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Based on the Specific Factors Model

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Unified Growth Based on the Specific Factors Model

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Abstract

The two-sector specific factor model is typically used in the theory of international trade where it helps to clarify the principle of comparative advantage. Instead, we use this model as explicit theoretical framework to explain major trends of long-run economic development. Combined with endogenous technical progress functions which assume that knowledge accumulates as a by-product of agricultural and manufacturing experience, the two-sector specific factors model can explain major historical trends and structural turnarounds. The technical progress functions establish the link between the agricultural and the manufacturing sector through the land-labour ratio, which is determined by the savings propensities of wage-earners, landlords and capitalists. This result is achieved by making use of the traditional investment = savings condition, without reference to complicated micro-based models of human capital accumulation.

Keywords: Economic development, growth, Industrial Revolution, income distribution.
JEL classification: E13, N1, O4

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1 Introduction and Motivation

Unified growth theory attempts to capture the entire process of economic development and its driving forces by a single theory. In conformity with the findings of economic historians, this process can be split into three consecutive phases: the epoch of Malthusian stagnation, the post-Malthusian transition, and the modern era of sustained growth.

Although unified growth theory is a relatively young sub-discipline of growth economics, the list of reference literature is vast, probably due to the nature of the subject, with its many streams of influence from multiple fields of inquiry. Particularly innovative contributions by economic theorists include Kremer (1993), Goodfriend and McDermott (1995), Galor and Weil (2000), and Hansen and Prescott (2002).

With few exceptions, e.g. Galor and Mountford (2006), models of unified growth theory are one-sector models with exogenous or endogenous technical change. In a recent contribution to this Journal, O'Rourke and Williamson (2005) referred to the specific factors model to highlight the dramatic reversal in distributional trends in the 19th century. As is well-known from the economics of international trade, the specific factors model is a two-sector model, where land is specific to agriculture and capital specific to manufacturing (industry). As such it appears to be particularly suitable for illustrating central features of unified growth theory, most notably the structural change from agricultural to industrial societies.

Without providing a fully-fledged theoretical analysis, O'Rourke and Williamson (2005) focus on five variables playing a role in the specific factors model. These variables include (1) the land-labour ratio, (2) the wage-rent ratio, (3) the price of agricultural in terms of manufactured output, (4) industrial productivity, and (5) the total factor productivity in agriculture. The historic development of these variables is portrayed for the British economy during two long-run periods, 1500-1840 and 1840-1936. For the somewhat shortened first period 1500-1750, linear regressions are derived which relate the wage-rent ratio (dependent variable) to the land-labour ratio, the total factor productivity in agriculture, and the industrial productivity (independent variables). The relative price of agricultural output is linked to the land-labour ratio only. Since, as a consequence of global market integration, the relative price of agriculture changes its character from a formerly dependent to an independent variable, the regression structure is different for the second period 1842-1936: The wage-rent ratio (dependent variable) becomes a linear function of industrial productivity and the relative price of agricultural output (independent variables). See the signs of the major regression estimates in Table 1.

The regressions for both periods support O'Rourke and Williamson's key message that major structural breaks in production and distribution after 1840 can be explained by the opening up of the European economy to international trade, which coincided with the Industrial Revolution.

Table 1: Signs of linear regressions in O'Rourke and Williamson (2005)

Periods		1500-1750		1842-1936
		Dependent Variables		
Regressions		P	$\omega_{L/X}$	$\omega_{L/X}$
	x	-	+	
Independent	A		-	
Variables	y_m		+	+
	P			-

P denotes the price of the agricultural good in terms of the manufactured good; x is the land-labour ratio; A is the level of technology in agriculture; y_m denotes the output-labour ratio in manufacturing; and $\omega_{L/X}$ is the factor-price ratio between labour and land.

In the following contribution, the specific factors model will be applied in a more rigorous way than in O'Rourke and Williamson (2005), where it was not formally specified. Additional relations will be taken into account, such as the share of labour employed in agriculture, the full set of factor-price ratios (not only the wage-rent ratio), and the capital intensity.

Unlike the prominent models of unified growth theory (e.g. by Galor) which focus on human capital investment and incorporate individual household maximisation rules, we concentrate on physical capital and introduce an investment = savings condition in the tradition of Kaldor, with the savings rates of workers, capitalists, and landowners as separate parameters.[2] Since the "demographic transition"[3] is not the subject of our analysis, we do not deal with the trade-off between child quantity and quality, which is a central feature of unified growth models. This does not mean that we deny its importance in explaining major historical trends and structural breaks. Instead, our argument is that these trends and reversals

[2]A micro-based unified growth model would have to distinguish between three types of households, workers, capitalists and landlords each having specific utility functions, budget constraints, time preference rates, etc. This would lead to enormous complexity, in particular when it comes to aggregation.

[3]In modern societies this trade-off has ultimately been resolved in favour of less, though better educated children by means of human capital investment.

can also be explained by simpler models such as the specific factors model.

Another ingredient of our model is the generalisation of Kremer's (1993) approach by introducing learning functions which explain the speed of technological progress in both manufacturing and agriculture. It is these technological learning functions that enable us to draw from the specific factors model empirically meaningful conclusions even beyond 1840. In our view, the specific factors model in combination with Kaldor's equilibrium condition and with Kremer's assumption on technological change provides a much simpler unified growth theory than the type which is known from the recent literature.

2 The Specific Factors Model

In the specific factors model referred to by O'Rourke and Williamson (2005) two commodities are produced: agricultural (= food) products using land and labour and manufactured (= non-food) goods using capital and labour. Hence, land is specific to the agricultural sector while capital is specific to the manufacturing sector. We assume that both consumption goods *and* capital goods are produced in the manufacturing sector.

The variables used should all include an index t for time. (See the summary of all variables in the appendix.) By convention, however, the subscript is omitted unless it is needed for clarification.

Production Functions

The production functions are of the Cobb-Douglas type and given by

$$Y_a = AL_a^{\alpha}X^{1-\alpha}, \quad 0 < \alpha < 1 \tag{1}$$

$$Y_m = ML_m^{\beta}K^{1-\beta}, \quad 0 < \beta < 1 \tag{2}$$

or in their intensive form:

$$y_a = Ax_a^{1-\alpha}, \tag{3}$$

$$y_m = Mk_m^{1-\beta}, \tag{4}$$

where Y_a, Y_m denote the output of agriculture and manufacturing, respectively. A, M are the levels of technology in the two sectors. L_a and L_m stand for the quantity of labour employed in agriculture and in manufacturing. X is the total quantity of land, while x_a is the land-labour ratio in agriculture. K denotes the quantity of capital and k_m the capital-labour ratio in manufacturing. y_a, y_m are the per capita output in agriculture and in manufacturing, respectively.

Full Employment/Full Capacity

Employment in agriculture (L_a) and manufacturing (L_m) is equal to total labour supply (L). Hence:

$$L_a + L_m = L. \tag{5}$$

Using ρ as the symbol indicating the share of agricultural labour in total labour supply (= demand), one can also write:

$$x = \rho x_a \tag{6}$$

and

$$k = (1 - \rho)k_m, \tag{7}$$

with x as the overall land-labour ratio and k as the economy-wide capital-labour ratio.

Efficiency Conditions

Under the assumption of commpetitive markets for labour, land and capital, the standard efficiency conditions require:

$$\begin{aligned}
\frac{\partial Y_a}{\partial L_a} &= \frac{W}{P}, \\
\frac{\partial Y_m}{\partial L_m} &= W, \\
\frac{\partial Y_a}{\partial X_a} &= \frac{R_X}{P}, \\
\frac{\partial Y_m}{\partial K_m} &= R_K,
\end{aligned} \tag{8}$$

where W is the (nominal) wage rate, R_X the rental rate of land, and R_K the rental rate of capital.

Taking account of (1) - (4) gives for the derivatives in (8):

$$W/P = A\alpha x_a^{1-\alpha},$$

$$
\begin{aligned}
&W = M\beta k_m^{1-\beta}, \\
&R_X/P = A(1-\alpha)x_a^{-\alpha}, \\
&R_K = M(1-\beta)k_m^{-\beta}.
\end{aligned} \tag{9}
$$

Therefore, after inserting (8) into the factor-price ratios:

$$\omega_{L/X} = \frac{W/P}{R_X/P} = \frac{\partial Y_a/\partial L_a}{\partial Y_a/\partial X_a}$$

and

$$\omega_{L/K} = \frac{W}{R_K} = \frac{\partial Y_m/\partial L_m}{\partial Y_m/\partial K_m},$$

two expressions eventually result for the specific factors model which relate the factor-price ratio between land and labour ($\omega_{L/X}$) to the land-labour ratio (x), and the factor-price ratio between labour and capital ($\omega_{L/K}$) to the capital intensity (k):

$$\omega_{L/X} = \frac{\alpha}{1-\alpha}x_a = \frac{\alpha}{1-\alpha}\frac{x}{\rho} \tag{10}$$

and

$$\omega_{L/K} = \frac{\beta}{1-\beta}k_m = \frac{\beta}{1-\beta}\frac{k}{1-\rho} \tag{11}$$

Equation (10) is the one which O'Rourke and Williamson (2005) implicitly refer to when they discuss the relationship between the land-labour ratio[4] and the wage-land rent ratio. This relationship is not as simple as it is the case in the static one-good world, where the positive correlation between $\omega_{L/X}$ and $x(= x_a)$ can be explained by diminishing returns in agriculture. Instead, in the two-sector specific factors model the development of $\omega_{L/X}$ is also influenced by ρ, the share of agricultural employment in total employment.

Differentiating the second expression in (10) with respect to time gives:

$$\dot{\omega}_{L/X} = \frac{\alpha x}{(1-\alpha)\rho}\left(\frac{\dot{x}}{x} - \frac{\dot{\rho}}{\rho}\right). \tag{12}$$

[4]Note that O'Rourke and Williamson (2005) define x as "land in agriculture" divided by the "total economy-wide labour force".

With (12) one can explain the dramatic reversal of long-run trends in the ratio of wages to land rents reported by O'Rourke and Williamson (2005): Prior to the 19th century, the ratio of wages to land rent had been declining for a long period because the land-labour ratio had also been falling. At the same time the impact of the structural change from agriculture to manufacturing, measured by the share of agricultural employment in total employment, had been weak, if anything, so that in (12) the expression in brackets had been negative.

This trend reversed at some point in the 19th century: The dramatic structural change brought by the Industrial Revolution was accompanied by an exodus of workers from agriculture to industry, which led to a massive drop in the growth rate of ρ up to a point where its (negative) growth rate overcompensated the (negative) growth rate of the land-labour ratio. Eventually, the parallel development of the factor-price ratio and the land-labour ratio was broken, because in (12) the expression in brackets became positive.

The figures in Table 2 illustrate the development of ρ, which had come down from a level of 70 per cent before 1700. As noted by Voth (2003), the British agricultural labour share in the 19th century was unusually low in comparison to other countries; e.g. in the USA (Japan) it was still as high 70 per cent in 1820 (1870). Note that between 1820 and 1870, i.e. the period when the dramatic turnaround described by O'Rourke and Williamson (2005) took place, the percentage change of ρ dropped by the factor 3.

Table 2: Agricultural labour share in the UK, 1700-2003

UK	1700	1820	1870	1890	1913	1929	1950	1973	2003
ρ	56	37.6	22.7	16.1	11.7	7.7	5.1	2.9	1.2
$\dot{\rho}/\rho$		-0.33%	-1.00%	-1.70%	-1.38%	-2.58%	-1.21%	-2.42%	-4.21%

Source of data: Maddison (2007)

The Relative Price of Agricultural Output

From the above efficiency conditions the relative price of agricultural output can easily be derived. Since

$$W = \partial Y_m/\partial L_m = P\partial Y_a/\partial L_a = M\beta k_m^{1-\beta} = PA\alpha x_a^{1-\alpha}, \tag{13}$$

we get

$$P = \frac{M\beta k_m^{1-\beta}}{A\alpha x_m^{1-\alpha}} = \frac{\beta y_m}{\alpha y_a}. \tag{14}$$

The price of the agricultural product, relative to the manufacturing good, is inversely proportional to the sectoral labour productivities. P declines (rises) if the growth rate of agricultural labour productivity exceeds (falls short of) the growth rate of labour productivity in the manufacturing sector. In the latter case, gains in living standards would occur primarily through the falling relative price of manufactured goods, as described by Broadberry and Gupta (2006) for the pre-1800 period.[5] P roughly doubled during 1500-1600, based on the original data by Phelps Brown and Hopkins (1957), partly because agricultural productivity fell by 24% as a result of strong population pressure.[6] Between 1600 and 1700 agricultural productivity recovered - it increased by 51% - , and P remained more or less constant, indicating that the productivity jump in agriculture must have been accompanied by a similar productivity gain in manufacturing.[7] Finally, P rose by 40% between 1700 and 1800, based on the original figures from Schumpeter (1938).[8] At the same time, agricultural productivity augmented by 24% suggesting an even stronger manufacturing productivity rise. Hence, the specific factors model supports the finding of economic historians (such as Crafts and Harley, 1992) that the Industrial Revolution did not appear as a sudden shock or "big bang", but had long been developed before its "official" beginning in 1760.

Endogenous Technical Progress

We have not yet discussed technological progress in the agricultural and in the manufacturing sector, which is represented by A_t and M_t, respectively. Inspired by Kremer (1993) the following "learning functions" are introduced:

$$\frac{A_{t+1} - A_t}{A_t} = \frac{\dot{A}}{A} = \lambda L_a, \tag{15}$$

$$\frac{M_{t+1} - M_t}{M_t} = \frac{\dot{M}}{M} = \mu L_m \tag{16}$$

with $\lambda, \mu > 0$. Equations (15) and (16) mean that technological change in both

[5] The positive sign of the regression between $\omega_{L/X}$ and y_m in O'Rourke and Williamson (2005) is not supported by the static specific factor model. Since $\omega_{L/X}$ is also equal to $\frac{\partial Y_m/\partial L_m}{P\partial Y_a/\partial X_a}$, or $\omega_{L/X} = \frac{\beta y_m}{PA(1-\alpha)x_a^{-\alpha}}$, a rise of y_m ceteris paribus increases both the nominator and the denominator (through P) by the same amount. However, the negative sign of the regression between $\omega_{L/X}$ and A (see Table 1) complies with the static specific factors model.

[6] The figures on agricultural productivity are from Allen (2000).

[7] O'Rourke and Williamson (2005) combine data from Phelps Brown and Hopkins (1957) with data from Schumpeter (1938) which are not totally consistent; e.g., for the period 1660-1700 P increased according to the former while it decreased according to the latter.

[8] If prices of consumption goods exclusive of cereals are considered, the price development of the latter was not significantly different from the development of producers' good prices.

sectors does not develop at constant rates but is proportional to the respective size of labour employment, which accumulates "knowledge" over time. Note that this type of endogenous technical progress is formally different from the type introduced by Matsuyama (1992) where the *absolute* change of total factor productivity in manufacturing is proportional to maufacturing output,[9] which in turn depends on the fraction of labour employed in manufacturing.

Taking the natural logarithm of the intensive form of the production functions (3) and (4) and differentiating with respect to time yields under consideration of (6) and (7):

$$\frac{\dot{y}_a}{y_a} = \frac{\dot{A}}{A} + (1-\alpha)\frac{\dot{x}_a}{x_a} = \frac{\dot{A}}{A} + (1-\alpha)\left(\frac{\dot{x}}{x} - \frac{\dot{\rho}}{\rho}\right), \tag{17}$$

$$\frac{\dot{y}_m}{y_m} = \frac{\dot{M}}{M} + (1-\beta)\frac{\dot{k}_m}{k_m} = \frac{\dot{M}}{M} + (1-\beta)\left(\frac{\dot{k}}{k} - \frac{\dot{\rho}}{\rho - 1}\right). \tag{18}$$

As equation (17) contains the same expression in brackets on the right hand as equation (10), we can repeat the above argument: Prior to the 19th century, the growth rate of x had exceeded that of ρ, with the consequence that the total factor productivity growth in agriculture (A) had surpassed the growth rate of agricultural output per worker in agriculture (y_a). Since, over a period of 300 years, output per worker in agriculture had risen by 43%[10], total factor productivity in agriculture had increased by an even higher rate, if one sticks to the assumptions of the specific factors model. However, at same point in the 19th century, this trend reversed and total factor productivity growth in agriculture fell short of growth of output per worker in agriculture.

Mutatis mutandis, the above way of reasoning can be applied to the manufacturing sector, where in the early phase of development total factor productivity growth had remained below manufacturing output growth per worker, before the former gained momentum during the Industrial Revolution and eventually surpassed the latter.

Inserting functions (15) and (16) into (17) and (18) helps understand the structural change initiated by the Industrial Revolution:

$$\frac{\dot{y}_a}{y_a} = \lambda L_a + (1-\alpha)\frac{\dot{x}_a}{x_a} = \lambda\rho L + (1-\alpha)\left(\frac{\dot{x}}{x} - \frac{\dot{\rho}}{\rho}\right), \tag{19}$$

[9] Strulik and Weisdorf (2008) generalise Matsuyama's approach in a recent contribution to *this* Journal.

[10] According to figures by Allen (2000), British output per worker in agriculture declined by 24% from 1500 until 1600, increased by 51% during 1600 and 1700, followed by a rise of 34% between 1700 and 1750 and a decline of 7% from 1750 until 1800.

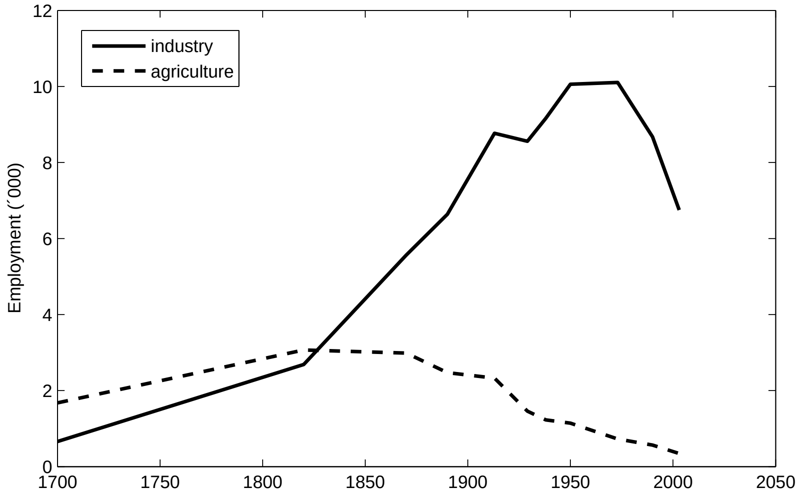

Figure 1: Agricultural and industrial employment in the UK [source of data: Maddison (2007)]

$$\frac{\dot{y}_m}{y_m} = \mu L_m + (1-\beta)\frac{\dot{k}_m}{k_m} = \mu(1-\rho)L + (1-\beta)\left(\frac{\dot{k}}{k} - \frac{\dot{\rho}}{\rho - 1}\right). \tag{20}$$

Learning-by-doing effects in agriculture reached their maximum between 1820 and 1870 when agricultural employment was highest (more than 3 million workers; see Figure 1). On the other hand, the stream of agricultural workers into industry created a new potential of learning-induced technological change boosting manufacturing productivity. While the learning-by-doing effects in agriculture had been persisting for hundreds of years and had gradually lifted agricultural labour productivity, the respective effects in industry were more abrupt and powerful, since the forces of the Industrial Revolution were amplified by global market integration, as pinpointed by O'Rourke and Williamson (2005).

Saving and Investment

With the growth equations in the previous section defining the growth of agricultural output per worker and of manufacturing output per worker, the description of the growth process is not yet complete. What is still missing is the development of capital accumulation. To complete the picture, we refer to a central building block of standard two-sector growth models in the tradition of Hahn (1965), the S = I condition.

Under the assumption that the gross savings propensities of wage earners (s_w), capitalists (s_c) and landlords (s_x) are nonnegative parameters in the range between 0 and 1 (not necessarily constant), total gross savings in the economy are:

$$s_w WL + s_c R_K K + s_x R_X X = S.$$

Gross investment is defined as

$$I = \dot{K} + \delta K.$$

In equilibrium with S = I, after making use of (6)-(9), the following expression holds:

$$i_m = \frac{I}{L_m} = \frac{\beta M k_m^{1-\beta} \left(s_w + s_c \frac{1-\beta}{\beta}(1-\rho) + s_x \frac{1-\alpha}{\alpha}\rho \right)}{1-\rho}. \tag{21}$$

Note that $\beta M k_m^{1-\beta}$ is manufacturing output per worker or y_m in our notation. The instantaneous change in the capital intensity of the economy

$$\dot{k} = i_m(1-\rho) - (\delta + n)k \tag{22}$$

may therefore be written as:[11]

$$\dot{k} = \beta M \left(\frac{k}{1-\rho} \right)^{1-\beta} \left(s_w + s_c \frac{1-\beta}{\beta}(1-\rho) + s_x \frac{1-\alpha}{\alpha}\rho \right) - (\delta + n)k. \tag{23}$$

More generally, for a given development of population and given parameter values of $\alpha, \beta, s_w, s_c, s_x$, and δ, the change in the capital-labour ratio is a function of k, ρ, and μ footnotesince M is a function of μ; see equation (16).

$$\dot{k} = \dot{k}(k, \rho, \mu). \tag{24}$$

In the special case in which the manufacturing sector produces only capital goods, i.e. $y_m = i_m$, ρ becomes a function of the savings propensities:

$$\rho = \frac{1 - [\beta s_w + (1-\beta)s_c]}{1 - \beta \left(s_c \frac{1-\beta}{\beta} - s_x \frac{1-\alpha}{\alpha} \right)} \tag{25}$$

For given parameter values of α and β, the structural transformation of the economy is driven by the savings propensities of workers, capitalists and landlords.

[11] If there is only one sector, then $\rho = 0$ and $k_m = k$, so that (23) takes the form of the basic Solow model.

During the "Malthusian epoch", bondsmen in the countryside and workers in the cities did not save; their incomes stayed on subsistence level. Capitalists were more or less engaged in trade or in the putting-out system, not in industry which did not exist at that time. Landlords did not save either. Their style of living was "consumption-oriented" [see e.g. Bloch (1939)]. Landlords scorned work and mocked the merchant's thriftiness. Rent was spent on luxurious castles and palaces, clothes and banquets. Rather small parts of rent were saved and if so, were mainly used to maintain buildings (stables, sheds, and palaces) and cattle. In England serfdom was abolished earlier than on the European continent, which compelled landlords to rent their land to farmers. These farmers (yeomen and commoners) introduced capitalistic norms into farming because they understood that they had to maximise the difference between revenue and cost (= rent plus wages). Additionally, they became interested in technical knowledge for improving the fertility of soil for crop enhancement. One innovation was the transition from the three-field system to the crop rotation system, which increased the cultivated area by one third.

If neither workers nor capitalists save, equation (25) is reduced to:

$$\rho = \frac{\alpha}{\alpha + (1-\alpha)\beta s_x}. \tag{26}$$

For realistic parameter values of α and β, ρ is relatively high but declines with an increase in the savings propensity of landlords (including gentry, yeomen, and commoners) since $\partial\rho/\partial s_x < 0$. Hence the declining trend of the fraction of labour employment in agriculture described in Tab. 1 must have been accompanied by a gradual increase in the savings propensity of land owners.

From (26) follows:[12]

$$\frac{\dot{\rho}}{\rho} = -\frac{\dot{s_x}}{s_x}. \tag{27}$$

The higher the growth rate of savings by landlords, the higher the exodus of agricultural workers is, who leave the sector with their technological know-how. The latter effect, however, is cushioned by the productivity gain in agriculture stemming from a smaller labour force employed on more fertile land, see equation (19).

[12] The negative correlation between the growth rate of the ratio of agricultural to total labour and the growth of the average savings propensity (or investment ratio) during 1700 and 1830 can easily be verified by combining data from Maddison (2007) on the former with data from Crafts (1985) on the latter.

3 Conclusion

In this paper we have presented a unified growth model composed of three building blocks: (a) a two-sector specific factors model covering the sectors agriculture and manufacturing; (b) endogenous technical progress functions for the two sectors, which introduce dynamics into the specific factors model; and (c) a conventional savings function which distinguishes between savings by workers, capitalists, and landlords. Despite its simplicity, the model can describe major trends in economic development over the very long run: It highlights (a) the parallel development of the land-labour ratio and the wage - land rent ratio before 1800; (b) the reversal of this relationship in the 19th century; and (c) the transformation of the U.K. from an economy based on agriculture to one dominated by industry - a creeping process which had gradually got under way, but which massively accelerated after the Industrial Revolution. This transformation becomes manifest in the development of the ratio of agricultural employment to total employment, which is a key variable of the model. Not only did its decreasing trend contribute to the dramatic reversal of the wage - land rent ratio, it also helped to thwart the slowing technical progress in agriculture caused by dissipating learning effects. Apart from this, the fraction of labour employment in agriculture plays an important role in the process of capital accumulation - besides the capital intensity, and the technological productivity of an industrial worker. In the special case in which the manufacturing sector produces only capital goods, the share of agricultural labour to total labour can be expressed as a function of the savings propensities of workers, capitalists, and landlords. What the model cannot confirm includes the apparently positive correlation between the labour productivity in manufacturing and the wage - land rent ratio before 1800, which remains an issue. In comparison to competing models, the above model is more conventional in two respects: It is a closed economy model which does not deal with the effects of global market integration, and it models physical instead of human capital accumulation. This does not mean that the growing openness to international trade and the behavioural changes in parents' preferences with regard to their offspring ("child quantity - quality trade-off) is denied. Instead, we simply confirm the more traditional view that technical progress and physical capital accumulation were the driving forces of economic development during the Industrial Revolution.

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Appendix: List of Variables

Y_a	:	output of agricultural good
Y_m	:	output of manufacturing goods
P	:	price of agricultural good relative to manufactured good ($P = P_a; P_m = 1$)
L_a	:	quantity of labour employed in agriculture
L_m	:	quantity of labour employed in manufacturing
X	:	total quantity of land
K	:	total quantity of capital
x	:	land-labour ratio = X/L
k	:	capital-labour ratio = K/L
x_a	:	land-labour ratio in agriculture =X/L_a
k_m	:	capital-labour ratio in manufacturing =K/L_m
A	:	level of technology in agriculture
M	:	level of technology in manufacturing
W	:	wage rate of homogenous labour
R_X	:	rental rate of land
R_K	:	rental rate of capital
$\omega_{L/X}$	:	factor-price ratio between labour and land
$\omega_{L/K}$	:	factor-price ration between labour and capital
y_j	:	output-labour ratio in sector j, j = a, m
n	:	growth rate of labour force
ρ	:	ratio of labour employed in agriculture = L_a/L
δ	:	rate of depreciation
s_w, s_c, s_x	:	gross savings propensities of workers, capitalists, and landlords
$\alpha, \beta, \lambda, \mu$	:	constant paramenters