to the representative household as a lump-sum transfer. The nominal transfer satisfies

$$\tau_t = M_t - M_{t-1} . \tag{23.}$$

Starting with the defintion of the growth rate of money, real balances ($m_t \equiv M_t / P_t$) can be expressed as

$$m_t = \frac{1+g_t}{1+\pi_t} m_{t-1} , \qquad (24.)$$

where g_t is the growth rate of the nominal money supply, π_t is the inflation rate. For the analysis at hand we define $\varpi_t = g_t - \overline{g}$ where "¯" denotes the steady state, i.e. ϖ_t is the deviation of the growth rate of money from its steady state. This deviation is formulated as a stochastic process (see Walsh, 1998, p. 69) which can be described in logarithmic form as:

$$\varpi_t = \psi_\varpi \varpi_{t-1} + \phi_z z_{t-1} + \varepsilon_t^\varpi , \qquad (25.)$$

where $0 \le \psi_\varpi < 1$ and $\varepsilon_t^\varpi \sim i.i.d.N\left(0, \sigma_\varpi^2\right)$.

It is assumed that the individual knows about the realisation of ϖ_t and z_t, when choosing its optimal values of consumption, leisure, real balance and capital in period *t*.

Symmetric equilibrium

The symmetric equlibrium is defined as a set of allocations that satisfies the following conditions:

- Taking prices as given, the representative household solves its optimization problem
- Taking all prices but his own as given, each intermediate goods producer solves the optimization problem (17.)
- Taking all prices as given, the final goods allocation solves the final goods optimization problem
- All markets clear, i. e.

$$\int_0^1 n_t^U(i)\,di = \int_0^1 n_t^H(i)\,di = n_t , \qquad (26.)$$

$$\int_0^1 k_{t-1}^U(i)\,di = \int_0^1 k_{t-1}^H(i)\,di = k_{t-1} , \qquad (27.)$$

$$\int_0^1 y_t^s(i)\,di = \int_0^1 y_t^d(i)\,di , \qquad (28.)$$

$$y_t^H = y_t^U = y_t. \tag{29.}$$

- The resource constraint for this economy holds

$$\begin{aligned} y_t &= c_t + k_t - (1-\delta)k_{t-1} + \frac{\phi_P}{2}\left[\frac{P_t(i)}{P_{t-1}(i)} - 1\right]^2 y_t \\ &= c_t + I_t + \frac{\phi_P}{2}\left[\frac{P_t(i)}{P_{t-1}(i)} - 1\right]^2 y_t. \end{aligned} \tag{30.}$$

Due to the symmetry of the model the price-level is $P_t(i) = P_t(j) = P_t$.

The model contains the following eleven variables: y_t, c_t, k_t, I_t, m_t, n_t, R_t, μ_t, π_t, z_t and ϖ_t.

Linear approximation

The focus of this paper is to examine whether two different forms of nominal rigidities influence the dynamics of the model. A natural way to explore the dynamics is to use simulated impulse response functions. In order to derive these impulse response functions it is necessary to solve the model. To do so, one can either use numerical methods to solve the nonlinear equations or, alternatively, approximate the model around its steady state and solve the (log-) linearized version of the model. In what follows, the equations of the model have been log-linearized around the steady state using the approximation technique of Uhlig (1999). The model can be denoted in linearized form as

$$\bar{y}\hat{y}_t = \bar{I}\hat{I}_t + \bar{c}\hat{c}_t + \tfrac{1}{2}\phi_P\bar{\pi}^2\bar{y}\hat{y}_t + \phi_P\bar{\pi}^2\bar{y}\hat{\pi}_t \tag{31.}$$

$$\bar{k}\hat{k}_t = \bar{I}\hat{I}_t + (1-\delta)\bar{k}\hat{k}_{t-1} \tag{32.}$$

$$\hat{y}_t = \alpha\hat{k}_{t-1} + (1-\alpha)\hat{n}_t + z_t \tag{33.}$$

$$\bar{R}\hat{R}_t = \frac{\alpha}{\bar{\mu}}\frac{\bar{y}}{\bar{k}}\left(\hat{y}_t - \hat{k}_{t-1} - \hat{\mu}_t\right) \tag{34.}$$

$$\hat{y}_t - \Phi\hat{c}_t + b(1-\Phi)\hat{m}_t - \hat{\mu}_t = \left(1 + \eta\frac{\bar{n}}{1-\bar{n}}\right)\hat{n}_t \tag{35.}$$

$$\hat{m}_t = \hat{m}_{t-1} - \frac{\overline{\pi}}{1+\overline{\pi}}\hat{\pi}_t + \frac{\overline{\pi}}{1+\overline{\pi}}\varpi_t \tag{36.}$$

$$E_t\hat{R}_{t+1} = E_t\left[\Phi(\hat{c}_{t+1} - \hat{c}_t) - b(1-\Phi)(\hat{m}_{t+1} - \hat{m}_t)\right] \tag{37.}$$

$$E_t\hat{R}_{t+1} + \frac{\overline{\pi}}{1+\overline{\pi}}E_t\hat{\pi}_{t+1} = \frac{\Theta-\beta}{\beta}(\hat{c}_t - \hat{m}_t) \tag{38.}$$

$$0 = \kappa_1\hat{\mu}_t + \kappa_2\hat{\pi}_t + \kappa_3 E_t\hat{\pi}_{t+1} + \kappa_4 E_t\left(\hat{y}_{t+1} - \hat{y}_t - \hat{R}_{t+1}\right) \tag{39.}$$

$$z_t = \psi_z z_{t-1} + \varepsilon_t^z \tag{40.}$$

$$\varpi_t = \psi_{\varpi}\varpi_{t-1} + \phi_z z_{t-1} + \varepsilon_t^{\varpi} \tag{41.}$$

The percentage deviation of a variable from its steady state value has been denoted as $\hat{x}_t$, i.e. $\hat{x}_t = \frac{x_t - \overline{x}}{\overline{x}}$. Using the toolkit of Uhlig (1999) the model is solved for the recursive equilibrium law of motion with the method of undetermined coefficients.

Calibration

The parameters of the models are common to the business cycle literature and most of the parameters have been adopted from Ireland (1997) and Walsh (1998). The calibration is consistent with the following scenario: the share of capital α is 30%, the discount factor β is chosen to guarantee a real interest rate of roughly 4% per year, the depreciation rate δ is 10% per year. The steady state share of labour is about 30%, i.e. 30% of time is market activity, the annual growth rate of the nominal money stock $\overline{g}$ is 5%, the value of *b* is compatible with a quotient of money to output of roughly 20%. θ is equal to 6, so that the gross steady state markup of price over marginal cost is 1.2. Being perhaps the parameter of most interest, ϕ_P, is equal to 3.95, so that the costs of adjusting the nominal price is about 0.030% of aggregate output.

In any case, simulation exercises indicate that the model is quite robust with regard to a variation of the parameters. This is especially true for the persistence of the variables after a monetary shock.[3]

The following table summerizes the parameters of the model

TABLE 1

BASELINE PARAMETER VALUES

α	β	δ	η	b	Φ	ψ_z	ψ_ϖ	σ	σ_ϖ
0.30	0.989	0.025	1	0.005	2.0	0.95	0.687	0.007	0.00216
ϕ_z	Θ	θ	ϕ_P						
-0.15	1.0125	6	3.95						

3 Monopolistic competition and staggered price-setting

In order to compare the staggered price setting mechanism and the adjustment cost specification the model is formulated similar to the previous one. Only those equations will be described explicitly which change due to the different nominal price inertia.

The specification of the household optimization problem and of the final goods producer remain the same. There is still a continuum of monopolistically competitive firms that produce differentiated products using capital and labour. These firms set nominal prices for a fixed number of periods and do so in a staggered fashion. In particular, each period *t*, $1/N$ of these firms choose new prices, which are then fixed for *N* periods. The intermediate goods producers are indexed so that producers indexed by $i \in \left[0, \frac{1}{N}\right]$ set new prices in *0*, *N*, *2N*, and so on, while producers indexed $i \in \left[\frac{1}{N}, \frac{2}{N}\right]$ set new prices in 1, $N + 1$, $2N + 1$, and so on, for the *N* cohorts of intermediate producers. In period *t*, each producer in a cohort chooses the $P_t(i)$ to maximize discounted profits from *t* to period $t + N - 1$ (see Chari/Kehoe/McGrattan, 2000, p. 1155).

[3] Simulation exercises have shown that augmenting the adjustment cost does not increase the persistence of the model, but increases the deviation from steady state.

The optimization problem can be stated as follows.[4] Each intermediate firm demands $k_{t-1}^{U}(i)$ and $n_t^U(i)$ in a way to minimize the cost of production, i.e. to minimize the unit cost. That is, each firm solves the problem

$$\min \frac{\lambda_t \mathrm{K}_t(i)}{P_t}, \tag{42.}$$

subject to

$$y_t^s(i) = z_t \left[k_{t-1}^U(i)\right]^{\alpha} \left[n_t^U(i)\right]^{1-\alpha} = 1. \tag{43.}$$

where λ_t / P_t is the marginal utility value to the representative household of an additional dollar of profits during period *t*. The nominal costs $\mathrm{K}_t(i)$ are defined as

$$\mathrm{K}_t(i) = P_t r_t k_{t-1}^U(i) + P_t w_t n_t^U(i). \tag{44.}$$

The first order conditions are

$$\xi_t f_k\left(k_{t-1}^U(i), n_t^U(i), z_t\right) = \lambda_t r_t^K, \tag{45.}$$

$$\xi_t f_n\left(k_{t-1}^U(i), n_t^U(i), z_t\right) = \lambda_t w_t, \tag{46.}$$

where ξ_t is Lagrange-multiplier of the constraint. The first order conditions (45.) and (46.) are exactly the same as in the previous model. As can be shown, ξ_t / λ_t can be interpreted as unit cost (and therefore is the inverse of the markup variable of the previous model). The unit costs will be denoted in the following as υ_t.

In period *t*, each producer in a cohort chooses ist price $P_t(i)$ to maximize discounted profits from period *t* to period *t* + *N* - 1. Each intermediate goods producer solves the problem

$$\max_{P_t(i)} \Pi_t(i) = E_t \sum_{j=0}^{t+N-1} \frac{1}{R_{t+j}^j} \left(P_t(i) - P_{t+j}\upsilon_{t+j}\right)\left(\frac{P_t(i)}{P_{t+j}}\right)^{-\theta} y_{t+j}.$$

The solution to this problem is

[4] One could, of course, state the problem completely analogous to the previous one.

$$P_t(i) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{j=0}^{t+N-1} R_{t+j}^{-j} P_{t+j}^{1+\theta} \upsilon_{t+j} y_{t+j}}{E_t \sum_{j=0}^{t+N-1} R_{t+j}^{-j} P_{t+j}^{\theta} y_{t+j}} . \tag{47.}$$

As already mentioned, implicit in equation (47.) are the demands for capital and labour of the intermediate goods producer.

Due to the staggering mechanism, the price level can now be written as

$$P_t = \left[\tfrac{1}{N} P_t(i)^{1-\theta} + \cdots + \tfrac{1}{N} P_{t-N+1}(i)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{48.}$$

As the remaining structure of the model is identical to the structure of the previous sticky price model we can skip the remaining details and go directly to the equlibrium of the model.

Symmetric equilibrium

The symmetric equlibrium is again defined by the decision rules of the representative household and the decision rules of the firms. In each period *t* there is a vector of factor prices that equates the supply of and the demand for labour and capital. There is a price vector $\{P_t(i)\},\ i \in [0, 1]$, that equates in every period *t* the market for intermediate goods. The price vector $\{P_t\}$ clears the market for finished goods. The resource constraint for this economy is

$$y_t = c_t + k_t - (1 - \delta)k_{t-1} = c_t + I_t . \tag{49.}$$

The model contains thirteen variables: y_t, c_t, k_t, I_t, m_t, n_t, R_t, μ_t, π_t, $P_t(i)$, P_t, z_t and ϖ_t. In comparison to the previous sticky price model two additional variables, $P_t(i)$ and P_t, have been introduced. To close the model we need one additional equation and introduce therefore the definition of the inflation rate, i.e.

$$\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{50.}$$

Linear approximation

Again, the model is log-linearized around the steady state. For $N = 2$ and $N = 4$ the model can be denoted as

$$\bar{y}\hat{y}_t = \bar{I}\hat{I}_t + \bar{c}\hat{c}_t \tag{51.}$$

$$\bar{k}\hat{k}_t = \bar{I}\hat{I}_t + (1-\delta)\bar{k}\hat{k}_{t-1} \tag{52.}$$

$$\hat{y}_t = \alpha\hat{k}_{t-1} + (1-\alpha)\hat{n}_t + z_t \tag{53.}$$

$$\bar{R}\hat{R}_t = \bar{\upsilon}\alpha\frac{\bar{y}}{\bar{k}}\left(\hat{y}_t + \hat{\upsilon}_t - \hat{k}_{t-1}\right) \tag{54.}$$

$$E_t\left[\Phi(\hat{c}_{t+1} - \hat{c}_t) - b(1-\Phi)(\hat{m}_{t+1} - \hat{m}_t) - \hat{R}_{t+1}\right] = 0 \tag{55.}$$

$$\left[\hat{y}_t + \hat{\upsilon}_t - \Phi\hat{c}_t + b(1-\Phi)\hat{m}_t\right] = \left(1 + \eta\frac{\bar{n}}{1-\bar{n}}\right)\hat{n}_t \tag{56.}$$

$$E_t\hat{R}_{t+1} + E_t\hat{p}_{t+1} - \hat{p}_t = \frac{\Theta - \beta}{\beta}\left(\hat{c}_t - \hat{m}_t\right) \tag{57.}$$

$$\hat{m}_t = \hat{m}_{t-1} - \frac{\bar{\pi}}{1+\bar{\pi}}\hat{\pi}_t + \frac{\bar{\pi}}{1+\bar{\pi}}\varpi_t \tag{58.}$$

$$\hat{p}_t(i) = \vartheta_1\hat{\upsilon}_t + \vartheta_2 E_t\hat{\upsilon}_{t+1} + \vartheta_1\hat{p}_t + \vartheta_2 E_t\hat{p}_{t+1} \qquad \text{(for } N = 2\text{)} \tag{59.}$$

$$\begin{aligned}\hat{p}_t(i) = {} & \chi_1\hat{\upsilon}_t + \chi_2 E_t\hat{\upsilon}_{t+1} + \chi_3 E_t\hat{\upsilon}_{t+2} + \chi_4 E_t\hat{\upsilon}_{t+3} \\ & + \chi_1\hat{p}_t + \chi_2 E_t\hat{p}_{t+1} + \chi_3 E_t\hat{p}_{t+2} + \chi_4 E_t\hat{p}_{t+3}\end{aligned} \qquad \text{(for } N = 4\text{)} \tag{60.}$$

$$\hat{p}_t = \tfrac{1}{2}\hat{p}_t(i) + \tfrac{1}{2}\hat{p}_{t-1}(i) \qquad \text{(for } N = 2\text{)} \tag{61.}$$

$$\hat{p}_t = \tfrac{1}{4}\hat{p}_t(i) + \tfrac{1}{4}\hat{p}_{t-1}(i) + \tfrac{1}{4}\hat{p}_{t-2}(i) + \tfrac{1}{4}\hat{p}_{t-3}(i) \qquad \text{(for } N = 4\text{)} \tag{62.}$$

$$\hat{\pi}_t = \frac{1+\bar{\pi}}{\bar{\pi}}\hat{p}_t - \frac{1+\bar{\pi}}{\bar{\pi}}\hat{p}_{t-1} \tag{63.}$$

$$z_t = \psi_z z_{t-1} + \varepsilon_t^z \tag{64.}$$

$$\varpi_t = \psi_\varpi\varpi_{t-1} + \phi_z z_{t-1} + \varepsilon_t^\varpi \tag{65.}$$

Calibration

The parameterization of the model is the same as in the prevoius model specification.

4 Transmission of a monetary shock

Adjustment costs

First, the monetary transmission mechanism of the first model is discussed where firms face quadratic costs of adjustments (see Figure 1 to 5). Interestingly, the persistence of the model does not increase, even when adjusment costs increase by a factor of 100 (see Figure 6 and 7). Second, the dynamics of the staggered price model for $N = 4$ are presented, i.e. firms choose their prices for one year (see Figure 8 to 12).

As the impulse-response-functions indicate, a 1% increase in the growth rate of the nominal money supply leads to an increase in aggregate output of about 0.3%. This value is considerably smaller than the simulation results in Hairault/Portier (1993) or Ireland (1997), who report deviations of about 0.8-1.0% and 1.6%, respectively. At first glance this might be puzzling, but can be explained by the fact that both Hairault/Portier (1993) and Ireland (1997) use a slightly different definition of the growth rate of money. Both define the (gross) rate of monetary growth as $g_t = M_t / M_{t-1}$ whereas here the growth rate has been defined as $g_t = (M_t - M_{t-1})/M_{t-1}$. Using their definition of the growth rate of money, the simulation results are similar in magnitude to Hairault/Portier (1993) (Ireland, 1997, does only report the impulse response function for output). Consumption, investment, labour, the real interest rate and inflation show a positive deviation from steady state, whereas the markup and the real balances show negative deviations from their steady state values.

As can be seen from the dynamics of the real balances (Figure 1) the model does not exhibit a textbook-style real-balance-effect that increases aggregate demand. Nevertheless, in the first step, due to the monetary shock, the funds of the representative household increase, because τ_t increases (by more than was expected the period before). Because of this wealth effect, the demand for consumption goods and leisure increase as long as both goods are superior goods. The increased demand for final good increases the demand of intermediate goods and therefore the demand of labour and capital. Consequently, the real wage and the real interest rate increase, and therefore marginal costs increase, too. This puts an upward pressure on prices, inflation and inflation expectations. Higher inflation, along with the associated reduction in real balances, increases the marginal utility of consumption, reinforcing the demand for consumption goods.[5] However, as can be seen from the first order conditions (9) and (10) (by inserting equation (9) in (10)) this is only compatible with an increase in labour supply given the real wage. Therefore, the response of the labour supply is ambigious.[6] As the representative household attempts to smooth its consumption profile the demand of investement goods (capital accumulation) increases, too.

Interestingly, as shown in Figure 6 and 7, the persistence of output does not increase, even when adjustment costs increase by a factor of 10 or 100, respectively. None of these impulse-responses show the degree of persistence that has been reported in the VAR literature of the monetary transmission mechanism.[7]

The impulse-responses reproduce one feature that has been emphasized in the New Keynesian literature on real rigidities, see e.g. Ball/Romer (1990).[8] Within this literature, it has been argued that the effects of sticky prices, due to some cost of adjustment, are modest, at least if one does not impose a very high level of adjustment costs. Nominal rigidities without real rigidities do not account for a non-neutrality of money. The explanation is straightforward. Although some impediments of price adjustment exists, prices are not sluggish, simply because the incentives of the firms to adjust their prices are too strong. Not adjusting the nominal price would be tantamount to a loss of profit. As long as this loss of profit is greater than the costs of adjustments there is no price sluggishness to be expected. The impulse-response-functions reproduce this insight in a dynamic context. The model does not exhibit real rigidity.

Especially the impulse response-function of the markup indicates the immediate response of the firms. Only one quarter after the monetary shock, the markup is back to its steady state level, indicating that firms have an incentive to adjust prices immediately to a new profit maximizing nominal price. Consequently, modest adjustment costs by themselves do not give

---

[5] This is because of $\Phi > 1$.

[6] Note, an additional amplification mechanism is due to increased inflation expectations, as the monetery shock exhibits serial correlation. However, this mechanism, which is crucial for the flex-price version of the model is quantitatively small, see Walsh (1998, p. 73) for a description.

[7] For instance, Christiano/Eichenbaum/Evans (1999) or Favero (2001).

[8] Ball/Romer (1990, S. 186) offer the following definition: *"We define a high degree of real rigidity as a ... small responsiveness of an agent's desired real price to changes in aggregate real spending"* Generally, two model

rise to a new quantitative important propagation mechanism that augments the effects of the monetary impulse and produces persistent effects. The endogenous markup adjusts very quickly and therefore does not influence the dynamics of the model.[9] Although an endogenous markup by itself can be understood as an additional transmission mechanism (see e.g. Rotemberg/Woodford, 1995, p. 244), the impulse response-functions illustrate that the quantitative effect of this transmission channel is negligible.

The immediate adjustment of the firms and therefore the lack of persistence is also very clearly illustrated by the adjustment of the inflation rate (see Figure 5). One period after the shock inflation is back to its steady state level.

In the end two reasons account for the lack of persistence. Persistence is small because the elasticity of substitution is constant $(-\theta)$[10] and because the conditional factor demands increase the marginal costs. The monetary shock is propagated by intra- and intertemporal substitution mechanisms and not, as might be expected, by a positive real balance effect. However, these substitution mechanisms are known to be weak (see Cogley/Nason, 1995).

Staggered price-setting

As in the previous specification, in the period of the monetary shock, consumption, investment, labour, the real interest rate and inflation show a positive deviation from steady state. Compatible with the negative deviation of the markup, unit cost increase whereas the the real balances again show negative deviations, although to a lesser extent. The size of the output deviation is bigger than in the previous specification. Again, none of these impulse-response-functions show a degree of persistence comparable to the results reported in the VAR literature. Interestingly, the adjustment back to steady state is different; most of the variables return back by oscillating around the steady state. The impulse-response-functions, illustrating a "negative endogenous persistence" (Bergin/Feenstra, 2000, p. 671), replicate the analytical results of Bergin/Feenstra (2000), Chari/Kehoe/McGrattan (2000) and Kiley (1998). The following interpretation is proposed: The expansionary shock increases output and the conditional factor demands. As can be seen in Figure 13, due to the increased factor demands, unit cost rises more than aggregate output. Because of the staggering, only the firms of the

---

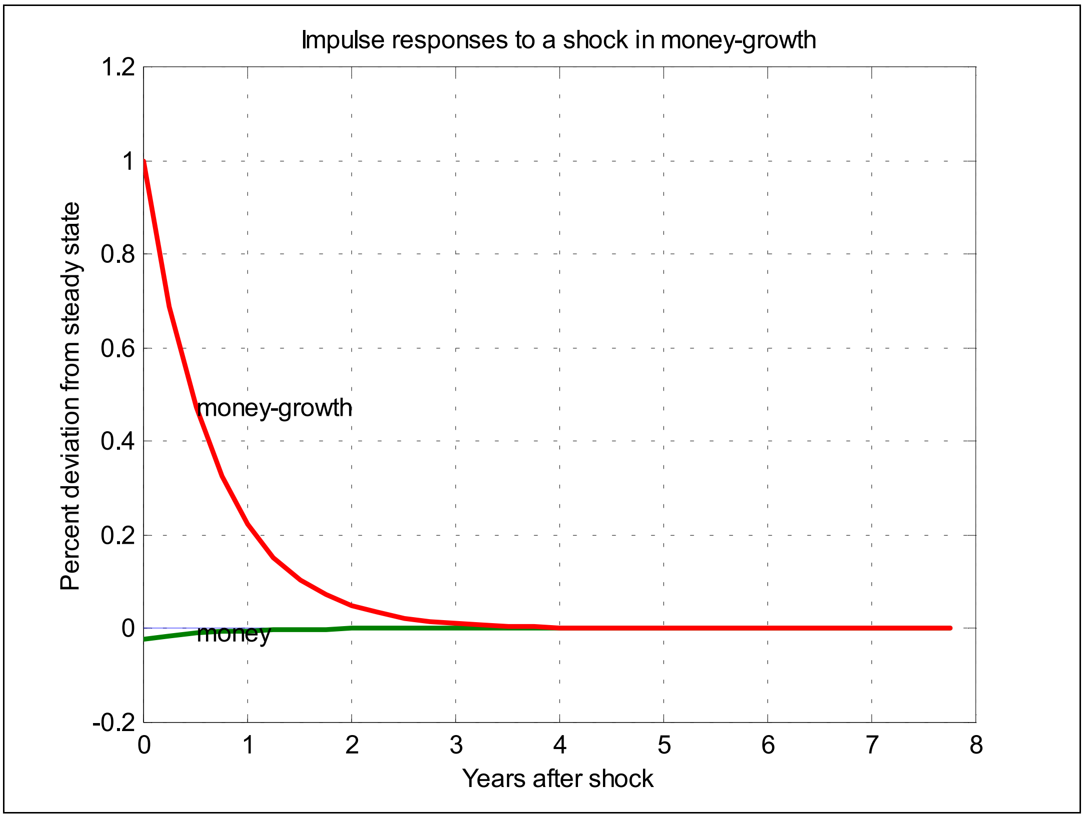
features can give rise to real rigidity: a small cyclical sensitivity of marginal cost or a large cyclical sensitivity of marginal revenue. See e.g. Romer (1993).

[9] The increase of both capital and labour demand increases the conditional factor demands and correspondingly the marginal costs. Increasing marginal costs reduce the markup.

[10] Therefore, prices are sensitive to changes in unit costs.

first cohort are able to compensate the increase of unit cost by increasing their price $\hat{p}_t(i)$. The firms of the other cohorts can only compensate the increased unit costs by reducing their factor demands; lower labour demand and therefore lower employment results in lower output. Whenever the firms of one cohort has an opportunity to adjust the price, the firms of this cohort rise their prices accordingly. Thus, after firms of the last cohort had an opportunity to adjust their prices (after four periods), output has fallen below steady state, and so has unit cost. Firms of the first cohort will find it worth to expand their output again, since they can compensate an increase of unit cost by adjusting their prices and the price adjustment process starts again.[11] However, these dynamics can hardly be reconciled with the empirical evidence.

Figure 1: Nominal rigidity due to adjustment costs



[11] For a similar reasoning, emphasizing the large increase of factor prices see Erceg (1997), Huang/Liu (1999) or Koenig (1999). Similar dynamics of the aggregate output can be found in Bergin/Feenstra (2000) or Huang/Liu (1999).

Figure 2: Nominal rigidity due to adjustment costs

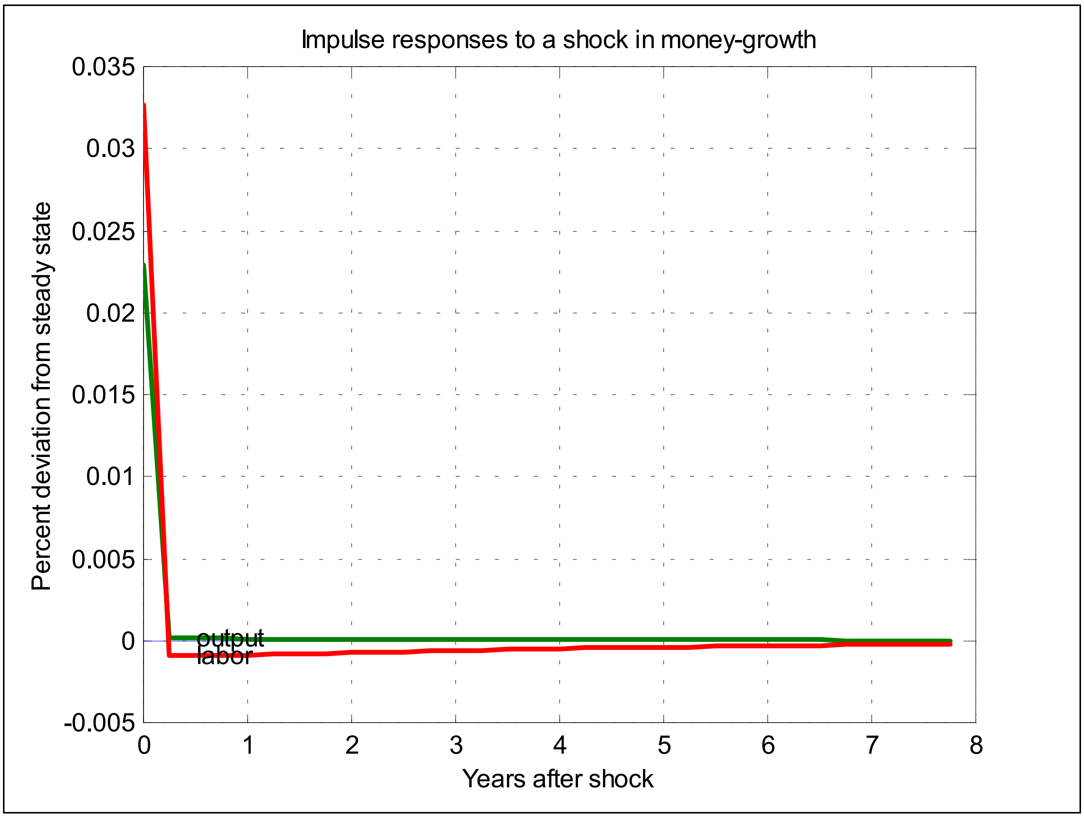


Figure 3: Nominal rigidity due to adjustment costs

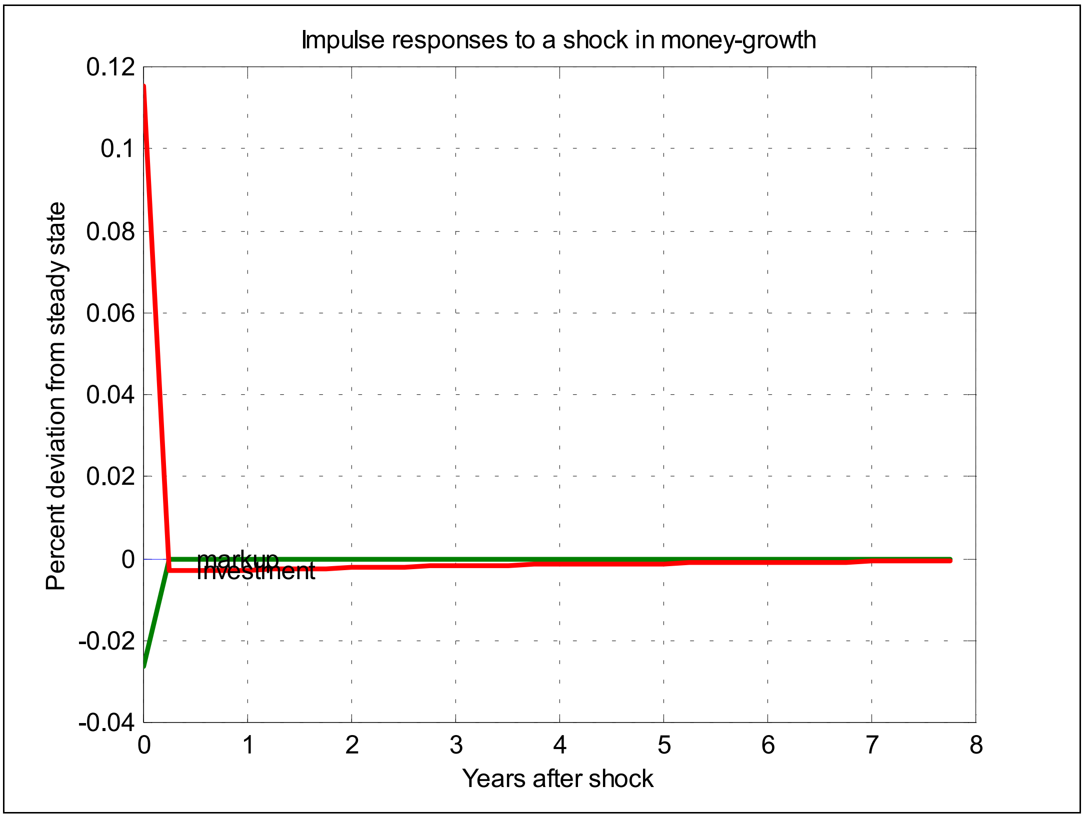


Figure 4: Nominal rigidity due to adjustment costs

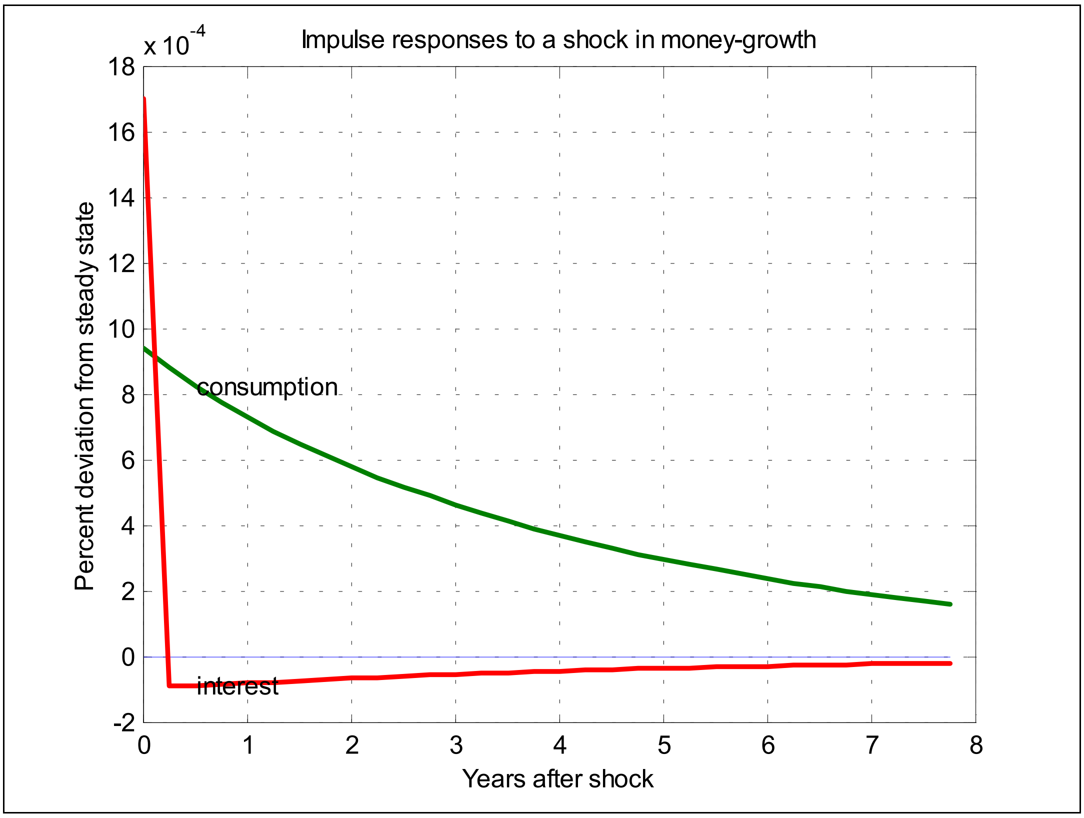


Figure 5: Nominal rigidity due to adjustment costs

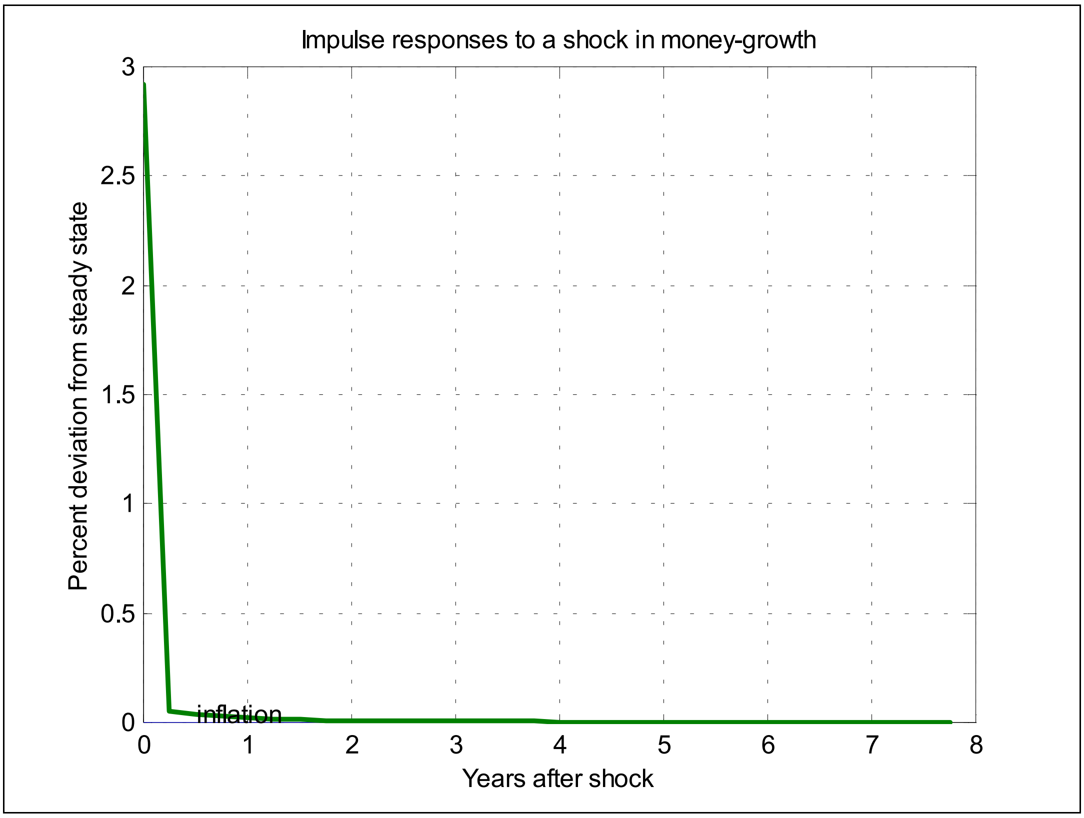


Figure 6: costs of adjusting the nominal price is about 0.3 % of aggregate output

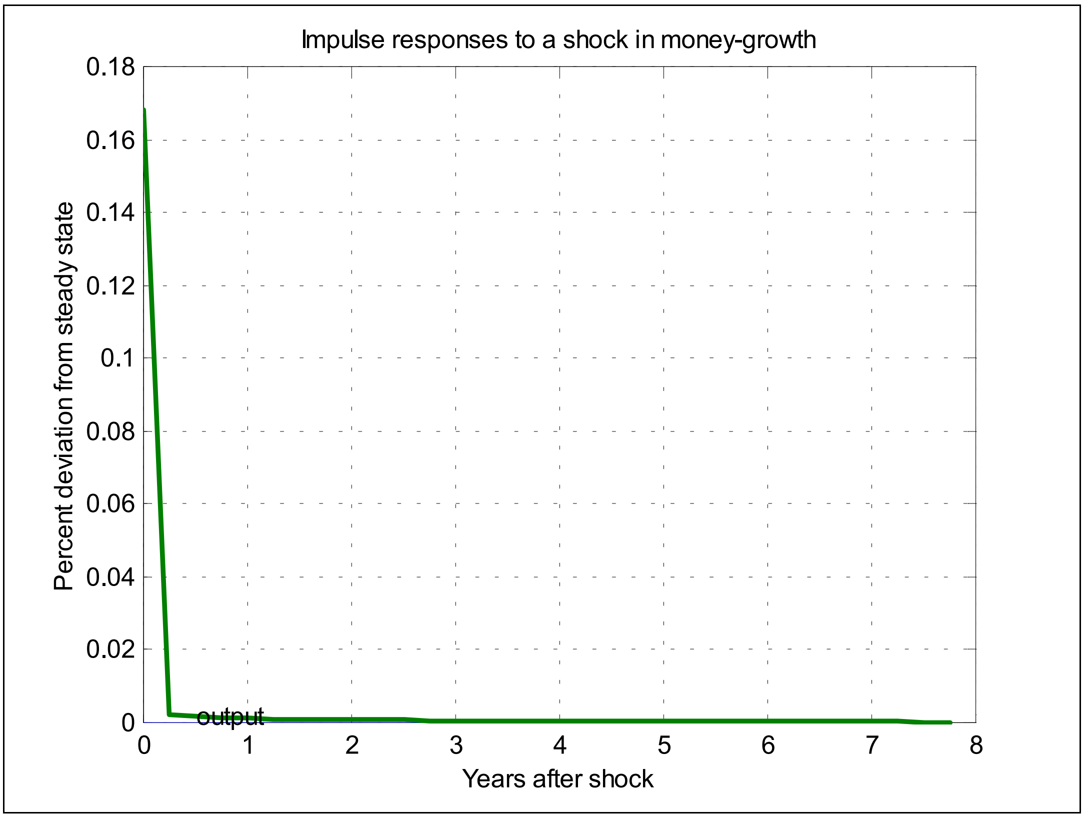


Figure 7: costs of adjusting the nominal price is about 3.0 % of aggregate output

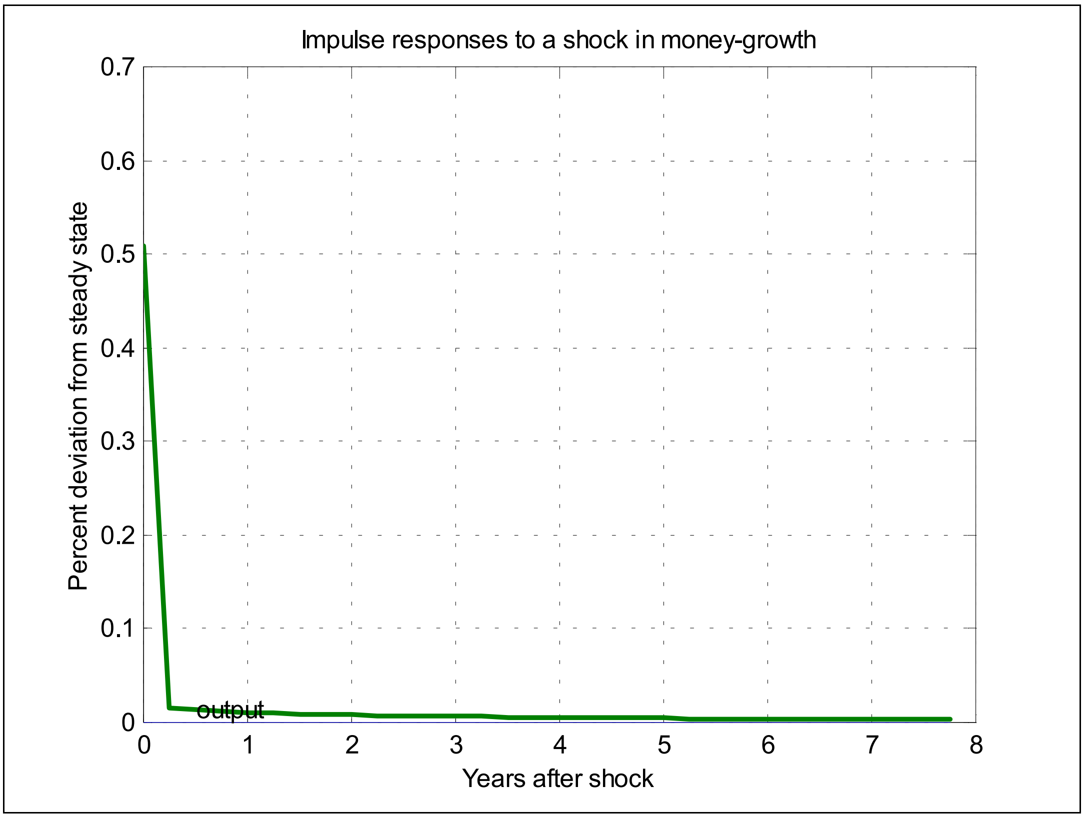


Figure 8: Nominal rigidity due to staggered price-setting

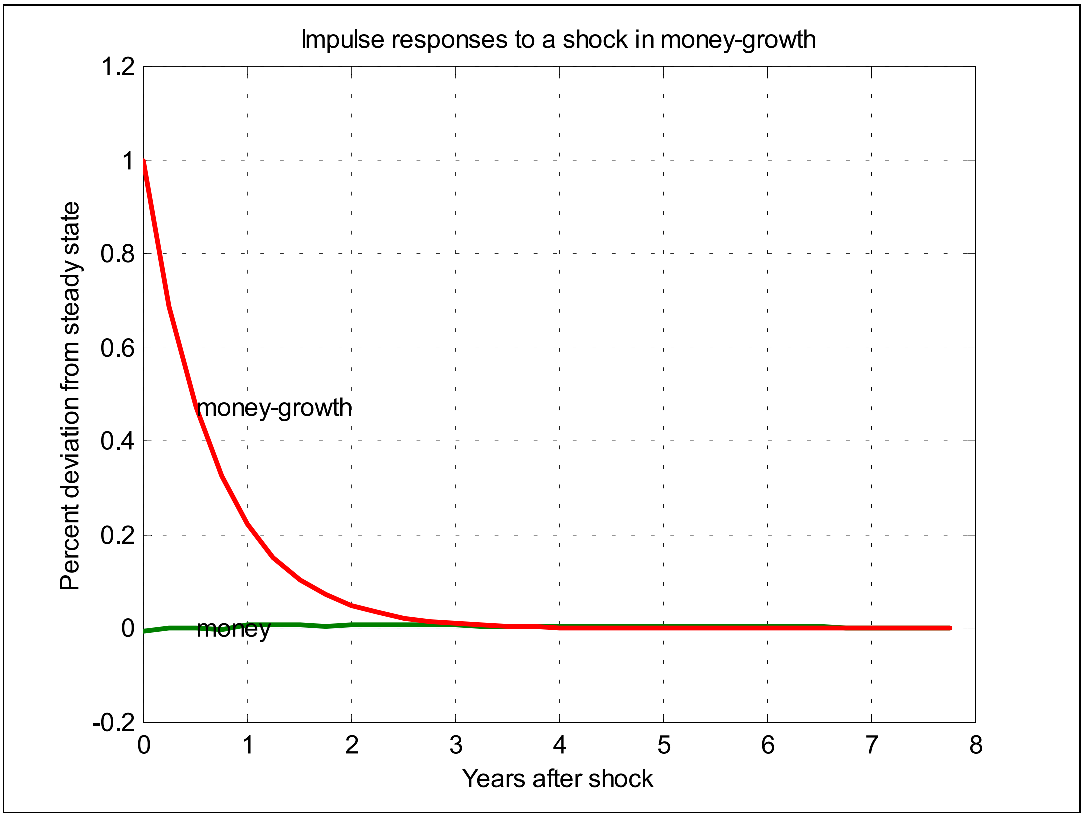


Figure 9: Nominal rigidity due to staggered price-setting

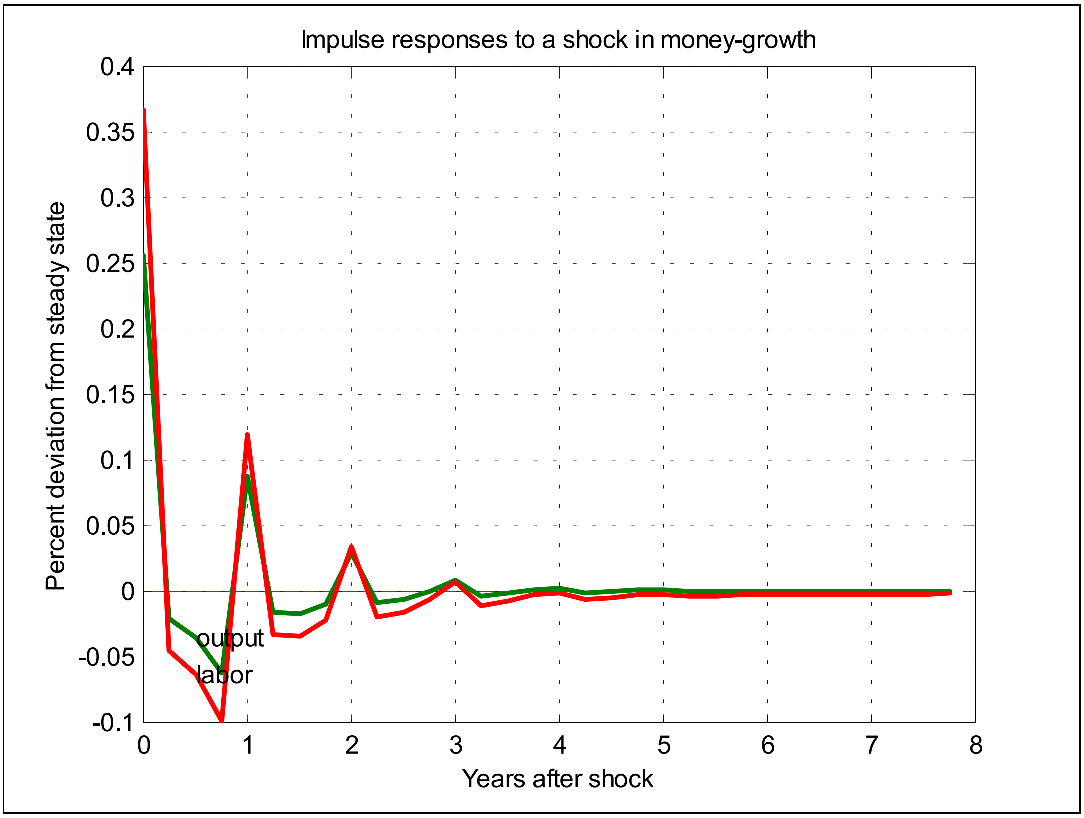


Figure 10: Nominal rigidity due to staggered price-setting

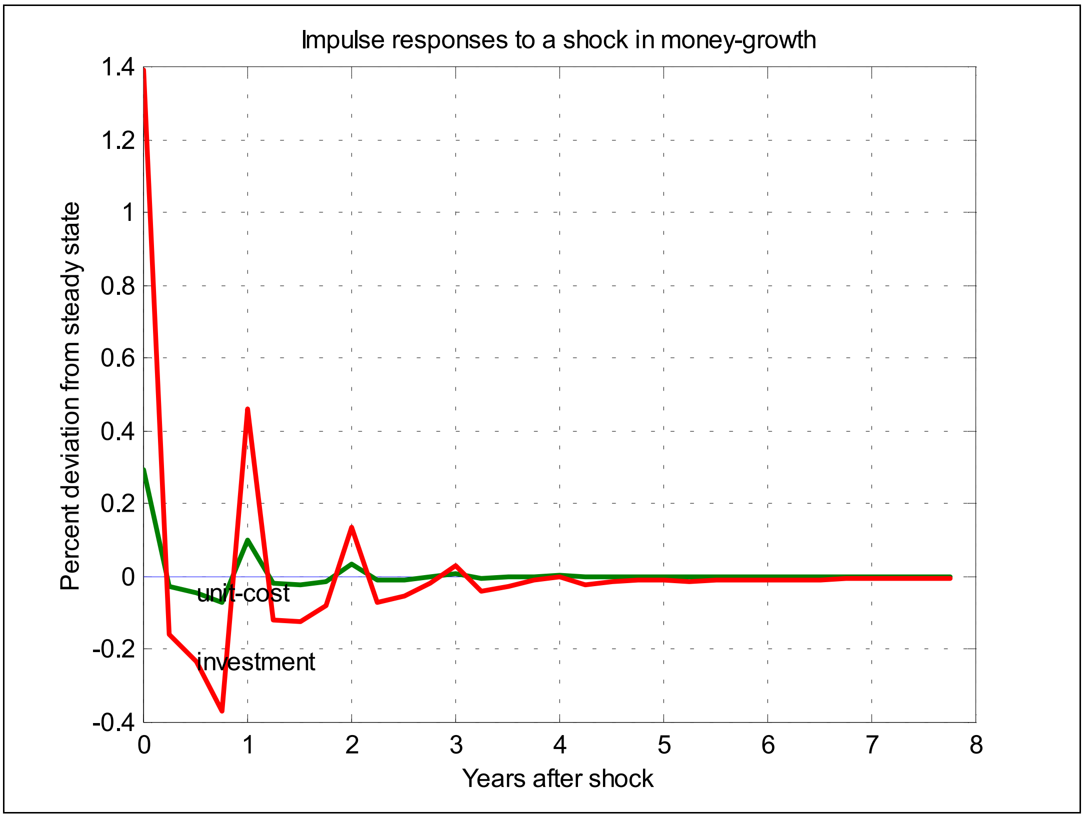


Figure 11: Nominal rigidity due to staggered price-setting

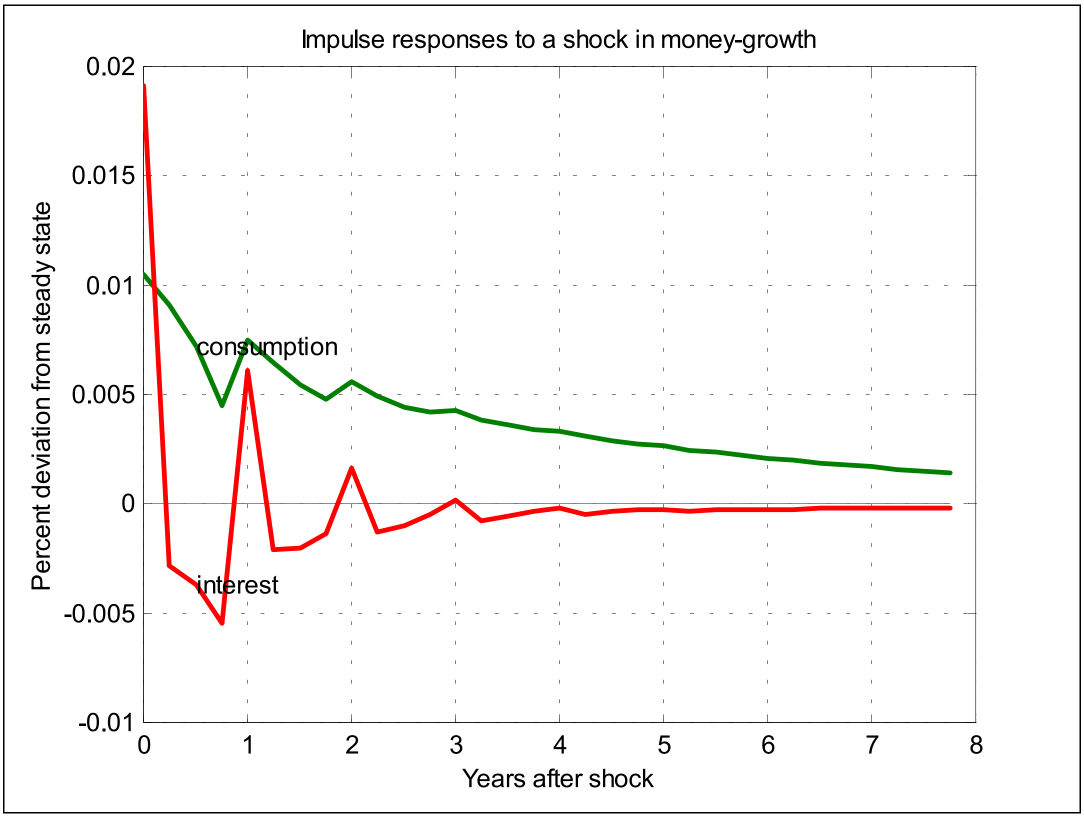


Figure 12: Nominal rigidity due to staggered price-setting

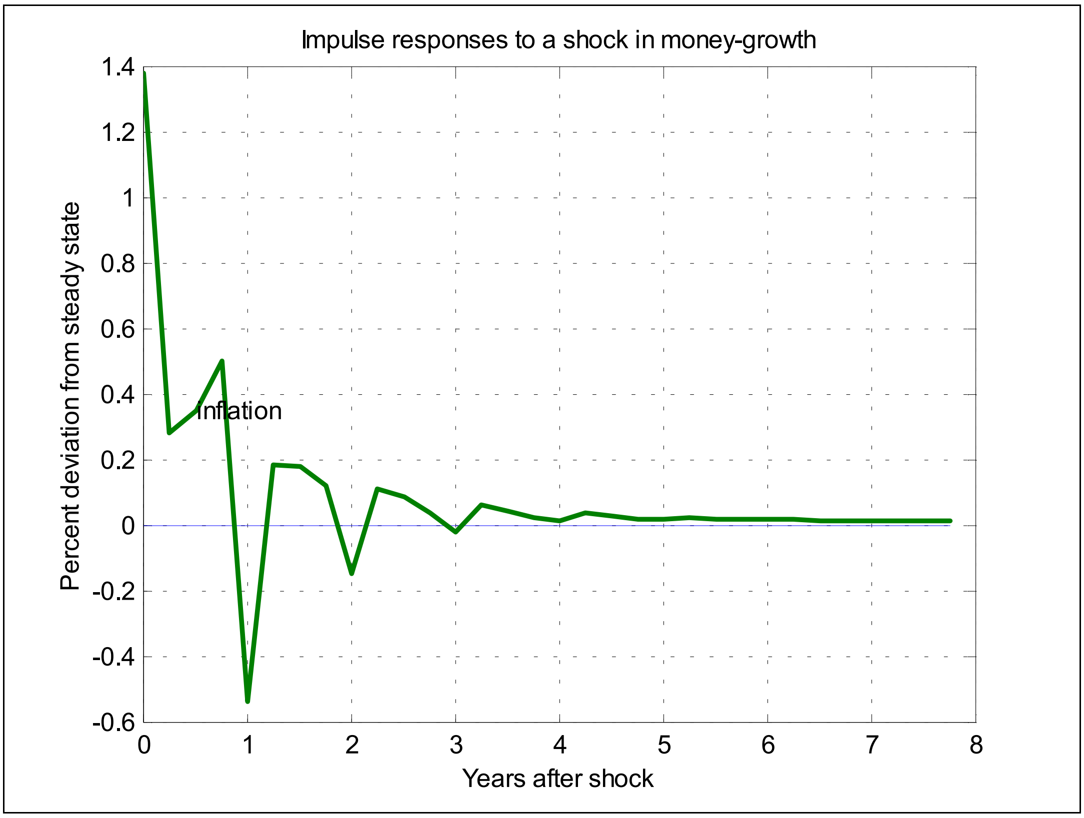
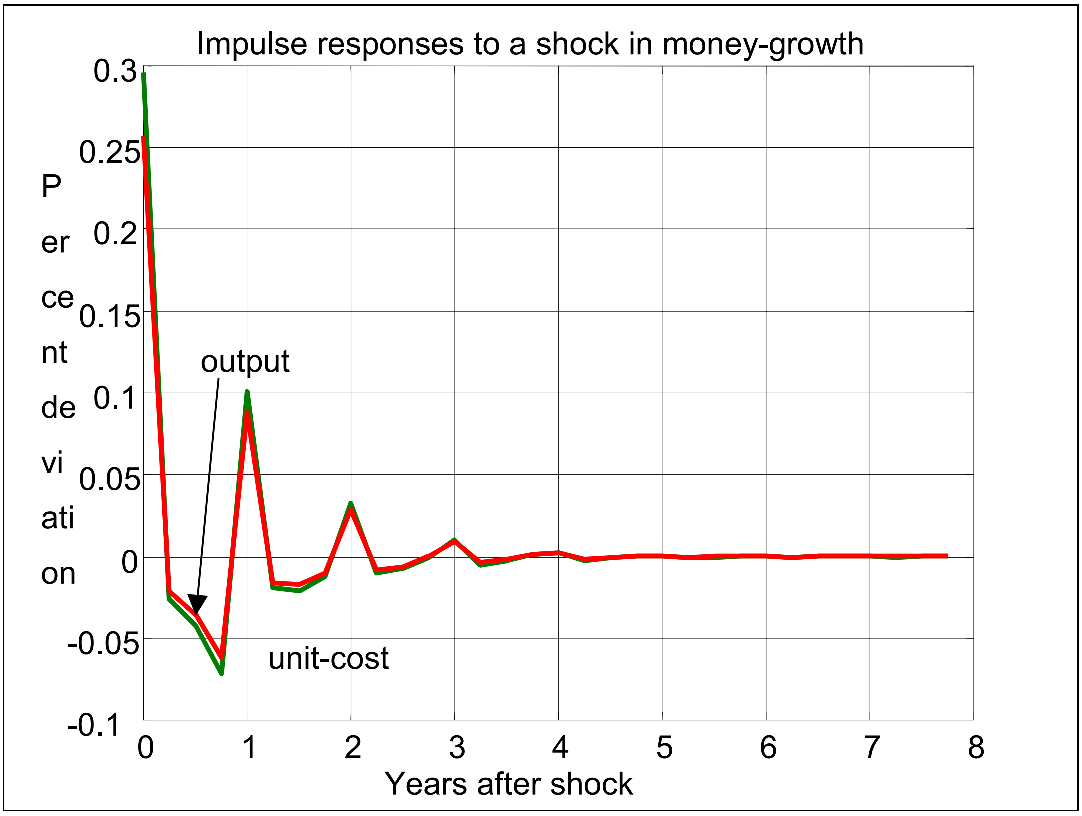


Figure 13: Nominal rigidity due to staggered price-setting



5 Conclusion

Introducing small degrees of nominal rigidities in otherwise standard dynamic general equilibrium models do not account for the observed persistence of output movements after a monetary shock. The main reason seems to be, that firms have incentives to keep their nominal prices close to the flex-price optimum. As emphasized in the New-Keynesian literature on real rigidity and reinfored by Chari/Kehoe/McGrattan (2000), persistence is absent in models, in which prices are sensitive to movements in marginal costs. A standard dynamic general equilibrium model with nominal rigidities is not capable to generate output persistence. However, qualitative differences between the dynamic effects of different forms of rigidities can be observed.

Recent work has started to implement additional features that reduce the sensitivity of marginal costs, such as real and nominal wage rigidity or variable capital utilization, see e.g. Erceg (1997), Jeanne (1998) or Christiano/Eichenbaum/Evans (2001). An alternative avenue is to implement features that reduce the price sensitivity to changes in marginal costs, such as convex demand or specific factor-inputs, see e.g. Kimball (1995), Bergin/Feenstra (2000) or Edge (2000).

Future work has to show whether the qualitative differences remain when dynamic general equlibrium models are enriched with additional features that propose to augment the persistence of the model.

6 References

Ball, L., Romer, D. (1990), Real rigidities and the Non-Neutrality of Money, *Review of Economic Studies*, 57, 183-203

Bergin, P. R., Feenstra, R. C. (2000), Staggered price setting, translog preferences, and endogenous persistence, *Journal of Monetary Economics*, 45, 657-680

Blanchard, O. J. (1983, [1991]), Price Asynchronization and Price-Level Inertia, in: Mankiw, N. G., Romer, D. (Eds.), *New-Keynesian Economics*, The MIT Press, Cambridge Massachussetts, 243-265

Blanchard, O. J., Kiyotaki, N. (1987), Monopolistic Competition and the Effects of Aggregate Demand, *American Economic Review*, 77, 4, 647-666

Chari, V. V., Kehoe, P. J., McGrattan, E. R. (2000), Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?, *Econometrica*, 68, 1151-1179

Cho, J.-O. (1993), Money and the business cycle with one period nominal contracts, *Canadian Journal of Economics*, XXVI, 1, 638-660

Christiano, L. J., Eichenbaum, M., Evans, C. (2001), Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *NBER Working Paper* 8403

Christiano, L. J., Eichenbaum, M., Evans, C. (1999), Monetary policy shocks: What have we learned and to what end?, in: Taylor, J., Woodford, M. (Eds.), *Handbook of Macroeconomics*, Elsevier, Amsterdam, 65-148

Cogley, T., Nason, J. (1995), Output dynamics in real-business-cycle models, *American Economic Review*, 85, 492-511

Edge, R. M. (2000), Time-to-build, time-to-plan, habit-persistence, and the liquidity effect, Board of Governors of the Federal Reserve System*, International Finance Discussion Papers.* No. 673

Erceg, C. J. (1997), Nominal Wage Rigidities and the Propagation of Monetary Disturbances, Board of Governors of the Federal Reserve System, *International Finance Discussion Papers.* No. 590

Favero, C. (2001), *Applied Macroeconometrics*, Oxford University Press, Oxford

Fischer, S. (1979a), Anticipations and the nonneutrality of money, *Journal of Political Economy,* 87, 2, 225-252

Fischer, S. (1979b), Capital Accumulation on the Transition Path in a Monetary Optimized Model, *Econometrica*, 47, 6, 1433-1439

Hairault, J., Portier, F. (1993), Money, New-Keynesian Macroeconomics and the Business Cycle, *European Economic Review*, 37, 1533-1568

Huang, K. D., Liu, Z. (1999), Staggered Contracts and Business Cycle Persistence, Institute for Empirical Macroeconomics, Federal Reserve Bank of Minneapolis, *Discussion Paper 127*

Ireland, P. N. (1997), A small, structural, quarterly model for monetary policy evaluation, *Carnegie-Rochester Conference Series on Public Policy*, 74, 83-108

Jeanne, O. (1998), Generating real persistent effect of monetary shocks: how much nominal rigidity do we really need?, *European Economic Review*, 42, 1009-1032

Kiley, M. T. (1998), Partial adjustment and Staggered Price Setting, Board of Governors of the Federal Reserve System, *Finance and Economics Discussion Series* No. 1999-01

Kimball, M. S. (1995), The Quantitative Analytics of the Basic Neomonetarist Model, *Journal of Money, Credit, and Banking*, 27, 4, 1241-1277

Koenig, E. F. (1999), Is There a Persistence Problem? Part1: Maybe, Federal Reserve of Dallas, *Economic and Financial Review*, 4^{th} Quarter, 10-17

Mankiw, N. G. (2001), The Inexorable and Mysterious Tradeoff between Inflation and Unemployment, *The Economic Journal*, 111, C45-C61

Roberts, J. M. (1995), New Keynesian Economics and the Phillips Curve, *Journal of Money, Credit, and Banking*, 27, 4, 975-984

Romer, D. (1993), The New Keynesian Synthesis, *Journal of Economic Perspectives*, 7, 1, 5-22

Rotemberg, J. J. (1982), Sticky Prices in the United States, *Journal of Political Economy*, 90, 6, 1187-1211

Rotemberg, J. J., Woodford. M. (1995), Dynamic General Equilibrium Models with Imperfectly Competitive Product Markets, in: Cooley, T. F. (Eds.), *Frontiers of Business Cycle Research*, Princeton University Press, Princeton New Jersey, 243-293

Taylor, J. B. (1980), Aggregate Dynamics and Staggered Contracts, *Journal of Political Economy*, 88, 1, 1-23

Uhlig, H. (1999), A toolkit for analysing nonlinear dynamic stochastic models easily, in: Marimon, R., Scott, A. (Eds.), *Computational Methods for the Study of Dynamic Economies*, Oxford University Press, Oxford, 30-61

Walsh, C. E. (1998), *Monetary Theory and Policy*, MIT Press, Cambridge

Yun, T. (1996), Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles, *Journal of Monetary Economics*, 37, 345-370