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## Investors Facing Risk: Loss Aversion and Wealth Allocation Between Risky and Risk-Free Assets


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# Investors Facing Risk:

## Loss Aversion and Wealth Allocation Between Risky and Risk-Free Assets*

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**Abstract**

This paper studies the impact of loss aversion on decisions regarding the allocation of wealth between risky and risk-free assets. We use a Value-at-Risk portfolio model with endogenous desired risk levels that are individually determined in an extended prospect theory framework. This framework allows for the distinction between gains and losses with respect to a subjective reference point as in the original prospect theory, but also for the influence of past performance on the current perception of the risky portfolio value. We show how the portfolio evaluation frequency impacts investor decisions and attitudes when facing financial losses and analyze the role of past gains and losses in the current wealth allocation. The perceived portfolio value exhibits distinct evolutions in two frequency segments delimitated by what we consider to be the optimal evaluation horizon of one year. Our empirical results suggest that previous research relying on VaR underestimates the aversion of real individual investors to financial losses.

*Keywords*: prospect theory, loss aversion, capital allocation, Value-at-Risk, portfolio evaluation

*JEL Classification*: C32, C35, G10

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# 1 Introduction

## 1.1 Background

Optimal portfolio allocation models represent important tools in helping investors to decide upon how to split their wealth among assets. The goal of such models is to find what is called the optimal allocation, i.e. the one that maximizes expected portfolio returns at a given risk level. The most well known and broadly used portfolio optimization setting is the mean-variance model introduced in Markowitz (1952). The employed risk measure is the variance of portfolio returns. Recent research suggests other ways of quantifying market risk, such as the so called Value-at-Risk (VaR), defined as the highest expected loss from financial investments over a specified time horizon and subject to a certain confidence level.

Acting on the VaR-concept, Campbell, Huisman, and Koedijk (2001) develop a model for maximizing expected returns subject to both a budget and a desired-VaR constraint. The latter requires the maximum expected loss to meet an exogenously specified VaR-limit (the so called desired VaR, henceforth VaR*). One important result of this model is that the so-called "two-fund separation theorem" applies, as in the classical mean-variance framework. In other words, neither the investors' initial wealth nor the desired VaR* affect the maximization procedure under the VaR constraint. Thus, investors interested in allocating wealth among risky assets can first determine the risky portfolio composition, and then decide upon an extra amount of money to be borrowed or lent (i.e. invested in risk-free assets). The latter takes place according to the individual degree of risk aversion measured by the selected VaR*. In practice, the former decision is often made by professional portfolio managers in charge of the construction of an optimal risky portfolio for their clients. These clients, usually non-professional investors, concentrate on the second decision step by choosing the amount of money to be invested in the risky portfolio as a whole, and implicitly fixing the level of the risk-free investment.[1] In this context, the focus of our paper is on the decisions of non-professional investors.[2]

[1]In other words, non-professional investors consider the risky portfolio as exogenously given (fixed by the manager). They are exclusively concerned with determining the final position in risky vs. riskless assets (i.e. how much money to put in the risky portfolio as a whole, while the rest is allocated to risk-free assets), according to their own level of risk aversion. We assume that, at the beginning of trading, non-professional investors already hold well diversified portfolios such as a market index (i.e. the empirical part of our paper considers the SP500 index as proxy for the risky portfolio). Thus, the problem they actually face reduces to the allocation of wealth between this risky portfolio (as a whole) and the risk-free investment alternative.

[2]The same idea is in keeping with the claim in Markowitz (1952) that the portfolio selection process develops in two steps. The first one consists of forming beliefs with respect to the future performance of the potential portfolio components. In the second step, these beliefs underlie the choice of portfolio. As most academic research has addressed the latter step, we attempt to complete the picture offered in Markowitz (1952) and try to answer how (non-professional) investors form beliefs about their acceptable level of risk.

This separation of the risky and risk-free investments complies with the concept of "mental accounting", as first introduced in Thaler (1980). According to Thaler (1992), people manifest the tendency to frame (i.e. code and evaluate) outcomes in several non-fungible mental categories or accounts (such as accounts for current income, current wealth or future income) with different consumption propensities. This mental categorization is decisive for the perceived utility of those outcomes.

Our paper comes in line with further findings regarding the influence of behavioral aspects on financial decisions. The prospect theory (abbr. PT) developed in Kahneman and Tversky (1979) and Tversky and Kahneman (1992) stresses that investors perceive gains and losses differently with respect to a subjective reference and warily avoid losses (which is denoted as "loss aversion"). Barberis, Huang, and Santos (2001) apply the main concepts of the PT to asset pricing, showing that investors derive utility not only from consumption but also from variations in the perceived value of financial investments. Moreover, they enrich the PT-formulation claiming that perception of losses appears to be affected by previous portfolio performance, i.e. by gains and losses accumulated from past trades and referred to as "cushions". The idea that past gains and losses may change the current risk aversion, hence financial decisions, is supported by an empirically observed phenomenon denoted as the "house money effect" and documented in Thaler and Johnson (1990). Accordingly, subjects who made money in past gambles appear to behave less risk aversely in subsequent bets. In other words, past gains make future losses less painful, while prior losses may increase the risk aversion. A neurobiological explanation of this human reaction is provided by the "somatic marker hypothesis" in Damasio (1994). Accordingly, preexisting somatic (i.e. bodily) states can influence new ones by inducing modifications in the level of activation (threshold) of the new state. As suggested in Bechara and Damasio (2005), prior somatic states (in our case generated by past series of gains or losses) can reinforce (impede) the perception of new ones (here, currently expected gains and losses) by congruous (incongruous) valence (i.e. positivity/negativity). Also, even when prior performance induces only weak somatic states (known as background states), it appears to exert an impact on risk aversion. For instance, negative background states diminish the risk aversion in face of sure losses (because the fear of experience one more loss after a series of past losses is higher and makes investors more risk loving in the hope of recovering those losses), while positive background states enhance risk aversion in face of sure gains (i.e. once several gains are experienced, investor predisposition to gambling diminishes).

Benartzi and Thaler (1995) develop a plausible explanation for the equity premium puzzle that relies on the interaction between loss aversion and frequent portfolio evaluations, denoted as "myopic loss aversion" (abbr. mLA). Their findings support the idea that, when investors review the performance of their portfolios yearly, the resulting empirical equity premium is consistent with the loss aversion values estimated in the standard PT framework. The occurrence of mLA has received support from numerous direct experimental tests, such as Thaler, Tversky, Kahneman, and Schwartz (1997a), Gneezy and Potters (1997), Gneezy, Kapteyn, and Potters (2003), or Haigh and List (2005). According to Barberis and Huang (2004a), myopia refers strictly to annual evaluations of gains and losses, hence the term of *narrow framing* is better suited to describing the underlying phenomenon. In a financial context, narrow framing illustrates the isolate evaluation of stock market risk (i.e. unrelated to overall wealth risk). As underlined in Barberis and Huang (2004b), this isolated evaluation entails an underestimation of the stock desirability, even though, viewed in a wide utility-risk frame, they represent a good diversification modality. Also, narrow framing can be interpreted as a consequence of regret at not having taken another decision (non-consumption utility explanation).[3] Another explanation relies on the (higher) accessibility of (financial) information that justifies its over-important role in final decisions. As referred to in Kahneman (2003), the easily accessible information is very appealing for the intuitive (i.e. spontaneous, effortless) way in which people use to make decisions. Our work draws upon the latter motivation, namely accessibility. We consider it as better suited to financial decisions because nowadays investors are exposed to a tremendously high quantity of financial information and need to make decisions in a fast changing financial environment. Consequently, they tend to perform more frequent checks on their investments.

## 1.2 Overview

Our model builds on the portfolio optimization setting with exogenous desired VaR* presented in Campbell, Huisman, and Koedijk (2001). We extend it by explicitly accounting for the formation of the individual VaR*-levels. These levels rely on the subjective perception of (non-professional) investors of the risky portfolio performance and of utility in general, that we formulate in line with PT. In other words, we analyze how non-professional investors set their subjective VaR* and how this (now endogenous) VaR* impacts on the wealth allocation between risk-free assets and the risky portfolio.[4]

[3] Clearly, this can be also related to the theory of cognitive dissonance introduced in Festinger (1957). Cognitive dissonance arises from the incompatibility of two cognitions that creates inner tension. It can exert a strong influence on decision making, being the source of several basic decision heuristics, such as representativeness, availability, and hindsight bias, as noted in Plous (1993). Specifically, the regret at not having chosen another alternative creates post-decisional dissonance.

[4] We consider the mean-VaR optimization framework better suited to combination with the prospect theory than the classic mean-variance approach. The reason is that the variance represents a symmetric measure of risk and hence equally accounts for (high) gains and losses, while VaR refers only to the left tail of the return distribution that corresponds to losses, as underlined in Krokhmal, Palmquist, and Uryasev (2001). Thus, the distinct subjective perception of gains and losses captured by the prospect

The first and most important decision of non-professional investors that we analyze refers to the formulation of VaR*. The value of VaR* determines the optimal portfolio composition and the sum of money to be invested in risk-free assets. As mentioned above, finding the optimal risky portfolio usually represents the task of professional portfolio managers and was extensively studied in previous research on portfolio optimization. However, the resulting values of the total risky vs. the risk-free investment as percentages of total wealth, directly concern non-professional investors. Thus, they become an object of study in the present work. In our setting, the desired Value-a-Risk (VaR*) is endogenously defined as the maximum expected loss perceived by individual investors and depends on past performance, loss aversion, current value of the risky investment and the expected return premium. We first compute the VaR* and then derive the desired level of investment in the risky portfolio relative to the risk-free allocation. This allows us to draw a conclusion about the investor risk aversion and to provide a comparison with the exogenous-VaR* setting in Campbell, Huisman, and Koedijk (2001).

The endogenous VaR* relies on the subjective perception of the value generated by one unit of risky project. In Kahneman and Tversky's (1979, 1992) PT, this value is captured by the so-called value function. Yet, following Barberis, Huang, and Santos (2001), we reconsider the original PT-definition of the value function in order to account for the the idea that individual risk perception is affected by the previous evolution of financial wealth. Specifically, past performances of the risky investment result in monetary cushions and the value function is assumed to be linear in both the gain and the loss domain, but steeper in the latter. Thus, facing past gains (losses) induces a more (less) aggressive behavior, hence an increase (decrease) in risky portfolio holdings. We present evidence for how different investment decisions of individual investors can be interpreted as a consequence of different financial performance histories, how these decisions change subject to the individual degree of loss aversion, and how our results conform with previously documented findings.

In our model, investors find the optimal solution to their decision problem by maximizing subjectively perceived utility. This utility is assumed to be derived merely from changes in financial wealth.[5] In line with the PT, the perceived value of risky investments is denoted as the prospective value. We design two ways of assessing the prospective value (which reduces in our setting to the expected value of the risky investment). One definition relies on PT, and another one answers what we call a "worst case scenario", where investors are assumed to be concerned with the maximum sustainable (and not

theory may be even more important when VaR is used as measure of risk.

[5]In other words, investors are interested only in the (perceived) value of their financial investments (and not in other determinants of utility, such as consumption). This could be due to the fact that investors narrowly frame, i.e. put excessive emphasis on the importance of financial investments and the utility they generate. According to Barberis, Huang, and Thaler (2003), this is a common situation in practice.

with the expected) loss in the risky investment. Moreover, we study how investor decisions change according to different market conditions, as captured by the PT-part of the utility function, and how different ways of representing loss aversion can influence utility. For instance, we expect that risk-averse investors reduce their risky holdings, shifting their positions to more secure investment alternatives. In addition, we consider two further measures that are in our view better suited to measuring the real investor attitude towards financial losses than the simple coefficient of loss aversion. Namely, we calculate the loss aversion index according to Köbberling and Wakker (2005) and introduce its counterpart in terms of the prospective value denoting it as global first-order risk aversion. In addition to the loss aversion coefficient, the first measure captures the influence of past losses and gains, while the second encompasses the expectations about future market conditions, which are aspects of practical importance for the non-professional financial decisions.

Acting on the mLA in Benartzi and Thaler (1995), we further study how investment decisions change under different portfolio evaluation horizons (such as one day, one month, two months up to one year, then two up to eight years). In other words, we investigate how the evaluation frequency exerts influence on the risk perception and wealth allocation. In this context, we estimate the evolution of the prospective value as well as of our two further measures of the investor attitude towards financial losses (i.e. the loss aversion index and the global first-order risk aversion), as functions of the evaluation frequency. Moreover, we address the problem of optimal evaluation horizons. Finally, we derive equivalent significance levels for VaR* at each trading time and compare them to the corresponding significance levels used in the original model of Campbell, Huisman, and Koedijk (2001).

The theoretical findings from the first part of our paper are implemented and amended in the subsequent empirical part. We rely on real market data between 1982-2006, such as the SP500 index as proxy for the risky portfolio and the US 10-year bond accounting for the risk-free investment alternative. Also, we analyze various specifications for the distribution of expected returns, cushions, and model parameters (such as the coefficient of loss aversion and the sensitivity to past losses). Our empirical findings lead to several interesting conclusions. First, the risky holdings of non-professional investors substantially vary subject to the portfolio evaluation frequency and to the horizon over which cushions accumulate. Thus, investors performing annual portfolio evaluations invest on average between $26 - 50\%$ of their wealth in risky assets, depending on the type of cushion (myopic or cumulative) and on the expected return distribution. These percentages decrease to values under 2% for the evaluation horizon of one day. Second, the cushions generated by past portfolio performance appear to drive the current perception of the risky prospect. Thus, when investors are unable to accumulate positive significant cushions, most of their wealth is directed to the risk-free investment. In other words, even when the coefficient of loss aversion remains constant over trading dates, financial wealth fluctuations determined by the success of previous decisions play a key role in the current portfolio allocation. Third, the creation of positive and significant cushions is inversely related to the portfolio evaluation frequency. As this frequency increases, the ability to accumulate profits decreases and a lower wealth portion will be invested in risky assets. Fourth, our results support the idea that one year appears to be the most plausible evaluation frequency used by non-professional loss-averse investors in practice, as suggested in Benartzi and Thaler (1995). Further estimations show that the evolution of the perceived portfolio utility (i.e. of the prospective value) for different evaluation frequencies can be decomposed into two distinct intervals, namely one for high evaluation frequencies (below one year), and a second one for low frequencies (above one year). The prospective value on the first interval can be analytically represented as a third-order polynomial. It increases subject to higher evaluation horizons, specifically at enhanced speed for horizons at the extreme quarters of the left one year interval. On the second segment, the prospective value appears to be upward-sloping and of the fourth degree. A similar segmentation can be observed for both measures of the actual investor attitude towards financial losses, namely the loss aversion index and the global first-order risk aversion. Their evolutions can be described equally well by third-degree polynomials for evaluation frequencies higher than one year, while for lower frequencies the global first-order risk aversion is approximatively linear. We argue that these two further measures of the loss attitude provide additional information on the investor sensitivity to financial losses subject to different performance histories, which can be of help in isolating practically relevant parameter values. Fifth, the VaR*-levels assessed within our setting on the basis of real market data point out that, in practice, the risk aversion of real non-professional investors may be higher than the values obtained for confidence levels commonly considered in previous theoretical papers, such as 90%, 95% and 99%. Finally, the average equivalent coefficients of loss aversion computed for fixed confidence levels of 99% and 90% lie far below the widely documented and empirically supported value of 2.25. Again, this implies that previous research considering these confidence levels underestimates the aversion to losses manifest in practice. The average coefficient of loss aversion lies around one, which implies that under exogenous VaR* constraints investors treat gains and losses in the same way.

The remainder of the paper is organized as follows. Section 2 presents the main theoretical considerations. We start with a brief review of the optimal portfolio selection model with exogenous VaR* as in Campbell, Huisman, and Koedijk (2001), on which we build our own theoretical structure. Section 2.2 takes on the reformulation of the value functions in Barberis, Huang, and Santos (2001), out of which we derive the the loss aversion index. In Section 2.3 we introduce the notions of VaR* and propose different ways of quantifying the endogenous VaR*. The subsequent Section 2.4 frames distinct ways of assessing the value of the risky portfolio as perceived by individual investors and adopts the notion of global first-order risk aversion. Section 2.5 treats the influence of variable portfolio evaluation frequencies on the prospective value and on our additional measures of the investor attitude towards financial losses. Section 3 illustrates the empirical implementation of our theoretical model. In particular, Section 3.1 discusses the impact of the evaluation frequency and of the cushion on the evolution of wealth percentages invested in the risky portfolio. In Section 3.2, we analyze the evolution of the prospective utility in time and in the evaluation frequency domain, an investigation that is replicated in Section 3.3 for the index of loss aversion and the global first-order risk aversion, subject to different revision frequencies. Finally, Section 3.4 restates our model in terms of previous research with exogenous VaR*, where equivalent significance levels of portfolio risk and of the loss aversion coefficient, that result from the average VaR* computed from our data and according to our model equations, are inferred. Section 4 summarizes the results and concludes. Graphics and further results are included in the Appendix.

# 2 Theoretical model

This section contains the main theoretical considerations of our work. We start by presenting the model of portfolio selection with VaR as the risk measure and an exogenous desired risk aversion (VaR*) of Campbell, Huisman, and Koedijk (2001). This model has motivated us to extend the analysis for the case with endogenous VaR*. Subsequently, we formulate our own setting by referring to the individual perception of risky projects and detailing the construction of the endogenous measure of risk aversion VaR* and its implications for individual investor decisions. More precisely, we show how the investor-desired VaR* can be formulated and how it flows into the prospective value of the risky investment that investors aim at maximizing. We also enrich the definition of the real investor attitude towards losses by first calculating the loss aversion index and next the so-called global first-order risk aversion. Moreover, we analyze how the prospective value and these two additional risk attitude measures vary subject to different portfolio evaluation frequencies.

## 2.1 Optimal portfolio selection with exogenous VaR*

Let us first refresh the portfolio selection model with exogenous VaR* introduced in Campbell, Huisman, and Koedijk (2001). Accordingly, financial assets are allocated by maximizing the expected return subject to the common budget constraint, as well as to an additional risk constraint, where risk is measured by the so-called Value-at-Risk (VaR). The optimal portfolio is derived such that the maximum expected loss does not exceed the VaR*-level indicated by non-professional investors. This VaR* represents the maximum acceptable loss for a chosen investment horizon and at a given confidence level. Additionally, investors can borrow or lend money at the fixed market interest rate.

We denote by $W_t$ the investor wealth at time $t$, by $B_t$ the amount of money to borrow ($B_t > 0$) or to lend ($B_t < 0$) at the fixed risk-free gross return rate $R_f$, and by VaR* the individually desired VaR (specified later in this section). Let the risky portfolio consist of $i = 1, \dots n$ financial assets with single time $t$ prices $p_{i,t}$ and define the set of portfolio weights at time $t$ as $[w_t \in R^n : \sum_{i=1}^{n} w_{i,t} = 1]$. Moreover, $x_{i,t} = w_{i,t}(W_t + B_t)/p_{i,t}$ represents the number of shares of the asset $i$ contained in the portfolio at time $t$. Obviously, the portfolio gross return at next trade ($R_{t+1}$) depends on the portfolio composition at the current date $w_t$. With the budget constraint:

$$W_t + B_t = \sum_{i=1}^{n} x_{i,t} p_{i,t} = x_t' p_t, \tag{2.1}$$

the value of the portfolio at $t + 1$ results in:

$$W_{t+1}(w_t) = (W_t + B_t) R_{t+1}(w_t) - B_t R_f. \tag{2.2}$$

As the investor desired-VaR (VaR*) is defined as the maximum expected loss over a given investment horizon and for a given confidence level 1-$\alpha$[6], we can write:

$$P_t[W_{t+1}(w_t) \leq W_t - \text{VaR}^*] \leq 1 - \alpha, \tag{2.3}$$

where $P_t$ is the conditional probability on the available information at time $t$. Equation (2.3) represents the risk constraint that (professional) investors have to take into account in addition to the budget constraint (2.1) when searching for optimal portfolio weights.

The portfolio optimization problem can be now expressed in terms of the maximization of expected portfolio returns $E_t[W_{t+1}(w_t)]$, subject to both the budget restriction and the VaR*-constraint:

$$w_t^* \equiv \arg\max_{w_t} \{(W_t + B_t) E_t[R_{t+1}(w_t)] - B_t R_f\}, \text{ s.t. (2.1) and (2.3).} \tag{2.4}$$

Here, $E_t[R_{t+1}(w_t)]$ represents the expected return of the portfolio given the information at time $t$.

[6] Note that VaR* is considered as the loss in absolute value, being hence positive.

The optimization problem can be rewritten in an unconstrained way, by replacing (2.1) in (2.2) and taking expectations:

$$E_t[W_{t+1}(w_t)] = x_t' p_t (E_t[R_{t+1}(w_t)] - R_f) + W_t R_f. \tag{2.5}$$

Equation (2.5) points out that risk-averse investors are going to put a fraction of their wealth in risky assets if the expected risky portfolio return is higher than the risk-free rate $E_t[R_{t+1}(w_t)] \geq R_f$.

Substituting (2.5) (before taking expectation) in (2.3) gives:

$$P[x_t' p_t (R_{t+1}(w_t) - R_f) + W_t R_f \leq W_t - \text{VaR}^*] \leq 1 - \alpha,$$

so that

$$P\left[R_{t+1}(w_t) \leq R_f - \frac{\text{VaR}^* + W_t(R_f - 1)}{x_t' p_t}\right] \leq 1 - \alpha \tag{2.6}$$

defines the quantile $q_t(w_t, \alpha)$ of the distribution of portfolio returns for a given confidence level $1 - \alpha$ (or probability of occurrence $\alpha$).

Thus, the budget constraint can be restated as:

$$x_t' p_t = \frac{\text{VaR}^* + W_t(R_f - 1)}{R_f - q_t(w_t, \alpha)}. \tag{2.7}$$

Finally, substituting (2.7) in (2.5) and dividing by the initial wealth $W_t$, we obtain a new expression to be maximized:

$$\frac{E_t[W_{t+1}(w_t)]}{W_t} = \frac{\text{VaR}^* + W_t(R_f - 1)}{W_t R_f - W_t q_t(w_t, \alpha)}(E_t[R_{t+1}(w_t)] - R_f) + R_f. \tag{2.8}$$

Given that at moment $t$ of maximization, $W_t$ is known and $R_f$ is fixed, the optimal portfolio composition can be derived as:

$$w_t^* \equiv \arg\max_{w_t} \frac{E_t[R_{t+1}(w_t)] - R_f}{W_t R_f - W_t q_t(w_t, \alpha)}. \tag{2.9}$$

Equation (2.9) shows that, similarly to the traditional mean-variance framework, the two-fund separation theorem applies, i.e. neither the (non-professional) investor's initial wealth nor the desired VaR* affect the maximization procedure. In other words, investors can first allocate wealth inside the risky portfolio (i.e. among different risky assets) and second fix the extra amount money to be borrowed or lent (i.e. invested in risk-free assets). The latter reflects by how much the portfolio VaR varies according to the investor degree of risk aversion, which is measured by the selected (desired) VaR* level. Replacing (2.1) in (2.7), we further derive:[7]

$$B_t = \frac{\text{VaR}^* + \text{VaR}_t}{R_f - q_t(w_t^*, \alpha)} \tag{2.10a}$$

$$\text{VaR}_t = W_t[q_t(w_t^*, \alpha) - 1]. \tag{2.10b}$$

Thus, the desired VaR* is imposed by the client *prior* to the portfolio formation and enters the portfolio optimization problem in form of a constraint. By contrast, the portfolio VaR is an *output* of this optimization and measures the actual maximum loss that can be incurred at time $t$ at the confidence level $1 - \alpha$ for the obtained optimal portfolio $w^*$.

## 2.2 The value function

Coming from the main ideas of the Campbell, Huisman, and Koedijk (2001) setting, our model goes a step further by asking how individual investors set their desired level of risk aversion VaR*. We elaborate on the construction of an endogenous VaR* and its implications for the wealth allocation between risky and the risk-free assets.

Investor desires depend on their perception of the value of financial investments. PT suggests how individual perceptions of financial performance can be formalized by means of the so-called *value function*. According to Kahneman and Tversky (1979) and Tversky and Kahneman (1992), human minds take for actual carriers of value not the absolute outcomes of a project, but their changes defined as departures from an individual reference point. The deviations above (below) this reference are labelled as gains (losses). Thus, the value function is kinked at the reference point and exhibits distinct evolution in the domains of gains and losses, i.e. it is steeper for losses (a property known as *loss aversion*). Also, it shows diminishing sensitivity in both domains (namely, it is concave for gains but convex for losses).

As noted in Barberis, Huang, and Santos (2001), the view of the original PT over individual perceptions of risky investments can be enriched by accounting for the potential impact of past performance (i.e. in addition to the mental distinction between gains and losses). Accordingly, the value function additionally reflects the influence of a so-called *cushion*, defined as the difference between the current value of the risky investment $S_t$ and a benchmark level from the past $Z_t$ (e.g. the purchasing price of the stock). When this difference is positive, investors made money from past risky investment, otherwise they accumulated losses.

[7] The expression $q_t(w_t^*, \alpha) - 1$ in Equation (2.10b) should be viewed as the quantile of the net returns $R_t - 1$ and corresponds to the quantile $q_t(w_t^*, \alpha)$ of the gross returns $R_t$. Equation (2.10a) can be restated in terms of net returns as: $B_t = (\text{VaR}^* + \text{VaR})/[(R_f - 1) - (q_t(w_t^*, \alpha) - 1)]$.

Our approach relies on the formulation of the value function proposed in Barberis, Huang, and Santos (2001). In their Equations (15) and (16), the reference point changes with the past performance (from $z_t R_{ft}$ for $z_t \leq 1$ to $R_{ft}$ for $z_t > 1$, where $z_t = Z_t/S_t$). We restate these definitions, in order to obtain identical reference points and similar courses in the loss domain for both considered cases with positive and negative cushions, as in the original PT formulation, where gains are defined as the difference between the value function argument (here $R_{t+1}$) and the reference point. Thus, we fix the reference value in both cases (with prior gains $z_t \leq 1$ and prior losses $z_t > 1$) to $R_{ft}$ and rearrange the terms in Equations (15) and (16) in Barberis, Huang, and Santos (2001), obtaining:

$$v_{t+1} = \begin{cases} S_t(R_{t+1} - R_{ft}) & \text{, for } R_{t+1} \geq R_{ft} \\ \lambda S_t(R_{t+1} - R_{ft}) + (\lambda - 1)(S_t - Z_t)R_{ft} & \text{, for } R_{t+1} < R_{ft} \end{cases} \quad \text{, for } z_t \leq 1 (\Leftrightarrow Z_t \leq S_t), \tag{2.11}$$

and

$$v_{t+1} = \begin{cases} S_t(R_{t+1} - R_{ft}) & \text{, for } R_{t+1} \geq R_{ft} \\ \lambda S_t(R_{t+1} - R_{ft}) + k(Z_t - S_t)(R_{t+1} - R_{ft}) & \text{, for } R_{t+1} < R_{ft} \end{cases} \quad \text{, for } z_t > 1 (\Leftrightarrow Z_t > S_t). \tag{2.12}$$

Here, $\lambda$ is denoted as the coefficient of loss aversion and the parameter $k > 0$ captures the influence of previous losses on the perception of current ones (i.e. the larger the previous loss is, the more painful the next losses become). We observe that, while the gain branches of both value functions are invariable to past performance $z_t$, the loss branches contain a first term that resembles the original PT, i.e. $\lambda S_t(R_{t+1} - R_{ft})$, but also a second one revealing the impact of the cushion $S_t - Z_t$. Moreover, the time $t+1$-value of the risky investment is derived as:

$$S_{t+1} = (W_t + B_t)R_{t+1}. \tag{2.13}$$

Of note is also the fact that the joint impact of the loss aversion coefficient $\lambda$ and of past losses $k$ changes the actual investor aversion to losses. This can be easily deduced by merging Equations (2.11) and (2.12) to:

$$v_{t+1} = \begin{cases} S_t x_{t+1} & \text{, for } x_{t+1} \geq 0 \\ [\lambda S_t - (1 - \pi_t)k(S_t - Z_t)]x_{t+1} + \pi_t(\lambda - 1)R_{ft}(S_t - Z_t) & \text{, for } x_{t+1} < 0, \end{cases}$$

where $\pi_t = P_t(z_t \leq 1)$ is the probability of experiencing past gains and $x_{t+1} = R_{t+1} - R_{ft}$ the equity return premium. Obviously, the loss branch of the above Equation (2.2) is more complex than the simple multiple of the gain branch with the loss aversion coefficient, as suggested by the PT. In order to capture this complexity, we follow the definition in Köbberling and Wakker (2005) (p. 121) and derive the *index of loss aversion* (abbr. LAi) as the ratio of the left and right derivatives of the value function at the reference point. In our case:

$$\frac{\partial v_{t+1}}{\partial x_{t+1}} = \begin{cases} S_t & \text{, for } x_{t+1} \geq 0 \\ \lambda S_t - (1-\pi_t)k(S_t - Z_t) & \text{, for } x_{t+1} < 0, \end{cases}$$

hence we can express the LAi as:

$$\tilde{\lambda}_t = \frac{\lambda S_t - (1-\pi_t)k(S_t - Z_t)}{S_t} = \lambda - (1-\pi_t)k(1-z_t). \tag{2.14}$$

Clearly, LAi contains more information than the simple loss aversion coefficient $\lambda$ introduced in the PT: The series of past gains (losses), i.e. $z_t \leq 1$ ($z_t > 1$), lower (increase) the actual investor aversion to losses, because they are more (less) confident in being able to cover prospective losses by past gains. Also, an increased sensitivity to past losses, i.e. a higher $k$, in the case with negative past performance $z_t > 1$ yields higher LAi-values. LAi is to be interpreted analogously to the simple coefficient of loss aversion, so that higher values point to an enhanced aversion towards financial losses. Henceforth, in line with the original PT formulation, we mostly refer with "loss aversion" to the loss aversion coefficient $\lambda$ and with "actual investor attitude towards financial losses" to the LAi (and the gRA from Section 2.4).

## 2.3 The endogenous VaR*

Our first goal is to formulate the maximum loss a-priori expected by individual investors, i.e. the *individual desired* VaR*. This value will subsequently enter the optimization problem and serve non-professional investors to decide between borrowing or lending.

To this end, we start from the literal definition of VaR* as viewed by non-experts, concentrating on the notions of "maximum", "loss", and "individual". First, VaR* quantifies losses. However, according to the PT, what actually counts for individual investors is not the absolute magnitude of a loss, but rather the subjectively perceived one, as captured by the value function. Hence, VaR* should rely on the subjective value (or utility) of losses expressed in the loss branches of the value functions (2.11) and (2.12). It depends on individual investor characteristics (originating in the subjective view over gains and losses) and can vary over time. Second, VaR* should represent a (subjective) expectation because the next period returns $R_{t+1}$, on which the evaluation of risky investments relies, are still unknown at the decision time $t$. Third, we are looking for a maximal value such that in calculating VaR* investors must ascribe a maximal occurrence probability $P_t(E_t[R_{t+1}] < R_{ft}) = 1$ to the losses in the value function.

Therefore, we propose that VaR* accounts for the maximum expectation of sustainable losses as resulting from individual valuations of the risky investment. However, we consider investors to be sophisticated enough in order to consider that not only the mean, but also the variation of prospective losses should be considered in order to accurately ascertain the maximum acceptable loss level. Thus, in a second approximation, we extend the VaR*-definition by adjusting for the loss variance.

Henceforth, we consider that value functions are weighted by the pure probabilities of occurrence (and not by non-linear probability functions as stated in the cumulative PT of Tversky and Kahneman (1992)). According to Equations (2.11) and (2.12), we then derive:

$$\begin{aligned} E_t[\text{loss-utility}_{t+1}] &= \pi_t[\lambda S_t(E_t[R_{t+1}] - R_{ft}) + (\lambda - 1)(S_t - Z_t)R_{ft}] \\ &+ (1 - \pi_t)[\lambda S_t(E_t[R_{t+1}] - R_{ft}) + k(Z_t - S_t)(E_t[R_{t+1}] - R_{ft})] \\ &= \lambda S_t(E_t[R_{t+1}] - R_{ft}) + [\pi_t(\lambda - 1)R_{ft} - (1 - \pi_t)k(E_t[R_{t+1}] - R_{ft})](S_t - Z_t) \end{aligned} \tag{2.15a}$$

$$\begin{aligned} Var_t[\text{loss-utility}_{t+1}] &= E_t[\text{loss-utility}^2_{t+1}] - E^2_t[\text{loss-utility}_{t+1}] \\ &= \pi_t[\lambda S_t(E_t[R_{t+1}] - R_{ft}) + (\lambda - 1)(S_t - Z_t)R_{ft}]^2 \\ &+ (1 - \pi_t)[\lambda S_t(E_t[R_{t+1}] - R_{ft}) + k(Z_t - S_t)(E_t[R_{t+1}] - R_{ft})]^2 \\ &- E^2_t[\text{loss-utility}_{t+1}] \\ &= [\lambda S_t(E_t[R_{t+1}] - R_{ft})]^2 \\ &+ [\pi_t(\lambda - 1)^2 R^2_{ft} - (1 - \pi_t)k^2(E_t[R_{t+1}] - R_{ft})^2](S_t - Z_t)^2 \\ &+ 2[\pi_t(\lambda - 1)R_{ft} - (1 - \pi_t)k(E_t[R_{t+1}] - R_{ft})]\lambda S_t(E_t[R_{t+1}] - R_{ft})(S_t - Z_t) \\ &- [\lambda S_t(E_t[R_{t+1}] - R_{ft})]^2 \\ &- [\pi_t(\lambda - 1)R_{ft} - (1 - \pi_t)k(E_t[R_{t+1}] - R_{ft})]^2(S_t - Z_t)^2 \\ &- 2[\pi_t(\lambda - 1)R_{ft} - (1 - \pi_t)k(E_t[R_{t+1}] - R_{ft})]\lambda S_t(E_t[R_{t+1}] - R_{ft})(S_t - Z_t) \\ &= \pi_t(1 - \pi_t)[(\lambda - 1)R_{ft} + k(E_t[R_{t+1}] - R_{ft})]^2(S_t - Z_t)^2. \end{aligned} \tag{2.15b}$$

Note that while the first term of the expected losses (2.15a) is similar to the loss-formulation in the PT, the remaining terms point out the influence of the cushion accumulated over past trades. In contrast, the variance of losses (2.15b) is exclusively dictated by the cushion-part, as individually perceived by investors, and depends on the probability of having made gains or losses in the past, on the variance of expected returns with respect to the reference risk-free rate, and on the squared cushion.

As mentioned above, in a first approximation we stick to the literal definition of VaR* as an *expectation* and design VaR* as the maximum expected loss:

$$\begin{aligned}\text{VaR}^{*1}_{t+1} &= E_t[\text{loss-utility}_{t+1}] \\ &= \lambda S_t(E_t[R_{t+1}] - R_{ft}) + [\pi_t(\lambda - 1)R_{ft} - (1-\pi_t)k(E_t[R_{t+1}] - R_{ft})](S_t - Z_t).\end{aligned} \tag{2.16}$$

However, investors may consider loss-variance as an equally important parameter for determining the maximal sustainable loss. Then, assuming that VaR* follows a certain distribution (i.e. normal or Student-t)[8] with the value $\varphi$, we introduce the second (variance-adjusted) VaR* definition:[9]

$$\text{VaR}^{*}_{t+1} = E_t[\text{loss-utility}_{t+1}] - \varphi\sqrt{Var_t[\text{loss-utility}_{t+1}]}, \tag{2.17}$$

which, according to Equations (2.15a) and (2.15b), results in:

$$\begin{aligned}\text{VaR}^{*}_{t+1} &= \lambda S_t(E_t[R_{t+1}] - R_{ft}) \\ &+ [(\pi_t - \varphi\sqrt{\pi_t(1-\pi_t)})(\lambda - 1)R_{ft} - (1 - \pi_t + \varphi\sqrt{\pi_t(1-\pi_t)})k(E_t[R_{t+1}] - R_{ft})](S_t - Z_t).\end{aligned} \tag{2.18}$$

Again, expression (2.18) encompasses the twofold loss effect stemming from the loss aversion coefficient of the original PT and from the cushion of the extended PT introduced in Barberis, Huang, and Santos (2001).

It is worth noting that, for sure gains (i.e. when $\pi_t = P_t(z_t \leq 1) = 1$), both VaR* expressions (2.16) and (2.18) reach a common upper bound:

$$\text{VaR}^{*1,\text{up}}_{t+1} = \text{VaR}^{*\text{up}}_{t} = \lambda S_t(E_t[R_{t+1}] - R_{ft}) + (\lambda - 1)R_{ft}(S_t - Z_t), \tag{2.19}$$

while for sure losses (i.e. when $\pi_t = P_t(z_t \leq 1) = 0$), the lowest value of:

$$\text{VaR}^{*1,\text{lo}}_{t+1} = \text{VaR}^{*\text{lo}}_{t} = \lambda S_t(E_t[R_{t+1}] - R_{ft}) - k(E_t[R_{t+1}] - R_{ft})(S_t - Z_t) \tag{2.20}$$

is attained.

The definition of VaR* serves to determine the optimal level of borrowing or lending ($B_t$) from Equation (2.10a). When VaR* lies "to the left" of the portfolio VaR (i.e. it is lower in absolute value than VaR), $B_t$ is negative, hence investors become more risk averse

[8] Although VaR is a very popular measure of risk, it has been criticized because it does not satisfy one of the four properties for coherent risk measure, namely subadditivity (see Artzner, Delbaen, Eber, and Heath (1999), Rockafellar and Uryasev (2000) and Szegö (2002)). However, according to Embrechts, McNeil, and Straumann (1999), VaR becomes subadditive and can be considered as a coherent risk measure, if used in the case of elliptic joint distributions, such as normal and Student-t with finite variance.

[9] Equation (2.17) results from the assumption that:
$(\text{VaR}^{*}_{t+1} - E_t[\text{loss-utility}_{t+1}])/(\sqrt{Var_t[\text{loss-utility}_{t+1}]}) = \varphi \sim N(0,1)$ or $t(5)$.

and save money. By contrast, for a VaR* higher than VaR in absolute value, investors augment their risky investment by borrowing extra money. Thus, an empirical analysis of the evolution of $B_t$ (as conducted in Section 3) can shed some light on the investor risk behavior.

Also, one interesting topic to investigate lies in estimating the equivalent loss aversion parameter $\lambda_t^{\bar{*}}$ that can be obtained for a $\text{VaR}_{t+1}^* = \overline{\text{VaR}^*}$ fixed for commonly used significance levels such as 1%, 5% or 10%. The result is immediate from Definition (2.18):

$$\lambda_{t+1}^{\bar{*}} = \frac{\overline{\text{VaR}^*} + [(\pi_t - v\sqrt{\pi_t(1-\pi_t)})R_{ft} + (1 - \pi_t + v\sqrt{\pi_t(1-\pi_t)})k(E_t[R_{t+1}] - R_{ft})](S_t - Z_t)}{S_t(E_t[R_{t+1}] - R_{ft}) + (\pi_t - v\sqrt{\pi_t(1-\pi_t)})R_{ft}(S_t - Z_t)}. \quad (2.21)$$

Moreover, since $\lambda_{t+1}^{\bar{*}}$ depends on the fixed (i.e. exogenous) $\overline{\text{VaR}^*}$, there should exist no further causal relationship between past and future losses, such that we can set $k = 0$. Accordingly, Equation (2.21) becomes:

$$\lambda_{t+1}^{\bar{*}} = \frac{\overline{\text{VaR}^*} + [(\pi_t - v\sqrt{\pi_t(1-\pi_t)})R_{ft}](S_t - Z_t)}{S_t(E_t[R_{t+1}] - R_{ft}) + (\pi_t - v\sqrt{\pi_t(1-\pi_t)})R_{ft}(S_t - Z_t)}. \quad (2.22)$$

## 2.4 The prospective value of the risky investment

The estimation of the maximum acceptable individual loss level represents only the first step in our analysis. As shown in Section 2.1, one of its consequences with direct impact on non-professional investors resides in the determination of the optimal borrowing level. This results as a byproduct of the optimization inside the risky portfolio undertaken by the professional manager. For the non-professional client it amounts to the optimal choice in terms of wealth percentages allocated between risky and riskless assets.

When investors decide on the optimal sum of money to be put in the risky portfolio (or equivalently in risk-free assets), they might not exclusively think in terms of VaR*, but sooner aim at maximizing the utility generated by their financial investments. This utility is encompassed in the PT by the so called *prospective value* of the risky investment $V_{t+1}$.[10] Denoting the expected equity return premium by $E_t[x_{t+1}] = E_t[R_{t+1}] - R_{ft}$ and the probability of a positive premium by $\omega_t = P_t(E_t[R_{t+1}] \geq R_{ft}) = P_t(E_t[x_{t+1}] \geq 0)$, the prospective value of the risky investment can be formulated as:

$$V_{t+1} = [\omega_t + (1-\omega_t)\lambda]S_t E_t[x_{t+1}] + (1-\omega_t)\{\pi_t(\lambda-1)R_{ft} - (1-\pi_t)kE_t[x_{t+1}]\}(S_t - Z_t). \quad (2.23)$$

Furthermore, we resolve to analyze the evolution of the prospective value for different portfolio evaluation frequencies, on the grounds that revising portfolio performance at

[10] Remember that our investors are not concerned with consumption and derive utility merely from financial wealth fluctuations.

different time intervals implies drawing back on distinct return values, hence on different return premia. This implicitly changes the values of several model parameters such as $S_t$, $Z_t$, $\pi_t$, or $\omega_t$ affecting the prospective value (2.23), a topic detailed in Section 2.5.

Yet, in practice, risk-averse investors may rely on a slightly different method for evaluating expected values of risky prospects. For instance, they may continue to consider gains as unsure events and account for them as "wishes" (i.e. expectations). However, losses would be assessed at their maximal impact, so to speak in a "worst case scenario". If this is the case, gains flow into the definition of the prospective value as expected gains, exactly as in Equation (2.23), while losses take the form of VaR*. In other words, investors are sufficiently wary as to take into account the possibility of experiencing a maximal loss, hence to put an upper bound (in absolute value) on expected losses. It is this upper bound that now generates utility (value) to the individual investor, and not the expected loss. These considerations entail an alternative definition $V^*_{t+1}$ of the prospective value:

$$\begin{aligned} V^*_{t+1} &= \omega_t S_t E_t[x_{t+1}] + \mathrm{VaR}^*_{t+1} \\ &= (\omega_t + \lambda S_t) E_t[x_{t+1}] \\ &+ [(\pi_t - \varphi\sqrt{\pi_t(1-\pi_t)})(\lambda - 1)R_{ft} - (1 - \pi_t + \varphi\sqrt{\pi_t(1-\pi_t)})kE_t[x_{t+1}]](S_t - Z_t), \end{aligned} \tag{2.24}$$

where the latter expression was derived according to Equation (2.18). In Section 3.2.2, we investigate the evolution and implications of both prospective value definitions stated here.

Before closing this section, we introduce a further notion that in our opinion provides additional information on the actual investor attitude towards financial risks compared to the simple coefficient of loss aversion. According to the original PT, loss aversion corresponds to risk aversion of first order in the loss domain. In this spirit, we denote the first derivative of the prospective value with respect to the expected equity premium as *global first-order risk aversion* (abbr. gRA) and formally define it as:

$$\Lambda_t = \frac{\partial V_{t+1}}{\partial E_t[x_{t+1}]} = [\omega_t + (1-\omega_t)\lambda]S_t - (1-\omega_t)(1-\pi_t)k(S_t - Z_t) = S_t[\omega_t + (1-\omega_t)\tilde{\lambda}_t]. \tag{2.25}$$

Specifically, the gRA reflects the sensitivity (in terms of first-order changes) of the prospective value (which can be rendered in traditional terms as investment utility) to the variation of expected returns (that yields to the variation of the expected equity premium). In our opinion, the gRA represents another way of quantifying the attitude of non-professional investors to financial losses that captures complementary features with respect to the LAi. While the LAi measures the differences in perception (of one unit risky investment) around the reference, the gRA is more general (and for this reason termed "global") and captures the slope of the aggregate individual view over both gain and losses. Yet formally the two measures are closely connected with each other, in the sense that the gRA per unit of risky investment yields a linear transformation of the LAi weighted by the probability of facing current losses. Consequently, the gRA varies similarly to LAi with respect to cushions and the sensitivity to past losses. Note however that LAi and gRA are to be differently interpreted. Specifically, as gRA directly reflects the changes in the prospective value that is proportional to the attractiveness of financial investments, higher gRA-values denote a more relaxed loss attitude.

## 2.5 The impact of the portfolio evaluation frequency

As shown in the previous sections, the expected portfolio returns $E_t[R_{t+1}]$ (hence the expected return premium $E_t[x_{t+1}]$) play a major role in the formulation of the value function and consequently of almost all other variables of interest in our model (such as VaR*, the prospective value, the optimal borrowing level, and also future cushions, gain probabilities, etc.). Therefore, it is essential to notice that the value of returns directly depends on the time horizon $\tau$ over which they are computed, i.e. or on the *portfolio evaluation frequency* $1/\tau$. We hypothesize that different evaluation frequencies impact on investor risk behavior and lead to different investment decisions. The main reason for this resides in the dependence of the computed performance of the risky portfolio on expected returns, which further gives rise to the dependency of the investor attitude towards the risky deposit and of the money invested in it on the portfolio evaluation frequency. The higher this frequency is, the less likely it is that risky returns lie above riskless ones, thus the more pronounced the investor disappointment concerning the risky portfolio performance. Since according to the PT registered losses are perceived as more painful than gains of similar size, risky investments become even less attractive.

The idea that the joint effect of narrow framing (myopia) over financial decisions and of the reluctance to make losses can dramatically impact risk perception and hence the subjective desirability of risky investments comes in line with the concept of mLA. The empirical part of our paper (Section 3) analyzes closely the impact of various evaluation horizons (ranging from one day to eight years) on the risk-free investment and on the prospective value, where the focus lies on high evaluation frequencies (the ones that are more plausible in practice), such as one day, one week, one month, two months and more, up to one year. In Section 3.2.1, we also plot and empirically assess the analytical forms of the LAi from Equation (2.14) and of the gRA from Equation (2.25) as functions of the evaluation frequency. Yet, in order to better understand how the evaluation frequency impacts the prospective value and the investor attitude towards risk, further explanations are necessary and the rest of this section is dedicated to detailing this problem.

We start by noting that the first variable affected by the evaluation horizon $\tau$ is the gross return value $R_t(\tau) = \log(P_t/P_{t-\tau})$ that accounts for the price variation over the time interval $\tau$. Therefore, the expected return premium $E_t[x_{t+1}(\tau)] = E_t[R_{t+1}(\tau)] - R_{ft}$ depends on the evaluation frequency.[11] For instance, if prices are highly volatile in the short run but do not change very much in mean in the long run, a higher $\tau$ should generate higher returns. However, even though there are more parameters (such as $S_t$, $Z_t$, $\pi_t$, etc.) that are computed from $R_t(\tau)$ being thus affected by $\tau$, in our (empirical) analysis we assume all $\lambda$, $k$, and $R_f$ as fixed (i.e. independent of $\tau$). Therefore, the changes of the prospective value $V_{t+1}$ documented in Section 3.2.1 are a consequence of a chain impact whose very first seed is the evaluation horizon, but that does *not* imply the loss aversion coefficient $\lambda$.[12] Obviously, although this chain reaction (hence its source, $\tau$) does not change the simple coefficient of loss aversion, it also affects our measures of the actual attitude towards financial losses LAi and gRA:

$$\tilde{\lambda}(\tau) = \lambda - [1 - \pi(\tau)]k[1 - z(\tau)]$$
$$\Lambda(\tau) = S(\tau)[\omega(\tau) + (1 - \omega(\tau))\tilde{\lambda}(\tau)].$$

As both LAi and gRA are computed as derivatives over the (expected) equity premium, the variation of $x(\tau)$ is excluded. Thus, potential changes of (one of) these measures subject to the evaluation horizon $\tau$ reflect the indirect impact of $\tau$ on other model variables such as $\pi$, $z$, or $\omega$.

In addition, we address a further theoretical issue which is closely related to the impact of the portfolio evaluation frequency discussed above. Given that this frequency appears to affect the investor risk perception, thus the level of risky investments, could the reverse causality hold as well? In other words, for a certain loss aversion value (at time $t$) is there an evaluation frequency that is optimal in terms of maximization of the prospective value? In order to answer this question, we first analyze the *direct* impact of $R_t(\tau)$ on the utility maximization problem of individual investors. To this end, the *c.p.* dependence of the prospective value $V(x)$ from Equation (2.23)[13] on $x(\tau)$ at time $t$ is taken into account. In other words, we study the direct dependence of utility on returns, but discard the indirect effects generated by other model parameters influenced by returns.[14] In Section 3.2.1, we search for a generally valid specification $V(\tau)$ that can be inverted in order to deliver

[11] For reasons of simplicity, we henceforth drop most of the time-indices at places where we discuss the dependence of the variables calculated at (the fixed) time $t$ on $\tau$.

[12] This chain reaction takes place in successive steps: (1) $\tau \to E_t[x_{t+1}] =: x(\tau)$, (2) $x(\tau) \to S_t =: S(\tau)$, (3) $S_1, S_2, ...S_t \to S_t - Z_t =: S(\tau) - Z(\tau)$.

[13] Or the corresponding $V^*(x)$ from Equation (2.24).

[14] In essence, this can be considered a plausible assumption. The choice of an optimal *current* $\tau$ takes place at the fixed time $t$ where the model parameters indirectly affected by $\tau$ (i.e. $S$, $Z$, $\pi$), depend on past values of $x$. The only exception is $\omega_t(\tau)$ that depends on $\tau$ through $E_t[x_{t+1}(\tau)]$, but assuming that investors also assess $\omega$ on the basis of past experience (e.g. as the frequency of past positive return premia), we can confine ourselves to analyze the isolate role of $E_t[x_{t+1}(\tau)]$ in the prospective value function.

the optimal $\tau$. Second, in the same empirical part of the paper we analyze the *indirect* dependence on the portfolio evaluation frequency by focusing on the two measures of the actual attitude towards losses LAi and gRA. Observing that they actually change with $\tau$, we infer analytical specifications of the type $\tilde{\lambda}(\tau)$ and $\Lambda(\tau)$ and derive the $\tau$-values at which they are minimized.

# 3 Empirical results

This chapter presents empirical findings complying with the theoretical results derived in Section 2.

The empirical analysis is based on daily data for the SP500 index and the 10-year nominal returns bond (considered as the risky and the risk-free investment alternative, respectively), ranging from 01/02/1962 to 03/09/2006 (11,005 observations).[15] From this data set, we construct daily, weekly, monthly (up to eleven months, increasing one month at the time), yearly and further lower frequency returns (ranging from two to eight years, with a one-year increment). We divide our sample into two parts on the basis of the fact that the early 80s mark the beginning of a new era of financial markets, due to the financial reform in 1979 that significantly changed the trading conditions. Consequently, we reckon that only the second part of the data (from 03/01/1982[16] to 03/09/2006, specifically 6,010 observations) is relevant for inferring current market evolutions and consider it as our "active" data set on which the subsequent empirical investigations are based. The first part of the sample (from 01/02/1962 to 03/01/1982) serves to estimate the empirical mean and the standard deviation of the portfolio returns at date zero of the trade (i.e. at 03/01/1982). Yet, the active data set contains an outlier corresponding to the October 1987 market crash which may distort the results. Because the real market data serves in our work merely as support for simulating trading behaviors, that we view as more general, this outlier is smoothened out by replacing it with the mean of the ten before and after data points.[17]

We consider that non-professional investors perceive risky investments according to the value functions in Equations (2.11) and (2.12), and calculate the maximum loss level according to Equation (2.18). The active data set allows us to run the model on the basis of Sections 2.1 and 2.2 and to derive the desired VaR*, as well as the wealth proportion invested in the risky portfolio (i.e. in the SP500 index). The remaining money is assumed to be automatically put in the risk-free 10-year bond. Moreover, investors are assumed

[15]Descriptive statistics can be found in Tables 5 and 6 of the Appendix.
[16]It took several years until the financial reform became operative.
[17]We consider that this method is appropriate for preserving some of the particularities of less probable market events such as crashes, while at the same time allowing for circumvention of excessive impacts due to extreme outliers.

to start trading with an even initial wealth allocation between the risky portfolio and the bond.[18] We also assume that the number of investors is constant, i.e. no investors can enter or exit the market during the trading interval (corresponding to the second part of the data).[19]

## 3.1 The evolution of the risky investment

In this section we address the interrelated questions of how risky investments develop subject to different portfolio evaluation frequencies and to distinct ways of assessing the cushion. Finally, we discuss the impact of applying the simpler definition $\text{VaR}^{*1}$ (that merely accounts for maximum expected losses) on the wealth percentages invested in the risky portfolio.

### 3.1.1 The impacts of the portfolio evaluation frequency and of the cushion

According to Benartzi and Thaler (1995), loss-averse investors who evaluate the performance of their portfolios once a year and employ a linear value function with conventional PT parameter values, give rise to a market evolution that can explain the equity premium observed in practice. In this context, we are interested in how varying the portfolio evaluation frequency can change investor decisions, hence the market evolution in our setting.

Furthermore, we ask which is the impact of different ways of assessing the cushion on investor decisions. First, the value $Z_t$ of past portfolio performance that impacts the valuation of current losses is taken to be identical to the last period risky asset holding $Z_t = S_{t-1}$. This applies to what we denote as *myopic cushions*. Second, as we assume that investors do not enter or exit the market during the entire trading interval, it is plausible to consider that they assess the investment performance starting with date zero. In other words, they amass what we call *cumulative cushions* by setting $Z_t = \sum_{i=0}^{t} S_i$, which comes in line with Barberis, Huang, and Santos (2001).

Following Campbell, Huisman, and Koedijk (2001), we start by computing the portfolio VaR in Equation (2.10b) for either (standard) normally or Student-t (with five degrees of freedom) distributed portfolio gross returns and for a significance level of 5%. In addition, we account for different ways of computing the expected portfolio returns, namely as the unconditional mean returns until the last date before the decision time, a zero mean process, or an AR(1) process. Then, taking $\lambda = 2.25$ and $k = 3$ as in Barberis, Huang,

[18] A similar assumption is made in Thaler, Tversky, Kahneman, and Schwartz (1997b).

[19] This assumption implies that the evaluation period is shorter than the lifetime of our loss averse agents or, equivalently, that investors are long-lived beyond the VaR horizon. Identical assumptions are made in Basak and Shapiro (2001), Berkelaar, Kouwenberg, and Post (2004), Berkelaar and Kouwenberg (2006).

and Santos (2001), as well as $\pi_t$ identical to the empirical frequency of the cases where $z_t \leq 1$ (i.e. of the past gains),[20] we derive $\text{VaR}^*_{t+1}$ according to Equation (2.18) on the basis of myopic or cumulative cushions. This value is then plugged into Equation (2.10a) in order to determine the optimal level $B_t$ of borrowing or lending.

Table 1 presents the average percentages of wealth $S_t/W_t$ invested in the risky portfolio, for both myopic and cumulative cushions, different portfolio evaluation horizons $\tau$, normally and Student-t distributed portfolio returns $R_t$, and expected returns $E_t[R_{t+1}]$ computed as unconditional mean of past returns.[21] Here, $S_t$ is derived according to Equation (2.13).

| | **myopic cushions** | | **cumulative cushions** | |
|---|---|---|---|---|
| **Evaluation frequency** | **Portfolio returns** | | **Portfolio returns** | |
| | Normal | Student-t | Normal | Student-t |
| 1 year | 32.65 | 26.18 | 49.31 | 44.83 |
| 6 months | 19.43 | 15.76 | 14.18 | 12.14 |
| 4 months | 16.12 | 13.14 | 19.38 | 14.96 |
| 3 months | 12.86 | 10.50 | 18.50 | 13.73 |
| 1 month | 7.49 | 6.18 | 2.05 | 1.88 |
| 1 week | 3.78 | 3.11 | 0.44 | 0.39 |
| 1 day | 1.88 | 1.54 | 0.15 | 0.15 |

Table 1: Wealth percentages invested in SP500

Accordingly, when investors are loss averse and use the $\text{VaR}^*_{t+1}$ from Equation (2.18) as measure of the maximal acceptable risk, higher portfolio evaluation frequencies entail lower investments in the risky portfolio, independent of the way of accounting for past performance (i.e. myopic or cumulative cushions). This result is consistent with previous findings, such as Benartzi and Thaler (1995) and Barberis, Huang, and Santos (2001), i.e. that loss-averse investors who perform high frequency evaluations and narrow-frame financial projects (by overly focusing on long series of past performances) become extremely risk averse. In particular for myopic cushions, the risky investment reduces to

[20]Figures 9 (10) in Appendix 5.1 illustrates the evolution of the probability of accumulating prior gains $\pi_t = P_t(z_t \leq 1)$ for myopic (cumulative) cushions and for yearly and daily evaluation horizons. When risky portfolios are evaluated once a year, there is almost no difference if investors use myopic or cumulative cushions. However, for more frequent evaluations the probability of past gains follows a similar pattern but remains lower, hence an increased degree of myopia manifested with respect to past performance diminishes the gain occurrence frequency. Specifically, the mean of $\pi_t$ amounts for myopic (cumulative) cushions to 0.7812 (0.7689) for yearly and 0.5261 (0.6344) for daily evaluations, reinforcing the idea that checking portfolios less often increases the percentage of gains made from risky investments.

[21]Similar results are obtained when expected returns are derived as zero mean or the AR(1) process, both for myopic and cumulative cushions. See Tables 7 and 8 in Appendix 5.2, as well as Footnote 22 below for further comments. For this reason and since unsophisticated investors (such as our non-professional traders) usually rely on simple descriptive statistics from past data (and less probable on more complex econometric models such as zero mean or AR(1)) for formulating return expectations, we henceforth concentrate on the case when expected returns are derived from average past returns.

half when switching from yearly to four-monthly evaluations. By contrast, investors using cumulative cushions turn out to be substantially less risk averse when evaluating their portfolios once a year. However, as the evaluation frequency increases (i.e. already for quarterly evaluations), the attractiveness of risky investment is perceived as lower when past results are accumulated over time than when merely the last period is accounted for. In essence, investors who use cumulative cushions end up by putting almost all their money in the risk-free asset (i.e. for daily evaluations).[22] Thus, the risk aversion appears to increase much faster when cushions are based on all previous trades relative to short-term cushions. Yet, independently of the way the cushions are computed, the risk aversion of investors appears to be lower for normally than for Student-t distributed portfolio returns. Interestingly, the yearly results with cumulative cushions under the normal distribution almost perfectly match the so called TIAA-CREF typical allocation (with slightly less than 50% as stock investment) mentioned in Benartzi and Thaler (1995).

Henceforth, we proceed in line with Barberis, Huang, and Santos (2001) and rely on cumulative cushions.[23] Given that VaR has been proven to be an adequate market risk measure for normal distributions, we mostly analyze the case with normally distributed gross returns.

In a next step, we are interested in the interdependence among risky portfolio returns, cushions and wealth percentages invested in the risky portfolio. In order to analyze this issue, we fix the evaluation frequency at one year and plot the annual returns of the index SP500, the evolution of the cushion $S_t - Z_t$ generated by series of past gains or losses, and the resulting yearly wealth percentages invested in the risky portfolio. As mentioned above, the past performance benchmark is set to be the risky investment value in the previous year $Z_t = S_{t-1}$, gross returns are considered as normally distributed, and expected returns are derived as the unconditional mean of past returns. Figure 1 points to a positive correlation of the three variables (SP500 yearly returns, yearly cushions, and yearly percentage investments in the risky portfolio).[24] Remember that the sample covers the last 24 years of analysis (from 03/01/1983 to 03/01/2006), such that every point on the horizontal time-axis corresponds to the 03/01 of each year. The proportion of wealth

[22]According to Table 8 in Appendix 5.2, the risky investment becomes exactly zero for daily evaluations when expected returns are derived as an AR(1) process. Note also that for an AR(1) process there is almost no difference between the allocations with myopic and cumulative cushions. For zero-mean expected returns and cumulative cushions, investors start with much higher risky allocations for yearly evaluations relative to the benchmark case with unconditional-mean return expectations, but at eleven months these allocations already resemble each other. In the same zero-mean case, but for myopic cushions, allocations are lower for yearly evaluations and decrease for higher evaluation frequencies but approach the benchmark more slowly.

[23]The results of identical tests performed for myopic cushions are available upon request.

[24]Indeed, the correlation between the SP500-returns and the yearly cushions amounts to 0.6607, the one between the cushions and the wealth percentage invested in risky assets to 0.6835, while the correlation between returns and the risky investment yields 0.5484.

invested in the risky portfolio appears to be mainly generated by the previous bull market observable in the SP500 returns.[25]

The importance of the cushion for investor decisions can be traced back to Equation (2.18) which reveals a twofold structure of the individual VaR*. The first term on the left-hand side accounts for the expectation of future portfolio returns weighted by the loss aversion coefficient $\lambda$, while the second term is responsible for the influence of previous performance (as encompassed by the cushion $S_t - Z_t$). We denote them as the *PT-term* and the *cushion term*, respectively. Accordingly, positive expectations with respect to the future evolution of the risky portfolio coupled with a positive cushion (i.e. past gains) should reduce investor aversion to financial losses. Consequently, given that VaR* directly enters $B_t$ and hence $S_t$, the wealth proportion invested in the risky portfolio $S_t/W_t$ increases, as illustrated in Figure 1. This effect is reversed when both return expectations and cushions become negative. Moreover, it is interesting to observe that small changes in the cushion at the beginning of the effective trade period[26] allow for high variations in the portfolio allocation. This first investor reaction turns strongly against investing money in risky assets, but the increase in cushions makes it smooth over time, so that it ends by following fairly close the cushion evolution. This result is again in line with the concept of loss aversion, i.e. the lower the cushion of wealth accumulated in past trades is, the more loss-averse investors become because they dispose of less back-up for later contingent losses. Moreover, this lowers the wealth fraction invested in risky assets.[27]

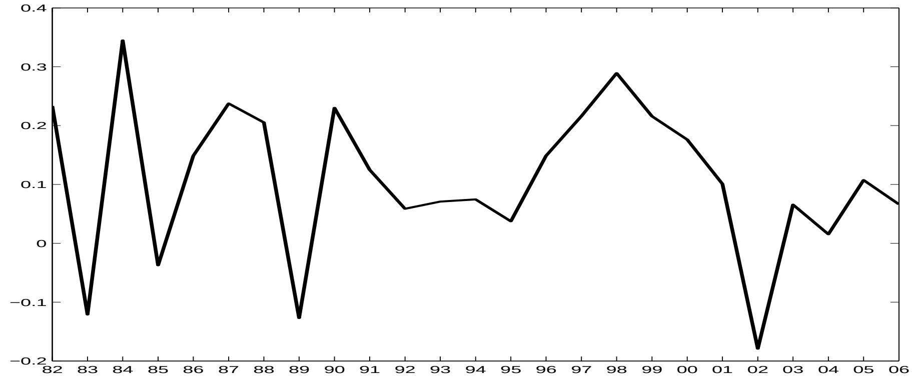
At this point, a further interesting empirical question arises: how long does it take for an investor performing frequent evaluations to quit the risky market? In order to answer this question, let us further assume that investors start with an initial investment in risky assets of 50% of the total wealth. Figure 2 points out the dramatic effect of high evaluation frequencies for investors who act upon cumulative cushions, i.e. when portfolio performance is checked every single day, investors get out of the risky market in not even half a year.[28] This behavior can also be explained in the context of Equation (2.18), according to which highly volatile SP500-returns and very low cumulative cushions (as generated by the daily change in position and apparent in Figure 2) result in an enhanced acceptable risk level VaR*. This captures the picture of an extremely risk-averse investor. However, investors with very short memory (one single day) concerning the past portfolio

[25]Specifically, this proportion reaches its maximum of 53.91% two periods after that SP500-returns attain a maximum value (which is in 1998), which coincides with the time when the yearly cushion are highest (i.e. 4775).

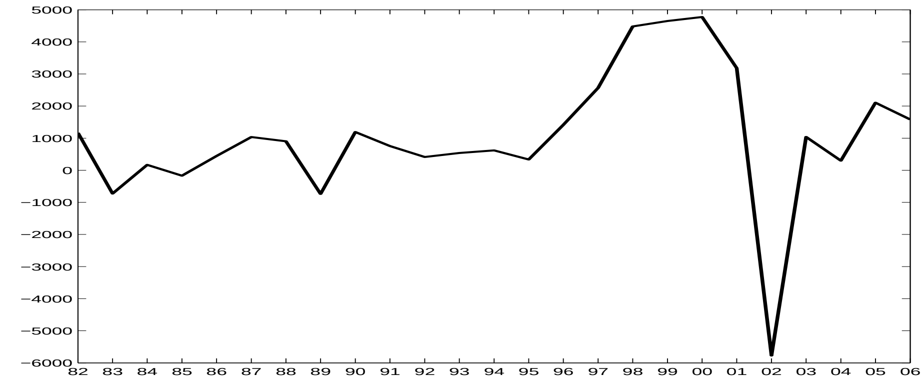
[26]Remember that the effective trade (i.e. the observations that effectively underlie the estimation procedure) begins at 03/01/1982.

[27]Gneezy and Potters (1997) test for the influence of experienced gains and losses on risk behavior, but find no significant effect. However, as noted on p. 641, their experimental framework deviates from real market settings, as considered in our model.

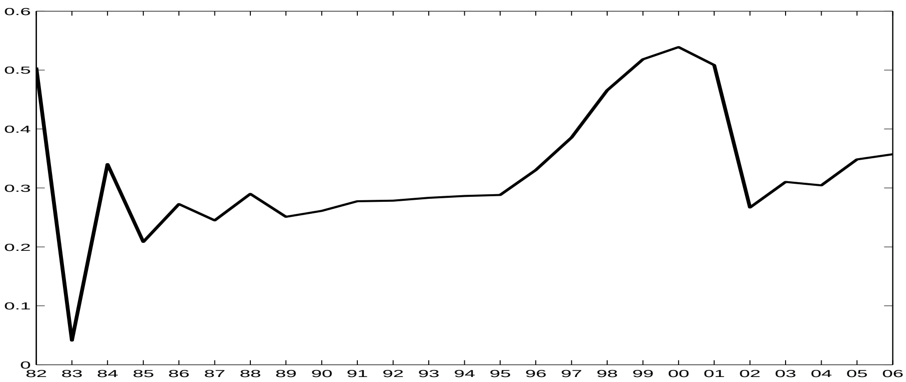
[28]In particular, their risky investments decrease from a maximum of 57.49% in the fifth day to 9.86% in the 14th day, and remain below 1.74% after the day 116.




(a) SP500 returns



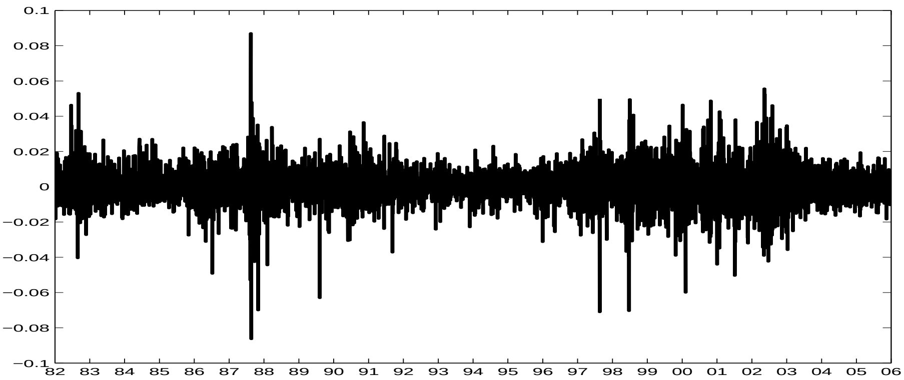

(b) Cushions



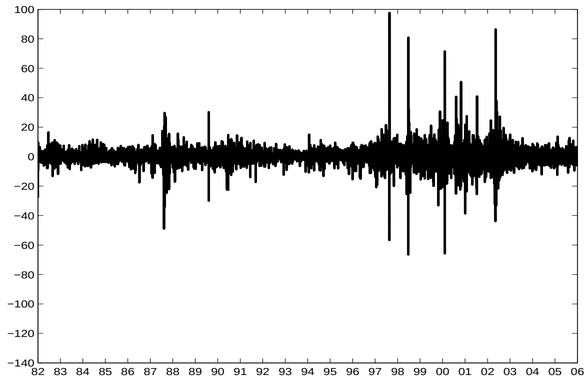

(c) Percentage investments in SP500

Figure 1: Evolution of SP500 returns, myopic and cumulative cushions, and percentages invested in the risky portfolio for yearly portfolio evaluations

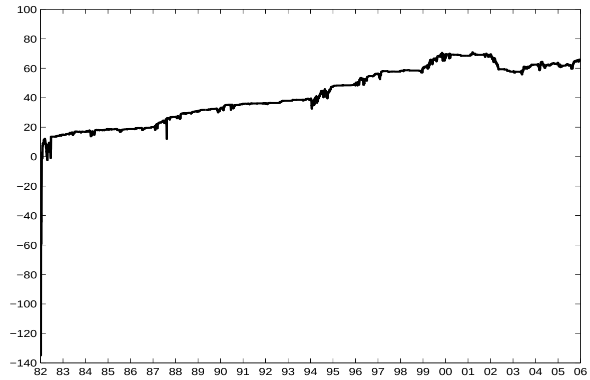
performance turn out to amass cushions that are higher in absolute value, but vary around an average of zero. Consequently, as each day can bring substantial change in the perceived past performance, they constantly allocate a low wealth percentage to risky assets.[29]



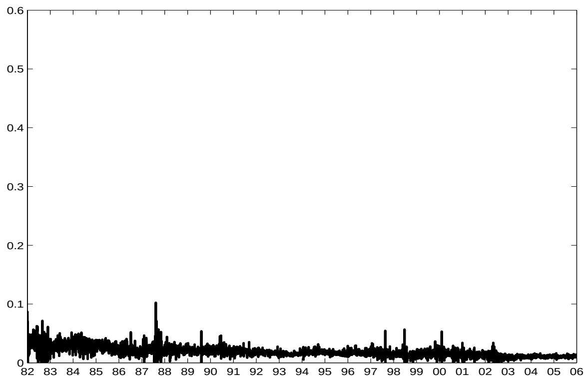
(a) SP500 returns



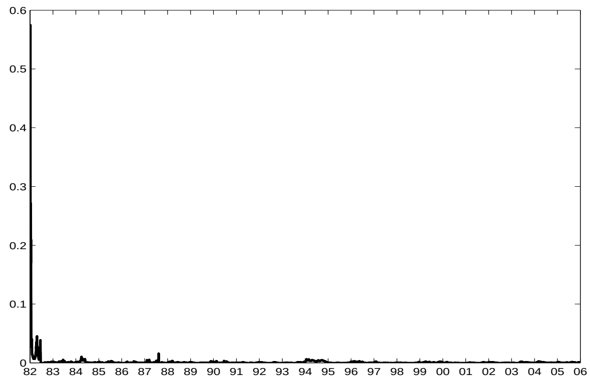
(b) Myopic cushions



(c) Cumulative cushions



(d) Percentage investments in SP500 for myopic cushions



(e) Percentage investments in SP500 for cumulative cushions

Figure 2: Evolution of SP500 returns, myopic and cumulative cushions, and percentages invested in the risky portfolio for daily portfolio evaluations

[29]The maximum risky investment is 10.26% and corresponds to the outlier found for the October '87 market crash.

### 3.1.2 An analysis with unadjusted VaR*

Finally, we analyze the investor behavior for a VaR* that exclusively accounts for maximum expected losses, as defined in Equation (2.16). The results confirm the mLA, in the sense that investors who merely account for the average expected losses in formulating individual risk constraints decrease their risky investments for higher evaluation frequencies. However, Table 9 in Appendix 5.2 points out that for myopic cushions, the use of $\text{VaR}^{*1}$ entails similar risky investments[30] relative to the adjusted VaR*. However, noticeable differences can be observed for cumulative cushions, when $\text{VaR}^{*1}$-investors start with lower risky allocations for yearly evaluations than their more sophisticated VaR*-peers, but reduce their risky investments subject to higher evaluation frequencies more slowly (faster) up to (above) five months, ending up by investing nothing in risky assets.[31] In essence, the evaluation frequency of one year, which is considered as standard in the literature, renders the adjustment in the VaR*-formula unimportant with respect to the wealth percentages dedicated to risky assets.

## 3.2 The evolution of the prospective value

This section first presents the influence of the evaluation frequency on the prospective value, then comments on the case when investors account for the "worst case scenario" in assessing the value of risky investments.

### 3.2.1 The impact of the portfolio evaluation frequency

According to the results in Section 3.1, the measured performance of the risky portfolio varies with the evaluation horizon $\tau$. In order to closer analyze the impact and to determine an optimal value of $\tau$ (on average over all decision times $t$), we first recall the observation made in Section 2.5 that $\tau$ exerts direct influence on the expected returns, thus on the expected return premium $E_t[x_{t+1}] = E_t[R_{t+1}] - R_{ft}$. Therefore, the evaluation time affects the prospective value of the risky investment from Equation (2.23).[32] Here, we distinguish between two terms with relevant contribution to the formation of $V(E_t[x_{t+1}])$, namely the first term on the right hand side of Equation (2.23) that stands for the prospective value as considered in the original PT (that we denote as the *PT-effect*), and the second one (called the *cushion effect*) generated by the cushions of past gains or losses suggested in Barberis, Huang, and Santos (2001).

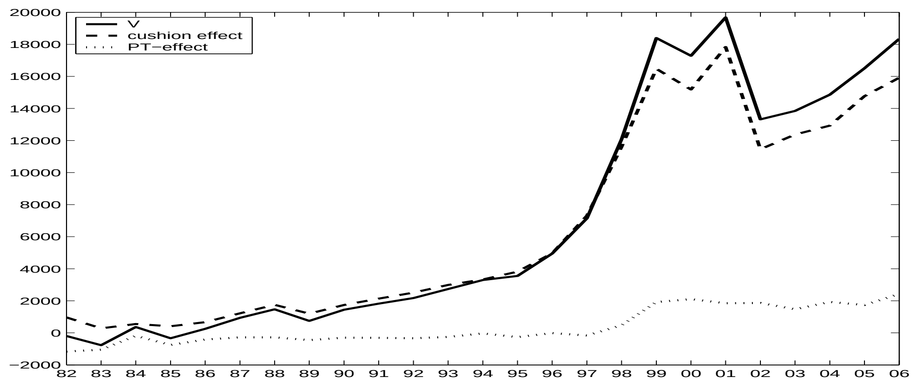
Figure 3 illustrates the evolution of the prospective value in Equation (2.23) and the contributions of these two effects, for evaluation frequencies of one year and one day,

[30]Specifically, these investments are only slightly lower on average.

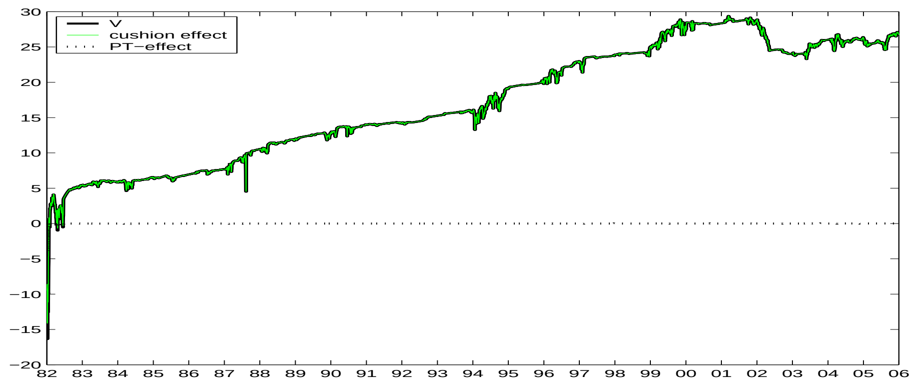
[31]The risky investment already amounts to zero for a two-month evaluation horizon.

[32]The effects of the alternative prospective value definition (2.24) are discussed in Section 3.2.2.

respectively. Recall that cushions cumulate from the beginning of the trade and investors anticipate normally distributed gross returns. On first inspection of the panel (a) in Figure 3, we find that the prospective value $V(E_t[x_{t+1}])$ relies on the PT-effect only at the beginning of the trade, as investors do not dispose of sufficient monetary provisions. Once positive cushions started to accumulate, the cushion effect clearly plays the lead role in the perceived risky value. This leading is even more pronounced for daily portfolio evaluations (panel b), where the cushion effect actually overlays the prospective value, the PT-effect being almost nil.[33]




(a) Yearly evaluations




(b) Daily evaluations

Figure 3: Prospective value evolution for daily and yearly evaluations

In the subsequent Figure 4, we now plot the prospective value and its two components (the PT- and the cushion effect) as functions of the evaluation horizon $\tau$, which ranges from one month to eight years, namely in monthly increments of up to one year and yearly increments thereafter.[34] As expected, the perceived riskiness of financial investments decreases as investors perform rarer evaluations. An apparent puzzling result is that $V$ looms negative at the frequency of two years. There is one particularity of our market

[33]Specifically, the mean PT-effect amounts to $-0.0066$.

[34]In order to obtain a suggestive graphic representation, we consider all frequencies from one to twelve months and discard the observations for one day and one week. An evaluation frequency of eight years implies that investors can only make three portfolio checks during our estimating sample. Therefore, a further increase of the evaluation time becomes senseless.

data that may have driven this result, namely that the two-year SP500-(log)returns turn out to be extremely variable (see Figure 11 in Appendix 5.3). These repeated changes of direction render investor decisions very difficult and result in negative values, because the non-professional investors are not able to cumulate positive cushions.[35]

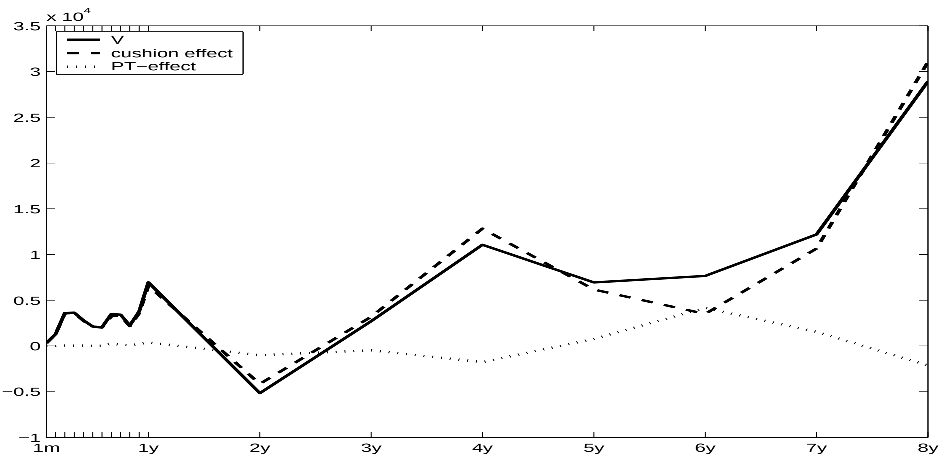
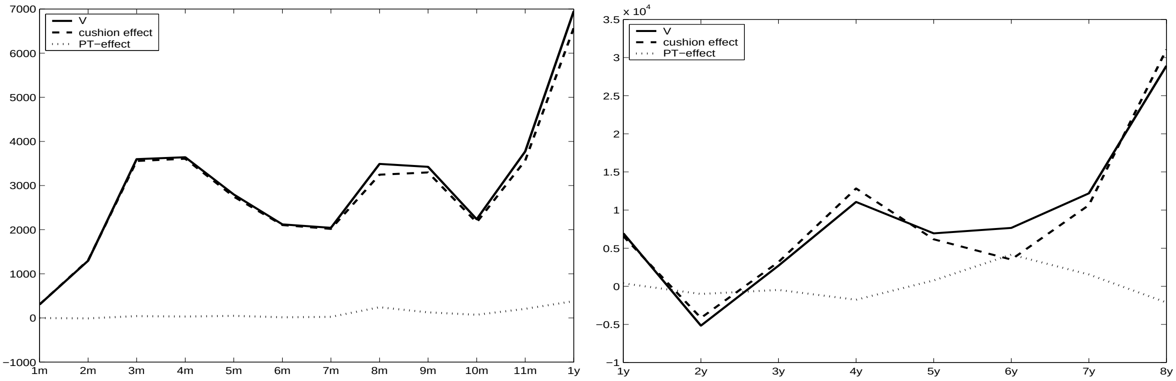



Figure 4: Prospective value evolution for different evaluation frequencies

The evolution of the prospective value in the evaluation horizon domain depicted in Figure 4 turns out to exhibit two distinct segments of different evolution, delimitated by an evaluation frequency of around one year, where a kink becomes apparent. This reinforces the idea that in practice, one year indeed represents a "critical" evaluation frequency. As documented in Benartzi and Thaler (1995), a decade ago (non-professional) investors actually used to perform yearly portfolios checks. Nowadays, due to the high amount of information available at almost no cost and to the enhanced dynamic of market events, we claim that a tendency to reconsider the problem of splitting their money between risky and risk-free assets more often becomes manifest. Thus, investor perceptions sooner lie in the left segment of the curve in Figure 4 (on which our subsequent analysis focus on as well). Yet, one year remains an important anchor in the investor minds given that, on one hand, various events (such as release of annual activity reports) take place with this frequency and, on the other hand, non-professional investors may not be sufficiently impatient (perhaps because they do not dispose of sufficient time and financial resources) to perform portfolio checks more often.

The apparent segmentation of the prospective value for evaluation frequencies lower vs. higher than one year motivates us to have a closer look at the two separate evaluation frequency segments illustrated in Figure 5. We attempt to finding an analytical form that underlies this evolution and that would allow us to conjecture upon an optimal evaluation frequency.

[35]Moreover, the problem gets worse since we have only twelve observations.




(a) $\tau \leq 1$ year (b) $\tau \geq 1$ year

Figure 5: Prospective value evolution on the two relevant evaluation frequency segments

For our usual case with $\lambda = 2.25$ and $k = 3$, the analytical functionals that best match the prospective value data $V(\tau)$ in Figure 5 consist of a third-order polynomial for $\tau \leq 1$ year and a fourth-order one for $\tau \geq 1$ year. The corresponding estimates of the curvature coefficients are given in Table 10 in Appendix 5.3. In addition, similar courses are found for further degrees of narrow framing such as $k \in \{0; 10; 20\}$, as illustrated in the subsequent Figure 12. In the left evaluation horizon segment, this analytical representation points out a three-stage evolution of the perceived risky value subject to higher evaluation frequencies (i.e. that reach from one month to one year). In other words, it appears that going from monthly to four-monthly evaluations entails substantial advances of the prospective value. Yet, a further increase in the evaluation frequency from five to ten months exhibits a much lower impact on the variation of $V$. Finally, when non-professional investors decrease the frequency of portfolio evaluation from ten months to one year, they perceive again higher and faster increasing prospective values. In the right segment, the evolution is more complex, but again middle-range evaluation frequencies (between four and seven years) demand lower variations of the prospective value. However, this more complex course may be in part determined by the negative $V$ obtained at the two-years evaluation horizon. We also note the resulting jump (kink) in the prospective value at what we consider to be the reference frequency of $\tau = 1$ year that complies with the idea of loss aversion.[36]

Section 5.3 in the Appendix summarizes some results of the various sensitivity checks performed for further values of the loss aversion coefficient $\lambda$ and of the past-losses sensitivity parameter $k$.[37] Tables 11, 12, and 13 present the prospective value evolution for the entire range of considered parameter values. As in practice it is less plausible that investors revise their portfolios less often than once a year, we briefly comment on the

[36]Specifically for $\lambda = 2.25$ and $k = 3$, $V(1 \text{ year}_-) = 6,519.56 \neq 6,273.3 = V(1 \text{ year}_+)$). Many authors consider the loss aversion to be defined by the kink of the value function at the reference point. See Tversky and Kahneman (1992), Berkelaar, Kouwenberg, and Post (2004), or Köbberling and Wakker (2005) among others.

[37]Further results are available on request.

findings for evaluation frequencies higher than one year (i.e. in the left evaluation horizon segment) in the sequel. First, the fitted curves for this segment depicted in Figures 12(a) and 13(a) emphasize the fact that while for $\lambda = 2.25$ the perceived risky value does not appear to change much subject to a higher sensitivity to past losses $k$, the reactions of investors showing different degrees of loss aversion (as measured by the index $\lambda$) are distinct for a fixed $k = 3$. In the latter case (i.e. $k = 3$), the third-order polynomial specification provides an acceptable fit for $\lambda \leq 2.25$ (namely, it explains more than 70% variation in the data as measured by the adjusted $R^2$). However, for high degrees of loss aversion ($\lambda = 3$) only a sixth-degree polynomial reaches an adjusted $R^2$ of over 40%. Moreover, while "veritable" loss-averse investors (with $\lambda > 1$) perceive risky investments to be more attractive as they perform evaluations less often, investors with $\lambda \leq 1$ manifest the opposed tendency towards a more favorable perception for more frequent evaluations. Clearly, the reversal takes place for the "neutral" case with $\lambda = 1$, where the prospective value turns out to be low and less variable. Also, the variation of $V$ over the evaluation horizon increased subject to higher values of the loss aversion coefficient $\lambda$. As expected in almost all cases with $\lambda \geq 1$[38], the maximum $V$ is to be found for the maximal evaluation horizon of the left segment, which is one year.

Returning to the question concerning the optimal evaluation frequency, it appears natural to assume that investors who are exclusively concerned with financial investments (and not with other sources of utility such as consumption) attempt to maximize the prospective value of their risky portfolios. Smart investors could look for an optimal evaluation frequency, i.e. one that maximizes the prospective value (at a given decision time $t$ or analogously on average).[39] The functional form fitted to our data set for the prospective value in Appendix 5.3 (see again Table 10 and Figure 12) for the left evaluation horizon segment (i.e. $\tau \leq 1$ year), show that loss-averse investors (with $\lambda \geq 1$) perceive the investment value as being maximal for the maximal evaluation time of this domain, i.e. one year. As mentioned above, we consider this segment as the sole one relevant in practice.[40] In the same spirit, the highest evaluation frequency of one day entails a minimal expected value of the risky portfolio, pushing investors to step out of the risky market and to allocate (almost) all their money to risk-free assets. In other words, loss-averse investors should check the performance of their risky investments as seldom as

[38]Only in the extreme case with $\lambda = 3$, the values of $V$ at ten monthly evaluations are higher for all considered $k$-values.

[39]Actually, the optimality of the evaluation frequency should be sooner understood from the viewpoint of portfolio managers, whose interest is to attract more clients willing to invest money in risky assets. Recommending these clients undertake performance checks in the "optimal" frequency should maximize the budget at managers' disposal. In the same context, Gneezy and Potters (1997) suggest that managers could manipulate the evaluation period of prospective clients.

[40]For the right segment (i.e. $\tau \geq 1$ year), the fitted polynomials exhibit a maximum at the lowest evaluation frequency of eight years and a local one between four and five years.

possible in order to maximize the corresponding prospective value of their investments. Under practical informational constraints that govern financial markets nowadays,[41] one year appears to be the most reasonable evaluation time that would increase the perceived returns of risky investments.

### 3.2.2 An analysis under the "worst case scenario"

For the "worst case scenario" described in Section 2.4, investors may use a slightly different definition of the prospective value, as suggested in Equation (2.24). Figure 6 plots the evolution of both $V_{t+1}$ in line with the original PT and the new $V^*_{t+1}$. Apparently, the latter is smaller and less variable but follows the same qualitative pattern.[42] Thus, the discussion on the evolution of the prospective value in the evaluation frequency domain conducted in Section 3.2.1 should also be valid in the "worst case scenario", at least in qualitative terms. Thus, the hypothesis that prudent investors perceive risks according to this "worst case" appears to be acceptably realistic (at least in the domain of evaluation frequencies) because it generates result patterns that are similar to those stemming from considerations of the original PT.

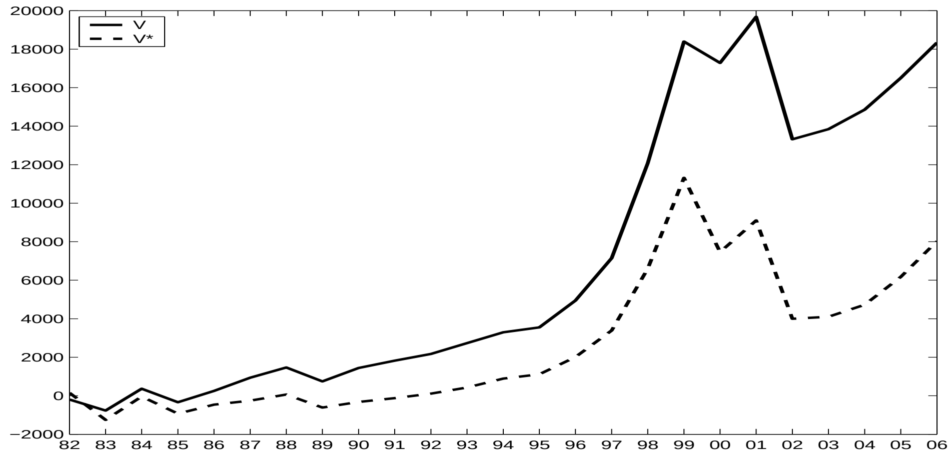



Figure 6: Prospective value $V$ vs. the "worst case scenario" $V^*$ for cumulative cushions and yearly evaluations

[41]Such as the huge amount of financial data available at almost no cost to each individual investor and the high interest raised by financial events in general. These natural market conditions entail an increase in the evaluation frequency below the limit of one year, such that investor perceptions lie sooner in the left evaluation-horizon domain and the crossover to the second segment is improbable.

[42]Indeed, the mean $\bar{V} = 6955.5$ while $\bar{V^*} = 2626.7$.

## 3.3 The evolution of the actual attitude towards financial losses

In this section, we address the question of how the attitude of non-professional investors, as captured by the LAi from Equation (2.14) and the gRA from Equation (2.25), vary subject to different portfolio evaluation frequencies. As discussed in Section 2.5, this variation reflects an indirect impact in the sense that it does not directly result from the changes of the equity premium with $\tau$, but on the collateral influence of $\tau$ on other model parameters such as the cushion $S_t - Z_t$ and the probability of past gains $\pi_t$. For both LAi and gRA, we conduct empirical investigations similar to the above analysis on the prospective value.

### 3.3.1 The impact of the portfolio evaluation frequency on the loss aversion index

In the traditional PT-framework, the index of loss aversion LAi reduces to the simple coefficient of loss aversion $\lambda$ (when the curvatures of the gain and loss branches of the value function are identical), hence it does not change with the portfolio evaluation frequency. The myopic loss aversion addresses the joint effect of this (fixed) index and the variation of returns due to more frequent evaluations. By contrast, in our extended framework the LAi itself fluctuates subject to the revision frequency and this effect overlaps the return variation resulting in perceptions of the risky investment that depend on the evaluation horizon. Thus, it is interesting to observe the evolution of the actual attitude towards losses subject to different portfolio evaluation frequencies. In particular, the LAi defined in Equation (2.14) exhibits the same twofold formal representation as the prospective value. A first term corresponding to the PT-effect consists of the coefficient of loss aversion $\lambda$ and a second one is analogous to the cushion effect and depends on the relative cushion $1 - z_t$, on the sensitivity to past losses $k$, and on the probability of past losses $1 - \pi_t$. Clearly, for positive but small cushions and low $k$-values (i.e. $k \leq 3$), the first term dominates and renders investors more reluctant to losses for higher $\lambda$.

We commence by analyzing the LAi-evolution for our benchmark case with $\lambda = 2.25$ and $k = 3$, the course of which is depicted in Figure 7. Apparently, LAi slightly increases on average for lower evaluation frequencies, a tendency that may appear counterintuitive at first. However, note that investors who check the performance of their risky portfolios less often (e.g. once every three years) and detect losses should become more averse to losses in general, as they have less flexibility in changing the portfolio composition to avoid future losses (e.g. the next evaluation will be undertaken only after three more years and investors have to bear the losses during the next three years).[43]

Figures 14 and 15 in Appendix 5.4 illustrate the LAi evolution for $\lambda = 2.25$ and different values of $k$, as well as for $k = 3$ and different $\lambda$, respectively. As expected, LAi does not vary much subject to the coefficient of loss aversion $\lambda$ for a fixed $k$ and for $k = 0$, it reduces to the simple coefficient of loss aversion $\lambda$. Yet, LAi becomes sensitive to the choice of the parameter describing the reaction intensity to past losses $k$. Surprisingly, it appears to diminish for increasing values of the sensitivity to past losses $k$. Also, for $k > 0$, LAi takes values that are always lower than the coefficient of loss aversion $\lambda$ (or equivalently than LAi in the case with $k = 0$). This reaction originates in Equation (2.14) and the fact that the absolute cushion $1-z_t$ is on average positive for almost all considered evaluation frequencies.[44] When non-professional investors impose high penalties on past losses (i.e. $k$ is big) and the current state is indeed a loss, they become extremely loss averse, which is formally equivalent to the fact that LAi substantially grows to exceed $\lambda$. However, when the current state is a gain and positive cushions have so far been accumulated (i.e. the average past performance is positive), investor perception cannot be characterized by extreme values of $k$. In fact, there is no meaningful interpretation of the case with high $k$ and past positive cushions, although it can be represented formally (as in Figure 14). The practical importance of our graphical illustrations refers to cases when negative cushions are coupled with current losses and we can observe how LAi grows for higher $k$-values.[45] Table 14 in Appendix 5.4 attempts to distinguish among practically relevant and irrelevant cases.

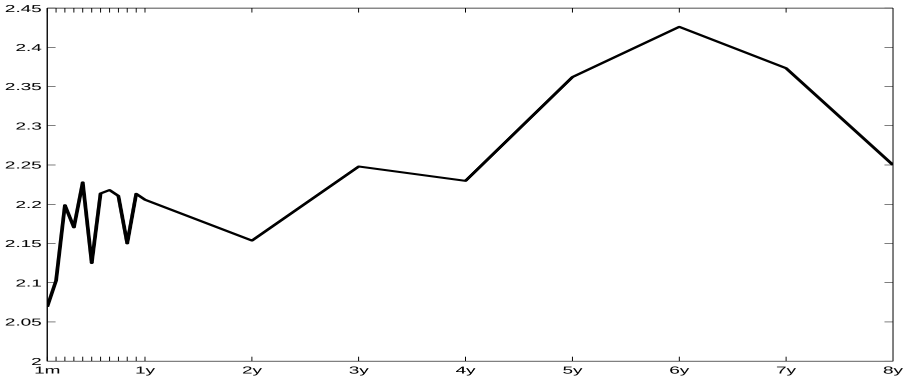
Moreover, in the left segment (i.e. high evaluation frequency) LAi turns out to be more variable the higher the $\lambda$-values and exhibits local minima (maxima) at one and ten (three and eight) months. This pattern extends over the right segment for portfolio evaluations more frequent than once every four years and is reversed for lower evaluation frequencies. A local minimum (maximum) becomes manifest around two (six) years. Note that for higher $k$-values, the LAi for ten months is almost as low as the global minimum of LAi obtained for the lowest evaluation horizon of one month, which meets the findings in Benartzi and Thaler (1995), who refer to ten months as to the evaluation period that explains the equity premium observed in practice when investors use a piecewise linear value function and linear probability weights.

As in Section 3.2.1, we perform separate fitting procedures for each of the two evalua-

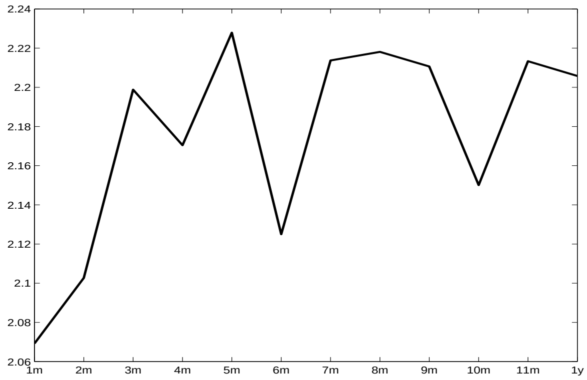
[43]Of course, this situation can lead the investors to increase their portfolio revision frequency. However, this is an open question that we left for future research.

[44]For $\lambda = 2.25$ and $k = 3$, there are only two negative mean values at five and six years. See Figure 16 in Appendix 5.4. At these two frequencies (which are in essence of no practical interest), LAi indeed increases for higher $k$, as apparent in Figure 14.

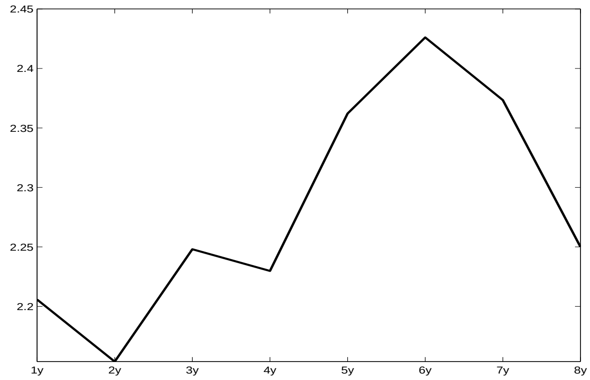
[45]As stressed above, this is the case for five and six yearly portfolio evaluations in Figure 14 for $\lambda = 2.25$ and $k = 3$.




(a) All evaluation frequencies




(b) $\tau \leq 1$ year




(c) $\tau \geq 1$ year

Figure 7: Evolution of the loss aversion index on the two relevant evaluation frequency segments

tion horizon segments (i.e. lower and higher than one year).[46] Table 15 in Appendix 5.4 presents the estimated fitting coefficients for $\lambda = 2.25$ and $k = 3$. The simplest specification describing quite well the evolution of LAi on each of the two evaluation frequency segments is a third-degree polynomial.[47] For $\lambda = 2.25$, while to the left of the evaluation horizon of one year the LAi grows with $\tau$, its form reminds of a sinusoid to the right of the reference point, where this pattern is more pronounced due to the enhanced sensitivity to past losses as described by $k$. The local extremal points are clearly not as pronounced as for the raw courses in Figure 14. However, the overall minimum is obtained at the maximum evaluation frequency of one month, then there is another local minimum between nine and ten months (between one and two years) in the left (right) segment.[48] Note that this evolution pattern fitted for different $k$-values in our usual case with $\lambda = 2.25$ persists for $\lambda = 3$, but is reverted for coefficients of loss aversion which are implausible according to PT (and denote the opposite of loss averse investors) $\lambda \leq 1$.[49]

### 3.3.2 The impact of the portfolio evaluation frequency on the global first-order risky aversion

Motivated by the above findings concerning the evolution of LAi as a function of the evaluation horizon $\tau$, we now turn our attention to the second measure of the loss attitude introduced in the theoretical part, namely the gRA. As stressed in the theoretical part of our paper, the gRA per unit of current risky holdings ($S_t$) represents a linear transformation of LAi. Being derived from the prospective value, it also encompasses some further more general elements such as the probabilities of current gains and losses. We expect that the same intuition holds and the gRA shows similar but somewhat less complex structure in the two evaluation frequency segments.

As illustrated in Figure 8 for our usual case with $\lambda = 2.25$ and $k = 3$, and in Figures 17 and 18 in Appendix 5.4 for further values of those two parameters, the gRA course follows the main evolution pattern observed for LAi. It increases slightly for lower portfolio evaluation frequencies, as well as subject to the coefficient of loss aversion $\lambda$ for a fixed sensitivity to past losses $k$. However, gRA turns to be more sensitive to the variation of $\lambda$

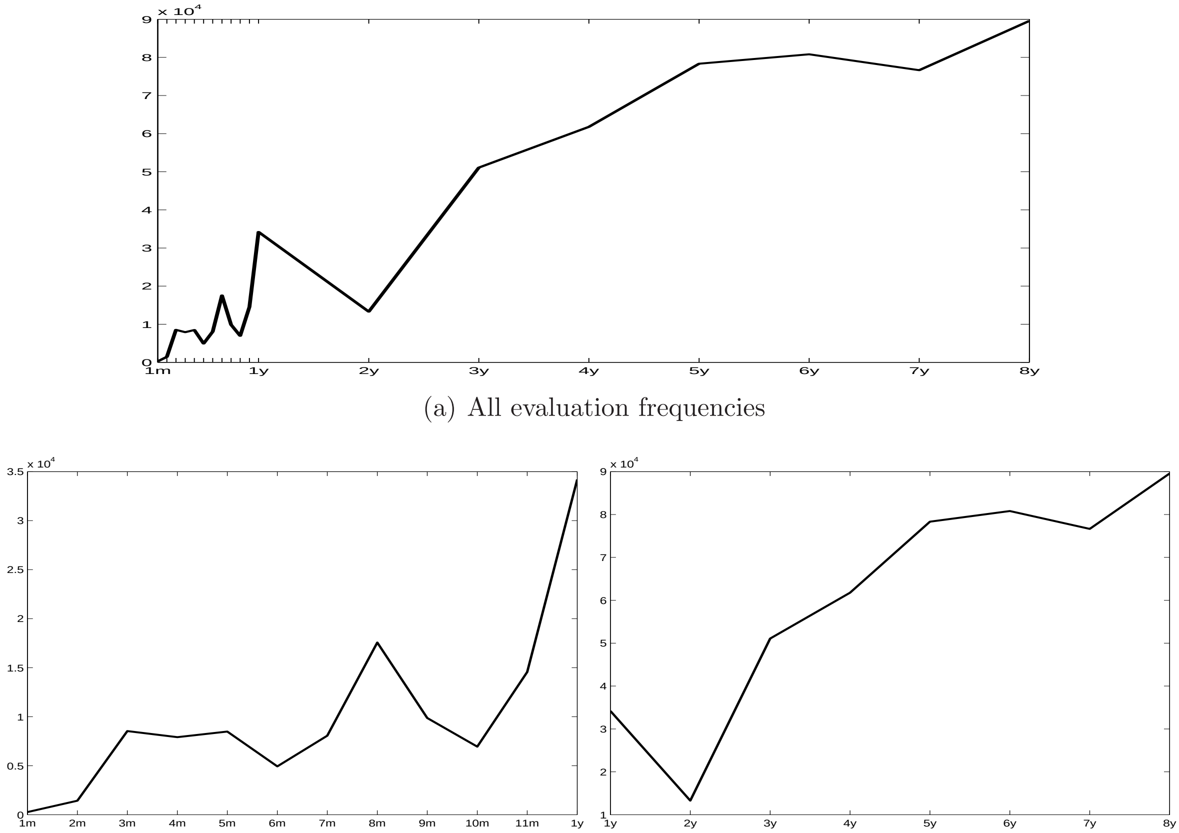
[46]Further numerical results are available upon request.

[47]In terms of the adjusted $R^2$, this specification explains over 70% of the data variation in each of the analyzed cases with moderate levels of $\lambda$ and $k$. Exceptions are some of the cases with very high $k$ in the high evaluation frequency segment, such as $\lambda = 0.5$ and $k = 20$, where the third-polynomial merely achieves an adjusted $R^2$ of 45.61%. Also, for $\lambda = 3$, the cubic polynomial provides the best fit but explains only between $50 - 60\%$ of the data variation for all $k$ in this segment. Interesting to note is that in the case that can be considered as "neutral" in terms of the loss aversion coefficient $\lambda = 1$, simple lines already provide a good description of the LAi for highly frequent portfolio revisions.

[48]In particular, the evaluation horizon (in months) at which the LAi in the left segment is minimal amounts to 9.6169 for $k = 3$, 9.4316 for $k = 10$, and 9.0406 for $k = 20$. The same values (in years) for the minima in the right segment are 1.9498 for $k = 3$, 1.8328 for $k = 10$, and 1.6204 for $k = 20$.

[49]Specifically for $\lambda \in \{0.5; 1\}$, LAi exhibits negative slope in the left segment.

compared to the LAi, given that it decreases in $k$ for a fixed $\lambda$ as long as portfolio revisions are performed more often than every four years. From this viewpoint, the gRA reflects more accurately the attitude of investors who heavily penalize past losses (i.e. exhibit a high $k$) and who become more averse to financial losses in general. For $\lambda = 2.25$, the global minimum of gRA is again attained for the highest evaluation horizon of one month and, local minima in the left (right) segment are found for two, six, and ten months (two years) portfolio evaluations for $k \leq 10$ and at four and ten months (one and four years) for $k = 20$. The negative values of gRA for $k = 20$ easily result from the Definition (2.25), where for positive cushions and high sensitivity to past losses $k$ but moderate loss aversion coefficient $\lambda$, the second term (in the middle expression) dominates the first PT-equivalent term and becomes negative. Hence, in practice investors may be hardly as averse to past losses as suggested by $k > 10$ (at least when experiencing past gains on average, as it is the case for our data set).




(a) All evaluation frequencies



(b) $\tau \leq 1$ year

(c) $\tau \geq 1$ year

Figure 8: Evolution of the global first-order risk aversion on the two relevant evaluation frequency segments

Third-degree polynomials appear to fit the left evaluation horizon segment acceptably well for our standard case with $\lambda = 2.25$, at least for moderate sensitivity to past losses ($k \leq 10$). For $k = 20$, the evolution of gRA turns to be more complex and variable, with a minimum at quarterly portfolio evaluations, but in line with the above arguments and with the corresponding considerations with respect to LAi we can consider this case as implausible in practice. These findings are similar to the evolution of the prospective value for evaluation frequencies higher than one year. Thus, the gRA appears to increase faster at the ends of the left segment, namely for evaluations performed more often than once every four months as well as between eight months and one year.[50] In the right segment, linear specifications are already sufficient for describing the evolution of gRA. The estimated coefficients for $k = 3$ are given in Table 15. These results hold for all the other considered values of the loss aversion coefficient.[51]

In sum, we can conclude that LAi and gRA effectively represent improvements over the common loss aversion coefficient $\lambda$, as they address additional factors that impact on the loss attitude such as past performance and expectations about the future market conditions. Thus, the above analysis offers a more complete picture over the causes and manifestations of how this attitude towards financial losses fluctuates subject to different portfolio evaluation frequencies.

## 3.4 A comparison with the portfolio optimization framework

This section proposes to translate the results obtained in our framework (where investors subjectively derive the maximum acceptable level of losses) in terms of the portfolio optimization "language" spoken by professional managers. To this end, we calculate equivalent significance levels and equivalent average indices of loss aversion that correspond to the VaR* derived according to our model and imposed as fixed risk constraints in the portfolio optimization problem.

### 3.4.1 VaR*-equivalent significance levels

One further question of interest arises from the use of the $\text{VaR}^*$ as a measure of risk in the portfolio optimization model in Section 2.1. Statistically, $\text{VaR}^*$ represents the lower quantile of portfolio returns at a given (i.e. fixed) significance level $\alpha$ (or confidence level $1 - \alpha$), where usually $\alpha \in [1, 10]\%$. The individually optimal $\text{VaR}^*_{t+1}$ that is previously derived by investors on the basis of subjective considerations according to Equation (2.18) is compared to the portfolio VaR in Equation (2.10b), in order to determine how investor wealth is going to be split between the risky portfolio and the risk-free bond (where the sum to be invested in risk-free assets is formalized in Equation (2.10a)). We denote by $\alpha^*_t$ the significance level that corresponds to the $\text{VaR}^*_{t+1}$ computed in our model. Thus, if the portfolio VaR at time $t$ corresponds to an $\alpha > \alpha^*_t$ (or equivalently, to a confidence

[50] The inflexion points of the fitted polynomial for $\lambda = 2.25$ and $k = 3$ lie at 4.4746 and 7.3503 months.

[51] For all considered values of $\lambda$, the adjusted $R^2$ lies over 75%. For $k = 20$, only fourth-degree polynomials achieve adjusted $R^2$ of over 40%. However, for $\lambda = 1$ simple lines explain more than 75% of the data variation.

level $1 - \alpha < 1 - \alpha_t^*$), then the sign of Equation (2.10a) is negative. In other words, too much risk would arise by putting the entire wealth in the risky portfolio, so that, in order to accommodate the desired (lower) risk level, a percentage of the investor wealth should be lent, i.e. invested in the risk-free asset ($B_t < 0$). On the contrary, if $\alpha < \alpha_t^*$, then the portfolio risk meets the individual risk requirements (being lower than the subjective risk threshold) and investors borrow extra money ($B_t > 0$) in order to increase their SP500-holdings.

In this section, we determine the significance levels corresponding to the values of $\mathrm{VaR}_{t+1}^*$ derived from Equation (2.18) for normally and Student-t distributed gross returns and cumulative cushions.

| **Evaluation frequency** | **Portfolio returns** | |
|---|---|---|
| | Normal | Student-t |
| 1 year | 0.00 | 0.00 |
| 6 months | 0.00 | 0.00 |
| 4 months | 0.00 | 0.00 |
| 3 months | 0.00 | 0.00 |
| 1 month | 0.00 | 0.00 |
| 1 week | 0.00 | 0.00 |
| 1 day | 0.00 | 0.00 |

Table 2: Portfolio-equivalent significance levels of the estimated $\mathrm{VaR}_{t+1}^*$ ($\alpha^*$).

Table 2 presents equivalent significance levels averaged over time ($\alpha^*$) and provides striking results. As stated above, classical portfolio selection models based on VaR assume that investors chose significance levels $\alpha$ in the interval $[1, 10]\%$. Our findings show that for any evaluation frequency higher than one year, this assumption does not comply with real market data as the equivalent significance level $\alpha^*$ lies below the theoretically acceptable interval (being practically zero). Thus, even the lowest significance level of 1% proposed in standard portfolio optimization models is not able to capture the risk aversion of non-professional investors acting according to our setting. In other words, investors may be substantially more risk averse in practice than considered in theory.

### 3.4.2 Portfolio-equivalent indices of loss aversion

The previous section shows that non-professional investors who are influenced by their personal history of gains and losses and in general behave according to the assumptions of our model are more risk averse than commonly described by in terms of significance levels $\alpha \in [1, 10]\%$. In the same context, we now address the impact of an exogenous $\mathrm{VaR}^*$ as originally employed in Campbell, Huisman, and Koedijk (2001), on the values of the loss aversion coefficient $\lambda_{t+1}^{\bar{*}}$, computed according to Equation (2.22) in our model. To this end, we go back to the conventional significance levels of 1% and 10% and estimate an homologous exogenous VaR* as derived from Equation (2.10b) that would correspond to the portfolio VaR at one of these two significance levels. This equivalent VaR* serves to compute $\lambda^{\bar{*}}_{t+1}$ according to Equation (2.22).

Tables 3 and 4 present equivalent wealth percentages that would be invested in the risky portfolio at the two significance levels mentioned above (1% and 10%, respectively) but are obtained imposing VaR*-values that result from our model. Remember that the portfolio VaR in Equation (2.10b) is estimated using a 5% significance level that is going to be considered as the benchmark for the values in these tables (i.e. it corresponds to 100% risky investments). The same tables also show the average equivalent coefficient of loss aversion $\lambda^{\bar{*}}$ and consider the cases with normally or Student-t distributed portfolio returns and cumulative cushions.

| | **Wealth %** | | $\lambda^{\bar{*}}$ | |
|---|---|---|---|---|
| | **Portfolio returns** | | **Portfolio returns** | |
| **Evaluation frequency** | Normal | Student-t | Normal | Student-t |
| 1 year | 69.10 | 47.65 | 0.91 | 0.83 |
| 6 months | 65.54 | 42.53 | 0.97 | 0.97 |
| 4 months | 64.48 | 41.04 | 0.97 | 0.97 |
| 3 months | 63.43 | 39.59 | 0.97 | 0.97 |
| 1 month | 61.64 | 37.12 | 0.98 | 0.99 |
| 1 week | 60.14 | 35.10 | 1.07 | 0.99 |
| 1 day | 59.35 | 34.05 | 1.00 | 1.00 |

Table 3: Wealth percentages invested in SP500 and the average $\lambda^{\bar{*}}$, for $\alpha = 0.01$

| | **Wealth %** | | $\lambda^{\bar{*}}$ | |
|---|---|---|---|---|
| | **Portfolio returns** | | **Portfolio returns** | |
| **Evaluation frequency** | Normal | Student-t | Normal | Student-t |
| 1 year | 116.47 | 120.91 | 0.68 | 1.12 |
| 6 months | 118.37 | 122.95 | 1.09 | 1.09 |
| 4 months | 118.94 | 123.55 | 1.12 | 1.09 |
| 3 months | 119.49 | 124.13 | 1.15 | 1.08 |
| 1 month | 120.45 | 125.11 | 1.09 | 1.07 |
| 1 week | 121.25 | 125.92 | 1.02 | 1.02 |
| 1 day | 121.67 | 126.34 | 1.01 | 1.02 |

Table 4: Wealth percentages invested in SP500 and the average $\lambda^{\bar{*}}$, for $\alpha = 0.10$

Accordingly, the equivalent recommendations from our model at 1% (10%) significance lie well below (above) the benchmark VaR at 5%. This points out a higher (lower) risk aversion in our endogenous VaR*-framework (after restating it in terms of the exogenous-

VaR model) relative to the portfolio risk measured by VaR. Comparing Tables 3 and 4, we can observe that the lower the significance level (or the higher the confidence level) the more risk averse is the non-professional investor, i.e. the proportion of wealth invested in the risky portfolio is smaller than 100%. However, even the lowest percentages in Table 3 are still much higher than those in Table 1, where VaR* is treated as endogenous. Interestingly, the results for $\alpha = 1\%$ are consistent with our previous findings supporting the mLA, as the wealth percentage invested in risky assets decreases for higher evaluation frequencies. By contrast, when $\alpha$ increases to 10%, this phenomenon is reversed and investors appear to allocate more money to the risky portfolio for more frequent evaluations. As mLA is a widely documented phenomenon, we can conclude that the traditional portfolio optimization framework fails once more to capture the real investor behavior in a consistent way.

Similar conclusions can be drawn for the loss aversion coefficient $\bar{\lambda}^*$ derived for conventional significance levels assumed in previous research, the values of which are much lower than 2.25, the empirical level estimated in the original PT and largely used in previous empirical research.[52] For the majority of the considered combinations of $\alpha$-values and evaluation frequencies, we obtain $\bar{\lambda}^* \simeq 1$, a level that indicates identical perception over gains and losses according to the value function from Equations (2.11) and (2.12) (and recalling that $k = 0$). Actually, this "neutral" level of one is exceeded merely for high evaluation frequencies, namely over one week (six months) for $\alpha = 1\%$ (10%). This reinforces our earlier claim that even assuming low significance levels (for example $\alpha = 1\%$ as is the common case in previous portfolio optimization research) entails an underestimation of the loss attitude of real investors captured by the specific coefficient $\lambda$.

# 4 Summary and conclusions

In this paper we investigate the risk behavior of non-professional investors facing problems of fixing a maximal acceptable level of financial losses and of splitting money between risk-free assets and a risky portfolio (capital allocation). We assume that these investors are loss averse, narrowly frame financial investments and perceive future portfolio returns as being influenced by past portfolio performance.

We extend the portfolio allocation model developed in Campbell, Huisman, and Koedijk (2001) in order to incorporate the effect of a desired VaR* that is now subjectively assessed by individual loss-averse investors. Thus, the first task of non-professional investors consists of fixing a VaR*-level that is subsequently communicated to professional portfolio managers in charge of finding the optimal portfolio composition. The portfolio optimiza-

[52]Such as Barberis, Huang, and Santos (2001), Benartzi and Thaler (1995).

tion procedure also delivers the optimal sum of money to be invested in risk-free assets, which represents another important decision variable for the non-professional investor.

In modeling the investor's perception over the risky investment that yields the subjective VaR*, we rely on the notion of myopic loss aversion introduced in Benartzi and Thaler (1995) and employ the extended subjective valuation of prospective risky investments proposed in Barberis, Huang, and Santos (2001). We integrate these behavioral explanations in the portfolio decision framework mentioned above, enriching the two models with original findings that stem both from theoretical consideration and empirical results obtained on the basis of real market data (such as SP500 and US 10-year bond price series).

Considering that investors are merely concerned with financial investments as source of utility, we theoretically model their perceptions regarding the utility of risky assets and define the maximum individually sustainable level of financial losses VaR*. This level serves in deciding upon the optimal amount of money to be invested in the risky portfolio. Also, we assess the utility of risky prospects captured by the prospective value and apply an extended definition of loss aversion (residing in the loss aversion index according to Köbberling and Wakker (2005) as well as a coefficient of global first-order risk aversion) that attempts to better capture the actual attitude towards financial losses of real investors. Moreover, we investigate the influence of different evaluation frequencies on the prospective value and on the actual loss attitude and point out a way to derive an optimal horizon of performance revisions under consideration of practical constraints.

The theoretical results are supported and extended by our empirical findings which, in sum, show that non-professional investors allocate the main part of their wealth to risk-free assets. A smaller sum is put into the risky portfolio for increased frequencies of revising its performance. Also, financial wealth fluctuations determined by the success of previous decisions exert a significant impact on the current portfolio allocation, making investors without substantial gain cushions firmly refuse holding risky assets. One year appears to be a critical evaluation frequency, optimal from the viewpoint of maximizing risky holdings and commonly used in practice. This evaluation frequency splits individual perceptions over risky investments (captured by the prospective value) and over financial losses in general (captured by the loss aversion index and the global first-order risk aversion) into two qualitatively different segments with distinct evolutions. Moreover, the computation of equivalent values of the significance level and of the loss aversion coefficient that correspond to confidence levels commonly assumed in previous research suggests an underestimation of the attitude of real non-professional investors to financial losses.

# 5 Appendix

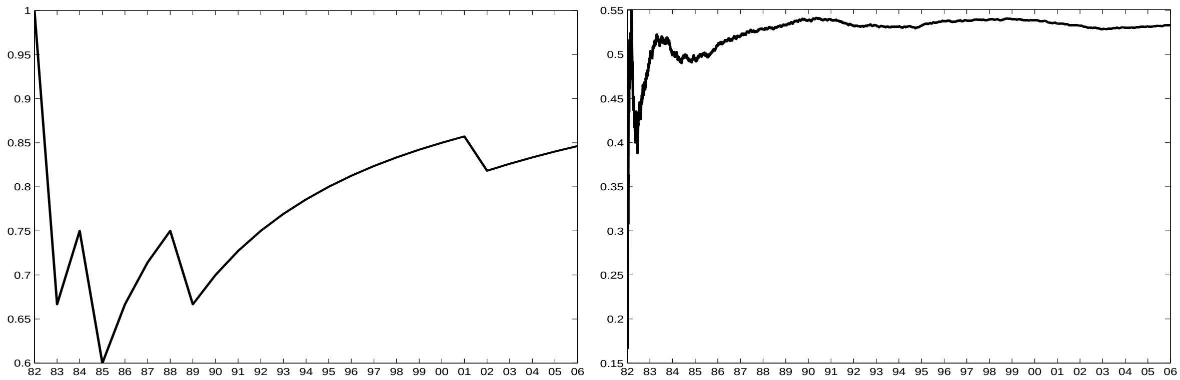
## 5.1 Descriptive statistics and the probability of prior gains

| **SP500** | **Evaluation frequency** | |
|---|---|---|
| | Quarterly | Yearly |
| Mean | 0.017 | 0.066 |
| Median | 0.018 | 0.071 |
| Std.Dev. | 0.079 | 0.136 |
| Kurtosis | 2.661 | -0.9659 |
| Skewness | -0.671 | -0.205 |
| Max. | 0.290 | 0.345 |
| Min. | -0.302 | -0.207 |
| Obs. | 175 | 43 |

Table 5: Log-difference of the SP500 index for quarterly and yearly portfolio evaluations

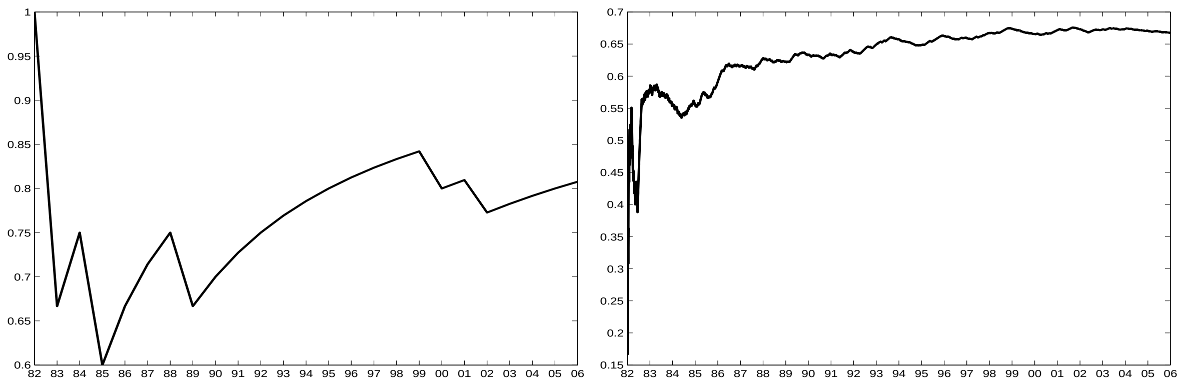
| **10-year** | **Evaluation frequency** | |
|---|---|---|
| | Quarterly | Yearly |
| Mean | 0.017 | 0.073 |
| Median | 0.017 | 0.070 |
| Std.Dev. | 0.006 | 0.026 |
| Kurtosis | 0.623 | 0.974 |
| Skewness | 0.951 | 1.042 |
| Max. | 0.036 | 0.142 |
| Min. | 0.009 | 0.037 |
| Obs. | 175 | 43 |

Table 6: 10-year bond return for quarterly and yearly portfolio evaluations




(a) Yearly evaluations (b) Daily evaluations

Figure 9: Evolution of the probability of prior gains $\pi_t = P_t(z_t \leq 1)$ for myopic cushions




(a) Yearly evaluations (b) Daily evaluations

Figure 10: Evolution of the probability of prior gains $\pi_t = P_t(z_t \leq 1)$ for cumulative cushions

## 5.2 The wealth percentages invested in SP500

| Evaluation frequency | myopic cushions | | cumulative cushions | |
|---|---|---|---|---|
| | Portfolio returns | | Portfolio returns | |
| | Normal | Student-t | Normal | Student-t |
| 1 year | 19.89 | 17.53 | 46.96 | 40.12 |
| 6 months | 13.17 | 11.43 | 14.75 | 12.65 |
| 4 months | 12.27 | 10.53 | 29.81 | 22.83 |
| 3 months | 10.23 | 8.72 | 28.05 | 20.65 |
| 1 month | 6.49 | 5.49 | 1.99 | 1.90 |
| 1 week | 3.52 | 2.94 | 0.43 | 0.38 |
| 1 day | 1.82 | 1.50 | 0.15 | 0.15 |

Table 7: Percentage investment in SP500 for expected returns = zero mean

| Evaluation frequency | myopic cushions | | cumulative cushions | |
|---|---|---|---|---|
| | Portfolio returns | | Portfolio returns | |
| | Normal | Student-t | Normal | Student-t |
| 1 year | 39.97 | 32.65 | 38.00 | 38.00 |
| 6 months | 26.16 | 20.52 | 26.68 | 20.13 |
| 4 months | 19.01 | 15.20 | 37.87 | 27.87 |
| 3 months | 14.33 | 11.62 | 34.86 | 23.67 |
| 1 month | 7.79 | 6.42 | 2.13 | 1.93 |
| 1 week | 3.82 | 3.13 | 0.00 | 0.00 |
| 1 day | 0.00 | 0.00 | 0.00 | 0.00 |

Table 8: Percentage investment in SP500 for expected returns = AR(1)

| Evaluation frequency | myopic cushions | | cumulative cushions | |
|---|---|---|---|---|
| | **Portfolio returns** | | **Portfolio returns** | |
| | Normal | Student-t | Normal | Student-t |
| 1 year | 25.34 | 21.18 | 47.04 | 43.41 |
| 6 months | 16.62 | 13.85 | 26.84 | 26.30 |
| 4 months | 14.45 | 12.09 | 8.27 | 7.80 |
| 3 months | 11.89 | 9.86 | 7.86 | 6.82 |
| 1 month | 7.20 | 5.96 | 0.00 | 0.00 |
| 1 week | 3.73 | 3.07 | 0.00 | 0.01 |
| 1 day | 1.91 | 1.55 | 0.00 | 0.00 |

Table 9: Wealth percentages invested in SP500 using VaR*1

## 5.3 The prospective value as a function of the portfolio evaluation frequency

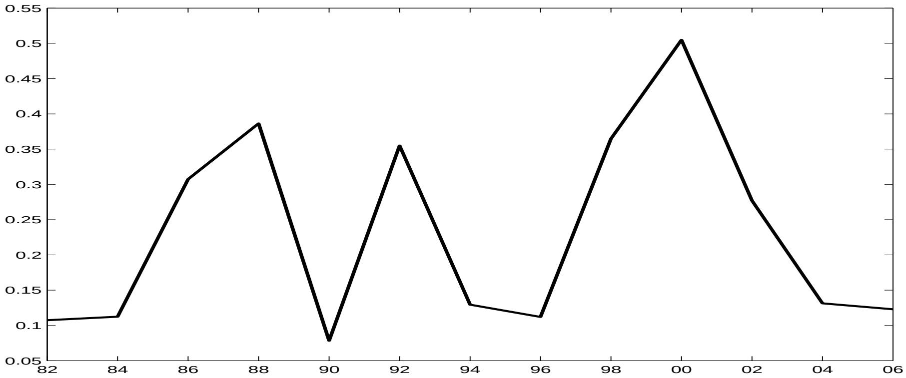
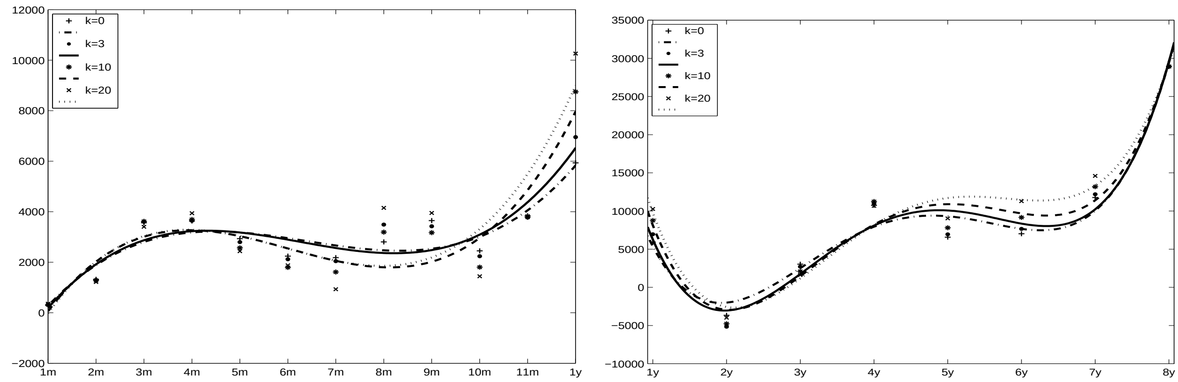



Figure 11: Two-year SP500 returns for $\lambda = 2.25$ and $k = 3$

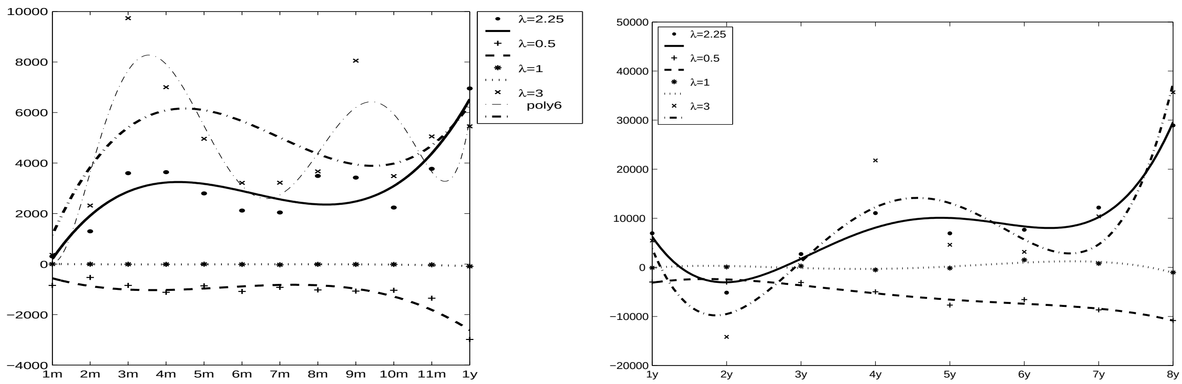
| Fitted model | Coefficient estimates | 95%-confidence interval | Goodness of fit |
|---|---|---|---|
| $\tau \leq 1$ year | | | |
| | $a_3 = 30.22$ | $(12.37, 48.08)$ | |
| $a_3\tau^3 + a_2\tau^2$ | $a_2 = -568.4$ | $(-920.5, -216.4)$ | $R^2$: 0.8143 |
| $+a_1\tau + a_0$ | $a_1 = 3,220$ | $(1,210, 5,230)$ | Adjusted $R^2$: 0.7446 |
| | $a_0 = -2,491$ | $(-5,634, 652.6)$ | RMSE: 833.1 |
| $\tau \geq 1$ year | | | |
| | $b_4 = 275.3$ | $(41.69, 508.9)$ | |
| $b_4\tau^4 + b_3\tau^3$ | $b_3 = -4,842$ | $(-9,067, -617.7)$ | $R^2$: 0.9558 |
| $+b_2\tau^2 + b^1\tau + b_0$ | $b_2 = 29,220$ | $(3,222, 55,220)$ | Adjusted $R^2$: 0.8968 |
| | $b_1 = -67,200$ | $(-129,500, -4,861)$ | RMSE: 3,123 |
| | $b_0 = 48,820$ | $(2,217, 95,420)$ | |

Table 10: Estimated prospective value evolution as a function of the portfolio evaluation frequency for $\lambda = 2.25$, $k = 3$




(a) $\tau \leq 1$ year: third-order polynomials (b) $\tau \geq 1$ year: fourth-order polynomials

Figure 12: Curve-fitting for the prospective value on the two relevant evaluation frequency segments for $\lambda = 2.25$ and $k \in \{0; 3; 10; 20\}$




(a) $\tau \leq 1$ year: third-order polynomials (b) $\tau \geq 1$ year: fourth-order polynomials

Figure 13: Curve-fitting for the prospective value on the two relevant evaluation frequency segments for $\lambda \in \{0.5; 1; 2.25; 3\}$ and $k = 3$

| | Evaluation frequency | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 day | 1 week | 1 month | 2 months | 3 months | 4 months | 5 months |
| $\lambda = 0.5$ | | | | | | | |
| $k = 0$ | -22.41 | -774.78 | -946.11 | -558.91 | -828.40 | -1072.19 | -871.46 |
| $k = 3$ | -23.55 | -809.63 | -846.89 | -532.06 | -843.84 | -1116.66 | -863.46 |
| $k = 10$ | -23.23 | -829.60 | -900.71 | -471.98 | -973.28 | -1386.02 | -916.78 |
| $k = 20$ | -27.13 | -873.80 | -666.05 | -567.40 | -1024.00 | -1876.70 | -923.65 |
| $\lambda = 1$ | | | | | | | |
| $k = 0$ | 0.00 | -0.04 | 0.46 | 2.23 | 6.20 | 11.23 | 21.66 |
| $k = 3$ | -0.03 | -0.25 | -2.14 | -4.69 | -5.89 | -11.38 | -5.37 |
| $k = 10$ | -0.09 | -0.72 | -7.54 | -17.87 | -30.84 | -54.14 | -54.62 |
| $k = 20$ | -0.18 | -1.37 | -12.87 | -25.24 | -45.00 | -87.87 | -78.91 |
| $\lambda = 2.25$ | | | | | | | |
| $k = 0$ | 17.03 | 61.20 | 294.14 | 1322.08 | 3563.08 | 3631.33 | 2933.67 |
| $k = 3$ | 17.11 | 62.11 | 305.26 | 1296.84 | 3597.88 | 3640.67 | 2796.66 |
| $k = 10$ | 17.48 | 64.56 | 319.82 | 1278.58 | 3618.99 | 3693.37 | 2569.44 |
| $k = 20$ | 17.65 | 67.39 | 360.72 | 1217.59 | 3404.81 | 3940.34 | 2437.73 |
| $\lambda = 3$ | | | | | | | |
| $k = 0$ | -44107.17 | 72.67 | 357.35 | 2296.12 | 8591.95 | 8271.97 | 5150.70 |
| $k = 3$ | -21085.72 | 74.13 | 364.20 | 2317.36 | 9735.66 | 7004.40 | 4961.41 |
| $k = 10$ | 32626.32 | 74.94 | 439.08 | 2341.81 | 12750.13 | 8266.93 | 4633.40 |
| $k = 20$ | 109346.83 | 78.37 | 491.56 | 2679.36 | 12763.28 | 6991.29 | 6449.35 |

Table 11: Prospective value evolution for evaluation frequencies up to five months and different parameter values

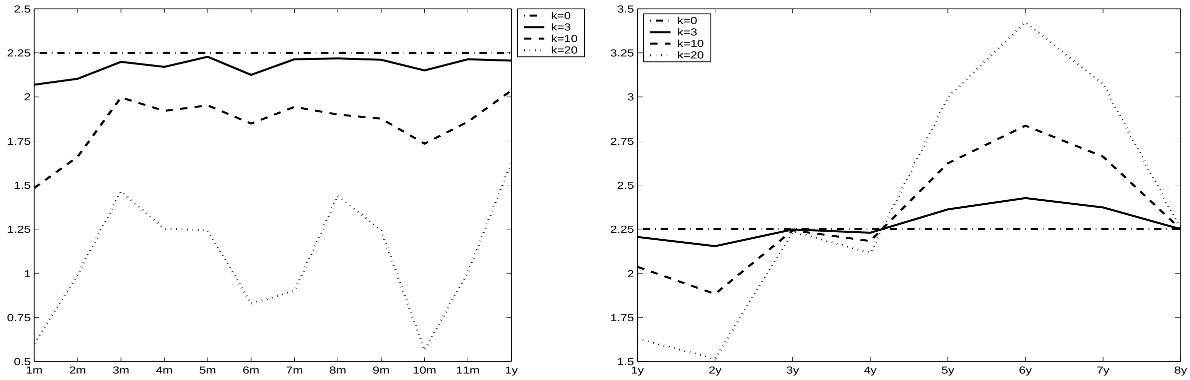
| | Evaluation frequency | | | | | | |
|---|---|---|---|---|---|---|---|
| | 6 months | 7 months | 8 months | 9 months | 10 months | 11 months | 12 months |
| $\lambda = 0.5$ | | | | | | | |
| $k = 0$ | -1115.90 | -993.57 | -1028.76 | -1097.26 | -1023.67 | -1268.19 | -2467.59 |
| $k = 3$ | -1083.11 | -920.89 | -1023.88 | -1071.85 | -1038.19 | -1350.60 | -2984.25 |
| $k = 10$ | -1034.21 | -603.16 | -1017.94 | -1074.05 | -1069.67 | -1541.96 | -4147.73 |
| $k = 20$ | -949.35 | -447.25 | -941.04 | -1218.97 | -929.38 | -1440.39 | -343.41 |
| $\lambda = 1$ | | | | | | | |
| $k = 0$ | 32.16 | 1.42 | 38.99 | 42.88 | 69.67 | 58.28 | 2.97 |
| $k = 3$ | -13.75 | -27.26 | -9.02 | -14.41 | -12.32 | -26.87 | -91.19 |
| $k = 10$ | -85.66 | -73.11 | -86.00 | -119.63 | -125.29 | -149.24 | -265.29 |
| $k = 20$ | -106.93 | -95.89 | -109.68 | -139.73 | -176.94 | -221.82 | -296.50 |
| $\lambda = 2.25$ | | | | | | | |
| $k = 0$ | 2241.31 | 2181.99 | 2805.10 | 3647.67 | 2449.35 | 3837.94 | 5933.59 |
| $k = 3$ | 2120.01 | 2042.33 | 3490.35 | 3423.85 | 2237.29 | 3772.57 | 6955.50 |
| $k = 10$ | 1793.25 | 1610.40 | 3192.06 | 3170.63 | 1804.37 | 3783.51 | 8750.43 |
| $k = 20$ | 1876.08 | 929.33 | 4151.86 | 3949.50 | 1441.88 | 3824.06 | 10263.26 |
| $\lambda = 3$ | | | | | | | |
| $k = 0$ | 3399.66 | 3414.29 | 4293.73 | 7173.57 | 3687.13 | 5054.86 | 7520.72 |
| $k = 3$ | 3214.24 | 3222.21 | 3669.12 | 8055.53 | 3483.86 | 5052.01 | 5454.59 |
| $k = 10$ | 2694.63 | 2682.72 | 5136.55 | 9095.14 | 2980.94 | 6798.98 | 9325.27 |
| $k = 20$ | 3153.57 | -654.15 | 6899.74 | 8882.01 | 2315.59 | 7142.13 | 8458.05 |

Table 12: Prospective value evolution for evaluation frequencies from six months $\tau \leq 1$ year and different parameter values

| | Evaluation frequency | | | | | | |
|---|---|---|---|---|---|---|---|
| | 2 years | 3 years | 4 years | 5 years | 6 years | 7 years | 8 years |
| $\lambda = 0.5$ | | | | | | | |
| $k = 0$ | -3011.96 | -3082.98 | -4964.06 | -7691.76 | -6554.58 | -8661.21 | -10830.26 |
| $k = 3$ | -3011.96 | -3082.98 | -4964.06 | -7691.76 | -6554.58 | -8661.21 | -10830.26 |
| $k = 10$ | -3011.96 | -3082.98 | -4964.06 | -7691.76 | -6554.58 | -8661.21 | -10830.26 |
| $k = 20$ | -3011.96 | -3082.98 | -4964.06 | -7691.76 | -6554.58 | -8661.21 | -10830.26 |
| $\lambda = 1$ | | | | | | | |
| $k = 0$ | 78.63 | 266.14 | -513.17 | -146.00 | 1526.51 | 796.55 | -1022.60 |
| $k = 3$ | 78.63 | 266.14 | -513.17 | -146.00 | 1526.51 | 796.55 | -1022.60 |
| $k = 10$ | 78.63 | 266.14 | -513.17 | -146.00 | 1526.51 | 796.55 | -1022.60 |
| $k = 20$ | 78.63 | 266.14 | -513.17 | -146.00 | 1526.51 | 796.55 | -1022.60 |
| $\lambda = 2.25$ | | | | | | | |
| $k = 0$ | -3661.13 | 3039.21 | 10741.69 | 6573.43 | 7016.68 | 11768.80 | 28926.98 |
| $k = 3$ | -5160.30 | 2706.18 | 11063.19 | 6942.36 | 7657.57 | 12192.57 | 28926.98 |
| $k = 10$ | -4767.64 | 2101.49 | 11245.85 | 7803.19 | 9152.97 | 13181.36 | 28926.98 |
| $k = 20$ | -3983.72 | 1605.80 | 10672.11 | 9032.94 | 11289.26 | 14593.92 | 28926.98 |
| $\lambda = 3$ | | | | | | | |
| $k = 0$ | -14104.02 | 945.54 | 19154.46 | 4220.01 | 2415.15 | 9731.33 | 35683.30 |
| $k = 3$ | -14158.12 | 1736.78 | 21797.33 | 4595.20 | 3163.75 | 10388.47 | 35683.30 |
| $k = 10$ | -14284.37 | 3042.36 | 30479.43 | 5470.66 | 4910.49 | 11921.81 | 35683.30 |
| $k = 20$ | -14464.71 | 10135.65 | 31160.75 | 6721.31 | 7405.84 | 14112.28 | 35683.30 |

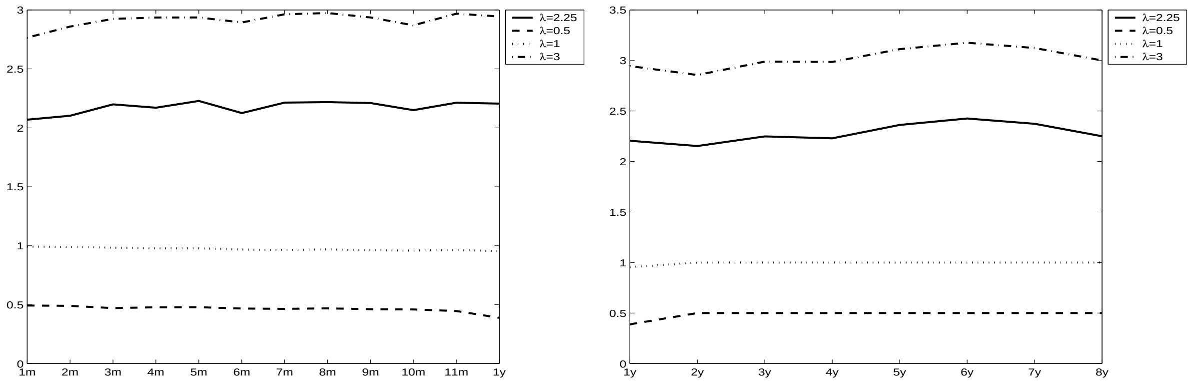
Table 13: Prospective value evolution for evaluation frequencies higher than one year and different parameter values

### 5.4 The actual attitude towards financial losses as a function of the portfolio evaluation frequency




(a) $\tau \leq 1$ year (b) $\tau \geq 1$ year

Figure 14: The loss aversion index on the two relevant evaluation frequency segments for $\lambda = 2.25$ and $k \in \{0; 3; 10; 20\}$




(a) $\tau \leq 1$ year (b) $\tau \geq 1$ year

Figure 15: The loss aversion index on the two relevant evaluation frequency segments for $\lambda \in \{0.5; 1; 2.25; 3\}$ and $k = 3$

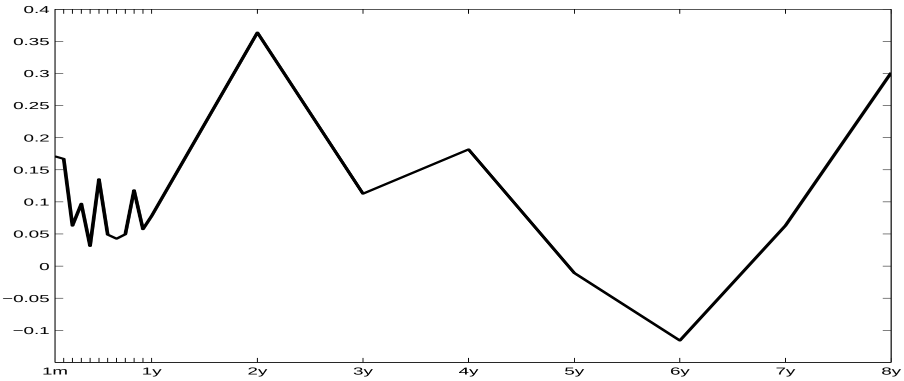


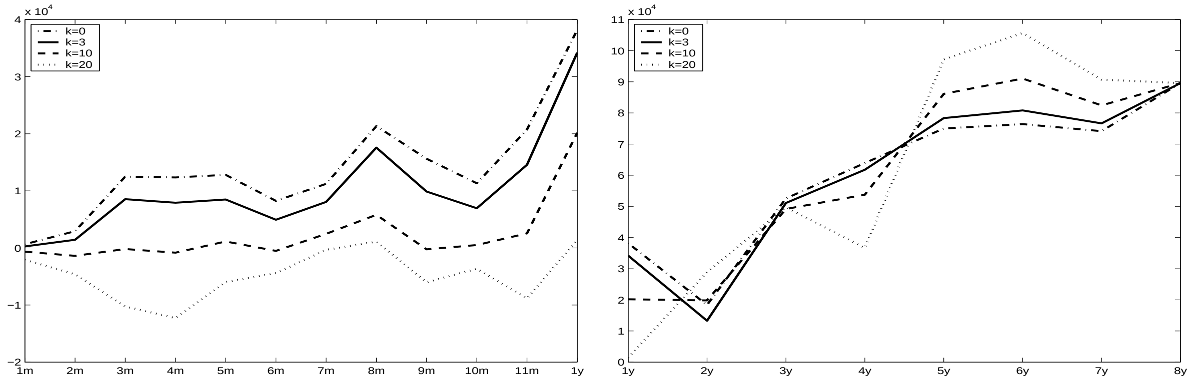

Figure 16: The average absolute cushion $1 - z$ for different evaluation frequencies, $\lambda = 2.25$, and $k = 3$

| **Cushions** | **Sensitivity to past losses** $k^s$ (small) | $k^l$ (large) |
|---|---|---|
| $z < 1$ (gains) | likely ($\tilde{\lambda}_t^s < \lambda$) | not likely ($\tilde{\lambda}_t^l < \tilde{\lambda}_t^s < \lambda$) |
| $z > 1$ (losses) | likely ($\tilde{\lambda}_t^s > \lambda$) | likely ($\tilde{\lambda}_t^l > \tilde{\lambda}_t^s > \lambda$) |

Table 14: Possible scenarios for LAi, where $\tilde{\lambda}_t^s$ ($\tilde{\lambda}_t^l$) stand for LAi under small (large) values of $k$

| **Fitted model** | **Coefficient estimates** | 95%-**confidence interval** | **Goodness of fit** |
|---|---|---|---|
| $\tau \leq 1$ year | *A first-order power specification performs identically well. | | |
| $a_3\tau^3 + a_2\tau^2 + a_1\tau + a_0$ | $a_3 = 0.0004866$ | $(-0.000337, 0.00131)$ | |
| | $a_2 = -0.01146$ | $(-0.0277, 0.004782)$ | $R^2$: 0.6055 |
| | $a_1 = 0.08541$ | $(-0.007317, 0.1781)$ | Adjusted $R^2$: 0.4576 |
| | $a_0 = 1.994$ | $(1.849, 2.139)$ | RMSE: 0.03844 |
| $\tau \geq 1$ year | | | |
| $b_3\tau^3 + b_2\tau^2 + b_1\tau + b_0$ | $b_3 = -0.006142$ | $(-0.01023, -0.00205)$ | |
| | $b_2 = 0.07538$ | $(0.01961, 0.1312)$ | $R^2$: 0.9177 |
| | $b_1 = -0.2239$ | $(-0.4462, -0.0015)$ | Adjusted $R^2$: 0.856 |
| | $b_0 = 2.361$ | $(2.115, 2.607)$ | RMSE: 0.03592 |

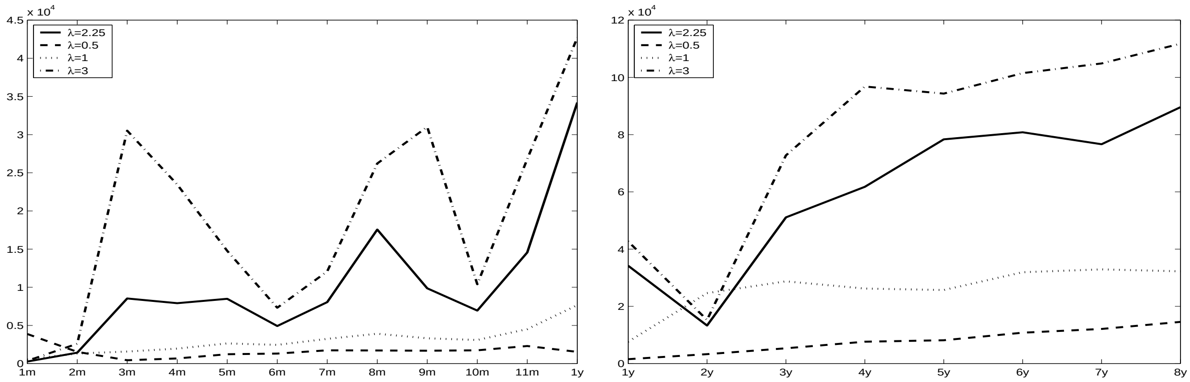
Table 15: Estimated evolution of the loss aversion index as a function of the portfolio evaluation frequency for $\lambda = 2.25$, $k = 3$




(a) $\tau \leq 1$ year

(b) $\tau \geq 1$ year

Figure 17: The global first-order risk aversion on the two relevant evaluation frequency segments for $\lambda = 2.25$ and $k \in \{0; 3; 10; 20\}$




(a) $\tau \leq 1$ year

(b) $\tau \geq 1$ year: lines

Figure 18: The global first-order risk aversion on the two relevant evaluation frequency segments for $\lambda \in \{0.5; 1; 2.25; 3\}$ and $k = 3$

| **Fitted model** | **Coefficient estimates** | **95%-confidence interval** | **Goodness of fit** |
|---|---|---|---|
| $\tau \leq 1$ year | | | |
| $a_3\tau^3 + a_2\tau^2$ $+a_1\tau + a_0$ | $a_3 = 112.7$<br>$a_2 = -1,999$<br>$a_1 = 11,120$<br>$a_0 = -10,940$ | $(4.283, 221.2)$<br>$(-4,138,140)$<br>$(-1,092,23,330)$<br>$(-30,030,8,161)$ | <br>$R^2$: 0.7668<br>Adjusted $R^2$: 0.6793<br>RMSE: 5062 |
| $\tau \geq 1$ year | | | |
| $b_1\tau + b_0$ | $b_1 = 9,644$<br>$b_0 = 17,310$ | $(4,917,14,370)$<br>$(-6,558,41,190)$ | $R^2$: 0.8059<br>Adjusted $R^2$: 0.7736<br>RMSE: 12,520 |

Table 16: Estimated evolution of the global first-order risk aversion as a function of the portfolio evaluation frequency for $\lambda = 2.25$, $k = 3$

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