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The Role of Money Demand in a Business Cycle Model with Staggered Wage Contracts*

by

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Abstract

The question of the main determinants of persistent responses due to nominal shocks captures, at least since Chari et al. (2000), a major part of the recent macroeconomic debate. However, the question whether sticky wages and/or sticky prices are sufficient for persistent reactions of key economic variables remains open.

In the present model we allow for nominal rigidities due to Taylor- like wage setting as well as price adjustment costs. However, as our analysis illustrates, smoothing marginal costs seems crucial to derive a contract multiplier, wage staggering alone is not sufficient. Without considering a more specific analysis of factor market frictions, we enforce a point made by Erceg (1997) by analyzing the structure of money demand. In particular, we analyze a 'standard' consumption based money demand function by varying the interest rate elasticity of money demand as well as the steady state rate of money holdings. Our results show that the persistency of the output/price dynamics can be affected crucially by the form of the implicit money demand function. In particular, it is shown that staggered wage contracts have to be accompanied by a sufficiently low interest rate elasticity, otherwise the model fails to reproduce reasonable responses of real variables.

JEL classification: E32, E41

Key Words: Monetary Policy Shocks, Sticky Prices, Staggered Wages, Money Demand

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1 Introduction

The question of the contribution of monetary shocks to the explanation of business cycles has been explored intensively in the recent past. For these shocks to play a significant role it is, however, necessary to overcome the classical neutrality result, that is, some nominal rigidity is required. An already voluminous literature has explored the extent to which imperfect information and adjustment costs can generate nominal stickiness.[1] While crucial aspects have been explored initially in partial equilibrium models, the recent literature places the analysis of monetary impulses and propagation mechanisms in dynamic general equilibrium settings and thereby implements this issue into a more genuine business cycle framework.

The finding that monetary shocks can have important real effects is, however, only half of the story. As a series of recent empirical studies has shown, see e.g. Christiano et al. (1999), persistence in output adjustment is an essential property of business cycle fluctuations and a model allowing a significant role of monetary shocks should be able to account for this observation.

A wide class of dynamic general equilibrium (DGE) models account for monetary non-neutrality through various mechanisms causing price and/or wage stickiness. These mechanisms include Fischer wage or price contracts (Cho and Cooley (1995)), Calvo staggered and overlapping price and wage contracts (Yun (1996), King and Watson (1995), Rotemberg and Woodford (1997), Woodford (2003)) or menu costs (Rotemberg (1996), Ireland (1997)). However, these models can be criticized either for building in a large exogenous component of price/wage stickiness or for failing to generate a reasonable degree of persistency, when only a modest exogenous degree of stickiness is incorporated.

In an influential paper Chari et al. (2000) have illustrated that a prototypical DGE model of the business cycle which includes money and price setting firms still lacks a quantitatively important transmission mechanism which propagates the monetary impact effect via (real) propagation mechanisms, i.e. such models do not show a contract multiplier, that is, output persistence does not go beyond the contract length. They exemplify this by implementing staggered price setting into a DGE model. In the spirit of Taylor (1980) Chari et al. (2000) incorporate an exogenous source of price stickiness by the assumption that contracts last for four

[1] For a detailed survey see Andersen (1998).

periods, and examine whether the model can account for endogenous output and price persistence. One of the main conclusion is that the inability of the DGE model to account for persistent output effects is basically due to the high procyclicality of marginal cost implied by standard assumptions about preferences, technology and factor market-clearing.

In a similar vein, several authors investigating sticky price models (Ball and Romer (1990), Romer (1993), Christiano et al. (1997), Huang and Liu (1999, 2002) Koenig (2000), Ascari (2003) or Andersen (2004)) infer that labour market frictions may play a key role in order to allow sticky price models to account for a contract multiplier without incorporating an implausibly large exogenous component of price stickiness.[2] More generally, the outcome of Chari et al. (2000) has engendered a growing literature aimed at developing alternative mechanisms for producing higher persistence. Examples include Bergin and Feenstra (2000), who combine a staggered-price mechanism with factor specificity and a non-CES production function or the study by Kiley (1997) who obtains the result of non-persistent real responses to monetary shocks by emphasizing compositional changes in output over the cycle. However, the most popular approach in this literature appears to be one in which staggered wage contracts are used as either an alternative or as a complement to they studies by Erceg (1997), Andersen (1998) or Huang and Liu (1999).[3]

Following Erceg (1997) we implement a dynamic wage-setting process that generates an empirically sensible degree of persistence into a DGE model. This process is derived from a household's optimization problem in a framework that is basically a dynamic version of Blanchard and Kiyotaki (1987). The labour inputs of different households are imperfect substitutes in production. Therefore, households behave as monopolistic suppliers of labour, taking their labour demand curve and the prevailing average wage as given. As in Taylor (1980), households fix their nominal wage for a given period of time, and agree to satisfy demand for their labour at this wage. In addition, wage setting is asynchronous, as only some of the households adjust their nominal wages in a given period. This wage setting process is embedded into a standard dynamic general equilibrium model, as, for instance, Ireland (1997).

[2] A recent example of a monetary DGE model where the labour market is characterized by job creation and destruction can be found in Walsh (2003a).

[3] See also Edge (2002) for model with staggered prices under the assumption of non-homogenous factor markets.

Our results show that a Taylor-like wage setting process derived from a household optimization problem can generate a high degree of nominal wage persistence, even though standard assumptions about preferences and technology are made. In other words, it is not, for example, necessary for the intertemporal elasticity of labour supply to be implausibly large. The model exhibits price and wage rigidity basically by smoothing the marginal costs of agents setting prices and wages relative to the marginal costs of the average firm or household. The smoothing is thereby attributable to substitution effects that reduce the relative demand of households readjusting their wages.

However, our analysis illustrates that while smoothing marginal costs seems crucial to derive a contract multiplier, wage staggering per se is not sufficient. In addition, we enforce a point made by Erceg (1997) by analyzing the structure of money demand which is implied by the household's optimization problem. As we will show below, necessary conditions for persistent output effects are a low interest rate elasticity and low rate of steady state money holdings. In line with Erceg (1997) our results reflect that output/price dynamics can be affected crucially by the form of the (implicit) money demand function.

In our model we assume that firms adjust their pricing facing quadratic adjustment costs (see e.g. Rotemberg (1982), Hairault and Portier (1993), or Ireland (1997)), in contrast to the assumption of staggered price setting. It has been already shown by Gerke (2001) that this kind of nominal price stickiness alone, in contrast to Calvo staggering, is not able to generate persistent reactions even when adjustment costs are unreasonably high. This result does not change when quadratic adjustments costs interact with staggered wage setting. However, as a benchmark, we allow in a basic formulation, that prices adjust without any frictions. Moreover, we allow for a secular trend in inflation as this is a feature shared by all industrialized countries and, as Ascari (2000) has shown, a positive inflation rate can crucially influence the model's ability in generating persistency: the higher the steady state rate of inflation the lower the degree of persistency.

Besides the question as to which kind of rigidity is crucial in order to generate persistency, we raise the question whether our monetary DGE model is capable of replicating stylized facts of the business cycle. In particular, the persistent response of inflation we find in our numerical examinations is able to fit the autocorrelation structure of inflation (and its cross correlation with output) found in the date (see e.g. Fuhrer and Moore (1995)). In addition, the model accounts, to some extent, for a procyclicality in the real wage which is, for instance, reported by Huang et al. (2004) for the U.S. post war period.[4]

The remainder of this paper is organized as follows. The following section outlines the market structure as well as the decision problems of the model. In the third section the equilibrium solution is derived. In sections four and five we present the results of our quantitative analysis. Section six concludes.

2 The Model

2.1 Market structure of the model

We assume that the economy consists of a continuum of households, a representative firm which produces a final good which either can be consumed or invested, a continuum of firms producing intermediate goods and a monetary authority. The intermediate goods firms produce capital goods with capital and labour as inputs for the use as inputs in the final good sector. Because of the assumption that intermediate goods are imperfect substitutes this market is characterized by monopolistic competition. Whereas, the final good and capital services are exchanged in perfectly competitive markets. The labour market is characterized by the existence of a representative agent (job bundler) who mediates labour supply and demand between households and intermediate goods firms. Wage staggering is introduced by assuming that the households have to fix their nominal wages for four periods after entering the mediation process.[5]

The household sector

The households are characterized by the maximization of their intertemporal utility with respect to a given budget constraint. The utility of an arbitrary household j is given by:

$$U_j = E_t\Big[\sum_{t=0}^{\infty} \beta^t u\big(c_t(j), l_t(j), m_t(j)\big)\Big], \tag{1}$$

[4]A more detailed survey of the cyclicality of real wages over the business cycle can be found in Abraham and Haltiwanger (1995).

[5]See Gerke (2003): 154/5.

where $\beta \in [0,1]$ denotes a discount factor. Furthermore, $c_t(j)$, $l_t(j)$, and $m_t(j) \equiv M_t(j)/P_t$ denote consumption, leisure and the demand of real cash balances, respectively, of the j-th household. We assume that total hours are normalized to one, i.e. $n_t(j) + l_t(j) = 1$.

We assume further, that the household owns and accumulates the capital stock. Capital is rented to the intermediate goods sector for a payment $P_t r_t k_{t-1}$ of nominal interest. The evolution of physical capital, k_t, is specified as

$$k_t(j) = (1-\delta)k_{t-1}(j) + I_t(j), \tag{2}$$

where $\delta \in (0,1)$ and $I_t(j)$ denote the depreciation rate and the household's investments, respectively.

Every period, the j-th household offers an amount of $n_t(j)$ of hours to the job - mediator under the condition that it earns a nominal wage $w_t(j)$. Furthermore, it is assumed that the household holds its nominal wage constant for four periods. Thus, every period only the j-th household is able to adjust its nominal wage rate.

Besides capital and wage income we assume that the j-th household receives a lump-sum transfer $\tau_t(j)$ of newly created money and a fraction $s_t(j)$ of the intermediate goods firms nominal profits $\Pi_t(j)$.

The households budget constraint follows as

$$\tau_t(j) + w_t(j)n_t(j) + r_t k_{t-1}(j) + M_{t-1}(j) + \int_0^1 s_t(j)\Pi_t(i)di = P_t c_t(j) + P_t I_t(j) + M_t(j), \tag{3}$$

where P_t denotes the price of the final good.

The nominal wage of the j-th household follows from its utility maximization with respect to its inverse labour demand function which ensures that the demand and supply of labour services equalize.

The representative job-bundler

It is assumed that every period the mediator buys $n_{j,t}$ hours from the j-th household. Afterwards, he cumulates individual hours to the aggregate hours, n_t. The aggregation of individual hours is determined by the following technology

$$n_t = \left[\int_0^1 n_t(j)^{\frac{\zeta-1}{\zeta}} dj\right]^{\frac{\zeta}{\zeta-1}}, \tag{4}$$

with $\xi > 1$. Given equation (4) the elasticity of substitution of individual labour input follows as $-\zeta$.

Final good producers

The production of the final good which either can be consumed or invested is described by the production technology

$$y_t = \Big[\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di\Big]^{\frac{\theta}{\theta-1}}, \tag{5}$$

where y_t denotes the final good and $y_t(i)$ represents the intermediate good of type i. Furthermore, it is assumed that $\theta > 1$ and the elasticity of substitution between intermediate goods is given by $-\theta$.

The intermediate goods sector

Each intermediate goods firm produces a distinct good $i \in [0, 1]$ with labour and capital as inputs. These intermediate goods are imperfect substitutes and are sold in a market under monopolistic competition. We assume a linear homogenous production technology with constant returns to scale:[6]

$$y_t(i) = f\big(k_{t-1}(i), n_t(i), z_t\big), \tag{6}$$

where $k_{t-1}(i)$ and $n_t(i)$ denote capital and labour employed by the i-th intermediate goods firm, furthermore, z_t represents a shock in total factor productivity which follows a stationary stochastic process

$$\log z_t = (1 - \psi_z) \log \bar{z} + \psi_z \log z_{t-1} + \epsilon_t^z, \tag{7}$$

with $\epsilon_t^z \sim i.i.d.\ \mathcal{N}(0, \sigma_z^2)$ and $\psi_z \in [0, 1]$. In every period the intermediate goods producer i demands for $n_t^U(i)$ units of labour. The respective nominal wage w_t^N is paid to the representative job bundler. In addition, they rent $k_{t-1}^U(i)$ units of capital and pay every period t $r_t^N k_{t-1}^U(i)$ to the households.

Following Rotemberg (1982) each intermediate goods producer is faced with a quadratic cost function which describes the adjustment of its nominal price. This cost function is expressed as

$$\frac{\phi_P}{2}\Big[\frac{P_t(i)}{P_{t-1}(i)} - 1\Big]^2 y_t. \tag{8}$$

Equation (8) highlights the notion that price changes might have negative effects on customer - firm relationships. These negative effects increase with the magnitude of the price change and the level of economic activity.

[6] Furthermore, it is assumed that the production function is twice continuously differentiable and satisfies the Inada conditions.

The monetary authority

The monetary authority determines the money supply of the economy. In every period t, nominal money supply grows at an exogenous rate g_t, i.e. $M_t = (1 + g_t)M_{t-1}$. The newly created money is paid to the household as a lump-sum transfer. The transfer satisfies:

$$\tau_t = M_t - M_{t-1} \tag{9}$$

By the definition of the growth rate of money, real balances ($m_t \equiv M_t/P_t$) can be expressed as

$$m_t = \frac{1+g_t}{1+\pi_t} m_{t-1}, \tag{10}$$

where π_t denotes the inflation rate at time t. With $\bar{g}$ as the steady state growth rate of money, we define $\bar{\omega}_t = g_t - \bar{g}$ as the deviation of the growth rate from its steady state. As in Walsh (2003b) $\bar{\omega}$ is formulated as a stochastic process[7]

$$\bar{\omega}_t = \psi_{\bar{\omega}}\bar{\omega}_{t-1} + \phi_z z_{t-1} + \epsilon_t^{\bar{\omega}}, \tag{11}$$

with $\psi_{\bar{\omega}} \in (0, 1]$ and $\epsilon_t^{\bar{\omega}} \sim i.i.d.\ \mathcal{N}(0, \sigma_{\bar{\omega}}^2)$. Furthermore, it is assumed that each household has knowledge of the realization of $\bar{\omega}_t$ and z_t when choosing its optimal values of consumption, leisure, real balances and capital in period t.

2.2 Decision Problems

The representative job mediator acts on perfect competitive markets. In every period t he chooses the profit maximizing amount of $n_t(j)\ \forall\ j \in (0, 1)$:

$$\max_{n_t(j)} w_t^N n_t - \int_0^1 w_t(j)^N n_t(j) dj, \tag{12}$$

where w_t^N describes the price of aggregate labour in monetary units, furthermore, $w_t^N(i)$ denotes the nominal wage earned by the j-th household. Given (12), the demand for labour services follows as

$$n_t^d(j) = \left[\frac{w_t^N(j)}{w_t^N}\right]^{-\zeta} n_t. \tag{13}$$

Because of the zero profit condition due to the assumption of perfect competition, the wage level for aggregate labour services results as:

$$w_t^N = \left[\int_0^1 w_t^N(j)^{1-\zeta} dj\right]^{\frac{1}{1-\zeta}}. \tag{14}$$

[7]See Walsh (2003b), p. 69. Note further that eqn. (11) is expressed in logs.

Profit maximizing behavior of the final good producer, given by

$$\max_{y_t(i)} P_t y_t - \int_0^1 P_t(i) y_t(i) di, \tag{15}$$

where $P_t(i)$ denotes the price of the intermediate good (i), leads to the following demand for intermediate goods:

$$y_t^d(i) = \Big[\frac{P_t(i)}{P_t}\Big]^{-\theta} y_t. \tag{16}$$

Because of the zero profit condition, the price level is determined as:

$$P_t = \Big[\int_0^1 P_t(i)^{1-\theta} di\Big]^{\frac{1}{1-\theta}}. \tag{17}$$

The optimization problem of the intermediate goods producers is to maximize the present value of profits[8]

$$\max \quad E_t \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t(j)\Pi_t(i)}{P_t}, \tag{18}$$

where $\beta^t \lambda_t / P_t$ denotes the marginal utility value of the representative household of an additional unit of profits during period t. The nominal profits of firm i, Π_t, are defined as:

$$\Pi_t(i) = P_t(i) y_t(i) - w_t^N n_{t-1} - r_t^N k_{t-1}^U(i) - P_t \frac{\phi_P}{2}\Big[\frac{P_t(i)}{P_{t-1}(i)} - 1\Big]^2 y_t. \tag{19}$$

Equation (18) is maximized subject to the following constraint:

$$y_t^s(i) = f\big(k_t(i), n_t(i), z_t\big) = \Big[\frac{P_t(i)}{P_t}\Big]^{-\theta} y_t^d. \tag{20}$$

Maximizing the intermediate goods producer's decision problem leads to the following first order conditions:[9]

$$\begin{aligned}
\lambda_t(j) r_t &= \xi_t f_k\big(k_{t-1}(i), n_t(i), z_t\big), && (21)\\
\lambda_t(j) w_t &= \xi_t f_n\big(k_{t-1}(i), n_t(i), z_t\big), && (22)\\
0 &= \lambda_t(j)(1-\theta)\Big[\frac{P_t(i)}{P_t}\Big]^{-\theta}\frac{y_t}{P_t} - \lambda_t(j)\phi_p\Big[\frac{P_t(i)}{P_{t-1}(i)} - 1\Big]\frac{y_t}{P_{t-1}} && (23)\\
&\quad + \xi_t\theta\Big[\frac{P_t(i)}{P_t}\Big]^{-\theta-1}\frac{y_t}{P_t} + \beta E_t\Big\{\lambda_{t+1}(j)\phi_p\Big[\frac{P_{t+1}(i)}{P_t(i)} - 1\Big] y_{t+1}\frac{P_{t+1}(i)}{[P_t(i)]^2}\Big\}.
\end{aligned}$$

[8]Note that $\beta^t \lambda_t(j)$ is a stochastic discount factor (pricing kernel). See Rotemberg and Woodford (1992), p. 1160 and 1168.

[9]Note that subscripts, except t and $t-1$, denote partial derivatives.

Note that ξ_t represents the discounted Lagrange - multiplier of the firm's maximization problem. From equations (21) and (22) follows that the respective marginal product of capital and labour inputs does not equal the factor payments. Furthermore, equation (23) gives the optimal price decision of intermediate goods firms.

The households choose their utility maximizing wages with respect to their labour demand function, but they do so by fixing the nominal wage for four periods, i.e. t, $t+1$, $t+2$, $t+3$. However, in every period the labour market is in equilibrium, i.e. $n_t^s(j) = n_t^d(j)$. Consequently, the households maximization problem reads as

$$\max_{c_t(j), w_t^N(j), k_t^H(j), m_t^H(j)} \quad E_t\Big\{\sum_{t=0}^{\infty} \beta^t u\Big(c_t(j), m_t(j), 1 - \Big[\frac{w_t^N(j)}{w_t^N}\Big]^{-\zeta} n_t\Big)\Big\} \tag{24}$$

subject to

$$\begin{aligned} w_t^N(j)\Big[\frac{w_t^N(j)}{w_t^N}\Big]^{-\zeta} n_t + P_t(1 + r_t - \delta)k_{t-1}^H(j) & \\ +\tau_t(j) + M_{t-1}(j) + \int_0^1 s_t(j)\Pi_t(i)di \quad &= \quad P_t c_t(j) + P_t k_t(j) + M_t(j). \end{aligned} \tag{25}$$

Maximization of equations (24) and (25) leads to the following first order conditions:

$$P_t\lambda_t(j) = u_c(\cdot) \tag{26}$$

$$\lambda_t(j) = u_M(\cdot)\frac{1}{P_t} + \beta E_t\lambda_{t+1}(j) \tag{27}$$

$$P_t\lambda_t(j) = \beta E_t\lambda_{t+1}(j)P_{t+1}(1 + r_{t+1} - \delta) \tag{28}$$

$$\begin{aligned} 0 = & \sum_{s=0}^{3} \beta^{t+s} E_t u_{w(j)}\Big(c_t(j), m_t(j), 1 - \Big[\frac{w_t^N(j)}{w_{t+s}^N}\Big]^{-\zeta} n_{t+s}\Big) \\ & \cdot\zeta\Big[\frac{w_t^N(j)}{w_{t+s}^N}\Big]^{-\zeta-1}\frac{n_{t+s}}{w_{t+s}^N} + E_t\lambda_{t+1}(j)(1-\zeta)\big(w_{t+s}^N(j)\big)^{-\zeta}\big(w_{t+s}^N\big)^{\zeta} n_{t+s} \end{aligned} \tag{29}$$

In order to simplify the solution we assume a perfect insurance market. Therefore, the optimal decision about the wage rate is independent of any idiosyncratic risks. Furthermore, because of the identical initial factor endowments and the separability of consumption and leisure, we assume that all households are identical in their demand for money and consumption goods as well as in their capital accumulation.[10]

[10]See, for example, Ascari (2000): 671, for similar assumptions.

3 Equilibrium Solution

In the symmetric equilibrium where $\int_0^1 P_t(i)di = P_t$ the following conditions hold:

$$\int_0^1 k_{t-1}(i)di = k_{t-1} \tag{30}$$

$$\int_0^1 y_t(i)di = y_t. \tag{31}$$

No symmetry is given for individual nominal wages, $w_t^N(j)$, real wages, $w_t(j)$, and for the individual labour supply $n_t(j)$.

The aggregate resource constraint follows as:

$$y_t = c_t + k_t - (1-\delta)k_{t-1} + \frac{\phi_P}{2}\Big[\frac{P_t(i)}{P_{t-1}(i)} - 1\Big]^2 y_t. \tag{32}$$

Because of the symmetry, equations (21) to (23) simplify to

$$\lambda_t r_t = \xi_t f_k(k_{t-1}, n_t, z_t) \tag{33}$$

$$\lambda_t w_t. = \xi_t f_n(k_{t-1}, n_t, z_t) \tag{34}$$

$$\begin{aligned} 0 &= \lambda_t(1-\theta) - \lambda_t\phi_P\Big[\frac{P_t}{P_{t-1}} - 1\Big]\frac{P_t}{P_{t-1}} + \xi_t\theta \\ &\quad \beta E_t\Big\{\lambda_{t+1}\phi_P\Big[\frac{P_{t+1}}{P_t} - 1\Big]\frac{y_{t+1}}{y_t}\frac{P_{t+1}}{P_t}\Big\}. \end{aligned} \tag{35}$$

Furthermore, from the first order conditions of the j-th household as well as the i-th intermediate goods producers follows in the symmetric equilibrium

$$u_c(\cdot) = u_M(\cdot) + \beta E_t\Big[u_c\Big(c_{t+1}, m_{t+1}, 1 - \Big(\frac{w_t(j)}{w_{t+1}}\Big)^{-\zeta} n_{t+1}\Big)\frac{1}{1+\pi_{t+1}}\Big], \tag{36}$$

$$u_c(\cdot) = \beta E_t R_{t+1} u_c\Big(c_{t+1}, m_{t+1}, 1 - \Big(\frac{w_t(j)}{w_{t+1}}\Big)^{-\zeta} n_{t+1}\Big) \tag{37}$$

$$R_t = \frac{1}{\mu_t} f_k(k_{t-1}, n_t, z_t) + 1 - \delta \tag{38}$$

$$\begin{aligned} 0 &= \sum_{s=0}^{3} \beta^{t+s} E_t u_{w(j)}\Big(c_t, m_t, 1 - \Big(\frac{w_t(j)}{w_{t+s}}\Big)^{-\zeta} n_{t+s}\Big)\zeta\Big(\frac{w_t(j)}{w_{t+s}}\Big)^{-\zeta-1}\frac{n_{t+s}}{w_{t+s}} \\ &\quad + E_t u_c\Big(c_t, m_t, 1 - \Big(\frac{w_t(j)}{w_{t+s}}\Big)^{-\zeta} n_{t+s}\Big)\frac{1}{P_{t+s}}(1-\zeta)\big(w_{t+s}(j)\big)^{-\zeta}\big(w_{t+s}\big)^{\zeta} n_{t+s}. \end{aligned} \tag{39}$$

An equilibrium of this economy is a set of variables

$$\Omega_t = \big\{y_t, c_t, k_t, I_t, m_t, n_t, R_t, \mu_t, w_t, w_t(j), P_t, \pi_t\big\},$$

which is described by equations (33) to (39), as well as (2), (7) , (10), (11), and (32).

Furthermore, the steady state is characterized by a positive inflation rate, which leads (in contrast to Erceg (1997) or Huang and Liu (2002)) to an asymmetric solution. Because of the staggered nominal wages which leads to a varying real wage, we obtain for labour supply and demand of different groups j, k in period t: $n_t(j) \neq n_t(k)\ \forall\ j \neq k$ and $j, k \in [0, 1]$.

For our calibrations we assume that the economy starts in an equilibrium with flexible prices and every group of households has determined their (identical) optimal nominal wages. The steady state of this economy is defined as a state in which the growth rate of money equals the growth rate of goods prices as well as nominal wages.[11] Therefore, the steady state is characterized by the constancy of aggregate variables.[12]

4 Calibration

For the calibration we assume the following function specifications:

$$y_t(i) = z_t\big[k_{t-1}(i)\big]^{\alpha}\big[n_t(i)\big]^{1-\alpha} \tag{40}$$

$$u(c_t, m_t, n_t) = \frac{\left(\left(bc_t^{\nu} + (1-b)m_t^{\nu}\right)^{\frac{1}{\nu}}\right)^{1-\Phi}}{1-\Phi} - \frac{n_t^{1-\eta}}{1-\eta} \tag{41}$$

The calibration are chosen in accordance with the literature. In our study we follow basically Ireland (1997) and Walsh (1998, 2003b). The calibration of our models is consistent with the following scenario: the capital share of total income, α, is 30 %, the discount factor, β, is assumed to be 4 % per year and the depreciation rate, δ, is 10 % per year. In the steady state, the households work 30 % of their time endowment. The annual growth rate of the nominal stock of money, $\bar{g}$, is assumed as 5 %. The markup of the monopolistic intermediate goods producers is 20 %, i.e. $\theta = 6$. $\phi_P = 3.95$ denotes that the costs of price adjustments correspond to a ratio of 0.03 % of GDP. Furthermore, according to Erceg (1997) the household's markup is assumed as 11 % which corresponds to a value of $\zeta = 10$. The intertemporal elasticity of substitution of consumption as well as of labour are assumed as 2.0, i.e. $\Phi = 0.5$ and $\eta = 1$, respectively. The parameters b and ν, which determine the

[11] Cf. Ascari and Rankin (2002): 661.

[12] A detailed description of the steady states can be found in Gerke (2003).

steady state holdings of money and the interest rate elasticity of money demand, are chosen analogous to Chari et al. (2000).

Table 1: Parameter Specifications

Model - I					
α	β	δ	η	b	Φ
0.30	0.99	0.025	1.0	0.98	0.5
ϕ_z	Θ	ψ_z	ψ_ω	σ	σ_ω
-0.15	1.0125	0.95	0.687	0.007	0.00216
θ	ϕ_P	ζ	ν		
6.0	3.95	10	-1.56		

For the subsequent analysis the model is log-linearized (first order approximation) and solved by the method of undetermined coefficients in order to get a recursive equilibrium law of motion (see Uhlig (1999) for details).

5 The Monetary Transmission Mechanism

An expansionary monetary impulse increases the household's resources. However, the aggregate wage level cannot increase proportionally to the monetary injection, because the speed of adjustment of nominal wages depends on the ability of the households adjust their individual wages. In addition, the price level does not adjust proportionally, too, because of the firms who set their optimal prices as a markup over marginal costs which are determined by real wages as well as rental costs. As a result, real aggregate demand increases, raising both households' income and firms' demand for labour services. Therefore, the marginal utility of income (consumption) decreases, whereas the marginal utility of leisure increases. Those households who are able to adjust their nominal wages choose their wages according to the maximization rule, i.e. the marginal rate of substitution between consumption and leisure equals real wages including a markup.[13] Thereby, the household recognizes

[13]Note that the j-th household sets its real wage such that it corresponds with the marginal rate of substitution of consumption and leisure weighted with the respective markup, i.e.

$$w_t(j) = \frac{\zeta}{\zeta - 1} MRS,$$

see also Ascari (2000):167.

that the possible increase of their nominal wages will trigger the job mediator to demand more labour from those households who are not able to adjust their wages. This substitution effect as well as the income effect (due to a higher relative wage) counteract the primary reduction of the marginal utility of income and the primary increase of the marginal utility of leisure and therefore dampen the increase of wages. The higher the elasticity of substitution among differentiated labour skills, the smaller the optimal percentage change in wages and thus the higher the persistence of output. The increase of nominal wages will therefore not be proportional to the increase of aggregate demand.

Assuming a positive steady state inflation rate gives a further incentive to the households to increase their nominal wages. Therefore, households try to set their nominal wages such that the real wage that prevails on average in the next four periods corresponds to the real wage in absence of inflation or, equivalently, which they would set in a flexible wage setup when he can adjust real wage every period. Due to the symmetry of the model the decision problem of the subsequent household group is identical to that of the first group which adjusts its nominal wage without time lag.

Because of the gradual price increase following the monetary injection the reactions of the households are dampened in periods $t+1$, $t+2$, ... At the end of this transmission process the real variables converge to the initial steady state.[14]

5.1 Quantitative Analysis

Our numerical analysis is structured as follows. In order to examine the factors which determine persistent reactions we compare a flex-price version, i.e. we assume $\phi_P = 0$, with a sticky price version, i.e. we set $\phi_P = 3.95$, of our model. Furthermore, we analyze the business cycle properties of the models and compare them to U.S. time series data.

The impulse response functions (see figures 1 and 2 below) reflect the optimal decisions of the households and firms. The variables labour, output and consumption (figure 1) react positively to the monetary impulse. Furthermore, the immediate response of real wages is negatively whereas we observe a positive response of real cash balances (figure 2).

[14] Note, that long run monetary neutrality still holds in this setup.

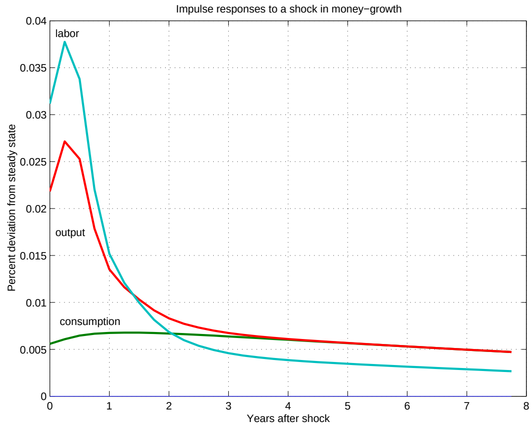

Figure 1: No adjustment costs (y, c, l)

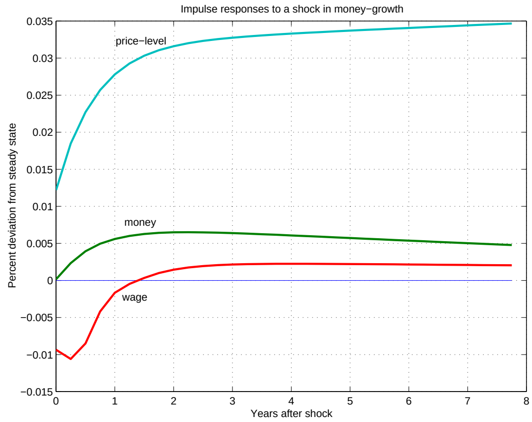

Figure 2: No Adjustment Costs (p, m, w)

After one year, when every household has used its possibility to adjust wages we observe a smooth adjustment process to the steady state. Furthermore, the introduction of price adjustment costs does not lead to a lower degree of persistence, as shown by figures 3 and 4 below.

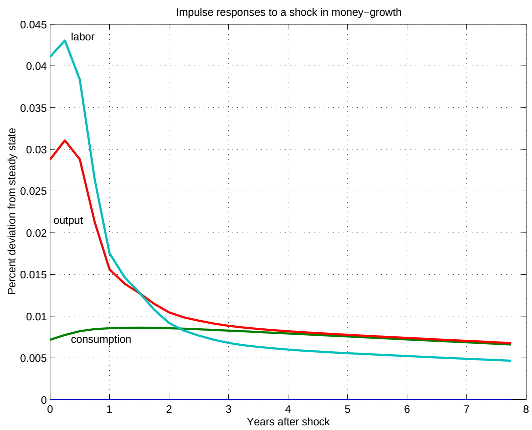

Figure 3: Adjustment costs (y, c, l)

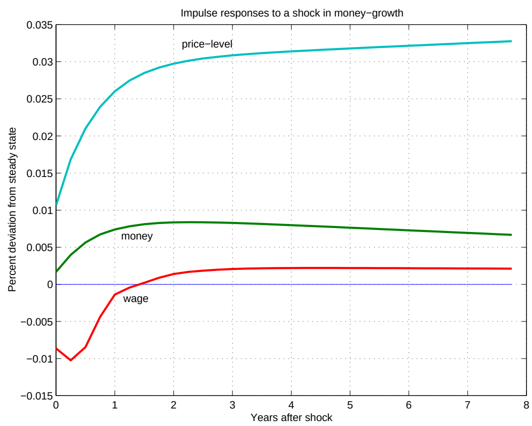

Figure 4: Adjustment costs (p, m, w)

Our results suggests that firms do not have a strong incentive to adjust their prices. This can be seen in figure 5 where inflation reacts both cases, i.e. with and without price adjustment costs, rather inertially.

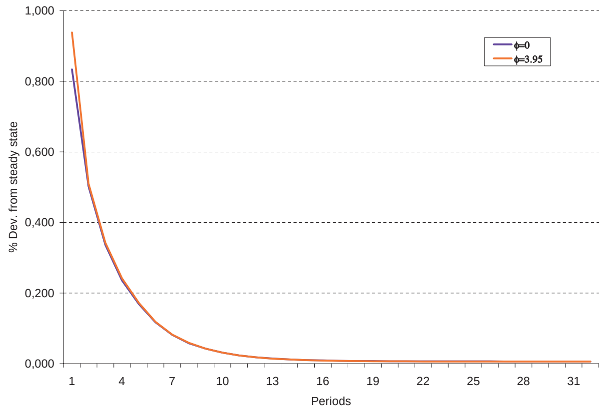

Figure 5: Inflation Response

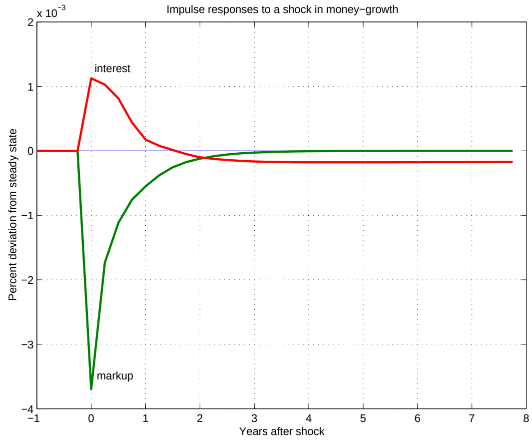

Figure 6: Interest Rate and Markup

Obviously firms do not have an incentive to adjust their prices because the unit costs or equivalently the markup do not react sensible, because the increase of rental price for capital is rather low (see figures 5 and 6). All in all, we do not find a significant difference between the flex-price (blue) and the sticky price (red) version the model.

The remaining question is, why do we observe the aforementioned adjustment pattern? Already Erceg (1997) has pointed out that the dynamic reactions of the variables after a monetary shock depend on the specification of money demand. In particular, when an income-based money demand function and, in addition, a high income elasticity is used, the model yields persistent reactions.[15] Given the evolution of prices the behavior of output is determined by the output elasticity of money demand. Therefore, in the case of a low income elasticity, output (consumption) has to rise by a large amount to allow the demand for real balances to adjust to the higher nominal supply. A consumption-based money demand function in general implies a very low income elasticity of money demand, as consumption varies much less than output in response to a transitory shock. Thus, given sticky prices in the short run, consumption must rise high enough in order to allow the demand for real balances to adjust to the higher nominal supply. After the period of the shock output declines subsequently because producers demand much larger price increases than occur in equilibrium to keep output close to its initial peak. This suggests that output persistency is increased by imposing a structure that makes consumption

[15] Cf. Erceg (1997):4

more responsive to income.

However, our calibration implies a consumption elasticity of money demand equal to one whereas the interest rate elasticity, i.e. $1/(1-\nu) = -.39$, is rather low. However, a utility specification analogous to Fischer (1979), where the interest rate elasticity equals the consumption elasticity of money demand, does not produce any persistent responses due to monetary shocks.[16] Following the reasoning above, the dynamics does depend on the interest rate sensitivity. If money demand reacts sensitive to an increase of the interest rate households diminish their money holdings by more as if this is the case when money demand is insensitive to interest rate fluctuations. In consequence, again, output has to rise enough to prompt the households to hold the additional money supply.

We conclude the discussion of the monetary transmission process by taking a glimpse on the business cycle properties of the model variants. Concentrating on the following variables, output (y), consumption (c), hours (n), inflation (π), and real wages (w) we obtain for the model variants and the U.S. :

Table 2: Business Cycle Properties

	volatility[a]	$x(t-5)$	$x(t-2)$	$x(t-1)$	$x(t)$	$x(t+1)$	$x(t+2)$	$x(t+5)$
Model[b]								
y	0.0206	0.01	0.69	0.89	1.00	0.89	0.69	0.01
π	42.169	-0.19	-0.26	-0.12	0.02	0.33	0.28	0.16
c	0.718	0.08	0.81	0.89	0.87	0.62	0.41	-0.02
n	1.077	-0.07	0.46	0.72	0.93	0.95	0.79	0.08
w	0.393	0.20	0.56	0.32	-0.01	-0.25	-0.31	-0.17
U.S.[c]								
y	0.0106	-0.31	0.42	0.72	1.00	0.72	0.42	-0.31
		(0.08)	(0.08)	(0.04)	—	(0.05)	(0.07)	(0.08)
π	18.837	-0.28	-0.37	-0.22	0.03	0.24	0.40	0.42
		(0.09)	(0.10)	(0.11)	(0.10)	(0.09)	(0.06)	(0.09)
c	0.767	-0.12	0.56	0.74	0.79	0.52	0.27	-0.30
		(0.07)	(0.07)	(0.05)	(0.03)	(0.06)	(0.07)	(0.08)
n	0.341	-0.08	0.48	0.65	0.73	0.47	0.20	-0.39
		(0.08)	(0.06)	(0.05)	(0.04)	(0.07)	(0.09)	(0.07)
w	0.663	0.12	0.54	0.55	0.46	0.27	0.11	-0.34
		(0.10)	(0.07)	(0.06)	(0.09)	(0.11)	(0.09)	(0.07)

[a] Measured as the standard deviation of the respective variable around trend relative to the standard deviation of Output.

[b] Note that the obtained correlations for both variants are based on the flex-price versions of the model. Applying the sticky-price versions leads to slight changes in the magnitudes of the obtained correlations, only, but not to changes in signs.

[c] Own calculations, with real quarterly data taken from OECD Main Economic Indicators 2004 for the period 1964.2-1999.4. The real wage is based on own calculations. Wages are measured as hourly wages taken from Bureau of Labor Statistics (2004). All variables are measured in per capita units and are HP-filtered. The empirical standard errors (noted in parentheses below the coefficients) are obtained from Bootstrap simulations.

[16] Cf. Walsh (2003b): 69.

Comparing the obtained autocorrelations of our simulations with U.S. time series data we observe that the model fits the empirical facts in several dimensions reasonably well, except the lagged correlations of the real wage do not coincide with the empirical findings.

5.2 Sensitivity Analysis

Of course, one could argue that the obtained results depend on the parameter values chosen in our calibration. However, as can be seen below, for reasonable variations of parameters that are crucial, for instance, the intertemporal elasticity of substitution of labour or consumption there is no significant difference in the strength of the contract multiplier.[17] Furthermore, a variation in ζ, the elasticity of substitution of labour, or the presence of price adjustment costs do not lead to a noteworthy increase in output persistency.

In a first step we examine the effects of price adjustment costs, in a second one we vary ζ in an interval between 5 and 20. Basic intuition would conclude that the higher (lower) elasticity of substitution among differentiated labour skills leads to a smaller percentage change in wages and ceteris paribus to a lower (higher) increase of unit costs and therefore dampen the incentive of firms to increase their goods prices. Similarly, an increase of the adjustment costs lowers potentially the incentive of firms to adjust prices. However, our quantitative examination do not reveal a notable increase of persistence.[18]

Significant changes due to parameter variations can be observed in our model when a high interest rate elasticity of money demand as well as a high steady state ratio of money per consumption is assumed. In contrast to our baseline calibration we set $\nu = -0.4$, that gives an interest rate elasticity of -2.5, and furthermore, we reduce the parameter b to $b = 0.35$.[19] In this setup the households give higher weight

[17] See, for example, Blanchard and Fischer (1989), Ascari (2000), or Chari et al. (2000).

[18] See figures 10 to 12 in appendix A.

[19] The money demand function is implied by eqns. (26) and (27) and follows as

$$\ln(m_t) = \frac{1}{1-\nu} \ln\left(\frac{1-b}{b}\right) - \frac{1}{1-\nu} \ln\left(\frac{i}{1+i}\right) + \ln c_t.$$

to real money balances which results in higher money holdings in the steady state. Furthermore, the low value of b determines the magnitude a monetary injection has on the marginal rate of substitution between consumption and real cash balances. In particular, the effect of the higher interest rate elasticity is strengthened because of a sharp reduction of the rate of substitution, i.e. households now have a higher incentive to consume more than to hold money. As shown by figures 7 and 8 below, the magnitudes of output and the inflation responses are higher when we allow for a higher interest rate elasticity of money demand as well as a higher share of money in utility function. However, we observe a sharp decrease of output persistence (see figure 7), but no qualitative difference in the persistence of inflation (figure 8).

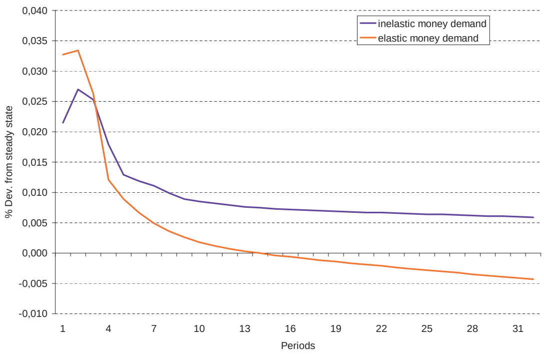

Figure 7: Output persistence

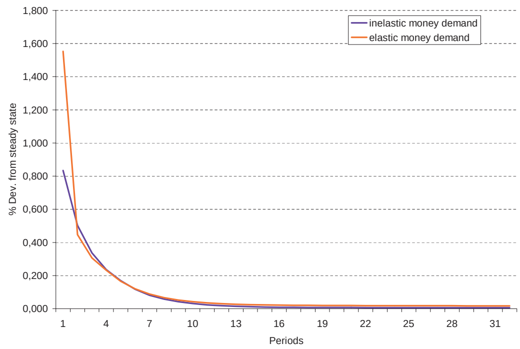

Figure 8: Inflation persistence

The higher response of the rental price of capital (figure 9) tempts the firms to adjust their goods prices. Therefore, the real wage converges rather fast to its steady value.

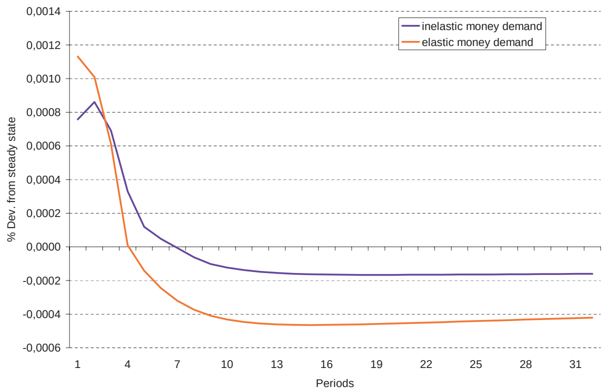

Figure 9: Interest Rate persistence

Given a setup in which a high interest rate elasticity of money demand is assumed which further goes at hand with a higher steady state share of money holdings per consumption (output) the observed degree of persistency of real variables decreases significantly.

6 Conclusion

In the recent macroeconomic debate, as pointed out by Chari et al. (2000), it is generally accepted that sticky price models alone are insufficient to generate persistent real effects of monetary shocks that are observed empirically. A greater degree of persistence is achieved, as shown by Andersen (1998) or Huang and Liu (1999), when staggered wage contracts are assumed. However, as, for example, noted by Edge (2002), the models offered by the latter strand of literature fail to reproduce, the extend of empirically observed persistence. In this line of research, Edge (2002) or Woodford (2003) have shown that a reasonable degree of persistency of real variables can be generated either by staggered wages or prices once one allows for specific factor market imperfections.[20]

Although we do not neglect the importance of factor market frictions, the question whether staggered wages alone are sufficient to generate persistent effects remained open. The present paper shows that staggered wage contracts are a necessary condition for reproducing persistent effects of real variables, whereby the the consideration of price adjustment costs as a source of price stickiness fails to improve the model's ability to generate reasonable responses. However, as shown in section 5.2, the assumption of staggered wage contracts has to be accompanied by a low rate of steady state money holdings as well as a sufficiently low interest rate elasticity of money demand.

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[20]Cf. Edge (2002):560 or Woodford (2003):230.

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A Sensitivity Analysis

The effects of price adjustment costs as well as variations in ζ are shown in figures 10 and 10 below:

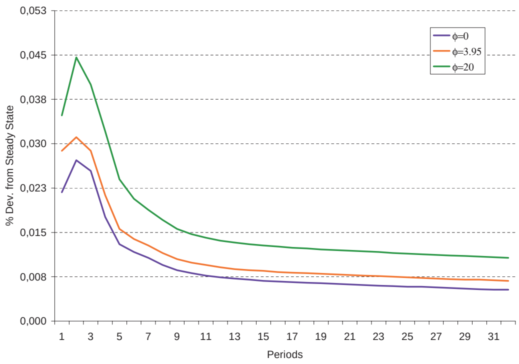

Figure 10: Variations in ϕ,

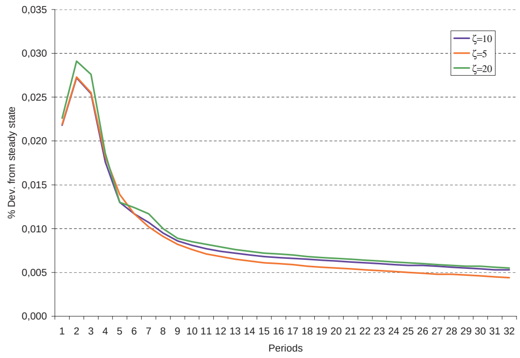

Figure 11: Variations in ζ, $\phi = 0$

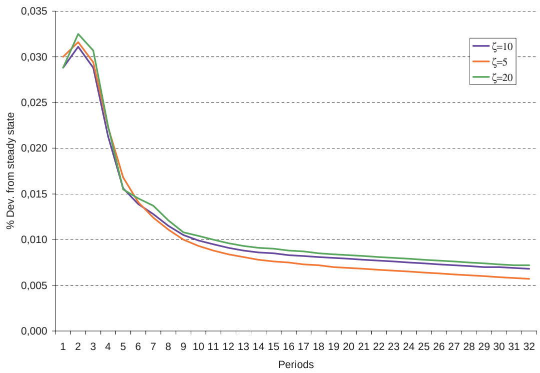

Figure 12: Variations in ζ, $\phi = 3.95$