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Thomas Fischer

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News Reaction in Financial Markets within a Behavioral Finance Model with Heterogeneous Agents

Thomas Fischer*
TU Darmstadt
fischer@vwl.tu-darmstadt.de

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Abstract

This paper presents a Heterogeneous Agent Model of a financial market with chartist and fundamentalist traders that exhibit bounded rationality and short-term thinking to explain the effect of under and overreaction to news. The existence of the Market Maker's finite price adjustment speed leads to the fact that prices do not adjust instantaneously to new information. Chartists use moving average rules to make their investment decisions. Chartist can transform an underreaction-only scenario into a market with overreaction. The use of long moving average rules might even make the market unstable. Furthermore, noise in financial markets can lead to long time decoupling from fundamental value. Higher market efficiency (low deviations from fundamental value), on the other hand, is achieved if high rationality and long-term thinking for the agents is assumed.
JEL classification: G14 - D84 - C62 - C15
Keywords: Heterogeneous Agent Model - stock market - under and overreaction to news - moving average rules - financial stability

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1 Introduction

This paper shows that the phenomenon of under and overreaction to news can be explained by a Heterogeneous Agent Model (HAM) of a financial market. This effect is only considered scantly in the literature on HAMs.[1] First, an analytical discussion of a simplified linearized version of the model without noise is presented. Instead of using Bifurcation Theory, the analytical framework of classical control theory is applied. We show that the emergence of overreaction and instability depends on the chartists' strategy. Underreaction occurs due to finite price adjustment speed and risk aversion by fundamental traders. It can be dampened by chartist behavior. In the case of a combined under and overreaction scenario high aggressiveness of chartists and high price adjustment speed can lead to instability. Secondly, a simulation-based approach of the complex model shows that a low degree of rationality of agents as well as short-term thinking increase the effect of both under and overreaction and therefore decrease market efficiency. Simulation also confirms that market noise leads to long-term decoupling from fundamental value.

HAMs dating back to Day and Huang (1990) have recently become very popular for discussing the behavior of stock markets. These models rely on two basic assumption: agents (i) exhibit bounded rationality and (ii) form heterogeneous beliefs. The HAMs in finance normally distinguish between fundamentalists, technical, and noise traders. The models have been applied to different markets such as commodities (Reitz and Westerhoff, 2007), foreign exchange (De Grauwe and Grimaldi, 2006), options (Frijns et al., 2010), and stocks (Westerhoff, 2008). The models are able to replicate several stylized facts found in actual financial markets such as excess volatility, random walk behavior (indicated by insignificant autocorrelations in returns), volatility clustering (as indicated by significant slowly decreasing autocorrelations in absolute returns), skewness as well as excess kurtosis of return distribution (Lux, 2009). In a mathematical sense, these models are represented by non-linear difference equations. Current research expands these models to incorporate realistic trading strategies (e.g.Westerhoff (2006)), whilst the mathematical analysis, mostly relying on the tools of Bifurcation Theory, is brought to a more sophisticated level (e.g. Hommes and Wagener (2009)). This analysis helps to understand which parameters or model features drive the stylized facts (e.g. He and Li (2007)) and correspond to the stability of the market (e.g. Chiarella et al. (2009)). Major factors seem to be the rules

[1] Boswijk et al. (2007) present a HAM of the S&P500 explaining the DotCom-bubble by the overreaction to good fundamental news.

used by chartist traders (Chiarella et al., 2006) and the noise in financial markets (Chiarella et al., 2011).

In this paper we use the HAM framework to examine the effect of under and/or overreaction. This effect is inconsistent with the Efficient Market Hypothesis assuming instant price reaction to news fundamentals. Nevertheless, several empirical studies seem to confirm these effects in real markets. Underreaction describes the idea that prices only sluggishly react to new information and is therefore also often referred to as the Momentum Effect. This effect implies that past price movements have predictive power for future prices, since they are followed by returns of the same sign. Overreaction on the other side states that markets overreact to good or bad news, but returns adjust to a mean in the long run. Therefore, this effect is also known as Mean Reversion. These effects seem to be contradictory. Note that underreaction is mostly measured in the short run, whilst overreaction is found in longer horizons of roughly three to five years (Beechey et al., 2000).

Several models explain the effects of under and overreaction based on findings of Behavioral Finance. Daniel et al. (1998) attribute these effects to overconfidence and biased self-attribution. Individuals overestimate the precision of private signals (overconfidence). By contrast, reaction to public events is asymmetrical: events that confirm the validity of private information are attributed to high forecast ability, while public information that disconfirms private information is blamed on noise or sabotage (biased self-attribution). Daniel et al. (1998) provide simulations that show short-run Momentum followed by long-run reversals. This is also measured by short-run positive and long-run negative autocorrelations in returns. Note that the model predicts initial overreaction followed by even more overreaction. Another approach for explaining both effects in a unified theoretical framework is presented by Barberis et al. (1998). They assume the two psychological effects of representativeness and conservatism. The former refers to the effect that market participants tend to see patterns based on few observations, while the latter refers to the slow updating of beliefs. The combination of these two effects is able to replicate the effect of short-term Momentum and long-run Mean Reversion. While these models rely on the idea of a single representative agent, the approach of Hong and Stein (1999) introduces the interaction of different trader types as a key to understand both effects. Due to slow diffusion of private information among so-called Information Traders, there is underreaction and momentum in the prices, which evokes the action of Momentum Traders with positive feedback behavior creating the effect of overreaction. The authors present a hump-shaped price reaction function and are also able to measure the short-run positive and long-run negative autocorrelations. Under and overreaction are both stronger when low information diffusion is considered. Both Hong and Stein (1999) and Barberis et al. (1998) present a model with initial underreaction followed by subsequent overreaction.

In the remainder of this paper, we follow the rationale of Hong and Stein (1999) that combined under and overreaction can be explained by the interaction of heterogeneous agents with bounded rationality. Therefore, a very common representation of a HAM is presented in section 2. Based on a linearized version of the model, the conditions for under and overreaction are examined analytically in section 3. In line with Chiarella et al. (2006) it is assumed that technical traders use moving average rules. The window length of this rule proves to be crucial for systemic stability. Longer moving average rules might even lead to instability. Furthermore, we discuss the interaction of the parameters of chartist and fundamentalists aggressiveness as well as price reaction speed of the Market Maker. One key finding is that due to the fact that markets have a finite price adjustment speed and are therefore not cleared at any time as assumed by Walrasian aucutioneer, trend-following chartist traders emerge and eventually lead to overreaction or even instability. In section 4 the complex model is discussed on a simulation based approach. The model is able to replicate several effects found in empirical studies of under and overreaction. Both analytical and simulation-based approaches confirm that noise trading in combination with Momentum trading is a crucial factor that drives real markets and affects market stability. Section 5 concludes and gives directions for further research.

2 Basic model

This section presents the basic model. The model presented is closely related to well-known HAMs of financial markets as presented in recent surveys by Hommes and Wagener (2009) and Chiarella et al. (2009). We assume mean-variance portfolio optimization in a world with two assets: a risky asset with expected return $E_i(r_{t+1})$ and a risk-free asset with safe return of r_f[2]. The demand for risky asset is derived with mean-variance portfolio optimization (Hommes and Wagener, 2009):

$$D_t^i = \frac{E_i(r_{t+1}) - r_f}{RA \cdot \sigma_r^2} \tag{1}$$

[2]The effect of dividends is neglected since we assume day-trading behavior. Since dividends are normally only paid out once a year only, they do not matter for all but one trading period a year.

The demand of a certain group of agents i therefore crucially depends on the group's individual expectation of future returns.[3] Demand for risky assets increases with high expected excess returns (relative to risk-free rate). Inversely, demand is low in the case of high risk aversion RA and high volatility of returns σ_r^2.

The market-clearing in classic economic models is modeled as a Walrasian auctioneer. The key idea is that after determining the excess market demand, the auctioneer keeps announcing prices and interacts with the market feedback until the excess demand equals zero. This yields the classic *demand equals supply* equation:

$$\sum_{i=1}^{n} W_t^i D_t^i = N_t \tag{2}$$

In this case, $0 < W_t < 1$ represents the market weight of a specific group of agents. The aggregate demand should equal the supply N_t. Since agents can go short in stocks in the case that they expect prices to fall, they can also supply stocks ($D_t^i < 0$). Thus, no external supply N_t is necessary. This case shall be referred to as Zero Net Supply.

As presented in Chiarella et al. (2009), this modeling approach, even though widely used in economic analysis, only plays a part in one real market (the market for silver in London). Therefore, it is convenient to model a so-called Market Maker mechanism for market-clearing (e.g., Chiarella et al. (2006), Westerhoff (2008)). Even though this approach is still very simplified, it comes closer to price determination in actual markets. The key idea here is that an institution named Market Maker takes an offsetting long or short position to assure that excess demand in period t equals zero. In the next period, the Market Maker announces a new log-price p_{t+1} to reduce excess demand[4]:

$$p_{t+1} = p_t + \mu(\sum_{i=1}^{n} W_t^i D_t^i - N_t) \tag{3}$$

In this case, $\mu > 0$ represents the price reaction speed of the Market Maker. If we assume infinite reaction speed, this approach reduces to a Walrasian auctioneer:

$$\lim_{\mu \to \infty} \left(\frac{p_t - p_{t+1}}{\mu} + \sum_{i=1}^{n} W_t^i D_t^i - N_t \right) = \sum_{i=1}^{n} W_t^i D_t^i - N_t = 0 \tag{4}$$

[3] To improve the processing of the demand in the computational model I apply a slightly different formation for the demand, which is presented in the appendix in section A.

[4] The model uses log-prices p_t instead of real prices P_t. This is briefly discussed in appendix A.

This result will be of interest when the dynamic properties of the system are analyzed in the following section. Furthermore, the parameter μ can be interpreted as the liquidity of the market. In time of illiquid markets μ is high and prices react severely to excess demand.

In the basic model, the weights of the different agents vary in time. This represents the empirical fact pointed out by Menkhoff and Taylor (2007) that traders do not stick to a certain rule, but instead use a combination of both technical and fundamental analysis. The weights of the groups are derived using a Multinominal Logit Model as presented in Manski and McFadden (1981):

$$W_t^i = \frac{e^{\gamma A_t^i}}{\sum_{i=1}^n e^{\gamma A_t^i}} \tag{5}$$

Due to the construction of the equation, the individual weights sum up to one. The parameter γ presents a degree of rationality in choosing a strategy. In case γ equals zero, the weights of the groups are constant and amount to $1/n$. The other extreme case with γ converging to infinity represents the case in which all individuals choose the optimal forecast. De Grauwe and Grimaldi (2006) therefore interpret this parameter as a model of the behavioral effect of Status Quos Bias as presented in Kahneman et al. (1991). This effect implies that individuals find it difficult to change a decision rule they used in the past. In a more general way, this parameter can also be considered as a value for bounded rationality in the sense of Simon (1955). Due to the limited resources of time and money, individuals use suboptimal rules.

The weight of a strategy W_t^i in the market is evaluated by its attractiveness A_t^i in a period t. This parameter is modeled in the following way[5]:

$$\begin{aligned} A_t^i = D_{t-1}^i \cdot (r_t - r_f) + \eta A_{t-1}^i &\approx D_{t-1}^i \cdot (\ln(1+r_t) - \ln(1+r_f)) + \eta A_{t-1}^i \\ &= D_{t-1}^i \cdot (p_t - p_{t-1} - \ln(1+r_f)) + \eta A_{t-1}^i \end{aligned} \tag{6}$$

It considers the profits a strategy yielded between period $(t-1)$ and t. Note that a profit is made in the case where risky assets are bought when returns are higher than risk-free returns, or risky assets are sold when their return is lower than the return of the risk-free asset. The parameter $0 < \eta < 1$ represents the memory of the agent. If it is set to zero, myopic traders that only value the very last success of the strategy are considered. In the case $\eta = 1$, instead of profits the accumulated wealth of a group is taken into account. This modeling approach enables us to investigate the effect

[5]This equation builds on results presented in appendix A.

of short-term focusing in financial markets. The parameters γ and η are therefore the key to measuring the degree of irrationality in markets.

The model investigates four different strategies: (i) fundamentalism, (ii) chartism using moving average rules, (iii) noise trading, and (iv) a passive investment strategy. *Fundamental* traders know the true fundamental log-value of an asset f_t and expect the prices to converge to it. Their expectations can therefore be modeled in the following way:

$$E_F(p_{t+1}) - p_t = \alpha(f_t - p_t) \tag{7}$$

The parameter $0 < \alpha < 1$ measures the speed at which fundamentalist traders expect prices of stock to converge to their true underlying value. This strategy can be interpreted as the the Hedge Fund strategy of so-called Alpha Seeking trying to buy undervalued and to sell overvalued securities in the market (securities whose α, representing the deviation from the Security Market Line, are positive, respectively negative). Their action contributes to higher market efficiency.

Chartists on the other hand do not consider fundamental prices, but derive order signals from past prices. There are several studies indicating widespread use of technical analysis (even) among professional traders in particular in foreign exchange markets. Chartism is especially important for short-term forecast horizons.[6] Hong and Stein (1999) show that chartism can be useful in exploiting the general underreaction of markets. Chartism is often also referred to as Technical Trading, since it derives its trading signals from clear rules that can be automated. For this reason it is also very easy to implement these rules in a HAM. One of the easiest rules to implement is the moving average rule:

$$E_C(p_{t+1}) - p_t = \beta \left[\frac{1}{N_s} \sum_{i=0}^{N_s-1} p_{t-i} - \frac{1}{N_l} \sum_{i=0}^{N_l-1} p_{t-i} \right] \tag{8}$$

This strategy compares a long to a short-moving average ($N_s < N_l$). The use of the moving averages can be explained by market noise: it filters fluctuations around a long-run trend (Menkhoff and Taylor, 2007).[7] Normally, an intersection of the two moving averages is required to generate a trading signal. If we neglect this condition, this rule can generate a trading signal in each trading period implying that traders are always in the market (Brock et al., 1992). Another important feature of this rule is that it shows Momentum behavior by generating buying signals in case of increasing prices and

[6] For a survey the reader is referred to Menkhoff and Taylor (2007).

[7] In a control theory sense, a moving average acts as a low-pass filter, which filters away high-frequency noise.

selling signals in case of decreasing prices (Menkhoff and Taylor, 2007)[8]. The parameter $0 < \beta < 1$ measures the aggressiveness with which the chartist traders take positions in the market.

A crucial factor in market trading is *noise trading*. According to Black (1986), noise traders trade on noise as if it were information. Noise is modeled as an i.i.d. process with mean zero and variance σ_i^2. This is consistent with the consideration of Shleifer (2000) that noise should, on mean, cancel itself out. Noise trading can also be explained by the need for liquidity (here the need to raise capital for other reasons (Bouchaud et al., 2009)). In line with Westerhoff (2008), noise is considered in three parts of the model. First, there is a demand of pure noise traders a_t which is included in the Market Maker equation:

$$p_{t+1} = p_t + \mu(\sum_{i=1}^{n} W_t^i D_t^i - N_t) + a_t \tag{9}$$

On the other side, both fundamentalist and chartist traders have features of noise traders. Therefore their expectations formation is also superimposed by noisy processes b_t and c_t:

$$E_F(p_{t+1}) - p_t = \alpha(f_t - p_t) + b_t \tag{10}$$

$$E_C(p_{t+1}) - p_t = \beta \left[\frac{1}{N_s} \sum_{i=0}^{N_s-1} p_{t-i} - \frac{1}{N_l} \sum_{i=0}^{N_l-1} p_{t-i} \right] + c_t \tag{11}$$

Since chartists exhibit more irrational behavior, it is assumed that $\sigma_c > \sigma_b$.

The last remaining group are *passive traders*. Since they only invest in the risk-free asset, the attractiveness of their strategy is always zero, implying that they do not earn excess return relative to the risk-free rate. Note that if fundamentalists or chartists fail to predict future price movements correctly, their attractiveness can become negative. Accordingly, the weight of the passive agents increases. Apart from that, high risk-free rates, high risk aversions and high volatility of stocks contribute to the attractiveness of the passive strategy modeling a flight to quality. Since passive traders do not take orders in the market, they do not have an impact on the prices.

[8] The opposite is the case for a Mean Reversion strategy, which is heavily used by Hedge Funds. If a short moving average is below a long moving average, a buying signal is perceived. The long-moving average in the Mean Reversion strategy therefore can therefore be considered a proxy for the fundamental value derived upon historic data.

3 Analytical approach in a linearized version of the model

The analytical approach applies the techniques of control theory in the frequency domain. These rules have been developed for linear differential equations. The use of linear differential equations for the modeling of stock market behavior dates back to Beja and Goldman (1980) and is still widely used in models such as Chiarella et al. (2011). Since the model consists of non-linear difference equations, several simplifications have to be made. First, we assume that prices are described by a continuous time function $p(t)$ instead of a discrete function p_t with the following property:

$$p_{t+1} - p_t \approx \frac{dp(t)}{dt} = \dot{p} \tag{12}$$

Furthermore, the simplified model assumes risk-neutral investors and a risk-free rate of zero, which leads to the fact that demand equals the expected change of log-prices of each group[9]:

$$D_t^i = E_i(p_{t+1}) - p_t \tag{13}$$

If we now consider the case of $\gamma = 0$ for the weighting equation (equation 5), we model totally irrational individuals who stick to a rule, even though it is not profitable. Taking into account equation 5 this results in the fact that all three rules have the same market share. If we neglect the scaling behavior of the weighting factor, the following continuous time Market Maker equation can be derived:

$$\dot{p} = \mu(\frac{1}{3} \cdot D_C + \frac{1}{3} D_F) \approx \mu(D_C + D_F) \tag{14}$$

This result is identical to the one of Chiarella et al. (2011). The same result can be derived if totally rational (γ converging to infinity) but myopic investors ($\eta = 0$) are considered[10]. Thereby the analytical results in this chapter apply for total irrational as well as extremely myopic investors. The noise terms in the models are set to their expected value of zero (De Grauwe and Grimaldi, 2006).

[9] This formation of demand is based on the results presented in appendix A. Higher risk aversion can be considered if low values for the aggressiveness of a strategy as measured in the parameters α and β are assumed. Since the daily risk-free rate is close to zero it is usually neglected (e.g. Fama (1998)).

[10] The derivation of the law of motion of prices assuming $\eta = 0$ and $\gamma \to \infty$ is presented in appendix B.

First, we want to examine the fundamentalist-only case ($\beta = 0$). This results in the following law of motion for log-prices p:

$$\dot{p} = \mu(\alpha(f - p)) \tag{15}$$

If we transfer this equation into the frequency domain, the following response function $F(s)$ to a step-shock in fundamental value can be derived:

$$F(s) = \frac{p(s)}{f(s)} = \frac{1}{1 + \frac{s}{\mu\alpha}} \tag{16}$$

By assuming a step function, we examine the effect of prices in the case where the log-fundamental value f suddenly changes from zero to one. The result resembles the classic PT_1-behavior of control theory (Unbehauen, 2008):

$$F(s) = \frac{K}{1 + Ts} \tag{17}$$

The system converges to a final value of K with a speed of T (see figure 1). Since in this case $K = 1$, the model converges to its fundamental value. In

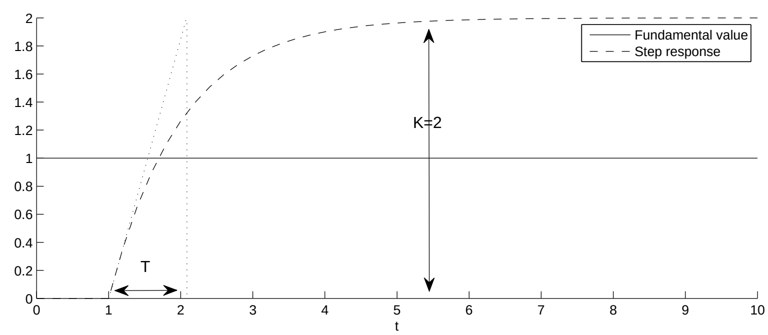

Figure 1: Response of a PT_1 system to a step function

this case an underreaction-only scenario is produced. The effect of underreaction is stronger for high values of T:

$$T = \frac{1}{\mu\alpha} \Rightarrow s = -\frac{1}{T} = -\frac{1}{\mu\alpha} < 0 \tag{18}$$

The effect of underreaction is therefore stronger in case the case of low price adjustment speed μ (i.e. high market liquidity) as well as the low aggressiveness of fundamental agents α (i.e. high risk aversion of fundamentalists). The system is always stable since the eigenvalue is always negative. If we furthermore take into account a Walrasian auctioneer as a special case of Market

Maker with infinite conversion speed, there is no underreaction ($T = 0$). The same result can be derived for the case of risk-neutral fundamental trader (α converging to infinity). This is consistent with the idea of the EMH that prices adjust instantaneously to news (Menkhoff and Taylor, 2007).

Now, the effect of different technical rules on the behavior of prices is investigated. We start by assuming the very simple case of $N_s = 1$ and $N_l = 2$. This yields the following demand for chartists:

$$D_C = \beta(p_t - \frac{1}{2}(p_t - p_{t-1})) = \frac{\beta}{2}(p_t - p_{t-1}) \tag{19}$$

This modeling for the demand of chartists is frequently used in HAMs (e.g. Westerhoff (2008)). The main idea is that chartists expect the most recent trend to continue at a speed of $\frac{\beta}{2}$. Considering differential instead of difference equation chartist demand can be presented as follows[11]:

$$D_C = \frac{\beta}{2}(p_t - p_{t-1}) = \frac{\beta}{2}(\dot{p} - \ddot{p}) \tag{20}$$

If we insert this into the Market Maker equation and transfer it into the frequency domain, the following response function $F(s)$ to a step in fundamental value can be derived[12]:

$$F(s) = \frac{p(s)}{f(s)} = \frac{1}{\frac{\beta}{2\alpha}s^2 + \frac{2-\mu\beta}{2\mu\alpha}s + 1} \tag{21}$$

This behavior represents the so-called PT_2 function of control theory (Unbehauen, 2008):

$$F(s) = \frac{K}{\frac{1}{\omega_0^2}s^2 + \frac{2D}{\omega_0}s + 1} \tag{22}$$

The eigenvalues of the system are defined by the following equation:

$$s_{1/2} = \omega_0(-D \pm \sqrt{D^2 - 1}) \tag{23}$$

In this case, the variables D and ω_0 are given as follows:

$$\omega_0 = \sqrt{\frac{2\alpha}{\beta}} \tag{24}$$

[11] A derivation of this result is presented in appendix C.

[12] A short introduction to the analysis of linear differential equation in the frequency domain is given in the appendix C. This section also presents the derivation and the discussion of the transfer functions for the different presented cases.

$$D = \frac{2 - \mu\beta}{2\sqrt{2}\mu\sqrt{\alpha\beta}} \tag{25}$$

Depending on the value of D three cases can be distinguished (see figure 2). In the first case $D > 1$, the system converges in a slow process of underreaction to its fundamental value like the PT_1 transfer function. The condition for underreaction-only therefore is as follows:

$$2 > \mu\sqrt{\beta}(\sqrt{\beta} + 2\sqrt{2}\sqrt{\alpha}) \tag{26}$$

Low values of price adjustment speed μ as well as low aggressiveness of agents α and β therefore lead to the underreaction-only scenario. Keeping in mind that low aggressiveness can also be interpreted as high risk aversion by agents this leads to the result that underreaction is promoted in a scenario with high risk aversion. Furthermore, low values of μ can be interpreted as high liquidity. This implies that overreaction tends to occur more frequently in illiquid markets. Note that in the presence of chartists high aggressiveness by both chartist and fundamental traders lead to overreaction.

Overreaction on the other side occurs in second case of $0 < D < 1$. As presented in Hommes (2011), the effect of overreaction can only be produced in the case where chartist traders with autoregressive behavior of at least second order (AR(2) behavior) are assumed. The simulation shows the well-known hump-shaped price pattern as presented in Daniel et al. (1998) and Hong and Stein (1999) of underreaction in the first instance followed by subsequent overreaction (see figure 2). In the long-run, the system converges

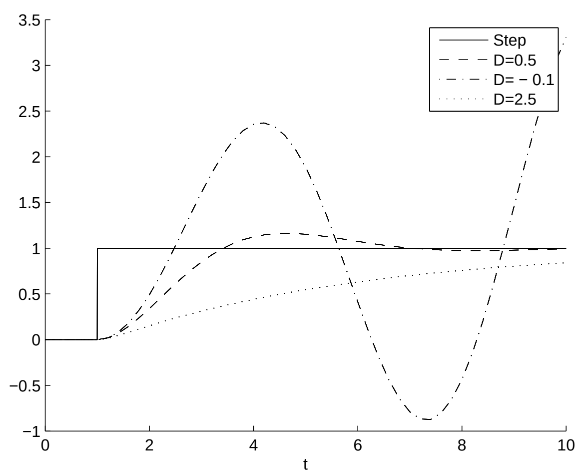

Figure 2: Response of a PT_2 system to a step function with exemplary values for D

to its underlying fundamental value.

This does not hold in the third case ($D < 0$). For $\mu\beta > 2$ we have an unstable system. High price adjustment speed and the high aggressiveness of chartist traders therefore lead to instability.

The parameter ω_0 represents the frequency of price behavior. In the case of underreaction-only, high values of ω_0 therefore indicate fast conversion to fundamental value, whilst in the case of combined under and overreaction they lead to faster swings between under and overreaction. High value for fundamentalist aggressiveness α relative to the aggressiveness of chartists β therefore at first sight might therefore lead to less underreaction. On the other side, as presented in equation 26, higher values of α lead to overreaction. In other words, high aggressiveness of fundamentalists in order to reduce underreaction leads to the effect of overreaction of market prices to news.

If we now assume $L_s = 1$ and $L_l = 3$, the following chartist demand can be derived:

$$D_c = \beta \left[p_t - \frac{1}{3} \sum_{i=0}^{2} p_{t-i} \right] = \beta \left[\frac{2}{3} p_t - \frac{1}{3} p_{t-1} - \frac{1}{3} p_{t-2} \right] \tag{27}$$

The price reaction function is described by the following equation:[13]

$$F(s) = \frac{1}{-\frac{\beta}{\alpha 3} s^3 + \frac{4\beta}{3\alpha} s^2 + (\frac{1-\mu\beta}{\mu\alpha}) s + 1} \tag{28}$$

This system is always unstable. Therefore the theoretical results of Chiarella et al. (2009) which show that longer moving average rules destabilize the market are confirmed.

4 Simulation of the complex model

As discussed in section 3, the analytical approach required some simplifications. Therefore, the simulation results of the complex model are compared with the linearized model. In the process, we also want to investigate the parameter of autocorrelation intensely discussed in empirical studies of under and overreaction.

Applying a shock of $\ln(2) \approx 0.69$ in log-fundamental value f_t is identical to a doubling of real fundamental value F_t. Figure 3 shows simulation results for the case with $L_s = 1$ and $L_l = 2$ in a zero-noise-framework. The parameters are set to $\mu = 1$ and $\alpha = \beta = 0.6$ implying overreaction for

[13] The determination of this equation is presented in appendix C.

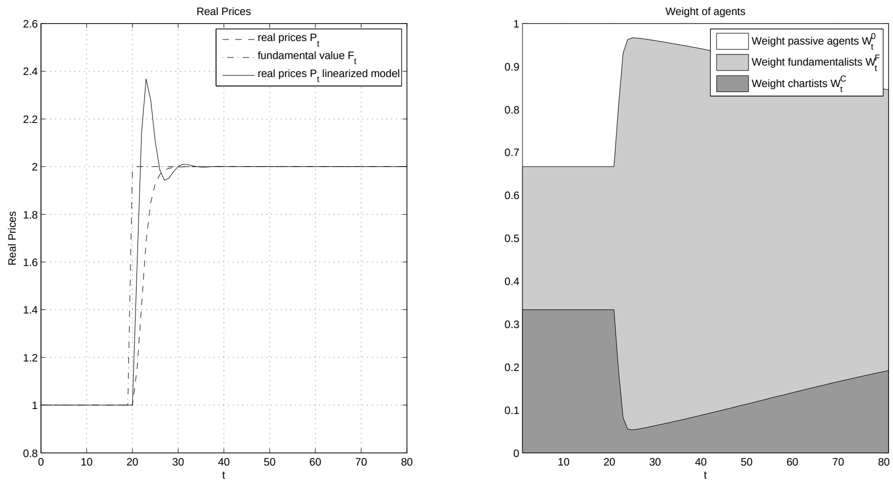

Figure 3: (left:) Price reaction to a step shock in news in both linearized and complex model; (right:) Weight of agents in complex model

the linearized model. Furthermore, the values of $\eta = 0.985$ for memory and $\gamma = 20$ for rationality are assumed.[14] Note that the hump-shaped pattern is only produced in the linearized case. This can be explained by the fact that the linearized version assumes constant weights of agents ($W_C = W_F = 1$).

As presented in figure 3, the shock in news fundamentals is accompanied by a higher weight of fundamentalist traders. Since the weight of Momentum Traders is less important than in the linearized model there is no overreaction. Even though there is no overreaction in the complex case, it also exhibits the negative autocorrelation for higher time lags (see figure 4). For that reason, long-run negative autocorrelations do not have to signify overreaction.

[14]The risk aversion is assumed as $RA = 10$ and the risk-free rate as $r_f = 0.01\%$ (equals an annual rate of approximately $r_f = 2.5\%$).

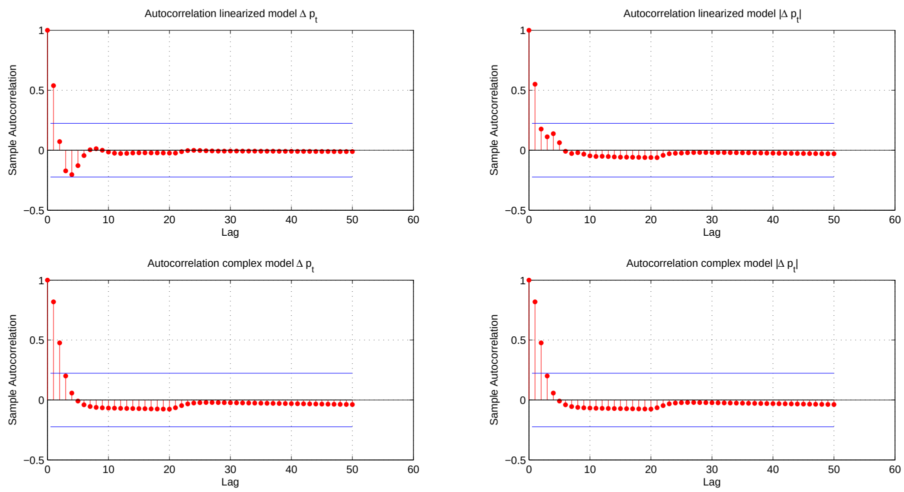

Figure 4: Autocorrelation of raw and absolute returns in both linearized and complex model after a step shock in news fundamentals

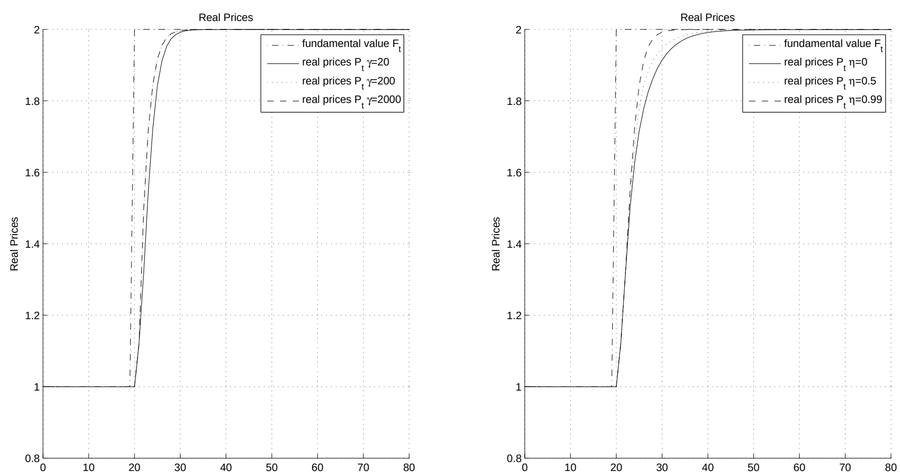

Figure 5: Price reaction to a step shock in news fundamentals in the complex model with $L_l = 3$ and variation of rationality γ and memory η

The $L_l = 3$ is always unstable in the linearized version, whilst the complex case produces underreaction. In figure 5 the parameters for rationality γ and memory η are varied for this case. High values of rationality γ and memory η lead to lower underreaction and therefore to higher market efficiency.

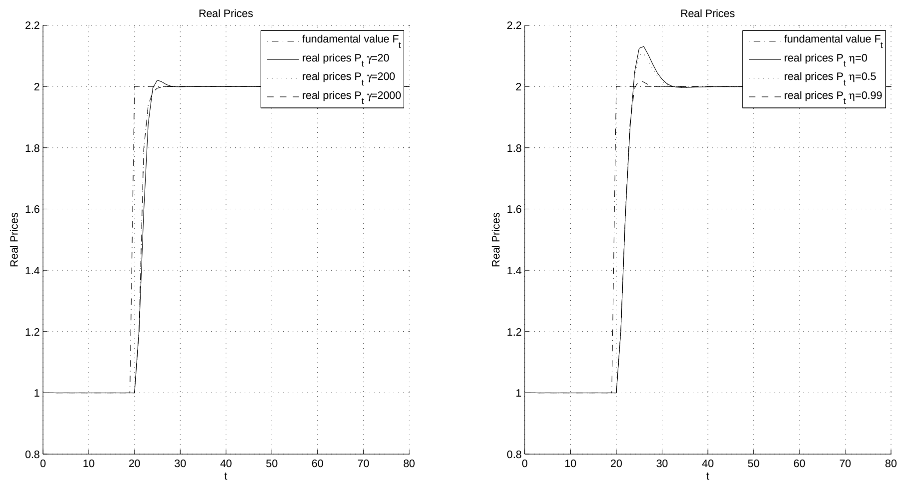

Figure 6: Price reaction to a step shock in news fundamentals in the complex model with $L_l = 4$, $\mu = 1.4$, and variation of rationality γ and memory η

When simulating the case with $L_l = 4$ in the complex model and further also assuming $\mu = 1.4$, the classic hump-shaped pattern of overreaction is produced. As shown in figure 6, a higher degree of rationality γ leads to lower overreaction. Short-term thinking (low values for η) on the other side amplifies the effect of overreaction. Simulation confirmed that the effect of memory is more important than the effect of rationality. Since high memories and high rationality lead to both lower under and overreaction they contribute to higher market efficiency.

Simulation assumed zero noise. Now, different forms of noise are applied for the $L_l = 3$ case. As shown in figure 7, pure noise trading noise σ_a and fundamentalist noise σ_b only lead to noise-induced swings around the true fundamental value. Chartist noise σ_c on the other hand leads to a permanent trend away from the fundamental value.

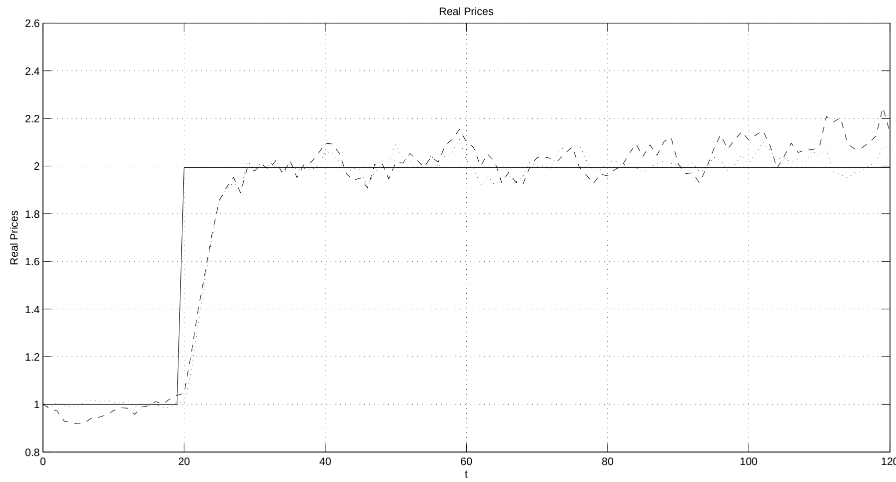

Figure 7: Reaction to a step shock in news fundamentals in the complex model with $L_l = 3$ and variation of noise: solid line fundamental value; dotted line $\sigma_a = 0.01$, $\sigma_b = 0.02$, $\sigma_c = 0$; dashed line $\sigma_a = 0.01$, $\sigma_b = 0.02$, $\sigma_c = 0.05$

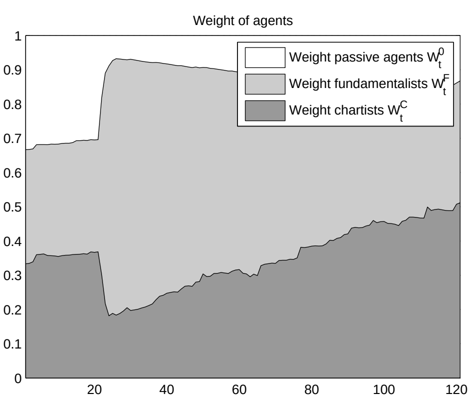

Figure 8: Weights of agents with step shock to news fundamentals and noise

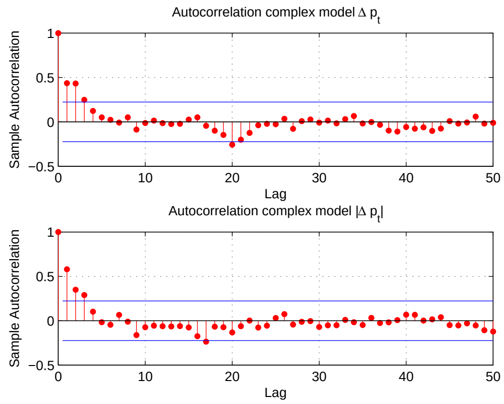

Figure 9: Autocorrelation of raw and absolute returns in the complex model after a shock in news fundamentals in the case of noise

This can be explained by the fact that high noise makes the chartist's process attractive. As shown in figure 8, for the case with all three forms of noise, after the fundamental shock in period 20 which makes fundamentalism attractive, chartist traders take over the market and destabilize it. Noise also leads to the fact that the autocorrelations of the returns become insignificant and lose their patterns (see figure 9). Therefore, it is difficult to derive results from empirical studies of autocorrelation.

5 Conclusion

In this paper the phenomenon of under and overreaction to news in financial markets is discussed within the framework of a Heterogeneous Agent Model. This model relies on the idea that market prices are the result of the interaction of fundamental and technical traders both subject to bounded rationality as well as short-term thinking. Furthermore, there is noise in the trading process. An analytical approach of the linearized model confirmed that the existence of finite price adjustment speed and risk-aversion of fundamental traders leads to underreaction. A fundamental-only scenario with infinite price adjustment speed (Walrasian auctioneer) on the other hand can replicate the instantaneous adjustment to news fundamentals as predicted by the Efficient Market Hypothesis. Chartist behavior transform an underreaction-only scenario into a scenario with under and overreaction. Consistent with Chiarella et al. (2006), the use of longer moving average rules also leads to systemic instability.

Based on a simulation this paper also shows that the analytical approach overestimates the effect of financial fragility by assuming constant agent weights. In the simulation, news leads to a higher weight of fundamental agents that transforms the system back to its fundamental value. The simulation is able to reproduce the short-run positive and long-run negative autocorrelations in returns shown in empirical studies. Apart from that, the simulation confirms that high degrees of rationality and long-term thinking decrease the effect of underreaction. In a scenario with overreaction, high rationality can decrease the effect of overreaction. Short-term thinking with low values for memory on the other hand, worsens the effect of overreaction.

The simulation also considers the effect of noise. First of all, noise in financial markets makes it difficult to derive results from empirical studies of autocorrelation. Moreover, in combination with chartist trading noise leads to further decoupling from fundamental value.

Further research therefore should analyze the effect of noise in more detail. This paper only presents a very simplified analytical approach. Deeper insights might be gained in the case where the model is analyzed in the so-called z-domain developed for difference equations (e.g. Juang (1994)). Furthermore, more realistic moving-average rules, as presented in Brock et al. (1992), should be examined in a simulation-based approach. Further research should also discuss the effect of these rules on statistical properties commonly investigated in HAMs.

A Demand in the computational model

The computational model processes log-prices p_t instead of real prices P_t. This has the advantage that in contrast to real prices, which cannot fall below zero, log-prices are not bounded. Recall the following mathematical connection for log-prices:

$$E(p_{t+1}) - p_t = \ln(E(P_{t+1})) - \ln(P_t) = \ln\left(\frac{E(P_{t+1})}{P_t}\right) = \ln(1 + E(r_{t+1})) \quad (29)$$

Recall the first-order Taylor approximation for the ln function:

$$\ln(1 + x) = x + O(x^2) \quad (30)$$

If we now use these results, the demand of group i can be displayed in the following way:

$$\begin{aligned} D_t^i = \frac{E_i(p_{t+1}) - p_t - \ln(1 + r_f)}{RA \cdot \sigma_r^2 + 1} &= \frac{\ln(1 + E_i(r_{t+1})) - \ln(1 + r_f)}{RA \cdot \sigma_r^2 + 1} \\ &\approx \frac{E_i(r_{t+1}) - r_f}{RA \cdot \sigma_r^2 + 1} \end{aligned} \quad (31)$$

Note that the value of one is added up in the denominator. This happens for two reasons: (i) As the variance at the beginning of computing time is zero, simulation would otherwise run into problems of zero division. (ii) In this modeling approach RA acts as a scaling factor. By setting this parameter to zero we can account for risk-neutral individuals.

B A different linearization approach for the model

The linearization approach presented in the text is independent of the memory of agents η. A similar result can be derived if we assume optimal weighting (γ converging to infinity) but short-term thinking due to zero memory ($\eta = 0$). We simplify the weighting equation by using the first-order Taylor approach for the exponential function:

$$e^x = 1 + x + 0(x^2) \quad (32)$$

The weighting in this case only depends on the attractiveness:

$$W_i = \frac{1 + \gamma A_i}{\sum_{i=1}^{3}(1 + \gamma A_i)} = \lim_{\gamma \to \infty}\left(\frac{\frac{1}{\gamma} + A_i}{\frac{3}{\gamma} + \sum_{i=1}^{3} A_i}\right) = \frac{A_i}{\sum_{i=1}^{3} A_i} \quad (33)$$

By also assuming a zero risk-free rate, the attractiveness can be determined by the following differential equation:

$$A_t^i = (p_t - p_{t-1})D_t^i + 0 \cdot A_{t-1}^i \Rightarrow A_i = (\dot{p} - \ddot{p})D_i \tag{34}$$

This results in the following market equation, which can be simplified by assuming small values for demand so that it represents the one presented in equation 14.

$$\begin{aligned}\dot{p} &= \mu\left[\frac{A_C}{A_C + A_F} \cdot D_C + \frac{A_F}{A_C + A_F} \cdot D_F\right] \\ &= \mu\left[\frac{(\dot{p} - \ddot{p}) \cdot D_C \cdot D_C + (\dot{p} - \ddot{p}) \cdot D_F \cdot D_F}{(\dot{p} - \ddot{p})(D_C + D_F)}\right] \\ &= \mu\left[\frac{D_C^2 + D_F^2}{D_C + D_F}\right] \\ &= \mu\left[D_C + D_F - \frac{2D_C \cdot D_F}{D_C + D_F}\right] \approx \mu(D_C + D_F)\end{aligned} \tag{35}$$

Therefore, this Market Maker equation can be derived if bounded rationality due to suboptimal rules or myopic thinking is considered.

C Derivation of the different transfer functions

The transformation from the time domain t to the frequency domain s is given by the solution of the Fourier integral (Unbehauen (2008)):

$$y(s) = \int_0^\infty y(t)e^{-st}dt \tag{36}$$

It can be described by the following symbolism:

$$y(t) \circ\!\!-\!\!\!-\!\!\bullet\, y(s) \tag{37}$$

One of the most important transformations is the one for derivatives (Unbehauen (2008)):

$$\frac{d^n y(t)}{dt^n} \circ\!\!-\!\!\!-\!\!\bullet\, s^n y(s) - \sum_{i=1}^{n} s^{n-i}\left(\frac{d^{(i-1)}f(t)}{dt^{i-1}}\right)_{t=0+} \tag{38}$$

The transfer function $F(s)$ describes the behavior of a dynamic system y to the input u and is defined in the following way (Unbehauen (2008, p. 60)):

$$F(s) = \frac{y(s)}{u(s)} = \frac{b_0 + b_1 s + \cdots + b_m s^m}{a_0 + a_1 s + \cdots + a_n s^n} = \frac{N(s)}{D(s)} \tag{39}$$

By setting the denominator to zero ($D(s) \overset{!}{=} 0$) we can derive the so-called poles or eigenvalues of the system $s_1, s_2, \cdots, s_n$, which describe the homogeneous solution of the system in the time domain (Unbehauen (2008)):

$$y_{hom}(t) = \sum_{i=1}^{n} C_i e^{s_i \cdot t} \tag{40}$$

The stability condition is that the real part of the eigenvalue is negative ($Re\{s_i\} < 0$) (Unbehauen (2008, p. 140)).

The chartist demand for the case $L_s = 1$ and $L_l = 2$ can be derived if we consider the following assumption for the second order derivative:

$$\ddot{p} \approx \dot{p}(t) - \dot{p}(t-1) \approx (p_{t+1} - p_t) - (p_t - p_{t-1}) = p_{t+1} - 2p_t + p_{t-1} \tag{41}$$

This results in the following chartist demand D_C:

$$D_C = \frac{\beta}{2}(p_t - p_{t-1}) = \frac{\beta}{2}((p_{t+1} - p_t) - (p_{t+1} - 2p_t + p_{t-1})) = \frac{\beta}{2}(\dot{p} - \ddot{p}) \tag{42}$$

Using these results the following transfer function can be derived:

$$\begin{aligned} \dot{p} &= \mu \left[\frac{\beta}{2} (\dot{p} - \ddot{p}) + \alpha(f - p) \right] \\ \circ\!\!-\!\!\bullet\, p(s) \left(\frac{\mu\beta}{2} s^2 + (1 - \frac{\mu\beta}{2}) s + \mu\alpha \right) &= f(s) \cdot \mu\alpha \\ \Rightarrow F(s) = \frac{\mu\alpha}{\frac{\mu\beta}{2} s^2 + (1 - \frac{\mu\beta}{2}) s + \mu\alpha} &= \frac{1}{\frac{\beta}{2\alpha} s^2 + \frac{(2-\mu\beta)}{2\mu\alpha} s + 1} \end{aligned} \tag{43}$$

Now, the case of $L_s = 1$ and $L_l = 3$ is presented. The transformation of the difference equation into a differential equation requires the following connection:

$$\begin{aligned} \dddot{p}_t &\approx \ddot{p}(t) - \ddot{p}(t-1) \\ \approx (p_t - 2p_{t-1} + p_{t-2}) - (p_{t-1} - 2p_{t-2} + p_{t-3}) &= p_t - 3p_{t-1} + 3p_{t-2} - p_{t-3} \end{aligned} \tag{44}$$

The chartist demand can therefore be described by the following equation:

$$\begin{aligned} &\frac{1}{3}\dddot{p} - \frac{4}{3}\ddot{p} + \dot{p} \\ = \left(\frac{1}{3}p_{t+1} - p_t + p_{t-1} - \frac{1}{3}p_{t-2} \right) + &\left(-\frac{4}{3}p_{t+1} + \frac{8}{3}p_t - \frac{4}{3}p_{t-1} \right) + (p_{t+1} - p_t) \\ &= \frac{2}{3}p_t - \frac{1}{3}p_{t-1} - \frac{1}{3}p_{t-2} - \frac{1}{3}p_{t-3} \end{aligned} \tag{45}$$

Using this result the transfer function is calculated:

$$\begin{aligned}
\dot{p} &= \mu\left[\beta(\frac{1}{3}\dddot{p} - \frac{4}{3}\ddot{p} + \dot{p}) + \alpha(f - p)\right] \\
\circ\!\!-\!\!\!-\!\!\bullet\, p(s)\left(-\frac{\mu\beta}{3}s^3 + \frac{4}{3}\mu\beta s^2 + (1 - \mu\beta)\,s + \mu\alpha\right) &= f(s)\cdot\mu\alpha \qquad (46)\\
\Rightarrow F(s) &= \frac{1}{-\frac{\beta}{\alpha 3}s^3 + \frac{4\beta}{3\alpha}s^2 + (\frac{1-\mu\beta}{\mu\alpha})s + 1}
\end{aligned}$$

This system is always unstable. This system is a so-called PT_3 system, which can be described as serial connection of three PT_1 systems. Mathematically, this can be done by multiplying PT_1 functions:

$$F(s) = \left(\frac{K}{sT + 1}\right)^3 = \frac{K^3}{s^3T^3 + 3s^2T^2 + 3sT + 1} \qquad (47)$$

Since the stability condition for the PT_1 system requires $T > 0$ all coefficients of the denominator of the PT_3 function have to be positive as well. In this case the coefficient of s^3 is always negative, thus rendering the system unstable.

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