



Rigorous analysis of the interaction problem of sea ice with a rigid body

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Abstract

Consider the set of equations modeling the motion of a rigid body enclosed in sea ice. Using Hibler’s viscous-plastic model for describing sea ice, it is shown by a certain decoupling approach that this system admits a unique, local strong solution within the L^p -setting.

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1 Introduction

It is a classical problem in fluid mechanics to study the movement of rigid or elastic bodies immersed in a fluid, see e.g. the works of Galdi [16], Hoffmann and Starovoitov [26], Desjardins and Esteban [11, 12], Gunzberger, Lee and Seregin [22], Feireisl, Hillairet and Nečasová [14], Cumsille, Takahashi and Tucsnak [7, 8], Geissert, Götze and Hieber [19], Maity, Raymond, Roy and Vanninathan [35, 43], and the recent work of Ervedoza, Maity and Tucsnak [13]. We also refer to the survey article [17] by Galdi and Neustupa for the stationary case. The very recent article [13] by Ervedoza, Maity and Tucsnak discusses the long-time behaviour of a system accounting for the motion of a rigid body enclosed in a viscous incompressible fluid.

Mathematically, interaction problems of rigid bodies can be described by a moving domain problem coupling a PDE with an ODE. The PDE comes from the geophysical

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equation which models the surrounding material, whereas the ODE is given by the balance equations for the momentum and angular momentum of the immersed rigid body. In this article, we investigate the problem of interaction of a rigid body in sea ice. Sea ice as a material exhibits a complex mechanical and thermodynamical behaviour. A composite of pure ice, liquid brine, air pockets and solid salt is formed by freezing sea water. As indicated by Feltham [15] or by Golden [20], the details of this formation depend on the laminar or turbulent environmental conditions. The response of this composite to heating, pressure or mechanical forces is for example different from the response of the (salt-free) glacial ice of ice sheets. For a recent survey in the Notices of the AMS, see [21].

We note that there is significant interest in understanding the interaction of ice and rigid structures. One particular application is e.g. the behaviour of a ship in sea ice. For numerical simulations of the movements of ships in an ice floe field, we refer e.g. to the work of Zhan, Agar, He, Spenced and Molyneux [50] and Kim, Sawamura [28]; the article [49] of Tuhkuri and Polojörvi provides a review of ice-structure interaction simulations.

W.D. Hibler suggested in 1979 the governing equations of large-scale sea ice dynamics in a seminal article [24]. These equations form the basis of many sea ice models in climate science. Sea ice is here modeled as a material with a very specific constitutive law combined with viscous-plastic rheology.

During the last decades, various communities have been investigating this set of equations numerically, see e.g. [9, 29, 31, 38–40, 45]. Unlike the equations describing atmospheric or oceanic dynamics as e.g. the primitive equations, see the seminal article by Cao and Titi [4], rigorous analysis of the sea ice equations started only very recently by the works of Brandt, Disser, Haller-Dintelmann and Hieber [3] as well as Liu, Thomas and Titi [32]. The underlying set of equations is a coupled degenerate quasilinear parabolic-hyperbolic system, whose analysis is delicate.

In [3], it is shown by means of the theory of quasilinear evolution equations that a suitable regularization of Hibler's model coupling velocity, thickness and compactness of sea ice is locally strongly well-posed and also globally strongly well-posed for initial data close to constant equilibria. The approach developed in [32] emphasizes the parabolic-hyperbolic character of Hibler's model and proves also local strong well-posedness by means of a different regularization and by energy estimates.

It is the aim of this article to present for the first time a rigorous analytical study of the interaction of sea ice with rigid structures. We develop an L^p -theory for strong solutions to the interaction problem of a rigid body trapped in sea ice in a bounded domain. Our main theorem shows local-in-time existence and uniqueness of the interaction problem of sea ice with a rigid body. Note that sea ice cannot be viewed as a generalized Newtonian fluid in the scope of [19] or a compressible fluid as in [25]. The stress tensor, see (2.1) or (2.3) for its regularized version, is of *quasilinear nature*, and it is not possible to express the factors in front of the deformation tensor ε and its trace $\text{tr}(\varepsilon)$ as a function of the Hilbert–Schmidt norm of the symmetric part of the gradient. In addition, the ice strength is a function of two unknowns here, namely of the mean ice thickness h and the ice compactness a , see (2.2) below. The linear theory as e.g. the maximal L^p -regularity or the bounded \mathcal{H}^∞ -calculus for the associated linearized

operator, also referred to as the *Hibler operator*, was established only very recently in the work of Brandt et al. [3].

The strategy of the proof of our main result is as follows. Since the domain of the sea ice equation is one of the unknowns, we first transform the problem to a fixed domain. Our approach in this article is “monolythic”, i.e., we deal with the entire transformed problem which is still coupled and analyze the linearized operator associated to the coupled sea ice-rigid body system. Using the results from [3, Sects. 4, 5 and 6], we establish maximal regularity of this linearized operator by means of a decoupling technique. In addition, the quasilinear terms in the system of PDEs for the sea ice dynamics are estimated by exploiting the regularity of the coordinate transform in conjunction with the respective estimates in [3, Sect. 6], and we estimate the nonlinear terms of the ODE. At this stage, the quasilinear nature of the stress tensor in Hibler’s sea ice model fully comes into play, and we emphasize that these estimates are novel and rely on a variant of nonlinear complex interpolation result due to Bergh [2]. In the spirit of our “monolythic” approach, we interpret the entire transformed problem as a non-autonomous quasilinear abstract Cauchy problem. It allows us to use classical local-in-time existence theory instead of developing a more sophisticated fixed point argument as in [19, 23] or [25]. Finally, we conclude existence and uniqueness of the original problem via performing the corresponding backwards coordinate transform.

The above transform to a fixed domain was first introduced by Inoue and Wakimoto [27] and then used by Conca, Cumsille, San Martín, Scheid, Takahashi and Tucsnak [6–8, 44, 46, 47] in the context of incompressible fluids, by Geissert et al. [19] in the situation of Newtonian and generalized Newtonian fluids, and by Hieber and Murata [25] for the study of compressible fluids. In the cited articles, this transform is the starting point to show the existence of a unique, local strong solution, or even a global solution in the case of small data, to the coupled system on bounded or unbounded fluid domains and in the situation of two or three space dimensions.

In the context of incompressible fluids, the above fluid–structure interaction problem was investigated by many other authors in the weak and strong setting by means of different types of transforms. The existence of local or global strong solutions has its roots in the works of Galdi [16], Galdi and Silvestre [18], Hoffmann and Starovoitov [26], Desjardins and Esteban [11, 12], Gunzberger et al. [22] as well as Feireisl et al. [14].

An abstract framework for linear fluid–solid interaction problems was introduced by Maity and Tucsnak [36]. Viewing the linearized fluid–solid interaction problems as boundary controlled fluid systems with dynamic boundary feedback and invoking the theory of extrapolation spaces to deal with the boundary conditions, they find that the interaction system can be regarded as a perturbation of the pure fluid equations. Consequently, they obtain maximal regularity of the linearized fluid–solid system by verifying \mathcal{R} -sectoriality by means of a perturbation argument.

Our approach is inspired by the work of Casarino et al. [5] for the question of the generation of a C_0 -semigroup. In our case, we need maximal regularity, and we use that maximal regularity is preserved under similarity transforms. In particular, the trace space for the initial data incorporates the boundary and coupling conditions in

our situation. It seems to us that this approach is quicker in our instance. For fluid–structure problems, both approaches work. However, in a general abstract framework, our approach allows to deal with a larger class of unbounded feedback operators.

The technique developed in [36] was later also used by Haak et al. [23] to show that the interaction of a compressible Navier–Stokes–Fourier fluid with a rigid body is locally strongly well-posed and also globally strongly well-posed for small initial data and by Maity et al. [34] in the context of a rigid structure floating in a viscous fluid.

This article is organized as follows. In Sect. 2, we briefly recall Hibler’s sea ice equations and introduce the interaction problem rigorously. Afterwards, we give a precise definition of a solution and formulate our main theorem. Section 3 is then devoted to the presentation of the coordinate transform and the computation of the transformed terms. In Sect. 4, we provide two auxiliary results concerning non-autonomous quasilinear evolution equations and nonlinear complex interpolation, respectively. In Sect. 5, we reformulate the transformed problem as a non-autonomous quasilinear abstract Cauchy problem. Further, we verify the assumptions of the local existence theorem: We show maximal regularity of the transformed operator by a decoupling argument and establish the Lipschitz estimates of the operator matrix and the right-hand sides. This results in the proof of the main theorem.

2 Sea ice interaction with a rigid body and main result

The present sea ice model is a 2D model, and we consider a bounded domain $\mathcal{O} \subset \mathbb{R}^2$ of class C^2 . Moreover, $0 < T \leq \infty$ represents a positive time. We denote the time-dependent bounded domain occupied by the rigid body at time $t \in [0, T]$ by $\mathcal{B}(t)$ and the remaining part of the domain filled by sea ice by $\mathcal{D}(t) = \mathcal{O} \setminus \overline{\mathcal{B}(t)}$. Moreover, the interface between the body and the sea ice is denoted by $\Gamma(t)$. To simplify the notation, we introduce $\mathcal{D} := \mathcal{D}(0)$, $\mathcal{B} := \mathcal{B}(0)$ as well as $\Gamma := \Gamma(0)$. The outer normal at $\Gamma(t)$ is denoted by $n(t)$, and the sets $\mathcal{Q}_{\mathcal{D}}$ and \mathcal{Q}_{Γ} consist of all points in spacetime where the spatial component is in $\mathcal{D}(t)$ and $\Gamma(t)$, respectively, i.e.,

$$\begin{aligned} \mathcal{Q}_{\mathcal{D}} &:= \{(t, x) \in \mathbb{R}^3 : t \in (0, T), x \in \mathcal{D}(t)\} & \text{and} \\ \mathcal{Q}_{\Gamma} &:= \{(t, x) \in \mathbb{R}^3 : t \in (0, T), x \in \Gamma(t)\}. \end{aligned}$$

Note that this can be written as

$$\mathcal{Q}_{\mathcal{D}} = \dot{\bigcup}_{t \in (0, T)} \{t\} \times \mathcal{D}(t) \quad \text{and} \quad \mathcal{Q}_{\Gamma} = \dot{\bigcup}_{t \in (0, T)} \{t\} \times \Gamma(t),$$

where $\dot{\bigcup}$ denotes the disjoint union. In the sequel, we also employ the notation $\mathcal{Q}_{\partial\mathcal{O}} = \{(t, x) \in \mathbb{R}^3 : t \in (0, T), x \in \partial\mathcal{O}\}$.

2.1 Hibler’s sea ice model

By $u : \mathcal{Q}_D \rightarrow \mathbb{R}^2$, we denote the horizontal velocity of the sea ice, while $h : \mathcal{Q}_D \rightarrow \mathbb{R}_+$ and $a : \mathcal{Q}_D \rightarrow [0, 1]$ represent the mean ice thickness and the ice compactness, respectively. Concerning the mean ice thickness, we impose the constraint that $h > \kappa$ for some small parameter $\kappa > 0$ which indicates the transition to open water. More precisely, a value of $h(t, x)$ less than κ means that at (t, x) , there is open water. In addition, we suppose that $a \in (\alpha, 1 - \alpha)$ holds for some small $\alpha > 0$.

Following [24], the constitutive law for the ice stress is given by

$$\sigma(u, h, a) = \frac{1}{e^2} \frac{P(h, a)}{\Delta(\varepsilon)} \varepsilon + \left(1 - \frac{1}{e^2}\right) \frac{P(h, a)}{2\Delta(\varepsilon)} \text{tr}(\varepsilon) \mathbf{I}_2 - \frac{P(h, a)}{2} \mathbf{I}_2, \tag{2.1}$$

where $\varepsilon = \varepsilon^{(u)} = \varepsilon(u) = \frac{1}{2} (\nabla u + (\nabla u)^T)$ is the deformation tensor, \mathbf{I}_2 denotes the unit matrix in $\mathbb{R}^{2 \times 2}$, $e > 1$ is the ratio of major to minor axes of the elliptical yield curve on which the principal components of the stress lie, and P represents the ice pressure and is defined by

$$P = P(h, a) = p^* h e^{-c(1-a)}, \tag{2.2}$$

for given constants $p^* > 0$ and $c > 0$. In addition, we have

$$\Delta^2(\varepsilon) := (\varepsilon_{11}^2 + \varepsilon_{22}^2) \left(1 + \frac{1}{e^2}\right) + \frac{4}{e^2} \varepsilon_{12}^2 + 2\varepsilon_{11}\varepsilon_{22} \left(1 - \frac{1}{e^2}\right).$$

Even though the above law describes an idealized viscous-plastic material, its viscosities become singular if Δ tends to zero. Following [3, 39], see also [30], we consider for $\delta > 0$ the regularization $\Delta_\delta(\varepsilon) := \sqrt{\delta + \Delta^2(\varepsilon)}$. Thus, we define the regularized internal ice stress by

$$\begin{aligned} \sigma_\delta(u, h, a) &:= S_\delta(u, h, a) - \frac{P(h, a)}{2} \mathbf{I}_2 \\ &:= \frac{1}{e^2} \frac{P(h, a)}{\Delta_\delta(\varepsilon)} \varepsilon + \left(1 - \frac{1}{e^2}\right) \frac{P(h, a)}{2\Delta_\delta(\varepsilon)} \text{tr}(\varepsilon) \mathbf{I}_2 - \frac{P(h, a)}{2} \mathbf{I}_2, \end{aligned} \tag{2.3}$$

where S_δ takes the shape

$$S_\delta(u, P) := S_\delta(\varepsilon, P) := \frac{P}{2} \frac{\mathbb{S}\varepsilon}{\Delta_\delta(\varepsilon)}. \tag{2.4}$$

Here $\mathbb{S} : \mathbb{R}^{2 \times 2} \rightarrow \mathbb{R}^{2 \times 2}$ is a map satisfying

$$\mathbb{S}\varepsilon = \begin{pmatrix} \left(1 + \frac{1}{e^2}\right)\varepsilon_{11} + \left(1 - \frac{1}{e^2}\right)\varepsilon_{22} & \frac{1}{e^2}(\varepsilon_{12} + \varepsilon_{21}) \\ \frac{1}{e^2}(\varepsilon_{12} + \varepsilon_{21}) & \left(1 - \frac{1}{e^2}\right)\varepsilon_{11} + \left(1 + \frac{1}{e^2}\right)\varepsilon_{22} \end{pmatrix}$$

for the deformation tensor $\varepsilon = \varepsilon(u)$. Identifying $\varepsilon \in \mathbb{R}^{2 \times 2}$ with the vector $(\varepsilon_{11}, \varepsilon_{12}, \varepsilon_{21}, \varepsilon_{22})^T$, we find that the map \mathbb{S} corresponds to the positive semi-definite matrix

$$\mathbb{S} = \left(\mathbb{S}_{ij}^{kl} \right) = \begin{pmatrix} 1 + \frac{1}{e^2} & 0 & 0 & 1 - \frac{1}{e^2} \\ 0 & \frac{1}{e^2} & \frac{1}{e^2} & 0 \\ 0 & \frac{1}{e^2} & \frac{1}{e^2} & 0 \\ 1 - \frac{1}{e^2} & 0 & 0 & 1 + \frac{1}{e^2} \end{pmatrix}.$$

Simple comparisons reveal that \mathbb{S} exhibits the symmetries

$$\mathbb{S}_{ij}^{kl} = \mathbb{S}_{ji}^{lk} = \mathbb{S}_{kj}^{il} = \mathbb{S}_{kl}^{ij} = \mathbb{S}_{il}^{kj}.$$

With $S_\delta(u, P)$ as introduced in (2.4), Hibler’s operator is defined by

$$A^H u := -\operatorname{div} S_\delta(u, P).$$

Following the calculations in [3, Sect. 3], we find that Hibler’s operator can be written as an operator in non-divergence form

$$\begin{aligned} (A^H u)_i &= - \sum_{j,k,l=1}^2 \frac{P}{2} \frac{1}{\Delta_\delta(\varepsilon)} \left(\mathbb{S}_{ij}^{kl} - \frac{1}{\Delta_\delta^2(\varepsilon)} (\mathbb{S}\varepsilon)_{ik} (\mathbb{S}\varepsilon)_{jl} \right) \partial_k \varepsilon_{jl} \\ &\quad - \frac{1}{2\Delta_\delta(\varepsilon)} \sum_{j=1}^2 (\partial_j P) (\mathbb{S}\varepsilon)_{ij} \\ &= \sum_{j,k,l=1}^2 \frac{P}{2} \frac{1}{\Delta_\delta(\varepsilon)} \left(\mathbb{S}_{ij}^{kl} - \frac{1}{\Delta_\delta^2(\varepsilon)} (\mathbb{S}\varepsilon)_{ik} (\mathbb{S}\varepsilon)_{jl} \right) D_k D_l u_j \\ &\quad - \frac{1}{2\Delta_\delta(\varepsilon)} \sum_{j=1}^2 (\partial_j P) (\mathbb{S}\varepsilon)_{ij} \end{aligned} \tag{2.5}$$

for $i = 1, 2$ and $D_m = -i\partial_m$.

Let us now introduce the external forces acting on the sea ice. For $m_{\text{ice}} = \rho_{\text{ice}}h$ denoting the mass of the sea ice, where $\rho_{\text{ice}} > 0$ is the density, the Coriolis force term is given by $m_{\text{ice}}c_{\text{cor}}n \times u$ with Coriolis parameter $c_{\text{cor}} > 0$ and unit vector $n: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ normal to the surface. Further, $m_{\text{ice}}g \nabla H$ represents the force due to changing sea surface tilt with sea surface dynamic height $H: (0, T) \times \mathbb{R}^2 \rightarrow [0, \infty)$ and gravity g . The atmospheric wind and oceanic forces are described by the terms τ_{atm} and $\tau_{\text{ocean}}(u)$, respectively, and they take the shape

$$\begin{aligned} \tau_{\text{atm}} &= \rho_{\text{atm}} C_{\text{atm}} |U_{\text{atm}}| R_{\text{atm}} U_{\text{atm}} \quad \text{and} \\ \tau_{\text{ocean}}(u) &= \rho_{\text{ocean}} C_{\text{ocean}} |U_{\text{ocean}} - u| R_{\text{ocean}} (U_{\text{ocean}} - u), \end{aligned}$$

where U_{atm} and U_{ocean} are the velocity of the surface winds and current, respectively. Furthermore, C_{atm} and C_{ocean} denote air and ocean drag coefficients, ρ_{atm} and ρ_{ocean}

represent the densities for air and sea water, and R_{atm} and R_{ocean} are rotation matrices acting on wind and current vectors. Except for the sea ice velocity u , the quantities involved in τ_{atm} and $\tau_{\text{ocean}}(u)$ are assumed to be constant in space and time for simplicity. In particular, τ_{atm} and $\tau_{\text{ocean}}(u)$ are independent of the mean ice thickness h and the ice compactness a .

In the sequel, we use f_1 to denote the external force terms in the momentum equation, so

$$f_1(u, h) = -m_{\text{ice}}c_{\text{cor}}n \times u - m_{\text{ice}}g\nabla H + \tau_{\text{atm}} + \tau_{\text{ocean}}(u). \tag{2.6}$$

For $f \in C_b^1([0, \infty); \mathbb{R})$ denoting an arbitrary function describing the ice growth rate, for instance the one proposed by Hibler [24, Sect. 3], the thermodynamic terms in the balance laws are given by

$$S_h = f\left(\frac{h}{a}\right)a + (1 - a)f(0) \tag{2.7}$$

and

$$S_a = \begin{cases} \frac{f(0)}{\kappa}(1 - a), & \text{if } f(0) > 0, \\ 0, & \text{if } f(0) < 0, \end{cases} + \begin{cases} 0, & \text{if } S_h > 0, \\ \frac{a}{2h}S_h, & \text{if } S_h < 0. \end{cases} \tag{2.8}$$

Recalling the shape of the regularized stress tensor σ_δ from (2.3), the external force term f_1 from (2.6) and the thermodynamic terms S_h and S_a from (2.7) and (2.8), respectively, we obtain that the system of equations accounting for the sea ice dynamics is given by

$$\begin{cases} m_{\text{ice}}(u_t + u \cdot \nabla u) = \text{div}\sigma_\delta(u, h, a) + f_1(u, h), & \text{in } \mathcal{Q}_{\mathcal{D}}, \\ h_t + \text{div}(uh) = d_h\Delta h + S_h(h, a), & \text{in } \mathcal{Q}_{\mathcal{D}}, \\ a_t + \text{div}(ua) = d_a\Delta a + S_a(h, a), & \text{in } \mathcal{Q}_{\mathcal{D}}, \\ u(0) = u_0, \quad h(0) = h_0, \quad a(0) = a_0, & \text{in } \mathcal{D}, \end{cases} \tag{2.9}$$

where Δ denotes the Laplacian, and $d_h > 0$ as well as $d_a > 0$ represent constants.

2.2 Sea ice interaction with a rigid body

For simplicity, we use $v = (u, h, a)$ to denote the principle variable of the sea ice equations in the sequel. Moreover, $\eta: (0, T) \rightarrow \mathbb{R}^2$ represents the translational velocity, while $\omega: (0, T) \rightarrow \mathbb{R}$ describes the angular velocity. With the regularized stress tensor σ_δ as in (2.3), the balance equations for the momentum and the angular momentum of the rigid body are

$$\begin{cases} m_{\text{B}}\eta'(t) + \int_{\Gamma(t)} \sigma_\delta(v)(t, x)n(t, x) \, dS = F(t), \quad t \in (0, T), \\ J\omega'(t) + \int_{\Gamma(t)} (x - x_c(t))^\perp \sigma_\delta(v)(t, x)n(t, x) \, dS = N(t), \quad t \in (0, T), \\ \eta(0) = \eta_0, \quad \omega(0) = \omega_0. \end{cases} \tag{2.10}$$

The constants m_B and J represent the body’s mass and inertia tensor. As in [8], the inertia tensor is given by

$$J = \int_{B(t)} \rho_B |x - x_c(t)|^2 dx = \int_{B(0)} \rho_B |y|^2 dy.$$

In particular, J is not time-dependent and $(J\omega)'(t) = J\omega'(t)$. The functions $F: (0, T) \rightarrow \mathbb{R}^2$ and $N: (0, T) \rightarrow \mathbb{R}$ represent external forces and torques. Further, x_c describes the position of the body’s center of gravity, where we suppose that $x_c(0) = 0$ for convenience. We observe that $x'_c(t) = \eta(t)$ for $t \in (0, T)$, so x_c can be deduced from η by

$$x_c(t) = \int_0^t \eta(s) ds. \tag{2.11}$$

For $y = (y_1, y_2)^T \in \mathbb{R}^2$, we write $y^\perp = (y_2, -y_1)^T$. The full velocity of the rigid body is then given by

$$u_B(t, x) := \eta(t) + \omega(t)(x - x_c(t))^\perp. \tag{2.12}$$

The velocity of the rigid body u_B as given in (2.12) coincides with the velocity of the sea ice u on their interface $\Gamma(t)$ for every time $t \in (0, T)$. This equation couples the system of PDEs of the sea ice dynamics with the ODEs accounting for the motion of the rigid body.

The coupled system of the sea ice dynamics as in (2.9) and the motion of the enclosed rigid body as in (2.10) is completed by boundary conditions for the unknowns u, h, a, η and ω to the following set of equations

$$\left\{ \begin{array}{ll} m_{ice}(u_t + u \cdot \nabla u) = \operatorname{div} \sigma_\delta(v) + f_1(u, h), & \text{in } \mathcal{Q}_D, \\ h_t + \operatorname{div}(uh) = d_h \Delta h + S_h(h, a), & \text{in } \mathcal{Q}_D, \\ a_t + \operatorname{div}(ua) = d_a \Delta a + S_a(h, a), & \text{in } \mathcal{Q}_D, \\ u(t, x) = \eta(t) + \omega(t)(x - x_c(t))^\perp, & \text{on } \mathcal{Q}_\Gamma, \\ \partial_\nu h = \partial_\nu a = 0, & \text{on } \mathcal{Q}_\Gamma, \\ u = 0, \partial_\nu h = \partial_\nu a = 0, & \text{on } \mathcal{Q}_{\partial\mathcal{O}}, \\ u(0) = u_0, h(0) = h_0, a(0) = a_0, & \text{in } \mathcal{D}, \\ m_B \eta'(t) = F(t) & \\ \quad - \int_{\Gamma(t)} \sigma_\delta(v)(t, x) n(t, x) dS, & t \in (0, T), \\ J \omega'(t) = N(t) & \\ \quad - \int_{\Gamma(t)} (x - x_c(t))^\perp \sigma_\delta(v)(t, x) n(t, x) dS, & t \in (0, T), \\ \eta(0) = \eta_0, \omega(0) = \omega_0. & \end{array} \right. \tag{2.13}$$

2.3 Main result

In the sequel, we need the following function spaces. For an arbitrary open set $O \subset \mathbb{R}^n$, $n \in \mathbb{N}$, $p, q \in (1, \infty)$, $k \in \mathbb{N}$ and $s \geq 0$, we denote by $L^q(O)$, $C(O)$, $C^k(O)$, $C^\infty(O)$, $W^{s,q}(O)$, $H^{s,q}(O)$ and $B_{qp}^s(O)$ the spaces of q -integrable functions, continuous functions, k -times continuously differentiable functions, smooth functions, (fractional) Sobolev spaces, Bessel potential spaces and Besov spaces, respectively. The definitions are standard and can e.g. be found in [48].

To make precise the concept of a solution of the system (2.14), we need the definition of function spaces on time-dependent domains $\mathcal{D}(t)$. Let $Z: \mathcal{Q}_{\mathcal{D}} \rightarrow \mathcal{D}$ be a map such that

$$\varphi: \mathcal{Q}_{\mathcal{D}} \rightarrow (0, T) \times \mathcal{D}, \quad (t, x) \mapsto (t, Z(t, x))$$

is a C^1 -diffeomorphism and $Z(\tau, \cdot): \mathcal{D}(\tau) \rightarrow \mathcal{D}$ are C^2 -diffeomorphisms for all $\tau \in [0, T]$. For any $p, q \in (1, \infty)$, $s \in \{0, 1\}$ and $l \in \{0, 1, 2\}$, we define then

$$W^{s,p}(0, T; W^{l,q}(\mathcal{D}(\cdot))) := \{f(t, \cdot): \mathcal{D}(t) \rightarrow \mathbb{R} : f \circ \varphi \in W^{s,p}(0, T; W^{l,q}(\mathcal{D}))\}.$$

We remark that $\mathcal{Q}_{\mathcal{D}}$ is a $C^{1,2}$ -manifold with spatial boundary $\mathcal{Q}_{\Gamma} \cup \mathcal{Q}_{\partial\mathcal{O}}$ where the only chart is $\varphi: \mathcal{Q}_{\mathcal{D}} \rightarrow (0, T) \times \mathcal{D}$. Our definition of time-dependent function spaces coincides with the well known definition of function spaces on a manifold with boundary.

Next, we introduce the set of suitable initial data for our problem. For $\kappa > 0$, $\alpha > 0$ sufficiently small as introduced at the beginning of Sect. 2.1, consider the set

$$\begin{aligned} V := \{ & (u_0, h_0, a_0, \eta_0, \omega_0) \in B_{qp}^{2-2/p}(\mathcal{D}; \mathbb{R}^2) \times B_{qp}^{2-2/p}(\mathcal{D})^2 \times \mathbb{R}^2 \times \mathbb{R} : \\ & h_0 > \kappa \text{ and } a_0 \in (\alpha, 1 - \alpha), \partial_\nu h_0 = \partial_\nu a_0 = 0 \text{ on } \partial\mathcal{D}, \\ & u_0 = 0 \text{ on } \partial\mathcal{O} \text{ and } u_0 = \eta_0 + \omega_0 x^\perp \text{ on } \Gamma \}. \end{aligned} \tag{2.14}$$

Since $\kappa > 0$ and $\alpha > 0$ are fixed throughout the article, we omit them in the notation of V . We also observe that we consider $p, q \in (1, \infty)$ such that

$$\frac{2}{p} + \frac{3}{q} < 1 \tag{2.15}$$

in the sequel. As a consequence, it is especially valid that $2/p + 1/q < 1$, or, equivalently, $2 - 2/p > 1 + 1/q$, so it is legit to consider Dirichlet or Neumann boundary conditions for elements in $B_{qp}^{2-2/p}(\mathcal{D})$, see e.g. [1, Sect. 5]. On the other hand, as (2.15) implies in particular that $2/p + 2/q < 1$, which is in turn equivalent to $2 - 2/p > 1 + 2/q$, we obtain the embedding

$$B_{qp}^{2-2/p}(\mathcal{D}) \hookrightarrow C^1(\overline{\mathcal{D}}) \tag{2.16}$$

from [48, Theorem 4.6.1], so it is justified to impose the pointwise conditions on h_0 and a_0 in the definition of V .

Now, we are in the position to formulate our main result on local-in-time existence and uniqueness of the coupled sea ice-rigid body system (2.14).

Theorem 2.1 *Let $p, q \in (1, \infty)$ be such that (2.15) is satisfied, let $\mathcal{O} \subset \mathbb{R}^2$ be a bounded domain of class C^2 , and consider the domains of the rigid body and the fluid at time zero, \mathcal{B} and \mathcal{D} , respectively. Moreover, let $w_0 = (u_0, h_0, a_0, \eta_0, \omega_0) \in V$, where V is defined precisely in (2.14), and suppose that $F \in L^p(0, T; \mathbb{R}^2)$ as well as $N \in L^p(0, T)$. If for some $d > 0$ it holds that $\text{dist}(\mathcal{B}, \partial\mathcal{O}) > d$, then there exists $T' \in (0, T]$ and a map $Z \in C^1([0, T']; C^2(\mathbb{R}^2))$ such that $Z(\tau, \cdot) : \mathcal{D}(\tau) \rightarrow \mathcal{D}$ are C^2 -diffeomorphisms for all $\tau \in [0, T']$, and (2.14) admits a unique solution (u, h, a, η, ω) such that*

$$\begin{aligned} u &\in W^{1,p}(0, T'; L^q(\mathcal{D}(\cdot); \mathbb{R}^2)) \cap L^p(0, T'; W^{2,q}(\mathcal{D}(\cdot); \mathbb{R}^2)), \\ h &\in W^{1,p}(0, T'; L^q(\mathcal{D}(\cdot))) \cap L^p(0, T'; W^{2,q}(\mathcal{D}(\cdot))), \\ a &\in W^{1,p}(0, T'; L^q(\mathcal{D}(\cdot))) \cap L^p(0, T'; W^{2,q}(\mathcal{D}(\cdot))), \\ \eta &\in W^{1,p}(0, T'; \mathbb{R}^2), \quad \omega \in W^{1,p}(0, T'). \end{aligned}$$

Remark 2.2 A solution (u, h, a, η, ω) in the regularity class as in Theorem 2.1 is called a strong solution.

3 Coordinate transform

We present the diffeomorphism accounting for the transform from the moving on the fixed domain in this section, and we also compute the transformed system of equations. It is important to note that the coordinate transform is an unknown part of the solution of our system. Throughout this section, for η and ω as in Sect. 2.2, we consider a fixed pair $(\eta, \omega) \in W^{1,p}(0, T; \mathbb{R}^2) \times W^{1,p}(0, T)$. Now, for the matrix

$$m(t) = \omega(t) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

satisfying $m(t)x = \omega(t)x^\perp$, we take the differential equation

$$\begin{cases} \partial_t Z_0(t, y) = m(t)(Z_0(t, y) - x_c(t)) + \eta(t), & (0, T) \times \mathbb{R}^2, \\ Z_0(0, y) = y, & y \in \mathbb{R}^2, \end{cases} \tag{3.1}$$

into account. As discussed in Sect. 2.2, the coordinates are chosen such that the center of gravity of the rigid body at time 0 is the origin, i.e., $x_c(0) = 0$. The corresponding solution then takes the shape $Z_0(t, y) = Q(t)y + x_c(t)$, where $Q(t) \in \text{SO}(2)$ and $Q \in W^{2,p}(0, T; \mathbb{R}^{2 \times 2})$ provided $(\eta, \omega) \in W^{1,p}(0, T; \mathbb{R}^2) \times W^{1,p}(0, T)$. It follows from (2.11) and (3.1) that Q is the unique solution of

$$\begin{cases} \partial_t Q(t) = m(t)Q(t), & t \in (0, T), \\ Q(0) = \text{Id}. \end{cases} \tag{3.2}$$

Further, the inverse $Y_0(t)$ of $Z_0(t)$ is given by

$$Y_0(t, x) = Q^T(t)(x - x_c(t)),$$

and it satisfies the differential equation

$$\begin{cases} \partial_t Y_0(t, x) = -\tilde{m}(t)Y_0(t, x) - \xi(t), & (0, T) \times \mathbb{R}^2, \\ Y_0(0, x) = x, & x \in \mathbb{R}^2, \end{cases}$$

with

$$\tilde{m}(t) := Q^T(t)m(t)Q(t), \quad \xi(t) := Q^T(t)\eta(t).$$

The next step is to modify the diffeomorphisms Z_0, Y_0 of $\mathcal{D}(t)$ and $\mathcal{B}(t)$ such that they rotate space only in an appropriate open neighborhood of the rotating and translating body, and they must not rotate or translate the outer boundary $\partial\mathcal{O}$. Here we follow the strategy of Geissert, Götze and Hieber for their consideration of a bounded fluid domain, see [19, Sect. 3 and Sect. 7]. However, it is not necessary to include a Bogovskiĭ operator in our case, as we are not in the situation of an incompressible fluid.

The new diffeomorphism is now defined implicitly, using an ODE of the form (3.1), namely

$$\begin{cases} \partial_t Z(t, y) = b(t, Z(t, y)), & (0, T) \times \mathbb{R}^2, \\ Z(0, y) = y, & y \in \mathbb{R}^2. \end{cases} \tag{3.3}$$

The right-hand side b in (3.3) determines the modified velocity of the change of coordinates. Close to the rigid body, b should be equal to the velocity of the body, while it is supposed to be zero further away. Additionally, considering that the rigid body starts from a position with some distance from the boundary of the sea ice domain and moves with a continuous velocity, we restrict the solution to a time that guarantees that a small distance remains. More precisely, in the statement of Theorem 2.1, we assume that $\text{dist}(\mathcal{B}, \partial\mathcal{O}) > d$, and we set b such that a distance of $\frac{d}{2}$ between the body and the outer boundary is maintained. It is also important that b is smooth in the space variables. To obtain these properties, we define a cut-off function $\chi \in C^\infty(\mathbb{R}^2; [0, 1])$ by

$$\chi(y) := \begin{cases} 1, & \text{if } \text{dist}(y, \partial\mathcal{O}) \geq d, \\ 0, & \text{if } \text{dist}(y, \partial\mathcal{O}) \leq \frac{d}{2}, \end{cases}$$

and a time-dependent vector field $b: [0, T] \times \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by

$$b(t, x) := \chi(x - x_c(t))[m(t)(x - x_c(t)) + \eta(t)]. \tag{3.4}$$

We observe that $b \in W^{1,p}(0, T; C_c^\infty(\mathbb{R}^2))$ by construction. As we assume that $\text{dist}(\mathcal{B}, \partial\mathcal{O}) > d$, i.e., the body starts with a positive distance from the outer sea ice boundary, and by virtue of $x_c(0) = 0$, we obtain that $\text{dist}(x, \partial\mathcal{O}) \geq d$ for every $x \in \Gamma$. Therefore,

$$b(0, x) = m(0)(x - x_c(0)) + \eta(0) = \omega_0 x^\perp + \eta_0$$

is valid for $x \in \Gamma$, i.e., $b|_\Gamma = \omega_0 x^\perp + \eta_0$.

For $(\eta, \omega) \in W^{1,p}(0, T; \mathbb{R}^2) \times W^{1,p}(0, T)$ the Picard–Lindelöf theorem implies that the Eq. (3.3) admits a unique solution $Z \in C^1(0, T; C^\infty(\mathbb{R}^2))$. Besides, the solution has continuous mixed partial derivatives $\frac{\partial^{|\alpha|+1} Z}{\partial t(\partial y_j)^\alpha}$ and $\frac{\partial^{|\alpha|} Z}{(\partial y_j)^\alpha}$, where $\alpha \in \mathbb{N}_0^3$ is a multi-index. Further, the elements of the Jacobi matrix J_Z of the diffeomorphism are of the form

$$(J_Z)_{ij}(t, y) = \partial_j Z_i(t, y) = \delta_{ij} + \int_0^t \frac{\partial b_i}{\partial y_j}(s, Z(s, y)) \, ds.$$

A proof of the following lemma can be found in [25, Sect. 2].

Lemma 3.1 *If either $T_0 \in (0, T]$ is small enough or $\|\nabla_y b\|_{L^\infty([0, T] \times \mathbb{R}^2; \mathbb{R}^{2 \times 2})} < c$ for some sufficiently small constant $c > 0$, then $J_Z(t, \cdot)$ is invertible for every $t \in (0, T_0)$ or even for every $t \in (0, T)$ in the second case.*

The inverse transform Y of Z satisfies the equation

$$\begin{cases} \partial_t Y(t, x) = b^{(Y)}(t, Y(t, x)), & (0, T_0) \times \mathbb{R}^2, \\ Y(0, x) = x, & x \in \mathbb{R}^2, \end{cases} \tag{3.5}$$

where

$$b^{(Y)}(t, y) := -J_Z^{-1}(t, y)b(t, Z(t, y)), \tag{3.6}$$

which is well-defined for $t < T_0$ in view of Lemma 3.1. It follows immediately that

$$J_Z(t, y)J_Y(t, Z(t, y)) = \text{Id}.$$

Note that by this definition, $b^{(Y)}$ and Y possess the same space and time regularity as b and Z . We emphasize that the diffeomorphism Z accounts for the transform from the moving domain to the fixed domain, and we observe that Z and Y coincide with Z_0 and Y_0 provided the rigid body is sufficiently far away from the boundary of the sea ice domain, while it holds that $\partial_t Z(t, y) = \partial_t Y(t, x) = 0$ if the rigid body comes close to the boundary.

For $(t, y) \in [0, T) \times \mathbb{R}^2$, Z as in (3.3) and Q as introduced in (3.2), we now define

$$\begin{aligned} \tilde{u}(t, y) &:= u(t, Z(t, y)), \\ \tilde{h}(t, y) &:= h(t, Z(t, y)), \\ \tilde{a}(t, y) &:= a(t, Z(t, y)), \\ \xi(t) &:= Q^T(t)\eta(t), \\ \Omega(t) &:= \omega(t), \\ \tilde{F}(t) &:= Q^T(t)F(t), \\ \tilde{N}(t) &:= N(t), \\ \mathcal{T}_\delta(\tilde{u}(t, y), \tilde{h}(t, y), \tilde{a}(t, y)) &:= Q^T(t)\sigma_\delta(\tilde{u}(t, y), \tilde{h}(t, y), \tilde{a}(t, y))Q(t), \\ I &:= J, \\ \tilde{n} &:= Q^T(t)n(t). \end{aligned}$$

Using that $Q(t) \in \text{SO}(2)$ as well as $\Omega(t) = \omega(t)$, we infer that

$$\tilde{m}(t)x = Q^T(t)m(t)Q(t)x = \omega(t) \det(Q(t)) \begin{pmatrix} x_2 \\ -x_1 \end{pmatrix} = \Omega(t)x^\perp. \tag{3.7}$$

The inertia tensor remains unchanged, so time-independence is also preserved for I . We remark that \tilde{n} represents the outer normal at \mathcal{B} . Moreover, we compute for the principal variables $v = (u, h, a)$ and $\tilde{v} = (\tilde{u}, \tilde{h}, \tilde{a})$ that

$$\int_{\Gamma(t)} \sigma_\delta(v)n(t) \, dS = Q \int_{\Gamma} \mathcal{T}_\delta(\tilde{v})\tilde{n} \, dS \quad \text{as well as}$$

$$\int_{\Gamma(t)} (x - x_c(t))^\perp \sigma_\delta(v)n(t) \, dS = \int_{\Gamma} y^\perp \mathcal{T}_\delta(\tilde{v})\tilde{n} \, dS,$$

where we made use of $Q(t) \in \text{SO}(2)$ to establish that $y^\perp Qz = (Q^T y^\perp)z$ holds true for all $y, z \in \mathbb{R}^2$.

Having introduced the transformed translational and angular velocities ξ and Ω , we summarize the procedure to construct Z and Y from given ξ and Ω .

Remark 3.2 Given the pair $(\xi, \Omega) \in W^{1,p}(0, T; \mathbb{R}^2) \times W^{1,p}(0, T)$, we first obtain $Q \in W^{2,p}(0, T; \mathbb{R}^{2 \times 2})$ by solving

$$\partial_t Q^T(t) = \tilde{m}(t)Q^T(t), \quad t \in (0, T), \quad Q^T(0) = \text{Id},$$

where $\tilde{m}(t)x = \Omega(t)x^\perp$ for $x \in \mathbb{R}^2$ as in (3.7). Next, we derive the original translational and angular body velocities from $\eta(t) = Q^T(t)\xi(t)$ and $\omega(t) = \Omega(t)$, and we then set, as in (3.4),

$$b(t, x) = \chi(x - x_c(t))[m(t)(x - x_c(t)) + \eta(t)],$$

where $x_c(t)$ is recovered from the aforementioned $\eta(t)$ via $x_c(t) = \int_0^t \eta(s) \, ds$, see (2.11). Having the right-hand side b at hand, we solve (3.3) to deduce Z . For T_0 as in Lemma 3.1 and $t \in (0, T_0)$, we set $b^{(Y)}(t, y) = -J_Z^{-1}(t, y)b(t, Z(t, y))$ as in (3.6) and subsequently solve (3.5) to obtain the inverse Y of Z .

The following estimates can be concluded from the procedure described in Remark 3.2, see [19, Proposition 6.1] and the lemmas thereafter for a proof. For simplicity, we use $\|\cdot\|_{\infty, \infty}$ to denote the norm associated to $L^\infty(0, T; L^\infty(\mathbb{R}^2))$. In addition, the index $i \in \{1, 2\}$ means that the corresponding diffeomorphism is associated to ξ_i and Ω_i .

Proposition 3.3 *Let $T > 0$ and $(\xi_1, \Omega_1), (\xi_2, \Omega_2) \in W^{1,p}(0, T; \mathbb{R}^2) \times W^{1,p}(0, T)$. For $i \in \{1, 2\}$, it then holds that $Z_i, Y_i \in C^1(0, T_0; C^\infty(\mathbb{R}^2))$, and the estimates*

$$\|\partial^\alpha Z_i\|_{\infty, \infty} + \|\partial^\alpha Y_i\|_{\infty, \infty} \leq C(K_i) \quad \text{as well as}$$

$$\|\partial^\beta(Z_1 - Z_2)\|_{\infty, \infty} + \|\partial^\beta(Y_1 - Y_2)\|_{\infty, \infty} \leq C(K_i)T(\|\xi_1 - \xi_2\|_{L^\infty(0, T; \mathbb{R}^2)} + \|\Omega_1 - \Omega_2\|_{L^\infty(0, T)})$$

are valid for all multi-indices α, β such that $1 \leq |\alpha| \leq 3$ and $0 \leq |\beta| \leq 3$. The constants only depend on the norms $K_i := \|\xi_i\|_{L^\infty(0,T;\mathbb{R}^2)} + \|\Omega_i\|_{L^\infty(0,T)}$, but they do not depend on ξ_i or Ω_i directly.

Recalling Hibler’s operator A^H from (2.5), we denote the coefficients of the principal part of A^H by

$$a_{ij}^{kl}(\nabla u, P) := \frac{P}{2} \frac{1}{\Delta_\delta(\varepsilon)} \left(S_{ij}^{kl} - \frac{1}{\Delta_\delta^2(\varepsilon)} (\mathbb{S}\varepsilon)_{ik} (\mathbb{S}\varepsilon)_{jl} \right). \tag{3.8}$$

Then for sufficiently smooth initial data, its linearization at $v_0 = (u_0, h_0, a_0)$ is given by

$$\begin{aligned} [A^H(v_0)u]_i &= \sum_{j,k,l=1}^2 a_{ij}^{kl}(\varepsilon(u_0), P(h_0, a_0)) D_k D_l u_j \\ &\quad - \frac{1}{2\Delta_\delta(\varepsilon(u_0))} \sum_{j=1}^2 (\partial_j P(h_0, a_0)) (\mathbb{S}\varepsilon(u))_{ij}. \end{aligned} \tag{3.9}$$

We start by calculating the transformed symmetric part of the gradient. Using the chain rule and arguing similarly as in [19, Sect. 9], we obtain

$$\begin{aligned} 2\varepsilon_{ij}^{(u)}(t, x) &= (\partial_i u_j)(t, x) + (\partial_j u_i)(t, x) \\ &= \sum_{k=1}^2 (\partial_i Y_k)(t, Z(t, y)) \partial_k \tilde{u}_j(t, y) + (\partial_j Y_k)(t, Z(t, y)) \partial_k \tilde{u}_i(t, y) \\ &=: 2\tilde{\varepsilon}_{ij}^{(\tilde{u})}(t, y), \end{aligned}$$

where $\tilde{\varepsilon} = \tilde{\varepsilon}(\tilde{u}) = \tilde{\varepsilon}(\tilde{u})$ denotes the transformed symmetric part of the gradient.

Inserting $\tilde{\varepsilon}(\tilde{u})$ as well as \tilde{h} and \tilde{a} into the Hibler operator from (2.5) and computing the transformed derivatives, we calculate that the transformed Hibler operator is given by

$$\begin{aligned} \tilde{A}^H(t, \tilde{w})\tilde{u} &= - \sum_{j,k,l,m=1}^2 a_{ij}^{klm}(\tilde{\varepsilon}(\tilde{u}), P(\tilde{h}, \tilde{a})) \partial_m \tilde{\varepsilon}_{jl}(\tilde{u}) \\ &\quad - \frac{1}{2\Delta_\delta(\tilde{\varepsilon}(\tilde{u}))} P^* e^{-c(1-\tilde{a})} \sum_{j,k=1}^2 \partial_j Y_k (\partial_k \tilde{h} + c\partial_k \tilde{a}) (\mathbb{S}\tilde{\varepsilon}(\tilde{u}))_{ij}, \end{aligned} \tag{3.10}$$

where $\tilde{w} = (\tilde{u}, \tilde{h}, \tilde{a}, \xi, \Omega)$, $a_{ij}^{klm}(\tilde{\varepsilon}(\tilde{u}), P(\tilde{h}, \tilde{a})) = (\partial_k Y_m) a_{ij}^{kl}(\tilde{\varepsilon}(\tilde{u}), P(\tilde{h}, \tilde{a}))$ and

$$\begin{aligned} \partial_m \tilde{\varepsilon}_{jl}(\tilde{u}) &= \frac{1}{2} \sum_{n=1}^2 ((\partial_m \partial_j Y_n) \partial_n \tilde{u}_l + (\partial_j Y_n) \partial_m \partial_n \tilde{u}_l \\ &\quad + (\partial_m \partial_l Y_n) \partial_n \tilde{u}_j + (\partial_l Y_n) \partial_m \partial_n \tilde{u}_j). \end{aligned}$$

Note that the right-hand side in (3.10) can be regarded as a function depending on $(t, \tilde{u}, \tilde{h}, \tilde{a}, \xi, \Omega) = (t, \tilde{w})$ by virtue of Remark 3.2. Recalling $P = p^* h e^{-c(1-a)}$, we find that the lower order terms corresponding to $\operatorname{div} \frac{P}{2} \mathbf{I}_2$ transform to

$$\begin{aligned} (\tilde{\mathbf{B}}_1(t, \tilde{w}) \tilde{h})_i &= \frac{p^* e^{-c(1-\tilde{a})}}{2} \sum_{j=1}^2 (\partial_i Y_j) \partial_j \tilde{h} \quad \text{and} \\ (\tilde{\mathbf{B}}_2(t, \tilde{w}) \tilde{a})_i &= \frac{cp^* \tilde{h} e^{-c(1-\tilde{a})}}{2} \sum_{j=1}^2 (\partial_i Y_j) \partial_j \tilde{a}, \end{aligned} \tag{3.11}$$

where the dependence on (t, \tilde{w}) is again implied by Remark 3.2. Introducing the metric contravariant tensor

$$g^{ij} := g^{ij}(t, \xi, \Omega) = \sum_{k=1}^2 (\partial_k Y_i) (\partial_k Y_j), \tag{3.12}$$

we determine the transformed Laplacian operators to be given by

$$\tilde{\mathbf{L}} \tilde{h} := \tilde{\mathbf{L}}(t, \xi, \Omega) \tilde{h} = \sum_{j=1}^2 (\Delta Y_j) \partial_j \tilde{h} + \sum_{j,k=1}^2 g^{jk} \partial_k \partial_j \tilde{h}, \tag{3.13}$$

and the right-hand side can be regarded as a function of (t, \tilde{w}) thanks to Remark 3.2. The shape of $\tilde{\mathbf{L}} \tilde{a} := \tilde{\mathbf{L}}(t, \xi, \Omega) \tilde{a}$ is completely analogous.

Next, we observe that the force terms f_1 , S_h and S_a do not contain any derivatives. Therefore, we get the respective transformed terms by simply inserting the transformed variables, i.e., $f_1(\tilde{u}, \tilde{h})$, $S_h(\tilde{h}, \tilde{a})$ and $S_a(\tilde{h}, \tilde{a})$, where we recall the respective shapes from (2.6), (2.7) and (2.8).

We continue by determining the transformed bilinear terms. To this end, we calculate

$$\begin{aligned} ((u \cdot \nabla) u)_k &= \sum_{i,j=1}^2 \tilde{u}_i (\partial_i Y_j) \partial_j \tilde{u}_k \quad \text{and} \\ \operatorname{div}(uh) &= \sum_{i,j=1}^2 (\partial_i Y_j) (\tilde{u}_i \partial_j \tilde{h} + \tilde{h} \partial_j \tilde{u}_i). \end{aligned} \tag{3.14}$$

The transformed time derivative of a function b is given by

$$b_t = \tilde{b}_t + \sum_{j=1}^2 \dot{Y}_j \partial_j \tilde{b}, \tag{3.15}$$

where \dot{Y}_j denotes the time derivative of Y_j .

Using the shapes of the transformed time derivative from (3.15) and the transformed transport terms from (3.14), respectively, and recalling for $\tilde{w} = (\tilde{u}, \tilde{h}, \tilde{a}, \xi, \Omega)$ the transformed Hibler operator $\tilde{A}^H(t, \tilde{w})$ from (3.10), the transformed lower order terms $\tilde{B}_1(t, \tilde{w})$ and $\tilde{B}_2(t, \tilde{w})$ from (3.11), f_1 from (2.6), \tilde{L} from (3.13), S_h from (2.7) as well as S_a from (2.8), we obtain that (2.14) rewrites as the following system on a cylindrical domain $(0, T) \times \mathcal{D}$ when applying the transform to a fixed domain:

$$\left\{ \begin{aligned} \tilde{u}_t + \sum_{j=1}^2 \dot{Y}_j \partial_j \tilde{u} &= - \sum_{i,j=1}^2 \tilde{u}_i (\partial_i Y_j) \partial_j \tilde{u} \\ &\quad + \frac{1}{\rho_{\text{ice}} \tilde{h}} (-\tilde{A}^H(t, \tilde{w}) \tilde{u} + \tilde{B}_1(t, \tilde{w}) \tilde{h} \\ &\quad + \tilde{B}_2(t, \tilde{w}) \tilde{a} + f_1(\tilde{u}, \tilde{h})), && \text{in } (0, T) \times \mathcal{D}, \\ \tilde{h}_t + \sum_{j=1}^2 \dot{Y}_j \partial_j \tilde{h} &= - \sum_{i,j=1}^2 (\partial_i Y_j) (\tilde{u}_i \partial_j \tilde{h} + \tilde{h} \partial_j \tilde{u}_i) \\ &\quad + d_h \tilde{L}(t, \xi, \Omega) \tilde{h} + S_h(\tilde{h}, \tilde{a}), && \text{in } (0, T) \times \mathcal{D}, \\ \tilde{a}_t + \sum_{j=1}^2 \dot{Y}_j \partial_j \tilde{a} &= - \sum_{i,j=1}^2 (\partial_i Y_j) (\tilde{u}_i \partial_j \tilde{a} + \tilde{a} \partial_j \tilde{u}_i) \\ &\quad + d_a \tilde{L}(t, \xi, \Omega) \tilde{a} + S_a(\tilde{h}, \tilde{a}), && \text{in } (0, T) \times \mathcal{D}, \\ \tilde{u}(t, y) &= \xi(t) + \Omega(t) y^\perp, && \text{on } (0, T) \times \Gamma, \\ \partial_\nu \tilde{h} &= \partial_\nu \tilde{a} = 0, && \text{on } (0, T) \times \Gamma, \\ \tilde{u} &= 0, \quad \partial_\nu \tilde{h} = \partial_\nu \tilde{a} = 0, && \text{on } (0, T) \times \partial \mathcal{O}, \\ \tilde{u}(0) &= u_0, \quad \tilde{h}(0) = h_0, \quad \tilde{a}(0) = a_0, && \text{on } \mathcal{D}, \\ m_B \xi'(t) &= \tilde{F}(t) + m_B \Omega \xi^\perp \\ &\quad - Q \int_\Gamma \mathcal{T}_\delta(\tilde{u}, \tilde{h}, \tilde{a})(t, y) \tilde{n}(t, y) \, dS, \quad t \in (0, T), \\ I \Omega'(t) &= \tilde{N}(t) \\ &\quad - \int_\Gamma y^\perp \mathcal{T}_\delta(\tilde{u}, \tilde{h}, \tilde{a})(t, y) \tilde{n}(t, y) \, dS, \quad t \in (0, T), \\ \xi(0) &= \eta_0, \quad \Omega(0) = \omega_0. \end{aligned} \right. \tag{3.16}$$

Our main result, Theorem 2.1, can now be rephrased as follows.

Theorem 3.4 *Let $p, q \in (1, \infty)$ be such that (2.15) is fulfilled, let $\mathcal{O} \subset \mathbb{R}^2$ be a bounded domain of class C^2 , and consider the domains of the rigid body and the fluid at time zero, \mathcal{B} and \mathcal{D} , respectively. Moreover, let $\tilde{w}_0 = (\tilde{u}_0, \tilde{h}_0, \tilde{a}_0, \xi_0, \Omega_0) \in V$, where V is defined precisely in (2.14), and suppose that $F \in L^p(0, T; \mathbb{R}^2)$ as well as $N \in L^p(0, T)$. If for some $d > 0$ it holds that $\text{dist}(\mathcal{B}, \partial\mathcal{O}) > d$, then there exists $T' \in (0, T]$ and a map $Z \in C^1(0, T'; C^2(\mathbb{R}^2))$ such that $Z(\tau, \cdot) : \mathcal{D}(\tau) \rightarrow \mathcal{D}$ are C^2 -diffeomorphisms for all $\tau \in [0, T']$, and (3.16) admits a unique strong solution*

$$\begin{aligned} \tilde{u} &\in W^{1,p}(0, T'; L^q(\mathcal{D}; \mathbb{R}^2)) \cap L^p(0, T'; W^{2,q}(\mathcal{D}; \mathbb{R}^2)), \\ \tilde{h} &\in W^{1,p}(0, T'; L^q(\mathcal{D})) \cap L^p(0, T'; W^{2,q}(\mathcal{D})), \\ \tilde{a} &\in W^{1,p}(0, T'; L^q(\mathcal{D})) \cap L^p(0, T'; W^{2,q}(\mathcal{D})), \\ \xi &\in W^{1,p}(0, T'; \mathbb{R}^2), \text{ and } \Omega \in W^{1,p}(0, T'). \end{aligned}$$

4 Auxiliary results

In this section, we present the tools needed for the proof of our main result, namely the local existence theorem and the variant of a nonlinear complex interpolation result due to Bergh.

4.1 A local well-posedness result for non-autonomous quasilinear evolution equations

Throughout this subsection, we denote by A the quasilinear operator, by F the nonlinear right-hand side and by u the principle variable of the evolution equation. The non-autonomous quasilinear abstract Cauchy problem takes the shape

$$\begin{cases} \dot{u} + A(t, u)u = F(t, u), & t \in [0, T], \\ u(0) = u_0. \end{cases} \tag{4.1}$$

By X_0 and X_1 , we denote the ground space and the regularity space, respectively, and we assume that $X_1 \hookrightarrow X_0$ is dense, $\mathbb{F}_T := L^p(0, T; X_0)$ is the data space, while $\mathbb{E}_T := L^p(0, T; X_1) \cap W^{1,p}(0, T; X_0)$ is the maximal regularity space. Moreover, we denote by X_γ the trace space, and it is well known that $X_\gamma = (X_0, X_1)_{1-1/p, p}$, see e.g. [42, Proposition 3.4.4]. The set V represents an open subset of X_γ . Additionally, we assume the following structure conditions on the nonlinearities as well as on the linearized operator.

Assumption 4.1 (A1) We have $A \in C([0, T] \times V, \mathcal{L}(X_1, X_0))$. Given $u_0 \in V$, there is $R_0 > 0$ such that $\overline{B_{X_\gamma}(u_0, R_0)} \subset V$, and for all $R \in (0, R_0)$, there exists a Lipschitz constant $L(R) > 0$ independent of τ with

$$\|A(\tau, u_1)v - A(\tau, u_2)v\|_{X_0} \leq L(R)\|u_1 - u_2\|_{X_\gamma} \cdot \|v\|_{X_1}$$

for all $\tau \in [0, T]$, $v \in X_1$ and all $u_1, u_2 \in X_\gamma$ with $\|u_i - u_0\|_{X_\gamma} \leq R, i = 1, 2$.

(A2) For the map $F : [0, T] \times X_\gamma \rightarrow X_0$, we assume

- (i) $F(\cdot, u)$ is measurable for every $u \in V$,
- (ii) $F(\tau, \cdot) \in C(V, X_0)$ for almost all $\tau \in [0, T]$,
- (iii) $F(\cdot, u) \in \mathbb{F}_T$ holds for every $u \in V$,
- (iv) given $u_0 \in V$, for all $R > 0$ with $\overline{B}_{X_\gamma}(u_0, R) \subset V$, there is $\varphi_R \in L^p(0, T)$ such that

$$\|F(\tau, u_1) - F(\tau, u_2)\|_{X_0} \leq \varphi_R(\tau) \cdot \|u_1 - u_2\|_{X_\gamma}$$

for almost all $\tau \in [0, T]$ and all $u_1, u_2 \in X_\gamma$ with $\|u_i - u_0\|_{X_\gamma} \leq R, i = 1, 2$.

(A3) The operator $A(0, u_0)$ has maximal regularity on X_0 for every $u_0 \in V$.

The following proposition yields local existence of a unique strong solution to the above evolution Eq.(4.1) under the assumptions presented in Assumption 4.1. We remark that a result of this type is well known in the autonomous case, see e.g. [42, Theorem 5.1.1], and the non-autonomous case also follows by mimicking and slightly adjusting the arguments therein. We refer to [41, Sect. 2] and [10, Sect. 7.1] for a discussion of the non-autonomous situation.

Proposition 4.2 *Let $p \in (1, \infty)$, assume that $u_0 \in V$, and make Assumption 4.1. Then there exists $T' \in (0, T]$ such that (4.1) has a unique solution $u \in \mathbb{E}_{T'}$ in $(0, T')$.*

4.2 A variant of a nonlinear complex interpolation result

This subsection is dedicated to stating a variant of a nonlinear complex interpolation result due to Bergh [2], and we first recall the underlying setting. Let (E_0, E_1) be a couple of complex Banach spaces that are both embedded in a common Hausdorff topological vector space. By Σ_E , we denote the sum of the Banach spaces E_0 and E_1 , i.e., $\Sigma_E = E_0 + E_1$, and by $\mathcal{F}(E_0, E_1)$, we denote the Banach space of all functions f defined on the strip $0 \leq \text{Re } z \leq 1$ in the complex plane such that

- (i) $f(z) \in \Sigma_E$ and f is continuous in Σ_E on $0 \leq \text{Re } z \leq 1$,
- (ii) $f(j + it) \in E_j, j = 0, 1$, and $f(j + i\cdot)$ is continuous in E_j and satisfies that $\lim_{|t| \rightarrow \infty} f(j + it) = 0$ in E_j ,
- (iii) f is analytic in Σ_E on $0 < \text{Re } z < 1$, and
- (iv) $\|f\|_{\mathcal{F}} := \max_{j=0,1} (\sup_{t \in \mathbb{R}} \|f(j + it)\|_{E_j})$.

The complex interpolation space E_θ is the Banach space of all $f(\theta)$ such that $f \in \mathcal{F}(E_0, E_1)$, where we have chosen $0 < \theta < 1$, and the norm of $a \in E_\theta$ is given by $\|a\|_\theta := \inf \|f\|_{\mathcal{F}}$, where $f(\theta) = a$.

A slight modification of the standard arguments in [2] then yields the following result.

Proposition 4.3 *Let $E^1 = (E_0^1, E_1^1), E^2 = (E_0^2, E_1^2)$ and $F = (F_0, F_1)$ be three couples of complex Banach spaces. Assume in addition that the (not necessarily linear) operator $N : \Sigma_{E^1} \times \Sigma_{E^2} \rightarrow \Sigma_F$ fulfills*

- (i) $f_k \in \mathcal{F}(E_0^k, E_1^k)$ implies $Nf = N(f_1, f_2) \in \mathcal{F}(F_0, F_1)$, and

$$(ii) \|N(a_1, a_2)\|_{F_j} \leq \mu(\|a_1\|_{E_j^1}, \|a_2\|_{E_j^2}), a_1 \in E_j^1, a_2 \in E_j^2, j = 0, 1,$$

where μ is continuous and positive on $\mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0}$, and it additionally satisfies the following monotonicity property

$$\mu(x, y) \leq \mu(\tilde{x}, y) \quad \text{and} \quad \mu(x, y) \leq \mu(x, \tilde{y})$$

for all $x, y, \tilde{x}, \tilde{y} \in \mathbb{R}_{\geq 0}$ such that $x \leq \tilde{x}$ and $y \leq \tilde{y}$.

Then, $a = (a_1, a_2) \in E_\theta^1 \times E_\theta^2$ implies that $N(a_1, a_2) \in F_\theta$, and it is valid that

$$\|N(a_1, a_2)\|_{F_\theta} \leq \mu(\|a_1\|_{E_\theta^1}, \|a_2\|_{E_\theta^2}).$$

5 Proof of the main result

This section is dedicated to showing the main result. Let us first explain the general strategy.

In order, to apply Proposition 4.2 to (3.16), we reformulate (3.16) as a non-autonomous quasilinear abstract Cauchy problem in Sect. 5.1. Afterwards, we verify the conditions of Assumption 4.1. We start by checking that the transformed operator has the property of maximal L^p -regularity in Sect. 5.2, for which we use decoupling techniques to account for the coupling condition on the interface. Besides, employing the structure and regularity of the coordinate transform as explained in Remark 3.2 and Proposition 3.3, we establish Lipschitz estimates for the transformed operator in Sect. 5.3 and for the transformed right-hand side in Sect. 5.4. The latter aspect relies on an application of Proposition 4.3. Finally, the local well-posedness of (3.16) follows from Proposition 4.2.

In order to conclude Theorem 3.4, recall from Remark 3.2 that the matrix Q and the transforms Z and Y can be constructed from (ξ, Ω) . As a consequence of Theorem 3.4, we obtain our main result Theorem 2.1 by performing the backward change of variables and coordinates given in Sect. 3.

5.1 Reformulation of (3.16) as a non-autonomous quasilinear abstract Cauchy problem

We start by initializing the Banach spaces

$$\tilde{X}_0 = \tilde{X}_0 \times \mathbb{R}^3 = L^q(\mathcal{D}; \mathbb{R}^2) \times L^q(\mathcal{D}) \times L^q(\mathcal{D}) \times \mathbb{R}^2 \times \mathbb{R}. \tag{5.1}$$

For fixed $t \in \mathbb{R}_+$ and $\tilde{w}_0 = (\tilde{u}_0, \tilde{h}_0, \tilde{a}_0, \xi_0, \Omega_0)$, let $\tilde{A}_m^H(t, \tilde{w}_0)$ be given by

$$\begin{aligned} \tilde{A}_m^H(t, \tilde{w}_0)\tilde{u} &:= \frac{1}{\rho_{ice}\tilde{h}_0}\tilde{A}^H(t, \tilde{w}_0)\tilde{u}, \\ D(\tilde{A}_m^H(t, \tilde{w}_0)) &:= \{\tilde{u} \in W^{2,q}(\mathcal{D}; \mathbb{R}^2) : \tilde{u} = 0 \text{ on } \partial\mathcal{O}\}, \end{aligned} \tag{5.2}$$

where the transformed Hilber operator $\tilde{A}^H(t, \tilde{w}_0)\tilde{u}$ is defined in (3.10). Note that its domain is independent of t and \tilde{w}_0 . Further, the lower order terms lead to the operator

$$\tilde{B}(t, \tilde{w}_0) \begin{pmatrix} \tilde{h} \\ \tilde{a} \end{pmatrix} := \frac{1}{\rho_{ice}\tilde{h}_0} \tilde{B}_1(t, \tilde{w}_0)\tilde{h} + \frac{1}{\rho_{ice}\tilde{h}_0} \tilde{B}_2(t, \tilde{w}_0)\tilde{a}, \tag{5.3}$$

where $\tilde{B}_1(t, \tilde{w}_0)$ and $\tilde{B}_2(t, \tilde{w}_0)$ are given in (3.11). Finally, the transformed Laplacians $\tilde{D}(t, \tilde{w}_0)$ are given by

$$\begin{aligned} \tilde{D}(t, \tilde{w}_0) \begin{pmatrix} \tilde{h} \\ \tilde{a} \end{pmatrix} &:= \begin{pmatrix} d_h \tilde{L}(t, \xi_0, \Omega_0)\tilde{h} \\ d_a \tilde{L}(t, \xi_0, \Omega_0)\tilde{a} \end{pmatrix}, \\ D(\tilde{D}(t, \tilde{w}_0)) &:= \left\{ (\tilde{h}, \tilde{a}) \in W^{2,q}(\mathcal{D})^2 : \partial_\nu \tilde{h} = \partial_\nu \tilde{a} = 0 \text{ on } \partial\mathcal{D} \right\}, \end{aligned} \tag{5.4}$$

where $\tilde{L}(t, \xi_0, \Omega_0)$ is defined as in (3.13). Moreover, we denote by $\text{tr}_\Gamma : W^{2,q}(\mathcal{D}; \mathbb{R}^2) \rightarrow W^{2-1/q,q}(\Gamma; \mathbb{R}^2)$ the trace operator on Γ , and we define the coupling operator

$$R : \mathbb{R}^3 \rightarrow W^{2-1/q}(\Gamma, \mathbb{R}^2), \quad \begin{pmatrix} \xi \\ \Omega \end{pmatrix} \mapsto (\Omega y^\perp + \xi)\mathbb{1}_\Gamma, \tag{5.5}$$

where $\mathbb{1}_\Gamma$ denotes the constant 1-function on Γ .

For $\tilde{w}_0 = (\tilde{u}_0, \tilde{h}_0, \tilde{a}_0, \xi_0, \Omega_0)$ we consider $\tilde{A}_m^H(t, \tilde{w}_0)$ as in (5.2), $\tilde{B}(t, \tilde{w}_0)$ as in (5.3), $\tilde{D}(t, \tilde{w}_0)$ as in (5.4) and R as in (5.5), and we introduce the family of operator matrices $\tilde{\mathcal{A}}(t, \tilde{w}_0)$ given by

$$\tilde{\mathcal{A}}(t, \tilde{w}_0) := \begin{pmatrix} \tilde{A}_m^H(t, \tilde{w}_0) & -\tilde{B}(t, \tilde{w}_0) & 0 \\ 0 & -\tilde{D}(t, \tilde{w}_0) & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{5.6}$$

with domain

$$\begin{aligned} \tilde{\mathcal{X}}_1 &:= D(\tilde{\mathcal{A}}(t, \tilde{w}_0)) \\ &:= \left\{ \tilde{w} = (\tilde{u}, \tilde{h}, \tilde{a}, \xi, \Omega) \in D(\tilde{A}_m^H(t, \tilde{w}_0)) \times D(\tilde{D}(t, \xi_0, \Omega_0)) : \right. \\ &\quad \left. \text{tr}_\Gamma \tilde{u} = R \begin{pmatrix} \xi \\ \Omega \end{pmatrix} \right\}. \end{aligned} \tag{5.7}$$

Again, we observe that the domain is independent of t and \tilde{w}_0 . We will see in Sect. 5.2 that $\tilde{\mathcal{A}}(t, \tilde{w}_0)$ is a closed operator and hence $\tilde{\mathcal{X}}_1 = D(\tilde{\mathcal{A}}(t, \tilde{w}_0))$ is a Banach space with the graph norm. Furthermore, we denote by $\tilde{\mathcal{X}}_\gamma = (\tilde{\mathcal{X}}_0, \tilde{\mathcal{X}}_1)_{1-1/p,p}$ the trace space, which we will determine in terms of Besov spaces in Lemma 5.3.

Next, we summarize the transformed nonlinear terms coming from the sea ice equations in (3.16) in

$$\tilde{\mathcal{G}}_1(t, \tilde{w}) = \left(\begin{array}{c} \frac{1}{\rho_{ice}\tilde{h}} f_1(\tilde{u}, \tilde{h}) - \sum_{i,j=1}^2 \tilde{u}_i (\partial_i Y_j) \partial_j \tilde{u} \\ S_h(\tilde{h}, \tilde{a}) - \sum_{i,j=1}^2 (\partial_i Y_j) (\tilde{u}_i \partial_j \tilde{h} + \tilde{h} \partial_j \tilde{u}_i) \\ S_a(\tilde{h}, \tilde{a}) - \sum_{i,j=1}^2 (\partial_i Y_j) (\tilde{u}_i \partial_j \tilde{a} + \tilde{a} \partial_j \tilde{u}_i) \end{array} \right), \tag{5.8}$$

with f_1 is as in (2.6), S_h as in (2.7) and S_a as in (2.8), where Remark 3.2 guarantees that the right-hand side is a function depending only on (t, \tilde{w}) . Similarly, we introduce $\tilde{\mathcal{G}}_2$ for the transformed nonlinear terms in the equations for the motion of the rigid body, i.e.,

$$\tilde{\mathcal{G}}_2(t, \tilde{w}) = \left(\begin{array}{c} \frac{1}{m_B} \tilde{F} - \frac{1}{m_B} \int_{\Gamma} \mathcal{T}_{\delta}(\tilde{u}, \tilde{h}, \tilde{a}) \tilde{n} \, dS \\ I^{-1} \tilde{N} - I^{-1} \int_{\Gamma} y^{\perp} \mathcal{T}_{\delta}(\tilde{u}, \tilde{h}, \tilde{a}) \tilde{n} \, dS \end{array} \right) \tag{5.9}$$

for $\tilde{w} = (\tilde{u}, \tilde{h}, \tilde{a}, \xi, \Omega)$. We then set $\tilde{\mathcal{G}}(t, \tilde{w}) := \begin{pmatrix} \tilde{\mathcal{G}}_1(t, \tilde{w}) \\ \tilde{\mathcal{G}}_2(t, \tilde{w}) \end{pmatrix}$. Taking into account Remark 3.2, we introduce the notation

$$\tilde{\mathcal{M}}_1(t, \xi, \Omega) \tilde{v} = \begin{pmatrix} \sum_{j=1}^2 \dot{Y}_j \partial_j \tilde{u} \\ \sum_{j=1}^2 \dot{Y}_j \partial_j \tilde{h} \\ \sum_{j=1}^2 \dot{Y}_j \partial_j \tilde{a} \end{pmatrix} \tag{5.10}$$

for $\tilde{v} = (\tilde{u}, \tilde{h}, \tilde{a})$ and $\tilde{\mathcal{M}}(t, \xi, \Omega) := \begin{pmatrix} \tilde{\mathcal{M}}_1(t, \xi, \Omega) \\ 0 \end{pmatrix}$. Finally, we define

$$\tilde{\mathcal{F}}(t, \tilde{w}) := \tilde{\mathcal{G}}(t, \tilde{w}) - \tilde{\mathcal{M}}(t, \xi, \Omega) \tilde{w} + (0, 0, 0, \Omega \xi^{\perp}, 0)^T. \tag{5.11}$$

Therefore, (3.16) can be written as a non-autonomous quasilinear abstract Cauchy problem

$$\begin{cases} \tilde{w}_t + \tilde{\mathcal{A}}(t, \tilde{w}) \tilde{w} = \tilde{\mathcal{F}}(t, \tilde{w}), & t \in [0, T], \\ \tilde{w}(0) = \tilde{w}_0, \end{cases} \tag{5.12}$$

with initial data $\tilde{w}_0 = (\tilde{u}_0, \tilde{h}_0, \tilde{a}_0, \xi_0, \Omega_0) \in V$, where V is as made precise in (2.14). In the next subsections, we verify that (5.12) is in the framework of Sect. 4.1, i.e., we check that $\tilde{\mathcal{A}}$ as in (5.6) and $\tilde{\mathcal{F}}$ as in (5.11) satisfy the conditions (A1), (A2) and (A3) from Assumption 4.1.

5.2 Maximal regularity of the linearized operator matrix

In this subsection, we give an abstract tool to verify the assumption that for $w_0 \in V$ fixed, $\tilde{\mathcal{A}}(0, w_0)$ admits maximal L^p -regularity on the ground space $\tilde{\mathcal{X}}_0$ as made precise in (5.1). Throughout this subsection, we employ the notation $b = (h, a)$, $z = (\eta, \omega)$.

Recalling $A^H(w_0)$ from (3.9) as well as $\tilde{A}_m^H(0, w_0)$ and $D(A_m^H) = D(\tilde{A}_m^H(0, w_0))$ from (5.2), we introduce

$$\begin{aligned}
 A_m^H(w_0)u &:= \tilde{A}_m^H(0, w_0)u = \frac{1}{\rho_{\text{ice}}h_0} A^H(w_0)u, \\
 D(A_m^H) &:= D(\tilde{A}_m^H(0, w_0)), \\
 Db &:= \text{diag}(d_h \Delta, d_a \Delta)b, \\
 D(D) &:= \{b \in W^{2,q}(\mathcal{D})^2 : \partial_\nu h = \partial_\nu a = 0 \text{ on } \partial\mathcal{D}\}, \\
 B(w_0)b &:= \frac{\partial_h P(h_0, a_0)}{2\rho_{\text{ice}}h_0} \nabla h + \frac{\partial_a P(h_0, a_0)}{2\rho_{\text{ice}}h_0} \nabla a, \text{ for } b \in D(D).
 \end{aligned}$$

Here we used that the transform Y satisfies $Y(0) = \text{Id}$ by definition, and therefore, the transformed variables and the usual variables coincide. We then get

$$\mathcal{A} := \tilde{\mathcal{A}}(0, w_0) = \begin{pmatrix} \tilde{A}_m^H(0, w_0) & -\tilde{B}(0, w_0) & 0 \\ 0 & -\tilde{D}(0, w_0) & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} A_m^H(w_0) & -B(w_0) & 0 \\ 0 & -D & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

with coupled domain $\tilde{\mathcal{X}}_1$ as in (5.7). To simplify the notation, we omit the entry w_0 , since it is fixed. For the trace tr_Γ and the coupling operator R as in (5.5), the coupled domain then rewrites as

$$\tilde{\mathcal{X}}_1 = D(\mathcal{A}) = \{(u, b, z) \in D(A_m^H) \times D(D) \times \mathbb{R}^3 : \text{tr}_\Gamma u = Rz\}.$$

Next, we introduce the *Hibler operator with homogeneous Dirichlet boundary conditions* A_D^H on $L^q(\mathcal{D}; \mathbb{R}^2)$ as investigated in [3, Sects. 4 and 6]. In the present framework, it is given by

$$A_D^H u := A_m^H u, \quad D(A_D^H) := \ker(\text{tr}_\Gamma) = W^{2,q}(\mathcal{D}; \mathbb{R}^2) \cap W_0^{1,q}(\mathcal{D}; \mathbb{R}^2).$$

Now, the *decoupled operator matrix* $\mathcal{A}_0 : D(\mathcal{A}_0) \subset \tilde{\mathcal{X}}_0 \rightarrow \tilde{\mathcal{X}}_0$ is given by

$$\mathcal{A}_0 = \begin{pmatrix} A_D^H & -B & 0 \\ 0 & -D & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad D(\mathcal{A}_0) = D(A_D^H) \times D(D) \times \mathbb{R}^3, \tag{5.13}$$

and we equip $D(\mathcal{A}_0)$ with the graph norm. By [3, Theorem 4.4 and Lemma 6.1], there is $\lambda_0 \in \mathbb{R}$ such that for all $\lambda > \lambda_0$ it holds that $A_D^H + \lambda$ has the property of maximal L^p -regularity, and it follows that $A_D^H + \lambda$ then is also invertible. Therefore, for such λ , we introduce the translated versions of \mathcal{A} and \mathcal{A}_0 as $\mathcal{A}_\lambda := \mathcal{A} + \text{diag}(\lambda, 0, 0)$ and $\mathcal{A}_{0,\lambda} := \mathcal{A}_0 + \text{diag}(\lambda, 0, 0)$ which does not affect the domains of the operators, i.e., $D(\mathcal{A}_\lambda) = D(\mathcal{A}) = \tilde{\mathcal{X}}_1$ and $D(\mathcal{A}_{0,\lambda}) = D(\mathcal{A}_0)$.

The aim now is to show that \mathcal{A}_λ admits maximal L^p -regularity on $\tilde{\mathcal{X}}_0$ by invoking the maximal regularity of $A_D^H + \lambda$ and exploiting the upper triangular structure of the operator matrix. To this end, we use a method of decoupling, and we first argue that

$$L_0 = \left(\text{tr}_\Gamma \Big|_{\ker(A_m^H + \lambda)} \right)^{-1} \tag{5.14}$$

is well-defined and continuous:

Lemma 5.1 *The operator $\text{tr}_\Gamma|_{\ker(A_m^H + \lambda)} \in \mathcal{L}(W^{2,q}(\mathcal{D}; \mathbb{R}^2), W^{2-1/q,q}(\Gamma; \mathbb{R}^2))$ is continuously invertible, and L_0 defined in (5.14) is thus bounded from $W^{2-1/q,q}(\Gamma; \mathbb{R}^2)$ to $W^{2,q}(\mathcal{D}; \mathbb{R}^2)$.*

Proof We make use of [48, Theorem 4.7.1] to deduce that tr_Γ is a retraction from $W^{2,q}(\mathcal{D}; \mathbb{R}^2)$ onto $W^{2-1/q,q}(\Gamma; \mathbb{R}^2)$, so it is in particular surjective and continuous. The closed graph theorem, or, equivalently, the bounded inverse theorem, yields that the graph norm of A_m^H is equivalent to the $W^{2,q}$ -norm, and closedness of A_m^H follows by the fact that it is an elliptic differential operator of second order. In particular, we have argued that $\text{tr}_\Gamma \in \mathcal{L}(W^{2,q}(\mathcal{D}; \mathbb{R}^2), W^{2-1/q,q}(\Gamma; \mathbb{R}^2))$. The latter observation and closedness of A_m^H imply that

$$\begin{pmatrix} A_m^H \\ \text{tr}_\Gamma \end{pmatrix} : D(A_m^H) \rightarrow L^q(\mathcal{D}; \mathbb{R}^2) \times W^{2-1/q,q}(\Gamma; \mathbb{R}^2)$$

is closed. The invertibility of $A_D^H + \lambda$ yields the existence and continuity of the above L_0 in view of [5, Lemma 2.2]. Note that tr_Γ plays the role of L in the abstract framework of [5, Section 2]. □

The shape of R reveals that it is especially bounded and $\text{Im}(R) \subset W^{2-1/q,q}(\Gamma; \mathbb{R}^2)$, so it follows that L_0R is bounded as the product of two bounded operators. Consequently, the operator

$$\begin{aligned} S &= \begin{pmatrix} \text{Id} & 0 & -L_0R \\ 0 & \text{Id} & 0 \\ 0 & 0 & \text{Id} \end{pmatrix} \quad \text{is bounded with inverse} \\ S^{-1} &= \begin{pmatrix} \text{Id} & 0 & L_0R \\ 0 & \text{Id} & 0 \\ 0 & 0 & \text{Id} \end{pmatrix}. \end{aligned} \tag{5.15}$$

The following result establishes maximal regularity in the coupled setting.

Proposition 5.2 *Let $p, q \in (1, \infty)$ be such that (2.15) holds true. Then for $w_0 \in V$, given in (2.14), and $\lambda > \lambda_0$, the operator matrix \mathcal{A}_λ admits maximal L^p -regularity on $\tilde{\mathcal{X}}_0$.*

Proof Using $\text{Im}(L_0) \subset D(A_m^H)$, we derive that

$$\begin{aligned} SD(\mathcal{A}) &= \left\{ \begin{pmatrix} u - L_0Rz \\ b \\ z \end{pmatrix} : u \in D(A_m^H), b \in D(D), z \in \mathbb{R}^3, \text{tr}_\Gamma u = Rz \right\} \\ &= \left\{ \begin{pmatrix} \hat{u} \\ b \\ z \end{pmatrix} : \hat{u} \in D(A_m^H), b \in D(D), z \in \mathbb{R}^3, \text{tr}_\Gamma \hat{u} = 0 \right\} \\ &= D(A_D^H) \times D(D) \times \mathbb{R}^3 = D(\mathcal{A}_{0,\lambda}). \end{aligned}$$

On the other hand, employing that $\text{Im}(L_0) \subset \ker(A_m^H + \lambda)$, we deduce that

$$S\mathcal{A}_\lambda S^{-1} = \begin{pmatrix} A_m^H + \lambda & -B & (A_m^H + \lambda)L_0R \\ 0 & -D & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} A_m^H + \lambda & -B & 0 \\ 0 & -D & 0 \\ 0 & 0 & 0 \end{pmatrix} = \mathcal{A}_{0,\lambda},$$

where we used that $SD(\mathcal{A})$ is precisely the domain of $\mathcal{A}_{0,\lambda}$. The upper triangular structure of $\mathcal{A}_{0,\lambda}$, the well known fact that the Neumann Laplacian operator on $L^q(\mathcal{D})$ has the property of maximal L^p -regularity and the aforementioned result from [3] imply that $\mathcal{A}_{0,\lambda}$ admits maximal L^p -regularity. The claim follows now from the identity $\mathcal{A}_\lambda = S^{-1}\mathcal{A}_{0,\lambda}S$. □

We next verify that the trace space in the coupled setting can be represented in terms of the matrix S^{-1} and the trace space $\tilde{\mathcal{Y}}_\gamma$ in the decoupled setting, and we characterize the coupled trace space in terms of Besov spaces. The ground space in the decoupled setting is also given by $\tilde{\mathcal{X}}_0$, while $D(\mathcal{A}_0)$ as in (5.13) represents the regularity space.

Lemma 5.3 *For S and S^{-1} as in (5.15), $\tilde{\mathcal{X}}_0$ as in (5.1), $\tilde{\mathcal{X}}_1$ as in (5.7) and $D(\mathcal{A}_0)$ as in (5.13), $\tilde{\mathcal{X}}_\gamma = (\tilde{\mathcal{X}}_0, \tilde{\mathcal{X}}_1)_{1-1/p,p}$ and $\tilde{\mathcal{Y}}_\gamma = (\tilde{\mathcal{X}}_0, D(\mathcal{A}_0))_{1-1/p,p}$, it holds that*

$$\begin{aligned} \tilde{\mathcal{X}}_\gamma &= S^{-1}(\tilde{\mathcal{Y}}_\gamma) \\ &= \left\{ (u, b, z) \in B_{qp}^{2-2/p}(\mathcal{D}; \mathbb{R}^2) \times B_{qp}^{2-2/p}(\mathcal{D})^2 \times \mathbb{R}^3 : u = \eta + \omega(x - x_c)^\perp \right\}. \end{aligned}$$

Proof It is well known that $(L^q(\mathcal{D}), W^{2,q}(\mathcal{D}))_{1-1/p,p} = B_{qp}^{2-2/p}(\mathcal{D})$, see e.g. [48] or [1, Chapter 5].

We recall from the above arguments that \mathcal{A}_0 and \mathcal{A} generate analytic semigroups $T_{\mathcal{A}_0}$ and $T_{\mathcal{A}}$ on $\tilde{\mathcal{X}}_0$. Moreover, $\mathcal{A} = S^{-1}\mathcal{A}_0S$, and it also follows that $T_{\mathcal{A}} = S^{-1}T_{\mathcal{A}_0}S$. By definition of the trace space, an insertion of the relation of the operators and semigroups, and using $x = S^{-1}y$ for some $y \in \tilde{\mathcal{X}}_0$ in conjunction with easy functional analytic arguments involving the boundedness of S , we have

$$\begin{aligned} \tilde{\mathcal{X}}_\gamma &= \left\{ x \in \tilde{\mathcal{X}}_0 : [x]_{1-1/p,p} := \left(\int_0^\infty \|t^{1/p} \mathcal{A}T_{\mathcal{A}}x\|_{\tilde{\mathcal{X}}_0}^p dt/t \right)^{1/p} < \infty \right\} \\ &= \left\{ S^{-1}y \in \tilde{\mathcal{X}}_0 : [S^{-1}y]_{1-1/p,p} := \left(\int_0^\infty \|t^{1/p} S^{-1}\mathcal{A}_0T_{\mathcal{A}_0}y\|_{\tilde{\mathcal{X}}_0}^p dt/t \right)^{1/p} < \infty \right\} \\ &= S^{-1}(\tilde{\mathcal{Y}}_\gamma), \end{aligned}$$

so the proof is completed by recalling the shape of S^{-1} . □

5.3 Lipschitz estimates of the operator matrix

We start verifying the aspect (A1) of Assumption 4.1.

Lemma 5.4 *Let $p, q \in (1, \infty)$ satisfy (2.15), consider V as in (2.14) and recall \tilde{A} from (5.6), $\tilde{\mathcal{X}}_0$ from (5.1) as well as $\tilde{\mathcal{X}}_1$ from (5.7). Then the map*

$$[0, T] \times V \rightarrow \mathcal{L}(\tilde{\mathcal{X}}_1, \tilde{\mathcal{X}}_0), (t, \tilde{w}_0) \mapsto \tilde{A}(t, \tilde{w}_0)$$

is continuous.

Proof To show continuity of $[0, T] \times V \rightarrow \mathcal{L}(\tilde{\mathcal{X}}_1, \tilde{\mathcal{X}}_0), (t, \tilde{w}_0) \mapsto \tilde{A}(t, \tilde{w}_0)$, it suffices to show continuity of $(t, \tilde{w}_0) \mapsto \tilde{A}_m^H(t, \tilde{w}_0), (t, \tilde{w}_0) \mapsto -\tilde{B}(t, \tilde{w}_0)$ and $(t, \tilde{w}_0) \mapsto -\tilde{D}(t, \tilde{w}_0)$ separately.

First, we observe that for $\tilde{w}_0 = (\tilde{u}_0, \tilde{h}_0, \tilde{a}_0, \xi_0, \Omega_0) \in V$, the angular and translational velocities ξ_0 and Ω_0 are independent of time, so it is in particular valid that $(\xi_0, \Omega_0) \in W^{1,p}(0, T; \mathbb{R}^2) \times W^{1,p}(0, T)$. Consequently, we are in the framework of Remark 3.2 as well as Proposition 3.3. As a result, we obtain continuity of

$$\begin{aligned} [0, T] \times \mathbb{R}^3 &\rightarrow L^\infty(\mathbb{R}^2): (t, \xi_0, \Omega_0) \mapsto \partial_i Y \quad \text{as well as} \\ [0, T] \times \mathbb{R}^3 &\rightarrow L^\infty(\mathbb{R}^2): (t, \xi_0, \Omega_0) \mapsto \partial_i \partial_j Y, \quad \text{and then also of} \quad (5.16) \\ [0, T] \times \mathbb{R}^3 &\rightarrow L^\infty(\mathbb{R}^2): (t, \xi_0, \Omega_0) \mapsto g^{ij}, \end{aligned}$$

where g^{ij} denotes the contravariant tensor as introduced in (3.12).

Using Hölder’s inequality in conjunction with the fact that $\tilde{L}(t, \xi_0, \Omega_0)$ from (3.13) is a differential operator of second order, we deduce from (5.16) the continuity of $[0, T] \times \mathbb{R}^3 \rightarrow \mathcal{L}(W^{2,q}(\mathcal{D}), L^q(\mathcal{D})): (t, \xi_0, \Omega_0) \mapsto \tilde{L}(t, \xi_0, \Omega_0)$. It readily follows from the shape of $-\tilde{D}(t, \xi_0, \Omega_0)$ from (5.4) that

$$[0, T] \times V \rightarrow \mathcal{L}(W^{2,q}(\mathcal{D})^2, L^q(\mathcal{D})^2): (t, \tilde{w}_0) \mapsto -\tilde{D}(t, \tilde{w}_0)$$

is continuous.

Next, we note that for $\tilde{w}_0 = (\tilde{u}_0, \tilde{h}_0, \tilde{a}_0, \xi_0, \Omega_0) \in V$, we get $\frac{1}{\rho_{ice}\tilde{h}_0} < \frac{1}{\rho_{ice}\kappa} < \infty$. In addition, we infer continuity of

$$\begin{aligned} L^\infty(\mathcal{D})^2 &\rightarrow L^\infty(\mathcal{D}), (\tilde{h}_0, \tilde{a}_0) \mapsto e^{-c(1-\tilde{a})} \quad \text{and} \\ L^\infty(\mathcal{D})^2 &\rightarrow L^\infty(\mathcal{D}), (\tilde{h}_0, \tilde{a}_0) \mapsto \tilde{h}_0 e^{-c(1-\tilde{a})}, \end{aligned} \quad (5.17)$$

so the embedding

$$V \hookrightarrow L^\infty(\mathcal{D})^4 \times \mathbb{R}^3, \quad (5.18)$$

following from (2.16) and Lemma 5.3, and Hölder’s inequality yield continuity of

$$[0, T] \times V \rightarrow \mathcal{L}(W^{1,q}(\mathcal{D})^2, L^q(\mathcal{D})^2): (t, \tilde{w}_0) \mapsto -\tilde{B}(t, \tilde{w}_0)$$

upon recalling the shape of $\tilde{B}(t, \tilde{w}_0)$ from (5.3).

Finally, we show the continuity of \tilde{A}_m^H : In view of (5.2), the above observation that $\frac{1}{\rho_{ice}\tilde{h}_0} < \frac{1}{\rho_{ice}\kappa} < \infty$ for $\tilde{w}_0 = (\tilde{u}_0, \tilde{h}_0, \tilde{a}_0, \xi_0, \Omega_0) \in V$ implies that it suffices to show the continuity of \tilde{A}^H defined in (3.10). By Hölder’s inequality, (5.16), (5.17), (5.18),

the shapes of a_{ij}^{kl} as in (3.8) and a_{ij}^{klm} as in Sect. 3 and the continuous dependence of the coefficients a_{ij}^{kl} on \tilde{u}_0, \tilde{h}_0 as well as \tilde{a}_0 , see [3, Sect. 6], we conclude continuity of

$$[0, T] \times V \rightarrow \mathcal{L}(W^{2,q}(\mathcal{D})^2, L^q(\mathcal{D})^2): (t, \tilde{w}_0) \mapsto \tilde{A}^H(t, \tilde{w}_0).$$

The assertion of the lemma then follows by concatenating the previous arguments and observing that

$$\tilde{\mathcal{X}}_1 \hookrightarrow W^{2,q}(\mathcal{D})^4 \times \mathbb{R}^3 \tag{5.19}$$

by virtue of $\tilde{\mathcal{X}}_1 = \mathcal{S}^{-1}(D(\mathcal{A}_0))$, see the proof of Proposition 5.2 for this relation of $D(\mathcal{A}_0)$ as in (5.13), and the classical embedding $D(\mathcal{A}_0) \hookrightarrow W^{2,q}(\mathcal{D})^4 \times \mathbb{R}^3$. \square

Now we show the Lipschitz continuity of the system matrix.

Lemma 5.5 *Let $p, q \in (1, \infty)$ be such that (2.15) is valid, and consider $\tilde{w}_0 \in V$ fixed. Then there exists $R_0 > 0$ and a constant $L > 0$ independent of τ such that $\overline{B}_{\tilde{\mathcal{X}}_\gamma}(\tilde{w}_0, R_0) \subset V$ and*

$$\|\tilde{A}(\tau, \tilde{w}_1)\tilde{u} - \tilde{A}(\tau, \tilde{w}_2)\tilde{u}\|_{\tilde{\mathcal{X}}_0} \leq L \cdot \|\tilde{w}_1 - \tilde{w}_2\|_{\tilde{\mathcal{X}}_\gamma} \cdot \|\tilde{u}\|_{\tilde{\mathcal{X}}_1}$$

holds for all $\tilde{w} \in \tilde{\mathcal{X}}_1$, for all $\tilde{w}_1, \tilde{w}_2 \in \tilde{\mathcal{X}}_\gamma$ with $\|\tilde{w}_i - \tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma} \leq R_0, i = 1, 2$, and for $\tau \in [0, T]$, with $T > 0$ sufficiently small.

Proof We recall that $\tilde{w}_i = (\tilde{u}_i, \tilde{h}_i, \tilde{a}_i, \xi_i, \Omega_i)$ for $i \in \{0, 1, 2\}$. First, it follows from $\tilde{w}_1, \tilde{w}_2 \in V$ that ξ_i and $\Omega_i, i = 1, 2$, are independent of time, so it holds in particular that $(\xi_i, \Omega_i) \in W^{1,p}(0, T; \mathbb{R}^2) \times W^{1,p}(0, T)$, and we are thus in the setting of Remark 3.2 and Proposition 3.3. Using that

$$\begin{aligned} K_i &= \|\xi_i\|_{L^\infty(0,T;\mathbb{R}^2)} + \|\Omega_i\|_{L^\infty(0,T)} \\ &= |\xi_i| + |\Omega_i| \leq c(R_0 + \|\tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma}), \quad i = 1, 2, \end{aligned}$$

where we additionally exploited that (ξ_i, Ω_i) are independent of time. Employing this observation in conjunction with Proposition 3.3, we argue that

$$\begin{aligned} &\|\partial^\alpha Z_i\|_{\infty,\infty} + \|\partial^\alpha Y_i\|_{\infty,\infty} \\ &\leq \tilde{c}(R_0 + \|\tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma}), \quad i = 1, 2, \quad \text{and} \\ &\|\partial^\beta(Z_1 - Z_2)\|_{\infty,\infty} + \|\partial^\beta(Y_1 - Y_2)\|_{\infty,\infty} \\ &\leq \tilde{c}(R_0 + \|\tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma})T(|\xi_1 - \xi_2| + |\Omega_1 - \Omega_2|) \end{aligned} \tag{5.20}$$

for all multi-indices α, β such that $1 \leq |\alpha| \leq 3$ and $0 \leq |\beta| \leq 3$. As above, the index i indicates that Z_i and Y_i correspond to $(\xi_i, \Omega_i), i = 1, 2$.

As in the proof of Lemma 5.4, it is sufficient to verify the Lipschitz properties of the entries $\tilde{A}_m^H(t, \tilde{w}), -\tilde{B}(t, \tilde{w})$ and $-\tilde{D}(t, \tilde{w})$ separately. First, using Hölder’s inequality,

for $\tilde{w}_1, \tilde{w}_2 \in \overline{B}_{\tilde{\mathcal{X}}_\gamma}(\tilde{w}_0, R_0) \subset V$, $\tilde{w} \in \tilde{\mathcal{X}}_1$ and $\tau \in [0, T]$, we get for \tilde{L} as in (3.13) that

$$\begin{aligned} & \|\tilde{L}(\tau, \xi_1, \Omega_1)\tilde{h} - \tilde{L}(\tau, \xi_2, \Omega_2)\tilde{h}\|_{L^q(\mathcal{D})} \\ & \leq \sum_{j=1}^2 \|\Delta Y_j(\tau, \xi_1, \Omega_1) - \Delta Y_j(\tau, \xi_2, \Omega_2)\|_{L^\infty(\mathcal{D})} \cdot \|\partial_j \tilde{h}\|_{L^q(\mathcal{D})} \\ & \quad + \sum_{j,k=1}^2 \|g^{jj}(\tau, \xi_1, \Omega_1) - g^{jj}(\tau, \xi_2, \Omega_2)\|_{L^\infty(\mathcal{D})} \cdot \|\partial_k \partial_j \tilde{h}\|_{L^q(\mathcal{D})} \\ & \leq C(R_0 + \|\tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma})T \|\tilde{w}_1 - \tilde{w}_2\|_{\tilde{\mathcal{X}}_\gamma} \|\tilde{w}\|_{\tilde{\mathcal{X}}_1} \end{aligned}$$

by virtue of (5.20) and the embedding from (5.19). We emphasize that the Lipschitz constant is independent of $\tau \in [0, T]$. Furthermore, we find a similar estimate for $\|\tilde{L}(\tau, \xi_1, \Omega_1)\tilde{a} - \tilde{L}(\tau, \xi_2, \Omega_2)\tilde{a}\|_{L^q(\mathcal{D})}$, so for $\tilde{D}(t, \xi_i, \Omega_i)$ as in (5.4), we get

$$\begin{aligned} & \left\| \tilde{D}(\tau, \xi_1, \Omega_1) \begin{pmatrix} \tilde{h} \\ \tilde{a} \end{pmatrix} - \tilde{D}(\tau, \xi_2, \Omega_2) \begin{pmatrix} \tilde{h} \\ \tilde{a} \end{pmatrix} \right\|_{L^q(\mathcal{D})^2} \\ & \leq C(R_0 + \|\tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma})T \|\tilde{w}_1 - \tilde{w}_2\|_{\tilde{\mathcal{X}}_\gamma} \|\tilde{w}\|_{\tilde{\mathcal{X}}_1}. \end{aligned}$$

Next, making use of Hölder’s inequality, (5.20), $\tilde{w}_i \in V$, with V as in (2.14), resulting in $e^{-c(1-\tilde{a}_i)}$ being finite, the embedding (5.18) as well as the Lipschitz estimates of terms associated to sea ice, see [3, Lemma 6.2], for $\tilde{B}_1(t, \tilde{w}_i)$ as in (3.11), we infer that

$$\begin{aligned} & \|\tilde{B}_1(\tau, \tilde{w}_1)\tilde{h} - \tilde{B}_1(\tau, \tilde{w}_2)\tilde{h}\|_{L^q(\mathcal{D})^2} \\ & \leq C \sum_{j=1}^2 \|e^{-c(1-\tilde{a}_1)} \partial_i Y_{1,j} - e^{-c(1-\tilde{a}_2)} \partial_i Y_{2,j}\|_{L^\infty(\mathcal{D})^2} \|\tilde{h}\|_{W^{1,q}(\mathcal{D})} \\ & \leq C \left(\|e^{-c(1-\tilde{a}_1)} (\partial_i Y_1 - \partial_i Y_2)\|_{L^\infty(\mathcal{D})^2} \right. \\ & \quad \left. + \| (e^{-c(1-\tilde{a}_1)} - e^{-c(1-\tilde{a}_2)}) \partial_i Y_2 \|_{L^\infty(\mathcal{D})^2} \right) \|\tilde{h}\|_{W^{1,q}(\mathcal{D})} \\ & \leq C(R_0 + \|\tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma})(T + 1) \|\tilde{w}_1 - \tilde{w}_2\|_{\tilde{\mathcal{X}}_\gamma} \|\tilde{w}\|_{\tilde{\mathcal{X}}_1}. \end{aligned}$$

Similar arguments lead to an analogous estimate for $\tilde{B}_2(\tau, \tilde{w}_1)\tilde{h} - \tilde{B}_2(\tau, \tilde{w}_2)\tilde{h}$. Additionally invoking that $\frac{1}{\rho_{ice}\tilde{h}_i} < \frac{1}{\rho_{ice}\kappa} < \infty$ by $\tilde{w}_i \in V$ and employing the embedding from (5.19), we deduce that for $\tilde{B}(t, \tilde{w}_i)$ as in (5.3), it holds that

$$\|\tilde{B}(\tau, \tilde{w}_1) - \tilde{B}(\tau, \tilde{w}_2)\|_{L^\infty(\mathcal{D})^2} \leq C(R_0 + \|\tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma})(T + 1) \|\tilde{w}_1 - \tilde{w}_2\|_{\tilde{\mathcal{X}}_\gamma} \|\tilde{w}\|_{\tilde{\mathcal{X}}_1}.$$

Recalling the shapes of \tilde{A}_m^H and \tilde{A}^H from (5.2) and (3.10), respectively, and exploiting Hölder’s inequality, (5.20), the embeddings (5.18) and (5.19), $\tilde{w}_i \in V$ as well as the estimates of the quasilinear terms in [3, Lemma 6.2], we find by means of a similar procedure as in the estimates above that

$$\|\tilde{A}_m^H(\tau, \tilde{w}_1)\tilde{w} - \tilde{A}_m^H(\tau, \tilde{w}_2)\tilde{w}\|_{L^q(\mathcal{D})^2} \leq C(R_0 + \|\tilde{w}_0\|_{\tilde{\mathcal{X}}_Y})(T + 1)\|\tilde{w}_1 - \tilde{w}_2\|_{\tilde{\mathcal{X}}_Y} \|\tilde{w}\|_{\tilde{\mathcal{X}}_1}.$$

Concatenating the above estimates and observing that the associated Lipschitz constants do only depend on R_0 , $\|\tilde{w}_0\|_{\tilde{\mathcal{X}}_Y}$ and T , but not on $\tau \in [0, T]$, we conclude the statement of the lemma. □

5.4 Lipschitz properties of the right-hand side

This subsection is devoted to checking the aspect (A2) of Assumption 4.1.

Lemma 5.6 *Let $p, q \in (1, \infty)$ be such that (2.15) holds true, and let V be as in (2.14), $\tilde{\mathcal{F}}$ as in (5.11), $\tilde{\mathcal{X}}_0$ as in (5.1) and $\tilde{\mathcal{X}}_1$ as in (5.7), and suppose that $F \in L^p(0, T; \mathbb{R}^2)$ as well as $N \in L^p(0, T)$. Then*

- (i) $\tilde{\mathcal{F}}(\cdot, \tilde{w}_0)$ is measurable for every $\tilde{w}_0 \in V$,
- (ii) $\tilde{\mathcal{F}}(\tau, \cdot) \in C(V, \tilde{\mathcal{X}}_0)$ for almost all $\tau \in [0, T]$, and
- (iii) $\tilde{\mathcal{F}}(\cdot, \tilde{w}_0) \in L^p(0, T; \tilde{\mathcal{X}}_0)$ is valid for every $\tilde{w}_0 \in V$.

Proof The assumption $\tilde{w}_0 \in V \subset \tilde{\mathcal{X}}_Y$ ensures that the terms involved are well-defined. In addition, for $\tilde{w}_0 = (\tilde{u}_0, \tilde{h}_0, \tilde{a}_0, \xi_0, \Omega_0) \in V$, we deduce that ξ_0 and Ω_0 are independent of time, so $(\xi_0, \Omega_0) \in W^{1,p}(0, T; \mathbb{R}^2) \times W^{1,p}(0, T)$. Consequently, it is justified to use Remark 3.2 as well as Proposition 3.3.

The shape of $\tilde{\mathcal{F}}$ as seen in (5.11), see also (5.8), (5.9) as well as (5.10), in conjunction with the regularity of the Z and Y in time and space as discussed in Proposition 3.3 and the measurability of the initial objects on the moving sea ice domain yield that (i) is satisfied.

Employing similar arguments as in the proof of Lemma 5.4, we find that (ii) is also valid. Property (iii) follows by similar arguments as (i), and in particular, we take the assumptions concerning F and N from Theorem 2.1 into account and observe the way they are transformed in Sect. 3 as well as $Q \in W^{2,p}(0, T; \mathbb{R}^{2 \times 2}) \hookrightarrow L^\infty(0, T; \mathbb{R}^{2 \times 2})$, see Remark 3.2. □

It remains to verify the Lipschitz property of $\tilde{\mathcal{F}}$, i.e., aspect (A2)(iv) in Assumption 4.1. Using a similar strategy as for the treatment of $\tilde{\mathcal{A}}$ in Sect. 5.3, i.e., making use of Remark 3.2 as well as Proposition 3.3 and invoking the respective estimates of the sea ice terms from [3, Lemma 6.2], we get the following result for $\tilde{\mathcal{G}}_1$ as in (5.8).

Lemma 5.7 *Let $p, q \in (1, \infty)$ be such that (2.15) is satisfied, and let $\tilde{w}_0 \in V$ be fixed. Then there is $R_0 > 0$ and a constant $L > 0$ such that $\overline{B}_{\tilde{\mathcal{X}}_Y}(\tilde{w}_0, R_0) \subset V$, and*

$$\|\tilde{\mathcal{G}}_1(\tau, \tilde{w}_1) - \tilde{\mathcal{G}}_1(\tau, \tilde{w}_2)\|_{\tilde{\mathcal{X}}_0} \leq L \cdot \|\tilde{w}_1 - \tilde{w}_2\|_{\tilde{\mathcal{X}}_Y}$$

holds for all $\tilde{w}_1, \tilde{w}_2 \in \tilde{\mathcal{X}}_\gamma$ with $\|\tilde{w}_i - \tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma} \leq R_0, i = 1, 2$, and for $\tau \in [0, T]$, where $T > 0$ is sufficiently small.

The treatment of $\tilde{\mathcal{G}}_2$ as in (5.9) is more involved, relying on a nonlinear complex interpolation result, see Proposition 4.3.

Lemma 5.8 *Let $p, q \in (1, \infty)$ satisfy (2.15), and consider $\tilde{w}_0 \in V$ fixed. Then there exists $R_0 > 0$ and a constant $L > 0$ so that $\bar{B}_{\tilde{\mathcal{X}}_\gamma}(\tilde{w}_0, R_0) \subset V$ and*

$$|\tilde{\mathcal{G}}_2(\tau, \tilde{w}_1) - \tilde{\mathcal{G}}_2(\tau, \tilde{w}_2)| \leq L(R_0, \|\tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma}) \cdot \|\tilde{w}_1 - \tilde{w}_2\|_{\tilde{\mathcal{X}}_\gamma}$$

is valid for all $\tilde{w}_1, \tilde{w}_2 \in \tilde{\mathcal{X}}_\gamma$ with $\|\tilde{w}_i - \tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma} \leq R_0, i = 1, 2$, and for $\tau \in [0, T]$, where $T > 0$ is sufficiently small.

Proof To simplify the notation, we define the operator

$$\mathcal{J} : \mathbf{W}^{\varepsilon+1/q, q}(\mathcal{D}; \mathbb{R}^{2 \times 2}) \rightarrow \mathbb{R}^3, \quad g \mapsto \left(\frac{1}{\text{m}_B} \int_\Gamma g \tilde{n}(y) \, dS, \int_{I^{-1}} \int_\Gamma y^\perp g \tilde{n}(y) \, dS \right),$$

for $0 < \varepsilon < 1 - \frac{1}{q}$. The boundedness of the trace $\gamma : \mathbf{W}^{\varepsilon/2+1/q, q}(\mathcal{D}) \rightarrow \mathbf{L}^q(\partial\mathcal{D})$ yields that

$$|\mathcal{J}(g)| \leq C \|g\|_{\mathbf{W}^{\varepsilon+1/q, q}(\mathcal{D}; \mathbb{R}^{2 \times 2})}, \quad g \in \mathbf{W}^{\varepsilon+1/q, q}(\mathcal{D}; \mathbb{R}^{2 \times 2}). \tag{5.21}$$

For $p, q \in (1, \infty)$ with $\frac{2}{p} + \frac{3}{q} < 1 - \varepsilon$, we consider $s \in (1, 2)$ with $s > \frac{2}{q} + 1$, and the resulting embedding

$$\mathbf{B}_{qp}^s(\mathcal{D}) \hookrightarrow \mathbf{C}^1(\bar{\mathcal{D}}) \hookrightarrow \mathbf{W}^{1, q}(\mathcal{D}),$$

see also (2.16), yields that

$$\begin{aligned} & \|\sigma_\delta(\tilde{v}_1) - \sigma_\delta(\tilde{v}_2)\|_{\mathbf{L}^q(\mathcal{D}; \mathbb{R}^{2 \times 2})} \\ & \leq C(1 + \|\tilde{v}_1\|_{\mathbf{C}^1(\mathcal{D})^4} + \|\tilde{v}_2\|_{\mathbf{C}^1(\mathcal{D})^4}) \|\tilde{v}_1 - \tilde{v}_2\|_{\mathbf{C}^1(\mathcal{D})^4} \\ & \leq C(1 + \|\tilde{v}_1\|_{\mathbf{B}_{qp}^s(\mathcal{D})^4} + \|\tilde{v}_2\|_{\mathbf{B}_{qp}^s(\mathcal{D})^4}) \|\tilde{v}_1 - \tilde{v}_2\|_{\mathbf{B}_{qp}^s(\mathcal{D})^4}. \end{aligned} \tag{5.22}$$

Analogously, we obtain that

$$\begin{aligned} \|\sigma_\delta(\tilde{v}_i)\|_{\mathbf{L}^q(\mathcal{D}; \mathbb{R}^{2 \times 2})} & \leq C(1 + \|\tilde{v}_i\|_{\mathbf{C}^1(\mathcal{D})^4}) \|\tilde{v}_i\|_{\mathbf{C}^1(\mathcal{D})^4} \\ & \leq C(1 + \|\tilde{v}_i\|_{\mathbf{B}_{qp}^s(\mathcal{D})^4}) \|\tilde{v}_i\|_{\mathbf{B}_{qp}^s(\mathcal{D})^4} \end{aligned} \tag{5.23}$$

Moreover, for $(\xi_i, \Omega_i) \in \mathbb{R}^3$, we have again $(\xi_i, \Omega_i) \in \mathbf{W}^{1, p}(0, T; \mathbb{R}^2) \times \mathbf{W}^{1, p}(0, T)$, so we are in the framework of Remark 3.2. Invoking the proof of [19, Lemma 6.1] and especially equation (6.2) in the aforementioned reference, we infer that

$$\|Q_i\|_{\mathbf{L}^\infty(0, T; \mathbb{R}^{2 \times 2})} + \|Q_i^T\|_{\mathbf{L}^\infty(0, T; \mathbb{R}^{2 \times 2})} \leq C \left(1 + T(|\xi_i| + |\Omega_i|)^2 \right), \quad \text{and}$$

$$\|Q_1 - Q_2\|_{L^\infty(0,T;\mathbb{R}^{2 \times 2})} \leq CT|\Omega_1 - \Omega_2|. \tag{5.24}$$

Recalling from Sect. 3 that

$$\mathcal{T}_\delta(\tilde{u}(t, y), \tilde{h}(t, y), \tilde{a}(t, y)) = Q^T(t)\sigma_\delta(\tilde{u}(t, y), \tilde{h}(t, y), \tilde{a}(t, y))Q(t)$$

and making use of (5.22), (5.23) as well as (5.24), we derive that

$$\begin{aligned} & \|\mathcal{T}_\delta(\tilde{w}_1) - \mathcal{T}_\delta(\tilde{w}_2)\|_{L^q(\mathcal{D};\mathbb{R}^{2 \times 2})} \\ & \leq Cp(\|\tilde{w}_1\|_{\mathbf{B}_{qp}^s(\mathcal{D})^4 \times \mathbb{R}^3}, \|\tilde{w}_2\|_{\mathbf{B}_{qp}^s(\mathcal{D})^4 \times \mathbb{R}^3})\|\tilde{w}_1 - \tilde{w}_2\|_{\mathbf{B}_{qp}^s(\mathcal{D})^4 \times \mathbb{R}^3} \end{aligned} \tag{5.25}$$

for a suitable polynomial $p(x_1, x_2)$ with nonnegative coefficients in view of (5.22), (5.23) and (5.24).

Similarly, it follows that

$$\begin{aligned} & \|\mathcal{T}_\delta(\tilde{w}_1) - \mathcal{T}_\delta(\tilde{w}_2)\|_{W^{1,q}(\mathcal{D};\mathbb{R}^{2 \times 2})} \\ & \leq Cp(\|\tilde{w}_1\|_{\mathbf{B}_{qp}^{s+1}(\mathcal{D})^4 \times \mathbb{R}^3}, \|\tilde{w}_2\|_{\mathbf{B}_{qp}^{s+1}(\mathcal{D})^4 \times \mathbb{R}^3})\|\tilde{w}_1 - \tilde{w}_2\|_{\mathbf{B}_{qp}^{s+1}(\mathcal{D})^4 \times \mathbb{R}^3} \end{aligned} \tag{5.26}$$

for the same polynomial $p(x_1, x_2)$ as in (5.25).

We define $S(\tilde{w}_1, \tilde{w}_2) := \mathcal{T}_\delta(\tilde{w}_1 + \tilde{w}_2) - \mathcal{T}_\delta(\tilde{w}_2)$, and we set $F_0 = L^q(\mathcal{D}; \mathbb{R}^{2 \times 2}) \times \mathbb{R}^3$, $F_1 = W^{1,q}(\mathcal{D}; \mathbb{R}^{2 \times 2}) \times \mathbb{R}^3$, $E_0 = \mathbf{B}_{qp}^s(\mathcal{D})^4 \times \mathbb{R}^3$ and $E_1 = \mathbf{B}_{qp}^{s+1}(\mathcal{D})^4 \times \mathbb{R}^3$ for suitable $s \in (1, 2)$ with $s > \frac{2}{q} + 1$. Making use of (5.25) and (5.26), we infer that for $\tilde{w}_1, \tilde{w}_2 \in E_j$ it holds that

$$\begin{aligned} \|S(\tilde{w}_1, \tilde{w}_2)\|_{F_j} &= \|\mathcal{T}_\delta(\tilde{w}_1 + \tilde{w}_2) - \mathcal{T}_\delta(\tilde{w}_2)\|_{F_j} \\ &\leq Cp(\|\tilde{w}_1 + \tilde{w}_2\|_{E_j}, \|\tilde{w}_2\|_{E_j})\|\tilde{w}_1\|_{E_j} \\ &\leq C\tilde{p}(\|\tilde{w}_1\|_{E_j}, \|\tilde{w}_2\|_{E_j})\|\tilde{w}_1\|_{E_j} \end{aligned}$$

for another polynomial $\tilde{p}(x_1, x_2)$ with nonnegative coefficients by virtue of the respective property of $p(x_1, x_2)$ from (5.25) and the triangle inequality. Consequently, we set

$$\mu(\|\tilde{w}_1\|_{E_j}, \|\tilde{w}_2\|_{E_j}) := C\tilde{p}(\|\tilde{w}_1\|_{E_j}, \|\tilde{w}_2\|_{E_j})\|\tilde{w}_1\|_{E_j}. \tag{5.27}$$

The shape of μ from (5.27) as a polynomial with nonnegative coefficients yields directly that it is continuous and positive on $\mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0}$ and that it satisfies the monotonicity property from Proposition 4.3, so it lies within the scope of the latter proposition.

In order to apply this proposition, it remains to verify that $S(\tilde{w}_1, \tilde{w}_2) \in \mathcal{F}(F_0, F_1)$ holds for $\tilde{w}_1, \tilde{w}_2 \in \mathcal{F}(E_0, E_1)$. If S would only depend on one variable, then it would suffice to show that S is Fréchet-differentiable from E_0 to F_0 and continuous from E_1 to F_1 by virtue of $E_1 \hookrightarrow E_0$ and $F_1 \hookrightarrow F_0$, see [37, Sect. 2]. The latter embeddings also result in $\Sigma_E = E_0 + E_1 = E_0$ as well as $\Sigma_F = F_0 + F_1 = F_0$, where the norm of the respective sums are equivalent to the norm of E_0 and F_0 , respectively.

Let now $\tilde{w}_1, \tilde{w}_2 \in \mathcal{F}(E_0, E_1)$. Similarly as in Sects. 6 and 7 of [3], we argue that σ_δ is Fréchet-differentiable from E_0 to F_0 . Additionally recalling the shape of \mathcal{T}_δ from Sect. 3 and observing that Q and Q^T only depend on time and are independent of the spatial variables, we argue that \mathcal{T}_δ is also Fréchet-differentiable from E_0 to F_0 . It readily follows that S is Fréchet-differentiable from $\tilde{E}_0 := E_0 \times E_0$ to F_0 provided \tilde{E}_0 is equipped with a suitable norm. A simple argument also reveals that $\tilde{w} = (\tilde{w}_1, \tilde{w}_2)$ is holomorphic in \tilde{E}_0 on $0 < \text{Re } z < 1$. As the composition of a holomorphic function and a Fréchet-differentiable operator is holomorphic, we deduce that $S(\tilde{w}_1, \tilde{w}_2)$ is holomorphic in F_0 on $0 < \text{Re } z < 1$. Likewise, one can verify the other aspects of $S(\tilde{w}_1, \tilde{w}_2) \in \mathcal{F}(F_0, F_1)$.

It is thus justified to employ Proposition 4.3, and doing so, we infer that

$$\|S(\tilde{w}_1, \tilde{w}_2)\|_{F_\theta} \leq C \tilde{p}(\|\tilde{w}_1\|_{E_\theta}, \|\tilde{w}_2\|_{E_\theta}) \|\tilde{w}_1\|_{E_\theta}.$$

Reproducing the above estimate with $S(\tilde{w}_1 - \tilde{w}_2, \tilde{w}_2)$, we get for another polynomial $p_1(x_1, x_2)$ with nonnegative coefficients

$$\begin{aligned} \|\mathcal{T}_\delta(\tilde{w}_1) - \mathcal{T}_\delta(\tilde{w}_2)\|_{F_\theta} &= \|S(\tilde{w}_1 - \tilde{w}_2, \tilde{w}_2)\|_{F_\theta} \\ &\leq Cp_1(\|\tilde{w}_1\|_{E_\theta}, \|\tilde{w}_2\|_{E_\theta}) \|\tilde{w}_1 - \tilde{w}_2\|_{E_\theta}. \end{aligned} \tag{5.28}$$

Next, we observe that $F_\theta = H^{\theta,q}(\mathcal{D}; \mathbb{R}^{2 \times 2})$ and $E_\theta = B_{qp}^{s+\theta}(\mathcal{D})^4 \times \mathbb{R}^3$. The assumptions on p and q imply that $q > 2$, so it is also valid that $H^{\theta,q} \hookrightarrow W^{\theta,q}$, see e.g. Example 2.18 in [33] for the embedding on \mathbb{R}^n and use the boundary regularity to transfer this result to the present setting via an extension operator. Plugging $\theta = \varepsilon + \frac{1}{q}$ into (5.28), we infer that

$$\begin{aligned} &\|\mathcal{T}_\delta(\tilde{w}_1) - \mathcal{T}_\delta(\tilde{w}_2)\|_{W^{\varepsilon+1/q,q}(\mathcal{D}; \mathbb{R}^{2 \times 2})} \\ &\leq C p_1(\|\tilde{w}_1\|_{B_{qp}^{s+\varepsilon+1/q}(\mathcal{D})^4 \times \mathbb{R}^3}, \|\tilde{w}_2\|_{B_{qp}^{s+\varepsilon+1/q}(\mathcal{D})^4 \times \mathbb{R}^3}) \|\tilde{w}_1 - \tilde{w}_2\|_{B_{qp}^{s+\varepsilon+1/q}(\mathcal{D})^4 \times \mathbb{R}^3}. \end{aligned} \tag{5.29}$$

It remains to argue that

$$\tilde{\mathcal{X}}_\gamma \hookrightarrow B_{qp}^{2-2/p}(\mathcal{D})^4 \times \mathbb{R}^3 \hookrightarrow B_{qp}^{s+\varepsilon+1/q}(\mathcal{D})^4 \times \mathbb{R}^3.$$

In fact, the second embedding is implied provided $s \leq 2 - \frac{2}{p} - \frac{1}{q} - \varepsilon$. On the other hand, we recall that $s > 1 + \frac{2}{q}$ has to be ensured, so we can find such $s \in (1, 2)$ if

$$1 + \frac{2}{q} < 2 - \frac{2}{p} - \frac{1}{q} - \varepsilon, \text{ or, equivalently, } \frac{2}{p} + \frac{3}{q} < 1,$$

as ε is arbitrarily small. The above condition is guaranteed by assumption. The first embedding is valid in view of Lemma 5.3.

Thus, we deduce from (5.29) that

$$\|\mathcal{T}_\delta(\tilde{w}_1) - \mathcal{T}_\delta(\tilde{w}_2)\|_{W^{\varepsilon+1/q,q}(\mathcal{D}; \mathbb{R}^{2 \times 2})} \leq Cp_1(\|\tilde{w}_1\|_{\tilde{\mathcal{X}}_\gamma}, \|\tilde{w}_2\|_{\tilde{\mathcal{X}}_\gamma}) \|\tilde{w}_1 - \tilde{w}_2\|_{\tilde{\mathcal{X}}_\gamma}. \tag{5.30}$$

Combining (5.21) as well as (5.30) and employing that $\|\tilde{w}_i\|_{\tilde{\mathcal{X}}_\gamma} \leq R_0 + \|\tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma}$, we conclude that

$$\begin{aligned} |\tilde{\mathcal{G}}_2(\tau, \tilde{w}_1) - \tilde{\mathcal{G}}_2(\tau, \tilde{w}_2)| &\leq C \|\mathcal{T}_\delta(\tilde{w}_1) - \mathcal{T}_\delta(\tilde{w}_2)\|_{\mathbf{W}^{\varepsilon+1/q, q}(\mathcal{D}; \mathbb{R}^{2 \times 2})} \\ &\leq Cp_1(\|\tilde{w}_1\|_{\tilde{\mathcal{X}}_\gamma}, \|\tilde{w}_2\|_{\tilde{\mathcal{X}}_\gamma}) \|\tilde{w}_1 - \tilde{w}_2\|_{\tilde{\mathcal{X}}_\gamma} \\ &\leq C \tilde{p}_1(R_0, \|\tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma}) \cdot \|\tilde{w}_1 - \tilde{w}_2\|_{\tilde{\mathcal{X}}_\gamma} \end{aligned}$$

for another polynomial \tilde{p}_1 with nonnegative coefficients. We emphasize that $\tilde{p}_1(R_0, \|\tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma})$ may depend on T , but it is independent of τ . □

We then get the desired Lipschitz estimate for the complete term $\tilde{\mathcal{F}}$.

Corollary 5.9 *Let $p, q \in (1, \infty)$ be such that (2.15) is satisfied. Then for $\tilde{\mathcal{F}}$ as in (5.11), for $\tilde{w}_0 \in V$ and for every $R > 0$ such that $\overline{B}(\tilde{w}_0, R) \subset V$, there exists $\varphi_R \in L^p(0, T)$ with*

$$\|\tilde{\mathcal{F}}(\tau, \tilde{w}_1) - \tilde{\mathcal{F}}(\tau, \tilde{w}_2)\|_{\tilde{\mathcal{X}}_0} \leq \varphi_R(\tau) \|\tilde{w}_1 - \tilde{w}_2\|_{\tilde{\mathcal{X}}_\gamma}$$

for almost all $\tau \in [0, T]$ for $T > 0$ sufficiently small and all $\tilde{w}_1, \tilde{w}_2 \in \tilde{\mathcal{X}}_\gamma$ with $\|\tilde{w}_i - \tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma} \leq R$.

Proof Let $\tilde{w}_0 \in V$ and consider $R > 0$ arbitrary such that $\overline{B}(\tilde{w}_0, R) \subset V$. Moreover, we take $\tilde{w}_1, \tilde{w}_2 \in V$ with $\|\tilde{w}_i - \tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma} \leq R$ into account. Simple calculations reveal that

$$|\Omega_1 \xi_1^\perp - \Omega_2 \xi_2^\perp| \leq C(R + |\xi_0| + |\Omega_0|) \left| \begin{pmatrix} \xi_1 \\ \Omega_1 \end{pmatrix} - \begin{pmatrix} \xi_2 \\ \Omega_2 \end{pmatrix} \right|. \tag{5.31}$$

Before proceeding, we remark again that we are in the setting of Remark 3.2 and Proposition 3.3 by virtue of (ξ_i, Ω_i) independent of time, implying the validity of $(\xi_i, \Omega_i) \in \mathbf{W}^{1,p}(0, T; \mathbb{R}^2) \times \mathbf{W}^{1,p}(0, T)$ for $\tilde{w}_i = (\tilde{u}_i, \tilde{h}_i, \tilde{a}_i, \xi_i, \Omega_i) \in V$. With regard to $\tilde{\mathcal{M}}_1$ from (5.10), we can then argue similarly as in the proof of Lemma 5.5 to obtain

$$\begin{aligned} &\|\tilde{\mathcal{M}}_1(\tau; \xi_1, \Omega_1) \tilde{w}_1 - \tilde{\mathcal{M}}_1(\tau, \xi_2, \Omega_2) \tilde{w}_2\|_{\tilde{\mathcal{X}}_0} \\ &\leq C(R + \|\tilde{w}_0\|_{\tilde{\mathcal{X}}_\gamma})(T + 1) \|\tilde{w}_1 - \tilde{w}_2\|_{\tilde{\mathcal{X}}_\gamma}. \end{aligned} \tag{5.32}$$

In total, concatenating Lemmas 5.7, 5.8, (5.31) and (5.32), we derive the assertion of the corollary, where we observe that the Lipschitz constants in the aforementioned lemmas and equations are independent of τ . □

5.5 Proof of Theorem 2.1

Combining Lemma 5.4 and Lemma 5.5, we deduce that (A1) from Assumption 4.1 is satisfied. By virtue of Lemma 5.6 and Corollary 5.9, aspect (A2) from Assumption 4.1 is also fulfilled. Invoking the maximal regularity as established in Proposition 5.2, showing (A3) from Assumption 4.1, we conclude by Proposition 4.2 the existence and uniqueness of a local strong solution to (5.12) on the fixed domain. The solution lies in the maximal regularity space $L^p(0, T; \tilde{\mathcal{X}}_1) \cap \mathbf{W}^{1,p}(0, T; \tilde{\mathcal{X}}_0)$.

From $(\xi, \Omega) \in W^{1,p}(0, T, \mathbb{R}^2 \times W^{1,p}(0, T))$, we obtain η, ω and Z as described in Remark 3.2. In total, this shows Theorem 3.4.

Performing the backward change of variables and coordinates given in Sect. 3 and explained in Remark 3.2, we derive our main result Theorem 2.1 from Theorem 3.4. The solution must be unique as a consequence of the uniqueness implied by the quasilinear existence result and the uniqueness of the transform.

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