Darmstadt Discussion Papers in Economics

Investors Facing Risk II: Loss Aversion and Wealth Allocation When Utility Is Derived From Consumption and Narrowly Framed Financial Investments

Erick W. Rengifo, Emanuela Trifan

Nr. 181

Arbeitspapiere
des Instituts für Volkswirtschaftslehre
Technischen Universität Darmstadt

Investors Facing Risk II

Loss Aversion and Wealth Allocation When Utility Is Derived From Consumption and Narrowly Framed Financial Investments*

Erick W. Rengifo† Emanuela Trifan‡

February 2007

Abstract

This paper studies the attitude of non-professional investors towards financial losses and their decisions concerning wealth allocation among consumption, risky, and risk-free financial assets. We employ a two-dimensional utility setting in which both consumption and financial wealth fluctuations generate utility. The perception of financial wealth is modelled in an extended prospect-theory framework that accounts for both the distinction between gains and losses with respect to a subjective reference point and the impact of past performance on the current perception of the risky portfolio value. The decision problem is addressed in two distinct equilibrium settings in the aggregate market with a representative investor, namely with expected and non-expected utility. Empirical estimations performed on the basis of real market data and for various parameter configurations show that both settings similarly describe the attitude towards financial losses. Yet, the recommendations regarding wealth allocation are different. Maximizing expected utility results on average in low total-wealth percentages dedicated to consumption, but supports myopic loss aversion. Non-expected utility yields more reasonable assignments to consumption but also a high preference for risky assets. In this latter setting, myopic loss aversion holds solely when financial wealth fluctuations are viewed as the main utility source and in very soft form.

Keywords: prospect theory, Value-at-Risk, loss aversion, expected utility, non-expected utility

JEL Classification: C32, C35, G10

*The authors would like to thank Horst Entorf for helpful suggestions. The usual disclaimers apply.

†Fordham University, New York, Department of Economics. 441 East Fordham Road, Dealy Hall, Office E513, Bronx, NY 10458, USA, phone: +1(718) 817 4061, fax: +1(718) 817 3518, e-mail: rengifomina@fordham.edu

‡Darmstadt University of Technology, Institute of Economics, Department of Applied Economics and Econometrics, 15 Marktplatz, 64283 Darmstadt, Germany, phone: +49(0) 6151 162636, fax: +49 (0)6151 165652, e-mail: trifan@vwl.tu-darmstadt.de

1 Introduction

Optimal allocation of resources among different activities that generate utility represents a main decision problem in everyday life. For each individuum, the first and most important source of individual utility is consumption. Additionally, people who are active in financial markets derive utility from their investments. This paper addresses the attitude towards financial losses of individual (non-professional) investors who narrowly frame risky investments and change current perceptions subject to the past performance of these investments. Equally, we analyze how (non-professional) investors split their money between consumption and (risky and risk-free) financial projects as a consequence of their loss attitude.

Our work is based on the theoretical framework introduced in Barberis, Huang, and Santos (2001) and developed in Barberis, Huang, et al. (2003, 2004). Several ideas developed in these papers have already been incorporated by Rengifo and Trifan (2006) into the portfolio optimization setting developed in Campbell, Huisman, and Koedijk (2001). In particular, Rengifo and Trifan (2006) describe how individual perceptions of risky financial investments impact on the optimal composition of a risky portfolio and the money to be invested in risk-free assets. Their focus lies thus on capital allocation decisions of non-professional investors, where finding the optimal mix of *risky* assets is assumed to represent the task of professional portfolio managers. Meanwhile, non-professional investors are left with the choice between the risky portfolio as a whole and a risk-free asset. In this context, individual utility is considered to exclusively originate in financial wealth fluctuations. Thus, the designed utility setting in Rengifo and Trifan (2006) can be regarded as one-sided.

The present paper aims at enlarging this perspective to what we denote as two-dimensional utility. Specifically, our setting accounts for consumption as additional source of utility besides the utility derived from financial investments. Our non-professional investors have now to decide upon the optimal wealth allocation between consumption and financial investments in general, where the latter category offers a further choice between the risky portfolio and a risk-free asset. We adopt the formal views in Rengifo and Trifan (2006) regarding the subjectively perceived value of risky investments (denoted as the prospective value) and how it enters the decision problem of non-professional investors. In addition, we rely on two theoretical approaches proposed in Barberis and Huang (2004a,b), according to which investors decisions rely on the maximization of either expected utility or of recursive non-expected utility with first-order risk aversion. In both cases, the utility function is shaped in order to account for the narrow framing of financial investments and for the influence of past portfolio performance on current perceptions of risky investments. Moreover, for a better description of the individual attitude towards financial losses, two further measures used in Rengifo and Trifan (2006) are adopted, namely the loss aversion index (LAi) and the global first-order risk aversion (gRA).

In order to analyze the loss attitude and the wealth allocation in this context, we proceed in line with Barberis and Huang (2004a,b). Thus, we first derive necessary and sufficient equilibrium conditions on the aggregate market with a representative investor. Second, fixing general market parameters (such as the dynamics of consumption and of expected returns), these conditions serve to derive the equilibrium expressions of the variables of interest. These variables are either common to both considered approaches with expected and non-expected utility (such as the prospective value, the loss aversion coefficient, and further measures of the attitude towards losses) or specific (such as the time-discounting factor of the utility function in the expected-utility setting, or the percentages of total wealth dedicated to consumption and of post-consumption wealth invested in risky assets in the non-expected utility setting).

In the sequel, the theoretical part developed so far is empirically implemented and amended. To this end, we rely on the same data set as in Rengifo and Trifan (2006), consisting of the SP500 and the 10-year bond nominal returns, as proxies for a well diversified risky portfolio and the risk-free investment, respectively. In addition, we employ aggregate per-capita consumption data that allow us to analyze the investor loss attitude and the decisions concerning the wealth allocation for two different evaluation frequencies of the risky-portfolio performance (of one year and three months). We simulate how non-professional investors who follow the logic of our model make decisions in an environment where consumption and financial markets are characterized by general parameters derived from the sets of real data at hand. In so doing, we consider various configurations of the individual (behavioral) parameters (such as the consumption-based risk aversion, the narrow-framing degree, the penalties imposed on past financial losses, and the initial coefficient of loss aversion). Moreover, we attempt to avoid the impossibility of covering current consumption needs from financial revenues over the entire investing interval, by considering that investors periodically dispose of exogenous additional incomes. In order to investigate the sensitivity of our model to this assumption, we further consider different levels of the additional income that are further shaped to render comparable the two approaches with expected and non-expected utility.

In both considered utility settings, we estimate the prospective values ascribed to risky investments and derive the corresponding loss aversion coefficients in equilibrium. These estimates also serve to compute the equilibrium values of the extended measures of individual attitude to financial risk, namely the loss aversion index and the global first-order risk aversion. In addition, the discounting factor of the utility function is additionally estimated from the equilibrium conditions in the expected-utility setting.

Here, the wealth percentages dedicated to consumption, risky, and riskless assets can be assessed on average only. By contrast, the non-expected utility approach delivers direct estimates of the proportions of total wealth allocated to consumption and of post-consumption wealth invested in risky assets.

The empirical findings can be summarized as follows. On the one hand, maximizing expected or non-expected utility derived from consumption and narrowly framed financial investments, generates similar attitudes towards financial losses. In particular, the loss aversion coefficient in equilibrium lies close to the "neutral" value of 1, which shows that investors who derive utility from twofold source are more relaxed towards financial losses than suggested by the original prospect theory (that proposes the value of 2.25 for the same coefficient). Moreover, the same coefficient increases for either higher degrees of narrow framing, or when no penalties are imposed on past losses, other things being equal. Mostly, it also decreases c.p. as the risky performance is more frequently evaluated, but the direction of this variation is more sensitive with respect to the magnitude of the additional income in the non-expected utility setting. In addition, the loss aversion index in equilibrium follows closely in value and the evolution pattern of the simple loss aversion coefficient. The global first-order risk aversion exhibits similar development and appears to more consistently describe the actual investor attitude towards financial losses. Moreover, both approaches entail negative estimates of the prospective value in equilibrium, suggesting that financial investments decrease the overall utility. A final common finding refers to the fact that both an excessive consumption-related risk aversion and the lack of narrow framing of financial investments entail implausible equilibrium-estimates.

On the other hand, the two approaches provide different recommendations regarding the wealth allocation between consumption and (risky and riskless) financial assets. Note that in both settings, this allocation substantially varies with the magnitude of the additional income. Investors who maximize expected utility appear to be on average very open to financial investments, as they consume only small fractions of their total wealth (less than one-fifth). The myopic loss aversion holds in this setting, as the total-wealth percentages invested in risky assets diminish to almost one-third when the risky performance is evaluated every three-months instead of once a year. By contrast, reaching the aggregate equilibrium with non-expected utility requires more reasonable wealth percentages dedicated to consumption (namely over one-third). In exchange, investors are now very open to risky investments and even borrow money to increase the value of their risky portfolios. The resulting parts of total wealth that flow into risky investments are substantially higher but also more variable subject to the magnitude of the additional income, relative to the expected-utility setting. Finally, myopic loss aversion holds under the maximization of non-expected utility only when investors regard narrowly framed financial investments as a more important source of utility relative to consumption. Even in such situations the sums allocated to risky assets are only minimally reduced as their performance is revised more often.

The remainder of the paper is organized as follows. The theoretical framework is presented in Section 2. In this context, Section 2.1 provides a brief review of the general purpose of the model in Rengifo and Trifan (2006) and refreshes the definitions of variables of interest for the present paper. The extension of this model to the two-dimensional utility setting is undertaken in Section 2.2, which details the approaches with expected and non-expected utility. Section 3 presents the empirical implementation of our theoretical model for the expected-utility framework in Section 3.2, and the non-expected utility setting in Section 3.3. The main findings under these two theoretical approaches are confronted in the subsequent Section 3.4. Finally, Section 4 summarizes our findings. Further numerical results are included in the Appendix.

2 Theoretical model

This section presents the theoretical framework describing how non-professional investors perceive financial risks and accordingly allocate their wealth between consumption and financial investments in order to maximize perceived utility. In line with Barberis and Huang (2004a,b), we adopt two distinct formulations of the maximization problem, first around expected utility and, second, around recursive non-expected utility with first-order risk aversion. Both settings account for narrow framing of financial projects and for the influence of past performance on the perceived prospective value of risky investments.

2.1 A one-dimensional utility framework

One of the most important decisions in everyday life is how to allocate money among different type of activities. These activities may be either necessary and/or can generate further revenues. In the latter category, investing in financial assets has nowadays become one of the most popular alternatives. This widespread trend of ordinary people turning into "investors" is due at least in part to the formidable accessibility of information concerning financial markets, at almost no cost and in almost no time. Under the pressure of the huge amount and intensity of such information, even non-professional investors may have no choice but to become overly concerned with their financial investments. This phenomenon of putting excessive emphasis on financial investments is denoted in technical terms as "narrow framing". Under narrow framing, allocating the "right" amount of money first to financial investments in general, then across different financial assets, turns into a central decision. Also, financial investments are perceived as distinct and overly important generators of individual utility. This dissociates the decisions upon wealth allocation from the naturally larger context with multiple utility generators (such as consumption or other factors that do not exclusively apply to financial markets).

Drawing on this idea, Rengifo and Trifan (2006) model the attitude towards financial risks and the decision making of non-professional investors regarding the optimal wealth allocation among different financial assets. In so doing, they assume that investors derive utility merely from financial investments. Their theoretical analysis develops in the portfolio optimization framework suggested in Campbell, Huisman, and Koedijk (2001), where market risk is measured by the Value-at-Risk (VaR) and enters the optimization problem in form of a specific constraint. In particular, individual non-professional investors fix in a first step their desired VaR-levels (denoted VaR*) according to their subjective preferences and perceptions. The choice of VaR* takes place *outside* of the risky portfolio so to speak, in the sense that it precedes and hence it is not influenced by its composition. The individually chosen VaR* is then communicated to professional portfolio managers. They are in charge of the optimal money allocation among risky assets, in other words of decisions *inside* of the risky portfolio. In a second step, considering the given VaR*, portfolio managers determine the optimal composition of the risky portfolio and implicitly the sum to be invested in risk-free assets. In so doing, professional managers help their non-professional clients to solve the problem of optimally allocating money between risky and risk-free assets.

The present work builds on the model in Rengifo and Trifan (2006) and extends it for the case when investors derive utility not only from financial investments, but also from consumption. Before entering into the details of this extended framework, we briefly review the main model structure and the notions defined and applied in Rengifo and Trifan (2006).

In order to quantify the formation of the desired VaR*-level in investor minds, Rengifo and Trifan (2006) adopt the extended prospect theory framework developed in Barberis, Huang, and Santos (2001). This framework models individual perceptions of risky projects. Accordingly, the subjective perception of one unit of risky investment is captured by an *extended value function* v_{t+1}. As in the original prospect theory of Kahneman and Tversky (1979, 1992), this value function accounts for the distinct perception of gains and losses with respect to a subjective reference point and the higher reluctance to losses. In addition, it is designed to capture the possible influence of past performance on the current risk perception. We apply the definitions of the value function proposed in Section

2.2 of Rengifo and Trifan (2006), namely:

$$v_{t+1} = \begin{cases} S_t(R_{t+1} - R_{ft}) & , \text{for } R_{t+1} \geq R_{ft} \\ \lambda S_t(R_{t+1} - R_{ft}) + (\lambda - 1)(S_t - Z_t)R_{ft} & , \text{for } R_{t+1} < R_{ft} \end{cases} \quad , \text{for } z_t \leq 1 \qquad (1)$$

and

$$v_{t+1} = \begin{cases} S_t(R_{t+1} - R_{ft}) & , \text{for } R_{t+1} \geq R_{ft} \\ \lambda S_t(R_{t+1} - R_{ft}) + k(Z_t - S_t)(R_{t+1} - R_{ft}) & , \text{for } R_{t+1} < R_{ft} \end{cases} \quad , \text{for } z_t > 1, \quad (2)$$

where $k > 0$ represents the sensitivity to past losses and $\lambda > 0$ the individual loss aversion coefficient (with $\lambda \geq 1$ indicating loss aversion in strict sense). Moreover, R_{t+1} stands for the next-period portfolio returns and R_{ft} for the risk-free returns. Finally, S_t denotes the current value of the risky investment, while Z_t is a benchmark level for past portfolio performance, so that $S_t - Z_t$ accounts for the so-called *cushion* of past gains and/or losses generated by the risky portfolio.

Denoting the equity premium by $x_{t+1} = R_{t+1} - R_{ft}$ and the probability of experiencing past gains by $\pi_t = P(Z_t \leq S_t)$, Equations (1) and (2) can be pooled to a single expression that accounts for both cases with prior gains ($z_t \leq 1$) and prior losses ($z_t > 1$), namely:

$$v_{t+1} = \begin{cases} S_t x_{t+1} & , \text{for } x_{t+1} \geq 0 \\ [\lambda S_t - (1 - \pi_t)k(S_t - Z_t)]x_{t+1} + \pi_t(\lambda - 1)R_{ft}(S_t - Z_t) & , \text{for } x_{t+1} < 0, \end{cases}$$

On the basis of the perception of financial investments captured in the value functions from Equations (1) and (2), Rengifo and Trifan (2006) define the *maximum risk level* desired (accepted) by individual (non-professional) investors VaR* as:

$$\begin{aligned} \text{VaR}^*_{t+1} &= \lambda S_t E_t[x_{t+1}] \\ &+ [(\pi_t - \varphi\sqrt{\pi_t(1 - \pi_t)})(\lambda - 1)R_{ft} - (1 - \pi_t + \varphi\sqrt{\pi_t(1 - \pi_t)})kE_t[x_{t+1}]](S_t - Z_t), \end{aligned} \qquad (3)$$

where $E_t[x_{t+1}] = E_t[R_{t+1}] - R_{ft}$ represents the expected equity premium.

As mentioned above, once this VaR* has formed in investor minds it is communicated to the portfolio manager in the form of a fixed (so to speak "portfolio-exogenous") risk level. It hence flows into the problem of optimization inside the risky portfolio as risk constraint. According to Campbell, Huisman, and Koedijk (2001), finding the optimal capital allocation implies the derivation of the *optimal investment in risk-free assets*. This is referred as the optimal amount of money to be borrowed or lent and yields:

$$B_t = \frac{\text{VaR}^* + \text{VaR}}{R_{ft} - q_t(w_t^*, \alpha)}, \tag{4}$$

where $B_t > 0$ ($B_t < 0$) stands for borrowing (lending), VaR=$W_t[q_t(w_t^*, \alpha) - 1]$ represents the so-called *portfolio Value-at-Risk*, w_t^* the optimal portfolio weights, and $q_t(w_t, \alpha)$ the quantile of the distribution of portfolio returns at a given confidence level α.

Central to the analysis conducted in Rengifo and Trifan (2006), on which the present work is based, is the derivation of the so-called *prospective value* V_{t+1}. This variable captures the subjectively perceived utility of the risky portfolio, which non-professional investors aim at maximizing.[1] According to Equation (2.23) in Rengifo and Trifan (2006), the prospective value can be formally defined as:

$$V_{t+1} = [\omega_t + (1-\omega_t)\lambda]S_t E_t[x_{t+1}] + (1-\omega_t)[\pi_t(\lambda - 1)R_{ft} - (1-\pi_t)kE_t[x_{t+1}]](S_t - Z_t), \tag{5}$$

where $\omega_t = P(E_t[R_{t+1}] \geq R_{ft})$ is the probability of having a positive expected equity premium $E_t[x_{t+1}] \geq 0$. Moreover, Rengifo and Trifan (2006) distinguish in Equation (5) between the *PT-effect* and the *cushion effect.* The former refers to the first term on the right-hand side and stems from the value function formulation in the original prospect theory. The latter effect denotes the second term on the right-hand side of Equation (5) and has as its source the prior gains and losses accumulated from trading the risky portfolio. The present work aims at determining the prospective value ascribed by the representative investor who derives utility from both consumption and financial investments in the market equilibrium.

From the prospective value in equilibrium, we subsequently determine the equivalent coefficient of loss aversion λ_{t+1}. Formally, this yields:

$$\lambda_{t+1} = \frac{V_{t+1} - \omega_t S_t E_t[x_{t+1}] + (1-\omega_t)\{\pi_t R_{ft} + (1-\pi_t)kE_t[x_{t+1}]\}(S_t - Z_t)}{(1-\omega_t)\{S_t E_t[x_{t+1}] + \pi_t R_{ft}(S_t - Z_t)\}}. \tag{6}$$

This coefficient plays a central role in our model, as it represents a measure of the attitude towards financial losses and previous research suggests concrete values for it (such as 2.25 in the prospect theory).

As Rengifo and Trifan (2006) note, the joint impact of the loss aversion coefficient λ and of past negative performance k changes the actual investor aversion to financial losses.

[1]Note that the term "maximization" shall be understood here in a general, lax sense (rather as "optimization"). In essence, the non-professional investors in Rengifo and Trifan (2006) are considered rather unsophisticated, and hence do not have to tackle any maximization problem in strict mathematical sense. Yet, these investors (as every person) attempt to perform most useful actions, given real constraints (such as their individual loss aversion, the past performance of their risky portfolios, etc.). Recall also that utility is assumed to exclusively emanate from investment decisions.

Further measures then appear necessary to capture additional and more realistic aspects of this loss attitude. The first of them applied in Rengifo and Trifan (2006) is the *index of loss aversion* (shortly LAi). Introduced in Köbberling and Wakker (2005), this index represents the ratio of the left and right derivatives of the value function at the reference point:

$$\tilde{\lambda}_t = \frac{\lambda S_t - (1-\pi_t)k(S_t - Z_t)}{S_t} = \lambda - (1-\pi_t)k(1-z_t). \quad (7)$$

Second, as loss aversion stands for risk aversion of first order in the loss domain, Rengifo and Trifan (2006) introduce the notion of *global first-order risk aversion* (shortly gRA). It is defined as the first derivative of the prospective value with respect to the expected equity premium, namely:

$$\Lambda_t = \frac{\partial V_{t+1}}{\partial E_t[x_{t+1}]} = [\omega_t + (1-\omega_t)\lambda]S_t - (1-\omega_t)(1-\pi_t)k(S_t - Z_t) = S_t[\omega_t + (1-\omega_t)\tilde{\lambda}_t]. \quad (8)$$

It is important to note that LAi is to be interpreted analogously to the simple coefficient of loss aversion, namely that increasing values are equivalent with an enhanced aversion towards financial losses. By contrast, higher gRA-values denote a more relaxed loss attitude.[2] In the empirical part of the present work, we analyze the evolution of LAi and gRA in our extended equilibrium framework with two-dimensional utility.

2.2 A two-dimensional utility framework

As noted in Section 2.1, Rengifo and Trifan (2006) consider that investors are merely concerned with financial investments and the utility they generate. Specifically, investors exclusively aim at the maximization of the subjectively perceived next-period value of their financial investments. In practice, such considerations appear to be better suited to professional than to non-professional investors. The activity of the former demands a strictly investment-oriented perspective, and their main task reduces to making money that is going to be reinvested in financial markets. By contrast, non-professional investors sooner regard financial investments as a source of income dedicated to covering consumption needs.[3] In other words, for non-professional investors consumption should be the main generator of individual utility. Yet, as financial investments are usually risky and non-professional investors risk averse, the attention payed to financial results may be excessive. As already mentioned, this phenomenon is denoted as *narrow framing* and is driven by the fear of registering losses when facing financial risks. Narrow framing renders

[2] According to Equation (8), gRA directly reflects the changes in the prospective value V. Recall that the latter is proportional to the attractiveness of financial investments.

[3] Campbell, Huisman, and Koedijk (2001) note on p. 1800 that the simple VaR-framework without consumption is sufficiently informative for describing decision making of (non-professional) investors under risk.

the importance of financial investments as source of utility comparable to the relevance of consumption.

Based on these considerations, our work extends the setting in Rengifo and Trifan (2006) by allowing for *two sources of individual utility*, namely financial wealth fluctuations and consumption. The present section details the theoretical background of this original contribution.

In effect, the wealth allocation problem of non-professional investors tackled in the one-dimensional utility framework of Rengifo and Trifan (2006) is now augmented with an extra step. This consists of splitting money between consumption and financial investments. Strictly speaking, this step should be placed on a time axis *before* the decision about how much money to invest in different types of financial assets. The reason is that nobody can decide upon partitioning a certain sum among risky and riskless assets, without having already determined how much money has to be assigned to financial projects *in total* (i.e. after consumption). However, given that the performance of risky investments is measured in general (and in our approach in particular) with respect to risk-free assets,[4] we can formally merge these steps into a single decision issue. The common goal is then the maximization of total utility derived from consumption and risky (relative to risk-free) financial investments.

Following Barberis and Huang (2004b), we consider an aggregate market which lacks perfect substitution, hence we can focus on absolute pricing and avoid possible arbitrage opportunities generated by narrow framing. In this setting, the total utility is formulated in order to account for the above-mentioned twofold origin. Thus, we denote the utility derived from consumption by $U(C)$ and the one from financial wealth changes by $\tilde{V}_{t+1}$.[5] Accordingly, the total utility represents a sum of discounted utilities of consumption and of perceived values of financial investments:[6]

$$U = U(C) + \tilde{V} = \sum_{t=0}^{\infty}[\rho^t U(C_t) + \rho^{t+1} b_t \tilde{V}_{t+1}], \tag{9}$$

where $\rho < 1$ stands for the discounting factor.

According to the above Equation (9), the current consumption is discounted with ρ^t, while the prospective value needs to be discounted with ρ^{t+1} as it encompasses subjective perception of the the next-period performance.[7] In line with Barberis, Huang, and Santos

[4]Recall that the reference point of the perceived value of the risky prospect in Definitions (1) and (2) is R_{ft}.

[5]Strictly speaking, $\tilde{V}_{t+1}$ corresponds to the prospective value V_{t+1} as defined in Equation (5), *before* taking expectations, as in our framework the prospective value is generated by the value function weighted by pure probabilities hence reduces to an expected value.

[6]In the empirical part, we consider a finite investment duration T that is however sufficiently long in order to allow for reaching an equilibrium.

[7]Recall that the prospective value encompasses the future return R_{t+1}.

(2001), b_t represents an exogenous scaling factor designed to map the perceived value of gains and losses into consumption units. In our model, it follows the rule stated in their Equation (11): $b_t = b_0 \bar{C}_t^{-\gamma}$, where $\bar{C}_t$ represents the exogenous[8] aggregate per-capita consumption at time t and b_0 measures the degree of narrow framing. Finally, γ is the consumption-related coefficient of risk aversion.

In line with Barberis and Huang (2004a), it is now possible to develop an equilibrium framework in the aggregate market with a representative investor.[9] We derive the equilibrium conditions in two different settings, first when this investor maximizes expected utility, and second when a recursive non-expected utility function with first-order risk aversion is optimized. Throughout, we formally incorporate the assumptions of narrow framing and dependence of current decisions on past portfolio performance.

2.2.1 The expected-utility approach

First, we consider the approach adopted in Barberis, Huang, and Santos (2001), where the representative investor aims at maximizing total *expected utility* generated by both consumption and financial wealth changes.[10] We refer to the utility of consumption in traditional CRRA-terms, namely $U(C_t) = \dfrac{C_t^{1-\gamma}}{1-\gamma}$. The utility of financial investments is measured by the prospective value in Equation (5). Thus, the maximization problem in the above Equation (9) yields:

$$E_t[U] = E_t\left[\sum_{i=0}^{\infty}\left(\rho^i \frac{C_i^{1-\gamma}}{1-\gamma} + b_0 \rho^{i+1} \bar{C}_i^{-\gamma} v(G_{i+1})\right)\right] \underset{C_t,\theta_t}{\longrightarrow} \max., \tag{10}$$

where v is the value function from Equations (1) and (2) and

$$G_{t+1} = \theta_t (W_t - C_t)(R_{t+1} - R_{ft}) \tag{11}$$

represents the change in value of the risky investment. Moreover, W_t stands here for the *total* wealth and θ_t for the fraction of *post-consumption* wealth allocated to the risky portfolio.

The following equations provide for the formal compatibility of the one-dimensional utility framework in Rengifo and Trifan (2006) and the current two-dimensional utility framework. Specifically, the post-consumption wealth proportion put in risky assets θ_t,

[8] The exogeneity refers here to the subjective viewpoint of the individual investor. It points out the fact that b_t is independent of every individual feature related to risk or loss aversion.

[9] Henceforth, we use the denominations of "representative investor" and "investors" interchangeably, where the latter represent a group with homogenous preferences. In essence, the actions of all investors in equilibrium can be summarized by the corresponding choices of the representative investor.

[10] As demonstrated in Barberis, Huang, and Santos (2001), this framework can explain the emergence of equity premiums of the magnitude observed in practice.

the current value of the risky investment S_t, as well as the amount of money borrowed ($B_t > 0$) or lent ($B_t < 0$) are reformulated in order to correspond to the total wealth W_t, that now also comprises consumption and yield:

$$\theta_t = \frac{W_t - C_t + B_t}{W_t - C_t} \tag{12a}$$

$$S_t = \theta_t(W_t - C_t) \tag{12b}$$

$$B_t = (W_t - C_t)\frac{\text{VaR}^* + \text{VaR}}{(W_t - C_t)R_{ft} - \text{VaR}}. \tag{12c}$$

Note that the post-consumption wealth fraction allocated to risk-free assets becomes $1 - \theta_t = -B_t/(W_t - C_t)$. Also, the next period total results from the current financial investment and can be expressed as:[11]

$$W_{t+1} = (W_t - C_t)[\theta_t R_{t+1} + (1 - \theta_t)R_{ft}]. \tag{13}$$

Noting that the maximization in Equation (10) is carried out for both consumption C_t and the wealth fraction invested in risky assets θ_t (hence the value of the risky investment S_t), the corresponding Euler equations for optimality at equilibrium yield:[12]

$$\rho R_f E\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma}\right] = 1 \tag{14a}$$

$$\rho E\left[R_{t+1}\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma}\right] + b_0\rho\bar{C}_t^{-\gamma}E[\bar{v}(G_{t+1})] = 1. \tag{14b}$$

Moreover, $E[\bar{v}(G_{t+1})]$ is identical to the prospective value at equilibrium that we denote by $\bar{V}$. Our goal is to then provide an empirical value for $\bar{V}$ according to Equation (6) for the loss aversion parameter λ of the representative investor in equilibrium.

In order to perform the estimation of $\bar{V}$, additional assumptions concerning the consumption and return dynamics are needed. In line with Barberis and Huang (2004a), Equations (68)-(70), we take:[13]

---

[11] A part C_{t+1}/W_{t+1} is subsequently allocated to consumption, but consumption generates merely utility and not wealth.

[12] See Equation (27), (28) from Barberis, Huang, and Santos (2001).

[13] Note that Barberis, Huang, and Santos (2001) assume that returns develop following the dividends payed by the risky asset $R_{t+1} = (P_{t+1} + D_{t+1})/P_t$. If this can be considered as a sound assumption on an annual basis, an annoying problem emerges in terms of higher portfolio evaluation frequencies. While prices vary daily, dividends are released only once every three months or even at longer time intervals. (For instance, according to data from finance.yahoo.com, the mean frequency of dividends releases amounts to approximatively 4.5 months.) This generates a non-smooth dividend evolution that accounts on one hand for the dates of dividend release (when dividends truly change in value) and, on the other hand, for the in-between periods (when no dividends are distributed to investors, meaning they

$$\log\left(\frac{C_{t+1}}{C_t}\right) = c + \sigma_c \epsilon_{t+1} \tag{15a}$$

$$\log(R_{t+1}) = r + \sigma_r \eta_{t+1} \tag{15b}$$

$$\begin{pmatrix} \epsilon_{t+1} \\ \eta_{t+1} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \sigma_{cr} \\ \sigma_{cr} & 1 \end{pmatrix}\right), \text{ i.i.d. over time.} \tag{15c}$$

Thus, for a constant risk-free rate R_f the equilibrium Equations (14) entail:[14]

$$\exp\left(-\gamma c + \frac{\gamma^2 \sigma_c^2}{2}\right) = \frac{1}{\rho R_f} \tag{16a}$$

$$\exp\left(-\gamma c + r + \frac{\gamma^2 \sigma_c^2 + \sigma_r^2}{2} - \gamma \sigma_{cr}\right) + b_0 \bar{C}_t^{-\gamma} \bar{V} = \frac{1}{\rho}. \tag{16b}$$

2.2.2 The non-expected utility approach

Although the expected-utility maximization represents the most widespread theoretical approach so far, Barberis, Huang, and Thaler (2003) claim there is another specification that best captures the utility of decisions under risk. In particular, this is a *non-expected recursive utility with first-order risk aversion* (R-FORA). Yet, simple R-FORA specifications account merely for loss aversion and hence need to be extended in order to accommodate with both narrow framing and loss aversion. These phenomena appear to be crucial for explaining several stock market puzzles and constitute the core of our approach. Henceforth we refer to the R-FORA setting with narrow framing as the non-expected utility approach.

We rely on the approach proposed in Barberis and Huang (2004a) according to which investors maximize a recursive utility-function U_t, that is defined as:

$$U_t = \diamond \langle C_t, \mu(U_{t+1} + b_0 E_t[v(G_{t+1})] | I_t) \rangle, \tag{17}$$

can be considered as constant from one trade to the other). Formally, between two successive dividend releasing times $(u, u+1]$, we have:

$$D_{t+1} = \begin{cases} D_u, & \text{for } t \in (u, u+1) \\ D_{u+1}, & \text{for } t = u+1, \end{cases} \qquad \text{hence} \qquad \frac{D_{t+1}}{D_t} = \begin{cases} 1, & \text{for } t \in (u, u+1) \\ \dfrac{D_{u+1}}{D_u}, & \text{for } t = u+1. \end{cases}$$

[14] Here we used the fact that, for $x \sim N(\mu, \sigma^2)$, $E[\exp(x)] = \exp(\mu + \sigma^2/2)$. Also, for $x_i \sim N(\mu_i, \sigma_i^2)$, where $i = 1, 2$ i.i.d. over time and with covariance σ_{12}, $E[\exp(x_1 + x_2)] = \exp(\mu_1 + \mu_2 + (\sigma_1^2 + \sigma_2^2)/2 + \sigma_{12})$.

where

$$\diamond \langle C, x \rangle = [(1-\beta)C^{1-\gamma} + \beta x^{1-\gamma}]^{\frac{1}{1-\gamma}}, \text{ for } 0 < \beta < 1,\, 0 < \gamma \neq 1 \tag{18a}$$

$$\mu(x) = (E[x^{1-\gamma}])^{\frac{1}{1-\gamma}}, \text{ for } 0 < \gamma \neq 1 \tag{18b}$$

$$G_{t+1} = \theta_t (W_t - C_t)(R_{t+1} - R_{ft}) \tag{18c}$$

$$v(x) = \begin{cases} x, \text{ for } x \geq 0 \\ \lambda x, \text{ for } x < 0 \end{cases}, \text{ for } \lambda > 1. \tag{18d}$$

Here, $\diamond(.,.)$ is an aggregator function and μ the homogenous certainty equivalent of the distribution of future utility conditional on the information I_t at time t, the next-period value of the risky investment G_{t+1}, and the individual value function v.

We restrict our analysis to the general equilibrium for aggregate markets (with a representative investor), in line with Equations (60)-(62) and the subsequent Example 6.1 for the stock market in Barberis and Huang (2004b). Our focus remains on non-professional investors' decisions concerning the wealth allocation among consumption, the risky portfolio returning R_t, and the risk-free asset with the gross return R_{ft}. When a fraction θ_t of post-consumption wealth is invested in the risky portfolio and another fraction, now of the total wealth, $\alpha_t = C_t/W_t$ is consumed, the following (Euler) equations yield necessary and sufficient conditions at equilibrium:

$$\beta R_{ft} E_t \left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} \right] \left\{ \beta E_t \left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} R^{\text{tot}}_{t+1} \right] \right\}^{\frac{\gamma}{1-\gamma}} = 1 \tag{19a}$$

$$\frac{E_t \left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} (R_{t+1} - R_{ft}) \right]}{E_t \left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} \right]} + b_0 R_{ft} \left(\frac{\beta}{1-\beta} \right)^{\frac{\gamma}{1-\gamma}} \left(\frac{1-\alpha_t}{\alpha_t} \right)^{-\frac{\gamma}{1-\gamma}} E_t[v(R_{t+1} - R_{ft})] = 0 \tag{19b}$$

$$\frac{E_t \left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} (R^{\text{tot}}_{t+1} - R_{ft}) \right]}{E_t \left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\gamma} \right]} + b_0 R_{ft} \left(\frac{\beta}{1-\beta} \right)^{\frac{\gamma}{1-\gamma}} \left(\frac{1-\alpha_t}{\alpha_t} \right)^{-\frac{\gamma}{1-\gamma}} \theta_t E_t[v(R_{t+1} - R_{ft})] = 0, \tag{19c}$$

where $R^{\text{tot}}_{t+1} = \theta_t R_{t+1} + (1-\theta_t) R_{ft}$ is the total gross return of the combination between risky and risk-free assets. Equation (19c) is derived from (19b) by multiplication with θ_t.

The next period financial wealth formulated in Equation (13) can be now rewritten as

$W_{t+1} = (W_t - C_t)R^{\text{tot}}_{t+1}$. Thus,

$$R^{\text{tot}}_{t+1} = \frac{\alpha_t}{\alpha_{t+1}(1-\alpha_t)} \frac{C_{t+1}}{C_t}. \tag{20}$$

Assuming time constancy for the portfolio wealth fraction θ, the consumption ratio α, and the risk-free return R_f, the total gross return results in:

$$R^{\text{tot}}_{t+1} = \frac{1}{1-\alpha} \frac{C_{t+1}}{C_t} \Rightarrow \log(R^{\text{tot}}_{t+1}) = -\log(1-\alpha) + c + \sigma_c \epsilon_{t+1}. \tag{21}$$

Thus, the equilibrium Equations (19) yield:

$$\beta^{\frac{1}{1-\gamma}} (1-\alpha)^{-\frac{\gamma}{1-\gamma}} R_f E\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma}\right] \left\{E\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{1-\gamma}\right]\right\}^{\frac{\gamma}{1-\gamma}} = 1 \tag{22a}$$

$$\frac{E\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma} (R_{t+1} - R_f)\right]}{E\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma}\right]} + b_0 R_f \left(\frac{\beta}{1-\beta}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{1-\alpha}{\alpha}\right)^{-\frac{\gamma}{1-\gamma}} E[\bar{v}(R_{t+1} - R_f)] = 0 \tag{22b}$$

$$\frac{E\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma} (R^{\text{tot}}_{t+1} - R_f)\right]}{E\left[\left(\frac{\bar{C}_{t+1}}{\bar{C}_t}\right)^{-\gamma}\right]} + b_0 R_f \left(\frac{\beta}{1-\beta}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{1-\alpha}{\alpha}\right)^{-\frac{\gamma}{1-\gamma}} \theta E[\bar{v}(R_{t+1} - R_f)] = 0. \tag{22c}$$

We proceed similar to Section 2.2.1 by assuming the parameter dynamics of consumption and returns in Equations (15). Also, we consider that the value functions (1) and (2) are equivalent in expectation to the prospective value in Equation (5), i.e.

$\bar{V} = E[\bar{v}(R_{t+1} - R_f)]$. Thus, the equilibrium Equations (22) can be further restated as:

$$\beta^{\frac{1}{1-\gamma}}(1-\alpha)^{-\frac{\gamma}{1-\gamma}} R_f \exp\left(\frac{\gamma\sigma_c^2}{2}\right) = 1 \tag{23a}$$

$$-\exp\left(-\gamma c + r + \frac{\gamma^2\sigma_c^2 + \sigma_r^2}{2} - \gamma\sigma_{cr}\right) + R_f \exp\left(-\gamma c + \frac{\gamma^2\sigma_c^2}{2}\right)$$
$$= b_0 R_f \left(\frac{\beta}{1-\beta}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{1-\alpha}{\alpha}\right)^{-\frac{\gamma}{1-\gamma}} \exp\left(-\gamma c + \frac{\gamma^2\sigma_c^2}{2}\right)\bar{V} \tag{23b}$$

$$-\frac{1}{1-\alpha}\exp\left((1-\gamma)c + \frac{(1-\gamma)^2\sigma_c^2}{2}\right) + R_f \exp\left(-\gamma c + \frac{\gamma^2\sigma_c^2}{2}\right)$$
$$= b_0 R_f \left(\frac{\beta}{1-\beta}\right)^{\frac{\gamma}{1-\gamma}} \left(\frac{1-\alpha}{\alpha}\right)^{-\frac{\gamma}{1-\gamma}} \exp\left(-\gamma c + \frac{\gamma^2\sigma_c^2}{2}\right)\theta\bar{V}. \tag{23c}$$

3 Empirical results

This section presents empirical findings based on the theoretical framework presented in the above Section 2.2. We start off by describing the general assumptions made in order to facilitate the estimation procedure and to render the two settings with expected and non-expected utility comparable. Subsequently, the estimation results are illustrated and commented on for each of these settings, where the exposition focuses on two main aspects. First, we address the evolution of the attitude towards financial losses as described by the loss aversion coefficient and the extended measures LAi and gRA. Second, we examine the optimal wealth allocation among consumption, risky, and riskless assets, as well as the occurrence of myopic loss aversion.

3.1 General assumptions

Our estimations are based on the same data set as in Rengifo and Trifan (2006), that includes the SP500 and the 10-year bond nominal returns (as proxies for the risky and the risk-free investment, respectively) from 01/02/1962 to 03/09/2006 (11,005 daily observations). This data is divided into two parts, from which only the second one (from 03/01/1982 to 03/09/2006, specifically 6,010 observations) is considered to be the active set (and used for performing simulations). The observations before the "date zero" of the trade (03/01/1982) serve to estimate the empirical mean and the standard deviation of the portfolio returns at date zero. Additionally, aggregate per-capita consumption data between 01/02/1962 and 12/31/2005[15] provide a basis for the calculation of the

[15] This data was provided by the Department of Commerce, Bureau of Economic Analysis and Bureau of the Census.

log-consumption mean and variance.[16] Note that this data set allows us to assess consumption values corresponding to portfolio evaluations frequencies of merely one year and three months. Thus, we cannot replicate the more detailed analysis in Rengifo and Trifan (2006) regarding the impact of the evaluation frequency on investor decisions.

After smoothing out the outlier corresponding to the October 1987 market crash,[17] quarterly and yearly returns are constructed from the active data set and used to derive the optimal risky investment. In so doing, we assume that investors start by spreading their wealth equally between consumption and financial assets, where the latter fraction is further allocated equally to the risky portfolio and the bond. In addition, investors are considered to be long-lived beyond the VaR-horizon and are not allowed to quit the market during the trading period. Moreover, cushions are assumed to be cumulatively amassed from past trades (starting at date zero) $Z_t = \sum_{i=0}^{t} S_i$, portfolio gross returns to be normally distributed, and future portfolio returns to be estimated as the unconditional mean of past returns. In addition, we assess $\hat{R}_f = \text{mean}[R_{ft}]$, $\hat{c} = \text{mean}[\log(C_{t+1}) - \log(C_t)]$ in Equation (15a), and $\hat{r} = \text{mean}[\log(R_t)]$ in Equation (15b), where the means are computed throughout the active period (from 03/01/1982 to 03/09/2006).

A delicate issue that might emerge from our consideration that investors are long-lived investors and have financial investments as sole source of wealth,[18] forces us to make a final and more specific assumption. It is possible that financial investments do not generate sufficient revenues in order to cover investors' consumption needs over the entire investment interval. We attempt to circumvent this potential problem[19], by considering that at each time t investors dispose of additional incomes I_t. Such incomes represent, for instance, the wages earned by non-professional investors from their main employment.[20] These incomes are considered as exogenous, that is, they stem from *outside* of those investments that constitute the decision making object at hand. Under this assumption, the total wealth W_t in Equation (13) results from both financial investments and the

[16] Descriptive statistics can be found in Table 7 in Appendix 5.1.

[17] This outlier is replaced with the mean of the ten before and after data points

[18] Recall that both consumption and financial investments generate *utility*, yet only the latter is "productive" and can effectively augment *wealth*.

[19] Note that Barberis and Huang (2004a,b) avoid this problem by fixing the percentage of post-consumption wealth invested in risky assets θ. We do not consider this as appropriate in our framework for two reasons. First, Barberis and Huang (2004a,b) exclusively work with non-expected utility, while our aim is to render comparable two approaches, namely those with expected and non-expected utility. Second, our model rests on the idea that θ depends on the subjective VaR* (see Equations (12)). Hence, imposing constancy on this parameter would eliminate the whole analyzed mechanism of how individual perceptions of financial investments reflect in the wealth allocation.

[20] As their name indicates, *non-professional* investors mainly earn their living from other activities (developed for example as employees of a company) than from financial investments. The latter merely represent a secondary source of revenues.

additional income I_t and yields:

$$W_{t+1} - I_{t+1} = (W_t - C_t)[\theta_t R_{t+1} + (1 - \theta_t) R_{ft}]. \tag{24}$$

We assume that the additional income covers a part of the consumption needs of the current period and set:[21]

$$I_t = \frac{C_t}{\alpha\delta}, \tag{25}$$

where α represents the percentages of total wealth dedicated to consumption in the equilibrium of the non-expected utility setting and $\delta > 0$ is an arbitrary constant. Apparently, for $\delta \leq 1/\alpha$ ($\delta > 1/\alpha$) the extra income exceeds (does not entirely meet) the consumption needs of the period $I_t \geq C_t$ ($I_t < C_t$). We distinguish two particular cases with no practical meaning in the present framework. First for $\delta = 1$, the current extra income yields a fraction of the consumption needs $I_t = C_t/\alpha$ and investors should assign no money to financial assets in total $R^{\text{tot}}_{t+1} = 0$.[22] Second, for $\delta = 1/\alpha$ the extra income that covers exactly the current consumption $I_t = C_t$. As the total financial investment then becomes independent of α, $R^{\text{tot}}_{t+1} = C_{t+1}/C_t$, there is no further connection between investment decisions and the subjective perception of financial investments in the non-expected utility equilibrium.[23] Consequently, we henceforth set $\delta \in \mathbb{R}^+ \setminus \{0, 1, 1/\alpha\}$.[24]

We close this section by detailing the values of the behavioral parameters that underlie our simulations. First, we chose different values of the initial coefficient of loss aversion λ, namely in the set $\{0.5; 1; 2.25; 3\}$. The value of 2.25 recommended by the prospect theory is considered as the standard case and is mainly referred to in the text. Second, we consider different values of the additional income I_t. As α depends on β according to Equations (22), the choice of β and further of δ dictate the evolution of the additional income. Thus, we take $\beta \in \{0.2; 0.5; 0.8\}$ and $\delta \in \{0.6; 0.7; 0.8; 0.9\}$.[25] Our subsequent comments usually

[21] This assumption permits an easy formal manipulation and ensures the comparability of the two approaches. At the same time, it allows for sufficient flexibility with respect to the choice of the income magnitude.

[22] Moreover, in this case the percentage of post-consumption wealth assigned to risky assets in equilibrium from Equation (29b) yields: $\theta = R_f/[R_f - \exp(r + \sigma_r^2/2 - \gamma\sigma_{cr})] \geq 1$. This induces investors to throw caution to the wind, borrowing ever more money , which is then channelled into the risky portfolio.

[23] Recall first that V relies on subjective perceptions and is derived on the basis of behavioral parameters according to Equation (5). Then this connection is ensured by the interdependency of α and V from Equations (29). It is central to our work, as we assume that all decisions rely on individual perceptions and attempt to analyze how they change subject to different perception parameters. For $\delta = 1/\alpha$, the percentage of post-consumption wealth allocated to risky assets in equilibrium from Equation (29b) becomes independent of α, specifically $\theta = [R_f - \exp(c + (1 - 2\gamma)\sigma_c^2/2)/[R_f - \exp(r + \sigma_r^2/2 - \gamma\sigma_{cr})]$.

[24] Note that for the purpose of comparability, we consider identical additional incomes in both settings with expected and non-expected utility.

[25] We actually run simulations for all $\beta \in \{0.2; 0.5; 0.8; 0.98\}$ and $\delta \in \{0.2; 0.5; 0.6; 0.7; 0.8; 0.9; 2; 10; 100\}$. The final choice of the value-ranges mentioned in the text is motivated by several facts and findings. First, $\delta > 1$ entails negative values for the wealth percentages to be consumed in the expected-utility setting. Second, the value $\beta = 0.98$, which is in line with Barberis and Huang (2004b), yields implausible (that is negative) estimates of the same percentages in the

account for all considered (β, δ)-combinations. Yet the results illustrated in the tables of Sections 3.2 and 3.3 focus on the case with $\delta = 0.9$ and consider two different values of $\beta = 0.2$ and $\beta = 0.8$. This allows us to compare the reactions of investors who perceive one of the two utility sources (namely the consumption for $\beta = 0.2$ and the financial wealth fluctuations for $\beta = 0.8$) as dominant. All unreported numerical results are available upon request.

3.2 The expected-utility approach

In order to estimate the variables of interest in our model, we start by considering the expected-utility setting. Following Barberis, Huang, and Santos (2001) and Barberis and Huang (2004a), we chose three values of the parameter γ that express different degrees of the risk aversion of the consumption utility, namely $\gamma \in \{0.5; 1; 1.5\}$. In line with the same papers, we also account for no and moderate influence of past losses on the perception of risky investments and set $k \in \{0; 3\}$. In addition, we consider different degrees of narrow framing, specifically $b_0 \in \{0.001; 100; 1,000\}$ for $\gamma \geq 1$, and $b_0 \in \{0.001; 5; 10\}$ for $\gamma < 1$.[26] Finally, recall that in order to make the expected-utility setting comparable to the non-expected utility one in Section 3.3, we consider that investors dispose of periodical additional incomes. The magnitude of these incomes is dictated by a free-choice parameter δ and by a behavioral parameter β that is specific to the non-expected utility setting. As β stems from the non-expected utility framework, it has no intuitive meaning in the present setting with expected utility. Thus, we subsequently interpret the variation of our equilibrium estimates with respect to the changes in the additional income I_t (instead of the changes in β). This is possible, as I_t increases (decreases) subject to higher β (δ)-values.[27] In essence, higher additional incomes are equivalent to more relaxed requirements for financial investments.

Equation (16a) delivers an estimate of the discounting factor ρ in the aggregate equilibrium.[28] Plugging this estimate $\hat{\rho}$ into Equation (16b), we obtain an empirical equivalent for the prospective value in equilibrium $\bar{V}$. We can further assess the corresponding loss

non-expected equilibrium.

[26]First, in light of Equation (26), $b_0 \neq 0$. Second, recall that the weighting coefficient of the utility derived from risky investments in Equation (10) is obtained by multiplying the narrow framing parameter b_0 with the aggregate consumption $\bar{C}_t^{-\gamma}$. Consequently, as mean$[C_t] \approx 46,000$ in our sample, yet the highest $b_0 = 1,000$ entails for $\gamma = 1$ a still small (non-discounted) contribution of this source of utility to the total utility, specifically of less than 2.2%. Similarly, $b_0 = 10$ and $\gamma = 0.5$ result in a share of less than 4.7%.

[27]Due to our assumption that $I_t = C_t/(\alpha\delta)$ and according to $\alpha = 1 - \beta^{\frac{1}{\gamma}} R_f^{\frac{1-\gamma}{\gamma}} \exp \frac{(1-\gamma)\sigma_c^2}{2}$ in Equation (29a).

[28]Of course, we could also fix ρ and estimate γ. However, this procedure proves to be more delicate, given that Equation (16a) is a second-order equation in γ, such that the existence and number of real roots depends on the sign of its determinant $c^2 - 2\sigma_c^2 \log(\rho R_f)$. In the case with either none or two distinct real solutions, we cannot provide an economic interpretation of the equilibrium.

aversion coefficient $\hat{\lambda}$ according to Equation (6). In particular, the explicit expressions of the desired variables are based on the following reformulation of the equilibrium Equations (16):

$$\rho = \frac{1}{R_f} \exp\left(\gamma c - \frac{\gamma^2 \sigma_c^2}{2}\right) \tag{26a}$$

$$\bar{V} = \frac{\bar{C}_t^{\gamma}}{b_0} \left[\frac{1}{\rho} - \exp\left(-\gamma c + r + \frac{\gamma^2 \sigma_c^2 + \sigma_r^2}{2} - \gamma \sigma_{cr}\right)\right]. \tag{26b}$$

Unreported results show that in the absence of narrow framing (as approximated by the case with the lowest $b_0 = 0.001$), the estimates of the loss aversion coefficient λ are highly negative (positive) for yearly (quarterly) portfolio revisions, and thus implausible. In effect, the individual attitude to losses stemming from financial investments has no practical meaning (and is difficult to interpret in terms of our model) when investors pay no attention to the utility derived from these investments compared to the consumption utility. Similarly, an excessive consumption-based risk aversion $\gamma = 1.5$ results in implausible estimates of both ρ (that are slightly supra-unitary) and in part of λ (that are negative or too highly positive, with the exception of the case with $b_0 = 1000$), as intuitively expected according to Equations (26). In sum, plausible parameter combinations for reaching the aggregate market equilibrium in the expected-utility setting turn out to be ($\gamma = 0.5$, $b_0 \in \{5; 10\}$) and ($\gamma = 1$, $b_0 \in \{100; 1000\}$) and they are addressed below.

Table 1 (Table 8 in Appendix 5.2) presents the estimation results for these parameter combinations, yearly (quarterly) portfolio evaluations in our usual cases with $\lambda = 2.25$, $\delta = 0.9$, and $\beta \in \{0.2; 0.8\}$. Recall that the switch from $\beta = 0.2$ to $\beta = 0.8$ for a fixed $\delta = 0.9$ is equivalent with (hence interpreted as) an increase in additional incomes.

	$\beta = 0.8$				$\beta = 0.2$			
	$\gamma = 0.5$		$\gamma = 1$		$\gamma = 0.5$		$\gamma = 1$	
	$k = 0$	$k = 3$	$k = 0$	$k = 3$	$k = 0$	$k = 3$	$k = 0$	$k = 3$
$b_0 = 5$								
$\hat{\rho}$	0.95685	0.95685			0.95685	0.95685		
$\hat{V}$	-92.17895	-92.17895			-92.17895	-92.17895		
$\hat{\lambda}$	1.0853	1.0836			0.79627	0.99587		
$b_0 = 10$								
$\hat{\rho}$	0.95685	0.95685			0.95685	0.95685		
$\hat{V}$	-46.08948	-46.08948			-46.08948	-46.08948		
$\hat{\lambda}$	1.0867	1.0851			0.79024	0.99968		
$b_0 = 100$								
$\hat{\rho}$			0.98208	0.98208			0.98208	0.98208
$\hat{V}$			-1,155.37750	-1,155.37750			-1,155.37750	-1,155.37750
$\hat{\lambda}$			1.0639	1.0585			1.0625	0.86629
$b_0 = 1,000$								
$\hat{\rho}$			0.98208	0.98208			0.98208	0.98208
$\hat{V}$			-115.53775	-115.53775			-115.53775	-115.53775
$\hat{\lambda}$			1.0802	1.0752			1.2341	0.75677

Table 1: The main estimated parameters in the expected-utility setting for yearly data, $\lambda = 2.25$ and $\delta = 0.9$.

We first address the investors' attitude towards financial losses. One variable that captures this attitude is the loss aversion coefficient, the equilibrium estimates of which are denoted by $\hat{\lambda}$. From Tables 1 and 8 we observe that for higher additional incomes (formally corresponding to $\beta = 0.8$), $\hat{\lambda}$ takes values slightly above 1 for both yearly and quarterly portfolio evaluations. As 1 can be considered to be the "neutral" loss-aversion value, our representative investor appears to be rather indifferent between financial gains and losses.

Surprisingly, lower extra-money dedicated to consumption ($\beta = 0.2$) entails a decrease in the loss aversion coefficient in equilibrium. This variation is confirmed for further δ-values. At first glance, this conclusion appears to be counterintuitive. Accordingly, investors who are less sure of being able to cover current consumption needs (as their additional incomes are lower) become more relaxed towards financial losses (although such losses would reduce their revenues even more). One possible explanation is that agents tend toward taking the chance of investing in financial assets, as this chance (in other words gambling) hides not only the danger of losses, but also the promise of gains. However, note that the loss aversion coefficient is *not* a one-to-one measure of investor actions concerning wealth allocation. They represent the result of more complex phenomena. Thus, variations of the loss aversion coefficient due to changes in the additional income do not need to be proportionally reflected in the magnitude of the actual investment in financial assets. For instance, Tables 3 and 10 show that the lower additional incomes corresponding to $\beta = 0.2$ entail *smaller* fractions of the total wealth allocated to risky assets $(1 - \bar{C}/\hat{W})\hat{\theta}$. More detailed investigations referring to this issue are undertaken at the end of this section. Finally, note also that the loss aversion coefficient might be an *imperfect* measure of the attitude towards financial losses. Yet, the other two measures LAi and gRA do not appear to evolve differently (see the findings in this respect presented later in this section).

In general, $\hat{\lambda}$ does not reach the value of 2.25 suggested in the original prospect theory for any of the considered parameter configurations.[29] This is not very surprising, as the latter has been mostly obtained in experiments where subjects are faced only with financial decisions. Our results underline the fact that, when accounting for both consumption and financial investments as generators of individual utility, investor reluctance towards financial losses measured by the equilibrium-equivalent loss aversion coefficient $\hat{\lambda}$ is lower.

On average, $\hat{\lambda}$ is somewhat smaller when portfolios are evaluated more often. At first glance, this appears to be at odds with the idea of myopic loss aversion. However, two remarks can be made in this regard. First, note that the myopic loss aversion originates in, but is *not* measured by, the loss aversion coefficient. A more appropriate measure is

[29] One of the highest values, $\hat{\lambda} = 1.782$, is obtained for $\beta = 0.2$, $\delta = 0.7$, $k = 0$, and yearly evaluations.

the wealth fraction allocated to risky assets, the evolution of which will be subsequently detailed.[30] Second, the observed changes in $\hat{\lambda}$ are rather small, especially for incomes corresponding to $\beta \geq 0.5$. This points to a relative stability of the loss attitude in equilibrium with respect to the portfolio evaluation frequency for a higher certitude of covering current consumption needs (namely for middle-range to high additional incomes).

Moreover, for a fixed degree of narrow framing b_0, the variable $\hat{\lambda}$ resulting for the higher additional income corresponding to $\beta = 0.8$, appears to decrease when switching from $k = 0$ to $k = 3$. Note also that the respective changes in value are very small for quarterly portfolio evaluations although the inverse holds for $\lambda = 2.25$, $\beta = 0.2$ and $\gamma = 0.5$. This tendency is confirmed for most of the other considered parameter combinations, where the differences become substantial for low values of both β and δ.[31] However, we can conclude that when the additional income to be consumed is sufficiently high ($\beta \geq 0.5$), the reluctance towards past losses does not have a significant impact on the attitude towards current losses. It is still difficult to formulate clear-cut conclusions with respect to the variation of $\hat{\lambda}$ subject to the narrow framing coefficient b_0 when k is fixed. For the highest additional income values ($\beta = 0.8$), we notice a slight tendency to increase (decrease) when portfolios are evaluated once a year (every three months), the changes being yet very small.

In the same context of the investor attitude towards financial losses, it is interesting to follow the evolution of the two other measures introduced in the theoretical section, namely LAi in Equation (7) and gRA in Equation (8). Table 2 (Table 9 in Appendix 5.2) presents the corresponding estimates for yearly (quarterly) portfolio evaluations and the cases already analyzed in Table 1 (Table 8) above.

[30]This remark is in the same spirit as the above comments on the variation of $\hat{\lambda}$ subject to β.

[31]Specifically, one of the biggest differences is obtained for $\beta = 0.2$ and $\delta = 0.7$, where $\hat{\lambda}$ decreases for $k = 3$ to almost one half of the value for $k = 0$.

	$\beta = 0.8$				$\beta = 0.2$			
	$\gamma = 0.5$		$\gamma = 1$		$\gamma = 0.5$		$\gamma = 1$	
	$k = 0$	$k = 3$	$k = 0$	$k = 3$	$k = 0$	$k = 3$	$k = 0$	$k = 3$
$b_0 = 5$								
LAi	1.0853	1.0408			0.79627	0.95421		
gRA	2,375,552.2	2,086,117.3			288,130	249,340		
$b_0 = 10$								
LAi	1.0867	1.0423			0.79024	0.95801		
gRA	2,375,720.2	2,086,278.7			288,100	249,450		
$b_0 = 100$								
LAi			1.0639	1.0157			1.0625	0.82444
gRA			4,126,211.6	3,625,896.5			489,230	416,970
$b_0 = 1,000$								
LAi			1.0802	1.0323			1.2341	0.71492
gRA			4,129,926.1	3,629,313.8			494,810	416,410

Table 2: The estimated index of loss aversion (LAi) and global first-order risk aversion (gRA) in the expected-utility setting for yearly data, $\lambda = 2.25$ and $\delta = 0.9$.

For the most part, the evolutions of LAi and gRA confirm the conclusions reached so far for $\hat{\lambda}$. That is, both measures of the attitude towards financial losses increase for enhanced narrow framing, decrease when past losses are penalized, and also diminish when portfolio performance is checked more often (where the changes in gRA are very high), other things being equal.

Moreover, LAi closely follows the values of the simple loss aversion coefficient in the market equilibrium, being somewhat (but almost always insignificantly) lower. We cannot yet detect a clear pattern of its variation with the additional income. In this regard, gRA behaves more consistently, as it clearly increases for higher additional incomes (specifically, for either higher β or lower δ-values).[32] Also, gRA better describes how the reluctance to losses extends across time, as it clearly drops when k increases from zero to three. In sum, gRA appears to better capture the changing attitude to losses in equilibrium subject to the variation of different model parameters.

Before approaching the problem of optimal wealth allocation, we investigate the other two variables for which estimates can be obtained in the expected-utility equilibrium. Tables 1 and 8 contain the respective values for the usual cases with $\delta = 0.9$ and $\beta \in \{0.2; 0.8\}$. As apparent from Equation (26a), the discounting factor ρ does not depend on the choice of either β, or δ, or b_0. Table 1 shows that for yearly portfolio evaluations and $\gamma = 1$, $\hat{\rho} \approx 0.98$, which complies with the assumptions of Barberis, Huang, and Santos (2001) and Barberis and Huang (2004a). As expected, lower consumption-based risk aversion $\gamma = 0.5$ entails lower preference for immediacy and our estimate $\hat{\rho} < 0.96$. The corresponding estimates from Table 8 for quarterly portfolio revisions are somewhat higher and less sensitive to changes in γ.

Finally, note that the estimated prospective value in equilibrium remains negative $\hat{V} < 0$ across all considered parameter configurations. Thus, financial investments appear to lower the overall utility of our representative investor. The evolution of $\hat{V}$ for different parameter combinations confirms the tendencies pointed out for $\hat{\lambda}$. In particular, independently of the penalty on past losses k, $\hat{V}$ increases subject to higher narrow framing b_0, as well as for more frequent portfolio evaluations, other things being equal.

In the sequel, we turn to the question of whether or not myopic loss aversion becomes manifest in our equilibrium setting with two-dimensional expected utility. The answer goes hand in hand with the wealth allocation in equilibrium. The subsequent comments refer to the findings for *all* considered additional income configurations (or all (β, δ)-combinations), while direct references to the values in Tables 3 and 10 are explicitly indicated.

In this context, we compute the mean of the proportions of total wealth dedicated to

[32] Recall that higher gRA-values point to lower aversion to financial losses.

consumption $\bar{C}/\hat{W}$ and the one of the remaining wealth (after consumption) assigned to risky assets $\hat{\theta}$, where the latter results from Equation (12a). They are illustrated in Table 3 (Table 10 in Appendix 5.2) for yearly (quarterly) portfolio evaluations and our standard cases with $\lambda = 2.25$, $\delta = 0.9$, and $\beta \in \{0.2; 0.8\}$. In essence, all variables that refer to the wealth allocation are robust[33] with respect to the sensitivity to past losses k and are independent of the degree of narrow framing b_0.

	$\beta = 0.8$				$\beta = 0.2$			
	$\gamma = 0.5$		$\gamma = 1$		$\gamma = 0.5$		$\gamma = 1$	
	$k = 0$	$k = 3$	$k = 0$	$k = 3$	$k = 0$	$k = 3$	$k = 0$	$k = 3$
$\bar{C}/\hat{W}$	0.05576	0.05573	0.04169	0.04167	0.20578	0.20634	0.14703	0.14733
$\hat{I}/\bar{C}$	3.5459	3.5459	5.5556	5.5556	1.1609	1.1609	1.3889	1.3889
$\hat{\theta}$	0.41549	0.41445	0.42767	0.42704	0.27721	0.27135	0.33236	0.32822
$(1 - \bar{C}/\hat{W})\hat{\theta}$	0.39232	0.39135	0.40984	0.40925	0.22017	0.21536	0.28349	0.27986

Table 3: The estimated wealth allocation in the expected-utility setting for yearly data, $\lambda = 2.25$ and $\delta = 0.9$.

First, our overall results show that investors who maximize expected utility from both consumption and financial investments and who evaluate the latter once every year (three months) turn out to dedicate between $2 - 21\%$ $(1 - 13\%)$ of their total wealth to consumption. These percentages clearly depend on the additional income (i.e. on the chosen values of β and δ).[34] Thus, the mean values $\bar{C}/\hat{W}$ decrease for higher additional incomes (formally for either higher β-values or lower higher δ-s), as covering the current consumption needs from financial revenues is not so stringent.[35] This becomes apparent in Tables 3 and 10 for the particular case with $\delta = 0.9$. The highest overall value is obtained for the lowest additional income corresponding to $\beta = 0.2$ and $\delta = 0.9$. It is indeed reasonable to think that, when investors dispose of sufficient additional revenues, they develop a more relaxed attitude towards financial investments, specifically they allocate more money to these financial ventures (and proportionally less to consumption). However, the average portions of total wealth dedicated to consumption in equilibrium $\bar{C}/\hat{W}$ are very small. At first glance, this appears to attest to an unexpected openness towards financial investments in general. It should however be recalled that the total wealth (the mean of which is in equilibrium $\hat{W}$) includes not only financial revenues, but also the additional incomes. As the latter is sufficient for fulfilling consumption needs, the remaining part becomes available for being invested in financial assets. This makes the total wealth increase faster than consumption (and even more for higher β-values), resulting in small ratios $\bar{C}/\hat{W}$.

[33]Changes are in the third or fourth decimal point.

[34]Note that total wealth includes the initial wealth invested in the financial markets plus the additional income.

[35]In particular, all $\bar{C}/\hat{W}$ lie under 6% (14%) when β exceeds the threshold of 0.5.

The coverage rate of consumption from additional incomes is on average reflected by the ratio $\hat{I}/\bar{C}$, the evolution of which can be observed in the same Tables 3 and 10 for $\delta = 0.9$. In this case, the additional income covers on average more than the consumption needs, as $\hat{I}/\bar{C} > 1$.[36]

Second, between $27 - 45\%$ $(2 - 17\%)$ of the remaining wealth after consumption is allocated to risky assets when yearly (quarterly) portfolio checks are performed. These percentages correspond to the estimates $\hat{\theta}$ derived across all considered parameter configurations. The values of $\hat{\theta}$ grow with the magnitude of the additional income (namely subject to higher β or to lower δ-values).[37] As stressed above, when investors have more money at their disposal, they may also allocate higher sums to risky assets. Note that the estimated percentages of post-consumption wealth invested in the risky portfolio are lower than the respective values found in the one-dimensional utility setting in Rengifo and Trifan (2006)[38], where the estimates $\hat{\theta}$ obtained for high β coupled with low δ-values come closest. Again, the mean $\hat{\theta}$ is not significantly different when investor penalize or not past losses.

Finally, recall that while in the one-dimensional setting the total wealth is exclusively assigned to financial investments, in the present two-dimensional setting $\hat{\theta}$ stands for percentages of *post-consumption* wealth invested in risky assets. The corresponding percentages of *total* wealth assigned to risky investments can be obtained by multiplying $(1 - \bar{C}/\hat{W})$ by $\hat{\theta}$. Across all considered additional income values, $\hat{\theta}$ amounts to between approximately $22 - 43\%$ $(2 - 17\%)$ for yearly (quarterly) portfolio evaluations. They are higher for a more pronounced consumption-based risk aversion γ, for higher extra incomes (which is equivalent to higher β and/or lower δ-values), and for more frequent portfolio evaluations. These conclusions are illustrated for the case with $\delta = 0.9$ in the same Tables 3 and 10. We can hence conclude that myopic loss aversion continues to manifest when consumption is incorporated as an additional source of utility besides financial investments and when expected utility is maximized. In particular, investors remain highly averse towards financial risks and their reluctance increases for more frequent evaluations of the risky portfolio. In addition, the reluctance towards risky assets is more pronounced than in the one-sided utility framework in Rengifo and Trifan (2006), and as expected, grows as the additional income covers less of the current consumption needs.

We close this section with several remarks concerning the result replication for further

[36] For all considered $\delta < 1$-values, $\hat{I}/\bar{C}$ lies in the interval $1.16 - 8.33$ for both frequencies of the risky-performance evaluations. The respective values increase for (higher β, lower δ)-combinations, as well as for $\gamma = 1$.

[37] In particular, all values lie above 34% (14%) when β crosses over the neutral value of 0.5.

[38] According to Table 1 in Rengifo and Trifan (2006), for cumulative cushions and normally distributed expected returns they find that 49% (18.5%) of wealth is invested in risky assets when portfolios are evaluated yearly (quarterly).

values of the initial loss-aversion coefficient λ. Recall that this coefficient is employed in order to derive VaR* from Equation (3) and the prospective value from Equation (5), and the main analysis conducted so far relies on $\lambda = 2.25$. We run further sensitivity tests for $\lambda \in \{0.5; 1; 3\}$, the results of which show that all evolution patterns found above remain valid. While neither the estimated discounting factor $\hat{\rho}$ nor the derived prospective value $\hat{V}$ depend on the initial loss-aversion coefficient, the equivalent $\hat{\lambda}$ varies as expected, proportionally to λ. Moreover, the equilibrium-equivalent estimates of LAi and gRA increase in λ, where the variations of LAi are rather minor but those of gRA more pronounced. Naturally, the initial attitude towards losses should be proportionally reflected in the attitude of the representative investor, required in order to reach the aggregate equilibrium. Furthermore, the mean percentages of total wealth dedicated to consumption in equilibrium $\bar{C}/\hat{W}$ are rather robust with respect to the initial coefficient of loss aversion. The percentages of post-consumption wealth invested in risky assets $\hat{\theta}$ slightly increase in λ, but the changes are already minor for $\lambda \geq 1$. The same holds for the resulting percentages of total wealth dedicated to risky investments $(1-\bar{C}/\hat{W})\hat{\theta}$. This suggests that investors who are initially more averse to financial losses and maximize two-dimensional expected utility may exhibit slightly enhanced reluctance towards financial investments in general, and towards risky investments in particular.

3.3 The non-expected utility approach

In order to derive parameter estimates in the equilibrium setting with non-expected utility, we proceed analogously to Section 3.2. We again follow Barberis, Huang, and Santos (2001) and Barberis and Huang (2004a), as well as our model restrictions in Equations (18a) and (18b), and choose the risk aversion parameter of the consumption utility $\gamma \in \{0.5; 1.5\}$[39] and the sensitivity to prior losses $k \in \{0; 3\}$. Given the particular form of Equation (18a), the contribution of the risky prospect to the total utility decreases in the narrow-framing coefficient b_0 for $\gamma > 1$, such that we now take $b_0 \in \{0.001; 0.01; 0.1; 0.5; 1\}$.

Under the assumption of periodical additional incomes of $I_t = C_t/(\alpha\delta)$, the total gross return from financial investments in Equation (21) results in:

$$
\begin{aligned}
R_{t+1}^{\text{tot}} = \frac{1}{1-\alpha}\frac{C_{t+1} - \alpha I_{t+1}}{C_t} = \frac{1}{1-\alpha}\frac{\delta - 1}{\delta}\frac{C_{t+1}}{C_t} \\
\Rightarrow \log(R_{t+1}^{\text{tot}}) = \log(\delta - 1) - \log(\delta) - \log(1-\alpha) + c + \sigma_c \epsilon_{t+1}.
\end{aligned}
\tag{27}
$$

[39]Recall that according to Equation (18a), $\gamma \neq 1$.

Consequently, the equilibrium Equation (23c) changes to:

$$
\begin{aligned}
&-\frac{\delta-1}{\delta(1-\alpha)}\exp\left((1-\gamma)c+\frac{(1-\gamma)^2\sigma_c^2}{2}\right)+R_f\exp\left(-\gamma c+\frac{\gamma^2\sigma_c^2}{2}\right)\\
&\quad=b_0R_f\left(\frac{\beta}{1-\beta}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{1-\alpha}{\alpha}\right)^{-\frac{\gamma}{1-\gamma}}\exp\left(-\gamma c+\frac{\gamma^2\sigma_c^2}{2}\right)\theta\bar{V}.
\end{aligned} \tag{28}
$$

Thus, for a fixed subjective weight β in the aggregator function in Equation (18a), Equations (23a), (23b), and (28) deliver estimators for the percentages of total wealth dedicated to consumption α, the portion of post-consumption wealth invested in risky assets θ and the prospective value $\bar{V}$ in equilibrium.[40] Specifically, dividing Equations (23b) and (28) and plugging the result into Equation (23a), we derive α and θ. Finally, we restate Equation (23b) in order to obtain an expression for $\bar{V}$. This yields the following expressions of the parameters that can be estimated in the equilibrium of the non-expected utility setting:

$$
\alpha=1-\beta^{\frac{1}{\gamma}}R_f^{\frac{1-\gamma}{\gamma}}\exp\left(\frac{(1-\gamma)\sigma_c^2}{2}\right) \tag{29a}
$$

$$
\begin{aligned}
\theta&=\frac{R_f-\dfrac{\delta-1}{\delta(1-\alpha)}\exp\left(c+\dfrac{(1-2\gamma)\sigma_c^2}{2}\right)}{R_f-\exp\left(r+\dfrac{\sigma_r^2}{2}-\gamma\sigma_{cr}\right)}\\
&=\frac{\delta\beta^{\frac{1}{\gamma}}R_f^{\frac{1}{\gamma}}-(\delta-1)\exp\left(c-\dfrac{\gamma\sigma_c^2}{2}\right)}{\delta\beta^{\frac{1}{\gamma}}R_f^{\frac{1-\gamma}{\gamma}}\left[R_f-\exp\left(r+\dfrac{\sigma_r^2}{2}-\gamma\sigma_{cr}\right)\right]}
\end{aligned} \tag{29b}
$$

$$
\begin{aligned}
\bar{V}&=\frac{1}{b_0R_f}\left[\frac{\alpha\beta}{(1-\alpha)(1-\beta)}\right]^{-\frac{\gamma}{1-\gamma}}\left[R_f-\exp\left(r+\frac{\sigma_r^2}{2}-\gamma\sigma_{cr}\right)\right]\\
&=\frac{1}{b_0}\beta(1-\beta)^{\frac{\gamma}{1-\gamma}}\exp\left(\frac{\gamma\sigma_c^2}{2}\right)\frac{R_f-\exp\left(r+\dfrac{\sigma_r^2}{2}-\gamma\sigma_{cr}\right)}{\left[1-\beta^{\frac{1}{\gamma}}R_f^{\frac{1-\gamma}{\gamma}}\exp\left(\dfrac{(1-\gamma)\sigma_c^2}{2}\right)\right]^{\frac{\gamma}{1-\gamma}}}.
\end{aligned} \tag{29c}
$$

Note that in Equation (29b), the percentage of post-consumption wealth invested in risky assets in equilibrium θ is inversely proportional on each additional income parameter δ and the total-wealth percentage dedicated to consumption in equilibrium α. Then, according to Equation (29a) and to our assumption $I_t = C_t/(\alpha\delta)$, θ increases c.p. for higher weights β, as well as for higher δ-values. Henceforth, we refer high β-values as the case where

[40] Again, it is possible to proceed in the opposite way: by fixing α and θ, an estimator for γ can be derived from Equation (23a). However, the double or possible non-real roots of quadratic equations are difficult to interpret from an economical point of view. Equation (23a) has real roots if and only if $\beta R_f \geq \exp(-(2\log(1-\alpha)+2r_f-\sigma_c^2)^2/(8\sigma_c^2))$.

investors consider the utility derived from narrowly framed financial investments more important than the consumption utility, according to the aggregator function in Equation (18a).[41]

Unreported results show that an extreme consumption-based risk aversion $\gamma = 1.5$ again yields implausible values of the equilibrium estimates $\hat{V}$ and $\hat{\lambda}$. Consequently, our subsequent comments focus on the case with $\gamma = 0.5$. Table 4 (Table 11 in Appendix 5.3) presents the estimates from Equations (29) for yearly (quarterly) evaluations and our standard cases with $\lambda = 2.25$, $\delta = 0.9$, and $\beta \in \{0.2; 0.8\}$.

	$\beta = 0.8$		$\beta = 0.2$	
	$k = 0$	$k = 3$	$k = 0$	$k = 3$
$b_0 = 0.001$				
$\hat{\alpha}$	0.31335	0.31335	0.95708	0.95708
$\hat{\theta}$	-0.6726	-0.6726	-2.0563	-2.0563
$\hat{V}$	-943.98	-943.98	-309.06	-309.06
$\hat{\lambda}$	1.0537	1.0502	0.83444	0.97344
$b_0 = 0.01$				
$\hat{\alpha}$	0.31335	0.31335	0.95708	0.95708
$\hat{\theta}$	-0.6726	-0.6726	-2.0563	-2.0563
$\hat{V}$	-94.398	-94.398	-30.906	-30.906
$\hat{\lambda}$	1.0846	1.0829	0.78922	1.0005
$b_0 = 0.1$				
$\hat{\alpha}$	0.31335	0.31335	0.95708	0.95708
$\hat{\theta}$	-0.6726	-0.6726	-2.0563	-2.0563
$\hat{V}$	-9.4398	-9.4398	-3.0906	-3.0906
$\hat{\lambda}$	1.0877	1.0861	0.7847	1.0032
$b_0 = 0.5$				
$\hat{\alpha}$	0.31335	0.31335	0.95708	0.95708
$\hat{\theta}$	-0.6726	-0.6726	-2.0563	-2.0563
$\hat{V}$	-1.888	-1.888	-0.61811	-0.61811
$\hat{\lambda}$	1.088	1.0864	0.7843	1.0034
$b_0 = 1$				
$\hat{\alpha}$	0.31335	0.31335	0.95708	0.95708
$\hat{\theta}$	-0.6726	-0.6726	-2.0563	-2.0563
$\hat{V}$	-0.94398	-0.94398	-0.30906	-0.30906
$\hat{\lambda}$	1.088	1.0865	0.78425	1.0035

Table 4: The main estimated parameters in the non-expected utility setting for yearly data, $\lambda = 2.25$, $\gamma = 0.5$ and $\delta = 0.9$.

We first note that similarly to the expected-utility findings, the prospective value

[41] Recall however that the weights of financial wealth in the (final) utility function are also influenced by the narrow framing coefficient, as apparent from Equation (17). However, β gives a clearer and more readily interpretable picture of how investors balance between consumption and financial utility.

estimates $\hat{V} < 0$ (and this across all considered parameter configurations). This reinforces the idea that maximizers of two-dimensional utility perceive financial investments as one that deteriorates the overall utility in the market equilibrium. However, $\hat{V}$ substantially increases subject to the intensity of narrow framing. Naturally, investors who narrow-frame financial investments perceive risky investments as more favorable or, equivalently, as less destructive for the total utility.

Regarding the attitude towards financial losses, we first note that the estimates $\hat{\lambda}$ of the loss aversion coefficient in equilibrium lie very close to the respective estimates in the expected-utility setting.[42] It is also interesting to observe that switching between (almost) no narrow framing $b_0 = 0.001$ to narrow framing $b_0 > 0.001$ entails substantial variations in $\hat{\lambda}$. By contrast, in the presence of narrow framing $b_0 > 0.001$ the loss aversion coefficient does not significantly vary subject to the degree of narrow framing. This supports the findings under expected-utility maximization that the absence of narrow framing with respect to financial projects cannot impact the aversion towards financial losses.

Interestingly, the estimates $\hat{\lambda}$ slightly diminish for more frequent portfolio evaluations and in the presence of narrow framing $b_0 > 0.001$, but only as long as financial investments are perceived as the main utility source $\beta = 0.8$. When, on the contrary, consumption receives the highest weight in the utility aggregation as $\beta = 0.2$, this tendency is reverted. Thus, more frequent checks of the risky-portfolio performance appear either to compensate for or to enhance the loss aversion in equilibrium, depending on the importance that is subjectively ascribed to financial utility relative to consumption utility.

Again, all obtained estimates $\hat{\lambda}$ lie below the originally proposed value of the loss aversion coefficient 2.25. For yearly evaluations they are slightly higher (lower) than the "neutral" value of 1, as $\beta = 0.8$ ($\beta = 0.2$). The highest value of $\hat{\lambda}$ is obtained for $\beta = 0.5$, $\delta = 0.9$, and $k = 3$.[43] Accordingly, investors behave loss aversely in the somewhat ambiguous situation where consumption and narrowly framed financial investments are aggregated with equal weights as utility sources. In the rest of the cases, the aggregate equilibrium with two-dimensional utility mostly requires that investors perceive financial losses in a more relaxed way.

Moreover, the loss aversion coefficient in equilibrium appears to be rather stable with respect to the penalty imposed on past losses k, especially when either risky performance is evaluated more often or the weights β are higher. The sole substantial changes (namely an increase) can be observed in Table 4 for $\beta = 0.2$, where the reluctance towards past losses appears to extend to current losses as well, as $k = 3$ entails higher values of $\hat{\lambda}$ compared to $k = 0$.

Finally, the loss aversion coefficient in equilibrium $\hat{\lambda}$ varies as expected proportionally

[42] Recall that we can compare the two settings for $\gamma = 0.5$ and $b_0 > 0.001$. See Tables 1 and 8.

[43] Specifically, this value is $\hat{\lambda} = 2.1797$.

to the initial attitude to losses given by λ. This conclusion is based on the findings for the entire range of $\lambda \in \{0.5; 1; 2.25; 3\}$). On average, the variations of the $\hat{\lambda}$ subject to λ are small. Moreover, for quarterly portfolio evaluations the evolution of $\hat{\lambda}$ subject to β and δ exhibits identical patterns to the original case with $\lambda = 2.25$. However, yearly portfolio revisions entail different changes in $\hat{\lambda}$, which grows for higher β and drops for higher δ as $\lambda = 0.5$, diminishes c.p. for both higher β and δ as $\lambda = 1$ and varies unsystematically for $\lambda = 3$. Accordingly, in the non-expected utility framework the evaluation frequency affects the way in which the initial attitude towards losses extends in equilibrium.

In the sequel, we proceed analogously to Section 3.2 by deriving equilibrium values of our two further measures of the actual investors' attitude to financial losses from Equations (7) and (8). The corresponding estimates for yearly (quarterly) data, and the usual cases with $\gamma = 0.5$, $\lambda = 2.25$, $\delta = 0.9$, and $\beta \in \{0.2; 0.8\}$ are included in Table 5 (Table 12 in Appendix 5.3).

	$\beta = 0.8$		$\beta = 0.2$	
	$k = 0$	$k = 3$	$k = 0$	$k = 3$
$b_0 = 0.001$				
LAi	1.0537	1.0074	0.83444	0.93177
gRA	2,372,400	2,083,000	288,530	248,880
$b_0 = 0.01$				
LAi	1.0846	1.0401	0.78922	0.95881
gRA	2,375,500	2,086,100	288,120	249,490
$b_0 = 0.1$				
LAi	1.0877	1.0433	0.7847	0.96152
gRA	2,375,900	2,086,400	288,080	249,550
$b_0 = 0.5$				
LAi	1.088	1.0436	0.7843	0.96176
gRA	2,375,900	2,086,400	288,070	249,550
$b_0 = 1$				
LAi	1.088	1.0437	0.78425	0.96179
gRA	2,375,900	2,086,400	288,070	249,550

Table 5: The estimated index of loss aversion (LAi) and global first-order risk aversion (gRA) in the expected-utility setting for yearly data, $\lambda = 2.25$, $\gamma = 0.5$ and $\delta = 0.9$.

First, note that the changes entailed by the different parameter combinations considered here go in similar directions for both measures of the actual attitude to financial losses LAi and gRA. Again, substantial changes in both LAi and gRA become apparent by switching between the cases without ($b_0 = 0.001$) and with narrow framing ($b_0 > 0.001$).

In general and similarly to the expected-utility setting, the equilibrium estimates of LAi closely follow the evolution of the simple coefficient of loss aversion $\hat{\lambda}$, where the former is somewhat smaller than the latter. LAi appears to be rather stable to the c.p.

variations of the degree of narrow framing b_0 (again as long as $b_0 > 0.001$), and of the penalty on past losses k. Again, the changes of LAi subject to the portfolio evaluation frequency remain unsubstantial as long as there is not penalty imposed on past losses and as financial investments are considered as the main source of utility $b_0 \geq 0.5$. By contrast, when investors are reluctant towards past losses $k = 3$ and when consumption and narrowly framed financial investments are viewed as equally important utility sources $\beta = 0.5$, somewhat higher variations of LAi between the two revision frequencies can be observed.

Finally, gRA develops in a direction that is intuitively to be expected for the attitude towards financial losses (where reductions in gRA are interpreted as higher loss aversion). In particular, it substantially drops for higher revision frequencies. It also clearly diminishes when past losses are penalized (i.e. by switching from $k = 0$ to $k = 3$), where the variation is more pronounced for quarterly portfolio evaluations. While the c.p. variation of LAi subject to the initial coefficient of loss aversion λ is weak and somewhat unsystematical, the equilibrium-equivalent estimates of gRA increase for higher λ-values. This confirms the results provided by the expected-utility approach, namely that gRA more consistently reflects the investor attitude towards financial losses.

Having analyzed the investor attitude towards losses in the non-expected utility equilibrium, we turn now to the problem of wealth allocation among consumption, risky, and risk-free financial assets. The results obtained for the percentages of both total wealth consumed α and post-consumption wealth invested in risky assets θ show that both these percentages are independent of the narrow framing coefficient b_0 and of the penalty on past losses k. They are illustrated for our usual cases with $\lambda = 2.25$, $\gamma = 0.5$, $\delta = 0.9$, and $\beta \in \{0.2; 0.8\}$, in the same above Tables 5 and 12.

First, as is apparent from Equation (29a), the estimates $\hat{\alpha}$ do not vary with δ and decrease for more pronounced weighting of the financial utility β, ranging from almost 96% (96%) for $\beta = 0.2$ to around 31% (35%) for $\beta = 0.8$ as portfolio performance is yearly (quarterly) evaluated. Thus, considering fluctuations of wealth from narrowly framed financial investments as the most important source of utility yields, higher wealth percentages allocated to financial assets (or equivalently, lower percentages dedicated to consumption) as expected. Moreover, $\hat{\alpha}$ grows for more frequent portfolio evaluations, where this tendency becomes manifest with a higher intensity as financial investments are considered as an important source of utility.[44] This corresponds to what we can denote as myopic aversion towards financial investments in general.

Second, note that $\hat{\theta} < 0$ for all chosen values of $\delta < 1$.[45] The explanation is provided

[44]Specifically, $\hat{\alpha}$ changes merely from 0.9578 to 0.9593 when switching from yearly to quarterly portfolio revisions for $\beta = 0.2$, while the corresponding variations for $\beta = 0.8$ are from 0.31335 to 0.34879.

[45]This situation changes for $\delta > 1$, as we obtain $\hat{\theta} > 1$. Thus, plausible values $0 < \hat{\theta} < 1$ appear to

by Equation (29b), where this denominator of the first-line expression is always negative. In essence, this denominator expresses the trade-off between the revenues from risk-free vs. risky investments[46] and *decreases* as risky investments become *more* profitable. In other words, the denominator of this equilibrium expression of θ reflects the profitability of risky with respect to risk-free assets taken with *contrary* sign. Recall now that θ stands for the percentages of post-consumption wealth put in risky assets, hence it is expected to *grow* as risky assets become *more* profitable. Therefore, only the absolute values of θ can be meaningfully interpreted, and the subsequent comments refer to $|\hat{\theta}|$.

As noted below Equations (29), higher β-values entail higher $\hat{\theta}$, hence lower $|\hat{\theta}|$. This tendency is slightly counterbalanced but never dominated by an increase in δ. Also recall that $|\hat{\theta}| > 1$ shows that investors borrow extra-money at the risk-free rate and invest it in the risky portfolio, which is the case for $\beta \leq 0.5$. Apparently, for lower coverage of consumption from additional incomes (i.e. low β-values, see for example Tables 6 and 13), agents make surprisingly large investments in risky assets, as they probably welcome the higher chances of gaining more money offered by risky investments. Across all considered (β, δ)-combinations, $|\hat{\theta}|$ varies between $67 - 944\%$ ($68 - 1008\%$) for yearly (quarterly) portfolio evaluations. Moreover, the estimates of θ taken in absolute value are generally lower for more frequent portfolio evaluations. Note also that $|\hat{\theta}|$ is higher compared to the mean percentages of post-consumption wealth invested in risky assets in the expected-utility setting.

	$\beta = 0.8$		$\beta = 0.2$	
	$k = 0$	$k = 3$	$k = 0$	$k = 3$
$\hat{I}/\bar{C}$	3.5459	3.5459	1.1609	1.1609
$(1 - \hat{\alpha})\|\hat{\theta}\|$	0.46184	0.46184	0.08826	0.08826

Table 6: The estimated wealth allocation in the non-expected utility setting for yearly data, $\lambda = 2.25$, $\gamma = 0.5$ and $\delta = 0.9$.

Finally, as θ does not reflect how *total* wealth is split between risky and riskless assets, it cannot tell us anything about the aversion to financial risks in a strict sense. Across all considered parameter-configurations, multiplying $1 - \hat{\alpha}$ by $|\hat{\theta}|$ amounts to around $9 - 78\%$ ($9 - 77\%$) for yearly (quarterly) portfolio evaluations. As expected, the highest values are obtained for maximal additional incomes (i.e. due to the fact that narrowly framed financial investments are considered the most important utility source $\beta = 0.8$ and/or

be possible only for $1 < \delta < 2$. However, $\delta > 1$ yields implausible values for other model parameters in equilibrium. In addition, the corresponding estimates in the expected-utility setting are also implausible. For this reason, we do not further analyze cases with $\delta > 1$.

[46]The denominator is close to the the expected equity premium taken with a negative sign, namely $-(E_t(R_{t+1}) - R_f) \simeq R_f - \exp(r + \sigma_r^2/2)$. Specifically, the risky returns are amended according to their correlation to consumption.

that δ is minimal). For $\beta = 0.8$, over 46% (44%) of the total wealth is dedicated to risky assets in equilibrium when portfolios are evaluated once a year (every three months). For $\beta = 0.2$, the respective percentages are very similar for both revision frequencies and lie below 9%. These results are captured in Tables 6 and 13 for $\delta = 0.9$. Thus, we can observe merely small changes in the percentages of total wealth invested in risky assets, as investors evaluate their portfolios more frequently. These changes are more evident for higher values of β. Based on this, we conclude that myopic loss aversion continues to hold in the two-dimensional utility setting with non-expected utility, but only as long as investors consider the utility derived form financial wealth fluctuations sufficiently important relative to the consumption utility. Even in this case, the myopic loss aversion holds in a softer form, as investors who evaluate their risky portfolios more often invest only slightly lower proportions of their total wealth in risky assets. Thus, maximizing non-expected utility appears to require in equilibrium a more relaxed attitude towards risky investments compared to the situation when expected utility is maximized. Moreover, relative to the findings in Rengifo and Trifan (2006) where financial investments are the sole source of utility, investors who maximize two-dimensional utility are more (less) open to risky investments, depending on whether narrowly framed financial investments are perceived as the most (least) important utility source.[47]

As a final remark, we note that both equilibrium percentages of total wealth dedicated to consumption $\hat{\alpha}$ and of post-consumption wealth invested in risky assets $\hat{\theta}$ are insensitive to the choice of the initial λ. Consequently, the same holds for the percentages of total wealth dedicated to risky assets $(1 - \hat{\alpha})|\hat{\theta}|$.

3.4 A summary comparison of the two approaches: expected and non-expected utility

We note that comparing the two considered settings with expected and non-expected utility is not an easy task. In spite of the "preventive measures" adopted in order to ensure the possibility of performing such comparisons (namely concerning the additional income and the values of the behavioral parameters), they employ distinct estimation procedures and hence deliver different parameter estimates. The former setting offers the additional advantage of being less complex and more intuitive, while the latter one allows us to directly estimate more variables of interest, especially those regarding the optimal wealth allocation. In spite of the claim in Barberis, Huang, and Thaler (2003) that non-expected utility better describes decision making under risk, it is difficult to reach

[47]In essence, the percentages of total wealth invested in risky assets in the one- and two-dimensional utility settings lie closest to each other for the somewhat ambiguous situation with $\beta = 0.5$ and middle-range δ-s, in sum for middle-range additional incomes.

a clear-cut conclusion about the adequacy of one or another of the settings developed in the present work, as they provide both common and specific results.

We start by noting the most important results that are similar under both approaches. First, excessively high values of the consumption-related risk aversion coefficient γ are incompatible with the idea of aggregate equilibrium under both approaches. Also, the lack of narrow framing of financial investments does not corroborate with the manifestation of an attitude towards financial investments (in particular towards losses) and effectively entails less plausible parameter estimates. Second, the aggregate markets where the representative investor maximizes expected and non-expected utility require similar attitudes towards financial losses. In particular, the loss aversion coefficient in equilibrium $\hat{\lambda}$ as well as the other two measures of this attitude LAi and gRA reach values that are very close to each other and exhibit similar evolutions subject to the variation of behavioral model parameters in both settings. The sole difference refers to the variation of $\hat{\lambda}$ subject to the revision frequency that appears to be dictated in the non-expected utility setting by the importance ascribed to financial relative to consumption utility. Also, under both approaches LAi closely follows $\hat{\lambda}$ and gRA appears to offer a more intuitive support of how the attitude towards losses changes.

On the other hand, maximizing expected and non-expected utility entail different results with respect to the wealth allocation among consumption, risky, and risk-free financial assets. This can in part be due to the above mentioned discrepancies in methodology. Specifically, in the expected-utility setting this wealth allocation can only be assessed on average, on the basis of the equilibrium estimates of other variables. By contrast, the non-expected utility approach prescribes exact values of the allocation variables in the market equilibrium. In particular, our empirical results show that investors who maximize non-expected utility appear to be substantially more reluctant towards financial investments in general, but more open towards investing in risky assets. The respective fractions of total wealth to be consumed in the aggregate equilibrium attain more plausible (namely higher) values than under the expected-utility approach. The variation of the total-wealth percentages invested in risky assets is more pronounced and their overall values higher. Moreover, myopic loss aversion clearly holds under expected-utility maximization. Yet, it becomes manifest in the non-expected utility framework only when investors consider the financial utility sufficiently important relative to the consumption utility. Even in this case, the wealth percentages dedicated to risky assets decrease only slightly for higher evaluation frequencies. Finally, investors with an additional source of utility appear to behave differently when maximizing expected utility or non-expected utility, with respect to their peers who derive utility exclusively from financial investments. Thus, while maximizing expected two-dimensional utility renders them more circumspect towards risky investments, maximizing non-expected two-dimensional utility yields the same only when consumption utility is considered more important than financial one. Otherwise, investors put substantially more money in risky assets relative to the one-sided utility approach.

4 Summary and conclusions

This paper extends the model in Rengifo and Trifan (2006) by analyzing loss attitude and wealth allocation in a two-dimensional utility framework. Specifically, one question we attempt to answer is how non-professional investors who derive utility from both consumption and (narrowly framed) financial investments behave when faced with financial risk. A related problem we tackle refers to how these investors consequently split their money between consumption, and risky vs. risk-free financial assets.

Following Barberis, Huang, and Thaler (2003) and Barberis and Huang (2004a,b), we consider two different ways of estimating prospective utility, namely either as expected or as non-expected utility. In both situations, we explicitly account for the narrow framing of financial investments, as well as for the impact of past performance on current perceptions. In this broader setting, we derive the necessary conditions for equilibrium in the aggregate market with a representative investor. Fixing general market parameters (such as the dynamics of consumption and of the expected returns), and several behavioral parameters specific to our model (such as the degree of narrow framing and the risk aversion to consumption), these conditions provide a basis for the derivation of the prospective value ascribed to risky investments in equilibrium. An equivalent loss aversion coefficient can then be estimated from the definition of this prospective value. In addition, the expected-utility setting allows for the estimation of the coefficient by which utility is discounted in time. The byproducts of the estimation procedure in the non-expected utility setting are the percentages of total wealth allocated to consumption and of post-consumption wealth invested in risky assets.

The theoretical results are subsequently tested and extended in an empirical context. We assess the equilibrium values and investigate the sensitivity of the variables of interest for various configurations of the behavioral parameters and for two distinct portfolio evaluation horizons (one year and three months). In order to avoid the impossibility of covering current consumption needs from financial revenues throughout the entire investing period, we also consider that investors periodically dispose of exogenous additional incomes. These incomes are shaped in order to ensure the equivalency of the two settings with expected and non-expected utility. We also analyze the variation of the equilibrium estimates with respect to changes in the magnitude of additional incomes.

The common and specific empirical findings under the two approaches with expected and non-expected utility can be summarized as follows. First, in both settings we can isolate equilibrium-incompatible parameter constellations. One the one hand, they are characterized by too high values of the consumption-related risk aversion coefficient. On the other hand, the lack of narrow framing entails implausible estimates in the expected-utility setting and unusual ones (relative to the values obtained for the remaining cases) in the non-expected utility framework. These cases are then discarded from the subsequent analysis.

Second, the loss aversion coefficients are very close to each other and also close to the "loss-neutral" value of 1 in both expected- and non-expected utility settings. Specifically, they lie slightly above (below) 1 for yearly (quarterly) portfolio evaluations. Thus, the coefficient of loss aversion required in order to attain the market equilibrium is clearly lower than 2.25, the value suggested by the original prospect theory. This indicates that the aversion towards financial losses is less pronounced when consumption is considered as an additional source of utility besides financial wealth fluctuations. Other things being equal, the loss aversion coefficients required to attain the aggregate equilibrium in both settings grow subject to higher degrees of narrow framing and drop when investors penalize past losses. Yet, in the expected-utility setting the same coefficient of loss aversion diminishes for more frequent risky performance checks. By contrast, under non-expected utility the corresponding variation is dictated by the importance ascribed to financial utility relative to consumption utility (specifically, for a higher importance we can observe the same reduction in the loss aversion coefficient for more frequent evaluations).

Furthermore, the other two measures of the investor attitude towards financial losses employed evolve in equilibrium similarly to the estimated loss aversion coefficients. The loss aversion index follows closely the simple loss aversion coefficient, being slightly lower. The global first-order risk aversion appears to describe more consistently and more intuitively the actual attitude towards financial losses and its variations subject to changes in the behavioral model parameters.

Third, the prospective value in equilibrium is always negative, suggesting that the overall utility diminishes with financial investments. This decrease in utility substantially shrinks as investors pay more attention to financial investments (i.e. by more intense narrow framing), as this attention is coupled with a positive image of the opportunities they might offer.

Fourth, the allocation of wealth between consumption and (risky and riskless) financial assets can be explicitly determined in the non-expected utility setting, where the equilibrium equations deliver the necessary estimates. Under the maximization of expected utility, this optimal allocation can only be assessed on average.

In the expected-utility framework, investors appear to allocate modest percentages of their total wealth to consumption, which also vary in high extent with the additional income meant to cover current consumption needs. The respective values amount to between $2 - 21\%$ $(1 - 13\%)$ for yearly (quarterly) portfolio evaluations, where the highest ones are obtained for minimal additional incomes. These small values should be understood in terms of our assumption that the total wealth includes the additional incomes, which implies that it increases faster than the consumption subject to higher additional incomes. By contrast, attaining the aggregate equilibrium when investors maximize non-expected utility requires substantial allocations for consumption purposes. The respective percentages yield $31 - 96\%$ $(35 - 96\%)$ for yearly (quarterly) performance checks and vary much less subject to the portfolio evaluation frequency. In neither of the two settings does the fractions of total wealth dedicated to consumption vary either with the degree of narrow framing or with the penalties imposed on past losses.

Moreover, in the expected-utility setting the percentages of post-consumption wealth invested in risky assets increase as the additional income covers more than current consumption needs (specifically, as it provides extra money to be invested in financial assets after accounting for consumption). The respective overall values lie around $27 - 45\%$ $(2 - 17\%)$ for yearly (quarterly) portfolio evaluations. The non-expected equilibrium delivers corresponding estimates that have to be interpreted in absolute value. For lower additional incomes, investors become very interested in risky assets, as they even borrow extra money for the purpose of increasing the risky investment. The minimal proportions of post-consumption wealth in the non-expected utility equilibrium lie above 67% (68%) for yearly (quarterly) portfolio evaluations and often exceed 100%. In both settings, the fractions of post-consumption wealth invested in risky assets are invariable with respect to the degree of narrow framing, but they slightly diminish when past losses are penalized in the expected-utility framework.

Finally, the resulting percentages of total wealth put in risky assets yield $22 - 43\%$ $(2 - 17\%)$ when expected utility is maximized and portfolio performance is evaluated yearly (quarterly). These values remain below the values obtained in the one-dimensional utility framework of Rengifo and Trifan (2006). The respective percentages in the non-expected equilibrium vary much more subject to the magnitude of the additional income, but appear to be less influenced by the changes in the evaluation frequency. Specifically, between $9 - 78\%$ $(9 - 77\%)$ of total wealth is invested in risky assets by investors maximizing non-expected utility. These values are hence higher (lower) than the average percentages which result when investors derive utility merely from financial investments, as the additional utility source, i.e. the consumption, becomes less (more) important.

In sum, these numbers support the idea that investors who check the performance of their portfolios more frequently invest less money in risky assets. Thus, myopic loss aversion continues to manifest when utility has twofold sources, albeit in a much softer form and merely for sufficiently high additional incomes when non-expected utility is maximized.

Fifth, our estimates appear to vary slightly and, as anticipated, proportionally with the initial loss aversion coefficient. The sole exception is obtained under the non-expected utility approach, where the loss aversion coefficient in equilibrium differently fluctuates subject to the initial loss aversion, depending on the magnitude of the the additional income. However, the corresponding variations remain low on average.

Concerning the expected-utility setting, the estimated discounting factor is found to lie close to the values proposed in the literature. Furthermore, it does not vary with the degree of narrow framing or the additional income, and is higher and less sensitive to changes in the consumption-based risk-aversion when portfolios are evaluated more often.

In sum, maximizing expected and non-expected utility when financial investments are narrowly framed reaches similar equilibrium results concerning the attitude to financial losses. By contrast, the two settings offer different pictures concerning the wealth allocation between consumption and risky and riskless assets in equilibrium. In particular, investors maximizing expected-utility appear to dedicate on average surprisingly small amounts of money to consumption and to behave on average myopically loss averse with respect to risky investments. The non-expected approach sets clear-cut target values for the wealth percentages to be consumed and invested in risky assets in order to reach the aggregate equilibrium. Specifically, both these percentages are substantially higher, but also less variable in the evaluation frequency than in the expected-utility setting. Consequently, under the non-expected utility approach, the aversion towards financial investments in general is more pronounced, but the attitude towards risky investments in particular is more relaxed, and myopic loss aversion holds merely in soft form and only when the financial wealth fluctuations are viewed as the major utility source.

5 Appendix

5.1 Descriptive statistics

	SP500		**10-year bond**		**Consumption**	
	Evaluation frequency					
	Quarterly	Yearly	Quarterly	Yearly	Quarterly	Yearly
Mean	0.017	0.066	0.017	0.073	0.016	0.052
Median	0.018	0.071	0.017	0.070	0.001	0.049
Std.Dev.	0.079	0.136	0.006	0.026	0.008	0.022
Kurtosis	2.661	-0.9659	0.623	0.974	0.673	-1.084
Skewness	-0.671	-0.205	0.951	1.042	-0.018	0.165
Max.	0.290	0.345	0.036	0.142	0.042	0.090
Min.	-0.302	-0.207	0.009	0.037	-0.010	0.011
Obs.	175	43	175	43	175	43

Table 7: Log-differences of the SP500 index and of the ten-year bond returns for quarterly and yearly portfolio evaluations.

5.2 The expected-utility approach

	$\beta = 0.8$				$\beta = 0.2$			
	$\gamma = 0.5$		$\gamma = 1$		$\gamma = 0.5$		$\gamma = 1$	
	$k = 0$	$k = 3$	$k = 0$	$k = 3$	$k = 0$	$k = 3$	$k = 0$	$k = 3$
$b_0 = 5$								
$\hat{\rho}$	0.99059	0.99059			0.99059	0.99059		
$\hat{V}$	-45.961	-45.961			-45.96085	-45.96085		
$\hat{\lambda}$	1.0104	1.0085			0.99221	0.99021		
$b_0 = 10$								
$\hat{\rho}$	0.99059	0.99059			0.99059	0.99059		
$\hat{V}$	-22.98	-22.98			-22.98042	-22.98042		
$\hat{\lambda}$	1.0033	1.0014			0.99085	0.98883		
$b_0 = 100$								
$\hat{\rho}$			0.99842	0.99842			0.99842	0.99842
$\hat{V}$			-308.13	-308.13			-308.13462	-308.13462
$\hat{\lambda}$			1.0663	1.0646			1.0196	1.0176
$b_0 = 1,000$								
$\hat{\rho}$			0.99842	0.99842			0.99842	0.99842
$\hat{V}$			-30.813	-30.813			-30.81346	-30.81346
$\hat{\lambda}$			1.0043	1.0024			0.99458	0.99252

Table 8: The main estimated parameters in the expected-utility setting for quarterly data, $\lambda = 2.25$ and $\delta = 0.9$.

	$\beta = 0.8$				$\beta = 0.2$			
	$\gamma = 0.5$		$\gamma = 1$		$\gamma = 0.5$		$\gamma = 1$	
	$k = 0$	$k = 3$	$k = 0$	$k = 3$	$k = 0$	$k = 3$	$k = 0$	$k = 3$
$b_0 = 5$								
LAi	1.0104	0.9908			0.99221	0.95704		
gRA	529,650	321,640			48,493	28,205		
$b_0 = 10$								
LAi	1.0033	0.98374			0.99085	0.95566		
gRA	529,700	321,690			48,528	28,238		
$b_0 = 100$								
LAi			1.0663	1.0469			1.0196	0.99993
gRA			1,091,600	664,180			102,750	61,130
$b_0 = 1,000$								
LAi			1.0043	0.98471			0.99458	0.97485
gRA			1,092,200	664,740			103,240	61,600

Table 9: The estimated index of loss aversion (LAi) and global first-order risk aversion (gRA) in the expected-utility setting for quarterly data, $\lambda = 2.25$ and $\delta = 0.9$.

	$\beta = 0.8$				$\beta = 0.2$			
	$\gamma = 0.5$		$\gamma = 1$		$\gamma = 0.5$		$\gamma = 1$	
	$k = 0$	$k = 3$	$k = 0$	$k = 3$	$k = 0$	$k = 3$	$k = 0$	$k = 3$
$\bar{C}/\hat{W}$	0.01866	0.01866	0.00975	0.00975	0.12545	0.12537	0.07108	0.07105
$\hat{I}/\bar{C}$	3.1856	3.1856	5.5556	5.5556	1.1583	1.1583	1.3889	1.3889
$\hat{\theta}$	0.15451	0.14439	0.1633	0.15316	0.03233	0.02182	0.09719	0.08692
$(1 - \bar{C}/\hat{W})\hat{\theta}$	0.15163	0.14170	0.16171	0.15167	0.02827	0.01908	0.09028	0.08074

Table 10: The estimated wealth allocation in the expected-utility setting for yearly data, $\lambda = 2.25$ and $\delta = 0.9$.

5.3 The non-expected utility approach

	$\beta = 0.8$		$\beta = 0.2$	
	$k = 0$	$k = 3$	$k = 0$	$k = 3$
$b_0 = 0.001$				
$\hat{\alpha}$	0.34879	0.34879	0.9593	0.9593
$\hat{\theta}$	-0.6799	-0.6799	-2.1644	-2.1644
$\hat{V}$	-803.47	-803.47	-292.13	-292.13
$\hat{\lambda}$	1.3553	1.3538	1.0335	1.0315
$b_0 = 0.01$				
$\hat{\alpha}$	0.34879	0.34879	0.9593	0.9593
$\hat{\theta}$	-0.6799	-0.6799	-2.1644	-2.1644
$\hat{V}$	-80.347	-80.347	-29.213	-29.213
$\hat{\lambda}$	1.0322	1.0303	0.99389	0.99185
$b_0 = 0.1$				
$\hat{\alpha}$	0.34879	0.34879	0.9593	0.9593
$\hat{\theta}$	-0.6799	-0.6799	-2.1644	-2.1644
$\hat{V}$	-8.0347	-8.0347	-2.9213	-2.9213
$\hat{\lambda}$	0.99699	0.99506	0.98992	0.98788
$b_0 = 0.5$				
$\hat{\alpha}$	0.34879	0.34879	0.9593	0.9593
$\hat{\theta}$	-0.6799	-0.6799	-2.1644	-2.1644
$\hat{V}$	-1.6069	-1.6069	-0.58427	-0.58427
$\hat{\lambda}$	1.0008	0.99928	0.98957	0.98753
$b_0 = 1$				
$\hat{\alpha}$	0.34879	0.34879	0.9593	0.9593
$\hat{\theta}$	-0.6799	-0.6799	-2.1644	-2.1644
$\hat{V}$	-0.80347	-0.80347	-0.29213	-0.29213
$\hat{\lambda}$	0.99663	0.9947	0.98953	0.98749

Table 11: The main estimated parameters in the non-expected utility setting for quarterly data, $\lambda = 2.25$, $\gamma = 0.5$ and $\delta = 0.9$.

	$\beta = 0.8$		$\beta = 0.2$	
	$k = 0$	$k = 3$	$k = 0$	$k = 3$
$b_0 = 0.001$				
LAi	1.3553	1.3361	1.0335	0.9983
gRA	528,060	320,079	48,151	27,871
$b_0 = 0.01$				
LAi	1.0322	1.0126	0.99389	0.95868
gRA	529,580	321,570	48,522	28,232
$b_0 = 0.1$				
LAi	0.99986	0.98027	0.98992	0.95472
gRA	529,730	321,720	48,559	28,268
$b_0 = 0.5$				
LAi	0.99699	0.97739	0.98957	0.95436
gRA	529,740	321,730	48,562	28,271
$b_0 = 1$				
LAi	0.99663	0.97703	0.98953	0.95432
gRA	529,740	321,730	48,563	28,271

Table 12: The estimated index of loss aversion (LAi) and global first-order risk aversion (gRA) in the expected-utility setting for quarterly data, $\lambda = 2.25$, $\gamma = 0.5$ and $\delta = 0.9$.

	$\beta = 0.8$		$\beta = 0.2$	
	$k = 0$	$k = 3$	$k = 0$	$k = 3$
$\hat{I}/\bar{C}$	3.1856	3.1856	1.1583	1.1583
$(1-\hat{\alpha})\lvert\hat{\theta}\rvert$	0.44276	0.44276	0.08809	0.08809

Table 13: The estimated wealth allocation in the non-expected utility setting for yearly data, $\lambda = 2.25$, $\gamma = 0.5$ and $\delta = 0.9$.

References

BARBERIS, N., AND M. HUANG (2004a): "The Loss Aversion/Narrow Framing Approach to Stock Market Pricing and Participation Puzzles," Working paper, University of Chicago, Stanford University.

——— (2004b): "Preferences with Frames: A New Utility Specification that Allows for the Framing of Risks," Working paper, University of Chicago, Stanford University.

BARBERIS, N., M. HUANG, AND T. SANTOS (2001): "Prospect Theory and Asset Prices," *Quarterly Journal of Economics*, 116, 1–53.

BARBERIS, N., M. HUANG, AND R. THALER (2003): "Individual Preferences, Monetary Gambles and the Equity Premium," Working paper, University of Chicago, Stanford University.

CAMPBELL, R., R. HUISMAN, AND K. KOEDIJK (2001): "Optimal Portfolio Selection in a Value-at-Risk Framework," *Journal of Banking and Finance*, 25, 1789–1804.

KÖBBERLING, V., AND P. WAKKER (2005): "An Index of Loss Aversion," *Journal of Economic Theory*, 122, 119–131.

RENGIFO, E., AND E. TRIFAN (2006): "Investors Facing Risk: Loss Aversion and Wealth Allocation Between Risky and Risk-Free Assets," Working paper, Darmstadt University of Technology.