

Robust Algorithms for Downlink Beamforming in the Conventional and Cognitive Radio Networks with Erroneous Channel State Information

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*To my parents,
brother and sisters*

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Zusammenfassung

In den letzten Jahrzehnten ist die Nachfrage nach drahtlosen Diensten sprunghaft angestiegen. Dies hat Forscher dazu angetrieben, neue und effiziente Wege zu suchen, die Datenraten in drahtlosen Netzwerken zu erhöhen und gleichzeitig eine verbesserte Reichweite und Betriebssicherheit zu gewährleisten. Mehrantennen-Verfahren haben sich als vielversprechende Lösung für diese Herausforderungen herausgestellt. Das liegt insbesondere an ihrem großen Potential, die Datenrate in einem Drahtlos-Netzwerk zu erhöhen. Die Verwendung von mehreren Antennen am Sender und/oder Empfänger bietet zusätzliche Diversität mit der sich Kanal-Fading und Mehrnutzerinterferenzen bekämpfen lassen. Mehrantennen-Verfahren erlauben uns, Beamforming (Strahlenformung) am Sender und/oder Empfänger zu nutzen, was erhebliche Gewinne in drahtlosen Netzwerken zur Folge haben kann.

In der letzten Zeit wurde zunehmend festgestellt, dass die übliche Verfahrensweise, welche den mobilen Nutzern feste Bandbreiten zuzuweist, eine ineffiziente Nutzung des zur Verfügung stehenden Frequenzspektrums zur Folge hat. Als Lösung dieses Problems hat das Konzept des Cognitive Radios (CRs) in den letzten Jahren stark an Popularität gewonnen. CR erlaubt, dass sich ein primäres Netzwerk (PN), dessen Bandbreite lizenziert ist, und ein sekundäres Netzwerk (SN) die Bandbreite teilen. Dem SN wird die opportunistische Nutzung des nicht ausgelasteten Frequenzspektrums gestattet unter der Bedingung, dass die Kommunikation in dem PN nicht beeinträchtigt wird. In dieser Hinsicht kann Beamforming am Sender des SN verwendet werden, um die Interferenzen zum PN einzuschränken.

In dieser Arbeit beschäftigen wir uns mit den Problemen des robusten Beamformens für Mehrnutzernetzwerken in der Abwärtsstrecke (engl. Downlink) von der Basisstation zu den

mobilen Endgeräten. In diesem Zusammenhang betrachten wir sowohl herkömmliche drahtlosen Netzwerke als auch drahtlosen CR Netzwerke. In beiden Fällen wird angenommen, dass dem Sender fehlerhafte Kanalzustandsinformationen (CSI) in Form von Schätzungen der zweiten Momente der Kanalkovarianzen zur Verfügung stehen. Ferner wird angenommen, dass die Grenzen der Fehlanpassung in den Kanalkovarianzmatrizen bekannt sind. Das Ziel des robusten Beamformens in der herkömmlichen Downlink Anwendung ist es, die Gesamtleistung des Senders zu minimieren unter der Nebenbedingung, dass die Servicequalität am Empfänger eine gewisse Mindestgüte nicht unterschreitet. Dieser Entwurfsansatz wird im Englischen allgemein als „Worst case Quality-of-Service (QoS)“ Ansatz bezeichnet. Der robuste Downlink-Beamformerentwurf für das CR-Netzwerk ist mathematisch ähnlich zum herkömmlichen Fall, wobei die Nebenbedingungen für Mindestanforderungen der Nutzer des SN gelten. Im CR-Fall existieren jedoch zusätzliche Nebenbedingungen, die den Zweck haben die Interferenz zu primären Nutzern auf eine Interferenz-Schwelle begrenzen.

In unserem ersten Downlink Beamformerentwurf, nehmen wir an, dass sich die Unbestimmtheit der Kanalkovarianzmatrizen mit Hilfe von Ellipsoiden von gegebener Größe und Form beschreiben lässt. Die Ellipsoide werden mittels gewichteter Frobenius-Norm modelliert, wobei die Koeffizienten der Gewichtungsmatrizen von der statistischen Verteilung der CSI-Fehlanpassung abhängen. Sowohl im herkömmlichen als auch im CR Downlink-Beamformerentwurf erhalten wir exakte, auf Lagrange-Dualität basierende Umformulierungen der Mindestgüte- und Interferenz-Nebenbedingungen. Somit können die groben Approximationen früherer Lösungen vermieden werden. Die endgültigen Problemformulierungen werden mittels Semi-Definiten Relaxation (SDR) in eine konvexe Form gebracht. Außerdem lösen wir die oben genannten robusten Probleme indem wir die auf Mindestgüte-basierenden Nebenbedingungen durch sogenannte „Outage-Wahrscheinlichkeits“-basierende Nebenbedingungen ersetzen. Wir beweisen zu dem die Äquivalenz zwischen beiden Entwurfsansätzen. Für die robuste CR Anwendung schlagen wir zudem einen iterativen Ansatz vor, der auf der Dualität zwischen der Abwärts- und der Aufwärtsstrecke der Übertragung in

drahtlosen Mehrnutzernetzwerken basiert. Dieser Ansatz vereint die Vorteile der vorgeschlagenen robusten Lösung mit dem recheneffizienten nicht-robusten iterativen Verfahren. Computersimulationen bestätigen die verbesserte Leistungsfähigkeit der vorgeschlagenen Verfahren gegenüber früheren robusten Beamformingverfahren sowohl im herkömmlichen als auch in den CR Szenarien.

Zuletzt betrachten wir für das herkömmliche Netzwerk einen alternativen Lösungsansatz für das robuste Beamforming Problem. Wir berücksichtigen, dass die CSI-Fehlanpassung aus einer Kombination unterschiedlicher Fehlerquellen, wie z. B. Schätzfehler, quantisierte Rückkopplung oder endliche Abtastung, hervorgehen kann. Die übliche Herangehensweise ist eine Grenze zu definieren innerhalb derer sich der kumulierte Fehler befindet, ohne die verschiedenen Fehlerquellen im Einzelnen zu berücksichtigen. Daher sind exakte Fehler-Grenzen für diese Herangehensweise oft schwer zu ermitteln. In unserem alternativen Ansatz schlagen wir vor, die CSI-Fehlanpassungen aus unterschiedlichen Quellen unabhängig voneinander zu begrenzen. Unser Fehlanpassungs-Modell berücksichtigt die physikalischen Phänomene, die den CSI-Fehlern zugrunde liegen. Dies ist hilfreich, um aussagekräftige Schwellwerte für die Fehler unterschiedlicher Quellen herzuleiten. In diesem Verfahren nutzen wir wieder die Lagrange-Dualität für die Reformulierung der Mindestgüte-Nebenbedingungen und wenden zuletzt die SDR Technik an, um das sich daraus ergebende Problem in eine konvexe Form zu bringen. Die Simulationsergebnisse bestätigen die Leistungsfähigkeit des vorgeschlagenen Verfahrens.

Abstract

In the last couple of decades, there has been an explosive growth in the demand of wireless services. This growth has urged the researchers to find new and efficient ways of increasing the data rates in wireless networks while providing improved coverage and reliability. Multi-antenna techniques have emerged as a promising solution to these new challenges due to the huge potential these techniques offer in improving the throughput in a wireless network. The use of multiple antennas at the transmitter and/or receiver provides us with additional diversity to fight against the channel fading. Multi-antenna techniques allow us to use beamforming at the transmitter and/or receiver that can result in significant gains in a wireless network.

Recently, it has been identified that the current fixed spectrum assignment policies result in the underutilization of the available radio spectrum. To solve this problem, the concept of cognitive radio (CR) has gained a lot of popularity in the recent years. CR allows the spectrum sharing between a primary network (PN), to which the spectrum is licensed, and a secondary network (SN). The SN is granted the opportunistic use of the underutilized spectrum of the PN under the condition that it does not harm the transmissions in the PN. In this regard, beamforming can be applied at the transmitter of the SN to control the interference leaked to the PN.

In this thesis, we address the problems of robust multiuser downlink beamforming for both the conventional and CR wireless networks. In both cases, the transmitter is assumed to have erroneous covariance-based channel state information (CSI) where the bounds on the mismatch in the channel covariance matrices are assumed to be available. The goal of

the robust problem in the conventional downlink scenario is to minimize the total transmitted power under the worst-case quality-of-service (QoS) constraints for the receivers. The robust downlink beamforming problem for CR network is mathematically similar to the conventional case where the QoS constraints apply to the users of the SN. However, additional constraints exist in CR case that limit the interference leaked to the primary users below a given interference threshold.

In our first approach of solving the robust downlink beamforming problems, we assume the uncertainties in the channel covariance matrices to be confined in ellipsoids of given sizes and shapes. The ellipsoids are modelled using weighted Frobenius norm, where the coefficients of weighting matrices depend on the statistical distribution of the CSI mismatch. In both the conventional and CR downlink beamforming problems, we obtain exact reformulations of the worst-case QoS and interference constraints based on Lagrange duality, avoiding the coarse approximations used by previous solutions. The final problem formulations are converted to convex forms using semidefinite relaxation (SDR). We also solve the aforementioned robust problems by replacing the worst-case based constraints with the probabilistic constraints and prove the equivalence between the worst-case and probabilistic robust designs. Additionally, for the robust CR scenario, we propose an iterative approach based on the uplink-downlink duality that combines the benefits of the proposed worst-case based solution with the computationally efficient non-robust iterative technique. Computer simulations verify the performance improvements of the proposed techniques over earlier robust downlink beamforming techniques for both conventional and CR scenarios.

Finally, for the case of conventional networks, we propose an alternative approach to the robust downlink beamforming problem. We consider the fact that the mismatch in the CSI is a combination of mismatches originating from different sources such as estimation errors, feedback quantization or finite sampling effects. The traditional approach is to put a bound on the cumulative error without taking the specific error sources into account and, therefore, exact error bounds for these approaches are generally difficult to obtain. In our alternative approach, we propose to bound the CSI mismatches resulting from different sources independently. Our mismatch model takes into account the physical phenomena, that cause the

errors in the CSI, which helps in deriving meaningful threshold values for the errors from various sources. In this approach, we again use Lagrange duality for the reformulation of the worst-case QoS constraints and finally SDR is applied to convert the resulting problem formulation to a convex form. The simulation results verify the effectiveness of the proposed technique.

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Notation

$ \cdot $	absolute value
\arg	argument
$(\cdot)^*$	complex conjugate
$\mathbf{R}_x \triangleq \text{E}\{\mathbf{x}\mathbf{x}^H\}$	covariance matrix of the vector \mathbf{x}
$\text{diag}\{\cdot\}$	denotes a diagonal matrix
erf	Error function (Gauss error function)
$\ \cdot\ $	Euclidean norm of a vector or Frobenius norm of a matrix
$\text{E}\{\cdot\}$	expected value
$(\cdot)^H$	Hermitian transpose
\inf	infimum
\otimes	Kronecker product
\max	maximum
$\lambda_{\max}(\cdot)$	maximum eigenvalue of a matrix
\min	minimum
$\lambda_{\min}(\cdot)$	minimum eigenvalue of a matrix
$(\cdot)^\star$	optimum
$\mathbf{X} \succeq 0$	denotes that the matrix \mathbf{X} is positive semidefinite
$\text{Pr}(\cdot)$	probability function
$(\cdot)^\dagger$	pseudoinverse of a matrix

$\text{rank}(\cdot)$	rank of a matrix
$\text{Tr}\{\cdot\}$	trace of a matrix
$(\cdot)^T$	transpose
$\text{vec}(\mathbf{X})$	stacks the columns of the matrix \mathbf{X} to form a vector
$\text{vec}^{-1}(\mathbf{x})$	takes a $K^2 \times 1$ vector \mathbf{x} as input and returns a $K \times K$ matrix \mathbf{X} such that $\text{vec}(\mathbf{X}) = \mathbf{x}$

Acronyms

CCI	co-channel interference
CSI	channel state information
CR	cognitive radio
FDD	frequency division duplex
GSM	Global System for Mobile Communications
ISI	inter-symbol interference
LTE	Long Term Evolution
PN	primary network
PU	primary user
QoS	quality-of-service
SDP	semidefinite programming
SDR	semidefinite relaxation
SINR	signal-to-interference-plus-noise ratio
SN	secondary network
SOCP	second-order cone programming
SU	secondary user
TDD	time division duplex
UMTS	Universal Mobile Telecommunications System
WiMAX	Worldwide Interoperability for Microwave Access

Chapter 1

Introduction

This thesis develops techniques for downlink beamforming in conventional and cognitive radio (CR) networks that are robust to the errors in the available channel state information (CSI). In this chapter, we first present the motivation of our work by discussing the challenges in modern wireless communication networks. Next, we give a general overview of the beamforming and its use in wireless communications. Finally, we give an outline of the thesis along with the contributions of the thesis.

1.1 Motivation

The wireless communication started with the invention of the photophone (or radiophone) invented by Alexander Graham Bell and Charles Sumner Tainter in 1880, at Bell's 1325 'L' Street laboratory in Washington, D.C [1]. However, it is only in the last few decades when wireless communications have seen a tremendous growth [2]. This growth has been driven by a number of factors. The technological advancements in the recent decades have resulted in a considerable reduction of the sizes of mobile devices. The current generations of mobile phones are much more compact and easy to carry around as compared to their earlier models. This along with the significant decrease in the tariffs of mobile services, has resulted in a tremendous increase in the usage of mobile phones in recent years. Additionally, the past few years have seen a rapid development in the segments of smartphones and tablet computers, attracting more and more people to replace their so-called dumb mobile phones

with modern smartphones and tablet computers [3]. The usage of these new generations of mobile devices for accessing data services, like accessing internet on the go, making video calls, or watching video on demand etc., has caused an explosive increase in the demand of wireless services. The growing demand for mobility and data services is driving the exponential development in the wireless communications and researchers are developing modern techniques to cope with this demand. Correspondingly, the data rates in modern wireless standards have increased significantly as can be seen in Fig. 1.1 (Source TeliaSonera [4]).

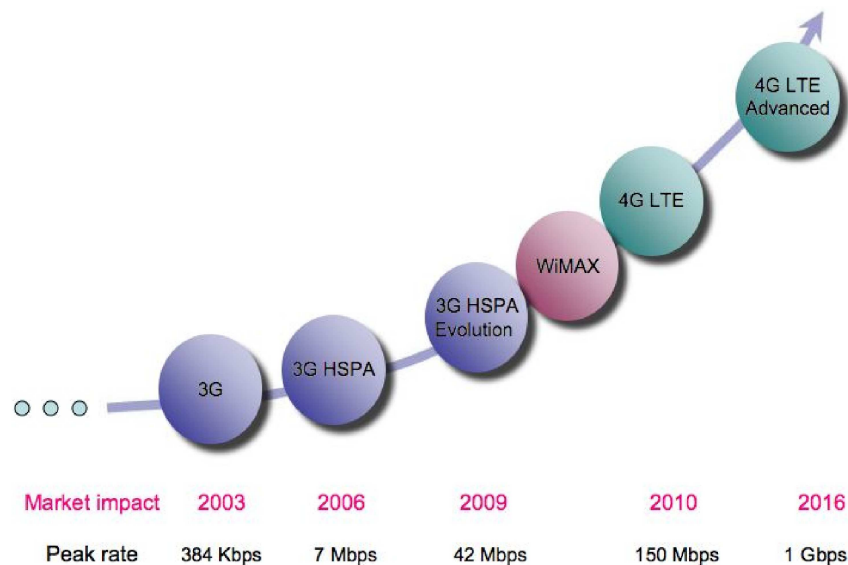


Figure 1.1: Data rates of different generations of mobile cellular systems.

One important consideration with the wireless communications is the fact that wireless channel, unlike the wireline channel, poses many challenges. The communication through a wireless channel suffers fading due to path loss, shadowing, scattering and multipath propagation [2, 5, 6]. These factors result in having multiple copies of the transmitted signals, that experience different fading, to arrive at the receiver. Although, this can lead to severe degradations of the transmitted signal, it also allows us to use different diversity

schemes, like frequency, time or spatial diversity, to combat the fading phenomena. One possible way of using fading to our advantage and improving the reliability and throughput of wireless communication systems is to use an array of multiple antennas at the transmitter and/or receivers. The deployment of multiple antennas provides us with spatial diversity and allows the use of transmit and/or receive beamforming techniques to achieve large gains in a communication network [7, 8]. We will discuss more about beamforming in Section 1.2.

The modern wireless standards utilize larger bandwidths per channel, as one of the ways to increase data rates and reliability, compared to the earlier wireless standards (e.g. LTE Advanced uses up to 100MHz while GSM uses 200kHz). The expansion in bandwidth to accommodate higher data rates is unfortunately not possible any more since virtually, all the available radio spectrum is allocated. Therefore, this increasing demand for wireless services has urged the researchers in the last few decades to develop modern techniques that allow more efficient utilization of the available radio spectrum. However, there is one interesting fact about the spectrum usage that can be manipulated to enable more efficient utilization of the available spectrum. In general, fixed spectrum assignment is employed in the current spectrum licensing policies. The bandwidth demand by the users, on the other hand, varies significantly with time and location. Therefore, very often the spectrum is not completely utilized. The underutilization of radio spectrum has been reported, for example, in [9]. It is therefore natural to seek for a more effective way of exploiting the unused bandwidth. The concept of CR, first presented by Mitola and Maguire in 1999 [10], has emerged as a promising solution for the efficient utilization of the radio spectrum. In a CR network, the secondary network (SN) is granted access to the licensed spectrum of the primary network (PN) under the condition that the transmissions in the SN do not adversely affect the operation of the PN. There are three different paradigms of interaction between the SN and PN, namely, interweave, underlay and overlay paradigms [11]. In interweave framework, a secondary user (SU) (user of the SN) uses spectrum sensing techniques [12, 13, 14, 15] to find spectrum holes and uses these holes for opportunistic communication [10, 16]. In overlay paradigm, the SU senses the message of the primary user (PU) (user of PN), and then achieves interference precancellation using advanced coding schemes [17, 18, 19]. In

the underlay CR, the SU can only use the spectrum of the PN when the interference caused to the PN, due to the transmissions in the SN, is below a considerable limit [20, 21].

The concept of CR helps in solving another very important issue, that most of us come across in our daily lives. Very often we face the degradation in the call quality while using our mobile phones indoor. Even at times our mobile phones show complete lack of signals from the service providers. Indoor coverage assurance is important and improving the reliability of indoor service coverage has become a major challenge in the current cellular systems. In modern state of the art wireless standards like UMTS, LTE and LTE-A, the deployment of femtocells in homes, offices and buildings helps in improving the indoor service coverage [22, 23, 24, 25, 26]. A femtocell base station usually utilizes the same frequency band as the surrounding macro network and is connected via a fixed line to the macro network. One major problem with the deployment of femtocells is that, unlike macrocells, careful cell planning cannot be carried out. Therefore, it is very important to control the interference caused due to the operation of femtocells so that macrocell users are not adversely affected. In this setup the macrocell, possessing the license of the spectrum, is treated as being privileged over the femtocell. This implies that an underlay CR framework can be adopted where the femtocell acts as the SN and the users it serves are referred as the SUs. The macrocell network acts as a PN and its users are the PUs that according to the underlay paradigm are assumed to be non-cooperative.

In the following, we give a brief introduction of beamforming and describe the two different kinds of beamforming approaches, namely the receive/uplink and transmit/downlink beamforming. We also discuss about the application of beamforming in a CR network.

1.2 Beamforming

Beamforming is a powerful approach used in sensor arrays that can be realized with the existence of multiple antennas at the transmitter and/or receiver. It can be used to improve the performance of a communication system by making use of the spatial diversity. It allows directional transmission and reception of the desired signals in the presence of noise together

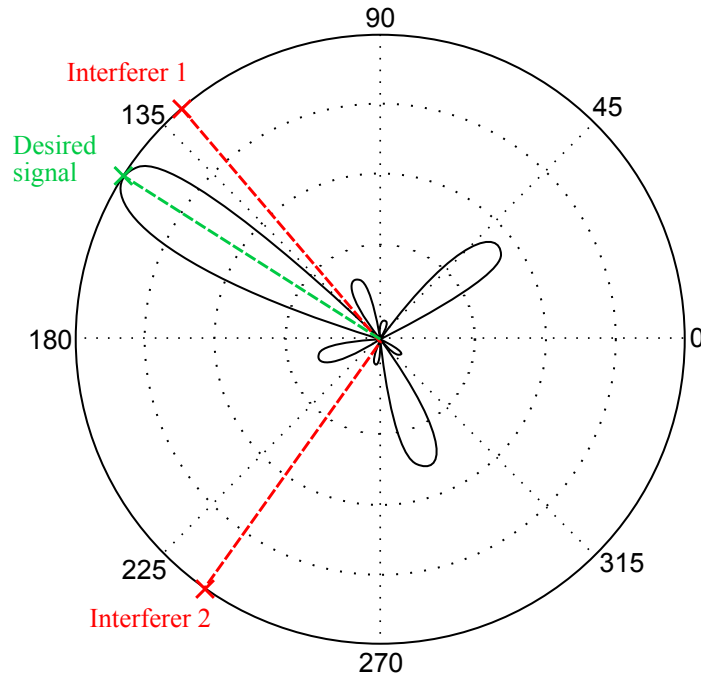


Figure 1.2: Polar plot illustration of receive/uplink beamforming. The beam pattern is adjusted to maximize the gain in the direction of desired signal and to place nulls in the direction of interferers.

with the directional rejection of the undesired signals. Beamforming can result in significant improvements, in terms of data rates and the utilized power, as compared to the traditional omni-directional reception/transmission, if accurate CSI is available [7, 8, 27, 28]. The basic idea of beamforming is to adjust the weight vectors (beamformers) of the multiple antennas such that a beam pattern of desired shape is formed. The shape of this beam pattern depends on the direction of desired receiver/transmitter and the directions of interference.

1.2.1 Receive (Uplink) Beamforming

Receive/uplink beamforming is a classical technique with many applications in different fields such as radar, sonar, communications, biomedicine, radio astronomy and seismology. Recently, multi-antenna techniques have gained increased attention of researchers due to their potential of bringing improvements in data rates and reliability of wireless networks. As a result, there has been a renewed interest in receive beamforming [7]. The receive

beamforming can improve the signal-to-interference-plus-noise ratio (SINR) at the receiver by suppressing the co-channel interference (CCI) and inter-symbol interference (ISI). This is achieved by adjusting the weight vector of the receive antenna array such that the nulls are placed in the directions of interference (in case of CCI) and multipath signals (in case of ISI), while providing the gain in the direction of desired signal [27, 28, 29, 30]. In the polar plot of Fig. 1.2, we show how a receiver uses receive beamforming to adjust its beam pattern in order to provide maximum gain in the direction of the desired signal while placing the nulls in the direction of the two interferers.

The traditional receive beamforming techniques show considerable decrease in performance when the information about the desired signal steering vector is erroneous [7, 31, 32, 33]. This has led to the development of many robust approaches based on the concept of worst-case performance optimization [34, 35, 36, 37, 38, 39, 40, 41]. The robust approaches based on worst-case error considerations try to design beamformers that satisfy the required constraints for the worst error within a given uncertainty set. This ensures that the constraints would be satisfied for all the possible errors in that set. The uncertainty set is defined such that the mismatch matrices are bounded by ellipsoids of given shapes and sizes without making any assumption on the distribution of the error. The robust techniques in [34, 35, 36, 37, 38, 39, 40, 41] use convex optimization theory that simplifies the complex robust problems by transforming these into tractable convex forms [7, 42]. There exists an alternative approach for developing robust techniques that is based on the design of the outage probabilities [43, 44, 45, 46]. In this case, the distribution of the CSI errors is assumed to be known and the worst-case constraints are replaced by probabilistic constraints. The beamformers are designed such that the outage probability of the constraints is below a predefined value. These methods also employ convex optimization to solve the resulting problems.

1.2.2 Transmit (Downlink) Beamforming

Unicast transmit/downlink beamforming is similar to the receive beamforming in the sense that the weight vector (beamformer) has to be matched to the steering vector of a single user of interest. However, in multiuser downlink beamforming, different weight vectors are used for the transmission to different users (or different group of users). The different weight vectors of the transmit antenna array are then adjusted to minimize the interference at each user and improve the user quality-of-service (QoS) [27, 28, 47]. Although uplink and downlink beamforming appear similar, there exist some important differences between the two. As the uplink beamforming is applied at the antenna array of one particular user, it does not affect the signals of other users and, therefore, can be implemented independently at each receiver [27, 28, 48, 49]. On the other hand the adjustment of beamformers at the transmitter antenna array affects the signal received by all the users. This means that the downlink beamforming has to consider the performance of the whole system jointly. In this case, the beamformers are designed such that the weight vector for a particular user has a large inner product with its own steering vector while maintaining smaller inner products with the steering vectors of all the other users (see for example [7, 50], and the references therein). The uplink and downlink beamforming also differ from each other in terms of the available CSI. In the case of uplink beamforming, the channel can be estimated at the receiver antenna array locally using different channel estimation methods. In contrast, a transmitter antenna array generally requires a feedback from the receivers to know the CSI [51, 52]. This does not hold true in case of Time Division Duplex (TDD) systems though, where due to reciprocity, the downlink channels can be estimated from the available uplink CSI. We show an example scenario of downlink beamforming in Fig. 1.3, where the multi-antenna transmitter transmits to two mobile users.

Downlink beamforming techniques have been implemented in many modern wireless communications standards such as the WiMAX, LTE, and LTE-A [8, 53]. As a result multiuser downlink beamforming has become an active area of research in recent years (see

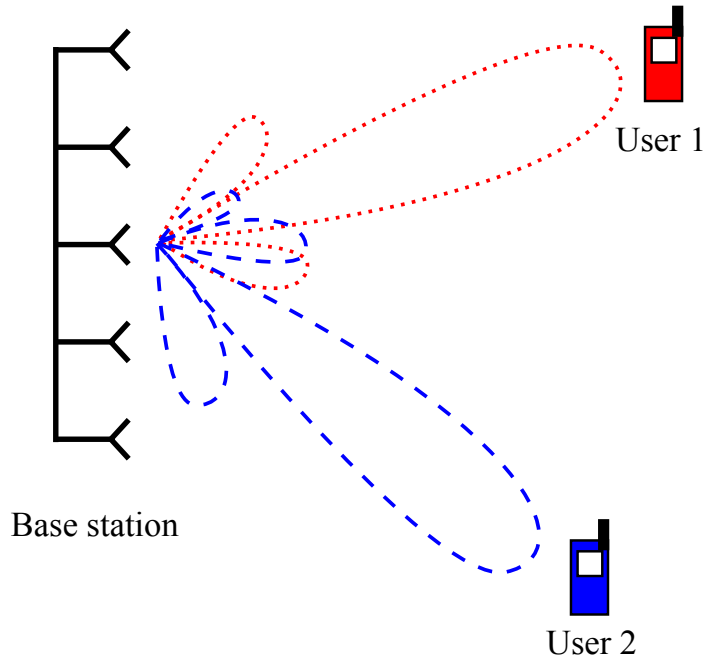


Figure 1.3: Schematic illustration of transmit/downlink beamforming where the base station transmits to two mobiles.

for example [7] and the references therein). There are a number of beamforming techniques that have been developed assuming the availability of perfect CSI. In the schemes of [27, 28, 54, 55, 56] the availability of perfect instantaneous CSI at the transmitter is considered. On the other hand, the techniques in [57, 58, 59] are based on the knowledge of perfect covariance-based CSI at the transmitter. The assumption in the above mentioned approaches, that the perfect CSI is available, however, is not practical. The available CSI at the transmitter is generally erroneous. The errors in the CSI could arise due to various reasons. The CSI could be estimated at the receiver using training sequences and then signaled back to the transmitter via feedback channels or, in the cases when channel reciprocity can be assumed, like in the TDD systems and certain frequency division duplex (FDD) systems, the uplink CSI can be used to determine the downlink channels [50]. This process of determining the CSI could result in the estimation errors [60] or the quantization errors due to the limited capacity of the feedback channels [61]. Furthermore, due to latencies in channel state feedback and short channel coherence time, CSI at the transmitter may be

imprecise. The presence of channel uncertainties can significantly affect the performance of the beamforming methods that are developed assuming perfect CSI. Therefore, several robust downlink beamforming techniques have been developed that are robust to the CSI errors. The methods of [62], [63], [64] are developed for instantaneous CSI, while the works in [50, 65, 66, 67, 68, 69] focus on the availability of erroneous covariance-based CSI. We would like to point out the fact that the rate of change of the second order statistics of the wireless channel is generally lower compared to the instantaneous CSI. This makes the use of covariance-based CSI attractive as it could significantly reduce the feedback requirements. This advantage of covariance-based CSI over the instantaneous CSI is much more significant in fast fading scenarios. Similar to the case of receive beamforming, some techniques in transmit beamforming are also proposed based on probabilistic design [70, 71]

1.2.3 Beamforming in an Underlay CR Framework

In an underlay CR network, the SN opportunistically uses the licensed spectrum of the PN. As mentioned earlier in Section 1.1, the interference caused to the PN due to the transmissions in the SN must be below a predefined threshold level. In case of downlink transmissions in the SN, the goal of the SN base station is to achieve a certain QoS at the receivers of SUs along with controlling the interference to the PN. Under these requirements, the use of beamforming in the SN by employing multiple antennas at its transmitter, allows spatial processing to enhance QoS at the SUs and to suppress the interference created to the users of PN. From a mathematical point of view, the CR beamforming problem can be viewed as a conventional beamforming problem with additional constraints on the interference to the PUs [20, 21, 72, 73]. Similar to the case of conventional beamforming, several techniques developed for the CR beamforming assume the availability of perfect CSI, see [73, 74, 75, 76, 77, 78]. On the other hand, a number of techniques consider the more practical case of erroneous CSI [79, 80, 81, 82, 83, 84, 85, 86]. We would also like to mention that a number of techniques have been developed for the uplink transmission in a CR network [87, 88, 89], that also make use of beamforming.

1.3 Thesis Overview and Contribution

In this thesis, we propose new robust downlink beamforming techniques for both the conventional and CR scenarios under the assumption that the transmitter has partial covariance-based CSI. In the proposed problem formulations and their solutions, we avoid the drawbacks of previous methods presented in [50, 79, 82]. This is achieved by considering more realistic models for defining the channel uncertainties and by avoiding the conservative steps, involved in the reformulations of the corresponding beamforming problems, that are used in the previous solutions.

The authors of [50] have considered the robust conventional downlink beamforming using the CSI based on channel covariance matrices. In [79], the technique of [50] has been extended for the robust CR downlink beamforming problem. However, in both [50] and [79], several conservative approximations have been used. In particular, these methods use conservative modifications of the QoS and PU interference constraints. Additionally, these approaches ignore the fact that the mismatched downlink channel covariance matrices must be positive semidefinite. For the conventional beamforming scenario, an improved robust approach is presented in [69] that avoids the approximations used in [50]. In [82] and [83], the idea of [69] is extended for the CR beamforming case. An important difference between the approaches of [82] and [83] is that the positive semidefiniteness constraints on the mismatched covariance matrices are not taken into account in [82].

The outline and the contributions of the thesis can be summarized as follows:

Chapter 2: In this chapter, we present the system model considered in this thesis along with the non-robust and robust downlink beamforming problem formulations for the conventional and CR wireless networks. We also discuss the previous solutions proposed for these problem formulations.

Chapter 3: We present the proposed worst-case optimization-based robust techniques for both the conventional and CR beamforming problems. In our approach, the weighted Frobenius norm is used to model the uncertainties in the channel covariance matrices, where

prior knowledge about the statistical distribution of the CSI mismatch is taken into account. We first present the robust downlink beamforming technique for the conventional downlink beamforming and then extend the developed robust approach to downlink beamforming in CR networks. We discuss the coarse approximations used by previous solutions [50, 79, 82] and avoid these in our technique by obtaining exact reformulations for both worst-case problems using Lagrange duality. The resulting problems can then be approximated using semidefinite relaxation (SDR). We also analyze the robust beamforming problem based on the probabilistic model and show that the final problem formulations for both the worst-case and probabilistic approaches are mathematically equivalent. In our simulations, we generally obtain rank-one solutions to the resulting semidefinite programming (SDP) problems implying that the obtained SDR solutions are optimal. However, we point out a class of specific examples in which all solutions violate the rank-one constraint and present detailed analysis of these examples. We then present an iterative robust approach based on the uplink-downlink duality for CR downlink beamforming that combines the benefits of the proposed worst-case based robust problem formulation with the non-robust iterative technique of [77]. Computer simulations show that the proposed techniques provide substantial performance improvements in terms of problem feasibility and transmitted power in comparison to the earlier robust downlink beamforming techniques for both the conventional and the CR scenarios.

Chapter 4: In this chapter, we present an alternative approach to the robust beamforming problem for the conventional downlink scenario. The general approach in the beamforming techniques, including the one used in Chapter 3, is that the robustness is usually incorporated using an error model in which the individual CSI mismatch matrices are bounded by Frobenius norm. As mentioned above, the imperfections in statistical CSI may result from estimation errors, feedback quantization or finite sampling. The specific error sources, however, are not taken into account by the previous robust approaches and therefore, it is generally difficult to obtain exact error bounds for these robust approaches. In the technique presented in this chapter, we propose a new approach of modeling the error, where

the bounding functions for specific CSI errors resulting from various sources are chosen independently. This new approach of modeling the CSI mismatches allows us to naturally derive meaningful threshold values for the different error sources. We present our proposed approach by first solving the robust downlink beamforming problem separately for each kind of error considered. Later, we combine the various kinds of errors in a single robust problem. The simulation results verify the effectiveness of the proposed technique.

Chapter 5: The concluding remarks are summarized in this chapter. Additionally, we describe some possible future extensions of the work presented in this thesis.

The work presented in this Thesis is based upon the following publications:

- K. L. Law, I. Wajid, and M. Pesavento, “Robust downlink beamforming using channel statistics information” *in preparation*.
- I. Wajid, M. Pesavento, Y. Eldar, and D. Ciochina, “Robust downlink beamforming with partial channel state information for conventional and cognitive radio networks” *accepted for publication in the IEEE Transactions on Signal Processing*.
- K. L. Law, I. Wajid, and M. Pesavento, “Robust downlink beamforming in multi-group multicasting using trace bounds on the covariance mismatches,” *Proceedings of the International Conference on Acoustics, Speech, Signal Processing (ICASSP’12)*, Kyoto, Japan, March 2012.
- I. Wajid, H. Nikolaeva, and M. Pesavento, “Iterative robust downlink beamforming in cognitive radio networks,” *Proceedings of the ICST Conference on Cognitive Radio Oriented Wireless Networks (CROWNCOM’11)*, pp. 375-379, Osaka, Japan, June 2011.
- I. Wajid, M. Pesavento, Y. Eldar, and A. B. Gershman, “Robust downlink beamforming for cognitive radio networks,” *Proceedings of the IEEE Global Communications Conference (GLOBECOM’10)*, pp. 1-5, Miami, USA, December 2010.

- I. Wajid, Y. Eldar and A. B. Gershman, “Robust downlink beamforming using covariance channel state information,” *Proceedings of the International Conference on Acoustics, Speech, Signal Processing (ICASSP’09)*, pp. 2285-2288, Taipei, Taiwan, April 2009.
- V. Sharma, I. Wajid, H. Chen, A. B. Gershman, and S. Lambotharan, “Robust downlink beamforming using positive semi-definite covariance constraints,” *Proceedings of the IEEE/ITG Workshop on Smart Antennas*, pp. 36-41, Darmstadt, February 2008.

Additionally, we would like to mention the following papers which were also developed during the course of this PhD. These papers laid the foundations of the work presented in this Thesis.

- A. Abdelkader, I. Wajid, A. B. Gershman, and N. D. Sidiropoulos, “Transmit beamforming for wireless multicasting using channel orthogonalization and local refinement,” *Proceedings of the International Conference on Acoustics, Speech, Signal Processing (ICASSP’09)*, pp. 2281-2284, Taipei, Taiwan, April 2009.
- I. Wajid, A. B. Gershman, S. A. Vorobyov, and Y. A. Karanouh, “Robust multi-antenna broadcasting with imperfect channel state information,” *Proceedings of the 2nd International Workshop on Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP’07)*, pp. 213-216, St. Thomas, Virgin Islands, USA, December 2007.

Chapter 2

System Model and Previous Work

2.1 Introduction

In this chapter, we present the system model considered in this thesis and give the formulation of the problems for which we propose new solutions. In Section 2.2, we discuss the non-robust scenario for conventional downlink beamforming and present its problem formulation as well as the existing solutions to this problem. We extend the non-robust problem to the CR scenario in Section 2.3 and discuss its state-of-the-art solutions. In Section 2.4, we present the robust problem formulation for both the conventional and CR downlink beamforming and, finally, the previous solutions to this problem are discussed in Section 2.5.

2.2 Non-Robust Conventional Downlink Beamforming

Let us consider a single-cell wireless network where the base station is assumed to have a single transmitter with N antenna elements. There are K single antenna receivers in the system with independent data stream to be transferred to each of these users. At time

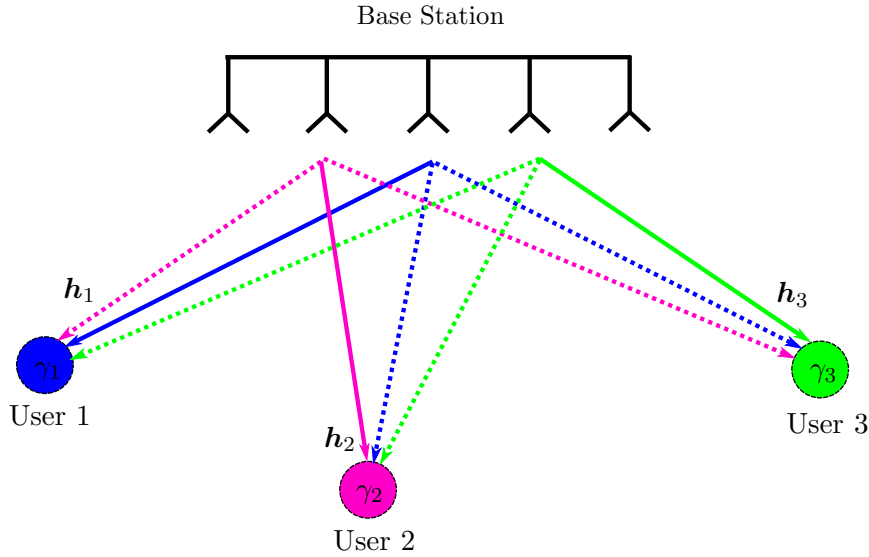


Figure 2.1: Schematic description of a conventional downlink beamforming network.

instance t , the base station transmits the $N \times 1$ vector

$$\mathbf{x}(t) = \sum_{i=1}^K s_i(t) \mathbf{w}_i \quad (2.1)$$

where $s_i(t)$ and \mathbf{w}_i are the intended signal and the $N \times 1$ complex weight vector for the i th user, respectively. The signal received by the k th user is then given by

$$y_k(t) = \mathbf{h}_k^H \mathbf{x}(t) + n_k(t) \quad (2.2)$$

where \mathbf{h}_k is the $N \times 1$ channel vector between the base station and the k th user and flat-fading scenario is assumed. The noise at the receiver of the k th user is denoted by $n_k(t)$ and is assumed to be zero-mean circularly symmetric white Gaussian with variance σ_k^2 . Assuming $E\{|s_i|^2\} = 1$ and using the covariance based CSI, the received SINR of the k th user can be expressed as [50]

$$\text{SINR}_k = \frac{\mathbf{w}_k^H \mathbf{R}_{\mathbf{h}_k} \mathbf{w}_k}{\sum_{\substack{i=1 \\ i \neq k}}^K \mathbf{w}_i^H \mathbf{R}_{\mathbf{h}_k} \mathbf{w}_i + \sigma_k^2}, \quad k = 1, \dots, K \quad (2.3)$$

where $\mathbf{R}_{\mathbf{h}_k} \triangleq E\{\mathbf{h}_k \mathbf{h}_k^H\}$ is the downlink channel covariance matrix for the k th. An example of such a system is shown in Fig. 2.1 where the transmitter has 5 antenna elements and

there are 3 single antenna receivers. The solid lines indicate the transmission to the desired receiver while the dotted lines indicate the interference received by the users due to the transmission intended for other receivers. The term γ_k represents the minimal acceptable SINR for the k th user.

To formulate the non-robust downlink beamformer design problem, it is assumed in [59] that the channel covariance matrices $\mathbf{R}_{h_1}, \dots, \mathbf{R}_{h_K}$ are perfectly known at the base station. The downlink beamformers are then designed to minimize the total transmitted power at the base station, subject to individual QoS constraints for all the users. The resulting beamforming problem can be written as

$$\begin{aligned} \min_{\{\mathbf{w}_i\}} & \sum_{i=1}^K \|\mathbf{w}_i\|^2 \\ \text{s.t.} & \frac{\mathbf{w}_k^H \mathbf{R}_{h_k} \mathbf{w}_k}{\sum_{\substack{i=1 \\ i \neq k}}^K \mathbf{w}_i^H \mathbf{R}_{h_k} \mathbf{w}_i + \sigma_k^2} \geq \gamma_k; \quad k = 1, \dots, K. \end{aligned} \quad (2.4)$$

For the special case of instantaneous CSI, i.e., when $\mathbf{R}_{h_k} = \mathbf{h}_k \mathbf{h}_k^H$, it has been shown in [59] that the problem (2.4) can equivalently be written as a second-order cone programming (SOCP) problem [90] and can be solved efficiently using convex optimization tools [91, 92]. To solve (2.4) for general rank \mathbf{R}_{h_k} , the authors of [59] introduce the variable transformation

$$\mathbf{W}_k \triangleq \mathbf{w}_k \mathbf{w}_k^H; \quad k = 1, \dots, K \quad (2.5)$$

resulting in the modified reformulation as

$$\begin{aligned} \min_{\{\mathbf{W}_i\}} & \sum_{i=1}^K \text{Tr}\{\mathbf{W}_i\} \\ \text{s.t.} & \gamma_k \sum_{\substack{i=1 \\ i \neq k}}^K \text{Tr}\{\mathbf{R}_{h_k} \mathbf{W}_i\} - \text{Tr}\{\mathbf{R}_{h_k} \mathbf{W}_k\} + \gamma_k \sigma_k^2 \leq 0 \\ & \mathbf{W}_k \succeq 0, \quad \text{rank}(\mathbf{W}_k) = 1; \quad k = 1, \dots, K. \end{aligned} \quad (2.6)$$

Then the non-convex constraints $\text{rank}(\mathbf{W}_k) = 1$ ($k = 1, \dots, K$) are removed following an approach, which in literature is referred to as SDR [50, 93]. As a result the original problem

(2.4) is relaxed to the following convex SDP problem [90]:

$$\begin{aligned}
& \min_{\{\mathbf{W}_i\}} \sum_{i=1}^K \text{Tr}\{\mathbf{W}_i\} \\
& \text{s.t.} \quad \gamma_k \sum_{\substack{i=1 \\ i \neq k}}^K \text{Tr}\{\mathbf{R}_{h_k} \mathbf{W}_i\} - \text{Tr}\{\mathbf{R}_{h_k} \mathbf{W}_k\} + \gamma_k \sigma_k^2 \leq 0 \\
& \quad \mathbf{W}_k \succeq 0; \quad k = 1, \dots, K.
\end{aligned} \tag{2.7}$$

It has been shown in [50] that the SDP problem (2.7) always has at least one rank-one solution (or linear combinations of rank-one solutions) for \mathbf{W}_k . In this case, problem (2.4) and (2.7) become equivalent and, therefore, the optimum beamforming vectors \mathbf{w}_k can be obtained from the principal eigenvectors of \mathbf{W}_k . However, in addition to one rank-one solution, the problem (2.7) might also have other higher rank-solutions. In [94, 95], an iterative algorithm has been proposed to obtain rank-one solutions from the corresponding higher rank solutions.

In [50], it is shown that the downlink beamforming problem (2.4) yields optimal beamformers that are identical to those of the virtual uplink problem

$$\begin{aligned}
& \min_{\{\mathbf{u}_i, q_i\}} \sum_{i=1}^K q_i \\
& \text{s.t.} \quad \frac{q_k \mathbf{u}_k^H \mathbf{R}_{h_k} \mathbf{u}_k}{\mathbf{u}_k^H \left(\sum_{\substack{i=1 \\ i \neq k}}^K q_i \gamma_i \mathbf{R}_{h_i} + \mathbf{I} \right) \mathbf{u}_k} \geq 1; \quad k = 1, \dots, K \\
& \quad q_k \geq 0; \quad \|\mathbf{u}_k\| = 1
\end{aligned} \tag{2.8}$$

where q_1, \dots, q_K are the virtual uplink powers with

$$\mathbf{w}_k \triangleq \sqrt{p_k} \mathbf{u}_k; \quad k = 1, \dots, K. \tag{2.9}$$

The authors of [50], then present an iterative algorithm that was derived independently in [28, 54], based on the uplink-downlink duality, to obtain the solution of problem (2.4).

The problem (2.4) represents one way of solving the downlink beamforming problem, that we consider in our thesis. However, there exist other ways of formulating the downlink

beamforming problems as well (see [96] and the references therein). One approach of formulating the downlink beamforming problem is to maximize the minimum received SINR subject to the constraints on the total transmitted power [55, 57, 58, 97]. Many works consider the downlink problem where, instead of the constraints on the total transmitted power, the constraints on the per-antenna transmitted power are considered [98, 99, 100].

2.3 Non-Robust CR Downlink Beamforming

In this section, we present the extension of the non-robust downlink beamforming problem (2.4) for the scenario of a CR network. Let us consider the case when the single-cell network described in Section 2.2 works as a SN and utilizes the licensed bandwidth of a PN that contains L single antenna PUs. In this scenario the K single antenna receivers of the SN are termed as SUs. This problem has been considered in [79], where additional PU interference constraints apply while designing the beamformer. The goal of the beamforming weight vector design is to minimize the total power transmitted by the SN base station, under the constraints that each SU receives a minimum SINR, while the interference leaked to the PUs does not exceed a predefined maximum threshold.

In Fig. 2.2, we present an example of such a CR network where the transmitter in the SN has 5 antenna elements. The SN has 3 single antenna SUs represented with circles and there exist 3 single antenna PUs represented with squares. The solid lines indicate the transmission to the desired SU while the dotted lines indicate the interference received by other SUs. The interference generated to the users of the PN is indicated by the dashed lines. The terms γ_k and ε_l denote the minimal acceptable SINR for the k th SU and the maximum allowable interference power at the l th PU, respectively. The CR downlink beamforming

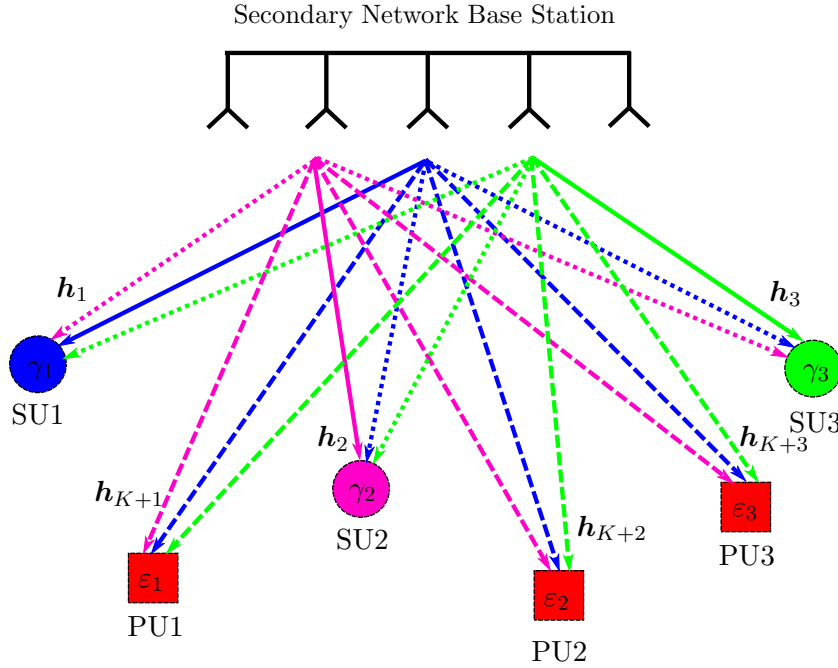


Figure 2.2: Schematic description of a CR downlink beamforming network.

problem can be formulated as

$$\begin{aligned}
 & \min_{\{\mathbf{w}_i\}} \sum_{i=1}^K \|\mathbf{w}_i\|^2 \\
 & \text{s.t.} \quad \frac{\mathbf{w}_k^H \mathbf{R}_{\mathbf{h}_k} \mathbf{w}_k}{\sum_{\substack{i=1 \\ i \neq k}}^K \mathbf{w}_i^H \mathbf{R}_{\mathbf{h}_k} \mathbf{w}_i + \sigma_k^2} \geq \gamma_k; \quad k = 1, \dots, K \\
 & \quad \sum_{i=1}^K \mathbf{w}_i^H \mathbf{R}_{\mathbf{h}_{K+l}} \mathbf{w}_i \leq \epsilon_l; \quad l = 1, \dots, L.
 \end{aligned} \tag{2.10}$$

Note that the problem (2.10) contains problem (2.4) as a special case for $L = 0$.

The solution to the problem (2.10) presented in [79] is basically an extension to the SDR approach developed in [59] for solving the conventional downlink beamforming problem (2.4). The authors of [79] use the transformation (2.5) and then relax the non-convex rank-one constraints. As a result the original problem (2.10) is relaxed to the following convex

form:

$$\begin{aligned}
& \min_{\{\mathbf{W}_i\}} \sum_{i=1}^K \text{Tr}\{\mathbf{W}_i\} \\
& \text{s.t.} \quad \gamma_k \sum_{\substack{i=1 \\ i \neq k}}^K \text{Tr}\{\mathbf{R}_{\mathbf{h}_k} \mathbf{W}_i\} - \text{Tr}\{\mathbf{R}_{\mathbf{h}_k} \mathbf{W}_k\} + \gamma_k \sigma_k^2 \leq 0 \\
& \quad \sum_{i=1}^K \text{Tr}\{\mathbf{R}_{\mathbf{h}_{K+l}} \mathbf{W}_i\} \leq \varepsilon_l \\
& \quad \mathbf{W}_k \succeq 0; \quad k = 1, \dots, K; \quad l = 1, \dots, L.
\end{aligned} \tag{2.11}$$

It has been shown in [94, 95] that when $L \leq 2$, a rank-one solution of (2.11) can always be constructed from higher rank solutions. In this case, the SDR step does not involve any approximation and the problems (2.10) and (2.11) become equivalent. Similar to the conventional beamforming problem (2.7), the optimum beamforming vectors \mathbf{w}_k can be obtained from the principal eigenvectors of \mathbf{W}_k , if the rank of the obtained \mathbf{W}_k is one. In case the rank of \mathbf{W}_k is not one and $L \leq 2$, the rank-reduction algorithm proposed in [94, 95] can be used to obtain the rank-one solutions. When $L > 2$ and a higher rank solution is obtained, the so-called *randomization techniques* [101] can be used to obtain an approximate rank-one solution.

In [77], following the approach of [28, 50, 54], it has been shown that the downlink beamforming problem (2.10) yields optimal beamformers that are identical to the corresponding virtual uplink problem. An iterative algorithm is then proposed, based on the uplink-downlink duality, to obtain the solution of problem (2.10).

2.4 Robust Downlink Beamforming for Conventional and CR Scenarios

The non-robust beamforming problems considered in Sections 2.2 & 2.3 assume the availability of perfect CSI. In practice, however, the CSI available at the base station is generally not perfect and the covariance matrices available at the transmitter, $\hat{\mathbf{R}}_{\mathbf{h}_k}$ and $\hat{\mathbf{R}}_{\mathbf{h}_{K+l}}$, are

erroneous. When the beamformers designed for these mismatched covariance matrices are applied to the true covariance matrices $\mathbf{R}_{\mathbf{h}_k}$ and $\mathbf{R}_{\mathbf{h}_{K+l}}$, it is not guaranteed anymore that the constraints in problem (2.10) would still be satisfied. Therefore, we modify the problem (2.10) such that it takes into account the possible mismatches in the covariance matrices. We consider that the available estimates of the true channel covariance matrices at the transmitter, for the k th SU and l th PU, are denoted as $\hat{\mathbf{R}}_{\mathbf{h}_k}$ ($k = 1, \dots, K$) and $\hat{\mathbf{R}}_{\mathbf{h}_{K+l}}$ ($l = 1, \dots, L$), respectively. The uncertainties in these estimates are modelled by the $N \times N$ Hermitian matrices Δ_k and Δ_{K+l} , respectively. The mismatch in the covariance matrices is assumed to be bounded such that $g^{(b)}(\Delta_k) \leq \alpha_k$ and $g^{(b)}(\Delta_{K+l}) \leq \alpha_{K+l}$ where $g^{(b)}(\cdot)$ represents a function that can characterize the set of the corresponding mismatch matrices and α_k and α_{K+l} represent the corresponding known bounds.

To design the robust beamformers, we follow a worst-case design approach. The robust problem is formulated such that the QoS and interference constraints for the SUs and PUs, respectively, are met for all the possible mismatched covariance matrices in the given bounds. The fact that the true covariances are always positive-semidefinite is also taken into account and as a result, the robust CR beamforming problem can be formulated as

$$\begin{aligned}
& \min_{\{\mathbf{w}_i\}} \sum_{i=1}^K \|\mathbf{w}_i\|^2 \\
& \text{s.t.} \quad \min_{\substack{g^{(b)}(\bar{\Delta}_k) \leq \alpha_k \\ \hat{\mathbf{R}}_{\mathbf{h}_k} - \bar{\Delta}_k \succeq 0}} \frac{\mathbf{w}_k^H (\hat{\mathbf{R}}_{\mathbf{h}_k} - \bar{\Delta}_k) \mathbf{w}_k}{\sum_{i=1, i \neq k}^K \mathbf{w}_i^H (\hat{\mathbf{R}}_{\mathbf{h}_k} - \bar{\Delta}_k) \mathbf{w}_i + \sigma_k^2} \geq \gamma_k; \quad k = 1, \dots, K \\
& \quad \max_{\substack{g^{(b)}(\bar{\Delta}_{K+l}) \leq \alpha_{K+l} \\ \hat{\mathbf{R}}_{\mathbf{h}_{K+l}} - \bar{\Delta}_{K+l} \succeq 0}} \sum_{i=1}^K \mathbf{w}_i^H (\hat{\mathbf{R}}_{\mathbf{h}_{K+l}} - \bar{\Delta}_{K+l}) \mathbf{w}_i \leq \varepsilon_l; \quad l = 1, \dots, L. \quad (2.12)
\end{aligned}$$

Similar to the problem (2.10), the the robust CR downlink beamforming problem (2.12) contains the conventional robust downlink beamforming problem as a special case for $L = 0$.

2.5 Previous Solutions for the Robust Downlink Beamforming Problem

The worst-case problem formulations for robust downlink beamforming have been considered in [50] and [79] for the conventional and the CR beamformer, respectively. In the formulations of [50] and [79], Frobenius norm is used to represent the bounding function $g^{(b)}(\cdot)$. The resulting problem formulation can be expressed as

$$\begin{aligned}
& \min_{\{\mathbf{w}_i\}} \sum_{i=1}^K \|\mathbf{w}_i\|^2 \\
& \text{s.t.} \quad \min_{\|\bar{\Delta}_k\| \leq \alpha_k} \frac{\mathbf{w}_k^H (\hat{\mathbf{R}}_{\mathbf{h}_k} - \bar{\Delta}_k) \mathbf{w}_k}{\sum_{i=1, i \neq k}^K \mathbf{w}_i^H (\hat{\mathbf{R}}_{\mathbf{h}_k} - \bar{\Delta}_{\mathbf{h}_k}) \mathbf{w}_i + \sigma_k^2} \geq \gamma_k; \quad k = 1, \dots, K \\
& \quad \max_{\|\bar{\Delta}_{K+l}\| \leq \alpha_{K+l}} \sum_{i=1}^K \mathbf{w}_i^H (\hat{\mathbf{R}}_{\mathbf{h}_{K+l}} - \bar{\Delta}_{K+l}) \mathbf{w}_i \leq \varepsilon_l; \quad l = 1, \dots, L.
\end{aligned} \tag{2.13}$$

Further, negative and positive diagonal loading [32, 102] is used to modify the QoS and PU interference constraints and the final problem formulations in [50] (with $L = 0$) and [79] appear as

$$\begin{aligned}
& \min_{\{\mathbf{w}_i\}} \sum_{i=1}^K \|\mathbf{w}_i\|^2 \\
& \text{s.t.} \quad \frac{\mathbf{w}_k^H (\hat{\mathbf{R}}_{\mathbf{h}_k} - \alpha_k \mathbf{I}) \mathbf{w}_k}{\sum_{i=1, i \neq k}^K \mathbf{w}_i^H (\hat{\mathbf{R}}_{\mathbf{h}_k} + \alpha_k \mathbf{I}) \mathbf{w}_i + \sigma_k^2} \geq \gamma_k; \quad k = 1, \dots, K \\
& \quad \sum_{i=1}^K \mathbf{w}_i^H (\hat{\mathbf{R}}_{\mathbf{h}_{K+l}} + \alpha_{K+l} \mathbf{I}) \mathbf{w}_i \leq \varepsilon_l; \quad l = 1, \dots, L.
\end{aligned} \tag{2.14}$$

Note that the problem (2.14) is mathematically similar to problem (2.10). Therefore, the authors in [50] and [79] propose the use of SDR for solving (2.14). The techniques of [50] and [79] ignore the positive definiteness constraints on the mismatched covariance matrices. Additionally, the transformations of QoS and PU interference constraints in (2.14) are also conservative.

In [80] and [81], a related robust CR downlink beamforming problem has been solved by maximizing the minimum received SINR among all the SUs with constraints on the total

transmitted power and the interference caused to the PUs where the technique of [81] is based on the instantaneous CSI. Another interesting robust CR approach, based on the instantaneous CSI, is proposed in [86], where the finite rate feedback is used as a partial CSI. It is assumed that finite feedback bits are used to represent an index of a predefined vector in the codebook, known to both the transmitter and the receiver. The receiver, assumed to have perfect knowledge of its channel, quantizes its channel vector into one of the vectors in the codebook and transmits the index to this vector back to the transmitter.

In [68], we have considered the problem (2.13) for the conventional beamforming scenario. In the proposed solution, the QoS constraints are treated conservatively in a similar manner as in the solutions of [50] and [79]. However, unlike the techniques of [50] and [79], the approach in [68] does take into account the positive semidefiniteness of the covariance matrices. This is achieved by modeling the channel uncertainties in $\hat{\mathbf{R}}_{h_k}^{1/2}$ rather than $\hat{\mathbf{R}}_{h_k}$, where the matrix $\hat{\mathbf{R}}_{h_k}^{1/2}$ is defined through the equation

$$\hat{\mathbf{R}}_{h_k} = \hat{\mathbf{R}}_{h_k}^{1/2H} \hat{\mathbf{R}}_{h_k}^{1/2}. \quad (2.15)$$

The solution to the robust problem (2.13) can then be found by solving the following SDP problem iteratively:

$$\begin{aligned} & \min_{\{\mathbf{W}_{k,l}, t_{k,l}\}} \sum_{i=1}^K \text{Tr}\{\mathbf{W}_{i,l}\} \\ \text{s.t.} \quad & \text{Tr}\{\mathbf{R}_{h_k} \mathbf{W}_{k,l}\} - \beta_k^2 \text{Tr}\{\mathbf{W}_{k,l}\} - t_{k,l} \geq 2\beta_k \sqrt{t_{k,l-1}} \sqrt{\text{tr}\{\mathbf{W}_{k,l-1}\}} \\ & t_{k,l} - \gamma_k \sum_{i \neq k}^K \text{Tr}\{(\mathbf{R}_{h_k} + \alpha_k \mathbf{I}) \mathbf{W}_{i,l}\} \geq \gamma_k \sigma_k^2 \\ & \mathbf{W}_{k,l} \succeq 0, \quad t_{k,l} \geq 0, \quad k = 1, \dots, K. \end{aligned} \quad (2.16)$$

Matrix $\mathbf{W}_{k,l}$ is the matrix \mathbf{W}_k , defined in (2.5), for the l th iteration and $t_{k,l}$ is an auxiliary variable. $\mathbf{W}_{i,l-1}$ and $t_{i,l-1}$ are the solutions obtained in the $(l-1)$ th iteration. β_k defines the bound on the uncertainty in the matrix $\hat{\mathbf{R}}_{h_k}^{1/2}$. The approach of [68] outperforms the method of [50] in terms of transmitted power.

Chapter 3

Robust Downlink Beamforming with Frobenius Norm Constraints on Channel Mismatch

3.1 Introduction

In this chapter, we present worst-case optimization-based robust techniques to solve the robust downlink beamforming problem (2.12) for both the conventional and CR beamforming problems where we use the weighted Frobenius norm to bound the channel uncertainties. Several approaches to solve this problem have been proposed in [50], for the conventional scenario, and in [79] and [82], for the CR network, which use the conventional Frobenius norm for bounding the channel uncertainties. We would like to mention that the weighted Frobenius norm is a better measure of the channel uncertainties compared to the conventional Frobenius norm as in practical systems the covariance matrix errors follow specific distributions which can, e.g., be estimated at the transmitter or the receivers. The use of weighted Frobenius norm can lead to significant improvements in practical implementations as it will be shown in the simulations. The techniques of [50], [79] and [82] further utilize

several conservative approximations in the modifications of the QoS and PU interference constraints, that we avoid in our proposed approaches. By using Lagrange duality, we present exact reformulations of the QoS and PU interference constraints. Next, applying SDR, the resulting problems are converted into convex SDP problems. We also analyze the robust beamforming problem based on the probabilistic model and show that the final problem formulations for both the worst-case and probabilistic approaches are mathematically equivalent. Interestingly, a similar relationship also exists for robust receive beamforming problems [46].

We then present an iterative robust approach for CR downlink beamforming, that combines the benefits of the obtained robust problem formulation with the method presented in [77]. The authors in [77] prove the uplink-downlink duality for non-robust CR beamforming problem and use this duality to propose an iterative algorithm to obtain the optimal beamformers and power allocations for the CR downlink beamforming problem. The non-robust algorithm in [77] is computationally efficient, that we adopt for the robust CR downlink beamforming problem. The so obtained robust iterative algorithm retains the benefit of computational efficiency and shows a convergence rate similar to the non-robust iterative technique. Note that another non-robust iterative algorithm, similar to the one in [77], is presented in [78].

In our simulations for the proposed non-iterative approach, we generally obtain rank-one solutions to the resulting SDP problems implying that the obtained SDR solutions are optimal. However, we point out a class of specific examples in which all solutions violate the rank-one constraint. These examples occur when uncertainty thresholds are large and/or covariance matrices are highly symmetric. We present a detailed analysis of these examples in Appendices B and C and conclude that the probability that these cases occur is very low in randomly fading channels and under reasonable assumptions regarding the size of the uncertainty sets. This result differs fundamentally from the non-robust beamforming case where rank one solutions can always be obtained if the number of interference constraints is small (≤ 2) [94, 95] and, therefore, provides additional insight to the robust beamforming problem. The proposed beamformers show substantial performance improvements in terms

of problem feasibility and transmitted power in comparison to those of [50] and [79].

3.2 Problem Formulation

Recall that the estimates of the true channel covariance matrices for the k th SU and l th PU are represented as $\hat{\mathbf{R}}_{h_k}$ ($k = 1, \dots, K$) and $\hat{\mathbf{R}}_{h_{K+l}}$ ($l = 1, \dots, L$), respectively, and $\mathbf{\Delta}_k$ and $\mathbf{\Delta}_{K+l}$ contain the corresponding uncertainties. Depending on the CSI estimation methods or the feedback quantization scheme, the uncertainty matrices $\mathbf{\Delta}_k$ and $\mathbf{\Delta}_{K+l}$ follow specific random distributions. We consider the case that the channel covariance are subject to colored noise. Let $\text{vec}(\mathbf{X})$ denote the operator that stacks the columns of the matrix \mathbf{X} to form a vector. We assume that

$$\mathbf{R}_{\mathbf{\Delta}_m} \triangleq \text{E}\{\text{vec}(\mathbf{\Delta}_m)\text{vec}(\mathbf{\Delta}_m)^H\}; \quad m = 1, \dots, K + L, \quad (3.1)$$

denotes the colored covariance matrix of the CSI error vector $\text{vec}(\mathbf{\Delta}_m)$. Assume that $\mathbf{u}_n(\mathbf{R}_{\mathbf{\Delta}_m})$ is the eigenvector of $\mathbf{R}_{\mathbf{\Delta}_m}$ with the corresponding eigenvalue $\lambda_n(\mathbf{R}_{\mathbf{\Delta}_m})$. Due to the coloring of the CSI errors, the errors along the dominant eigenvectors corresponding to the largest eigenvalues are more prominent than the errors along the minor eigenvectors. To take the effect of the colored CSI errors into account the uncertainty set is appropriately described by an ellipsoid with elliptic radii proportional to the square root of the eigenvalues $\lambda_n(\mathbf{R}_{\mathbf{\Delta}_m})$, as specified below.

Consider e.g., the weighting matrix defined as

$$\mathbf{Q}_m \triangleq \frac{\mathbf{R}_{\mathbf{\Delta}_m}}{\psi_m}; \quad m = 1, \dots, K + L \quad (3.2)$$

where ψ_m is a constant such that $\|\mathbf{Q}_m\| = 1$. Then we define the \mathbf{P} -weighted Frobenius norm, for some positive-semidefinite matrix \mathbf{P} as

$$\|\mathbf{X}\|_{\mathbf{P}} \triangleq \|\sqrt{\mathbf{P}}\text{vec}(\mathbf{X})\| = \sqrt{\text{vec}(\mathbf{X})^H \mathbf{P} \text{vec}(\mathbf{X})}. \quad (3.3)$$

In our beamforming approach we ensure robustness for all mismatch matrices that are bounded by ellipsoids of known elliptic radii, i.e., $\|\mathbf{\Delta}_k\|_{\mathbf{Q}_k^{-1}} \leq \alpha_k$ and $\|\mathbf{\Delta}_{K+l}\|_{\mathbf{Q}_{K+l}^{-1}} \leq \alpha_{K+l}$.

We remark that if $\lambda_n(\mathbf{Q}_k)$ denotes the n th eigenvalue of the weighting matrix \mathbf{Q}_k and $\|\mathbf{\Delta}_k\|_{\mathbf{Q}_k^{-1}} \leq \alpha_k$, then the n th elliptic radius is given by $\sqrt{\lambda_n(\mathbf{Q}_k)}\alpha_k$. In this sense, we consider the cases in which the true channel covariance matrices lie in the set of matrices defined as $\{\hat{\mathbf{R}}_{\mathbf{h}_k} - \mathbf{\Delta}_k : \|\mathbf{\Delta}_k\|_{\mathbf{Q}_k^{-1}} \leq \alpha_k\}$ ($k = 1, \dots, K$) and $\{\hat{\mathbf{R}}_{\mathbf{h}_{K+l}} - \mathbf{\Delta}_{K+l} : \|\mathbf{\Delta}_{K+l}\|_{\mathbf{Q}_{K+l}^{-1}} \leq \alpha_{K+l}\}$ ($l = 1, \dots, L$). In case of uniform weighting, hence $\mathbf{Q}_m = \mathbf{I}$, this approach of modeling the uncertainties becomes similar to the non-weighted techniques of [50] and [79].

Next, inserting the Frobenius norm based bounding function, i.e. $g^{(b)}(\mathbf{\Delta}_m) = \|\mathbf{\Delta}_m\|_{\mathbf{Q}_m^{-1}}$ ($m = 1, \dots, K + L$), in the robust CR beamforming problem (2.12), it can be reformulated as

$$\begin{aligned}
& \min_{\{\mathbf{w}_i\}} \sum_{i=1}^K \|\mathbf{w}_i\|^2 \\
\text{s.t.} \quad & \min_{\substack{\|\bar{\mathbf{\Delta}}_k\|_{\mathbf{Q}_k^{-1}} \leq \alpha_k \\ \hat{\mathbf{R}}_{\mathbf{h}_k} - \bar{\mathbf{\Delta}}_k \succeq 0}} \frac{\mathbf{w}_k^H (\hat{\mathbf{R}}_{\mathbf{h}_k} - \bar{\mathbf{\Delta}}_k) \mathbf{w}_k}{\sum_{\substack{i=1 \\ i \neq k}}^K \mathbf{w}_i^H (\hat{\mathbf{R}}_{\mathbf{h}_k} - \bar{\mathbf{\Delta}}_{\mathbf{h}_k}) \mathbf{w}_i + \sigma_k^2} \geq \gamma_k; \quad k = 1, \dots, K \\
& \max_{\substack{\|\bar{\mathbf{\Delta}}_{K+l}\|_{\mathbf{Q}_{K+l}^{-1}} \leq \alpha_{K+l} \\ \hat{\mathbf{R}}_{\mathbf{h}_{K+l}} - \bar{\mathbf{\Delta}}_{K+l} \succeq 0}} \sum_{i=1}^K \mathbf{w}_i^H (\hat{\mathbf{R}}_{\mathbf{h}_{K+l}} - \bar{\mathbf{\Delta}}_{K+l}) \mathbf{w}_i \leq \varepsilon_l; \quad l = 1, \dots, L. \tag{3.4}
\end{aligned}$$

3.3 Drawbacks of Previous Solutions

In Section 2.5, we have presented the solutions to the robust downlink beamforming problem (3.4). As previously mentioned, [50] and [79] consider the worst-case robust downlink beamforming problem formulations for the conventional and the CR beamformer, respectively. In the following, we discuss the drawbacks of these solutions.

One major disadvantage of the formulations of [50] and [79], i.e., problem (2.13), in contrast to problem (3.4), is the fact that these formulations ignore the positive definiteness constraints on the mismatched covariance matrices, which could in fact result in worst-case approaches that are unnecessarily conservative. Additionally, the robust beamforming solutions presented in [50] and [79] contain several conservative approximations of the QoS and PU interference constraints that adversely affect the performance of the beamformers.

Firstly, the worst-case QoS constraints of (2.13) are replaced in [50] and [79] by

$$\frac{\min_{\|\bar{\Delta}_k\| \leq \alpha_k} \mathbf{w}_k^H (\hat{\mathbf{R}}_{\mathbf{h}_k} - \bar{\Delta}_k) \mathbf{w}_k}{\sum_{\substack{i=1 \\ i \neq k}}^K \max_{\|\bar{\Delta}_k\| \leq \alpha_k} \mathbf{w}_i^H (\hat{\mathbf{R}}_{\mathbf{h}_k} - \bar{\Delta}_k) \mathbf{w}_i + \sigma_k^2} \geq \gamma_k. \quad (3.5)$$

The approximation (3.5), in fact, strengthens the QoS constraints in (2.13). This implies that when the constraints (3.5) are satisfied, the satisfaction of QoS constraints in (3.4) will be guaranteed. The reverse statement, however, is generally not true. The second conservative approximation is related to the PU interference constraints and, therefore, is used in the solution of [79]. In particular, the PU interference constraints are replaced by

$$\sum_{i=1}^K \max_{\|\bar{\Delta}_{K+l}\| \leq \alpha_{K+l}} \mathbf{w}_i^H (\hat{\mathbf{R}}_{\mathbf{h}_{K+l}} - \bar{\Delta}_{K+l}) \mathbf{w}_i \leq \varepsilon_l, \quad (3.6)$$

making these unnecessarily stricter. Both of the above mentioned approximations may lead to suboptimal beamforming solutions or even infeasibility of the strengthened problem.

Our proposed method in [68], that considers the problem (3.4) for the conventional beamforming scenario with $\mathbf{Q}_m = \mathbf{I}$, takes into account the positive semidefiniteness of the covariance matrices but retains the conservative reformulation of the QoS constraints. As a result, the approach of [68] outperforms the method of [50] in terms of transmitted power. However it has certain disadvantages. Firstly, as mentioned earlier it does not avoid the conservative reformulation of the QoS constraints. Secondly, it requires solving an SDP problem in each iteration which makes it more complex compared to the technique of [50].

In [69], we have presented an improved robust downlink beamforming approach for the conventional scenario that avoids the conservative modifications of the QoS constraints used in [50] and also takes into account the positive semidefiniteness constraints on the mismatched covariance matrices. The authors of [82] utilize the idea of [69] and extend it for the robust CR downlink beamforming. However, in [82], the positive semidefiniteness constraints on the mismatched covariance matrices are not taken into account. We also extended the approach of [69] to a robust CR scenario in [83] where we included the positive semidefiniteness constraints on the mismatched covariance matrices.

3.4 Proposed Solution Using Lagrange Duality

In this section, we develop a new approach to find the solution to the robust worst-case beamforming problems for both the conventional and CR scenarios. We utilize Lagrange duality to solve the inner minimizations, appearing in the QoS and PU interference constraints of the problem (3.4). By doing so, and also taking the positive semidefiniteness of the downlink covariance matrices into account, we avoid the aforementioned conservative approximations and come up with an exact reformulation of the problem (3.4). The proposed method shows an improved performance in terms of feasibility and transmitted power.

3.4.1 The Proposed Conventional Downlink Beamformer

Let us first consider the robust downlink beamforming problem for the conventional scenario, i.e., with $L = 0$. We reformulate the QoS constraints in (3.4) by solving the inner minimization on the left side of these constraints as a separate optimization problem. Towards this aim, let us rewrite the k th QoS constraint as

$$\min_{\substack{\|\bar{\Delta}_k\|_{\mathcal{Q}_k^{-1}} \leq \alpha_k \\ \hat{\mathbf{R}}_{h_k} - \bar{\Delta}_k \succeq 0}} \left(\mathbf{w}_k^H \hat{\mathbf{R}}_{h_k} \mathbf{w}_k - \gamma_k \sum_{i=1, i \neq k}^K \mathbf{w}_i^H \hat{\mathbf{R}}_{h_k} \mathbf{w}_i - \sigma_k^2 \gamma_k - \mathbf{w}_k^H \bar{\Delta}_k \mathbf{w}_k + \gamma_k \sum_{i=1, i \neq k}^K \mathbf{w}_i^H \bar{\Delta}_k \mathbf{w}_i \right) \geq 0. \quad (3.7)$$

Defining a new matrix variable \mathbf{A}_k for simplification of notation as

$$\mathbf{A}_k \triangleq \mathbf{w}_k \mathbf{w}_k^H - \gamma_k \sum_{i=1, i \neq k}^K \mathbf{w}_i \mathbf{w}_i^H, \quad (3.8)$$

the k th QoS constraint (3.7) can be written as

$$\min_{\substack{\|\bar{\Delta}_k\|_{\mathcal{Q}_k^{-1}} \leq \alpha_k \\ \hat{\mathbf{R}}_{h_k} - \bar{\Delta}_k \succeq 0}} \left(-\text{Tr}\{\bar{\Delta}_k \mathbf{A}_k\} + \text{Tr}\{\hat{\mathbf{R}}_{h_k} \mathbf{A}_k\} - \sigma_k^2 \gamma_k \right) \geq 0. \quad (3.9)$$

We can observe that the minimization on the left side of (3.9) can be substituted by the closed form solution of the following optimization problem:

$$\begin{aligned}
\min_{\bar{\mathbf{\Delta}}_k} \quad & -\text{Tr}\{\bar{\mathbf{\Delta}}_k \mathbf{A}_k\} + \text{Tr}\{\hat{\mathbf{R}}_{h_k} \mathbf{A}_k\} - \sigma_k^2 \gamma_k \\
\text{s.t.} \quad & \|\bar{\mathbf{\Delta}}_k\|_{\mathbf{Q}_k^{-1}} \leq \alpha_k \\
& \hat{\mathbf{R}}_{h_k} - \bar{\mathbf{\Delta}}_k \succeq 0.
\end{aligned} \tag{3.10}$$

If the matrix \mathbf{A}_k is given, then (3.10) is a convex problem in variable $\bar{\mathbf{\Delta}}_k$. Replacing the left side of the constraint (3.9) with the Lagrange dual of (3.10) could strengthen these constraints as the Lagrange dual of a given problem provides a lower bound to that problem [90]. However, as we would show later, strong duality exists between problem (3.10) and its Lagrange dual and, therefore, the modification of the constraint (3.9) would be exact. Therefore, we write the Lagrange dual function for problem (3.10) as

$$g(\tau_k, \mathbf{Z}_k) = \inf_{\bar{\mathbf{\Delta}}_k} f(\tau_k, \mathbf{Z}_k, \bar{\mathbf{\Delta}}_k) \tag{3.11}$$

where the Lagrangian function $f(\tau_k, \mathbf{Z}_k, \bar{\mathbf{\Delta}}_k)$ is given as

$$\begin{aligned}
f(\tau_k, \mathbf{Z}_k, \bar{\mathbf{\Delta}}_k) &= -\text{Tr}\{\bar{\mathbf{\Delta}}_k \mathbf{A}_k\} + \text{Tr}\{\hat{\mathbf{R}}_{h_k} \mathbf{A}_k\} - \sigma_k^2 \gamma_k + \tau_k (\|\bar{\mathbf{\Delta}}_k\|_{\mathbf{Q}_k^{-1}} - \alpha_k) - \text{Tr}\{(\hat{\mathbf{R}}_{h_k} - \bar{\mathbf{\Delta}}_k) \mathbf{Z}_k\} \\
&= \text{Tr}\{\bar{\mathbf{\Delta}}_k (\mathbf{Z}_k - \mathbf{A}_k)\} - \text{Tr}\{\hat{\mathbf{R}}_{h_k} (\mathbf{Z}_k - \mathbf{A}_k)\} - \sigma_k^2 \gamma_k + \tau_k (\|\bar{\mathbf{\Delta}}_k\|_{\mathbf{Q}_k^{-1}} - \alpha_k)
\end{aligned} \tag{3.12}$$

and, $\tau_k \geq 0$ and $\mathbf{Z}_k \succeq 0$ are the Lagrange variables. Next, utilizing the fact that the matrices \mathbf{Z}_k , \mathbf{A}_k , $\bar{\mathbf{\Delta}}_k$, and \mathbf{Q}_k are all Hermitian, we equate the derivative of (3.12) to zero in order to find its infimum as

$$\begin{aligned}
0 &= \frac{\partial f(\tau_k, \mathbf{Z}_k, \bar{\mathbf{\Delta}}_k)}{\partial \bar{\mathbf{\Delta}}_k^*} \\
\Leftrightarrow 0 &= -\frac{\partial \{\text{Tr}\{\bar{\mathbf{\Delta}}_k \mathbf{A}_k\}\}}{\partial \bar{\mathbf{\Delta}}_k^*} + \frac{\partial \{\text{Tr}\{\bar{\mathbf{\Delta}}_k \mathbf{Z}_k\}\}}{\partial \bar{\mathbf{\Delta}}_k^*} + \tau_k \frac{\partial \{\sqrt{\text{vec}(\bar{\mathbf{\Delta}}_k)^H \mathbf{Q}_k^{-1} \text{vec}(\bar{\mathbf{\Delta}}_k)}\}}{\partial \bar{\mathbf{\Delta}}_k^*} \\
\Leftrightarrow 0 &= -\mathbf{A}_k + \mathbf{Z}_k + \tau_k \frac{\text{vec}^{-1}(\mathbf{Q}_k^{-1} \text{vec}(\bar{\mathbf{\Delta}}_k))}{\|\bar{\mathbf{\Delta}}_k\|_{\mathbf{Q}_k^{-1}}} \\
\Leftrightarrow \mathbf{Z}_k - \mathbf{A}_k &= -\tau_k \frac{\text{vec}^{-1}(\mathbf{Q}_k^{-1} \text{vec}(\bar{\mathbf{\Delta}}_k))}{\|\bar{\mathbf{\Delta}}_k\|_{\mathbf{Q}_k^{-1}}}.
\end{aligned} \tag{3.13}$$

Taking the \mathbf{Q}_k -weighted Frobenius norm on both sides of the last equation in (3.13), we obtain that

$$\begin{aligned}\|\mathbf{Z}_k - \mathbf{A}_k\|_{\mathbf{Q}_k} &= \tau_k^* \frac{\sqrt{\text{vec}(\bar{\Delta}_k)^H \mathbf{Q}_k^{-1} \text{vec}(\bar{\Delta}_k)}}{\|\bar{\Delta}_k\|_{\mathbf{Q}_k^{-1}}} \\ &= \tau_k^*.\end{aligned}\quad (3.14)$$

Next, inserting $\mathbf{Z}_k - \mathbf{A}_k$ from (3.13) in the first term of (3.12), the Lagrange dual function (3.11) becomes

$$\begin{aligned}g(\tau_k, \mathbf{Z}_k) &= -\tau_k \frac{\text{Tr}\{\bar{\Delta}_k \text{vec}^{-1}(\mathbf{Q}_k^{-1} \text{vec}(\bar{\Delta}_k))\}}{\|\bar{\Delta}_k\|_{\mathbf{Q}_k^{-1}}} - \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}(\mathbf{Z}_k - \mathbf{A}_k)\} - \sigma_k^2 \gamma_k + \tau_k \|\bar{\Delta}_k\|_{\mathbf{Q}_k^{-1}} - \tau_k \alpha_k \\ &= -\tau_k \|\bar{\Delta}_k\|_{\mathbf{Q}_k^{-1}} - \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}(\mathbf{Z}_k - \mathbf{A}_k)\} - \sigma_k^2 \gamma_k + \tau_k \|\bar{\Delta}_k\|_{\mathbf{Q}_k^{-1}} - \tau_k \alpha_k \\ &= -\text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}(\mathbf{Z}_k - \mathbf{A}_k)\} - \sigma_k^2 \gamma_k - \tau_k \alpha_k\end{aligned}\quad (3.15)$$

where in the second equality we have used the fact that $\text{Tr}\{\mathbf{X}\mathbf{Y}\} = \text{vec}(\mathbf{X}^H)^H \text{vec}(\mathbf{Y})$ [103].

Inserting the optimal τ_k^* from (3.14) in (3.15), we obtain

$$g(\mathbf{Z}_k) = -\text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}(\mathbf{Z}_k - \mathbf{A}_k)\} - \sigma_k^2 \gamma_k - \alpha_k \|\mathbf{Z}_k - \mathbf{A}_k\|_{\mathbf{Q}_k}\quad (3.16)$$

and the dual problem corresponding to (3.10) can be written as

$$\begin{aligned}\max_{\mathbf{Z}_k} & -\alpha_k \|\mathbf{Z}_k - \mathbf{A}_k\|_{\mathbf{Q}_k} - \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}(\mathbf{Z}_k - \mathbf{A}_k)\} - \sigma_k^2 \gamma_k \\ \text{s.t.} & \quad \mathbf{Z}_k \succeq 0.\end{aligned}\quad (3.17)$$

As mentioned earlier, the problem (3.10) is convex in variable $\bar{\Delta}_k$ and is clearly bounded below. Furthermore, the error matrices $\bar{\Delta}_k = -\tilde{\alpha}_k \mathbf{I}$, with $0 < \tilde{\alpha}_k < \alpha_k / \sqrt{N}$, represent strictly feasible points of problem (3.10). Therefore, using [104, Th. 1.7.1], we can conclude that strong duality between (3.10) and (3.17) holds. This implies that replacing the k th QoS constraint (3.9) by the equivalent constraint

$$\max_{\mathbf{Z}_k \succeq 0} \left(-\alpha_k \|\mathbf{Z}_k - \mathbf{A}_k\|_{\mathbf{Q}_k} - \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}(\mathbf{Z}_k - \mathbf{A}_k)\} - \sigma_k^2 \gamma_k \right) \geq 0\quad (3.18)$$

does not involve any approximation. We observe that (3.18) (and correspondingly (3.9)) is satisfied if there exists some $\mathbf{Z}_k \succeq 0$ for which

$$-\alpha_k \|\mathbf{Z}_k - \mathbf{A}_k\|_{\mathbf{Q}_k} - \text{Tr}\{\hat{\mathbf{R}}_{h_k}(\mathbf{Z}_k - \mathbf{A}_k)\} - \sigma_k^2 \gamma_k \geq 0. \quad (3.19)$$

The QoS constraint (3.9) can, therefore, also be replaced by (3.19) along with the constraint $\mathbf{Z}_k \succeq 0$.

Using (3.19), along with the definition in (2.5), problem (3.4), for the case $L = 0$, can be modified as

$$\begin{aligned} & \min_{\{\mathbf{W}_i, \mathbf{Z}_i\}} \sum_{i=1}^K \text{Tr}\{\mathbf{W}_i\} \\ \text{s.t.} \quad & -\alpha_k \|\mathbf{Z}_k - \mathbf{A}_k\|_{\mathbf{Q}_k} - \text{Tr}\{\hat{\mathbf{R}}_{h_k}(\mathbf{Z}_k - \mathbf{A}_k)\} - \sigma_k^2 \gamma_k \geq 0 \\ & \mathbf{Z}_k \succeq 0, \mathbf{W}_k \succeq 0, \text{rank}(\mathbf{W}_k) = 1; k = 1, \dots, K. \end{aligned} \quad (3.20)$$

It is proven in Appendix A that $\mathbf{Z}_k^* = \mathbf{0}$ is a solution of (3.20). The remaining problem can be solved by dropping the rank-one constraint following the SDR approach. This results in the final problem formulation

$$\begin{aligned} & \min_{\{\mathbf{W}_i\}} \sum_{i=1}^K \text{Tr}\{\mathbf{W}_i\} \\ \text{s.t.} \quad & -\alpha_k \|\mathbf{A}_k\|_{\mathbf{Q}_k} + \text{Tr}\{\hat{\mathbf{R}}_{h_k} \mathbf{A}_k\} - \sigma_k^2 \gamma_k \geq 0 \\ & \mathbf{W}_k \succeq 0; k = 1, \dots, K. \end{aligned} \quad (3.21)$$

Problem (3.21) is a convex SDP and can be solved in polynomial time using interior-point algorithms [91, 92]. We discuss the rank of the solutions obtained from (3.21) in Section 3.6, and provide a procedure to find rank-one approximation of the solution of (3.21).

3.4.2 The Proposed CR Downlink Beamformer

After considering the robust downlink beamforming problem (3.4) with $L = 0$ for the conventional scenario, we solve the problem (3.4) for the more general case of CR downlink beamforming. To this end, we modify the PU interference constraints in (3.4), avoiding the

conservative reformulation of (3.6). Introducing the auxiliary matrix

$$\mathbf{C} \triangleq \sum_{k=1}^K \mathbf{w}_k \mathbf{w}_k^H, \quad (3.22)$$

the l th PU interference constraint in (3.4) can be written as

$$\begin{aligned} \max_{\substack{\|\bar{\Delta}_{K+l}\|_{\mathbf{Q}_{K+l}^{-1}} \leq \alpha_{K+l} \\ \hat{\mathbf{R}}_{\mathbf{h}_{K+l}} - \bar{\Delta}_{K+l} \succeq 0}} \text{Tr}\{\mathbf{C}(\hat{\mathbf{R}}_{\mathbf{h}_{K+l}} - \bar{\Delta}_{K+l})\} \leq \varepsilon_l. \end{aligned} \quad (3.23)$$

Following the approach used for the QoS constraint (3.9), we observe that the maximization on the left side of PU interference constraint (3.23) corresponds to the following optimization problem:

$$\begin{aligned} \min_{\bar{\Delta}_{K+l}} & -\text{Tr}\{\mathbf{C}(\hat{\mathbf{R}}_{\mathbf{h}_{K+l}} - \bar{\Delta}_{K+l})\} \\ \text{s.t.} & \quad \|\bar{\Delta}_{K+l}\|_{\mathbf{Q}_{K+l}^{-1}} \leq \alpha_{K+l} \\ & \quad \hat{\mathbf{R}}_{\mathbf{h}_{K+l}} - \bar{\Delta}_{K+l} \succeq 0. \end{aligned} \quad (3.24)$$

For a given matrix \mathbf{C} , (3.24) is a convex problem in variable $\bar{\Delta}_{K+l}$. Again using the fact that the Lagrange dual of (3.24) would provide a lower bound to its solution, we replace (3.24) by its dual. Therefore, we write the Lagrange dual function for problem (3.24) as

$$g_p(\tau_{K+l}, \mathbf{Z}_{K+l}) = \inf_{\bar{\Delta}_{K+l}} f_p(\tau_{K+l}, \mathbf{Z}_{K+l}, \bar{\Delta}_{K+l}) \quad (3.25)$$

where the Lagrangian function $f_p(\tau_{K+l}, \mathbf{Z}_{K+l}, \bar{\Delta}_{K+l})$ is given as

$$\begin{aligned} & f_p(\tau_{K+l}, \mathbf{Z}_{K+l}, \bar{\Delta}_{K+l}) \\ &= \text{Tr}\{\bar{\Delta}_{K+l} \mathbf{C}\} - \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_{K+l}} \mathbf{C}\} + \tau_{K+l} (\|\bar{\Delta}_{K+l}\|_{\mathbf{Q}_{K+l}^{-1}} - \alpha_{K+l}) - \text{Tr}\{(\hat{\mathbf{R}}_{\mathbf{h}_{K+l}} - \bar{\Delta}_{K+l}) \mathbf{Z}_{K+l}\} \\ &= \text{Tr}\{\bar{\Delta}_{K+l} (\mathbf{C} + \mathbf{Z}_{K+l})\} - \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_{K+l}} (\mathbf{C} + \mathbf{Z}_{K+l})\} + \tau_{K+l} (\|\bar{\Delta}_{K+l}\|_{\mathbf{Q}_{K+l}^{-1}} - \alpha_{K+l}) \end{aligned} \quad (3.26)$$

and, $\tau_{K+l} \geq 0$ and $\mathbf{Z}_{K+l} \succeq 0$ are the Lagrange variables. Utilizing the fact that the matrices \mathbf{Z}_{K+l} , \mathbf{C} , $\bar{\Delta}_{K+l}$, and \mathbf{Q}_{K+l} are all Hermitian, we equate the derivative of (3.26) to zero in

order to find its infimum as

$$\begin{aligned}
0 &= \frac{\partial f_p(\tau_{K+l}, \mathbf{Z}_{K+l}, \bar{\Delta}_{K+l})}{\partial \bar{\Delta}_{K+l}^*} \\
\Leftrightarrow 0 &= \frac{\partial \{\text{Tr}\{\bar{\Delta}_{K+l} \mathbf{C}\}\}}{\partial \bar{\Delta}_{K+l}^*} + \frac{\partial \{\text{Tr}\{\bar{\Delta}_{K+l} \mathbf{Z}_{K+l}\}\}}{\partial \bar{\Delta}_{K+l}^*} \\
&\quad + \tau_{K+l} \frac{\partial \{\sqrt{\text{vec}(\bar{\Delta}_{K+l})^H \mathbf{Q}_{K+l}^{-1} \text{vec}(\bar{\Delta}_{K+l})}\}}{\partial \bar{\Delta}_{K+l}^*} \\
\Leftrightarrow 0 &= \mathbf{C} + \mathbf{Z}_{K+l} + \tau_{K+l} \frac{\text{vec}^{-1}(\mathbf{Q}_{K+l}^{-1} \text{vec}(\bar{\Delta}_{K+l}))}{\|\bar{\Delta}_{K+l}\|_{\mathbf{Q}_{K+l}^{-1}}} \\
\Leftrightarrow \mathbf{C} + \mathbf{Z}_{K+l} &= -\tau_{K+l} \frac{\text{vec}^{-1}(\mathbf{Q}_{K+l}^{-1} \text{vec}(\bar{\Delta}_{K+l}))}{\|\bar{\Delta}_{K+l}\|_{\mathbf{Q}_{K+l}^{-1}}}. \tag{3.27}
\end{aligned}$$

Taking the \mathbf{Q}_{K+l} -weighted Frobenius norm on both sides of the last equation in (3.27), we obtain

$$\begin{aligned}
\|\mathbf{C} + \mathbf{Z}_{K+l}\|_{\mathbf{Q}_{K+l}} &= \tau_{K+l}^* \frac{\sqrt{\text{vec}(\bar{\Delta}_{K+l})^H \mathbf{Q}_{K+l}^{-1} \text{vec}(\bar{\Delta}_{K+l})}}{\|\bar{\Delta}_{K+l}\|_{\mathbf{Q}_{K+l}^{-1}}} \\
&= \tau_{K+l}^*. \tag{3.28}
\end{aligned}$$

Next, substituting $\mathbf{C} + \mathbf{Z}_{K+l}$ from (3.27) in the first term of (3.26), the Lagrange dual function (3.25) becomes

$$\begin{aligned}
g_p(\tau_{K+l}, \mathbf{Z}_{K+l}) &= -\tau_{K+l} \frac{\text{Tr}\{\bar{\Delta}_{K+l} \text{vec}^{-1}(\mathbf{Q}_{K+l}^{-1} \text{vec}(\bar{\Delta}_{K+l}))\}}{\|\bar{\Delta}_{K+l}\|_{\mathbf{Q}_{K+l}^{-1}}} - \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_{K+l}}(\mathbf{C} + \mathbf{Z}_{K+l})\} \\
&\quad + \tau_{K+l} \|\bar{\Delta}_{K+l}\|_{\mathbf{Q}_{K+l}^{-1}} - \tau_{K+l} \alpha_{K+l} \\
&= -\tau_{K+l} \|\bar{\Delta}_{K+l}\|_{\mathbf{Q}_{K+l}^{-1}} - \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_{K+l}}(\mathbf{C} + \mathbf{Z}_{K+l})\} + \tau_{K+l} \|\bar{\Delta}_{K+l}\|_{\mathbf{Q}_{K+l}^{-1}} - \tau_{K+l} \alpha_{K+l} \\
&= -\text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_{K+l}}(\mathbf{C} + \mathbf{Z}_{K+l})\} - \tau_{K+l} \alpha_{K+l}. \tag{3.29}
\end{aligned}$$

Inserting the optimal Lagrange multiplier τ_{K+l}^* from (3.28) in (3.29), we have

$$g_p(\mathbf{Z}_{K+l}) = -\text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_{K+l}}(\mathbf{C} + \mathbf{Z}_{K+l})\} - \alpha_{K+l} \|\mathbf{C} + \mathbf{Z}_{K+l}\|_{\mathbf{Q}_{K+l}} \tag{3.30}$$

and correspondingly the Lagrange dual to the problem (3.24) can be written as

$$\begin{aligned}
&\max_{\mathbf{Z}_{K+l}} -\alpha_{K+l} \|\mathbf{C} + \mathbf{Z}_{K+l}\|_{\mathbf{Q}_{K+l}} - \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_{K+l}}(\mathbf{C} + \mathbf{Z}_{K+l})\} \\
&\text{s.t. } \mathbf{Z}_{K+l} \succeq 0. \tag{3.31}
\end{aligned}$$

We have already mentioned the convexity of the problem (3.24) in variable $\bar{\Delta}_{K+l}$ and it is clear that the problem (3.24) is bounded below. Furthermore, the error matrices $\bar{\Delta}_{K+l} = -\tilde{\alpha}_{K+l}\mathbf{I}$, with $0 < \tilde{\alpha}_{K+l} < \alpha_{K+l}/\sqrt{N}$, represent strictly feasible points of problem (3.24). We can, therefore, use [104, Th. 1.7.1] to conclude that strong duality holds between (3.24) and (3.31). We can define the eigendecomposition of $\mathbf{C} + \mathbf{Z}_{K+l}$ similar to $\mathbf{A}_k + \mathbf{Z}_k$, as given in Appendix A. Using the fact that $\mathbf{C} \succeq 0$ and $\mathbf{Z}_{K+l} \succeq 0$, all the eigenvalues in this case are non-negative. Then using similar arguments as in Appendix A, it can be shown that $\mathbf{Z}_{K+l} = \mathbf{0}$ solves (3.31). Thus, the PU interference constraints (3.23) can be equivalently replaced with

$$\text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_{K+l}}\mathbf{C}\} + \alpha_{K+l}\|\mathbf{C}\|_{\mathbf{Q}_{K+l}} \leq \varepsilon_l. \quad (3.32)$$

Replacing the QoS and PU interference constraints in (3.4) with equivalent reformulations given in (3.19) and (3.32), respectively, problem (3.4) can be modified as

$$\begin{aligned} & \min_{\{\mathbf{W}_i, \mathbf{Z}_i\}} \sum_{i=1}^K \text{Tr}\{\mathbf{W}_i\} \\ \text{s.t.} \quad & -\alpha_k\|\mathbf{Z}_k - \mathbf{A}_k\|_{\mathbf{Q}_k} - \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}(\mathbf{Z}_k - \mathbf{A}_k)\} - \sigma_k^2\gamma_k \geq 0 \\ & \mathbf{Z}_k \succeq 0, \mathbf{W}_k \succeq 0, \text{rank}(\mathbf{W}_k) = 1; k = 1, \dots, K \\ & \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_{K+l}}\mathbf{C}\} + \alpha_{K+l}\|\mathbf{C}\|_{\mathbf{Q}_{K+l}} \leq \varepsilon_l; l = 1, \dots, L \end{aligned} \quad (3.33)$$

where we have also used the definition in (2.5). Similar to the problem (3.20), it can be proven that $\mathbf{Z}_k^* = \mathbf{0}$ at the optimum of (3.33). Finally, we drop the non-convex rank-one constraints, following the SDR approach, resulting in

$$\begin{aligned} & \min_{\{\mathbf{W}_i\}} \sum_{i=1}^K \text{Tr}\{\mathbf{W}_i\} \\ \text{s.t.} \quad & -\alpha_k\|\mathbf{A}_k\|_{\mathbf{Q}_k} + \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}\mathbf{A}_k\} - \sigma_k^2\gamma_k \geq 0 \\ & \mathbf{W}_k \succeq 0; k = 1, \dots, K \\ & \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_{K+l}}\mathbf{C}\} + \alpha_{K+l}\|\mathbf{C}\|_{\mathbf{Q}_{K+l}} \leq \varepsilon_l; l = 1, \dots, L. \end{aligned} \quad (3.34)$$

The final problem formulation in (3.34) represents a convex SDP problem that can be solved

in polynomial time using interior-point algorithms [91, 92] (see Section 3.6 for obtaining a rank-one solution from the solution of (3.34)).

We remark that our analysis in this section and in Appendix A reveals that for the conventional and the cognitive robust downlink problem at optimum the Lagrange multipliers \mathbf{Z}_k and \mathbf{Z}_{K+l} corresponding to the positive definiteness constraints in (3.10) and (3.24), respectively, are zero. This means that at optimum the positive definiteness constraints are inactive. In this sense our analysis rigorously proves the non-trivial result that the positive definiteness constraints can be neglected in the original problem (2.12) without introducing any conservative approximation.

Comparing the QoS constraints of the non-robust problem (2.11) with the robust problem (3.34) derived above, we observe that the robust formulation includes the additional term $-\alpha_k \|\mathbf{A}_k\|_{\mathbf{Q}_k}$. Similarly, an extra term $\alpha_{K+l} \|\mathbf{C}\|_{\mathbf{Q}_{K+l}}$ appears in the PU interference constraints in the robust case. These terms strengthen the constraints and can be viewed as a penalty for achieving the robustness against CSI errors. Further, it can be observed that with the choice $\mathbf{Q}_m = \mathbf{R}_{\Delta_m} / \|\mathbf{R}_{\Delta_m}\|$ ($m = 1, \dots, K + L$), the weighting matrices \mathbf{Q}_m have larger values in the components where the variance of the channel estimation error is large and smaller values for the components with smaller variance. Correspondingly, the penalty for robustness is also higher or smaller depending on these components.

3.5 Downlink Beamforming Based on the Probabilistic Approach and its Relation to the Worst-case Based Beamforming

In this section, we derive the robust downlink beamformers using a probabilistic model. The beamformers are designed such that the outage probability of QoS and PU interference constraints is below a certain given probability. We relate the results obtained in this section to the ones obtained using the worst-case approach in previous section. Replacing the QoS and PU interference constraints in (2.12) with corresponding probabilistic constraints, the

robust beamforming problem can then be formulated as

$$\begin{aligned}
& \min_{\{\mathbf{W}_i\}} \sum_{i=1}^K \text{Tr}\{\mathbf{W}_i\} \\
& \text{s.t. } \Pr(\text{SINR}_k \geq \gamma_k) \geq \pi_k; \quad k = 1, \dots, K \\
& \quad \Pr(\mathcal{I}_l \leq \varepsilon_l) \geq \pi_{K+l}; \quad l = 1, \dots, L \\
& \quad \text{rank}(\mathbf{W}_k) = 1, \mathbf{W}_k \succeq 0
\end{aligned} \tag{3.35}$$

where

$$\Pr(\text{SINR}_k \geq \gamma_k) = \Pr\left(\text{Tr}\{(\hat{\mathbf{R}}_{\mathbf{h}_k} - \mathbf{\Delta}_k)\mathbf{A}_k\} \geq \sigma_k^2 \gamma_k\right) \tag{3.36}$$

$$\Pr(\mathcal{I}_l \leq \varepsilon_l) = \Pr\left(\text{Tr}\{(\hat{\mathbf{R}}_{\mathbf{h}_{K+l}} - \mathbf{\Delta}_{K+l})\mathbf{C}\} \leq \varepsilon_l\right), \tag{3.37}$$

$\Pr(\cdot)$ denotes probability taken with respect to $\mathbf{\Delta}_m$, ($m = 1, \dots, K + L$) and π_k and π_{K+l} represent the desired non-outage probabilities for QoS and PU interference constraints, respectively.

The real-valued diagonal and complex-valued upper or lower triangle elements of $\mathbf{\Delta}_m$ are assumed to be zero-mean random variables and the covariance matrices of $\mathbf{\Delta}_m$ are as defined in (3.1). Let us define new real-valued random variables

$$y_k \triangleq \text{Tr}\{(\hat{\mathbf{R}}_{\mathbf{h}_k} - \mathbf{\Delta}_k)\mathbf{A}_k\} \tag{3.38}$$

$$y_{K+l} \triangleq \text{Tr}\{(\hat{\mathbf{R}}_{\mathbf{h}_{K+l}} - \mathbf{\Delta}_{K+l})\mathbf{C}\}. \tag{3.39}$$

The means of y_k and y_{K+l} are given, respectively, by

$$\mathbb{E}\{y_k\} = \mathbb{E}\{\text{Tr}\{(\hat{\mathbf{R}}_{\mathbf{h}_k} - \mathbf{\Delta}_k)\mathbf{A}_k\}\} = \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}\mathbf{A}_k\} \tag{3.40}$$

$$\mathbb{E}\{y_{K+l}\} = \mathbb{E}\{\text{Tr}\{(\hat{\mathbf{R}}_{\mathbf{h}_{K+l}} - \mathbf{\Delta}_{K+l})\mathbf{C}\}\} = \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_{K+l}}\mathbf{C}\}. \tag{3.41}$$

The variance of y_k can be computed as

$$\begin{aligned}
& \mathbb{E}\{\text{Tr}\{\mathbf{\Delta}_k\mathbf{A}_k\}\text{Tr}\{\mathbf{\Delta}_k\mathbf{A}_k\}^*\} \\
& = \mathbb{E}\{\text{vec}(\mathbf{A}_k^H)^H \text{vec}(\mathbf{\Delta}_k) \text{vec}(\mathbf{\Delta}_k)^H \text{vec}(\mathbf{A}_k^H)\} \\
& = \text{vec}(\mathbf{A}_k^H)^H \mathbb{E}\{\text{vec}(\mathbf{\Delta}_k) \text{vec}(\mathbf{\Delta}_k)^H\} \text{vec}(\mathbf{A}_k^H) \\
& = \psi_k \text{vec}(\mathbf{A}_k^H)^H \mathbf{Q}_k \text{vec}(\mathbf{A}_k^H) \\
& = \psi_k \|\mathbf{A}_k\|_{\mathbf{Q}_k}^2
\end{aligned} \tag{3.42}$$

where ψ_k is as defined in (3.2). Similarly, the variance of y_{K+l} can be computed as

$$\mathbb{E}\{\text{Tr}\{\mathbf{\Delta}_{K+l}\mathbf{C}\}\text{Tr}\{\mathbf{\Delta}_{K+l}\mathbf{C}\}^*\} = \psi_{K+l}\|\mathbf{C}\|_{\mathbf{Q}_{K+l}}^2. \quad (3.43)$$

Assuming that the random variables y_k and y_{K+l} have probability density functions $f_{Y_k}(y)$ and $f_{Y_{K+l}}(y)$, respectively, (3.36) and (3.37) can be written, respectively, as

$$\begin{aligned} & \Pr(\text{SINR}_k \geq \gamma_k) \\ &= \int_{\sigma_k^2 \gamma_k}^{\infty} f_{Y_k}(y) dy \\ &= \sqrt{2\psi_k} \|\mathbf{A}_k\|_{\mathbf{Q}_k} \int_{d_k}^{\infty} f_{Y_k} \left(\sqrt{2\psi_k} \|\mathbf{A}_k\|_{\mathbf{Q}_k} y + \text{Tr}\{\hat{\mathbf{R}}_{h_k} \mathbf{A}_k\} \right) dy \\ &\triangleq G_k(d_k) \end{aligned} \quad (3.44)$$

and

$$\begin{aligned} & \Pr(\mathcal{I}_l \leq \varepsilon_l) \\ &= \int_{-\infty}^{\varepsilon_l} f_{Y_{K+l}}(y) dy \\ &= \sqrt{2\psi_{K+l}} \|\mathbf{C}\|_{\mathbf{Q}_{K+l}} \int_{-\infty}^{d_{K+l}} f_{Y_{K+l}} \left(\sqrt{2\psi_{K+l}} \|\mathbf{C}\|_{\mathbf{Q}_{K+l}} y + \text{Tr}\{\hat{\mathbf{R}}_{h_{K+l}} \mathbf{C}\} \right) dy \\ &\triangleq G_{K+l}(d_{K+l}) \end{aligned} \quad (3.45)$$

where

$$d_k \triangleq \frac{\sigma_k^2 \gamma_k - \text{Tr}\{\hat{\mathbf{R}}_{h_k} \mathbf{A}_k\}}{\sqrt{2\psi_k} \|\mathbf{A}_k\|_{\mathbf{Q}_k}}; \quad d_{K+l} \triangleq \frac{\varepsilon_l - \text{Tr}\{\hat{\mathbf{R}}_{h_{K+l}} \mathbf{C}\}}{\sqrt{2\psi_{K+l}} \|\mathbf{C}\|_{\mathbf{Q}_{K+l}}}. \quad (3.46)$$

Using (3.44) and (3.45), the probabilistic constraints in (3.35) can be modified as

$$d_k \geq G_k^{-1}(\pi_k) \quad (3.47)$$

$$d_{K+l} \geq G_{K+l}^{-1}(\pi_{K+l}). \quad (3.48)$$

Let us next analyze in detail the special case when the real-valued diagonal and complex-valued upper or lower triangle elements of $\mathbf{\Delta}_m$, ($m = 1, \dots, K + L$) are zero-mean jointly Gaussian random variables. Then the functions G_k and G_{K+l} can be described, respectively,

in terms of the error function as

$$G_k(d_k) = \begin{cases} \frac{1}{2} + \frac{1}{2} \operatorname{erf} \left(\frac{\operatorname{Tr}\{\hat{\mathbf{R}}_{h_k} \mathbf{A}_k\} - \sigma_k^2 \gamma_k}{\sqrt{2\psi_k} \|\mathbf{A}_k\|_{\mathbf{Q}_k}} \right), & \text{if } \sigma_k^2 \gamma_k \leq \operatorname{Tr}\{\hat{\mathbf{R}}_{h_k} \mathbf{A}_k\} \\ \frac{1}{2} - \frac{1}{2} \operatorname{erf} \left(\frac{\sigma_k^2 \gamma_k - \operatorname{Tr}\{\hat{\mathbf{R}}_{h_k} \mathbf{A}_k\}}{\sqrt{2\psi_k} \|\mathbf{A}_k\|_{\mathbf{Q}_k}} \right), & \text{if } \sigma_k^2 \gamma_k \geq \operatorname{Tr}\{\hat{\mathbf{R}}_{h_k} \mathbf{A}_k\} \end{cases} \quad (3.49)$$

$$G_{K+l}(d_{K+l}) = \begin{cases} \frac{1}{2} - \frac{1}{2} \operatorname{erf} \left(\frac{\operatorname{Tr}\{\hat{\mathbf{R}}_{h_{K+l}} \mathbf{C}\} - \varepsilon_l}{\sqrt{2\psi_{K+l}} \|\mathbf{C}\|_{\mathbf{Q}_{K+l}}} \right), & \text{if } \varepsilon_l \leq \operatorname{Tr}\{\hat{\mathbf{R}}_{h_{K+l}} \mathbf{C}\} \\ \frac{1}{2} + \frac{1}{2} \operatorname{erf} \left(\frac{\varepsilon_l - \operatorname{Tr}\{\hat{\mathbf{R}}_{h_{K+l}} \mathbf{C}\}}{\sqrt{2\psi_{K+l}} \|\mathbf{C}\|_{\mathbf{Q}_{K+l}}} \right), & \text{if } \varepsilon_l \geq \operatorname{Tr}\{\hat{\mathbf{R}}_{h_{K+l}} \mathbf{C}\}. \end{cases} \quad (3.50)$$

Using (3.49) and (3.50) together with the fact that $\operatorname{erf}(-x) = -\operatorname{erf}(x)$, the QoS and PU interference constraints in (3.35) can be rewritten as

$$\operatorname{erf} \left(\frac{\operatorname{Tr}\{\hat{\mathbf{R}}_{h_k} \mathbf{A}_k\} - \sigma_k^2 \gamma_k}{\sqrt{2\psi_k} \|\mathbf{A}_k\|_{\mathbf{Q}_k}} \right) \geq 2\pi_k - 1 \quad (3.51)$$

$$\operatorname{erf} \left(\frac{\varepsilon_l - \operatorname{Tr}\{\hat{\mathbf{R}}_{h_{K+l}} \mathbf{C}\}}{\sqrt{2\psi_{K+l}} \|\mathbf{C}\|_{\mathbf{Q}_{K+l}}} \right) \geq 2\pi_{K+l} - 1. \quad (3.52)$$

After simple manipulations, we can modify (3.51) and (3.52) as

$$-\rho_k \|\mathbf{A}_k\|_{\mathbf{Q}_k} + \operatorname{Tr}\{\hat{\mathbf{R}}_{h_k} \mathbf{A}_k\} - \sigma_k^2 \gamma_k \geq 0 \quad (3.53)$$

$$\operatorname{Tr}\{\hat{\mathbf{R}}_{h_{K+l}} \mathbf{C}\} + \rho_{K+l} \|\mathbf{C}\|_{\mathbf{Q}_{K+l}} \leq \varepsilon_l \quad (3.54)$$

where $\rho_k \triangleq \sqrt{2\psi_k} \operatorname{erf}^{-1}(2\pi_k - 1)$, ($k = 1, \dots, K$) and $\rho_{K+l} \triangleq \sqrt{2\psi_{K+l}} \operatorname{erf}^{-1}(2\pi_{K+l} - 1)$, ($l = 1, \dots, L$).

Using (3.53) and (3.54), and applying the SDR approach, by removing the non-convex rank-one constraints, (3.35) can be rewritten as

$$\begin{aligned} & \min_{\{\mathbf{W}_i\}} \sum_{i=1}^K \operatorname{Tr}\{\mathbf{W}_i\} \\ & \text{s.t. } -\rho_k \|\mathbf{A}_k\|_{\mathbf{Q}_k} + \operatorname{Tr}\{\hat{\mathbf{R}}_{h_k} \mathbf{A}_k\} - \sigma_k^2 \gamma_k \geq 0 \\ & \quad \operatorname{Tr}\{\hat{\mathbf{R}}_{h_{K+l}} \mathbf{C}\} + \rho_{K+l} \|\mathbf{C}\|_{\mathbf{Q}_{K+l}} \leq \varepsilon_l; \quad l = 1, \dots, L \\ & \quad \mathbf{W}_k \succeq 0; \quad k = 1, \dots, K. \end{aligned} \quad (3.55)$$

We observe that the final formulation of the outage probability based problem in (3.55) is mathematically equivalent to the worst-case based formulation (3.34) for proper choices of α_k and α_{K+l} . The norm-bound coefficients α_k and α_{K+l} in (3.34), are replaced by new constants ρ_k and ρ_{K+l} in (3.55). The value of these new constants ρ_k and ρ_{K+l} depends on the outage probability chosen in the design.

3.6 Rank Properties of the Solutions Obtained from the Resulting SDPs

In general, the solutions obtained from the SDR-based problems do not have rank-one. In this case, when a higher rank solution is obtained, a typical approach is to use the so-called *randomization techniques* [101] to obtain an approximate rank-one solution of the original problem from the higher rank solution of the relaxed problem. Unfortunately, it is not possible to apply the standard randomization techniques in the case of problems (3.21), (3.34), and (3.55). The standard procedure for randomization involves the scaling of the candidate vectors such that the constraints are met. Simple scaling, however, in the case of problems (3.21), (3.34), and (3.55), generally results in the violation of the QoS and PU interference constraints. Fortunately, our simulations show that, generally, rank-one solutions for \mathbf{W}_k are obtained for the above mentioned problems. As a result, the weight vectors \mathbf{w}_k can be retrieved exactly from the principal eigenvectors of \mathbf{W}_k . There exist, however, certain specific scenarios in which higher rank solutions are obtained. This happens in the case of highly symmetric channels or very large uncertainties in the CSI. Whenever a higher rank solution is obtained, we approximate the corresponding rank-one solution using the principal eigenvectors of \mathbf{W}_k . The resulting solution, however, will not guarantee that all the constraints are satisfied. Later in this chapter, we derive efficient iterative robust techniques that guarantee rank-one solutions (when convergent).

In order to illustrate the conditions under which such higher rank solutions may be obtained, we discuss a simple example in the following. We consider the case with $N = 3$

transmit antennas, $K = 2$ users, and no PUs, i.e., $L = 0$. The channel covariance matrices of the users are assumed to be

$$\hat{\mathbf{R}}_1 = \begin{pmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{\mathbf{R}}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & r_3 \end{pmatrix}. \quad (3.56)$$

It is shown in the Appendix B that in this case the solution matrices \mathbf{W}_k have the diagonal structure, given as

$$\mathbf{W}_1 = \begin{pmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{pmatrix}, \quad \mathbf{W}_2 = \begin{pmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{pmatrix}, \quad (3.57)$$

and either a_{11} or a_{22} along with b_{33} must be non-zero for the QoS constraints to be satisfied. When either of a_{33} , b_{11} and b_{22} are non-zero, the solution (3.57) is clearly of higher rank. Further, it is shown in Appendix B that if a_{33} , b_{11} and b_{22} are all zero, which, e.g., is the case for $\gamma_1\gamma_2 \leq 1$, then the solution has the form

$$\mathbf{W}_1^* = \begin{pmatrix} \mu^*c & 0 & 0 \\ 0 & (1 - \mu^*)c & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{W}_2^* = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b_{33} \end{pmatrix} \quad (3.58)$$

where the values of b_{33} and c depend on the parameters of the problem (3.34) and $0 \leq \mu^* \leq 1$. In Appendix B, we also show that larger values of the uncertainty thresholds α_1 and α_2 favor a higher rank solution, while smaller values result in a rank-one solution. This observation is intuitively clear as, with the increase in channel uncertainty thresholds α_k , the norm term $-\alpha_k \|\mathbf{A}_k\|_{\mathbf{Q}_k}$, with \mathbf{A}_k defined in (3.8), becomes dominant in the QoS constraints in (3.34) and this term favors the higher rank solutions.

In Appendix C, we show that the simple class of diagonal channel covariances in (3.56), for which higher rank solutions are obtained, can be extended to more general, however still highly symmetric, channel cases.

3.7 Iterative Robust Downlink Beamforming

In this section we propose an iterative approach to solve the robust downlink beamforming problem (3.4). In the proposed approach, we combine the benefits of the methods presented in Section 3.4 and the non-robust iterative scheme developed in [77]. The authors of [77] have proved that uplink-downlink duality for non-robust CR beamforming problem (2.10) exists. Based on this duality, they have proposed an iterative algorithm to obtain the optimal beamformers and power allocations for the CR downlink beamforming problem (2.10).

3.7.1 Iterative Non-Robust Downlink Beamforming

In [77], the beamforming weight vector of the i th SU is defined using the definition given in (2.9) as $\mathbf{w}_i \triangleq \sqrt{p_i} \mathbf{u}_i$ where p_i denotes the transmitted power allocated to i th SU and \mathbf{u}_i is the $N \times 1$ normalized beamforming vector of the i th SU. The non-robust beamforming problem (2.10) is then written in terms of \mathbf{u}_i as

$$\begin{aligned}
 & \min_{\{\mathbf{u}_i, p_i\}} \sum_{i=1}^K p_i \\
 \text{s.t. } & \frac{p_k \mathbf{u}_k^H \mathbf{R}_{h_k} \mathbf{u}_k}{\sum_{\substack{i=1 \\ i \neq k}}^K p_i \mathbf{u}_i^H \mathbf{R}_{h_k} \mathbf{u}_i + \sigma_k^2} \geq \gamma_k; \quad k = 1, \dots, K \\
 & \sum_{i=1}^K p_i \mathbf{u}_i^H \mathbf{R}_{h_{K+l}} \mathbf{u}_i \leq \frac{1}{\gamma_{K+l}}; \quad l = 1, \dots, L
 \end{aligned} \tag{3.59}$$

where $\gamma_{K+l} \triangleq 1/\epsilon_l$.

In [77], a dual virtual uplink problem corresponding to the problem (3.59) is derived as

$$\begin{aligned}
 & \min_{\{\mathbf{u}_i, q_i\}} \sum_{i=1}^K \eta_i q_i - \sum_{l=1}^L p_{K+l} q_{K+l} \\
 \text{s.t. } & \frac{q_k \mathbf{u}_k^H \mathbf{R}_{h_k} \mathbf{u}_k}{\mathbf{u}_k^H \left(\sum_{\substack{i=1 \\ i \neq k}}^{K+L} q_i \gamma_i \mathbf{R}_{h_i} + \mathbf{I} \right) \mathbf{u}_k} \geq 1; \quad k = 1, \dots, K \\
 & q_k \geq 0; \quad q_{K+l} \geq 0; \quad p_{K+l} \leq 1; \quad \|\mathbf{u}_k\| = 1; \quad l = 1, \dots, L
 \end{aligned} \tag{3.60}$$

where q_1, \dots, q_K and q_{K+1}, \dots, q_{K+L} are the virtual uplink powers corresponding to the SUs and PUs, respectively, and $\eta_k \triangleq \sigma_k^2 \gamma_k$. It is shown in [77] that problem (3.60) yields optimal beamformers that are identical to those of problem (3.59). The authors then propose an iterative algorithm, based on the uplink-downlink duality, to obtain the solution of problem (3.59). Note that a similar iterative approach has been presented in [28, 50, 54] for the conventional beamforming problem (2.4). In contrast to the techniques of [79] and [59], these iterative approaches do not require any use of convex optimization tools, and, therefore, are simpler to implement.

3.7.2 Proposed Iterative Robust Downlink Beamforming

Let us first rewrite the robust problem (3.34) derived in Section 3.4 in an equivalent form as

$$\begin{aligned} & \min_{\{\mathbf{u}_i, p_i\}} \sum_{i=1}^K p_i \\ \text{s.t. } & \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k} \mathbf{A}_k\} \geq \eta_k + \alpha_k \|\mathbf{A}_k\|_{\mathbf{Q}_k}; \quad k = 1, \dots, K \\ & -\gamma_{K+l} \text{Tr}\{\mathbf{C} \hat{\mathbf{R}}_{\mathbf{h}_{K+l}}\} \geq \gamma_{K+l} \alpha_{K+l} \|\mathbf{C}\|_{\mathbf{Q}_{K+l}} - 1; \quad l = 1, \dots, L \end{aligned} \quad (3.61)$$

where we have used (2.9) along with the fact that $\mathbf{W}_k = p_k \mathbf{u}_k \mathbf{u}_k^H$. Correspondingly, the matrices \mathbf{A}_k and \mathbf{C} can also be rewritten as

$$\mathbf{A}_k = p_k \mathbf{u}_k \mathbf{u}_k^H - \gamma_k \sum_{\substack{i=1 \\ i \neq k}}^K p_i \mathbf{u}_i \mathbf{u}_i^H; \quad \mathbf{C} = \sum_{k=1}^K p_k \mathbf{u}_k \mathbf{u}_k^H. \quad (3.62)$$

As mentioned earlier, the difference between the robust problem (3.61) and the non-robust problem (2.10) is that the penalty terms $\|\mathbf{A}_k\|_{\mathbf{Q}_k}$ and $\gamma_{K+l} \alpha_{K+l} \|\mathbf{C}\|_{\mathbf{Q}_{K+l}}$ are added to the QoS and PU interference constraints, respectively, in the robust case. In the following, we consider an iterative algorithm, where in each iteration the penalty terms $\|\mathbf{A}_k\|_{\mathbf{Q}_k}$ and $\|\mathbf{C}\|_{\mathbf{Q}_{K+l}}$ are assumed to be known from the previous iteration. Let $\mathbf{A}_k(t)$ and $\mathbf{C}(t)$ denote the matrices, given in (3.62), obtained in the t th iteration. Then, in each iteration we have

the modified problem

$$\begin{aligned}
& \min_{\{\mathbf{u}_i(t), p_i(t)\}} \sum_{i=1}^K p_i(t) \\
\text{s.t. } & \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k} \mathbf{A}_k(t)\} \geq \hat{\eta}_k(t-1); \quad k = 1, \dots, K \\
& -\gamma_{K+l} \text{Tr}\{\mathbf{C} \hat{\mathbf{R}}_{\mathbf{h}_{K+l}}\} \geq -\hat{\eta}_{K+l}(t-1); \quad l = 1, \dots, L
\end{aligned} \tag{3.63}$$

where

$$\hat{\eta}_k(t) \triangleq \eta_k + \alpha_k \|\mathbf{A}_k(t)\|_{\mathbf{Q}_k} \tag{3.64}$$

and

$$\hat{\eta}_{K+l}(t) \triangleq 1 - \gamma_{K+l} \alpha_{K+l} \|\mathbf{C}(t)\|_{\mathbf{Q}_{K+l}}. \tag{3.65}$$

From (3.63), we observe that when the penalty terms in the t th iteration are fixed at $\|\mathbf{A}_k(t-1)\|_{\mathbf{Q}_k}$ and $\|\mathbf{C}(t-1)\|_{\mathbf{Q}_{K+l}}$, the robustness to the channel mismatch in the CR beamforming design is achieved by increasing the targets $\hat{\eta}_m(t)$ ($m = 1, \dots, K+L$) according to (3.64) and (3.65). Note that for fixed $\|\mathbf{A}_k(t-1)\|_{\mathbf{Q}_k}$ and $\|\mathbf{C}(t-1)\|_{\mathbf{Q}_{K+l}}$, problem (3.63) is mathematically equivalent to non-robust problem (2.10). The resulting iterative algorithm is described in Table 3.1.

Although, the algorithm given in Table 3.1 leads to a non-conservative rank-one solution of (3.4), it has a relatively high complexity as it requires to solve an SDP problem in each iteration. Simulation results show, that the algorithm generally converges after 4 to 6 iterations, but the number of iterations can go to over 100 for higher values of the SINR required by the SUs. In order to deal with the issue of complexity, we propose to use the iterative fixed-point algorithm given in [77] for solving the Step 1, instead of an SDP solver. In this case, Step 1 will consist of an inner loop to find the beamformers. We further propose to modify it by removing the outer iteration loop and updating the terms $\hat{\eta}_k(t)$ and $\hat{\eta}_{K+l}(t)$ in each iteration.

In the following we present the modified algorithm to solve the robust CR downlink beamforming problem (3.4). First, let us define $\mathbf{q} \triangleq [q_1, \dots, q_{K+L}]^T$ and $\mathbf{p} \triangleq [\mathbf{p}_1^T, \mathbf{p}_2^T]^T$ where $\mathbf{p}_1 \triangleq [p_1, \dots, p_K]^T$ and $\mathbf{p}_2 \triangleq [p_{K+1}, \dots, p_{K+L}]^T$ are the vectors containing the downlink powers and the virtual downlink powers of the K SUs and the L PUs, respectively. We

Initialization: Initialize $\mathbf{A}_k(0) = 0$ and $\hat{\eta}_k(0) = \eta_k$, for $k = 1, \dots, K$, $\mathbf{C}(0) = 0$, and $\hat{\eta}_{K+l}(0) = 1$ ($l = 1, \dots, L$).

Iterative loop: For $t = 1, 2, \dots$ until convergence, iterate the following steps:

- 1) **Beamformer computation:** Solve the problem (3.63) using some SDP solver to compute $p_k(t)$ and $\mathbf{u}_k(t)$ for $k = 1, \dots, K$.
- 2) **Targets update:** Update $\hat{\eta}_k(t)$ using (3.64) and $\hat{\eta}_{K+l}(t)$ as

$$\hat{\eta}_{K+l}(t) = \begin{cases} 1 - \gamma_{K+l} \alpha_{K+l} \|\mathbf{C}(t)\|_{\mathbf{Q}_{K+l}}, & \text{if } 1 - \gamma_{K+l} \alpha_{K+l} \|\mathbf{C}(t)\|_{\mathbf{Q}_{K+l}} > 0 \\ 1, & \text{otherwise.} \end{cases} \quad (3.66)$$

for $l = 1, \dots, L$.

- 3) **Convergence check:** For convergence, check the following condition:

$$\left| \sum_{i=1}^K p_i(t) - \sum_{i=1}^K p_i(t-1) \right| \leq \delta \quad (3.67)$$

where δ is a small positive constant.

Table 3.1: Iterative robust CR beamforming algorithm using SDP solver.

also introduce

$$[\mathbf{D}(t)]_{k,k} \triangleq \mathbf{u}_k(t)^H \hat{\mathbf{R}}_{\mathbf{h}_k} \mathbf{u}_k(t) \quad (3.75)$$

$$[\mathbf{G}_2(t)]_{l,j} \triangleq \frac{\gamma_{K+l}}{\hat{\eta}_{K+l}(t)} \mathbf{u}_j(t)^H \hat{\mathbf{R}}_{\mathbf{h}_{K+l}} \mathbf{u}_j(t) \quad (3.76)$$

$$[\mathbf{G}_1(t)]_{i,j} \triangleq \begin{cases} 0, & i = j \\ \gamma_i \mathbf{u}_j(t)^H \hat{\mathbf{R}}_{\mathbf{h}_i} \mathbf{u}_j(t), & i \neq j \end{cases} \quad (3.77)$$

$$\boldsymbol{\eta}(t) \triangleq [\hat{\eta}_1(t), \dots, \hat{\eta}_K(t)]^T \quad (3.78)$$

for all $l = 1, \dots, L$ and $k, i, j = 1, \dots, K$. The algorithm can then be described as given in Table 3.2. We remark that if we remove the Step 5, i.e. keeping $\hat{\eta}_k(t) = \eta_k(t)$ ($k = 1, \dots, K$) and $\hat{\eta}_{K+l}(t) = 1$ ($l = 1, \dots, L$), the algorithm reduces to the non-robust iterative algorithm of [77].

Initialization: Initialize $\mathbf{A}_k(0) = 0$, $\hat{\eta}_k(0) = \eta_k$, and $q_k(0) = 1$, for $k = 1, \dots, K$, $\hat{\eta}_{K+l}(0) = 1$ and $q_{K+l}(0) = 10^{-2}$ (can be chosen arbitrarily), for $l = 1, \dots, L$, and $\mathbf{C}(0) = 0$.

Iterative loop: For $t = 1, 2, \dots$ until convergence, iterate the following steps:

- 1) **Beamformer update:** Compute $\mathbf{u}_k(t)$, for $k = 1, \dots, K$, from the solution of the generalized eigenproblem as

$$\mathbf{u}_k(t) = \arg \max_{\|\mathbf{u}_k(t)\|=1} \frac{q_k(t-1)\mathbf{u}_k(t)^H \hat{\mathbf{R}}_{\mathbf{h}_k} \mathbf{u}_k(t)}{\mathbf{u}_k(t)^H (\mathbf{Q} + \mathbf{I}) \mathbf{u}_k(t)} \quad (3.68)$$

where

$$\mathbf{Q} \triangleq \sum_{i=1, i \neq k}^K q_i(t-1)\gamma_i \hat{\mathbf{R}}_{\mathbf{h}_i} + \sum_{l=1}^L \frac{q_{K+l}(t-1)\gamma_{K+l}}{\hat{\eta}_{K+l}(t-1)} \hat{\mathbf{R}}_{\mathbf{h}_{K+l}}. \quad (3.69)$$

- 2) **Transformation to the downlink domain:** For the beamforming vectors obtained in step 1 compute the equivalent virtual downlink powers $\mathbf{p}_1(t) = (\mathbf{D}(t) - \mathbf{G}_1(t))^{-1} \boldsymbol{\eta}$ and $\mathbf{p}_2(t) = \mathbf{G}_2(t)\mathbf{p}_1(t)$ with $\mathbf{D}(t)$, $\mathbf{G}_1(t)$, $\mathbf{G}_2(t)$ and $\boldsymbol{\eta}(t)$ defined in (3.75)-(3.78).

- 3) **Virtual uplink power allocation update (PUs):** Compute

$$q_{K+l}(t) = \begin{cases} p_{K+l}(t)q_{K+l}(t-1), & \text{if } \min\{p_l, \dots, p_K\} \geq 0 \\ q_{K+l}(t-1), & \text{otherwise} \end{cases} \quad (3.70)$$

for $l = 1, \dots, L$.

- 4) **Virtual uplink power allocation update (SUs):** Compute

$$q_k(t) = \frac{\mathbf{u}_k(t)^H (\hat{\mathbf{Q}} + \mathbf{I}) \mathbf{u}_k(t)}{\mathbf{u}_k(t)^H \hat{\mathbf{R}}_{\mathbf{h}_k} \mathbf{u}_k(t)} \quad (3.71)$$

where

$$\hat{\mathbf{Q}} \triangleq \sum_{i=1, i \neq k}^K q_i(t-1)\gamma_i \hat{\mathbf{R}}_{\mathbf{h}_i} + \sum_{l=1}^L \frac{q_{K+l}(t)\gamma_{K+l}}{\hat{\eta}_{K+l}(t-1)} \hat{\mathbf{R}}_{\mathbf{h}_{K+l}} \quad (3.72)$$

for $k = 1, \dots, K$.

- 5) **Targets update:** Compute $\hat{\eta}_k(t)$ and $\hat{\eta}_{K+l}(t)$ using (3.64) and (3.66), respectively.

- 6) **Convergence check:** For convergence, check the following conditions:

$$\|\mathbf{q}(t) - \mathbf{q}(t-1)\| \leq \delta_1; \quad p_{K+l}(t) - 1 \leq \delta_2 \quad (3.73)$$

$$\left| \sum_{i=1}^K p_i(t) - \sum_{i=1}^K p_i(t-1) \right| \leq \delta_3 \quad (3.74)$$

for $l = 1, \dots, L$ where δ_m ($m = 1, 2, 3$) are small positive constants.

Table 3.2: Iterative robust CR beamforming algorithm using uplink-downlink duality.

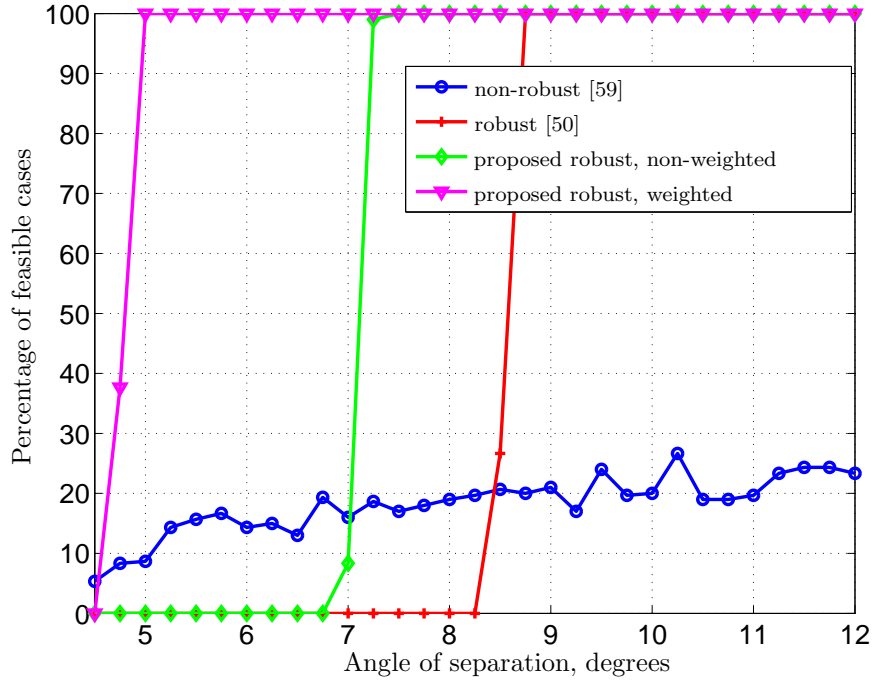
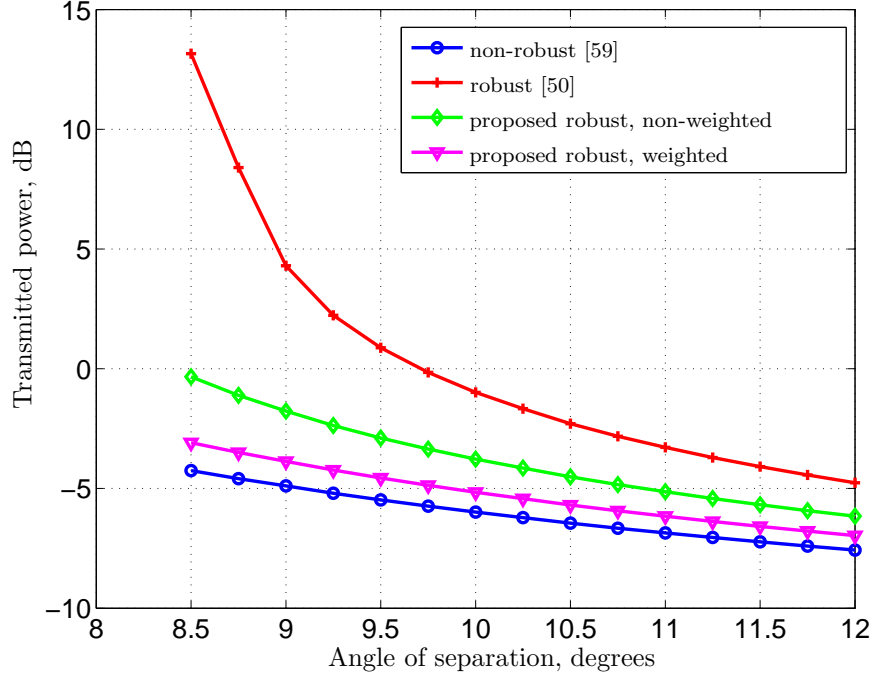


Figure 3.1: Feasibility percentage of the conventional beamforming schemes.

3.8 Simulations

In this section, we present simulation results for both the conventional and the cognitive beamforming approaches. First, in Figs. 3.1-3.7, we compare the performance of the proposed robust approach for the conventional downlink beamforming, presented in Section 3.4.1, to the non-robust and robust approaches of [59] and [50], respectively. In Figs. 3.8-3.11, we compare the proposed robust CR beamforming approaches, namely the SDR based technique from Section 3.4.2 and the iterative robust method presented in Table 3.2, with the non-robust and robust CR methods of [79].

For the conventional beamforming, we first consider the same scenario as in [59]. The base station is equipped with a uniform linear array of $N = 6$ sensors spaced half a wavelength apart and $K = 3$ single-antenna users are assumed. One of the users is located at $\theta_1 = 20^\circ$ relative to the array broadside, while the other two are located at $\theta_{2,3} = 20^\circ \pm \phi$, and ϕ is varied from 6° to 12° . The users are assumed to be surrounded by a large number

Figure 3.2: Total transmitted power versus angular separation ϕ .

of local scatterers corresponding to an angular spread of $\sigma_\theta = 5^\circ$, as seen from the base station. The channel covariance matrices \mathbf{R}_{h_k} , $k = 1, \dots, K$ are calculated in the same way as in [59]. To model the mismatch matrices Δ_k ($k = 1, \dots, K$), we considered the practically important case that the channel covariance matrices are estimated in the presence of colored noise, e.g., resulting from interference from neighboring base stations, such that the errors have a particular spatial signature. In this case the weighted Frobenius norm shall essentially whiten the estimation errors. In our simulations, we generate errors that are randomly distributed within ellipsoids with orientations and shapes corresponding to the eigenvectors and eigenvalues of interference-plus-noise covariance matrices from the neighboring cells, respectively. Let $\mathbf{R}_{k,I}$ denote the interference-plus-noise covariance matrix present during the estimation of \mathbf{R}_k with eigendecomposition $\mathbf{R}_{k,I} = \mathbf{U}_{k,I} \mathbf{\Gamma}_{k,I} \mathbf{U}_{k,I}^H$. Then we model the mismatch matrix as $\Delta_k = \mathbf{U}_{k,I} \mathbf{\Gamma}_{k,I}^{1/2} \mathbf{\Upsilon}_k \mathbf{\Gamma}_{k,I}^{1/2} \mathbf{U}_{k,I}^H$ where $\mathbf{\Upsilon}_k$ denotes a random matrix with $\|\mathbf{\Upsilon}_k\| \leq \alpha_k$ and $\mathbf{R}_k = \hat{\mathbf{R}}_k + \Delta_k \succeq 0$. Choosing the weighting matrix $\mathbf{Q}_k = \bar{\mathbf{R}}_{\Delta_k}^T \otimes \bar{\mathbf{R}}_{\Delta_k}$ with $\bar{\mathbf{R}}_{\Delta_k} = E[\Delta_k \Delta_k^H]$, where \otimes denotes the Kronecker matrix product, it can be shown that

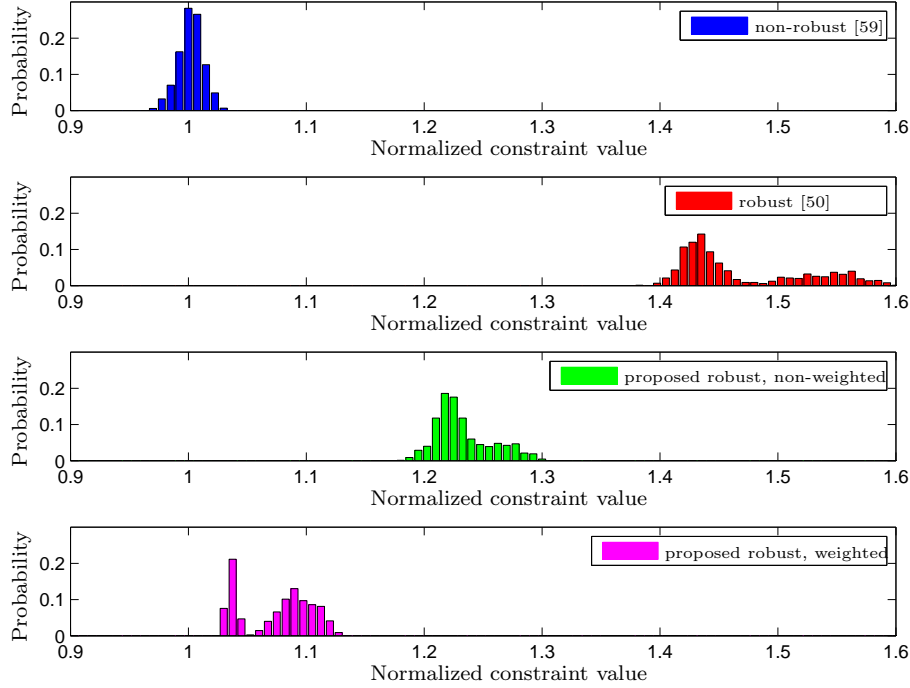


Figure 3.3: Histogram of normalized QoS constraints ($\phi = 9.5^\circ$).

$\|\Delta_k\|_{\mathbf{Q}_k^{-1}} = \sqrt{\text{Tr} \left\{ \Gamma_{k,I}^{-1/2} \mathbf{U}_{k,I}^H \Delta_k \bar{\mathbf{R}}_{\Delta_k}^{-1} \Delta_k \mathbf{U}_{k,I} \Gamma_{k,I}^{-1/2} \right\}} = \|\Upsilon_k\| \leq \alpha_k$, where we made use of the property $\text{vec}(\mathbf{A}\mathbf{X}\mathbf{B}) = (\mathbf{B}^T \otimes \mathbf{A})\text{vec}(\mathbf{X})$ for arbitrary matrices \mathbf{A} , \mathbf{B} , and \mathbf{X} of comfortable dimensions. For a fair performance comparison of the robust schemes with both weighted and non-weighted Frobenius norms, we chose for the non-weighted robust scheme a bound $\alpha_{k,n-w} = \sqrt{\lambda_{\Delta,max}}$, which follows from the observation that $\|\Delta_k\|_{\mathbf{Q}_k^{-1}} \leq \alpha_k$ implies $\|\Delta_k\|_{\mathbf{I}} \leq \sqrt{\lambda_{\Delta,max}} \alpha_k = \alpha_{k,n-w}$, where $\lambda_{\Delta,max}$ is the maximum eigenvalue of \mathbf{Q}_k . In our first simulation example the colored interference-plus-noise covariance matrices $\mathbf{R}_{k,I}$ are composed of an interference part according to the covariance model [59], with angles 50° , 22° and -10° , and an additive white noise term with variance 0.1.

We assume that $\gamma_k = \gamma$ and $\alpha_k = \alpha = 0.2$, for all $k = 1, \dots, K$. A total number of 500 Monte-Carlo runs has been used. In all simulations on conventional beamforming, the proposed robust technique is compared to the robust technique of [50]. As a benchmark we also display the results for the non-robust technique of [59].

In Fig. 3.1, we show the feasibility percentage, i.e., the percentage of channel realizations for which the different schemes under consideration yield feasible solutions. A beamforming solution is considered as feasible, if it satisfies all the constraints in (2.10) (for $L = 0$) for the true covariance matrices. The simulations are performed for $\gamma = 2\text{dB}$. We observe that the proposed robust approach with both weighted and non-weighted Frobenius norms outperforms the non-robust and robust approaches of [59] and [50], respectively, in terms of feasibility percentage. Note, that the feasibility of the non-robust approach is highly dependent on the structure of the error matrices $\mathbf{\Delta}_k$ and can change with the variations in the considered error distribution. Fig. 3.2 displays the transmitted power of all the techniques for the same scenario. For better comparison, here we consider only the cases for which the robust scheme of [50] yields feasible solutions. We remark that in all these instances the proposed approach is always feasible. Our robust approach is more power efficient compared to the robust approach of [50]. Recall that the proposed approach avoids the conservative approximations of [50] and, therefore, this result is in line with the expectations. Apparently, from Fig. 3.2, the non-robust approach of [59] appears to be more power efficient but, as can be observed from Fig. 3.1, it does not always provide a feasible solution.

For further comparison, we plot the histogram of the achieved normalized QoS in Fig. 3.3 for $\phi = 9.5^\circ$. We define the normalized QoS as

$$\zeta_k = \frac{\mathbf{w}_k^{*H} \mathbf{R}_{h_k} \mathbf{w}_k^*}{\sum_{\substack{i=1 \\ i \neq k}}^K \gamma_k \mathbf{w}_i^{*H} \mathbf{R}_{h_k} \mathbf{w}_i^* + \gamma_k \sigma_k^2}; \quad k = 1, \dots, K. \quad (3.79)$$

Due to the normalization (3.79), a value greater than one in Fig. 3.3 corresponds to satisfied QoS constraints. The proposed approach and the robust approach of [50] both satisfy all the constraints, as expected, but in the case of the non-robust approach of [59], a considerable number of constraints is not satisfied.

In order to further illustrate the benefits of using the weighted Frobenius norm, we show in Fig. 3.4 the transmitted power obtained for elliptic uncertainty sets with increasing eccentricity. We compare the performance of our weighted and non-weighted robust schemes. We fix the noise variance to 0.5 and vary the power of the interference part in the noise and interference matrix $\mathbf{R}_{k,I}$ from 0.01 to 5, in order to obtain error covariance matrices with

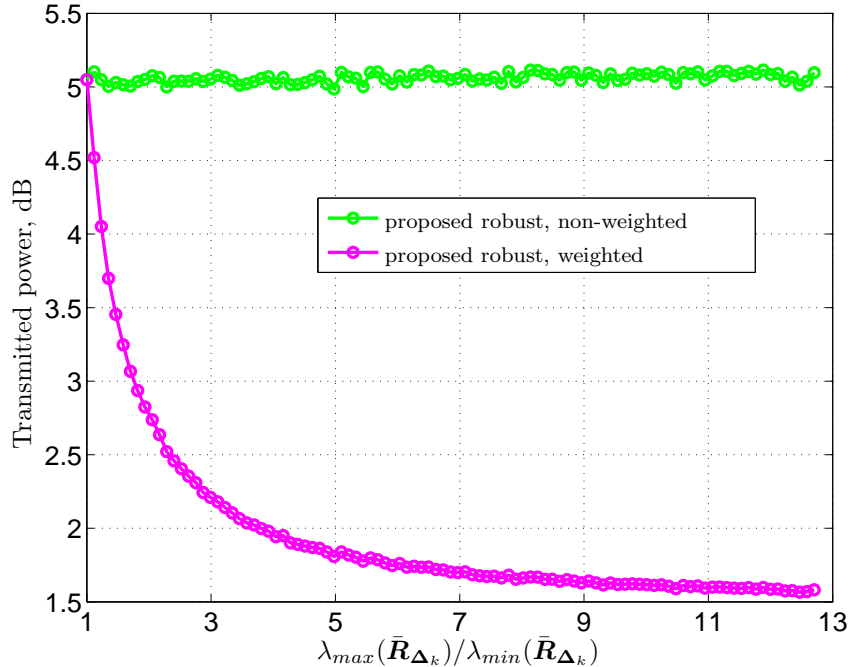


Figure 3.4: Total transmitted power versus the condition number of the error covariance matrix.

increasing eigenvalue spread.

Next, we consider the case with $N = 6$ sensors at the base station and $K = 4$ single-antenna users fixed at locations 10° , 40° , 55° , and 70° relative to the array broadside, while the QoS target γ is varied from -6dB to 3dB . Here, we assume the angular spread of $\sigma_\theta = 10^\circ$ and $\alpha_k = \alpha = 0.25$, for all $k = 1, \dots, K$. Similar as in the previous example, colored interference-plus-noise covariance matrices $\mathbf{R}_{k,I}$ corresponding to angles 60° , 60° , 40° , and 43° are assumed with an angular spread of $\sigma_\theta = 5^\circ$. The variance of the interfering signals is 0.25 and the variance of the noise is 0.1. We plot the feasibility percentage of all the schemes in Fig. 3.5 and observe that the proposed robust approach outperforms both the non-robust and robust approaches of [59] and [50], respectively, in terms of the feasibility percentage. In Fig. 3.6, we plot the corresponding transmitted power for all these techniques for the cases when the robust technique of [50] is feasible. The proposed approach remains more power efficient compared to the robust approach of [50] for this scenario as well. The

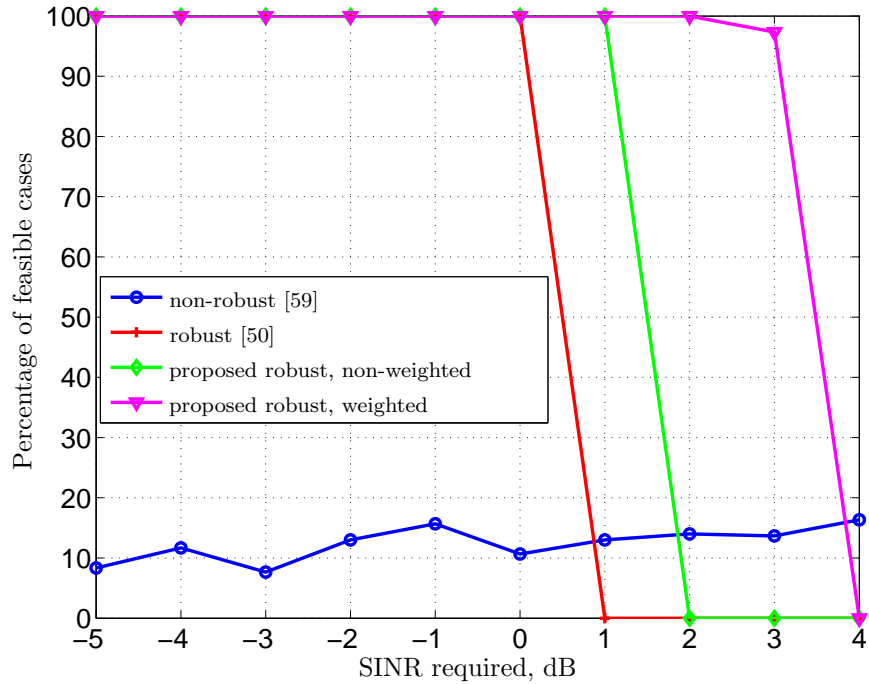
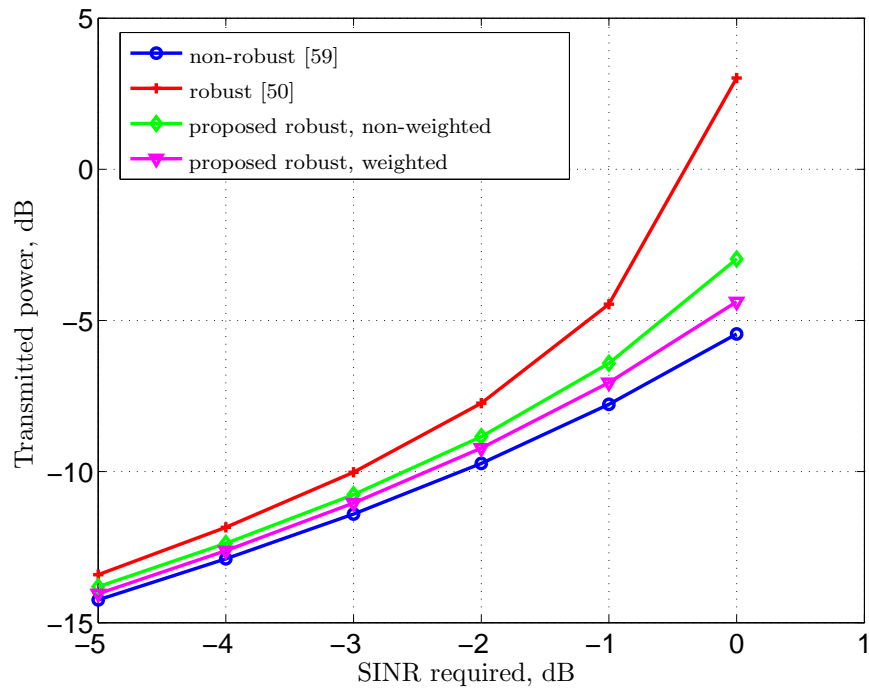


Figure 3.5: Feasibility percentage of the conventional beamforming schemes.

Figure 3.6: Total transmitted power versus required SINR γ .

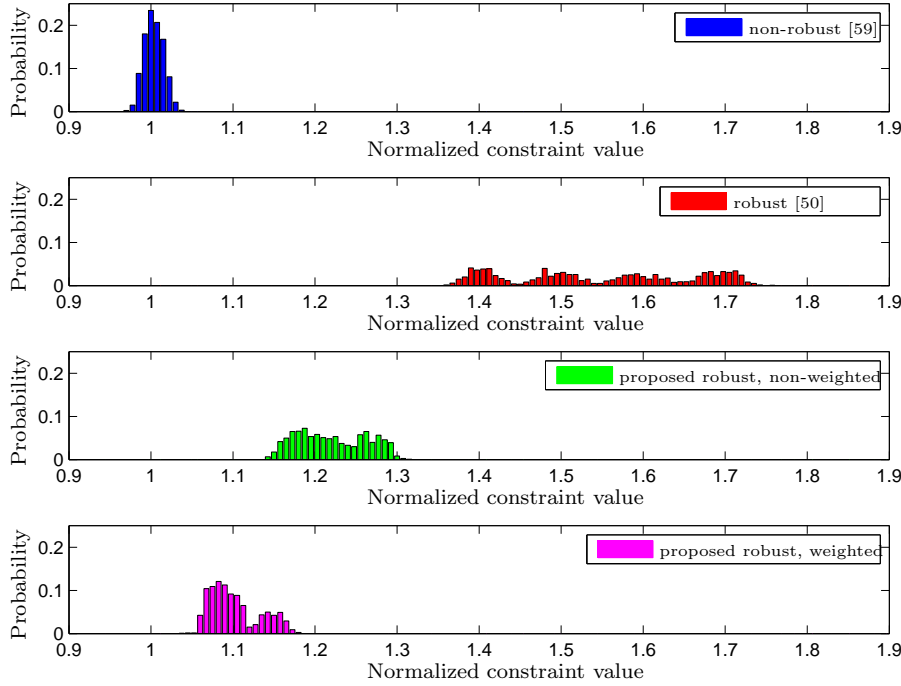


Figure 3.7: Histogram of normalized QoS constraints ($\gamma = 0$ dB).

non-robust approach of [59] exhibits reduced transmitted power but, as mentioned above, it cannot guarantee a feasible solution. In order to elaborate on this issue further, we plot the histogram of the normalized QoS constraints in Fig. 3.7 for $\gamma = 0$ dB. We observe from Fig. 3.7 that the non-robust approach does not satisfy all the constraints while all the constraints are satisfied for both the robust approaches.

In Figs. 3.8-3.11, we present simulation results for the CR beamformer. To reveal that in our proposed robust approach performance gains are obtained irrespective of the introduced weighting, we consider the case that the CSI errors are white and that the uncertainty sets are modeled by the non-weighted Frobenius norm. We compare the proposed robust approaches from Section 3.4.2 and Table 3.2 to the non-robust and robust techniques of [79]. We assume $N = 6$, $K = 4$, and $L = 3$. The channel covariance matrices for the SUs and PUs, $\mathbf{R}_{\mathbf{h}_k}$ and $\mathbf{R}_{\mathbf{h}_{K+l}}$, are generated in the same way as in the case of the conventional beamformer above. The SUs are located at 10° , 40° , 55° , and 70° relative to the array

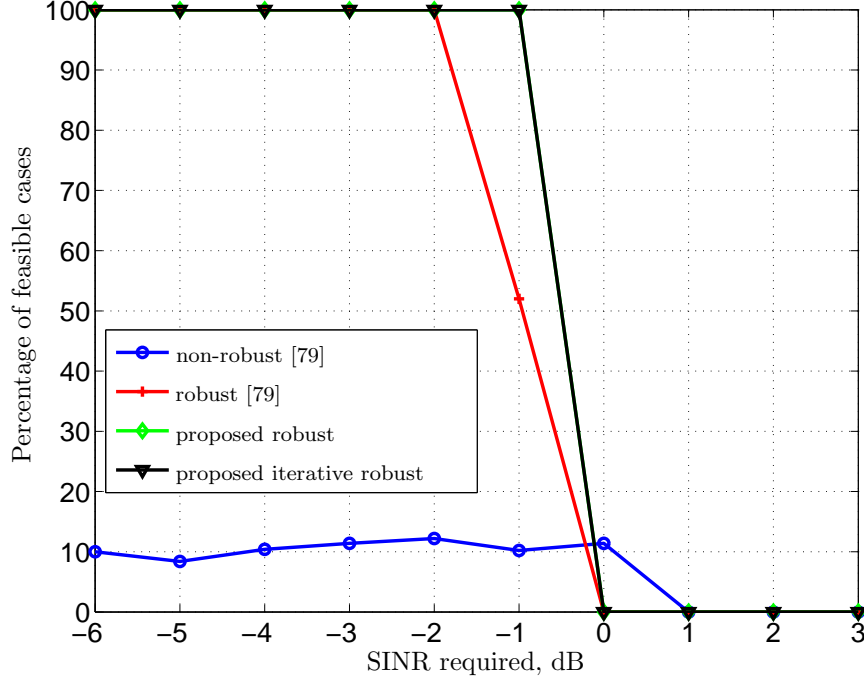


Figure 3.8: Feasibility percentage of the CR beamforming schemes.

broadside, while the PUs are located at 25° , 83° , and 86° relative to the array broadside. The users are assumed to be surrounded by a large number of local scatterers corresponding to an angular spread of $\sigma_\theta = 10^\circ$ in the case of SUs and $\sigma_\theta = 10^\circ$ for the PUs, as seen from the base station. The error matrices Δ_k and Δ_{K+l} are uniformly randomly generated in a sphere centered at zero with radii α_k and α_{K+l} , respectively. The resulting error matrices are then added to the covariance matrices to obtain the estimated covariance matrices $\hat{\mathbf{R}}_{h_k}$ and $\hat{\mathbf{R}}_{h_{K+l}}$, respectively. If $\hat{\mathbf{R}}_{h_k}$ or $\hat{\mathbf{R}}_{h_{K+l}}$ contain negative eigenvalues, then the corresponding eigenvalues are replaced by 0. We assume that $\gamma_k = \gamma$, $\alpha_k = \alpha_{K+l} = \alpha = 0.25$ and $\varepsilon_l = \varepsilon = 5$ dB, for all $k = 1, \dots, K$ and $l = 1, \dots, L$. A total number of 500 Monte-Carlo runs has been performed.

In Fig. 3.8, we plot the feasibility percentage of all the schemes for targets γ varying from -6 dB to 3 dB. Similar to the conventional beamforming case, a solution is considered feasible, if it satisfies all the constraints in (2.10) for the true covariance matrices. We observe that the percentage of feasible runs for all the robust approaches decreases monotonously

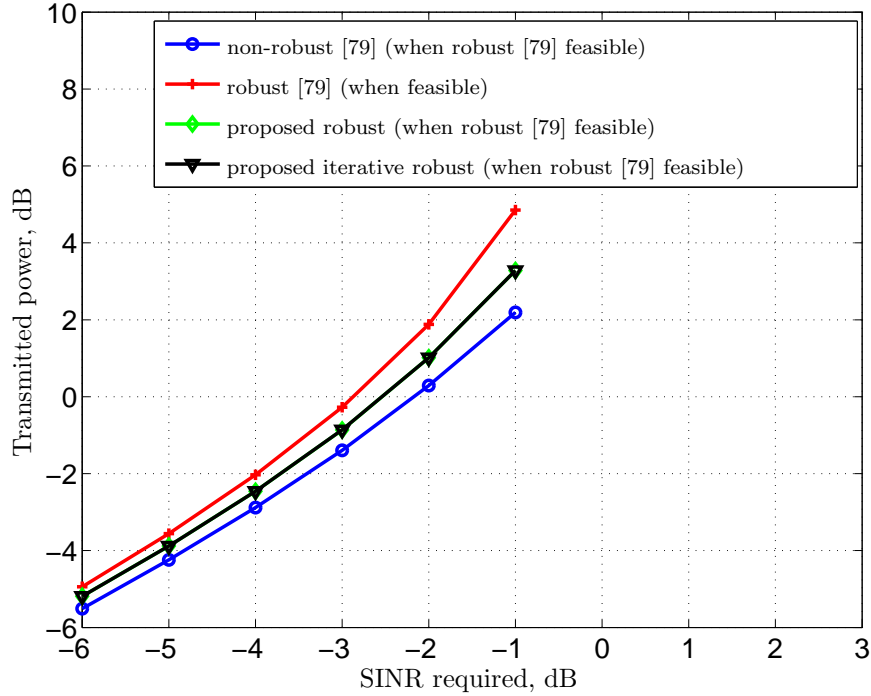
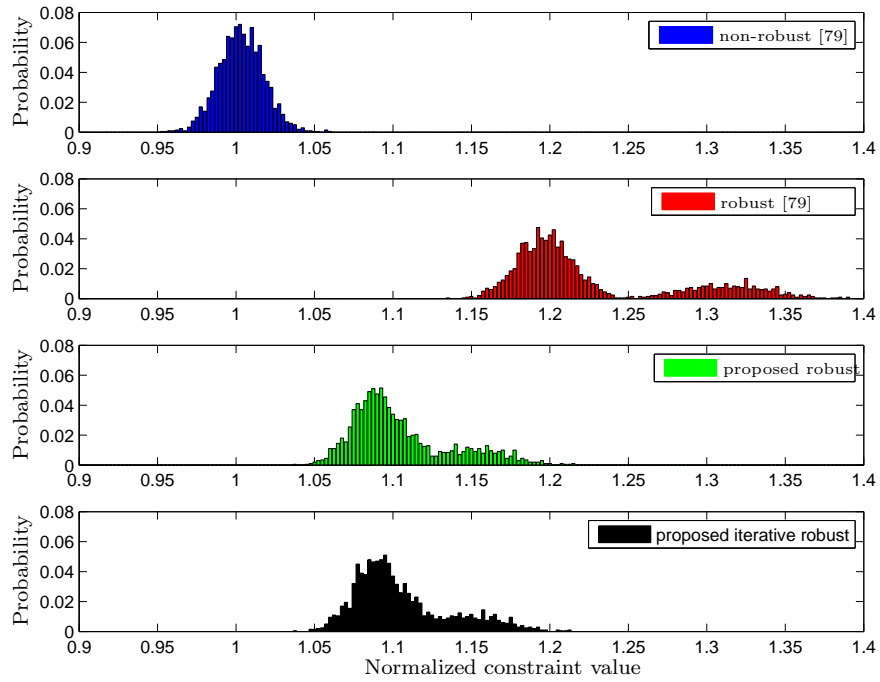
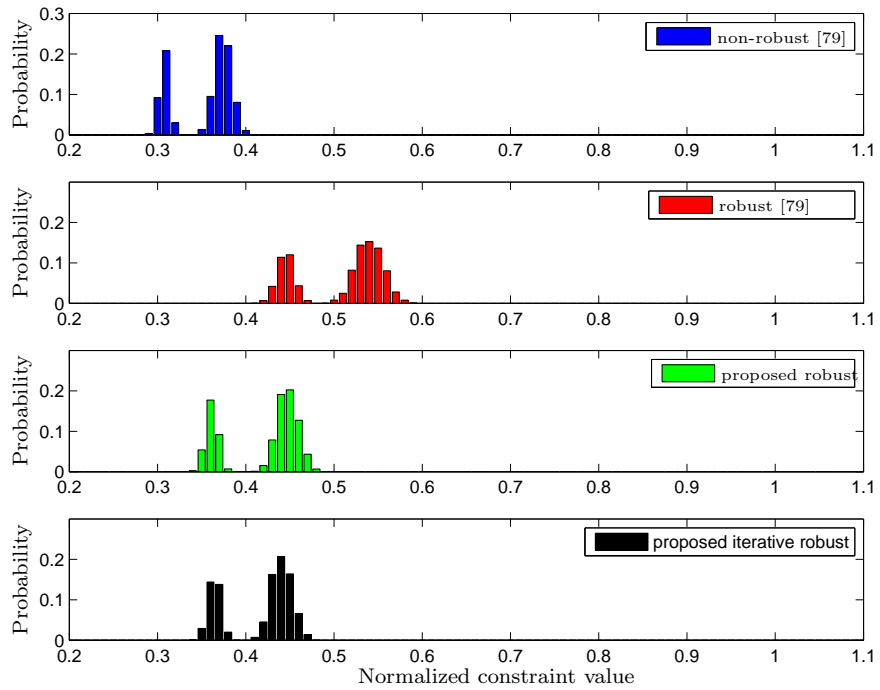


Figure 3.9: Total transmitted power versus SINR required by SUs.

as the target SINR increases. Both of the proposed robust approaches show an increased feasibility percentage as compared to the robust and non-robust methods of [79]. Note that the plots of proposed robust techniques overlap in this figure. It is important to note that similar to the case of the conventional beamforming problem, the feasibility percentage of the non-robust CR approach is highly dependent on the considered error distribution for matrices Δ_k and Δ_{K+l} . Fig. 3.9 displays the corresponding transmitted power versus the SINR required by the SUs for the four schemes under consideration. Similar to the conventional beamforming case, we consider only those cases for which the robust scheme of [79] yields feasible solutions. We remark that in all these cases, both the proposed approaches are always feasible. Both of our techniques are more power efficient (again the plots almost overlap in this figure) compared to the robust approach of [79]. The non-robust approach of [79] appears to be more power efficient, but the solutions it provides are not feasible when CSI errors are taken into account.

Figure 3.10: Histogram of normalized QoS constraints ($\gamma = -2\text{dB}$).Figure 3.11: Histogram of normalized PU interference constraints ($\gamma = -2\text{dB}$).

For further insight, we plot the histogram of the achieved normalized QoS and normalized PU interference power in Figs. 3.10 and 3.11, respectively for $\gamma = -2$ dB. The normalized QoS is defined as in (3.79), while the normalized PU interference power is defined as

$$\zeta_{K+l} = \frac{1}{\varepsilon_l} \sum_{i=1}^K \mathbf{w}_i^{*H} \mathbf{R}_{h_{K+l}} \mathbf{w}_k^*; \quad l = 1, \dots, L. \quad (3.80)$$

From (3.79), a value greater than one in Fig. 3.10 implies that the corresponding QoS constraint is satisfied. Similarly, from (3.80), in Fig. 3.11, a value less than one corresponds to satisfied PU interference constraint. The proposed robust approaches and the robust technique of [79] satisfy all the constraints, as expected, but for the non-robust approach of [79], a considerable number of constraints is left unsatisfied.

Chapter 4

Robust Downlink Beamforming using the Channel Statistics to Model the Channel Mismatch

4.1 Introduction

The prior art robust downlink beamforming techniques [50, 62, 63, 64, 67, 69], mentioned in Chapter 2, and the proposed approaches of Chapter 3, all consider that the mismatch matrices are bounded by ellipsoids of given shapes and sizes, by bounding the Frobenius norm of the mismatch matrices. In this setup, these approaches guarantee that the QoS constraints are satisfied for all possible mismatch matrices that lie within these ellipsoids. The error in the CSI could be a combination of errors arising due to various reasons. The CSI could be determined using training at the receivers and then fed back to the transmitter or, in the cases when channel reciprocity can be assumed, like in the FDD systems, the uplink CSI can be used to determine the downlink channels. This could result in the estimation errors [60] or the quantization errors due to the limited capacity of the feedback channels [61]. The latencies in channel state feedback, short channel coherence

time and finite sampling effects can introduce further errors in the CSI. However, the above mentioned robust approaches do not take into account the source of the error while modeling the uncertainties in the CSI and only put the bound on the cumulative error. The process to obtain meaningful error bounds for these robust approaches is generally unclear.

In this chapter, we look at the robust downlink beamforming problem (2.12) for the conventional scenario. We formulate a general robust downlink beamforming problem that bounds the errors resulting from different sources separately. We then separately consider the problem formulations when the errors in the CSI are either resulting from finite sampling or estimation. This allows us to model the uncertainty sets in each case appropriately. Later we combine these results to consider the case when the CSI errors are a sum of finite sampling and estimation errors. In this thesis, we do not consider the case when the error is resulting from the limited feedback. This can be done in a future work and then combined with the proposed robust problem to consider all these errors in a cumulative manner.

4.2 Generalized Robust Beamforming

In this section, we lay the foundations of our proposed approach by presenting a different formulation for the worst-case robust downlink beamforming (2.12) for the conventional scenario. In this new formulation, different kinds of errors are bounded separately, with respective bounds for each type of error. The resulting robust problem formulation is given as

$$\begin{aligned}
& \min_{\{\mathbf{w}_i\}} \sum_{i=1}^K \mathbf{w}_i^H \mathbf{w}_i \\
& \text{s.t.} \quad \min_{\bar{\Delta}_k^{(e)}, \bar{\Delta}_k^{(s)}} \frac{\mathbf{w}_k^H (\hat{\mathbf{R}}\mathbf{h}_k - \bar{\Delta}_k^{(s)} - \bar{\Delta}_k^{(e)}) \mathbf{w}_k}{\sum_{j \neq k} \mathbf{w}_j^H (\hat{\mathbf{R}}\mathbf{h}_k - \bar{\Delta}_k^{(s)} - \bar{\Delta}_k^{(e)}) \mathbf{w}_j + \sigma_k^2} \geq \gamma_k \\
& \quad g^{(s)}(\bar{\Delta}_k^{(s)}) \leq \alpha_k^{(s)}, \quad g^{(e)}(\bar{\Delta}_k^{(e)}) \leq \alpha_k^{(e)} \\
& \quad \hat{\mathbf{R}}\mathbf{h}_k - \bar{\Delta}_k^{(s)} \succeq 0, \quad \hat{\mathbf{R}}\mathbf{h}_k - \bar{\Delta}_k^{(s)} - \bar{\Delta}_k^{(e)} \succeq 0, \quad k = 1, \dots, K
\end{aligned} \tag{4.1}$$

where $\bar{\Delta}_k^{(s)}$ and $\bar{\Delta}_k^{(e)}$ are the mismatch error matrices representing the finite sample and estimation errors, respectively. $g^{(\cdot)}(\cdot)$ represents a function that can characterize the set of

the corresponding error matrices and $\alpha_k^{(\cdot)}$ is the corresponding bound.

In the Sections 4.3 & 4.4, we consider the problem (4.1) for the case when the CSI errors only originate from estimation errors, while in Section 4.5, we only look at the CSI errors resulting from finite sampling effects. Finally, in Section 4.6, the robust problem (4.1) is solved for the CSI errors caused by both estimation errors and finite sampling effects together.

4.3 Channel Error Model Based on Estimation Error

We present a new approach for modelling the error in the CSI that takes the statistics of the channels into account. Let \mathbf{h}_k and $\mathbf{h}_k(i)$ denote the actual random channel vector containing the $N \times 1$ flat-block fading complex channel coefficients of the k th user and the corresponding realization at the i th block, respectively. Defining $\mathbf{y}_k(i)$ to be the corresponding $T \times 1$ received training signal vector during the i th block where $T \geq N$, it can be expressed as

$$\mathbf{y}_k(i) = \mathbf{V}\mathbf{h}_k(i) + \mathbf{n}_k(i), \quad i = 1, \dots, n \quad (4.2)$$

where n is the number of sample training block, \mathbf{V} is the $T \times N$ training signal matrix at each block, $\mathbf{n}_k(i)$ is the $T \times 1$ complex zero-mean spatially and temporally white Gaussian noise vector with $E\{\mathbf{n}_k(i)\mathbf{n}_k(i)^H\} = \nu_k^2 \mathbf{I}_T$, and ν_k^2 denotes the mean power of the receiver noise during the estimation. Assuming the coefficients of $\mathbf{h}_k(i)$ and $\mathbf{n}_k(i)$ to be mutually statistically independent, the covariance matrix of the estimated channel can be expressed as

$$\mathbf{R}_{\mathbf{y}_k} = \mathbf{V}\mathbf{R}_{\mathbf{h}_k}\mathbf{V}^H + \nu_k^2 \mathbf{I}_T. \quad (4.3)$$

The received training signal can be rewritten in the matrix form as

$$\mathbf{Y}_k = \mathbf{V}\mathbf{H}_k + \mathbf{N}_k \quad (4.4)$$

where \mathbf{Y}_k , \mathbf{H}_k , and \mathbf{N}_k are $T \times n$, $N \times n$, and $T \times n$ matrices, respectively, defined as

$$\begin{aligned}\mathbf{Y}_k &\triangleq [\mathbf{y}_k(1), \mathbf{y}_k(2), \dots, \mathbf{y}_k(n)] \\ \mathbf{H}_k &\triangleq [\mathbf{h}_k(1), \mathbf{h}_k(2), \dots, \mathbf{h}_k(n)] \\ \mathbf{N}_k &\triangleq [\mathbf{n}_k(1), \mathbf{n}_k(2), \dots, \mathbf{n}_k(n)].\end{aligned}\quad (4.5)$$

Note that

$$\mathbf{R}_{\mathbf{y}_k} = \lim_{n \rightarrow \infty} \frac{1}{n} \mathbf{Y}_k \mathbf{Y}_k^H. \quad (4.6)$$

Furthermore, we have

$$\mathbf{Y}_k \mathbf{Y}_k^H = \mathbf{V} \mathbf{H}_k \mathbf{H}_k^H \mathbf{V}^H + \mathbf{N}_k \mathbf{H}_k^H \mathbf{V}^H + \mathbf{V} \mathbf{H}_k \mathbf{N}_k^H + \mathbf{N}_k \mathbf{N}_k^H. \quad (4.7)$$

Therefore, we can define the sample estimate of the covariance matrix $\mathbf{R}_{\mathbf{h}_k}$ as

$$\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} = \frac{1}{n} \mathbf{V}^\dagger \mathbf{Y}_k \mathbf{Y}_k^H (\mathbf{V}^\dagger)^H \quad (4.8)$$

where $\mathbf{V}^\dagger \triangleq (\mathbf{V}^H \mathbf{V})^{-1} \mathbf{V}^H$ is the pseudoinverse of \mathbf{V} . The sample estimate of the covariance matrix $\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)}$ in (4.8) can be rewritten as

$$\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} = \frac{1}{n} \sum_{j=1}^n \hat{\mathbf{h}}_k(j) \hat{\mathbf{h}}_k(j)^H \quad (4.9)$$

where we can see that

$$\begin{aligned}\hat{\mathbf{h}}_k(i) &\triangleq \mathbf{V}^\dagger \mathbf{y}_k(i) \\ &= \mathbf{h}_k(i) + \mathbf{V}^\dagger \mathbf{n}_k(i)\end{aligned}\quad (4.10)$$

is the $N \times 1$ estimated channel using the least square method [67]. Multiplying \mathbf{V}^\dagger and $(\mathbf{V}^\dagger)^H$ to the left and right of (4.3), the covariance of the estimated channel can be expressed as

$$\begin{aligned}\mathbf{R}_{\hat{\mathbf{h}}_k} &= \mathbf{V}^\dagger \mathbf{R}_{\mathbf{y}_k} (\mathbf{V}^\dagger)^H \\ &= \mathbf{R}_{\mathbf{h}_k} + \nu_k^2 (\mathbf{V}^H \mathbf{V})^{-1}.\end{aligned}\quad (4.11)$$

To minimize the power of the received training noise for a given training power \mathcal{P} , it is shown in [105] that any training matrix \mathbf{V} is optimal if $\mathbf{V}^H \mathbf{V} = \frac{\mathcal{P}}{N} \mathbf{I}_N$. Therefore, the true covariance of the channel can be rewritten as

$$\mathbf{R}_{\mathbf{h}_k} = \mathbf{R}_{\hat{\mathbf{h}}_k} - \frac{\nu_k^2 N}{\mathcal{P}} \mathbf{I}_N \quad (4.12)$$

where it is obvious that the part $\frac{\nu_k^2 N}{\mathcal{P}} \mathbf{I}_N$ represents the error in the estimate of $\mathbf{R}_{\mathbf{h}_k}$. Note that, using the maximum likelihood estimation [106], the estimator of the noise power is given by

$$\hat{\nu}_k^2 = \frac{\text{Tr}\{\mathbf{P}_{\mathbf{V}}^\perp \hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)}\}}{T - N} \quad (4.13)$$

where $\mathbf{P}_{\mathbf{V}}^\perp$ is the projection matrix onto the range of \mathbf{V} .

4.4 Robust Beamforming with Estimation Error

In this section, for simplicity, we assume that the true covariance of the estimated channel $\mathbf{R}_{\hat{\mathbf{h}}_k}$ is available and the uncertainty about the mean noise power ν_k^2 in (4.12) is the only error in the CSI. Using (4.12), the estimation error can be written as a scalar matrix $c_k \mathbf{I}_N$ where c_k represents the scaling factor. Suppose that c_k is bounded above and below by $(\bar{\nu}_k^2 + \xi_k)N/\mathcal{P}$ and $(\bar{\nu}_k^2 - \xi_k)N/\mathcal{P}$, respectively, where ξ_k is a confidence interval for an estimate $\bar{\nu}_k^2$ of the noise power with a given confidence level \tilde{p}_k such that

$$\Pr(|\eta_k - \bar{\nu}_k^2| \leq \xi_k) = \tilde{p}_k. \quad (4.14)$$

Here η_k is an instance of noise power and $\Pr(\cdot)$ is a given probability density function of the noise power.

The worst-case robust beamforming, that only considers the estimation error, can then be formulated as

$$\begin{aligned} & \min_{\{\mathbf{w}_i\}} \sum_{i=1}^K \mathbf{w}_i^H \mathbf{w}_i \\ & \text{s.t.} \quad \min_{|\bar{\eta}_k - \bar{\nu}_k^2| \leq \xi_k} \frac{\mathbf{w}_k^H (\mathbf{R}_{\hat{\mathbf{h}}_k} - \frac{\bar{\eta}_k N}{\mathcal{P}} \mathbf{I}_N) \mathbf{w}_k}{\sum_{j \neq k} \mathbf{w}_j^H (\mathbf{R}_{\hat{\mathbf{h}}_k} - \frac{\bar{\eta}_k N}{\mathcal{P}} \mathbf{I}_N) \mathbf{w}_j + \sigma_k^2} \geq \gamma_k \\ & \quad \mathbf{R}_{\hat{\mathbf{h}}_k} - \frac{\bar{\eta}_k N}{\mathcal{P}} \mathbf{I}_N \succeq 0, \quad k = 1, \dots, K. \end{aligned} \quad (4.15)$$

Problem (4.15) can be equivalently written as

$$\begin{aligned}
& \min_{\{\mathbf{w}_i\}} \sum_{i=1}^K \mathbf{w}_i^H \mathbf{w}_i \\
& \text{s.t.} \quad \min_{\alpha_{k2}^{(e)} \leq \delta_k \leq \alpha_{k1}^{(e)}} \frac{\mathbf{w}_k^H (\mathbf{R}_{\hat{\mathbf{h}}_k} - \delta_k \mathbf{I}_N) \mathbf{w}_k}{\sum_{j \neq k} \mathbf{w}_j^H (\mathbf{R}_{\hat{\mathbf{h}}_k} - \delta_k \mathbf{I}_N) \mathbf{w}_j + \sigma_k^2} \geq \gamma_k \\
& \quad \mathbf{R}_{\hat{\mathbf{h}}_k} - \delta_k \mathbf{I}_N \succeq 0, \quad k = 1, \dots, K
\end{aligned} \tag{4.16}$$

where

$$\alpha_{k1}^{(e)} \triangleq \frac{(\bar{\nu}_k^2 + \xi_k)N}{\mathcal{P}} \tag{4.17}$$

$$\alpha_{k2}^{(e)} \triangleq \frac{(\bar{\nu}_k^2 - \xi_k)N}{\mathcal{P}} \tag{4.18}$$

Recalling the definition of \mathbf{A}_k from (3.8) as given by

$$\mathbf{A}_k \triangleq \mathbf{w}_k \mathbf{w}_k^H - \gamma_k \sum_{j \neq i} \mathbf{w}_j \mathbf{w}_j^H, \tag{4.19}$$

and using it in problem (4.16) can be rewritten in a compact form as

$$\begin{aligned}
& \min_{\{\mathbf{w}_i\}} \sum_{i=1}^K \mathbf{w}_i^H \mathbf{w}_i \\
& \text{s.t.} \quad \min_{\alpha_{k2}^{(e)} \leq \delta_k \leq \alpha_{k1}^{(e)}} \text{Tr}\{(\mathbf{R}_{\hat{\mathbf{h}}_k} - \delta_k \mathbf{I}_N) \mathbf{A}_k\} \geq \sigma_k^2 \gamma_k \\
& \quad \mathbf{R}_{\hat{\mathbf{h}}_k} - \delta_k \mathbf{I}_N \succeq 0, \quad k = 1, \dots, K.
\end{aligned} \tag{4.20}$$

Taking an approach similar to the one used in [69], we consider the inner optimization problem in the worst-case QoS constraints of (4.20) as a separate optimization problem given by

$$\begin{aligned}
& \min_{\delta_k} \text{Tr}\{(\mathbf{R}_{\hat{\mathbf{h}}_k} - \delta_k \mathbf{I}_N) \mathbf{A}_k\} \\
& \text{s.t.} \quad \mathbf{R}_{\hat{\mathbf{h}}_k} - \delta_k \mathbf{I}_N \succeq 0, \quad \alpha_{k2}^{(e)} \leq \delta_k \leq \alpha_{k1}^{(e)}.
\end{aligned} \tag{4.21}$$

Supposing that the problem (4.21) is strictly feasible, the following lemma applies:

Lemma 1 (equivalence of worst-case QoS constraints): For a fixed non-zero weight vector \mathbf{w}_i (and correspondingly fixed matrices \mathbf{A}_k), the problem (4.21) and the problem

$$\begin{aligned} & \max_{c_{k1}, c_{k2}, \mathbf{Z}_k^{(e)}} \text{Tr}\{\mathbf{R}_{\hat{\mathbf{h}}_k}(\mathbf{A}_k - \mathbf{Z}_k^{(e)})\} - c_{k1}\alpha_{k1}^{(e)} + c_{k2}\alpha_{k2}^{(e)} \\ \text{s.t.} \quad & c_{k1} - c_{k2} - \text{Tr}\{\mathbf{A}_k - \mathbf{Z}_k^{(e)}\} \geq 0 \\ & \mathbf{Z}_k^{(e)} \succeq 0, \quad c_{k1} \geq 0, \quad c_{k2} \geq 0 \end{aligned} \quad (4.22)$$

have the same optimal value.

Proof: The Lagrangian associated with problem (4.21) is given by

$$\begin{aligned} \bar{f}_k(\delta_k, \alpha_{kj}, \mathbf{Z}_k^{(e)}) &= \text{Tr}\{(\mathbf{R}_{\hat{\mathbf{h}}_k} - \delta_k \mathbf{I}_N)\mathbf{A}_k\} + c_{k1}(\delta_k - \alpha_{k1}^{(e)}) \\ &\quad - c_{k2}(\delta_k - \alpha_{k2}^{(e)}) - \text{Tr}\{(\mathbf{R}_{\hat{\mathbf{h}}_k} - \delta_k \mathbf{I}_N)\mathbf{Z}_k^{(e)}\} \end{aligned} \quad (4.23)$$

where $\alpha_{kj} \geq 0, j = 1, 2$ are the Lagrange multipliers corresponding to the bound constraints on δ_k and the Hermitian matrix $\mathbf{Z}_k^{(e)}$ is the Lagrange multiplier for the positive semidefinite constraint in (4.21). Minimizing (4.23) with respect to δ_k and defining $y_k \triangleq c_{k1} - c_{k2} - \text{Tr}\{\mathbf{A}_k - \mathbf{Z}_k^{(e)}\}$ yields the Lagrange dual function as

$$\inf_{\delta_k} \bar{f}_k = \begin{cases} \text{Tr}\{\mathbf{R}_{\hat{\mathbf{h}}_k}(\mathbf{A}_k - \mathbf{Z}_k^{(e)})\} - c_{k1}\alpha_{k1}^{(e)} + c_{k2}\alpha_{k2}^{(e)}, & y_k \geq 0 \\ -\infty, & \text{otherwise.} \end{cases} \quad (4.24)$$

The dual problem of problem (4.21) can then be expressed as

$$\begin{aligned} & \max_{c_{k1}, c_{k2}, \mathbf{Z}_k^{(e)}} \text{Tr}\{\mathbf{R}_{\hat{\mathbf{h}}_k}(\mathbf{A}_k - \mathbf{Z}_k^{(e)})\} - c_{k1}\alpha_{k1}^{(e)} + c_{k2}\alpha_{k2}^{(e)} \\ \text{s.t.} \quad & c_{k1} - c_{k2} - \text{Tr}\{\mathbf{A}_k - \mathbf{Z}_k^{(e)}\} \geq 0 \\ & \mathbf{Z}_k^{(e)} \succeq 0, \quad c_{k1} \geq 0, \quad c_{k2} \geq 0 \end{aligned} \quad (4.25)$$

which is the same as problem (4.22). Note that the problem (4.21) is convex and clearly it is bounded below. Since the problem (4.21) is strictly feasible, using [104, Th. 1.7.1], we can conclude that strong duality exists between (4.21) and (4.22). \square

From (4.11), it is clear that $\mathbf{R}_{\hat{\mathbf{h}}_k} \succeq 0$. To ensure strict feasibility of problem (4.21), we can relax the lower bound of (4.18) by redefining $\alpha_{k2}^{(e)} \triangleq \min\{\lambda_{\min}(\mathbf{R}_{\hat{\mathbf{h}}_k}) - \epsilon_k, \frac{(\bar{\nu}_k^2 - \xi_k)N}{\mathcal{P}}\}$ where ϵ_k is a small positive value.

Using Lemma 1, problem (4.15) can be reformulated as

$$\begin{aligned}
& \min_{\{\mathbf{w}_i\}} \sum_{i=1}^K \mathbf{w}_i^H \mathbf{w}_i \\
& \text{s.t.} \quad \max_{c_{k1}, c_{k2}, \mathbf{Z}_k^{(e)}} \text{Tr}\{\mathbf{R}_{\hat{\mathbf{h}}_k}(\mathbf{A}_k - \mathbf{Z}_k^{(e)})\} - c_{k1}\alpha_{k1}^{(e)} + c_{k2}\alpha_{k2}^{(e)} \geq \sigma^2\gamma_k \\
& \quad c_{k1} - c_{k2} - \text{Tr}\{\mathbf{A}_k - \mathbf{Z}_k^{(e)}\} \geq 0 \\
& \quad \mathbf{Z}_k^{(e)} \succeq 0, \quad c_{k1} \geq 0, \quad c_{k2} \geq 0, \quad k = 1, \dots, K.
\end{aligned} \tag{4.26}$$

The inner problems in (4.26) are satisfied if there exists some $c_{k1} \geq 0, c_{k2} \geq 0$ and $\mathbf{Z}_k^{(e)} \succeq 0$ for which

$$\begin{aligned}
& \text{Tr}\{\mathbf{R}_{\hat{\mathbf{h}}_k}(\mathbf{A}_k - \mathbf{Z}_k^{(e)})\} - c_{k1}\alpha_{k1}^{(e)} + c_{k2}\alpha_{k2}^{(e)} \geq \sigma_k^2\gamma_k \\
& \quad c_{k1} - c_{k2} - \text{Tr}\{\mathbf{A}_k - \mathbf{Z}_k^{(e)}\} \geq 0.
\end{aligned} \tag{4.27}$$

Therefore, the problem (4.26) reduces to

$$\begin{aligned}
& \min_{\{\mathbf{w}_i, \alpha_{ij}, \mathbf{Z}_i^{(e)}\}} \sum_{i=1}^K \mathbf{w}_i^H \mathbf{w}_i \\
& \text{s.t.} \quad \text{Tr}\{\mathbf{R}_{\hat{\mathbf{h}}_k}(\mathbf{A}_k - \mathbf{Z}_k^{(e)})\} - c_{k1}\alpha_{k1}^{(e)} + c_{k2}\alpha_{k2}^{(e)} \geq \sigma_k^2\gamma_k \\
& \quad c_{k1} - c_{k2} - \text{Tr}\{\mathbf{A}_k - \mathbf{Z}_k^{(e)}\} \geq 0 \\
& \quad \mathbf{Z}_k^{(e)} \succeq 0, \quad c_{k1} \geq 0, \quad c_{k2} \geq 0, \quad k = 1, \dots, K,
\end{aligned} \tag{4.28}$$

which is a convex SDP problem and can be solved in polynomial time using interior-point algorithms [91, 92].

4.5 Robust Beamforming with Sample Error

In this section, we assume that the CSI is free from any estimation error, i.e., $\mathbf{R}_{\hat{\mathbf{h}}_k} = \mathbf{R}_{\mathbf{h}_k}$, and the error in the covariance matrices is only due to finite sampling effects. We first present the channel model in the following subsection and later we propose the robust beamforming problem based on this channel error model.

4.5.1 Sample Error Model

Let $\rho_k(i) = \hat{\mathbf{h}}_k(i)^H \hat{\mathbf{h}}_k(i)$ be the power of the estimated channel. Then we can define the two-sided confidence interval for the true mean m_{ρ_k} of the power of the estimated channel with probability p_k as

$$\Pr(|\hat{m}_{\rho_k} - m_{\rho_k}| \leq t_k \hat{v}_{\rho_k}) = p_k \quad (4.29)$$

for some positive t_k , where \hat{m}_{ρ_k} and $\hat{v}_{\rho_k}^2$ are the sample mean and the sample variance of the power of the estimated channel, respectively, given as

$$\hat{m}_{\rho_k} = \frac{1}{n} \sum_{j=1}^n \rho_k(j) \quad (4.30)$$

$$\hat{v}_{\rho_k}^2 = \frac{1}{n-1} \sum_{j=1}^n (\rho_k(j) - \hat{m}_{\rho_k})^2. \quad (4.31)$$

Note that the size of the confidence region depends on the sample variance of the power of estimated channel. In particular, using percentile-t bootstrapping techniques [107], [108], one can find a bootstrap two-sided confidence interval value \tilde{t}_k of the true expected power of the channel for a given confidence level p_k , i.e.,

$$\Pr(|\bar{m}_{\rho_k} - \hat{m}_{\rho_k}| \leq \tilde{t}_k \bar{v}_{\rho_k}) = p_k \quad (4.32)$$

where \bar{m}_{ρ_k} is the mean of the channel power calculated using bootstrap resampling and $(\bar{v}_{\rho_k})^2$ is the corresponding sample variance.

4.5.2 Proposed Robust Beamformer

We propose an alternative worst-case robust beamforming technique that only considers the mean channel power mismatches lying within the one-sided confidence interval t_k , for a fixed confidence level p_k , and ignores the mismatches outside this interval. Therefore, we consider that $|\hat{m}_{\rho_k} - E\{\|\hat{\mathbf{h}}_k\|^2\}| \leq t_k \hat{v}_{\rho_k}$. Let the random errors matrix $\mathbf{\Delta}_k^{(s)}$ model the difference between the sample estimate of the covariance matrix and the true covariance of the estimated channel, then

$$\mathbf{\Delta}_k^{(s)} = \hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \mathbf{R}_{\hat{\mathbf{h}}_k}. \quad (4.33)$$

Using (4.9), and (4.33), the mismatch can be bounded as

$$\begin{aligned} |\text{Tr}\{\Delta_k^{(s)}\}| &= |\text{Tr}\{\frac{1}{n} \sum_{j=1}^n \hat{\mathbf{h}}_k(j) \hat{\mathbf{h}}_k(j)^H - \mathbf{R}_{\hat{\mathbf{h}}_k}\}| \\ &= |\hat{m}_k - E\{\|\hat{\mathbf{h}}_k\|^2\}| \leq t_k \hat{v}_k \end{aligned} \quad (4.34)$$

where we have used the fact that $E\{\|\mathbf{v}\|^2\} = \text{Tr}\{\mathbf{R}_v\}$.

Let us first consider the case that the mismatch matrices $\Delta_k^{(s)}$ are positive semidefinite. Later, in this section, we show that the positive semidefinite constraint on $\Delta_k^{(s)}$ is inactive at the optimum and therefore the solution without this constraint remains the same. The worst-case based robust beamforming approach that considers the possible candidates of mismatches bounded by a given value $\alpha_k^{(s)} \triangleq t_k \hat{v}_k$ can then be formulated as

$$\begin{aligned} \min_{\{\mathbf{w}_i\}} & \sum_{i=1}^K \mathbf{w}_i^H \mathbf{w}_i \\ \text{s.t.} & \min_{\text{Tr}\{\Delta_k^{(s)}\} \leq \alpha_k^{(s)}} \frac{\mathbf{w}_k^H (\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\Delta}_k^{(s)}) \mathbf{w}_k}{\sum_{j \neq k} \mathbf{w}_j^H (\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\Delta}_k^{(s)}) \mathbf{w}_j + \sigma_k^2} \geq \gamma_k \\ & \hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\Delta}_k^{(s)} \succeq 0, \bar{\Delta}_k^{(s)} \succeq 0, \quad k = 1, \dots, K. \end{aligned} \quad (4.35)$$

The constraint $\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\Delta}_k^{(s)} \succeq 0$ guarantees that only the positive definite mismatched covariance matrices are considered in the robust problem (4.35). Using (3.8), problem (4.35) can be written in a compact form as

$$\begin{aligned} \min_{\{\mathbf{w}_i\}} & \sum_{i=1}^K \mathbf{w}_i^H \mathbf{w}_i \\ \text{s.t.} & \min_{\text{Tr}\{\Delta_k^{(s)}\} \leq \alpha_k^{(s)}} \text{Tr}\{(\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\Delta}_k^{(s)}) \mathbf{A}_k\} \geq \sigma_k^2 \gamma_k \\ & \hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\Delta}_k^{(s)} \succeq 0, \bar{\Delta}_k^{(s)} \succeq 0, \quad k = 1, \dots, K. \end{aligned} \quad (4.36)$$

Similar to the problem (4.20) of previous section, we consider the inner optimization problem in the worst-case QoS constraints of (4.36) as a separate problem given by

$$\begin{aligned} \min_{\Delta_k^{(s)}} & \text{Tr}\{(\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\Delta}_k^{(s)}) \mathbf{A}_k\} \\ \text{s.t.} & \hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\Delta}_k^{(s)} \succeq 0, \text{Tr}\{\bar{\Delta}_k^{(s)}\} \leq \alpha_k^{(s)}, \bar{\Delta}_k^{(s)} \succeq 0. \end{aligned} \quad (4.37)$$

Following lemma applies for problem (4.37):

Lemma 2 (equivalence of worst-case QoS constraints): For a fixed non-zero weight vector \mathbf{w}_i (and correspondingly fixed matrices \mathbf{A}_k), problem (4.37) and the problem

$$\begin{aligned} & \max_{\beta_k, \mathbf{Z}_k^{(s)}} \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)}(\mathbf{A}_k - \mathbf{Z}_k^{(s)})\} - \beta_k \alpha_k^{(s)} \\ & \text{s.t. } \beta_k \mathbf{I}_N - \mathbf{A}_k + \mathbf{Z}_k^{(s)} \succeq 0 \\ & \mathbf{Z}_k^{(s)} \succeq 0, \beta_k \geq 0 \end{aligned} \quad (4.38)$$

have the same optimal value.

Proof: The Lagrangian associated with problem (4.37) is given by

$$\begin{aligned} \tilde{f}_k(\bar{\Delta}_k^{(s)}, \beta_k, \mathbf{Z}_k^{(s)}) &= \text{Tr}\{(\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\Delta}_k^{(s)})\mathbf{A}_k\} + \beta_k(\text{Tr}\{\bar{\Delta}_k^{(s)}\} - \alpha_k^{(s)}) \\ &\quad - \text{Tr}\{(\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\Delta}_k^{(s)})\mathbf{Z}_k^{(s)}\} - \text{Tr}\{\bar{\Delta}_k^{(s)}\mathbf{J}_k\} \end{aligned} \quad (4.39)$$

where β_k is the Lagrange multiplier corresponding to the trace constraint and the Hermitian matrices $\mathbf{Z}_k^{(s)}$ and \mathbf{J}_k are the Lagrange multipliers corresponding to the positive semidefinite constraints in (4.37). Minimizing (4.39) with respect to $\bar{\Delta}_k^{(s)}$ yields the Lagrange dual function corresponding to (4.37) as

$$\inf_{\bar{\Delta}_k^{(s)}} \tilde{f}_k(\bar{\Delta}_k^{(s)}, \beta_k, \mathbf{Z}_k^{(s)}) = \begin{cases} \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)}(\mathbf{A}_k - \mathbf{Z}_k^{(s)})\} - \beta_k \alpha_k^{(s)}, & \beta_k \mathbf{I}_N - \mathbf{A}_k + \mathbf{Z}_k^{(s)} - \mathbf{J}_k \succeq 0 \\ -\infty, & \text{otherwise.} \end{cases} \quad (4.40)$$

and the dual problem of problem (4.37) can be expressed as

$$\begin{aligned} & \max_{\beta_k, \mathbf{Z}_k^{(s)}} \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)}(\mathbf{A}_k - \mathbf{Z}_k^{(s)})\} - \beta_k \alpha_k^{(s)} \\ & \text{s.t. } \beta_k \mathbf{I}_N - \mathbf{A}_k + \mathbf{Z}_k^{(s)} - \mathbf{J}_k \succeq 0 \\ & \mathbf{Z}_k^{(s)} \succeq 0, \mathbf{J}_k \succeq 0, \beta_k \geq 0. \end{aligned} \quad (4.41)$$

Note that problem (4.37) is convex and bounded below. Moreover, letting $\mathbf{U}_k \mathbf{\Lambda}_k \mathbf{U}_k^H$ to be the matrix decomposition of $\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)}$, there exists $\bar{\Delta}_k^{(s)} = \mathbf{U}_k \mathbf{D}_k \mathbf{U}_k^H$ such that $[\mathbf{D}_k]_{ij} = \min\{[\mathbf{\Lambda}_k]_{ij}, \bar{\alpha}_k\}$ with $0 < \bar{\alpha}_k < \alpha_k^{(s)}/N$ that is a strictly feasible point of problem (4.37).

Therefore, using [104, Th. 1.7.1], we can conclude that strong duality between (4.37) and (4.38) holds.

It is clear from (4.41) that when $(\beta_k^*, \mathbf{Z}_k^{(s)*}, \mathbf{J}_k^*)$ is an optimal solution of (4.41), then $(\beta_k^*, \mathbf{Z}_k^{(s)*}, \mathbf{0})$ is also a feasible point that yields the same objective function value. Therefore, we can set w.l.o.g. $\mathbf{J}_k^* = \mathbf{0}$, and, correspondingly, (4.41) reduces to (4.38). \square

The observation, that the Lagrange multipliers corresponding to the positive semidefinite constraint in (4.37) at the optimum of (4.38) can be chosen as $\mathbf{J}_k^* = \mathbf{0}$, leads to an interesting implication. It can be concluded from the complementary slackness conditions of the corresponding Karush-Kuhn-Tucker (KKT) optimality system [90], that the positive semidefinite constraint $\bar{\Delta}_k^{(s)} \succeq 0$ is inactive at optimum of the problem (4.37). This means that the solution of the problem (4.36) without the constraint $\bar{\Delta}_k^{(s)} \succeq 0$ does not differ from the case when this constraint is present.

Using Lemma 2, the problem (4.36) can be rewritten as

$$\begin{aligned} & \min_{\{\mathbf{w}_i\}} \sum_{i=1}^K \mathbf{w}_i^H \mathbf{w}_i \\ & \text{s.t.} \quad \max_{\mathbf{Z}_k^{(s)}, \beta_k} \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)}(\mathbf{A}_k - \mathbf{Z}_k^{(s)})\} - \beta_k \alpha_k^{(s)} \geq \sigma_k^2 \gamma_k \\ & \quad \beta_k \mathbf{I}_N - \mathbf{A}_k + \mathbf{Z}_k^{(s)} \succeq 0, \quad \mathbf{Z}_k^{(s)} \succeq 0 \\ & \quad \beta_k \geq 0, k = 1, \dots, K. \end{aligned} \quad (4.42)$$

Given that, for a $N \times N$ Hermitian matrix \mathbf{M} , the optimal value of the problem

$$\min_x x \quad \text{s.t.} \quad x \mathbf{I}_N - \mathbf{M} \succeq 0 \quad (4.43)$$

equals $\lambda_{\max}(\mathbf{M})$, the constraints in (4.42) can be compactly written as

$$\max_{\mathbf{Z}_k^{(s)} \succeq 0} \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)}(\mathbf{A}_k - \mathbf{Z}_k^{(s)})\} - \lambda_{\max}(\mathbf{A}_k - \mathbf{Z}_k^{(s)}) \alpha_k^{(s)} \geq \sigma_k^2 \gamma_k. \quad (4.44)$$

Here we have used the fact that $\sigma_k^2 \gamma_k \geq 0$, which implies that $\mathbf{A}_k - \mathbf{Z}_k^{(s)} \not\preceq 0$, and hence we have $\lambda_{\max}(\mathbf{A}_k - \mathbf{Z}_k^{(s)}) \geq 0$. Note that the constraint (4.44) is satisfied if there exists some $\mathbf{Z}_k^{(s)} \succeq 0$ for which

$$\text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)}(\mathbf{A}_k - \mathbf{Z}_k^{(s)})\} - \lambda_{\max}(\mathbf{A}_k - \mathbf{Z}_k^{(s)}) \alpha_k^{(s)} \geq \sigma_k^2 \gamma_k. \quad (4.45)$$

This allows us to modify the problem (4.42) as

$$\begin{aligned}
& \min_{\{\mathbf{w}_i, \mathbf{Z}_i^{(s)}\}} \sum_{i=1}^K \mathbf{w}_i^H \mathbf{w}_i \\
& \text{s.t.} \quad \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)}(\mathbf{A}_k - \mathbf{Z}_k^{(s)})\} - \lambda_{\max}(\mathbf{A}_k - \mathbf{Z}_k^{(s)})\alpha_k^{(s)} \geq \sigma_k^2 \gamma_k \\
& \quad \mathbf{Z}_k^{(s)} \succeq 0, \quad k = 1, \dots, K.
\end{aligned} \tag{4.46}$$

Problem 4.46 is a convex SDP problem and can be solved in polynomial time using interior-point algorithms [91], [92].

In the next section, we generalize the worst-case robust beamforming approach that considers the errors in CSI to be the result of both estimation errors and finite sampling effects.

4.6 Robust Beamforming Combining Different Kinds of Errors

Assuming that $\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} \succ 0$, the generalized robust problem can be formulated as

$$\begin{aligned}
& \min_{\{\mathbf{w}_i\}} \sum_{i=1}^K \mathbf{w}_i^H \mathbf{w}_i \\
& \text{s.t.} \quad \min_{\bar{\Delta}_k^{(s)}, \delta_k} \frac{\mathbf{w}_k^H (\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\Delta}_k^{(s)} - \delta_k \mathbf{I}_N) \mathbf{w}_k}{\sum_{j \neq k} \mathbf{w}_j^H (\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\Delta}_k^{(s)} - \delta_k \mathbf{I}_N) \mathbf{w}_j + \sigma_k^2} \geq \gamma_k \\
& \quad \text{Tr}\{\bar{\Delta}_k^{(s)}\} \leq \alpha_k^{(s)}, \quad \tilde{\alpha}_{k2}^{(e)} \leq \delta_k \leq \tilde{\alpha}_{k1}^{(e)}, \quad \hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\Delta}_k^{(s)} \succeq 0 \\
& \quad \hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\Delta}_k^{(s)} - \delta_k \mathbf{I}_N \succeq 0, \quad \bar{\Delta}_k^{(s)} \succeq 0, \quad k = 1, \dots, K
\end{aligned} \tag{4.47}$$

where

$$\tilde{\alpha}_{k1}^{(e)} \triangleq \frac{(\bar{\nu}_k^2 + \xi_k)N}{\mathcal{P}} \tag{4.48}$$

$$\tilde{\alpha}_{k2}^{(e)} \triangleq \min \left\{ \lambda_{\min}(\hat{\mathbf{R}}_k^{(s)}) - \epsilon_k, \frac{(\bar{\nu}_k^2 - \xi_k)N}{\mathcal{P}} \right\}. \tag{4.49}$$

Again considering the inner optimization problem in the QoS constraints of (4.47) as separate problem given by

$$\begin{aligned}
& \min_{\bar{\mathbf{A}}_k^{(s)}} \text{Tr}\{(\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\mathbf{A}}_k^{(s)} - \delta_k \mathbf{I}_N) \mathbf{A}_k\} \\
& \text{s.t.} \quad \hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\mathbf{A}}_k^{(s)} \succeq 0, \quad \hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \bar{\mathbf{A}}_k^{(s)} - \delta_k \mathbf{I}_N \succeq 0 \\
& \quad \text{Tr}\{\bar{\mathbf{A}}_k^{(s)}\} \leq \alpha_k^{(s)}, \quad \tilde{\alpha}_{k2}^{(e)} \leq \delta_k \leq \tilde{\alpha}_{k1}^{(e)}, \quad \bar{\mathbf{A}}_k^{(s)} \succeq 0,
\end{aligned} \tag{4.50}$$

we have the following equivalence of problems.

Lemma 3 (equivalence of worst-case QoS constraints): For a fixed non-zero weight vector \mathbf{w}_i (and correspondingly fixed matrices \mathbf{A}_k), problem (4.50) and the problem

$$\begin{aligned}
& \max_{\beta_k, c_{kj}, \mathbf{Z}_k^{(s)}, \mathbf{Z}_k^{(e)}} \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)}(\mathbf{A}_k - \mathbf{Z}_k^{(s)} - \mathbf{Z}_k^{(e)})\} - \beta_k \alpha_k^{(s)} - c_{k1} \tilde{\alpha}_{k1}^{(e)} + c_{k2} \tilde{\alpha}_{k2}^{(e)} \\
& \text{s.t.} \quad \beta_k \mathbf{I}_N - (\mathbf{A}_k - \mathbf{Z}_k^{(s)} - \mathbf{Z}_k^{(e)}) \succeq 0, \quad \mathbf{Z}_k^{(s)} \succeq 0 \\
& \quad c_{k1} - c_{k2} - \text{Tr}\{\mathbf{A}_k - \mathbf{Z}_k^{(e)}\} \geq 0, \quad \mathbf{Z}_k^{(e)} \succeq 0 \\
& \quad \beta_k \geq 0, \quad c_{k1} \geq 0, \quad c_{k2} \geq 0
\end{aligned} \tag{4.51}$$

have the same optimal value.

Proof: The proof, that the problem (4.51) is the dual of problem (4.50), is similar to the proofs of Lemma 1 and Lemma 2. The problem (4.50) is clearly convex and bounded below. Letting $\mathbf{U}_k \mathbf{\Lambda}_k \mathbf{U}_k^H$ to be the matrix decomposition of $\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)}$, there exists $\bar{\mathbf{A}}_k^{(s)} = \mathbf{U}_k \mathbf{D}_k \mathbf{U}_k^H$ and $\tilde{\delta}_k = \min\{[\mathbf{\Lambda}_k]_{ii}/2, \tilde{\alpha}_k\}$ such that $[\mathbf{D}_k]_{ij} = \min\{[\mathbf{\Lambda}_k]_{ij}/2, \bar{\alpha}_k\}$ with $0 < \bar{\alpha}_k < \alpha_k^{(s)}/N$ and $\tilde{\alpha}_{k2}^{(e)} < \tilde{\alpha}_k < \tilde{\alpha}_{k1}^{(e)}$, that represent a strictly feasible point of problem (4.51). Therefore, using [104, Th. 1.7.1], we have strong duality between (4.50) and (4.51). \square

Using Lemma 3, and arguments similar to those in the last two sections, the problem

(4.47) can be rewritten as

$$\begin{aligned}
& \min_{\{\mathbf{W}_i, c_{k1}, c_{k2}, \mathbf{Z}_i^{(s)}, \mathbf{Z}_i^{(e)}\}} \text{Tr}\left\{\sum_{i=1}^K \mathbf{W}_i\right\} \\
\text{s.t.} \quad & \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)}(\mathbf{A}_k - \mathbf{Z}_k^{(s)} - \mathbf{Z}_k^{(e)})\} - c_{k1}\tilde{\alpha}_{k1}^{(e)} + c_{k2}\tilde{\alpha}_{k2}^{(e)} \\
& - \lambda_{\max}(\mathbf{A}_k - \mathbf{Z}_k^{(s)} - \mathbf{Z}_k^{(e)})\alpha_k^{(s)} \geq \sigma_k^2\gamma_k \\
& c_{k1} - c_{k2} - \text{Tr}\{\mathbf{A}_k - \mathbf{Z}_k^{(e)}\} \geq 0 \\
& \mathbf{W}_k \succeq 0, \mathbf{Z}_k^{(s)} \succeq 0, \mathbf{Z}_k^{(e)} \succeq 0 \\
& c_{k1} \geq 0, c_{k2} \geq 0, k = 1, \dots, K.
\end{aligned} \tag{4.52}$$

Defining $\mathbf{W}_i \triangleq \mathbf{w}_i\mathbf{w}_i^H$, the problem (4.52) can be equivalently rewritten as

$$\begin{aligned}
& \min_{\{\mathbf{W}_i, c_{k1}, c_{k2}, \mathbf{Z}_i^{(s)}, \mathbf{Z}_i^{(e)}\}} \text{Tr}\left\{\sum_{i=1}^K \mathbf{W}_i\right\} \\
\text{s.t.} \quad & \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)}(\mathbf{B}_k - \mathbf{Z}_k^{(s)} - \mathbf{Z}_k^{(e)})\} - c_{k1}\tilde{\alpha}_{k1}^{(e)} + c_{k2}\tilde{\alpha}_{k2}^{(e)} \\
& - \lambda_{\max}(\mathbf{B}_k - \mathbf{Z}_k^{(s)} - \mathbf{Z}_k^{(e)})\alpha_k^{(s)} \geq \sigma_k^2\gamma_k \\
& c_{k1} - c_{k2} - \text{Tr}\{\mathbf{B}_k - \mathbf{Z}_k^{(e)}\} \geq 0, \mathbf{W}_k \succeq 0 \\
& \text{rank}(\mathbf{W}_k) = 1, \mathbf{Z}_k^{(s)} \succeq 0, \mathbf{Z}_k^{(e)} \succeq 0 \\
& c_{k1} \geq 0, c_{k2} \geq 0, k = 1, \dots, K.
\end{aligned} \tag{4.53}$$

where

$$\mathbf{B}_k \triangleq \mathbf{W}_k - \gamma_k \sum_{j \neq i} \mathbf{W}_j. \tag{4.54}$$

The objective function of the problem (4.53) is convex and the rank-one constraint is the only non-convex constraint. Following the SDR technique [50, 93], we remove the rank-one

constraint from problem (4.53) to obtain the following convex SDP problem:

$$\begin{aligned}
& \min_{\{\mathbf{W}_i, c_{kj}, \mathbf{Z}_i^{(s)}, \mathbf{Z}_i^{(e)}\}} \text{Tr}\left\{\sum_{i=1}^K \mathbf{W}_i\right\} \\
\text{s.t.} \quad & \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)}(\mathbf{B}_k - \mathbf{Z}_k^{(s)} - \mathbf{Z}_k^{(e)})\} - c_{k1}\tilde{\alpha}_{k1}^{(e)} + c_{k2}\tilde{\alpha}_{k2}^{(e)} \\
& -\lambda_{\max}(\mathbf{B}_k - \mathbf{Z}_k^{(s)} - \mathbf{Z}_k^{(e)})\alpha_k^{(s)} \geq \sigma_k^2\gamma_k \\
& c_{k1} - c_{k2} - \text{Tr}\{\mathbf{B}_k - \mathbf{Z}_k^{(e)}\} \geq 0, \mathbf{W}_k \succeq 0 \\
& \mathbf{Z}_k^{(s)} \succeq 0, \mathbf{Z}_k^{(e)} \succeq 0 \\
& c_{k1} \geq 0, c_{k2} \geq 0, k = 1, \dots, K.
\end{aligned} \tag{4.55}$$

The resulting problem (4.55) a convex SDP problem and can be solved efficiently using interior-point algorithms [91], [92].

4.7 Discussion

Note that the optimal Lagrange multiplier matrices $\mathbf{Z}_k^{(e)\star}$ and $\mathbf{Z}_k^{(s)\star}$ in (4.28), (4.42), and (4.52) are not necessarily equal to zero matrix in general. Therefore, the robust beamforming problem with constraints having these variables provides beamformers with a lower total transmitted power compared to the problem without these variables in the constraint. However, we claim that if $\hat{\mathbf{R}}_{\mathbf{h}_k} - \alpha_{k1}^{(e)}\mathbf{I}_N \succeq 0$, then we can choose $\mathbf{Z}_k^{(e)\star} = \mathbf{0}$ in (4.28). First, note that the second inequality constraint in (4.26) is active at the optimum and , therefore, we can set $c_{k1} = [c_{k2} + \text{Tr}\{(\mathbf{A}_k - \mathbf{Z}_k^{(e)})\}]^+$ where

$$[x]^+ \triangleq \begin{cases} x, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases} \tag{4.56}$$

Then the reasoning for our claim becomes clear from the fact that

$$\begin{aligned}
& \max_{c_{k2} \geq 0} \text{Tr}\{\mathbf{R}_{\hat{\mathbf{h}}_k} \mathbf{A}_k^*\} - [c_{k2} + \text{Tr}\{\mathbf{A}_k^*\}]^+ \alpha_{k1}^{(e)} + c_{k2} \alpha_{k2}^{(e)} \\
&= \max_{c_{k2} \geq 0, \mathbf{Z}_k^{(e)} \succeq 0} \text{Tr}\{\mathbf{R}_{\hat{\mathbf{h}}_k} \mathbf{A}_k^*\} - \text{Tr}\{(\mathbf{R}_{\hat{\mathbf{h}}_k} - \alpha_{k1}^{(e)} \mathbf{I}_N) \mathbf{Z}_k^{(e)}\} - [c_{k2} + \text{Tr}\{\mathbf{A}_k^*\}]^+ \alpha_{k1}^{(e)} + c_{k2} \alpha_{k2}^{(e)} \\
&= \max_{c_{k2} \geq 0, \mathbf{Z}_k^{(e)} \succeq 0} \text{Tr}\{\mathbf{R}_{\hat{\mathbf{h}}_k} (\mathbf{A}_k^* - \mathbf{Z}_k^{(e)})\} - ([c_{k2} + \text{Tr}\{\mathbf{A}_k^*\}]^+ - \text{Tr}\{\mathbf{Z}_k^{(e)}\}) \alpha_{k1}^{(e)} + c_{k2} \alpha_{k2}^{(e)} \\
&\geq \max_{c_{k2} \geq 0, \mathbf{Z}_k^{(e)} \succeq 0} \text{Tr}\{\mathbf{R}_{\hat{\mathbf{h}}_k} (\mathbf{A}_k^* - \mathbf{Z}_k^{(e)})\} - [c_{k2} + \text{Tr}\{\mathbf{A}_k^* - \mathbf{Z}_k^{(e)}\}]^+ \alpha_{k1}^{(e)} + c_{k2} \alpha_{k2}^{(e)} \\
&\geq \max_{c_{k2} \geq 0} \text{Tr}\{\mathbf{R}_{\hat{\mathbf{h}}_k} \mathbf{A}_k^*\} - [c_{k2} + \text{Tr}\{\mathbf{A}_k^*\}]^+ \alpha_{k1}^{(e)} + c_{k2} \alpha_{k2}^{(e)}. \tag{4.57}
\end{aligned}$$

Similarly, we also claim that if $\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \alpha_k^{(s)} \mathbf{I}_N \succeq 0$, then we can choose $\mathbf{Z}_k^{(s)*} = \mathbf{0}$ in (4.46). Using Weyl Theorem [103, p.181] (in particular, it shows that $\lambda_{\max}(\mathbf{M}_1 + \mathbf{M}_2) \leq \lambda_{\max}(\mathbf{M}_1) + \lambda_{\max}(\mathbf{M}_2)$ where \mathbf{M}_1 and \mathbf{M}_2 are Hermitian matrices), and the fact that $\tilde{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \alpha_k^{(s)} \mathbf{I}_N \succeq 0$, we have

$$\begin{aligned}
& \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} \mathbf{A}_k\} - \lambda_{\max}(\mathbf{A}_k) \alpha_k^{(s)} \\
&= \max_{\mathbf{Z}_k^{(s)} \succeq 0} \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} \mathbf{A}_k\} - \text{Tr}\{(\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} - \alpha_k^{(s)} \mathbf{I}_N) \mathbf{Z}_k^{(s)}\} - \lambda_{\max}(\mathbf{A}_k) \alpha_k^{(s)} \\
&= \max_{\mathbf{Z}_k^{(s)} \succeq 0} \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} (\mathbf{A}_k - \mathbf{Z}_k^{(s)})\} + (\text{Tr}\{\mathbf{Z}_k^{(s)}\} - \lambda_{\max}(\mathbf{A}_k)) \alpha_k^{(s)} \\
&\geq \max_{\mathbf{Z}_k^{(s)} \succeq 0} \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} (\mathbf{A}_k - \mathbf{Z}_k^{(s)})\} + (\lambda_{\max}(\mathbf{Z}_k^{(s)}) - \lambda_{\max}(\mathbf{A}_k)) \alpha_k^{(s)} \\
&\geq \max_{\mathbf{Z}_k^{(s)} \succeq 0} \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} (\mathbf{A}_k - \mathbf{Z}_k^{(s)})\} - \lambda_{\max}(\mathbf{A}_k - \mathbf{Z}_k^{(s)}) \alpha_k^{(s)} \\
&\geq \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k}^{(s)} \mathbf{A}_k\} - \lambda_{\max}(\mathbf{A}_k) \alpha_k^{(s)}. \tag{4.58}
\end{aligned}$$

The above results also apply to the problem (4.52).

Another interesting observation can be made by looking at the bound function used for the errors arising from finite sampling effects. In our proposed robust approach, the uncertainty sets of these errors are modelled as:

$$\text{Tr}\{\mathbf{\Delta}_k^{(s)}\} \leq \alpha_k^{(s)}, \quad \mathbf{\Delta}_k^{(s)} \succeq 0, \quad k = 1, \dots, K. \tag{4.59}$$

Comparing the uncertainty set in (4.59) with the one used in problem (3.4), it can be observed that in the latter set the Frobenius norm explicitly restricts all the entries of

$\mathbf{\Delta}_k^{(s)}$, whereas in our approach, only the diagonal elements of the mismatch matrices are explicitly restricted. However, with the added positive semidefinite constraint in (4.59) the off-diagonal entries of $\mathbf{\Delta}_k^{(s)}$ are also implicitly bounded, see [103, p. 398]. Although the addition of positive semidefiniteness constraint on the mismatch matrices appears to put unnecessary restriction on the uncertainty set, this positive semidefinite constraint is inactive at the optimum and can also be omitted without any effect on the solution of the robust beamforming problem, as discussed in Section 4.5.

Finally, we remark that for $\mathbf{\Delta}_k^{(s)} \succeq 0$, the inequality $\|\mathbf{\Delta}_k^{(s)}\| \leq \text{Tr}\{\mathbf{\Delta}_k^{(s)}\}$ holds, which shows that for the special choice $\alpha_k = \alpha_k^{(s)}$, the uncertainty set (4.59) is contained in the uncertainty set used in problem (3.4).

It can be seen from the problem formulations (4.16) and (4.35), that these problems reduce to the non-robust beamforming problem for $\tilde{\alpha}_{k2}^{(e)} = \tilde{\alpha}_{k1}^{(e)} = \alpha_k^{(s)} = 0$, and therefore, the terms $c_{k1}\tilde{\alpha}_{k1}^{(e)}$, and $\lambda_{\max}(\mathbf{A}_k - \mathbf{Z}_k^{(s)})\alpha_k^{(s)}$ in problems (4.28) and (4.46) quantify the penalty paid for achieving the robustness. From (4.22) and (4.38), we can see that the penalty terms in problems (4.28) and (4.46) are linear. On the other hand, the penalty terms in the problem formulation of [69] are quadratic. Therefore, our problem formulation is comparatively less complex.

As mentioned in Section 3.6 in Chapter 3, it is generally not guaranteed that the solutions to the SDR-based problems are rank-one. In such cases Gaussian randomization technique [97] can be used to obtain an approximate solution. However, in our simulations, we observe that the optimal \mathbf{W}_k^* of the problem (4.55), similar to the SDR approach in [69], are always rank-one and, consequently, the weight vectors \mathbf{w}_k^* can be retrieved exactly from the principal eigenvectors of \mathbf{W}_k^* .

4.8 Simulation Results

In our simulations, we consider a downlink beamforming network with $N = 8$ antennas at the transmitter and $K = 3$ users. The true covariance matrices $\mathbf{R}_{\mathbf{h}_k}$ are generated using the same channel model as used in [50] and [69] with the users located at $\theta_1 = 10^\circ$

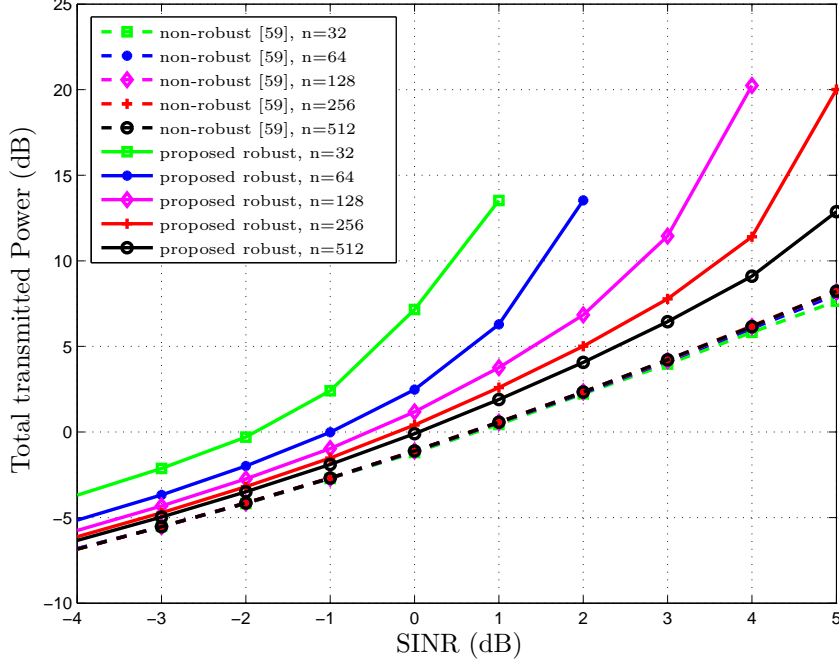


Figure 4.1: Total power versus required SINR with the training power $\mathcal{P} = 10$ dB.

and $\theta_{2,3} = 10^\circ \pm \phi$ relative to the array broadside. The angular spread of $\sigma_\theta = 2^\circ$ is considered. The noise at the user receiver is assumed to be Gaussian with variance $\sigma_k^2 = 1$ dB. We define $\mathbf{U}_k \mathbf{\Lambda}_k \mathbf{U}_k^H$ as the eigenvalue decomposition of $\mathbf{R}_{\mathbf{h}_k}$. Let $\mathbf{h}_k = \mathbf{U}_k \mathbf{\Lambda}_k^{1/2} \mathbf{u}_k$, and $\hat{\mathbf{h}}_k = \mathbf{h}_k + \mathbf{V}^\dagger \mathbf{n}_k$ be the realizations of the true channel and the estimated channel, respectively, where \mathbf{u}_k , and \mathbf{n}_k are $N \times 1$ vectors of i.i.d. circularly-symmetric zero-mean complex Gaussian random variables, respectively, with $E\{\mathbf{u}_k \mathbf{u}_k^H\} = \mathbf{I}_N$ and $E\{\mathbf{n}_k \mathbf{n}_k^H\} = \nu_k^2 \mathbf{I}_N$ where $\nu_k^2 = 1$ dB. Assuming that $T = N$, we choose \mathbf{V} to be the $N \times N$ discrete Fourier transform (DFT) matrix, which is given by

$$\mathbf{V} = \sqrt{\frac{\mathcal{P}}{N^2}} \begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & e^{j2\pi/N} & \dots & e^{j2\pi(N-1)/N} \\ \vdots & \vdots & & \vdots \\ 1 & e^{j2\pi(N-1)/N} & \dots & e^{j2\pi(N-1)^2/N} \end{pmatrix}. \quad (4.60)$$

We use two-sided percentile-t bootstrap and bootstrap principle [108] for calculating the confidence intervals for the means of the estimated channel power and noise power with

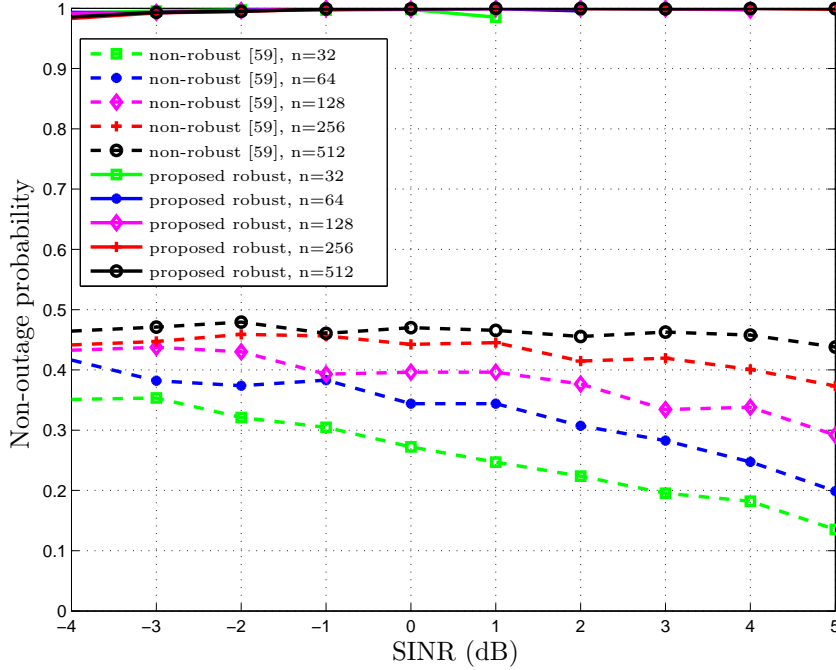


Figure 4.2: Average non-outage SINR probability for the true channel covariance versus required SINR with the training power $\mathcal{P} = 10$ dB.

given confidence levels $p_k = 0.95$, and $\tilde{p}_k = 0.95$, respectively. The numbers of bootstrap resamples and noise samples are chosen to be 3000 and 1000, respectively. It is assumed that $\gamma_k = \gamma$ for $k = 1, \dots, K$. For the non-robust method of [59], the estimated covariance matrix is chosen by subtracting the estimation error matrix from the sample estimate of the covariance matrix, i.e., $\hat{\mathbf{R}}_{h_k}^{(s)} - \frac{\nu_k^2 N}{\mathcal{P}} \mathbf{I}_N$. If any eigenvalue of the resultant matrix is negative, we replace it with zero as the covariance matrix must be a positive semidefinite matrix. We also utilize the normalized constraint value

$$\zeta_k = \frac{\mathbf{w}_k^{*H} \mathbf{R}_{h_k} \mathbf{w}_k^*}{\sum_{\substack{i=1 \\ i \neq k}}^K \gamma_k \mathbf{w}_i^{*H} \mathbf{R}_{h_k} \mathbf{w}_i^* + \gamma_k \sigma_k^2}; \quad k = 1, \dots, K. \quad (4.61)$$

which was defined in (3.79), as an abstract measure of the constraint satisfaction to evaluate the performance of our proposed method and the non-robust method of [59]. The corresponding QoS constraint is satisfied if and only if $\zeta_k \geq 1$. We calculate the SINR value using the actual channel covariance matrix \mathbf{R}_{h_k} and the computed optimal beamformers

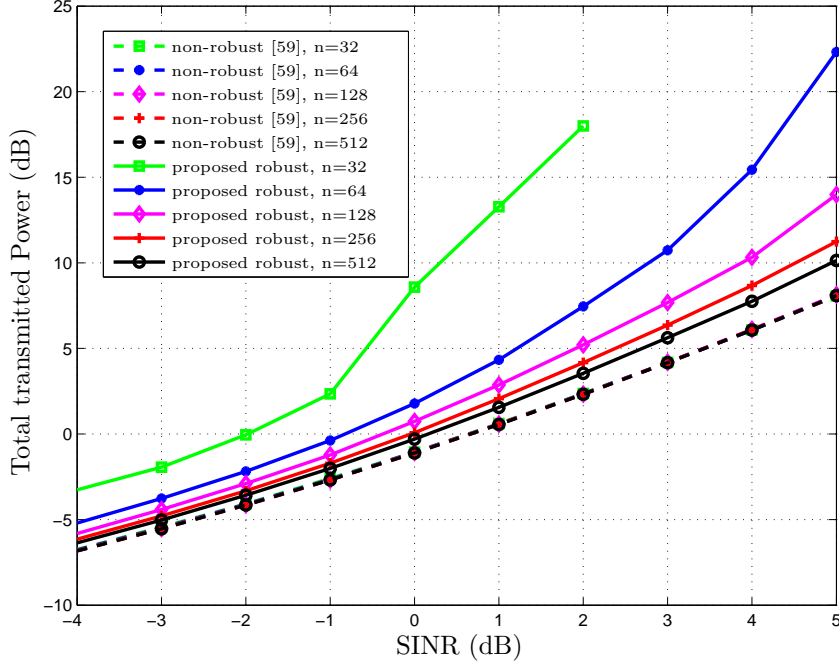


Figure 4.3: Total power versus required SINR with the training power $\mathcal{P} = 15$ dB.

corresponding to each technique. We then define the average non-outage SINR probability of each method by finding out the percentage of SINRs greater than γ_k for a large number of Monte Carlo runs.

In our first example, we compare the non-robust approach of [59] with the proposed method in (4.47) with $\phi = 7^\circ$ and training power $\mathcal{P} = 10$ dB. Fig. 4.1 and Fig. 4.2 plot the minimal total transmitted power and the non-outage probability, respectively, versus the SINR required for different numbers of sampling training blocks. It can be seen from these figures, that increasing the number of sampling training blocks increases the non-outage percentage and reduces the total transmitted power, which converges to the total power of the non-robust method. Moreover, compared to the non-robust approach of [59], our proposed approach substantially improves the performance in terms of non-outage probability, as clear from Fig. 4.2, especially in the region with low SINR values. We then increase the training power value keeping the remaining parameters unchanged. Fig. 4.3 and Fig. 4.4 display the minimal total transmitted power and the non-outage probability, respectively,

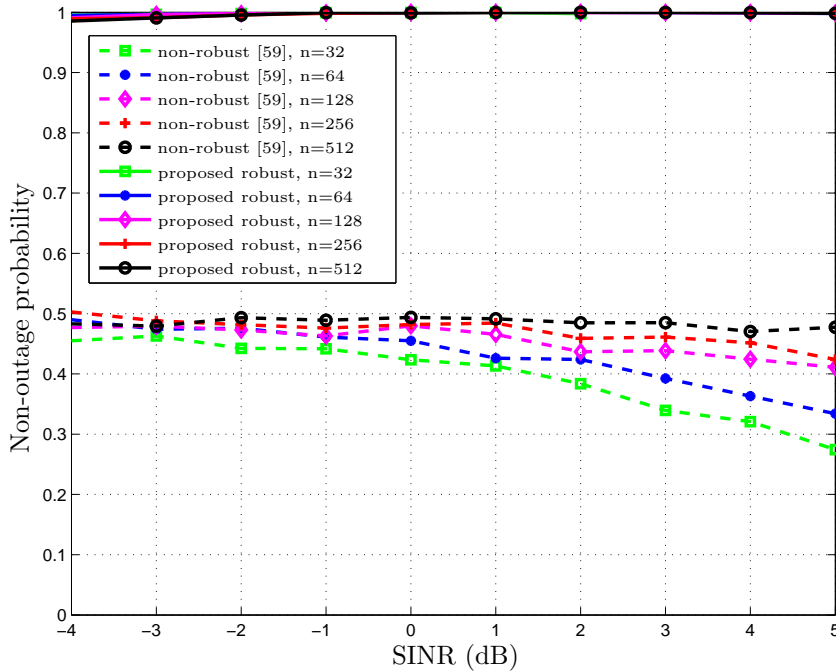


Figure 4.4: Average non-outage SINR probability for the true channel covariance versus required SINR with the training power $\mathcal{P} = 15$ dB.

versus γ for different numbers of sampling training blocks for $\mathcal{P} = 15$ dB. Similar to Figs. 4.1 & 4.2, we observe in Figs. 4.3 & 4.4, that our proposed technique substantially outperforms the non-robust technique of [59].

In the next scenario, we perform the simulations by varying the angle of separation between the users where ϕ is varied from 6° to 12° and we use $\gamma = 2$ dB and $\mathcal{P} = 15$ dB. Fig. 4.5 and Fig. 4.6 show the total transmitted power and the non-outage probability, respectively, versus the angular separation ϕ for different numbers of sampling training blocks. It can be observed from Fig. 4.5 and Fig. 4.6 that the proposed method can improve the performance in terms of non-outage probability compared to the non-robust method of [59] and the minimal total transmitted power of our proposed method approaches the total power of the non-robust method when the number of sampling training blocks is increased. Note that the transmitted power plot of robust approach for $n = 32$ in Fig. 4.5 has large variations. This is due to the fact that this plot is for a small number of sampling training blocks that

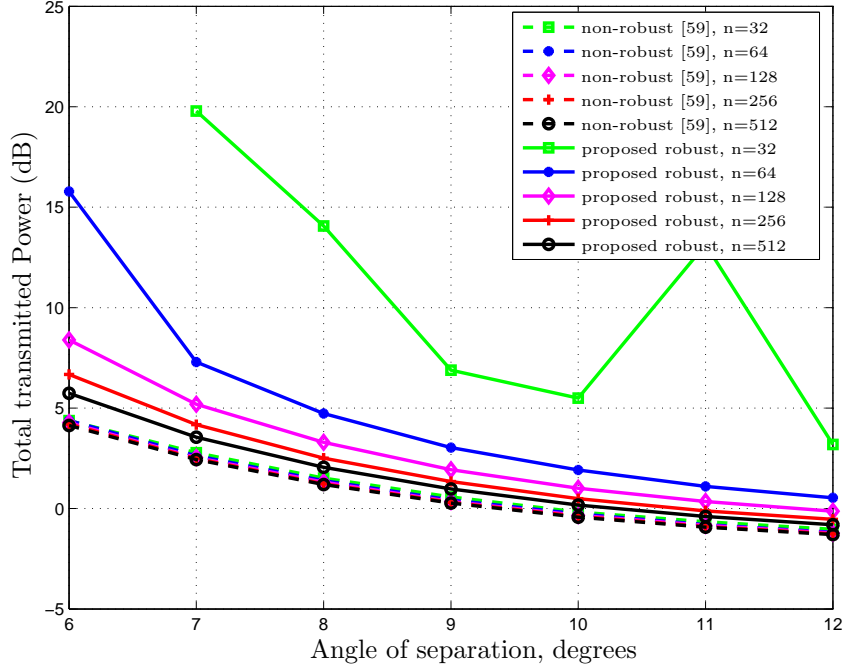


Figure 4.5: Total power versus angle of separation with the training power $\mathcal{P} = 15$ dB.

could lead to large variations in the resulting channel covariance matrices. Finally, we show the distribution of the the normalized constraint value ζ_k for the approaches of [59] and (4.47). Fig. 4.7 displays the histograms of ζ_k for $\phi = 7^\circ$, $\gamma_k = 2$ dB and $\mathcal{P} = 15$ dB. As can be observed from Fig. 4.7, the non-robust technique of [59] only satisfies about 50% of the constraints with a symmetric (normal-like) distribution. However, our proposed approach satisfies almost all of the constraints.

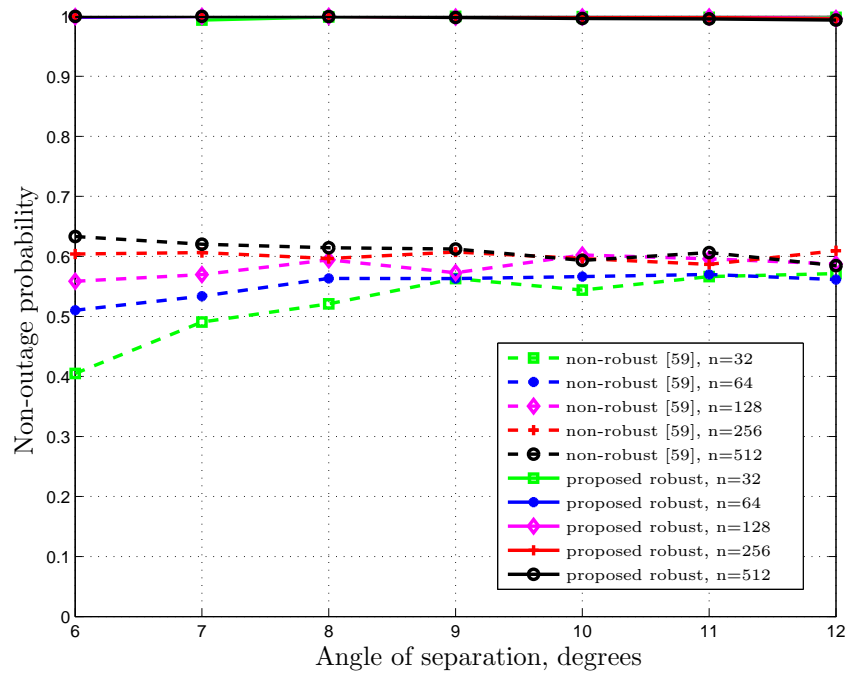


Figure 4.6: Average non-outage SINR probability for the true channel covariance versus angle of separation with $\gamma_k = 2$ dB and training power $\mathcal{P} = 15$ dB.

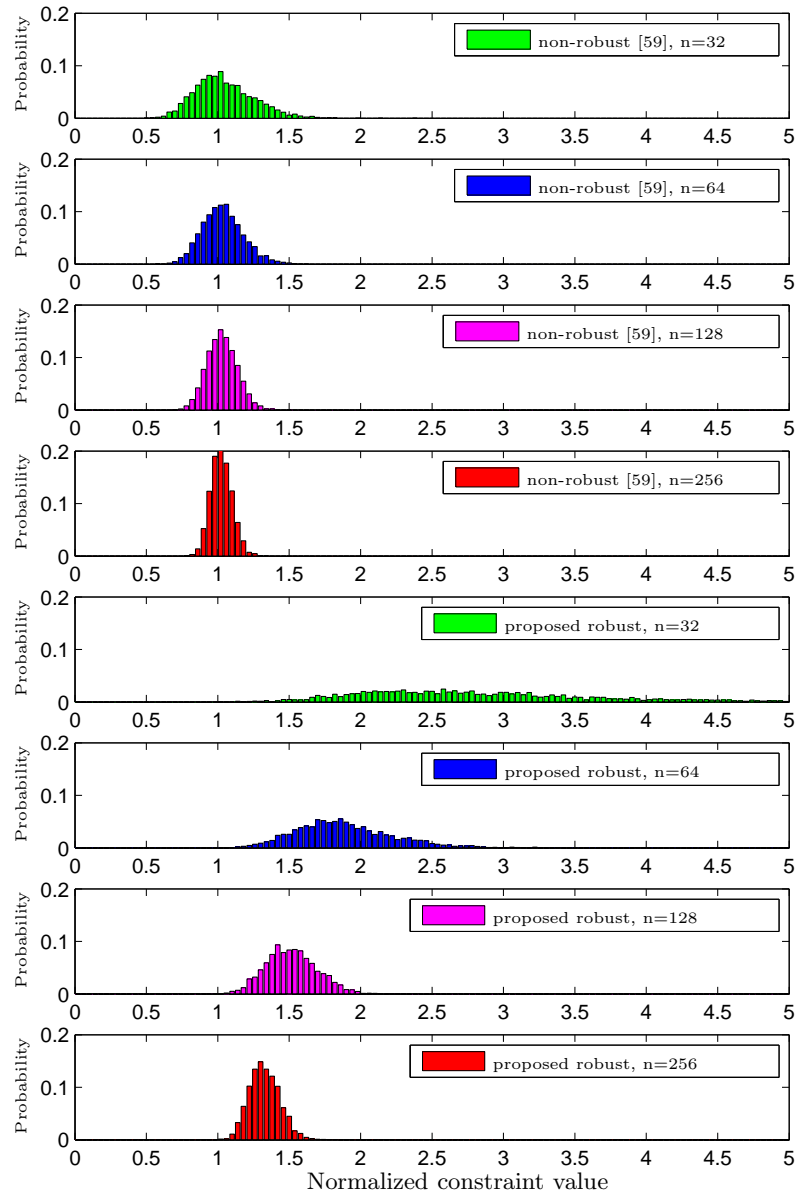


Figure 4.7: Average non-outage SINR probability for the true channel covariance versus angle of separation with $\gamma_k = 2$ dB and training power $\mathcal{P} = 15$ dB.

Chapter 5

Conclusions and Future Work

In this chapter, we present the conclusions drawn from the work presented in this thesis and, later, we discuss some of the possible future extensions to this work.

5.1 Conclusions

In this thesis, we have provided new worst-case based robust approaches for solving the problem of multiuser downlink beamforming using second order covariance-based CSI. Our proposed techniques consider the scenarios of both conventional downlink beamforming and the downlink beamforming in a CR Network.

In the first part of the thesis, we have considered the conventional and CR downlink beamforming problems for the case when the error bounds on the mismatched covariance matrices are defined using a weighted Frobenius norm. In our problem formulations and their derived solutions, we have avoided the coarse approximations used in the previous methods. We have derived exact reformulations of worst-case QoS and PU constraints using Lagrange duality. The resulting problems are then converted into convex SDP problems by applying SDR. The final problem formulations have additional terms in the QoS and PU constraints that represent the penalty being paid for achieving the robustness. We have also solved the aforementioned downlink beamforming problems using the probabilistic constraints, which interestingly leads to the same solutions as the worst-case approach. Generally, rank-one solutions are obtained for the resulting SDPs. However, there exist

special cases under which higher rank solutions are obtained for these SDPs. We have provided a detailed discussion of these special cases in this thesis. Additionally, we have also developed an iterative approach to solve the robust downlink beamforming problem for CR network, that is simpler to implement and, at the same time, retains the benefits of the proposed exact reformulation of the robust problem. We have verified the improvements in terms of the transmitted power and feasibility of the problem in the simulations.

In the second part of the thesis, we have developed an alternate approach for the formulation of robust downlink beamforming problem in the conventional scenario. In contrast to the general approach of putting a cumulative bound on the CSI mismatches using Frobenius norm, we have proposed to bound the CSI errors resulting from different sources independently. In particular, we have considered the CSI imperfections resulting due to the estimation errors and the finite sampling effects. By analyzing the source of error in each case, we have derived appropriate corresponding bounds for the CSI mismatches from different sources. Similar to the first part of this thesis, we have used Lagrange duality to derive the reformulations of worst-case QoS constraints and converted the resulting problems into convex SDP problems using SDR. We have also identified the terms that represent the penalty for achieving the robustness in the QoS constraints. The effectiveness of the proposed approach is verified through extensive simulations.

5.2 Future Work

There can be a few interesting extensions to the work presented in this thesis. The robust downlink beamforming problems presented in this thesis can be infeasible depending upon the number of transmit antennas, the number of users, channel conditions, value of noise power and the required thresholds for the QoS and PU interference constraints. In this case, however, some kind of admission control can be introduced that selects a number of users from the complete set of users and tries to solve the robust beamforming problem only for the selected users. Note that the admission control techniques for the non-robust conventional downlink beamforming have already been presented in [109, 110, 111, 112]

where the instantaneous CSI is used.

In Chapter 4, we have presented a new approach of bounding the CSI uncertainties, that bounds the errors resulting from different sources separately, to solve the robust downlink beamforming problem (2.12). However, in this thesis, we have only applied this approach to the conventional downlink beamforming scenario. This technique can be further extended to the CR scenario by modeling the CSI errors for the PUs using the similar approach. Furthermore, in Chapter 4, we have considered the CSI mismatches arising due to the estimation errors and the finite sampling effects. In this work, we have assumed that there is no error resulting due to the limited capacity of the feedback channels. In the future works, the robust design can be further improved by taking the feedback errors into account as well.

As a final remark, we would like to mention that our approach of modelling the CSI mismatches resulting from different sources independently, can also be applied to a number of robust problems in the field of uplink, downlink and network beamforming.

Appendices

Appendix A

Proof of $\mathbf{Z}_k^\star = \mathbf{0}$

Let $(\{\mathbf{W}_i^\star\}_{i=1}^K, \{\mathbf{Z}_i^\star\}_{i=1}^K)$ denote the optimal matrices solving (3.20). In the following we want to prove by contradiction that $\mathbf{Z}_k^\star = \mathbf{0}$ is optimal for (3.20). To show this, we define

$$\mathbf{Z}_k = \mathbf{Z}_k^\parallel + \mathbf{Z}_k^\perp \quad (\text{A.1})$$

where matrix \mathbf{Z}_k^\parallel is the component of \mathbf{Z}_k in the space spanned by the eigenvectors of \mathbf{A}_k , and \mathbf{Z}_k^\perp is the component in the null space of \mathbf{A}_k . Next, we define the eigendecomposition of $\mathbf{Z}_k - \mathbf{A}_k$ as

$$\mathbf{Z}_k - \mathbf{A}_k = \gamma_k \sum_{\substack{i=1 \\ i \neq k}}^K \mathbf{W}_i - \mathbf{W}_k + \mathbf{Z}_k = \mathbf{U}_k \mathbf{\Lambda}_k \mathbf{U}_k^H \quad (\text{A.2})$$

where

$$\mathbf{U}_k \triangleq [\mathbf{U}_{k+}, \mathbf{U}_{k-}, \mathbf{U}_{k\perp}]; \quad \mathbf{\Lambda}_k \triangleq \begin{bmatrix} \mathbf{\Lambda}_{k+} & 0 & 0 \\ 0 & -\mathbf{\Lambda}_{k-} & 0 \\ 0 & 0 & \mathbf{\Lambda}_{k\perp} \end{bmatrix}. \quad (\text{A.3})$$

The diagonal matrices $\mathbf{\Lambda}_{k+} \succeq 0$ and $\mathbf{\Lambda}_{k-} \succeq 0$ contain the positive and absolute values of the negative eigenvalues of $\mathbf{Z}_k^\parallel - \mathbf{A}_k$ on the main diagonal and the matrices \mathbf{U}_{k+} and \mathbf{U}_{k-} contain the corresponding eigenvectors, respectively. The matrices $\mathbf{\Lambda}_{k\perp} \succeq 0$ and $\mathbf{U}_{k\perp}$ contain the eigenvalues and corresponding eigenvectors of \mathbf{Z}_k^\perp , respectively.

Assuming first that $\mathbf{Z}_k^* = \mathbf{0}$, we can reformulate the first constraint in (3.20) as

$$\begin{aligned}
& -\alpha_k \|\mathbf{A}_k\|_{\mathbf{Q}_k} + \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k} \mathbf{A}_k\} - \sigma_k^2 \gamma_k \\
&= -\alpha_k \sqrt{\text{vec}(\mathbf{A}_k)^H \mathbf{Q}_k \text{vec}(\mathbf{A}_k)} + \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k} \mathbf{A}_k\} - \sigma_k^2 \gamma_k \\
&= -\alpha_k \sqrt{\text{vec}(\mathbf{U}_k \mathbf{\Lambda}_k \mathbf{U}_k^H)^H \mathbf{Q}_k \text{vec}(\mathbf{U}_k \mathbf{\Lambda}_k \mathbf{U}_k^H)} - \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k} \mathbf{U}_k \mathbf{\Lambda}_k \mathbf{U}_k^H\} - \sigma_k^2 \gamma_k \\
&= -\alpha_k \sqrt{\text{vec}(\mathbf{\Lambda}_k)^H (\mathbf{U}_k^* \otimes \mathbf{U}_k)^H \mathbf{Q}_k (\mathbf{U}_k^* \otimes \mathbf{U}_k) \text{vec}(\mathbf{\Lambda}_k)} - \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k} \mathbf{U}_{k+} \mathbf{\Lambda}_{k+} \mathbf{U}_{k+}^H\} \\
&\quad + \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k} \mathbf{U}_{k-} \mathbf{\Lambda}_{k-} \mathbf{U}_{k-}^H\} - \text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_k} \mathbf{U}_{k\perp} \mathbf{\Lambda}_{k\perp} \mathbf{U}_{k\perp}^H\} - \sigma_k^2 \gamma_k \geq 0. \tag{A.4}
\end{aligned}$$

Now we consider that $\mathbf{Z}_k^* \succeq \mathbf{0}$. If at optimum \mathbf{Z}_k^* has any rank-one component in the direction of a column of \mathbf{U}_{k+} , then the corresponding $\mathbf{\Lambda}_{k+}^*$ would have increased entries as compared to the case where $\mathbf{Z}_k^* = \mathbf{0}$. If we replace this non-zero component in \mathbf{Z}_k^* with zero, it would result in a loosened QoS constraint, as can be observed from the last inequality in (A.4). This contradicts optimality as we could then always find a solution with a reduced cost by scaling down \mathbf{W}_k^* without violating other constraints. This argument naturally extends to the case of higher rank components in the space spanned by \mathbf{U}_{k+} . Therefore, we conclude that \mathbf{Z}_k^* has no components in the space spanned by \mathbf{U}_{k+} .

Next, if an arbitrary rank-one component of \mathbf{Z}_k^* in the direction of a column of \mathbf{U}_{k-} is nonzero at optimum, then we could always find a different matrix $\mathbf{Z}_k^{*'}$ similar to \mathbf{Z}_k^* , but with that particular eigenvalue of \mathbf{Z}_k^* set to zero. In this case we could find a $\mathbf{W}_k^{*'}$ similar to \mathbf{W}_k^* where the corresponding component is reduced accordingly such that $\mathbf{Z}_k^{*'} - \mathbf{W}_k^{*'} = \mathbf{Z}_k^* - \mathbf{W}_k^*$. The resulting $\mathbf{W}_k^{*'}$ would yield a reduced objective function as $\text{Tr}\{\mathbf{W}_k^{*'}\} < \text{Tr}\{\mathbf{W}_k^*\}$ without violating other constraints in (3.20). This contradicts to the optimality assumption. This clearly can be extended for higher rank components and we conclude that \mathbf{Z}_k^* has no components in the space spanned by \mathbf{U}_{k-} .

From the last inequality in (A.4), it can be observed that any component of \mathbf{Z}_k^* orthogonal to \mathbf{A}_k , i.e., \mathbf{Z}_k^\perp , only makes the first and fourth terms more negative. This implies that at optimum all entries of $\mathbf{\Lambda}_{k\perp}$ must be zero as otherwise these could be reduced, resulting in a loosened QoS constraint. Again, this contradicts optimality as we could always find a solution with a reduced cost by scaling down \mathbf{W}_k^* without violating other constraints. \square

Appendix B

Example with Higher Rank

Solution

In this appendix, we show conditions under which the solution of (3.21) has higher rank when the covariance matrices are as given in (3.56). For simplicity, we consider the unweighted Frobenius norm to model the error matrices, i.e., $\mathbf{Q}_k = \mathbf{I}$. In this scenario, the inequality QoS constraints for the first and second users in (3.21), respectively, become

$$-\alpha_1 \|\mathbf{W}_1 + \gamma_1 \mathbf{W}_2\| + \text{Tr}\{\hat{\mathbf{R}}_{h_1} \mathbf{W}_1\} - \gamma_1 \text{Tr}\{\hat{\mathbf{R}}_{h_1} \mathbf{W}_2\} - \sigma_1^2 \gamma_1 \geq 0 \quad (\text{B.1})$$

$$-\alpha_2 \|\mathbf{W}_2 + \gamma_2 \mathbf{W}_1\| + \text{Tr}\{\hat{\mathbf{R}}_{h_2} \mathbf{W}_2\} - \gamma_2 \text{Tr}\{\hat{\mathbf{R}}_{h_2} \mathbf{W}_1\} - \sigma_2^2 \gamma_2 \geq 0. \quad (\text{B.2})$$

Let the solution of (3.21) be

$$\mathbf{W}_1 = \begin{pmatrix} a_{11} & a_{21}^* & a_{31}^* \\ a_{21} & a_{22} & a_{32}^* \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad \mathbf{W}_2 = \begin{pmatrix} b_{11} & b_{21}^* & b_{31}^* \\ b_{21} & b_{22} & b_{32}^* \\ b_{31} & b_{32} & b_{33} \end{pmatrix}. \quad (\text{B.3})$$

Then the cost function in (3.21) is given by

$$\text{Tr}\{\mathbf{W}_1 + \mathbf{W}_2\} = a_{11} + a_{22} + a_{33} + b_{11} + b_{22} + b_{33} \quad (\text{B.4})$$

and the terms in the constraint (B.1) become

$$\text{Tr}\{\hat{\mathbf{R}}_{h_1} \mathbf{W}_1\} = r_1 a_{11} + r_2 a_{22}, \quad \text{Tr}\{\hat{\mathbf{R}}_{h_1} \mathbf{W}_2\} = r_1 b_{11} + r_2 b_{22}, \quad (\text{B.5})$$

and

$$\| -\mathbf{W}_1 + \gamma_1 \mathbf{W}_2 \| = \left\| \begin{array}{ccc} -a_{11} + \gamma_1 b_{11} & -a_{21}^* + \gamma_1 b_{21}^* & -a_{31}^* + \gamma_1 b_{31}^* \\ -a_{21} + \gamma_1 b_{21} & -a_{22} + \gamma_1 b_{22} & -a_{32}^* + \gamma_1 b_{32}^* \\ -a_{31} + \gamma_1 b_{31} & -a_{32} + \gamma_1 b_{32} & -a_{33} + \gamma_1 b_{33} \end{array} \right\|. \quad (\text{B.6})$$

The off-diagonal entries of \mathbf{W}_1 and \mathbf{W}_2 have no contribution in $\text{Tr}\{\hat{\mathbf{R}}_{h_1} \mathbf{W}_1\}$, $\text{Tr}\{\hat{\mathbf{R}}_{h_1} \mathbf{W}_2\}$ and the cost function (B.4). However, the norm term (B.6), which is required to be as small as possible, is larger when these entries are non-zero as compared to the case when these are zero. Similarly for the second QoS constraint (B.2), the off-diagonal entries of \mathbf{W}_1 and \mathbf{W}_2 have no effect on the values of terms $\text{Tr}\{\hat{\mathbf{R}}_{h_2} \mathbf{W}_1\}$ and $\text{Tr}\{\hat{\mathbf{R}}_{h_2} \mathbf{W}_2\}$, but make the norm term $\| -\mathbf{W}_2 + \gamma_2 \mathbf{W}_1 \|$ larger. Therefore, at the optimum, the off-diagonal entries of matrices \mathbf{W}_1 and \mathbf{W}_2 must compensate each other, to reduce both the terms (B.6) and $\| -\mathbf{W}_2 + \gamma_2 \mathbf{W}_1 \|$. This implies that, if possible, we should have

$$-a_{ij} + \gamma_1 b_{ij} = 0; \quad i \neq j \quad (\text{B.7})$$

$$-b_{ij} + \gamma_2 a_{ij} = 0; \quad i \neq j. \quad (\text{B.8})$$

If $\gamma_1 = \gamma_2 = 1$, then equations (B.7) and (B.8) have a solution $a_{ij} = b_{ij}$ for $i \neq j$. However, for general γ_1 and γ_2 , both (B.7) and (B.8) are satisfied only if $a_{ij} = b_{ij} = 0$ for $i \neq j$. Therefore, we conclude that the solution of (3.21) has the form

$$\mathbf{W}_1 = \text{diag}\{a_{11}, a_{22}, a_{33}\}, \quad \mathbf{W}_2 = \text{diag}\{b_{11}, b_{22}, b_{33}\}. \quad (\text{B.9})$$

Using (B.9), the QoS constraints (B.1) and (B.2), respectively, can be modified as

$$\begin{aligned} & -\alpha_1 \| -\mathbf{W}_1 + \gamma_1 \mathbf{W}_2 \| + \text{Tr}\{\hat{\mathbf{R}}_{h_1} \mathbf{W}_1\} - \gamma_1 \text{Tr}\{\hat{\mathbf{R}}_{h_1} \mathbf{W}_2\} - \sigma_1^2 \gamma_1 \\ & = -\alpha_1 \sqrt{(a_{11} - \gamma_1 b_{11})^2 + (a_{22} - \gamma_1 b_{22})^2 + (a_{33} - \gamma_1 b_{33})^2} \\ & \quad + r_1(a_{11} - \gamma_1 b_{11}) + r_2(a_{22} - \gamma_1 b_{22}) - \sigma_1^2 \gamma_1 \geq 0 \end{aligned} \quad (\text{B.10})$$

$$\begin{aligned} & -\alpha_2 \| -\mathbf{W}_2 + \gamma_2 \mathbf{W}_1 \| + \text{Tr}\{\hat{\mathbf{R}}_{h_2} \mathbf{W}_2\} - \gamma_2 \text{Tr}\{\hat{\mathbf{R}}_{h_2} \mathbf{W}_1\} - \sigma_2^2 \gamma_2 \\ & = -\alpha_2 \sqrt{(b_{11} - \gamma_2 a_{11})^2 + (b_{22} - \gamma_2 a_{22})^2 + (b_{33} - \gamma_2 a_{33})^2} \\ & \quad + r_3(b_{33} - \gamma_2 a_{33}) - \sigma_2^2 \gamma_2 \geq 0. \end{aligned} \quad (\text{B.11})$$

The only positive terms on the left side of (B.10) are a_{11} or a_{22} . Therefore, when (3.21) is feasible, either a_{11} or a_{22} must be positive. Similarly from (B.11), the term b_{33} must also be positive for a feasible point of (3.21). Now if at optimum, either of the terms a_{33}^* , b_{11}^* , and b_{22}^* is non-zero, we clearly have a higher rank solution for (3.21).

Let us further investigate the case when $a_{33}^* = 0$, $b_{11}^* = 0$, and $b_{22}^* = 0$. This in fact is true for $\gamma_1\gamma_2 \leq 1$ as we prove in the following. Towards this aim, let us first assume that, at the optimum, $a_{33}^* \neq 0$, $b_{11}^* \neq 0$ and $b_{22}^* \neq 0$. If we choose $\bar{b}_{11} = b_{11}^* - \delta_b$ and $\bar{a}_{11} = a_{11}^* - \gamma_1\delta_b$, for some $\delta_b > 0$, and replace a_{11}^* by \bar{a}_{11} and b_{11}^* by \bar{b}_{11} , then the left side of (B.10) remains unchanged while the cost function (B.4) decreases. Next, we note that

$$\begin{aligned}\bar{b}_{11} - \gamma_2\bar{a}_{11} &= b_{11}^* - \delta_b - \gamma_2a_{11}^* + \gamma_1\gamma_2\delta_b \\ &= b_{11}^* - \gamma_2a_{11}^* + (\gamma_1\gamma_2 - 1)\delta_b.\end{aligned}\tag{B.12}$$

For $\gamma_1\gamma_2 \leq 1$, $\bar{b}_{11} - \gamma_2\bar{a}_{11} \leq b_{11}^* - \gamma_2a_{11}^*$ and the left side of (B.11) becomes even more positive. This leads to the conclusion that $\mathbf{W}_1 = \text{diag}\{\bar{a}_{11}, a_{22}^*, a_{33}^*\}$ and $\mathbf{W}_2 = \text{diag}\{\bar{b}_{11}, b_{22}^*, b_{33}^*\}$ denote a feasible point of (3.21) with lower value of cost function (B.4) as compared to $\mathbf{W}_1 = \text{diag}\{a_{11}^*, a_{22}^*, a_{33}^*\}$ and $\mathbf{W}_2 = \text{diag}\{b_{11}^*, b_{22}^*, b_{33}^*\}$. This contradicts optimality and, therefore, we can conclude that at optimum $b_{11}^* = 0$ for $\gamma_1\gamma_2 \leq 1$. Using similar arguments, it can easily be shown that both b_{22}^* and a_{33}^* are also equal to zero at the optimum in this scenario. We can therefore deduct that, for $\gamma_1\gamma_2 \leq 1$, the solution of (3.21) has the form

$$\mathbf{W}_1 = \text{diag}\{a_{11}, a_{22}, 0\}, \quad \mathbf{W}_2 = \text{diag}\{0, 0, b_{33}\}.\tag{B.13}$$

We next derive conditions under which, for $a_{33}^* = b_{11}^* = b_{22}^* = 0$, we still obtain higher rank solutions of (3.21). With \mathbf{W}_2 as given in (B.13), we can see that $\text{Tr}\{\hat{\mathbf{R}}_{h_1}\mathbf{W}_2\} = 0$. Let us assume that at optimum $\text{Tr}\{\mathbf{W}_1\} = c$ where c is a constant. This allows us to introduce the parameterization $a_{11} + a_{22} = c$ such that

$$a_{11} = \mu c, \quad a_{22} = (1 - \mu)c; \quad 0 \leq \mu \leq 1.\tag{B.14}$$

The value of μ can affect both the norm term and the term $\text{Tr}\{\hat{\mathbf{R}}_{h_1}\mathbf{W}_1\}$ and from (B.13), it is obvious that only $\mu = 0$ or 1 corresponds to a rank-one solution. Using (B.13) and the

parameterization (B.14), the QoS constraint (B.1) can be rewritten as

$$\begin{aligned}
& -\alpha_1 \|\mathbf{W}_1 + \gamma_1 \mathbf{W}_2\| + \text{Tr}\{\hat{\mathbf{R}}_{h_1} \mathbf{W}_1\} - \gamma_1 \text{Tr}\{\hat{\mathbf{R}}_{h_1} \mathbf{W}_2\} - \sigma_1^2 \gamma_1 \\
&= -\alpha_1 \sqrt{a_{11}^2 + a_{22}^2 + \gamma_1^2 b_{33}^2} + r_1 a_{11} + r_2 a_{22} - \sigma_1^2 \gamma_1 \\
&= -\alpha_1 \sqrt{\mu^2 c^2 + (1 - \mu)^2 c^2 + \gamma_1^2 b_{33}^2} + r_1 \mu c + r_2 (1 - \mu) c - \sigma_1^2 \gamma_1 \\
&= -\alpha_1 \sqrt{(2\mu^2 + 1 - 2\mu)c^2 + \gamma_1^2 b_{33}^2} + ((r_1 - r_2)\mu + r_2)c - \sigma_1^2 \gamma_1 \geq 0. \tag{B.15}
\end{aligned}$$

Similarly, the QoS constraint (B.2) becomes

$$\begin{aligned}
& -\alpha_1 \|\mathbf{W}_2 + \gamma_2 \mathbf{W}_1\| + \text{Tr}\{\hat{\mathbf{R}}_{h_2} \mathbf{W}_2\} - \gamma_2 \text{Tr}\{\hat{\mathbf{R}}_{h_2} \mathbf{W}_1\} - \sigma_2^2 \gamma_2 \\
&= -\alpha_2 \sqrt{\gamma_2^2 a_{11}^2 + \gamma_2^2 a_{22}^2 + b_{33}^2} + r_3 b_{33} - \sigma_2^2 \gamma_2 \\
&= -\alpha_2 \sqrt{\gamma_2^2 (2\mu^2 + 1 - 2\mu)c^2 + b_{33}^2} + r_3 b_{33} - \sigma_2^2 \gamma_2 \geq 0. \tag{B.16}
\end{aligned}$$

It is easy to show that at optimum both the QoS constraints (B.1) and (B.2) are satisfied at equality. Therefore, to obtain the optimal b_{33}^* and c^* , we set (B.15) and (B.16) equal to zero. This, respectively, results in

$$b_{33}^* = \sqrt{\frac{(((r_1 - r_2)\mu + r_2)c - \sigma_1^2 \gamma_1)^2 - \alpha_1^2 (2\mu^2 + 1 - 2\mu)}{\alpha_1^2 \gamma_1^2}} \tag{B.17}$$

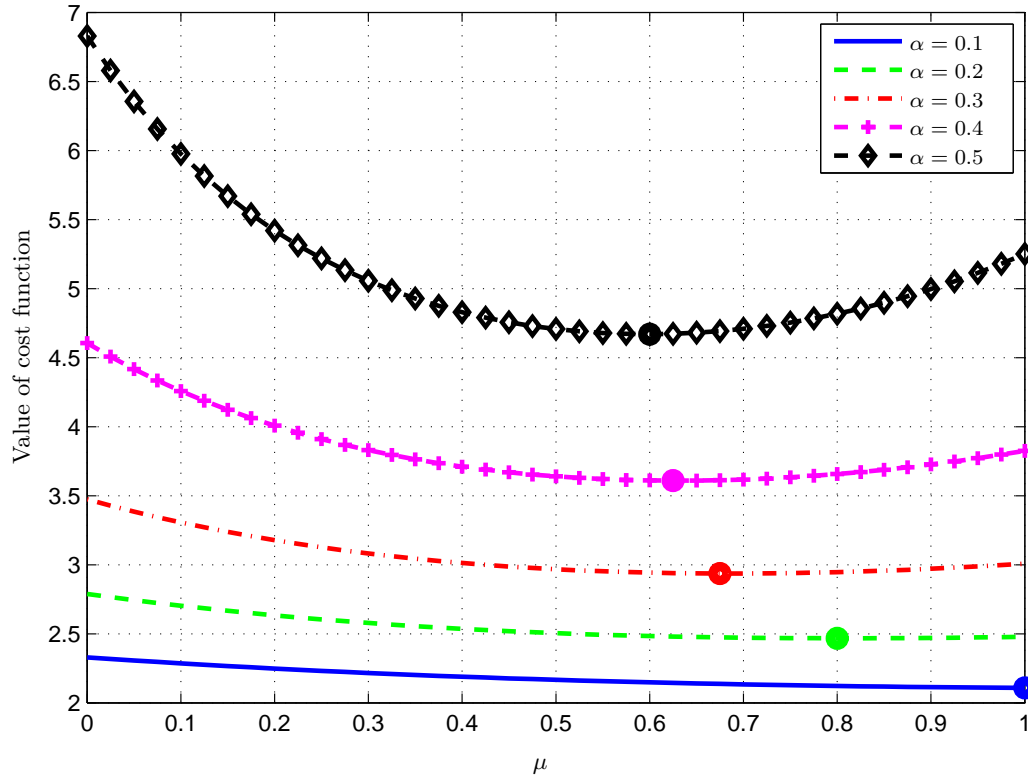
$$c^* = \sqrt{\frac{r_3^2 b_{33}^2 + \sigma_2^4 \gamma_2^2 - 2r_3 b_{33} \sigma_2^2 \gamma_2 - \alpha_2^2 b_{33}^2}{\alpha_2^2 \gamma_2^2 (2\mu^2 + 1 - 2\mu)}}. \tag{B.18}$$

Next, inserting (B.18) in (B.15) and (B.17) in (B.16), and then setting both (B.15) and (B.16) to zero, respectively, we get the decoupled equations as

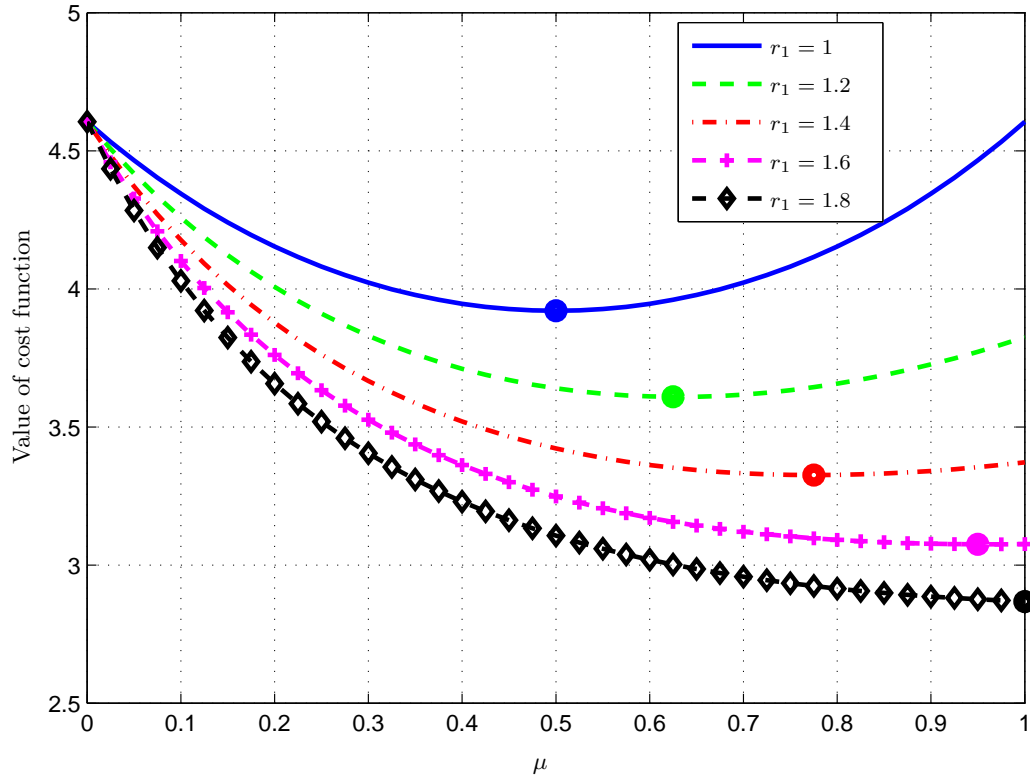
$$-\alpha_1 \sqrt{(2\mu^2 + 1 - 2\mu)c^{*2} + \gamma_1^2 b_{33}^{*2}} + ((r_1 - r_2)\mu + r_2)c^* - \sigma_1^2 \gamma_1 = 0 \tag{B.19}$$

$$-\alpha_2 \sqrt{\gamma_2^2 (2\mu^2 + 1 - 2\mu)c^{*2} + b_{33}^{*2}} + r_3 b_{33}^* - \sigma_2^2 \gamma_2 = 0. \tag{B.20}$$

To find the value of optimal μ^* , we numerically solve both (B.19) and (B.20) by varying μ from 0 to 1, and then select the value of μ that results in the least transmitted power $c + b_{33}$. In Fig. B.1, the value of cost function, i.e., transmitted power, is plotted against μ for different values of α (we consider $\alpha_1 = \alpha_2 = \alpha$). We use $r_1 = 1.2$, $r_2 = 1$, $r_3 = 1$, $\gamma_1 = \gamma_2 = 1$ and $\sigma_1^2 = \sigma_2^2 = 1$ for this plot. Each curve is marked with a circle that represents

Figure B.1: Value of cost function vs μ with varying α .

the minimum for that curve. It can be observed from Fig. B.1 that, with the increase in the uncertainty parameter α , optimal value of μ^* changes from 1 (rank-one solution) towards 0.5 (higher rank solution). This implies that higher rank solutions are favored in the case of larger values of error bounds. Next, we vary the value of r_1 from 1 to 1.8 with $\alpha = 0.4$ in Fig. B.2. As can be seen in Fig. B.2, the optimal μ^* varies from 0.5 towards 1 with the increase in r_1 . It shows that as the covariance matrices become less symmetric, the rank-one solution is favored.

Figure B.2: Value of cost function vs μ with varying r_1 .

Appendix C

Generalization of Solutions

In this appendix, similar to Appendix B, we consider the case when unweighted Frobenius norm is used to model the error matrices. Let \mathbf{W}_k^* ($k = 1, \dots, K$) be the solution of the problem (3.34) when the channel covariance matrices are given as $\hat{\mathbf{R}}_{h_k}$ ($k = 1, \dots, K$). We introduce a new set of covariance matrices, while keeping remaining parameters of problem (3.34) unchanged, as

$$\tilde{\mathbf{R}}_{h_k} = \mathbf{U}^H \hat{\mathbf{R}}_{h_k} \mathbf{U}; \quad k = 1, \dots, K \quad (\text{C.1})$$

where \mathbf{U} is some unitary matrix. With the covariance matrices as defined in (C.1), the modified QoS and PU interference constraints of (3.34), respectively, can be written as

$$-\alpha_k \|\mathbf{A}_k\| - \text{Tr}\{\hat{\mathbf{R}}_{h_k} \mathbf{U} \mathbf{A}_k \mathbf{U}^H\} - \sigma_k^2 \gamma_k \geq 0 \quad (\text{C.2})$$

$$\text{Tr}\{\hat{\mathbf{R}}_{h_{K+l}} \mathbf{U} \mathbf{C} \mathbf{U}^H\} + \alpha_{K+l} \|\mathbf{C}\| \leq \varepsilon_l. \quad (\text{C.3})$$

Defining the matrices

$$\tilde{\mathbf{W}}_k \triangleq \mathbf{U} \mathbf{W}_k \mathbf{U}^H; \quad \tilde{\mathbf{A}}_k \triangleq \mathbf{U} \mathbf{A}_k \mathbf{U}^H; \quad \tilde{\mathbf{C}} \triangleq \mathbf{U} \mathbf{C} \mathbf{U}^H, \quad (\text{C.4})$$

we obtain that $\|\tilde{\mathbf{A}}_k\| = \|\mathbf{A}_k\|$ and $\|\tilde{\mathbf{C}}\| = \|\mathbf{C}\|$. Note that the value of the cost function also remains unchanged as $\sum_{i=1}^K \text{Tr}\{\tilde{\mathbf{W}}_i\} = \sum_{i=1}^K \text{Tr}\{\mathbf{W}_i\}$. Next, the QoS constraints (C.2) and the PU interference constraints (C.3) can be modified as

$$-\alpha_k \|\tilde{\mathbf{A}}_k\| - \text{Tr}\{\hat{\mathbf{R}}_{h_k} \tilde{\mathbf{A}}_k\} - \sigma_k^2 \gamma_k \geq 0 \quad (\text{C.5})$$

$$\text{Tr}\{\hat{\mathbf{R}}_{\mathbf{h}_{K+l}}\tilde{\mathbf{C}}\} + \alpha_{K+l}\|\tilde{\mathbf{C}}\| \leq \varepsilon_l. \quad (\text{C.6})$$

This implies that the solution for the problem with the modified covariance matrices (C.1) can be obtained as $\mathbf{U}^H \mathbf{W}_k^* \mathbf{U}$.

Bibliography

- [1] W. Ackroyd, “The photophone,” in Science for All, Cassell & Co., London, ca. 1884, vol. 2 (R. Brown, ed.), pp. 307– 312.
- [2] G. B. Giannakis, Z. Liu, X. Ma, and S. Zhou, Space-Time Coding for Broadband Wireless Communications, John Wiley & Sons, Inc., Hoboken, New Jersey, 2007.
- [3] “Smartphone market hits all-time quarterly high due to seasonal strength and wider variety of offerings,” Feb. 2012, <http://www.idc.com/getdoc.jsp?containerId=prUS23299912>.
- [4] “Teliasonera annual report 2008-company description,” Available at <http://reports.teliasonera.com/2008/annualreport2008/en/>.
- [5] T. S. Rappaport, Wireless Communications: Principles and Practice, Prentice Hall, Upper Saddle River, NJ, 1996.
- [6] J. G. Proakis, Digital Communications, McGraw-Hill, New York, 2001.
- [7] A. B. Gershman, N. D. Sidiropoulos, S. Shahbazpanahi, M. Bengtsson, and B. Ottersten, “Convex optimization-based beamforming: From receive to transmit and network designs,” IEEE Signal Processing Magazine, vol. 27, no. 3, pp. 62–75, May 2010.
- [8] E. Dahlman, S. Parkvall, and J. Skold, 4G LTE/LTE-Advanced for Mobile Broadband, Elsevier, 2011.

- [9] Federal Communications Commission Spectrum Policy Task Force, “Report: Et docket no. 02-135,” Nov. 2002, www.fcc.gov/sptf/reports.html.
- [10] J. Mitola and G. Q. Maguire, “Cognitive radio: making software radios more personal,” IEEE Personal Communications, vol. 6, no. 4, pp. 13–18, Aug. 1999.
- [11] N. Yi, Y. Ma, and R. Tafazolli, “Underlay cognitive radio with full or partial channel quality information,” International Journal of Navigation and Observation, vol. 2010, 2010.
- [12] W. A. Gardner, “Exploitation of spectral redundancy in cyclostationary signals,” IEEE Signal Processing Magazine, vol. 8, no. 2, pp. 14–36, Apr. 1991.
- [13] Kay S. M., Fundamentals of Statistical Signal Processing: Detection Theory, vol. 2, Prentice-Hall, Englewood Cliffs, NJ, 1998.
- [14] R. Tandra and A. Sahai, “Fundamental limits on detection in low SNR under noise uncertainty,” in International Conference on Wireless Networks, Communications and Mobile Computing, 2005, June 2005, vol. 1, pp. 464–469.
- [15] D. Cabric, A. Tkachenko, and R. W. Brodersen, “Spectrum sensing measurements of pilot, energy, and collaborative detection,” in IEEE Military Communications Conference, MILCOM’06., Oct. 2006, pp. 1–7.
- [16] Y.-C. Liang, Y. Zeng, E. C. Y. Peh, and A. T. Hoang, “Sensing-throughput tradeoff for cognitive radio networks,” IEEE Transactions on Wireless Communications, vol. 7, no. 4, pp. 1326–1337, Apr. 2008.
- [17] T. Cover, “An achievable rate region for the broadcast channel,” IEEE Transactions on Information Theory, vol. 21, no. 4, pp. 399–404, Jul. 1975.
- [18] T. Han and K. Kobayashi, “A new achievable rate region for the interference channel,” IEEE Transactions on Information Theory, vol. 27, no. 1, pp. 49–60, Jan. 1981.

- [19] M. H. M. Costa, “Writing on dirty paper,” IEEE Transactions on Information Theory, vol. IT-29, no. 3, pp. 439441, 1983.
- [20] S. Haykin, “Cognitive radio: Brain-empowered wireless communications,” IEEE Journal on Selected Areas in Communications, vol. 23, no. 2, pp. 201–220, Feb. 2005.
- [21] G. Scutari and D. P. Palomar, “MIMO cognitive radio: A game theoretical approach,” IEEE Transactions on Signal Processing, vol. 58, no. 2, pp. 761–780, Feb. 2010.
- [22] picoChip, “The case for home base stations,” April 2007, Technical white paper.
- [23] Analysys Mason, “Picocells and femtocells: Will indoor base stations transform the telecoms industry?,” March 2007, Available at <http://www.analysismason.com/Research/Custom/Reports/>.
- [24] V. Chandrasekhar, J. G. Andrews, and A. Gatherer, “Femto networks: A survey,” IEEE Communications Magazine, vol. 46, no. 9, pp. 5967, Sep. 2008.
- [25] D. Calin, H. Claussen, and H. Uzunalioglu, “On femto deployment architectures and macrocell offloading benefits in joint macro-femto deployments,” IEEE Communications Magazine, vol. 48, no. 1, pp. 26–32, Jan. 2010.
- [26] D.-C. Oh, H.-C. Lee, and Y.-H. Lee, “Power control and beamforming for femtocells in the presence of channel uncertainty,” IEEE Transactions on Vehicular Technology, vol. 60, no. 6, pp. 2545–2554, July 2011.
- [27] F. R. Farrokhi, L. Tassiulas, and K. J. R. Liu, “Joint optimal power control and beamforming in wireless networks using antenna arrays,” IEEE Transactions on Communications, vol. 46, no. 10, pp. 1313–1324, Oct. 1998.
- [28] F. R. Farrokhi, K. J. R. Liu, and L. Tassiulas, “Transmit beamforming and power control for cellular wireless systems,” IEEE Journal on Selected Areas in Communications, vol. 16, no. 8, pp. 1437–1450, Oct 1998.

- [29] T. W. Monzingo, R. A. and Miller, Introduction to Adaptive Arrays, SciTech Publishing, New York, 1980.
- [30] H. L. Van Trees, Optimum Array Processing, Wiley, New York, 2002.
- [31] H. Cox, R. Zeskind, and M. Owen, “Robust adaptive beamforming,” IEEE Transactions on Acoustics, Speech and Signal Processing, vol. 35, no. 10, pp. 1365–1376, Oct. 1987.
- [32] H. Cox and R. Pitre, “Robust DMR and multi-rate adaptive beamforming,” in Conference Record of the Thirty-First Asilomar Conference on Signals, Systems & Computers, 1997, Nov. 1997, vol. 1, pp. 920–924.
- [33] A. B. Gershman, “Robust adaptive beamforming in sensor arrays,” AEU International Journal of Electronics and Communications, vol. 53, pp. 305–314, Dec. 1999.
- [34] S. Shahbazpanahi, A. B. Gershman, Z.-Q. Luo, and K. M. Wong, “Robust adaptive beamforming for general-rank signal models using worst-case performance optimization,” in Sensor Array and Multichannel Signal Processing Workshop Proceedings, 2002, Aug. 2002, pp. 13–17.
- [35] S. A. Vorobyov, A. B. Gershman, and Z.-Q. Luo, “Robust adaptive beamforming using worst-case performance optimization: a solution to the signal mismatch problem,” IEEE Transactions on Signal Processing, vol. 51, no. 2, pp. 313–324, Feb. 2003.
- [36] S. Shahbazpanahi, A. B. Gershman, Z.-Q. Luo, and K. M. Wong, “Robust adaptive beamforming for general-rank signal models,” IEEE Transactions on Signal Processing, vol. 51, no. 9, pp. 2257–2269, Sep. 2003.
- [37] S. A. Vorobyov, A. B. Gershman, Z.-Q. Luo, and N. Ma, “Adaptive beamforming with joint robustness against mismatched signal steering vector and interference non-stationarity,” IEEE Signal Processing Letters, vol. 11, no. 2, pp. 108–111, Feb. 2004.

- [38] R. G. Lorenz and S. P. Boyd, “Robust minimum variance beamforming,” IEEE Transactions on Signal Processing, vol. 53, no. 5, pp. 1684–1696, May 2005.
- [39] A. El-Keyi, T. Kirubarajan, and A. B. Gershman, “Wideband robust beamforming based on worst-case performance optimization,” in IEEE SSP Workshop, Bordeaux, France, July 2005, pp. 265–270.
- [40] S.-J. Kim, A. Magnani, A. Mutapcic, S. P. Boyd, and Z.-Q. Luo, “Robust beamforming via worst-case SINR maximization,” IEEE Transactions on Signal Processing, vol. 56, no. 4, pp. 1539–1547, Apr. 2008.
- [41] M. Rubsamen and A. B. Gershman, “Robust presteered broadband beamforming based on worst-case performance optimization,” in 5th IEEE Sensor Array and Multichannel Signal Processing Workshop, SAM’08., July 2008, pp. 340–344.
- [42] Z.-Q. Luo and W. Yu, “An introduction to convex optimization for communications and signal processing,” IEEE Journal on Selected Areas in Communications, vol. 24, no. 8, pp. 1426–1438, Aug. 2006.
- [43] Y. Rong, S. A. Vorobyov, and A. B. Gershman, “A robust linear receiver for uplink multi-user MIMO systems based on probability-constrained optimization and second-order cone programming,” in IEEE Sensor Array and Multichannel Signal Processing Workshop Proceedings, 2004, July 2004, pp. 153–157.
- [44] Y. Rong, S. A. Vorobyov, and A.B. Gershman, “Robust linear receiver design for multi-access space-time block coded MIMO systems using stochastic optimization,” in IEEE Workshop Statistical Signal Processing, Bordeaux, France, Jul. 2005, pp. 65–70.
- [45] S. A. Vorobyov, Y. Rong, and A. B. Gershman, “Robust minimum variance adaptive beamformers and multiuser MIMO receivers: From the worst-case to probabilistically constrained designs,” in IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP’06, May 2006, vol. 5.

- [46] S. A. Vorobyov, H. Chen, and A. B. Gershman, "On the relationship between robust minimum variance beamformers with probabilistic and worst-case distortionless response constraints," IEEE Transactions on Signal Processing, vol. 56, no. 11, pp. 5719–5724, Nov. 2008.
- [47] C. Farsakh and J. A. Nossek, "Spatial covariance based downlink beamforming in an SDMA mobile radio system," IEEE Transactions on Communications, vol. 46, no. 11, pp. 1497–1506, Nov. 1998.
- [48] L. C. Godara, "Applications of antenna arrays to mobile communications. I. Performance improvement, feasibility, and system considerations," Proceedings of the IEEE, vol. 85, no. 7, pp. 1031–1060, July 1997.
- [49] L. C. Godara, "Application of antenna arrays to mobile communications. II. Beamforming and direction-of-arrival considerations," Proceedings of the IEEE, vol. 85, no. 8, pp. 1195–1245, Aug. 1997.
- [50] M. Bengtsson and B. Ottersten, "Optimal and suboptimal transmit beamforming," in Handbook of Antennas in Wireless Communications, L. C. Godara, Ed., chapter 18. CRC Press, Aug. 2001.
- [51] D. Gerlach and A. Paulraj, "Adaptive transmitting antenna arrays with feedback," IEEE Signal Processing Letters, vol. 1, no. 10, pp. 150–152, Oct. 1994.
- [52] D. Samardzija and N. Mandayam, "Unquantized and uncoded channel state information feedback in multiple-antenna multiuser systems," IEEE Transactions on Communications, vol. 54, no. 7, pp. 1335–1345, July 2006.
- [53] S. P. Alex and L. M. A. Jalloul, "Performance evaluation of MIMO in IEEE802.16e/WiMAX," IEEE Journal of Selected Topics in Signal Processing, vol. 2, no. 2, pp. 181–190, Apr. 2008.
- [54] E. Visotsky and U. Madhow, "Optimum beamforming using transmit antenna arrays," in IEEE 49th Vehicular Technology Conference, 1999, Jul 1999, vol. 1, pp. 851–856.

- [55] E. Karipidis, N.D. Sidiropoulos, and Zhi-Quan Luo, “Quality of service and max-min fair transmit beamforming to multiple cochannel multicast groups,” IEEE Transactions on Signal Processing, vol. 56, no. 3, pp. 1268–1279, March 2008.
- [56] N. Bornhorst and M. Pesavento, “An iterative convex approximation approach for transmit beamforming in multi-group multicasting,” in IEEE 12th International Workshop on Signal Processing Advances in Wireless Communications, SPAWC’11, June 2011, pp. 426–430.
- [57] M. Schubert and H. Boche, “Solution of the multiuser downlink beamforming problem with individual SINR constraints,” IEEE Transactions on Vehicular Technology, vol. 53, no. 1, pp. 18–28, Jan. 2004.
- [58] M. Schubert and H. Boche, “Iterative multiuser uplink and downlink beamforming under SINR constraints,” IEEE Transactions on Signal Processing, vol. 53, no. 7, pp. 2324–2334, July 2005.
- [59] M. Bengtsson and B. Ottersten, “Optimal downlink beamforming using semidefinite optimization,” in 37th Annual Allerton Conference on Communications, Control and Computing, Sep. 1999, pp. 987–996.
- [60] B. Hassibi and B. M. Hochwald, “How much training is needed in multiple-antenna wireless links?,” IEEE Transactions on Information Theory, vol. 49, no. 4, pp. 951–963, April 2003.
- [61] D. J. Love, R. W. Heath, W. Santipach, and M. L. Honig, “What is the value of limited feedback for MIMO channels?,” IEEE Communications Magazine, vol. 42, no. 10, pp. 54–59, oct. 2004.
- [62] M. B. Shenoouda and T. N. Davidson, “Convex conic formulations of robust downlink precoder designs with quality of service constraints,” IEEE Journal of Selected Topics in Signal Processing, vol. 1, no. 4, pp. 714–724, Dec. 2007.

- [63] A. Mutapcic, S.-J. Kim, and S. Boyd, “A tractable method for robust downlink beamforming in wireless communications,” in Conference Record of the Forty-First Asilomar Conference on Signals, Systems and Computers, ACSSC’07, Nov. 2007, pp. 1224–1228.
- [64] N. Vucic and H. Boche, “Downlink precoding for multiuser MISO systems with imperfect channel knowledge,” in IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP’08, April 2008, pp. 3121–3124.
- [65] M. Biguesh, S. Shahbazpanahi, and A. B. Gershman, “Robust power adjustment for transmit beamforming in cellular communication systems,” in IEEE International Conference on Acoustics, Speech, and Signal Processing, ICASSP’03, April 2003, vol. 5, pp. 1520–6149.
- [66] M. Biguesh, S. Shahbazpanahi, and A. B. Gershman, “Robust downlink power adjustment in cellular communication systems with antenna arrays at base stations,” in 4th IEEE Workshop on Signal Processing Advances in Wireless Communications, SPAWC’03, June 2003, pp. 634–638.
- [67] M. Biguesh, S. Shahbazpanahi, and A. B. Gershman, “Robust downlink power control in wireless cellular systems,” EURASIP Journal on Wireless Communications and Networking, vol. 2004, no. 2, pp. 261–272, 2004.
- [68] V. Sharma, I. Wajid, A. B. Gershman, Haihua Chen, and S. Lambbotharan, “Robust downlink beamforming using positive semi-definite covariance constraints,” in International ITG Workshop on Smart Antennas, WSA’08., Feb. 2008, pp. 36–41.
- [69] I. Wajid, Y. C. Eldar, and A. B. Gershman, “Robust downlink beamforming using covariance channel state information,” in IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP’09., April 2009, pp. 2285–2288.

- [70] B. K. Chalise and A. Czylik, "Robust uplink beamforming based upon minimum outage probability criterion," in IEEE Global Telecommunications Conference, GLOBECOM '04, Dec. 2004, vol. 6, pp. 3974–3978.
- [71] B. K. Chalise, S. Shahbazpanahi, A. Czylik, and A. B. Gershman, "Robust downlink beamforming based on outage probability specifications," IEEE Transactions on Wireless Communications, vol. 6, no. 10, pp. 3498–3503, October 2007.
- [72] G. Scutari, D. P. Palomar, and S. Barbarossa, "MIMO cognitive radio: A game theoretical approach," in IEEE 9th Workshop on Signal Processing Advances in Wireless Communications, SPAWC'08, July 2008, pp. 426–430.
- [73] H. Islam, Y.-C. Liang, and A. T. Hoang, "Joint beamforming and power control in the downlink of cognitive radio networks," in IEEE Wireless Communications and Networking Conference, WCNC'07, Mar. 2007, pp. 21–26.
- [74] X. Fu, J. Wang, and S. Li, "Joint power management and beamforming for base stations in cognitive radio systems," in 6th International Symposium on Wireless Communication Systems, ISWCS'09, Sep. 2009, pp. 403–407.
- [75] K. Cumanan, L. Musavian, S. Lambotharan, and A. B. Gershman, "SINR balancing technique for downlink beamforming in cognitive radio networks," IEEE Signal Processing Letters, vol. 17, no. 2, pp. 133–136, Feb. 2010.
- [76] S. You, G. Noh, J. Lee, H. Wang, and D. Hong, "Joint beamforming and power control algorithm for cognitive radio network with the multi-antenna base station," in IEEE Wireless Communications and Networking Conference, WCNC'10, Apr. 2010, pp. 1–6.
- [77] M. Pesavento, D. Ciochina, and A. B. Gershman, "Iterative dual downlink beamforming for cognitive radio networks," in Fifth International Conference on Cognitive Radio Oriented Wireless Networks Communications, CROWNCOM'10, June 2010, pp. 1–5.

- [78] F. Negro, I. Ghauri, and D. T. M. Slock, “Beamforming for the underlay cognitive MISO interference channel via UL-DL duality,” in Fifth International Conference on Cognitive Radio Oriented Wireless Networks Communications, CROWNCOM’10, June 2010.
- [79] K. Cumanan, R. Krishna, V. Sharma, and S. Lambotharan, “Robust interference control techniques for multiuser cognitive radios using worst-case performance optimization,” in 42nd Asilomar Conference on Signals, Systems and Computers, 2008, Oct. 2008, pp. 378–382.
- [80] G. Zheng, K.-K. Wong, and B. Ottersten, “Robust cognitive beamforming with bounded channel uncertainties,” IEEE Transactions on Signal Processing, vol. 57, no. 12, pp. 4871–4881, Dec. 2009.
- [81] M. F. Hanif, P. J. Smith, and M. Alouini, “SINR balancing in the downlink of cognitive radio networks with imperfect channel knowledge,” in Fifth International Conference on Cognitive Radio Oriented Wireless Networks Communications, CROWNCOM’10, June 2010, pp. 1–5.
- [82] E. A. Gharavol, Y.-C. Liang, and K. Moutaana, “Robust downlink beamforming in multiuser MISO cognitive radio networks with imperfect channel-state information,” IEEE Transactions on Vehicular Technology, vol. 59, no. 6, pp. 2852–2860, July 2010.
- [83] I. Wajid, M. Pesavento, Y. C. Eldar, and A. B. Gershman, “Robust downlink beamforming for cognitive radio networks,” in IEEE Global Telecommunications Conference GLOBECOM’10, Dec. 2010.
- [84] Y. Huang, Q. Li, W.-K. Ma, and S. Zhang, “Robust secondary multicast transmit beamforming for cognitive radio networks under imperfect channel state information,” in IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP’11, May 2011, pp. 2816–2819.

- [85] I. Wajid, H. Nikolaeva, and M. Pesavento, "Iterative robust downlink beamforming in cognitive radio networks," in Sixth International ICST Conference on Cognitive Radio Oriented Wireless Networks and Communications CROWNCOM'11, June 2011, pp. 375–379.
- [86] J.-H. Noh and S.-J. Oh, "Beamforming in cognitive radio with partial channel state information," in IEEE Global Telecommunications Conference, GLOBECOM'11, Dec. 2011, pp. 1–6.
- [87] L. Zhang, Y.-C. Liang, and Y. Xin, "Joint beamforming and power allocation for multiple access channels in cognitive radio networks," IEEE Journal on Selected Areas in Communications, vol. 26, no. 1, pp. 38–51, Jan. 2008.
- [88] H. Islam, Y.-C. Liang, and A. T. Hoang, "Joint power control and beamforming for cognitive radio networks," IEEE Transactions on Wireless Communications, vol. 7, no. 7, pp. 2415–2419, July 2008.
- [89] N. Noori, S. M. Razavizadeh, and A. Attar, "Limiting harmful interference to the primary users through joint power allocation and beamforming in the uplink of cognitive radio networks," in IEEE 20th International Symposium on Personal, Indoor and Mobile Radio Communications, 2009, Sep. 2009, pp. 1246–1250.
- [90] S. Boyd and L. Vandenberghe, Convex Optimization, Cambridge University Press, 2004.
- [91] J. F. Sturm, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones," 1998.
- [92] M. Grant, S. Boyd, and Y. Ye, "CVX: MATLAB software for disciplined convex programming (web page and software)," June 2009, <http://www.stanford.edu/boyd/cvx/V.1.0RC3>.

- [93] Z.-Q. Luo, W.-K. Ma, A.M.-C. So, Y. Ye, and S. Zhang, “Semidefinite relaxation of quadratic optimization problems,” IEEE Signal Processing Magazine, vol. 27, no. 3, pp. 20–34, May 2010.
- [94] Y. Huang and D. P. Palomar, “Rank-constrained separable semidefinite programming with applications to optimal beamforming,” IEEE Transactions on Signal Processing, vol. 58, no. 2, pp. 664–678, Feb. 2010.
- [95] Y. Huang and D. P. Palomar, “A dual perspective on separable semidefinite programming with applications to optimal downlink beamforming,” IEEE Transactions on Signal Processing, vol. 58, no. 8, pp. 4254–4271, Aug. 2010.
- [96] K. T. Phan, S. A. Vorobyov, N. D. Sidiropoulos, and C. Tellambura, “Spectrum sharing in wireless networks via QoS-aware secondary multicast beamforming,” IEEE Transactions on Signal Processing, vol. 57, no. 6, pp. 2323–2335, June 2009.
- [97] N. D. Sidiropoulos, T. N. Davidson, and Z.-Q. Luo, “Transmit beamforming for physical-layer multicasting,” IEEE Transactions on Signal Processing, vol. 54, no. 6, pp. 2239–2251, June 2006.
- [98] W. Yu and T. Lan, “Downlink beamforming with per-antenna power constraints,” in IEEE 6th Workshop on Signal Processing Advances in Wireless Communications, 2005, June 2005, pp. 1058–1062.
- [99] W. Yu and T. Lan, “Transmitter optimization for the multi-antenna downlink with per-antenna power constraints,” IEEE Transactions on Signal Processing, vol. 55, no. 6, pp. 2646–2660, June 2007.
- [100] Y. K. Yazarel and D. Aktas, “Downlink beamforming under individual SINR and per antenna power constraints,” in IEEE Pacific Rim Conference on Communications, Computers and Signal Processing, PacRim’07., Aug. 2007, pp. 422–425.
- [101] S. Zhang, “Quadratic maximization and semidefinite relaxation,” Mathematical Programming, vol. 87, no. ser. A, pp. 453–465, 2000.

- [102] Y. I. Abramovich, “Controlled method for adaptive optimization of filters using the criterion of maximum SNR,” Radio Eng. Electron. Phys., vol. 26, pp. 87–95, Mar. 1981.
- [103] R. A. Horn and C. R. Johnson, Matrix Analysis, Cambridge University Press, 1985.
- [104] A. Nemirovski, Lectures on Modern Convex Optimization, Class Notes, Atlanta, GA: Georgia Institute of Technology, 2005.
- [105] M. Biguesh and A. B. Gershman, “Training-based MIMO channel estimation: a study of estimator tradeoffs and optimal training signals,” IEEE Transactions on Signal Processing, vol. 54, no. 3, pp. 884–893, Mar. 2006.
- [106] P. Stoica, B. Ottersten, M. Viberg, and R. L. Moses, “Maximum likelihood array processing for stochastic coherent sources,” IEEE Transactions on Signal Processing, vol. 44, no. 1, pp. 96–105, Jan. 1996.
- [107] B. Efron, “Bootstrap methods: Another look at the Jackknife,” The Annals of Statistics, vol. 7, no. 1, pp. 1–26, Jan. 1979.
- [108] A. M. Zoubir and B. Boashash, “The bootstrap and its application in signal processing,” IEEE Signal Processing Magazine, vol. 15, no. 1, pp. 56–76, Jan. 1998.
- [109] M. Butussi and M. Bengtsson, “Low complexity admission in downlink beamforming,” in IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP’06, May 2006, vol. 4, p. IV.
- [110] E. Matakani, N. D. Sidiropoulos, and L. Tassiulas, “On multicast beamforming and admission control for UMTS-LTE,” in IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP’08, April 2008, pp. 2361–2364.
- [111] E. Matakani, N. D. Sidiropoulos, Z.-Q. Luo, and L. Tassiulas, “Convex approximation techniques for joint multiuser downlink beamforming and admission control,” IEEE Transactions on Wireless Communications, vol. 7, no. 7, pp. 2682–2693, July 2008.

- [112] E. Matskani, N. D. Sidiropoulos, Z.-Q. Luo, and L. Tassiulas, “Efficient batch and adaptive approximation algorithms for joint multicast beamforming and admission control,” IEEE Transactions on Signal Processing, vol. 57, no. 12, pp. 4882–4894, Dec. 2009.

Wissenschaftlicher Werdegang

Studium

- Promotion, “Robuste Algorithmen für Downlink-Beamforming in konventionellen und Cognitive Radio Netzwerken mit fehlerhafter Kanalzustandsinformationen”, Technische Universität Darmstadt, seit 10/2006.
- MS (Elektrotechnik), Fachrichtung Nachrichtentechnik, Muhammad Ali Jinnah Universität, Islamabad, Pakistan, 01/2003 - 08/2005.
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Masterarbeit: “Unterdrückung der Interferenz von Nachbarkanälen für MSK-Signale unter Ausnutzung der Antennendiversität in Rayleigh Fading Kanälen.”
- Bachelor of Engineering (Informationstechnik), Hamdard Universität Karachi, Islamabad Campus, Pakistan, 09/1998 - 10/2002.
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Berufserfahrung

- Projektingenieur Fahrerassistenzsysteme bei der Firma Mobis Parts Europe N.V., seit 08/2012.
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