Received: 9 February 2023 | Revised: 21 March 2024 | Accepted: 13 June 2024

DOI: 10.1111/jems.12603

ORIGINAL ARTICLE

Journal of Economics & Management Strategy WILEY

Social networks, promotions, and the glass-ceiling effect

Michael Neugart[1] | **Anna Zaharieva**[2]

[1]Department of Law and Economics, Technische Universität Darmstadt, Darmstadt, Germany

[2]Faculty of Business Administration and Economics and Center for Mathematical Economics (IMW), Bielefeld University, Bielefeld, Germany

Correspondence
Anna Zaharieva, Faculty of Business Administration and Economics and Center for Mathematical Economics (IMW), Bielefeld University, Universitätsstrasse 25, 33615 Bielefeld, Germany.
Email: anna.zaharieva@uni-bielefeld.de

Funding information
Center for Interdisciplinary Research (ZIF) Bielefeld; Deutsche Forschungsgemeinschaft, Grant/Award Number: ZA 827/2-1

Abstract

Empirical studies show that women have lower chances of reaching top management positions, known as the glass-ceiling effect. To study women's careers, we develop a search and matching model where job ladders consist of three hierarchical levels and workers can progress in the career by means of internal promotions or by transitioning to another firm. Both, formal applications and referral hiring via endogenous social networks can be used for moving between firms. We show that when female workers are minority in the labor market and social link formation is gender-biased (homophilous), there are too few female contacts in the social networks of their male colleagues. This disadvantage implies that female workers are referred less often and, thereby, become underrepresented in top-level management positions of firms relative to their fraction in the market. Our main theoretical results are consistent with the empirical evidence based on the German Socio-Economic Panel.

1 | INTRODUCTION

There is ample evidence that climbing up a job ladder and reaching managerial and executive positions in hierarchical organizations is more difficult for women compared with men (McKinsey&Company, 2017). This phenomenon is known as the glass-ceiling effect. The most common explanations for worse labor market outcomes of women in the literature include occupational segregation, taste-based discrimination, social and cultural norms, as well as more frequent career interruptions by women (Erosa et al., 2022; Fernandez, 2013; Gneezy et al., 2003; Petit, 2007; Winter-Ebmer & Zweimüller, 1997). We consider another aspect of the problem and focus on differences in social networks of men and women as a potential barrier preventing women from advancing in their careers. Even though several articles mention the fact that women have a lack of "old-boys-club" connections as an important reason for the worse positioning of women in the job ladders and lower wages (Bertrand, 2018; Cassidy et al., 2016; Milgrom & Oster, 1987), a proper analytical investigation of the underlying mechanism and empirical evidence is largely missing in the literature.[1]

In this paper, we address the question of how differences in the structure of social networks, in conjunction with low participation of women in the labor market, contribute to stronger gender inequality reflected in different career paths of men and women. We show that gender homophily in the process of network formation leads to the underrepresentation of women in the networks of men, whereby the term homophily reflects a lower probability of creating social links with individuals from another sociodemographic group and is a robust observation in the literature on social networks (McPherson et al., 2001; Montgomery, 1991). A glass-ceiling arises because senior managers, considering workers for vacant positions that cannot be filled with internal candidates, refer workers from their social networks. Although senior managers randomly choose workers from their network, the progression of women to management positions is slowed down because they are underrepresented in the networks of the majority group of men. In addition to presenting a formal model of our argument, we use data from the German Socio-Economic Panel (SOEP) to quantify the mechanism and assess its relative importance.

Our argument rests on a nontrivial chain of reactions, and, therefore, a formal model may help clarify the proposed mechanism for why a glass-ceiling emerges. We develop a model with two gender groups and job ladders consisting of three hierarchical levels: simple jobs, junior management, and senior management. Open positions in senior management can be filled by promoting experienced junior managers or by hiring an external candidate, whereby external recruitment is subject to search and matching frictions (as in Dawid et al., 2023). Open positions in junior management can also be filled through external recruitment, where firms post vacancies and external candidates apply—this is the formal search channel. In addition, we allow for referral hiring. In this case, senior managers draw on their network of social contacts and refer one candidate at random, irrespective of gender. A novel feature of our model is that social networks evolve endogenously, exhibit homophily, and depend on the gender composition of the occupational labor force. This allows us to capture that many social contacts are created in the workplace (Rivera & Soderstrom, 2010) or college (Chetty et al., 2022) and are concentrated in the same occupation, making them relevant for referral hiring.

Interestingly, based on the developed framework, we find that network homophily gives rise to gender-biased referring and generates a glass-ceiling effect in isolation from other possible factors, such as occupational segregation or taste-based discrimination. For the proposed mechanism to arise, two prerequisites are necessary: the participation rate of women in the occupational labor market should be low, and network formation has to be homophilous. If these are given, we can show that there is a bias in referral hiring favoring men and that the gender gap in becoming a manager is decreasing in the participation rate of female workers. Moreover, the gender gap in the probability of obtaining a job referral can become nonmonotonic in the participation rate of women, being small when too few women participate in the labor market, or when there is an equal representation of men and women in the occupation. However, for the intermediate values of the participation rate, social networks of men and women diverge, giving rise to a substantial gap in the probability of a job referral.

Several model predictions can be tested with German SOEP data. The SOEP provides information on survey participants' labor market status, how workers found their jobs, and how workers' networks are characterized. The information on social networks is limited to the group of close friends, but also includes their gender. First, we can document different sizes of networks by gender and network homophily, which is in line with our modeling assumptions. The data shows that, on average, 70% of a person's friends are of the same gender as this person. Second, we can show that the incidence of a referral is lower for women and that women get fewer referrals compared with men for jobs in low and middle management. Third, we find that a part of the gender gap in referral hiring can be attributed to a different composition of women's networks, with a larger fraction of other women and a lower fraction of men. This finding is compatible with the theoretical model and with the idea in Marsden and Gorman (2001) that "women's networks are less likely to include contacts in possession of valuable job information … [because] there are more other women (who, when employed, are less likely to hold ownership or authority positions) in women's networks than in men's" (p. 476). Finally, the data confirms a nonmonotonic relationship between the share of women in the occupation and the gender gap in referral hiring. More specifically, the gender gap in the probability of entering a job via the referral is maximal when women comprise $30\% - 35\%$ of employees in the occupation. However, it is substantially higher when this share is relatively low or when there is an equal representation of men and women.

In Section 2, we proceed with a review of the related literature. In Section 3, we introduce our analytical apparatus, and in Section 4, we present the results with respect to the network formation and the gender distribution along the hierarchical levels of the firm. In Section 5, we conduct an empirical analysis confronting important predictions of our theoretical model with data from the German SOEP. Section 6 summarizes our results and derives policy implications. We also discuss the limits and possible extensions of our analysis there.

2 | LITERATURE REVIEW

We are not aware of formal analyses of how referrals, social networks and career progression of different gender groups are linked in frictional labor markets.[2] Each of these topics has received considerable attention on its own, however. Since Granovetter's (1995, p. 85) assertion that "Careers are not made up of random jumps from one job to another, but rather that individuals rely on contacts acquired at various stages of their work-life, and before." various empirical studies confirmed that a large fraction—sometimes close to and above 50%—of the employees found their jobs via personal contacts.[3]

From a theoretical perspective the seminal model on referral hiring was developed by Montgomery (1991) who formalized the idea of network homophily. In particular, he described homophily by ability, when high-ability employees recommend high-ability contacts from their network. However, the study did not consider gender homophily in the context of hierarchical firms. A considerable part of the literature on labor markets and social networks assumes that the use of contacts is exogenous to labor market conditions. Notable exceptions are Boorman (1975), Calvó-Armengol (2004), Galeotti and Merlino (2014), or Galenianos (2021). For instance, Galeotti and Merlino (2014) explore the effect of labor market conditions on the use of social networks and their effectiveness in matching job seekers to vacancies, and Galenianos (2021) shows that the welfare effects of referrals depend on the nature of workers' heterogeneity. Our study adds to this strand of literature, notably in a context where the formation of homophilous networks is related to gender differences in labor market participation rates.

To the best of our knowledge there are only three studies that analyze the implications of social networks in the market with on-the-job search and across firm mobility. These are Horvath (2014), Zaharieva (2015), and Arbex et al. (2019). In our work, if one worker group moves faster in the job ladder, it reduces the number of senior positions available to the other group, because the two groups are directly competing for a fixed number of jobs. This effect, known as a slot constraint, is absent in previous work.

Two studies most closely related to our work are by Rubineau and Fernandez (2013, 2015), who also investigate job segregating effects stressing the role of referrers within organizations. In addition to their setup, networks form endogenously in our framework. Furthermore, we model firms in a labor market with frictions, where firms may promote internal candidates or recruit externally in conjunction with a recommendation coming from an incumbent worker. Our paper is also related to the literature on gender inequality and internal promotions. Two recent contributions in this field include Cassidy et al. (2016) and Guertler and Guertler (2019). This literature follows the idea of promotions as signals of ability focusing on direct discrimination and unequal promotion chances. Even though the role of these mechanisms should be acknowledged our study investigates another source of gender inequality stemming from differences in the size and composition of social connections combined with referral hiring.

Further, we contribute to the scarce empirical literature on gender differences in referral hiring. One early study by Corcoran et al. (1980) reports that men in their sample were more likely than women to first hear about a job through a contact, to have known others at the workplace before being hired, and to have received aid in getting the job from an influential person in the workplace. A more recent overview by Topa (2011) comes to the conclusion that empirical "evidence suggests that women are less likely to use informal contacts than men (with regard to both information about vacancies and direct influence" (p. 1201) even though several studies find insignificant effects. Also Behtoui (2008) and Alaverdyan and Zaharieva (2022) support the view that informal methods of job-finding are less frequent among women in Sweden and Germany.

Empirical studies also show that there remains a substantial unexplained gender wage gap even after controlling for most of the conventional determinants, including human capital, tenure, industry, and other variables (see, e.g., Albrecht et al., 2003; Blau & Kahn, 2017). Typically, explanations for the gender wage gap and worse career outcomes of women relate to occupational segregation whereby women are sorted into low-wage occupations with poor opportunities for career development (Erosa et al., 2022), to more frequent job interruptions by women or taste-based discrimination (Petit, 2007; Winter-Ebmer & Zweimüller, 1997). Other studies emphasize the role of social norms for gender differences (Fernandez, 2013) and psychological factors, such as higher risk and competition aversion among women (Gneezy et al., 2003). Our study explores how an additional channel leads to the segregation of men and women to different hierarchical positions within organizations and contributes to the residual gender wage gap.

Finally, the search and matching framework introduced by Diamond (1982), Mortensen (1982), and Pissarides (1985) within which we model firms' recruitment behavior and workers' network formation has become one of the workhorse models in labor economics.

3 | THE MODEL

The primary goal of our theoretical approach is to analyze possible differences in the career progression of men and women stemming from differences in their social networks. In particular, we build on the observation that social networks serve as an important source of information about vacancies for job candidates (e.g., Granovetter, 1995), since workers connected in a network may exchange information about job openings. Also firms may benefit from the social connections of their employees when filling positions with recommended candidates. Note, however, that searching for information is only relevant in the presence of search frictions when locating vacancies (applicants) is a costly time-consuming process for workers (firms). Moreover, a study of career progression is only possible if firm hierarchies are modeled explicitly. Hence, in the following, we develop a search and matching model with two gender groups, firm hierarchies and referral hiring via social networks.

In a nutshell, our model has the following characteristics: time is continuous, and workers enter and exit the market at an exogenous rate ρ. The total population size is normalized to 1, and there is no population growth.[4] There are two types of individuals: female workers (F) and male workers (M). Fractions $h \leq 0.5$ of workers are of type F, and fractions $1 - h \geq 0.5$ are of type M. This parameter allows us to capture the lower participation rate of women in the labor market, especially in professional full-time jobs. All workers are identical with respect to their education and productivity.

Job ladders consist of three hierarchical levels: simple (nonmanagement) jobs, junior management, and senior management. Positions on the first hierarchical level are freely available to all workers without search frictions; these workers form a pool of applicants for junior management jobs. Since there are infinitely many firms offering simple jobs, we do not model these firms explicitly. All career firms consist of one junior management job and one senior management job. This is the simplest firm structure allowing to study internal promotions from junior to senior positions. Another advantage of having two-position firms is that senior managers may recommend their social contacts for junior jobs in the same firm which opens room for modeling referral hiring. In addition, this firm structure gives rise to two separate submarkets with search frictions: in the first submarket young workers employed in simple jobs apply for positions in junior management. In these positions, workers accumulate experience x and as they have reached a sufficient level ($\bar{x}$) they qualify for a senior managerial job. They are promoted by their current employer, if there is a vacancy on the senior level in their firm, otherwise they start applying for senior positions in the second submarket. Having a sufficient level of experience $\bar{x}$ is a necessary requirement for an application to the senior position, which leads to the complete segregation of the two submarkets. We assume that experience level ($\bar{x}$) is identical for all workers irrespective of their gender. This assumption allows us to isolate the effect of social networks from other possible sources of gender inequality such as unequal chances of internal promotions within firms.[5]

Further, hiring for junior management positions is based on the pool of formal applicants and candidates recommended by the senior manager of the firm. Referral hiring is based on social networks which form endogenously in the sense that they are not fixed from the beginning but links between workers form as they randomly meet. Importantly, when referring a worker to a job, referrers choose randomly among the members of their network, that is, they do not discriminate by gender.[6] We do not make a sharp distinction between private and professional networks and talk about a social network of the person including both types of contacts. The reason is that the two groups are likely to be mixing over time since close friends could become colleagues by getting a job with the same employer, whereas some colleagues may become close friends. What is relevant for our analysis is a subset of social contacts working in the same occupation/profession. The model implicitly assumes that this subset is relatively large, leading to the possibility of referring each other for open jobs. In this respect, we rely on the evidence from Rivera and Soderstrom (2010) and Chetty et al. (2022) that many social contacts are created in the workplace or college and are concentrated in the same occupation.

3.1 | Social networks

At rate ϕ every worker can be randomly matched with another worker. Formation of social links is subject to (gender) homophily, that is, workers are more likely to create social links with others of the same type (see McPherson et al., 2001). Let τ_0 denote the probability of creating a social link with a worker of a different type and $\tau \geq \tau_0$ be the probability of creating a link with a worker of the same type (conditional on matching). Note that the special case when $\tau = \tau_0$ corresponds to the situation without homophily. We consider directed links. This means that, if two workers A and B are randomly matched, it is possible that B becomes a social contact of A but not necessarily the other way round. Some justification for this assumption can be found in Plug et al. (2018), who show that less than a half of friendship ties are reciprocated (undirected) in their data. Also professional links are likely to be directed. Furthermore, this assumption simplifies the model. In addition, we assume that every link can be destroyed at an exogenous rate δ.

Let ξ_k^{ij} denote a fraction of type i workers with exactly k social contacts of type j, $i, j \in \{M, F\}$. This is a fraction out of all type i workers. Consider some type M worker without contacts of his type. With our notation, this worker belongs to the group ξ_0^{MM}. At rate ϕ this worker is matched with some other worker. With probability $1 - h$ this worker is of the same type M, and the social link is created with probability τ. Next, consider a worker of type M with only one contact of his type belonging to the group ξ_1^{MM}. This person may lose his contact at rate δ. In the steady state (when variables ξ_k^{MM} are constant), the propensity for the worker to make transition between the two states $k-1$ and k will be equalized, this means:

$$\xi_0^{MM}\phi(1-h)\tau = \delta\xi_1^{MM} \Rightarrow \xi_1^{MM} = \frac{\xi_0^{MM}\phi(1-h)\tau}{\delta},$$
$$\xi_1^{MM}\phi(1-h)\tau = 2\delta\xi_2^{MM} \Rightarrow \xi_2^{MM} = \xi_0^{MM}\left(\frac{\phi(1-h)\tau}{\delta}\right)^2\frac{1}{2},$$
$$\xi_{k-1}^{MM}\phi(1-h)\tau = k\delta\xi_k^{MM} \Rightarrow \xi_k^{MM} = \xi_0^{MM}\left(\frac{\phi(1-h)\tau}{\delta}\right)^k\frac{1}{k!}.$$

Let $\varphi = \phi/\delta$ and $\psi_{MM} \equiv \varphi(1-h)\tau$ to simplify the notation. Repeating the same steps for links with female workers F and defining $\psi_{MF} \equiv \varphi h\tau_0$ we obtain the following result:

Result 1. *The number of type M contacts in the network of a male worker has a Poisson distribution with parameter ψ_{MM}, whereas the number of type F contacts of a male worker has a Poisson distribution with parameter ψ_{MF}. So ψ_{MM} and ψ_{MF} are the average numbers of types M and F contacts in the social network of a male worker.*

Further, let n_M denote the average network size for type M workers including both types of social contacts and γ_M be the fraction of type M contacts in the network, so we get

$$n_M = \psi_{MM} + \psi_{MF} = \varphi[(1-h)\tau + h\tau_0], \quad \gamma_M = \frac{(1-h)\tau}{(1-h)\tau + h\tau_0}.$$

This equation shows the following. If $\tau_0 < \tau$, then a larger fraction of female workers in the market h is reducing the average size of social networks for men since $\partial n_M/\partial h = \varphi(\tau_0 - \tau) < 0$. The reason is that cross-gender links are less likely to be formed with gender homophily.

Next, we repeat the same approach for female workers and denote $\psi_{FF} \equiv \varphi h\tau$ and $\psi_{FM} \equiv \varphi(1-h)\tau_0$—average numbers of types F and M contacts in the social network of a female worker. The total number of contacts in the network of a female worker can be found as

$$n_F = \psi_{FF} + \psi_{FM} = \varphi[h\tau + (1-h)\tau_0], \quad \gamma_F = \frac{h\tau}{h\tau + (1-h)\tau_0},$$

where n_F is the total network size of female workers and γ_F is a fraction of type F contacts in their network. One can see that $\partial n_F/\partial h = \varphi(\tau - \tau_0)/ > 0$ if $\tau_0 < \tau$, so the average size of female social networks is increasing with a larger fraction of females h in the labor market. Moreover, the case of full homophily ($\tau_0 = 0$) leads to the complete segregation of social networks between the two genders, that is, $\gamma_M = \gamma_F = 1$. In the opposite case without homophily ($\tau_0 = \tau$), the fraction of contacts of the same type is equal to the fraction of this type in the total population, that is, $\gamma_M = 1 - h$ and $\gamma_F = h$. Comparing the average sizes of social networks for male and female workers one can show the following:

$$n_M - n_F = \varphi[(1-h)\tau + h\tau_0 - h\tau - (1-h)\tau_0] = \varphi(1-2h)(\tau - \tau_0).$$

This equation allows us to formulate the second result:

Result 2. *Social networks of women are smaller compared with men if women are the minority in the labor market ($h < 0.5$) and social connections exhibit some degree of homophily ($\tau_0 < \tau$). Under these conditions, female contacts are underrepresented in the networks of male workers ($1 - \gamma_M < h$), whereas they are overrepresented in the networks of female workers ($\gamma_F > h$).*

This finding forms the ground for gender-biased referrals presented in Section 3.2.

3.2 | Labor market

There are three types of jobs in the market: simple jobs, junior, and senior management jobs. Let e_0^i denote the measure/number of type i workers employed in simple jobs, $i = M, F$. Workers do not gain any professional experience by performing simple jobs. Junior and senior management jobs are provided by firms operating in a frictional market. The total number of these firms is fixed and denoted by d. All these firms are identical and every firm is a dyad consisting of two positions: one junior manager position and one senior manager position. Thus, job ladders consist of three hierarchical levels in total. Here we build on the model by Dawid et al. (2023).

At rate ρ every worker may exit the market for exogenous reasons and is substituted with a new agent of the same gender in simple job $e_0 = e_0^M + e_0^F$. So the total population and the gender shares are constant over time. From the perspective of firms, ρ is the job destruction shock. Let e_1^i denote the number of type i workers employed in junior jobs and e_2^i—the number of type i workers employed in senior jobs, $i = M, F$, so

$$e_0^F + e_1^F + e_2^F = h \quad \text{and} \quad e_0^M + e_1^M + e_2^M = 1 - h.$$

Once accepted in the junior position, workers start accumulating professional experience $x \geq 0$ with $\dot{x} = 1$. All workers in junior jobs have to accumulate an exogenously given experience level $\bar{x}$ to become eligible for senior positions. Accumulation of experience is costly for workers so it stops at $\bar{x}$ since there are no incentives for workers to accumulate more human capital on the job than required by firms. If the senior position is open, firms commit to promoting their employees with experience $\bar{x}$ to senior positions. If there is no worker eligible for promotion, the firm is posting an open senior vacancy on the external market. If there is no open senior position in the firm, the worker with experience $\bar{x}$ starts applying to senior positions in other firms. This is the process of on-the-job search. Experience $\bar{x}$ is observable and can be transferred to other firms when the worker changes the job voluntarily. Workers with experience $x < \bar{x}$ are not eligible for senior positions in any company.

There are two separate matching markets in our model, one where firms post junior positions and look for inexperienced workers with $x = 0$, and another one where firms post senior positions and look for workers with experience $x = \bar{x}$. Variable d_{00} denotes the stock of empty firms in the market, whereas d_{01} is the stock of firms with a senior manager but no junior manager. Since all these firms have an open junior position the total stock of open junior positions available for matching is equal to $d_{00} + d_{01}$. These positions are randomly matched with $z_1 e_0$ searching workers employed in simple jobs, where z_1 denotes the exogenous search effort of workers applying to junior jobs. More precisely, z_1 is the fraction of searching workers who prepare and send an application at every point in time.

To determine the number of matches in the submarket for junior positions, we use an urn-ball matching mechanism (see, e.g., Petrongolo & Pissarides, 2001). This means that every worker (ball) applies randomly with probability z_1 to one vacancy (urn). This matching function has solid microeconomic foundations and standard macroeconomic properties, for example, increasing in both arguments (the number of searching workers and vacancies) and constant returns to scale in the limit when the numbers of searching workers and vacancies are sufficiently large. This yields the formal job-arrival rate q_1 for firms:

$$q_1 = 1 - \left(1 - \frac{1}{d_{00} + d_{01}}\right)^{z_1 e_0} \approx \frac{z_1 e_0}{d_{00} + d_{01}}.$$

Here $(1 - \frac{1}{d_{00} + d_{01}})$ is a probability that the firm does not receive a particular random application of one worker. Taken to the power $z_1 e_0$ this term corresponds to the probability that the firm does not receive any of the $z_1 e_0$ independent applications submitted by workers in a given period of time. Thus q_1 is a probability that the firm receives at least one formal application, which is sufficient to fill the position. The job-finding rate from the perspective of workers is then the probability of submitting an application z_1 multiplied with a matching rate $q_1(d_{00} + d_{01})/(z_1 e_0)$, so that $\lambda_1 \approx z_1$.

Next, consider a candidate who was matched and chosen for the junior position. With probability $\alpha_1 = e_0^M/e_0$ this candidate is of type M, and with a counterprobability $(1 - \alpha_1) = e_0^F/e_0$ the candidate is of type F. Note that

$$\alpha_1 = \frac{(1-h)\mu_M}{(1-h)\mu_M + h\mu_F},$$

where $\mu_i = e_0^i/(1-h)$ is the equilibrium fraction of type i workers employed in simple jobs, $i = M, F$. This equation shows the following. If the distribution of workers across the hierarchical levels is identical for male and female workers, then $\mu_M = \mu_F$ and $\alpha_1 = 1 - h$. So, the probability that the hired job candidate is of type M is equal to the population average $1 - h$. However, if female workers are overrepresented at the bottom ($\mu_F > \mu_M$), a randomly matched job candidate is more likely to be a female and $\alpha_1 < 1 - h$.

In addition to the formal application process, junior positions in firms of type d_{01} can be filled by referrals. These firms consist of d_{0F} and d_{0M}, depending on the type of the senior manager. With probability s in both types of firms the senior manager is asked to recommend a contact for the open junior position. Consider a male senior manager with an average network composition, meaning $n_M\gamma_M$ male and $n_M(1-\gamma_M)$ female contacts in the network.[7] So with probability $(1-\mu_M)^{n_M\gamma_M}$ this senior manager does not know any type M candidate for the junior position. In addition, but with probability $(1-\mu_F)^{n_M(1-\gamma_M)}$ all female contacts are already employed in professional jobs. With this information, we obtain the following probability that there is at least one social contact recommended by the male senior manager:

$$\tilde{q}_1^M = s\left(1 - (1-\mu_M)^{n_M\gamma_M}(1-\mu_F)^{n_M(1-\gamma_M)}\right).$$

If the manager has several social contacts employed in simple jobs, the manager randomly chooses one of them *independent of the gender* and refers this contact for the open position in his firm. The referred candidate is of type M with probability $\tilde{\alpha}_1^M$ and of type F with probability $1 - \tilde{\alpha}_1^M$, where $\tilde{\alpha}_1^M$ depends on the composition of the network:

$$\tilde{\alpha}_1^M = \frac{\gamma_M\mu_M}{\gamma_M\mu_M + (1-\gamma_M)\mu_F} > \alpha_1 \quad \text{for } \tau_0 < \tau.$$

Intuitively, this means that a candidate referred by the male manager is more likely to be a male worker compared with the formal channel even if the manager does not have any taste for discrimination and randomizes between all of his social contacts interested in the junior job. Following the same logic, we define $\tilde{q}_1^F$—the probability that there is at least one social contact recommended by the female senior manager with an average network and $\tilde{\alpha}_1^F$—probability for a type F manager of recommending a type M candidate from the network, so that

$$\tilde{q}_1^F = s\left(1 - (1-\mu_M)^{n_F(1-\gamma_F)}(1-\mu_F)^{n_F\gamma_F}\right), \quad \tilde{\alpha}_1^F = \frac{(1-\gamma_F)\mu_M}{(1-\gamma_F)\mu_M + \gamma_F\mu_F} < \alpha_1 \quad \text{for } \tau_0 < \tau.$$

We summarize these results in the following way:

Result 3. *For $\tau_0 < \tau$ a job candidate referred by the male senior manager with an average network is more likely to be a male worker compared with the formal channel ($\tilde{\alpha}_1^M > \alpha_1$) even if the manager does not have any taste for discrimination and randomizes between all of his social contacts interested in the junior job. A candidate recommended by the female manager with an average network is less likely to be a male worker compared with the formal channel ($\tilde{\alpha}_1^F < \alpha_1$).*

Next, we can see that firms with an open junior position and a type j senior manager will fill their position with a type M candidate at rate $q_1\alpha_1$ via the formal application process and via the network at rate $\tilde{q}_1^j\tilde{\alpha}_1^j$. We do not assume that the recommended candidate is preferred to the external candidates. Rather all applicants for a given midlevel position are pooled together and a random draw is made. So the recommended applicant has the same chances as external applicants given that all of them have the same qualification. Assuming preference for the recommended candidate would amplify the network effect. Let the total job-filling rate with a type M candidate be denoted by $\bar{q}_1^{Mj}$ and with a female applicant by $\bar{q}^{Fj}$. These rates are given by

$$\bar{q}_1^{Mj} = q_1\alpha_1 + \tilde{q}_1^j\tilde{\alpha}_1^j \quad \text{and} \quad \bar{q}_1^{Fj} = q_1(1-\alpha_1) + \tilde{q}_1^j\left(1-\tilde{\alpha}_1^j\right).$$

Notice the following, when referral hiring is not used, that is, $s = 0$, the rate at which a male candidate is hired is equal to $q_1\alpha_1$, and the rate at which a female candidate is hired is equal to $q_1(1-\alpha_1)$. Both are independent of the gender of the senior manager.

Further, let $d_{10} = d_{F0} + d_{M0}$ denote firms with a junior manager but no senior manager. This means that the total number of open managerial positions at the senior level is given by $d_{00} + d_{10}$. Finally, let $d_{11}^N = d_{MF}^N + d_{MM}^N + d_{FF}^N + d_{FM}^N$ denote the stock of full firms with both employees, where the worker in the junior position is not yet eligible for promotion ($x < \bar{x}$). In a similar way, $d_{11}^S = d_{MF}^S + d_{MM}^S + d_{FF}^S + d_{FM}^S$ is the stock of full firms, where the junior worker is already eligible for senior positions ($x = \bar{x}$) and searching on the job. This means that the stock of applicants in the senior managerial market is given by $z_2 d_{11}^S$, where z_2 is the exogenous search intensity of experienced workers. So the urn-ball matching rate in the managerial market for firms q_2 is given by

$$q_2 = 1 - \left(1 - \frac{1}{d_{00} + d_{10}}\right)^{z_2 d_{11}^S} \approx \frac{z_2 d_{11}^S}{d_{00} + d_{10}}.$$

Note that $\lambda_2 \approx z_2$ is the job-finding rate in the managerial market from the perspective of workers. With probability α_2 the firm will be matched with a type M experienced junior worker and hire him for the manager position and with a counter-probability $1 - \alpha_2$ the firm will be matched with a type F experienced junior worker and hire her as a manager:

$$\alpha_2 = \frac{d_{MF}^S + d_{MM}^S}{d_{11}^S}.$$

So the total job-filling rate with a type M candidate is given by $q_2\alpha_2$ and $q_2(1-\alpha_2)$ with a type F candidate. Note, that we assume that workers do not recommend their contacts for senior positions, so there are no referrals to senior jobs. Moreover, there is no favoritism and gender-based discrimination in formal hiring since at this stage we seek to identify a separate effect of homophilous networks on labor market outcomes in isolation from other factors.

3.3 | Firm dynamics

In this subsection, we analyze the evolution of the stocks of firms as workers enter and exit jobs as well as the steady-state distributions of workers and firms. In total, we have 13 possible states for firms $\left\{d_{00}, d_{F0}, d_{M0}, d_{0F}, d_{0M}, d_{FM}^N, d_{FF}^N, d_{MF}^N, d_{MM}^N, d_{FM}^S, d_{FF}^S, d_{MF}^S, d_{MM}^S\right\}$. Recall, that the upper index N refers to junior workers who are not yet eligible for promotion and are not searching for another job, whereas S refers to junior workers with sufficient experience searching for senior positions. Rather than giving a detailed derivation of all the differential equations and their solutions here, we exemplify the general procedure with state d_{00} and give a detailed solution to the firm dynamics in Appendix A.

Consider changes in the stock of empty firms d_{00}, that is, firms at which both positions are vacant. Since every empty firm posts both the junior and the senior position in the respective submarkets it exits the state d_{00} whenever it finds the first employee. So the outflow of firms from d_{00} takes place at rate $q_1 + q_2$. The inflow into this state consists of all firms with only one employee experiencing the job destruction/exit shock ρ. These are the firms $d_{F0}d_{M0}$, d_{0F}, and d_{0M}. The change in the share of firms in state d_{00} becomes

$$\dot{d}_{00} = \rho(d_{F0} + d_{M0} + d_{0F} + d_{0M}) - (q_1 + q_2)d_{00}.$$

As we restrict our analysis to the steady states and consider a stationary distribution of workers and firms across states, we are looking for a solution $\dot{d}_{00} = 0$. In addition, we restrict the total number of firms in the market by an exogenous parameter d, which yields

$$d_{00} + d_{F0} + d_{M0} + d_{0F} + d_{0M} + d_{MM}^N + d_{MF}^N + d_{FM}^N + d_{FF}^N + d_{MM}^S + d_{MF}^S + d_{FM}^S + d_{FF}^S = d.$$

Solving for the steady state, we find the equilibrium distribution of firms $\{d_{00}, d_{F0}, d_{M0}, d_{0F}, d_{0M}, d_{FM}^N, d_{FF}^N, d_{MF}^N, d_{MM}^N, d_{FM}^S, d_{FF}^S, d_{MF}^S, d_{MM}^S\}$. On the basis of the distribution of firms, we can immediately calculate the distribution of male and female workers in different job levels. Recall that the absolute numbers of workers in different job levels are denoted by e_0^j, e_1^j, and e_2^j, $j = M, F$, so we get

$$e_1^F = d_{FF}^N + d_{F0} + d_{FM}^N + d_{FM}^S + d_{FF}^S, \quad e_1^M = d_{MM}^N + d_{M0} + d_{MF}^N + d_{MF}^S + d_{MM}^S,$$
$$e_2^F = d_{FF}^N + d_{0F} + d_{MF}^N + d_{MF}^S + d_{FF}^S, \quad e_2^M = d_{MM}^N + d_{0M} + d_{FM}^N + d_{FM}^S + d_{MM}^S.$$

Finally, let variables p_0^j, p_1^j, and $p_2^j, j = M, F$ denote the distribution of workers across different hierarchical levels, that is, $p_2^M = e_2^M/(1-h), p_1^M = e_1^M/(1-h), p_0^M = 1 - p_1^M - p_2^M$. And the same for female workers: $p_2^F = e_2^F/h, p_1^F = e_1^F/h, p_0^F = 1 - p_1^F - p_2^F$. So p_2^F is the fraction of female workers in senior managerial positions, whereas p_2^M is the corresponding fraction for male workers.

4 | NUMERICAL RESULTS

Given that our framework leads to a general equilibrium model based on a large set of steady-state equations, we characterize our results with a numerical analysis. As a starting point, we set the parameters to reproduce several stylized empirical facts observed in the German labor market. In the next step, we use these parameter values to highlight the main mechanism of the model. The first objective of the numerical analysis is to shed light on how the participation rate of women in the labor market h influences the composition of social networks of both worker groups and their chances of entering the job via a referral. The second objective is to study how gender differences in networks and referral probabilities affect the equilibrium employment shares of workers along the firm hierarchies. Later in Section 5 we confront these predictions with empirical data.

4.1 | Parameters

For parametrizing the model a vector of exogenous parameters has to be determined that includes a set of labor market characteristics $\{\rho, h, d, \bar{x}, z_1, z_2\}$ and network formation parameters $\{\varphi, \tau, \tau_0, s\}$.

For most of the parameters, we use data from the German SOEP for the years 2000–2018 (Goebel et al., 2019). We consider a sample of high-skill workers employed in full-time jobs, since women in this group are more likely to suffer from the "glass-ceiling." The data includes information about workers employed in top management, middle management (heads of departments), low management (heads of branches), as project managers and supervisors, as well as in nonmanagement jobs. Given that our model has three hierarchical levels, we merge top and middle managers into one category "senior managers." We merge heads of branches, project managers and supervisors into one category that we call "junior managers."

The data shows that the share of workers in nonmanagement, p_0, is equal to 52%. At the same time, the share of workers performing senior management jobs, p_2, is 20%. Targeting this cross-sectional distribution of workers across hierarchical levels, we find $d = 0.28$ and $z_2 = 0.09$.

Furthermore, the data shows that the fraction of high-skill women in full-time jobs varies between 10% and 16% in construction and mining, and 40%–44% in retail trade and nonfinancial services. In the middle range, there are agriculture, energy, manufacturing, financial services, and transportation, with 23%–31% of women. Following this evidence, we set $h = 0.3$, but we also consider a larger range of values $h \in [0 \ldots 0.5]$ for the purpose of comparative statics.

For several remaining labor market parameters, we use the same values as in Dawid et al. (2023). In particular, we set $z_1 = 0.014$ and $\rho = 0.015$. For this parameter setting, Dawid et al. (2023) find that the optimal promotion time $\bar{x}$ is in the range of $[40 \ldots 60]$ depending on the productivity of workers. Thus, we take a middle value and set $\bar{x} = 50$. Given that one period of time is interpreted as one quarter, $\bar{x} = 50$ implies that firms require at least $50/4 = 12.5$ years of professional experience for promoting junior workers to senior positions.

Next, we turn to determining the network parameters $\{\varphi, \tau, \tau_0, s\}$ and start with parameter s, which is the probability that the senior manager is asked to recommend a contact for the open junior position. This parameter is driving the frequency of referral hiring among junior managers. Our data shows that the frequency of referrals in full-time jobs is equal to 33%. This estimate is close to the lower bound of the range of 30%–50% reported in existing studies (Addison & Portugal, 2002; Bentolila et al., 2010; Pellizzari, 2010; Pistaferri, 1999; Rebien et al., 2020). When we additionally restrict the sample to high-skill junior managers, the share of referral hires falls to 24% indicating a lower incidence of referrals in these positions. We target this value and set $s = 0.15$.[8]

SOEP (2020) does not include information about a full network of a person, but it contains a variable corresponding to the number of best friends outside of one's household and their gender. Even though the network of best friends is only a part of the total social network, it can be used to obtain a preliminary estimate of gender homophily. The data shows that the fraction of men in the networks of men is equal to 0.66, whereas the fraction of women in the networks of women is equal to 0.74. Taking both groups together, we find that the average fraction of the same-gender contacts is equal to 0.7. Next, recall that in the model the shares of the same-gender contacts in the networks of male and female workers are given by variables γ_M and γ_F, respectively, so the average in the population can be obtained as

$$\gamma_M(1-h) + \gamma_F h = \frac{(1-h)^2}{1-h+h\frac{\tau_0}{\tau}} + \frac{h^2}{h+(1-h)\frac{\tau_0}{\tau}}.$$

Note that this average homophily value only depends on the compound parameter τ_0/τ rather than separate values of τ_0 and τ. On the basis of this equation and given that $h = 0.5$ in the networks of best friends, we find a value $\tau_0/\tau = 0.43$ that can reproduce the average number of the same-gender contacts observed in the data (equal to 0.7). Yet, this estimate is only preliminary since the homophily level in the remaining part of the social network could be different. So we take into account the evidence from Fernandez and Sosa (2005) on gender-biased referrals. They find that both genders tend to overrecommend their own types by about 10% compared with the fraction of their gender among external applicants. In our model, this is compatible with an estimate of $\tau_0/\tau = 0.6$. Given the two indirect estimates of the gender homophily, we take a middle value and set $\tau_0/\tau = 0.5$, but we also consider a full range of $\tau_0/\tau \in [0 \ldots 1]$ for the purpose of comparative statics.

Further, we consider the network size. For male workers it is given by variable n_M. So the total number of contacts in the network of a male worker is given by $n_M = \varphi\tau[(1-h) + \frac{\tau_0}{\tau}h]$. The average network size of female workers is given by $n_F = \varphi\tau[h + \frac{\tau_0}{\tau}(1-h)]$. Then the average network size in the population is given by

$$n_M(1-h) + n_F h = \varphi\tau\left[1 - 2h(1-h)\left(1 - \frac{\tau_0}{\tau}\right)\right].$$

Note, that this variable only depends on the compound parameters $\varphi\tau$ and relative network homophily $\frac{\tau_0}{\tau}$. The first variable $\varphi\tau$ is a maximum network size, which would be observed in the absence of homophily, that is, when $\tau_0 = \tau$. Given that our data does not have information about the total network size, we rely on related studies. For example, Cingano and Rosolia (2012) report that the number of social connections between individuals in their data is about 32. Glitz (2017) reports a similar number. We capture this information and set the average network size equal to 32. Given that $h = 0.3$ and $\tau_0/\tau = 0.5$, we find from the above equation that the maximum network size is $\varphi\tau = 40$. Finally, we set $\tau = 1$ without loss of generality, which gives the following estimates $\{\varphi = 40, \tau_0 = 0.5, \tau = 1\}$. This completes the choice of parameters summarized in Table 1.

4.2 | Networks

To illustrate how the network parameters φ and τ_0 as well as the share of female workers in the market h are shaping differences in the social networks of male and female workers, we use the following example. We start by considering equal proportions of male and female workers $h = 0.5$ and no gender homophily ($\tau_0 = 1$) meaning that half of the links are with workers of the same (opposite) gender. This is illustrated in panel (A) of Table 2. Next, we reduce participation of female workers, so that $h = 0.3$. This case is illustrated in panel (B). Since there is still no homophily in this setting, lower participation of female workers leads to a lower fraction of female workers in the networks of males and other

TABLE 1 Parameter values.

Parameter	Value	Definition and explanation
h	0.3	Fraction of female workers (SOEP, 2020).
ρ	0.015	Worker exit rate. Following Dawid et al. (2023).
$\bar{x}$	50	Experience level necessary for promotion. Following Dawid et al. (2023).
z_1	0.014	Search intensity of workers in low-level jobs. Following Dawid et al. (2023).
z_2	0.090	Search intensity of workers in junior jobs. Targeting the fraction of workers in senior jobs $p_2 = 0.20$ (SOEP, 2020).
d	0.28	Total number of firms per worker. Targeting the fraction of workers in simple jobs $p_0 = 0.52$ (SOEP, 2020).
φ	40	Maximum network size. Consistent with Fontaine (2008), Cingano and Rosolia (2012), and Glitz (2017).
τ	1	Normalization.
τ_0	0.5	Gender homophily ratio SOEP (2020) and Fernandez and Sosa (2005).
s	0.15	Probability of recommendation. Share of referral hires equal to 24% (SOEP, 2020).

TABLE 2 Network composition for different parameter values.

(A) $h = 0.5, \tau_0 = 1$				(C) $h = 0.5, \tau_0 = 0.5$			
	M	**F**	**Total**		**M**	**F**	**Total**
M	20	20	40	M	20	10	30
F	20	20	40	F	10	20	30
(B) $h = 0.3, \tau_0 = 1$				(D) $h = 0.3, \tau_0 = 0.5$			
	M	**F**	**Total**		**M**	**F**	**Total**
M	28	12	40	M	28	6	34
F	28	12	40	F	14	12	26

females. For example, we can see that both genders now have only a fraction $1 - \gamma_M = \gamma_F = 12/40$ of female workers in their networks, which is exactly 30%.

Further, we return to the setting with equal participation $h = 0.5$ and analyze the implications of gender homophily, illustrated in panel (C). Here, we set $\tau_0 = 0.5$, which is the benchmark value. Panel (C) reveals that gender homophily eliminates half of the cross-gender links and their average number falls from 20 down to 10. This also reduces the average total number of links in the social network down to 30.

Finally, we combine the two effects and consider the situation with $h = 0.3$ (lower participation of females) and $\tau_0 = 0.5$ (gender homophily). This case is contained in part (D) of Table 2. We can see that lower participation of female workers leads to $\psi_{MM} = 28$ and $\psi_{FF} = 12$ as in case (B). At the same time gender homophily is, as in case (C), erasing half of the potential cross-gender contacts, so that $\psi_{MF} = 6$ and $\psi_{FM} = 14$. One direct consequence of the combined effect is that female workers end up with smaller social networks ($n_F = 26 < n_M = 34$). Moreover, the fraction of female contacts in the networks of males is only 6/34, that is, 17.6%, which is well below the population average of 30%. The reason is twofold, on the one hand, female workers are the minority and, on the other hand, it is more difficult to create connections with the opposite gender. At the same time, the fraction of male contacts in the networks of females is 14/26, that is, 53.8%, which is well below the population average of males equal to 70%.

4.3 | Equilibrium employment across hierarchies

The small example in Section 4.2 shows how parameters τ_0 and h are shaping the composition of networks. In the following, we use our parameter choices from Table 1 to illustrate the implications of network structures for the equilibrium distributions of male and female workers across the hierarchical levels. These results are summarized in Table 3. We start with Model 0 presented in columns (1) and (2), which corresponds to the case without homophily $\tau_0 = 1$ (case B in Table 2). This is the case for which the network size of male and female networks is identical, despite the fact that female workers are the minority. For this reason, the probability that a male candidate is recommended by the senior manager is equal to the fraction of male candidates among external hires (α_1) and it does not depend on the gender of the senior manager, that is, $\alpha_1 = \tilde{\alpha}_1^M = \tilde{\alpha}_1^F = 0.7$. Also the distributions of male and female managers across the three hierarchical levels coincide ($p_0^M = p_0^F, p_1^M = p_1^F, p_2^M = p_2^F$), meaning that being a minority for women does not give rise to unequal opportunities. Thus, Model 0 reveals that neither differences in the participation rates nor referral hiring give rise to a glass-ceiling effect.

Further, we investigate the consequences of network hiring with homophilous social networks in Model I (columns 3 and 4 of Table 3). Conditional on the probability of referral hiring, male senior managers are more likely to recommend male applicants, which happens with probability $\tilde{\alpha}_1^M = 0.8209$. This probability is substantially higher than the share of male workers among external hires $\alpha_1 = 0.6962$. In a similar way, female senior managers tend to overrecommend female job candidates with the corresponding probability $1 - \tilde{\alpha}_1^F = 0.4660$, which is well above the share of female candidates among external hires equal to $1 - \alpha_1 = 0.3038$. Note, that women are slightly overrepresented among external candidates compared with their share in the population ($h = 0.3$) since their probability of moving from simple jobs to junior management is lower due to the network disadvantage. These results

TABLE 3 Equilibrium values of endogenous variables.

		Model 0 $\tau_0 = 1$		**Model I** $\tau_0 = 0.5$	
		(1)	**(2)**	**(3)**	**(4)**
Fractions of males and females among new hires					
α_1	$1 - \alpha_1$	0.7000	0.3000	0.6962	0.3038
α_2	$1 - \alpha_2$	0.7000	0.3000	0.7046	0.2954
$\tilde{\alpha}_1^M$	$1 - \tilde{\alpha}_1^M$	0.7000	0.3000	0.8209	0.1791
$\tilde{\alpha}_1^F$	$1 - \tilde{\alpha}_1^F$	0.7000	0.3000	0.5340	0.4660
Job-finding and job-filling rates					
λ_1	λ_2	0.0108	0.0896	0.0108	0.0896
q_1	q_2	0.4718	0.0215	0.4718	0.0215
$\tilde{q}_1^{MM}$	$\tilde{q}_1^{FM}$	0.4353	0.1865	0.4516	0.1702
$\tilde{q}_1^{MF}$	$\tilde{q}_1^{FF}$	0.4353	0.1865	0.4086	0.2132
Distribution of workers across job levels					
p_2^M	p_2^F	0.2064	0.2064	0.2076	0.2035
p_1^M	p_1^F	0.2680	0.2680	0.2696	0.2642
p_0^M	p_0^F	0.5256	0.5256	0.5227	0.5323

Note: Parameters $h = 0.3, s = 0.15, \rho = 0.015, \bar{x} = 50, z_1 = 0.014, z_2 = 0.09, d = 0.28$.

are consistent with the findings in Fernandez and Sosa (2005) that both genders tend to overrecommend their own types.

By now we know that male managers in the model are overrecommended by the senior male managers ($\tilde{\alpha}_1^M > \alpha_1$) but they are underrecommended by the senior female managers ($\tilde{\alpha}_1^F < \alpha_1$). But what can we say about the average probability of a referred candidate being a male or a female worker? Let this probability be denoted by $\tilde{\alpha}_1$, it can be calculated as

$$\tilde{\alpha}_1 = \tilde{\alpha}_1^M \cdot \frac{d_{0M}}{d_{0F} + d_{0M}} + \tilde{\alpha}_1^F \cdot \frac{d_{0F}}{d_{0F} + d_{0M}},$$

since $d_{0M}/(d_{0F} + d_{0M})$ is a probability that the senior manager in the company is a male worker, whereas the opposite probability $d_{0F}/(d_{0F} + d_{0M})$ stands for a female senior manager. For our parameter choices, this number is given by 0.736, which is above the probability of hiring a male worker in a formal way given by $\alpha_1 = 0.6962$, so that $\tilde{\alpha}_1 > \alpha_1$. Thus the advantage from overrecommendations is dominating for male workers. However, the opposite is true for female workers since the average probability of a referred candidate being a female worker is equal to $1 - \tilde{\alpha}_1 = 0.2639 < 1 - \alpha_1 = 0.3038$. Hence referral hiring exhibits a male bias compared with the formal channel.

Figure 1, left panel, illustrates both probabilities α_1 and $\tilde{\alpha}_1$ depending on the female participation rate in the labor market (h). Obviously $\alpha_1 = \tilde{\alpha}_1 = 1$ for $h = 0$, so the gender gap in the referral probability becomes negligible as $h \to 0$. In this case, there are no female workers in the market, so all new hires are male workers irrespective of the hiring channel. In addition, $\alpha_1 = \tilde{\alpha}_1 = 0.5$ when $h = 0.5$ since both gender groups are equally large in this case and their group-specific biases associated with referral hiring offset each other. However, for all $0 < h < 0.5$ referral hiring is biased towards male workers since $\alpha_1 < \tilde{\alpha}_1$.

Result 4. *A combination of two factors—lower participation of females ($h < 0.5$) and network homophily ($\tau_0 < 1$)—generates a bias in referral hiring favouring male workers, that is, $\tilde{\alpha}_1 > \alpha_1$. This gender gap in referral hiring is nonmonotonic in the female participation rate h: it is small for $h \to 0$ and for $h \to 0.5$, but it is large for intermediate values of h, such that $h \in (0 \ldots 0.5)$.*

To understand the reason for a gender bias in referral hiring, consider Model I in Table 3. If the senior manager in the firm is a male worker, then the rate at which the open junior job is filled with a male applicant is equal to $\bar{q}_1^{MM} = 0.4516$, which is higher compared with Model 0, and the rate at which this position is filled with a female worker is lower since $\bar{q}_1^{FM} = 0.1702$. In contrast, if the senior manager is a female worker, then the rate at which the job

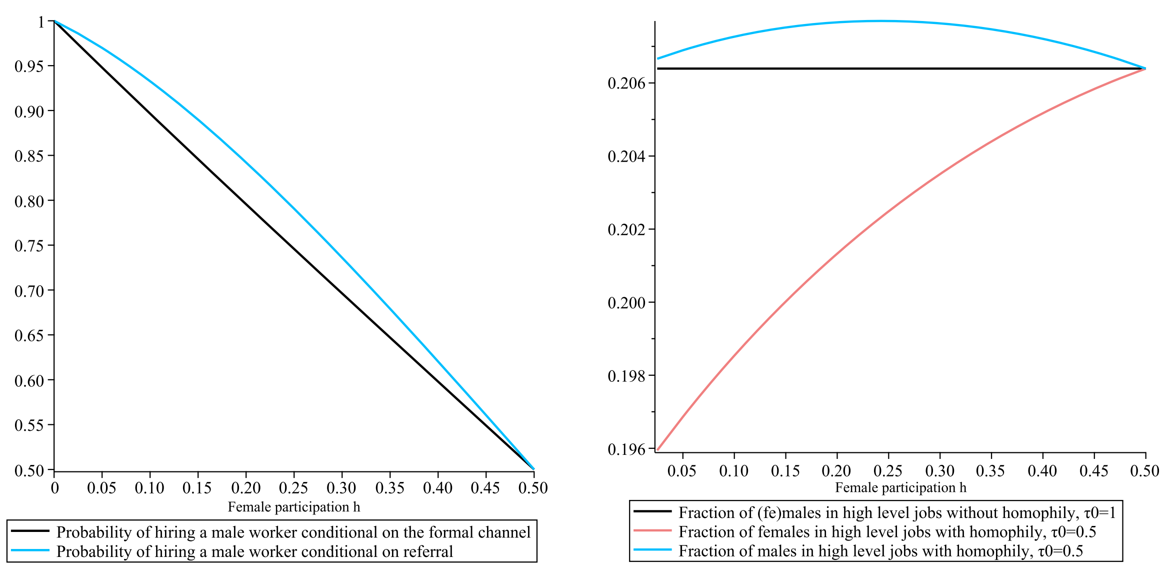

FIGURE 1 Referral bias favouring male workers. (Left) $\tilde{\alpha}_1 > \alpha_1$ and (Right) Fraction of employees on level 2 within their gender group, p_2^F and p_2^M. [Color figure can be viewed at wileyonlinelibrary.com]

is filled with a male candidate is lower ($\bar{q}_1^{MF} = 0.4086$), while the rate at which this position is filled with a female candidate is higher ($\bar{q}_1^{FF} = 0.2132$) compared with Model 0. One important point to be emphasized is that there are relatively many firms with a male senior manager $d_{0M} = 0.0067$ and relatively few firms with a female senior manager $d_{0F} = 0.0028$. This is due to the fact that female workers are the minority. Thus, job recommendations are issued more frequently by the senior male employees giving rise to the referral bias in favor of male workers.

Overall, Model I in Table 3 shows that lower participation of female workers, combined with intensive referral hiring and network homophily benefits and improves career opportunities of the majority male group, while worsening the chances of the minority female group. This leads to the more beneficial distribution of male workers with 52.27% of male workers remaining in simple jobs on average and 20.76% becoming senior managers compared with 53.23% of female workers at the bottom and 20.35% in senior management. Summarizing, we find the following:

Result 5. *A combination of two factors—lower participation of females ($h < 0.5$) and network homophily ($\tau_0 < 1$)—violates the equal career opportunities for men and women leading to the glass-ceiling effect: $p_2^F < p_2^M$ and $p_0^F > p_0^M$ (Model I). The glass-ceiling effect can arise even in the absence of direct discrimination.*

Note also that the glass-ceiling effect described in *Result 5* reinforces and amplifies the gender gap in referral hiring. This means that female job applicants are less likely to enter the job by recommendation not only because their social networks are smaller, but also because there is a larger fraction of other female contacts in their network situated in less influential positions compared with the male applicants.

To illustrate that the result obtained in Table 3 is present for a larger range of variable h, we perform comparative statics analysis and calculate the fractions of male and female employees in senior positions (p_2^F and p_2^M) for $h \in [0 \ldots 0.5]$. This is illustrated in the right panel of Figure 1. We can see that there are no differences between the two gender groups as long as $h = 0.5$. The gap is generated for $h < 0.5$ and it is increasing with lower h reaching the levels $p_2^F = 0.2035$ and $p_2^M = 0.2076$ for $h = 0.3$. These numbers correspond to Model I in Table 3. Overall, we conclude that lower participation of female workers generates a stronger disadvantage in terms of network contacts for female workers and amplifies the glass-ceiling effect. The same results apply to the probability of becoming a manager, so that $p_1^F + p_2^F < p_1^M + p_2^M$. We formulate the following result:

Result 6. *In Model I the probability of becoming a (senior) manager is lower for female workers, but it is increasing rapidly with a higher participation rate of females h, so that the gender gap in becoming a (senior) manager is decreasing with h.*

5 | EMPIRICAL ANALYSIS

Within the context of our search and matching model featuring a hierarchical structure of firms, endogenous networks, and referral hiring, we derived several results concerning the characteristics of the endogenously forming networks and equilibrium employment across hierarchies by gender. Recall that *Result 5* postulates that the glass-ceiling effect is an outcome of the interplay between the two factors: network homophily and low participation of female workers in the occupation. Due to its overarching nature, this result cannot be tested directly since a large chain of connections in the model generates it. However, we can confront Results 4 and 6 with data from the German SOEP (Goebel et al., 2019). In particular, we test whether lower female participation and network homophily generate a bias in referral hiring favoring men (*Result 4*) and whether the gender gap in becoming a manager is decreasing in the participation rate of female workers (*Result 6*). In testing these predictions, we use variation in female participation rates across occupations.

5.1 | Data

The German SOEP is an extensive longitudinal survey that started in 1984. For the purpose of our analysis, we use information between 2000 and 2018 since before 2000 the labor market was strongly influenced by the unification of

TABLE 4 Descriptive statistics.

	Observations	Mean	SD	Minimum	Maximum
Ref	28,403	0.33	0.46	0	1
Fem	28,403	0.52	0.50	0	1
Age	28,403	37.17	10.31	17	67
Education	28,403	12.69	2.81	7	18
Part-time	28,403	0.29	0.45	0	1
Nationality	28,403	0.14	0.35	0	1
ISEI	28,403	45.46	17.13	16	90
Rural	28,403	0.33	0.47	0	1
East	28,403	0.24	0.43	0	1

Note: Observations relate to the sample underlying regression in column (1) of Table 6. See Table A1 for definitions of variables and years for which particular variables are given. Data source: SOEP (2020).

Germany. In 2018 more than 32,000 individuals in more than 18,000 households were interviewed. In addition to information on the socioeconomic background of workers, SOEP provides information on whether workers hold management positions, whether they received their current positions via a referral, and how their social networks look like.

In Table 4, we summarize descriptive statistics on the successful channel of job search and the socioeconomic background of workers. The possible job search channels include referrals from social contacts, private and public employment agencies, newspapers and journals as well as internet job postings. We use this information to construct a referral dummy (Ref) and merge all other alternatives into one formal channel. Table 4 shows that 33% of workers report that they received their current job via a referral. This number is very close to the estimate by Rebien et al. (2020) equal to 37% and based on the German firm survey. Our main dependent variable—finding a job by referral—is attached to a rare event, that is, when the individual finds a new job. Given that the overall job stability is high in Germany, multiple observations for the same individual are very rare. Furthermore, questions related to the type of work (management or not) and the composition of social networks are only asked in selected years, which reduces the number of observations for these variables accordingly. Thus, although SOEP has a panel structure, our final sample is a cross-section of workers entering new jobs, therefore, we use pooled logistic regressions for the estimation.

In our regression analysis, we include a large set of controls to cope with the omitted variable bias. In particular, we control for workers holding part-time and full-time jobs, the socioeconomic status of a worker's occupation (international socioeconomic index [ISEI]), the geographical location, individuals' gender, age, education, and nationality.[9] Referral hiring is more common for part-time jobs and these jobs are frequently taken by women, which could reduce the estimated referral gender gap if working time was not controlled for. To account for possible differences in the job search across regions, we include a dummy variable for rural versus urbanized areas, the state of residence (16 categories,) and an additional binary variable for the East/West dimension. Our final sample includes workers between 17 and 67 years of age, who are not self-employed. Although a large set of control variables is included in our analysis, we interpret the findings as correlations and not being causal.

Several data waves include information about the type of job change and whether the worker holds a management position or not. The data shows that about a third of the respondents (32%) hold management positions. Among those who have management positions 38% are in senior management (top managers and heads of departments), while 62% are junior managers (heads of branch offices as well as project managers and supervisors).[10] Concerning the type of job change, the largest category includes workers changing jobs and moving to a new employer (58%), followed by workers reentering the labor market after a spell of nonemployment (21%), young individuals entering the job for the first time (5.8%), and other minor categories.

Additionally, the data includes information on two characteristics of friendship networks, that is, the number of close friends and gender of the three best friends (see Table 5). We are aware that this is only limited information as individuals' networks relevant for finding new jobs will typically extend beyond best friends. Nevertheless, even though best friends only form a subsample of all contacts, we believe that this variable can be used to provide first insights

TABLE 5 Social network structure by gender.

	Observations	Number of close friends	SD	Observations	Number and share of female friends 0	1	2	3	Share
Males	4641	4.58	4.97	2200	0.27	0.44	0.25	0.04	0.35
Females	5360	4.11	3.11	2515	0.01	0.15	0.45	0.38	0.73

Note: The number of close friends (left part of the table) is based on a sample of individuals from column (5) of Table 6. The number/share of female friends (right part of the table) is based on a sample of individuals from column (7) of Table 6. Data source: SOEP (2020).

about gender differences in the network composition, if they exist. The data shows that networks of men are larger (4.58) compared with women (4.11), moreover, there is strong gender homophily: the fraction of the same-gender friends is 65% among men and 73% among women. Both findings support our modeling setup formulated in *Result 2*.

5.2 | Results

Our theory suggests that the glass-ceiling effect arises only if two effects are present simultaneously: gender homophily and low participation of women. In the presence of these factors, *Result 4* postulates that referral hiring exhibits a bias favoring male applicants compared with the formal channel, that is, $\tilde{\alpha}_1 > \alpha_1$. We split this result into various testable hypotheses (H1–H4) in the following.

A first hypothesis we test is:

Hypothesis H1. The probability of being recommended for a job is lower for female workers compared with male workers.

To test this result, we run the following logistic regressions, with the dependent variable Ref_{ij} taking value 1 if the person entered the job by referral and 0 otherwise:

$$Ref_{ij} = \beta_0 + \beta_1 Fem_i + \eta_0 ISEI_j + X_i'\eta_1 + \varepsilon_{ij}. \tag{1}$$

Here subindex j stands for the occupation, $ISEI_j$ is the index of occupational prestige capturing occupational heterogeneity, X_i is a vector of individual characteristics and η_1 is a corresponding vector of coefficients. In relation to H1, we expect $\beta_1 < 0$.[11]

From Table 4, we already know that on average 33% of workers in the sample enter the job following a referral from a social contact. If we split the sample by gender, we find that this incidence is 35.1% for men, but it is only 31.2% for women. Thus, in the raw data, we can already see a gap in the probability of entering the job by referral between men and women approximately 4%. In Table 6, we investigate if this gap remains once we control for observable worker and job characteristics. Column (1) of Table 6 shows that the coefficient corresponding to the female dummy is significantly negative ($\beta_1 = -0.212 < 0$) supporting Hypothesis H1. This coefficient is translated into a gender gap in referral hiring equal to 4.6%. Other regressors indicate that immigrant workers and part-time workers rely more often on referral hiring, whereas positions with higher occupational prestige (ISEI) are less often filled by referrals. In relation to education, we find a U-shaped pattern with a minimum at 16.9 years of schooling. This means that referral hiring is a frequent matching channel for workers with very low and very high skills, but it is less frequent among workers with average skills. An interpretation for this finding could be that low-skill workers use referrals from family members and relatives as a channel of last resort, since their chances of finding a job in a formal way are very low. At the same time, high-skill workers often rely on professional recommendations from former supervisors and senior colleagues (Stupnytska & Zaharieva, 2015).

In our theoretical model, referrals are used when workers move from a simple job on level 0 to their first management job on level 1. This change can be interpreted as a job-to-job transition, but it can also be interpreted as finding a first career-oriented job. In the following, we use information in our data on the type of job change of a worker. We split the sample so that in column (2) of Table 6 *Type of job change* = 1 corresponds to workers who entered the job for the first time, and in column (3) *Type of job change* = 3 corresponds to workers who moved to a new position

TABLE 6 Logistic regressions: H1 and H2.

Dependent variable	H1 incidence of referral (1)	(2)	(3)	H2 incidence of referral (4)	(5)	(6)	(7)
Fem	−0.212***	−0.384**	−0.205***	−0.225***	−0.216***	−0.282***	−0.185*
	(0.052)	(0.182)	(0.066)	(0.061)	(0.062)	(0.079)	(0.103)
Num. close friends					0.019***		
					(0.006)		
Share fem. friends							
1/3							−0.124
							(0.107)
2/3							−0.154
							(0.118)
3/3							−0.315**
							(0.134)
Part-time	0.208***	0.690***	0.277***	0.094	0.091	0.297***	0.309***
	(0.032)	(0.150)	(0.046)	(0.065)	(0.065)	(0.084)	(0.080)
Nationality	0.323***	0.139	0.145***	0.365***	0.350***	0.090	0.078
	(0.037)	(0.233)	(0.056)	(0.071)	(0.072)	(0.103)	(0.105)
Age	−0.026***	−0.013	−0.021	−0.02*	−0.020*	−0.009	−0.008
	(0.008)	(0.069)	(0.013)	(0.011)	(0.011)	(0.022)	(0.022)
Age sq.	0.000***	−0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Education	−0.305***	0.074	−0.254***	−0.293***	−0.297***	−0.241***	−0.240***
	(0.029)	(0.135)	(0.044)	(0.045)	(0.045)	(0.071)	(0.069)
Education sq.	0.009***	−0.005	0.008***	0.009***	0.009***	0.008***	0.008***
	(0.001)	(0.005)	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)
ISEI	−0.008***	−0.010***	−0.007***	−0.009***	−0.009***	−0.012***	−0.013***
	(0.001)	(0.004)	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)
Rural	0.044	0.100	0.023	0.026	0.027	−0.128	−0.125
	(0.040)	(0.233)	(0.064)	(0.057)	(0.057)	(0.107)	(0.107)
East	0.081***	0.987***	−0.018	0.002	−0.013	−0.088***	−0.099***
	(0.009)	(0.083)	(0.015)	(0.009)	(0.009)	(0.020)	(0.023)
Constant	2.404***	0.001	2.520***	2.324***	2.250***	1.314**	1.379**
	(0.331)	(1.056)	(0.528)	(0.347)	(0.352)	(0.626)	(0.609)
Observations	28,403	1504	14,949	10,001	10,001	4715	4715
Wald χ^2	710,23	1208,37	2346,06	1267,41	1200,42	100,10	387,96
Pseudo-R^2	0.0447	0.104	0.0311	0.0456	0.0466	0.0431	0.0441
Log likelihood	−17,157	−845.2	−9675	−6139	−6133	−2810	−2807
Fixed effects	Y	Y	Y	Y	Y	Y	Y

Note: All regressions include fixed effects for year, industry, state of residence, and firm size. The underlying sample includes full- and part-time workers who are not self-employed. Workers are less than 68 years old. Standard errors in parenthesis are clustered at the state level. Wald χ^2 statistic refers to testing the joint significance of the key regressors excluding fixed effects. Data source: SOEP (2020).

*$P < .1$; **$P < .05$; ***$P < .01$.

with a different employer. Column (3) shows that the coefficient corresponding to the female variable remains almost unchanged indicating that the referral disadvantage of women is observed in job-to-job transitions. At the same time, the gender coefficient in column (2) indicates even stronger disadvantage when workers enter their first job early in the career. The value of this coefficient (−0.384) is translated into a gender gap in referral hiring equal to 8.1%. Furthermore, the indicator variable on "East" implies that referral hiring for the first labor market entry is more widespread in East Germany.

Next, we investigate the relationship between network characteristics and the probability of finding a job by referral. The referral disadvantage of women in our model stems from the fact that women have smaller networks and a larger fraction of other women in the network, who are situated in less influential positions compared with men (see panel D of Table 2). We formulate this prediction as Hypothesis H2:

Hypothesis H2. The gender gap in the probability of being recommended for a job is due to the fact that female workers have smaller social networks and a larger fraction of females in the network.

We test this hypothesis by using variables *Num. close friends* and *Num. female friends*. Both variables are jointly available only for one observation year (2011), making the sample size inadequately small. Thus, we present our results including each of the two network characteristics separately. Information about the number of close friends is available for 7 years since 2000, whereas information about the three best friends is available for 4 years (see Appendix A).

First, we estimate regression (1) only for those 7 years where the number of close friends is available, this is column (4) in Table 6. We use this regression as a reference point and add the number of close friends in column (5). This regression suggests that workers with larger networks enter jobs more frequently with the help of their network contacts compared with workers with smaller social networks. This finding indicates that social networks have some influence on the labor market opportunities for workers. However, the gender coefficient remains virtually unchanged (compare −0.225 in column 4 and −0.216 in column 5), showing that differences in the number of close friends do not contribute substantially to the gender gap in the frequency of referral hiring.

Next, we test the second part of Hypothesis H2 and include the fraction of female friends in the regression. This variable takes values {0/3, 1/3, 2/3, 3/3}. We use category 0/3 as a reference point. The coefficients for this regression are presented in column (7) of Table 6. The data on the gender composition of the network is only available for the years 2001, 2006, 2011, and 2016. So in column (6), we restricted the sample to these observation years to have a point of comparison. Column (7) shows that a segregated network with a large share of female friends (3/3) is associated with a lower probability of entering a job by referral. This finding implies that one reason for a lower probability of obtaining a referral by women could be a higher fraction of female friends in their social networks, which also means a lower fraction of men, and supports Hypothesis H2. Our model suggests that one explanation for this finding is that men are situated in more influential positions compared with women and are more frequently in a situation when they can recommend their contacts. We conclude that the gender composition of the network is quantitatively more important than the total size of the social network in explaining the gender gap in the probability of a job referral.

Summarizing, the analysis presented in Table 6 support Hypotheses H1 and H2 and the first part of the theoretical *Result 4*, except that differences in the number of close friends between men and women seem to not contribute to the gender gap in referral hiring. Before we proceed with testing the second part of the theoretical *Result 4*, we exploit information about the management level of the job and its relation to referral hiring. Our model is based on the assumption that referral hiring takes place on the junior management level rather than on the senior management level, and so the gender gap in the probability of a referral is also attributed to junior management. Our data allows one to test this assumption. We formulate it as Hypothesis H3:

Hypothesis H3. The gender gap in the probability of a referral is generated at the lower management level rather than at the senior management level.

The raw data shows that the frequency of referrals in junior management jobs is 30%. For women this number is 26.07%, whereas for men it is 33, 21%, so the raw gender gap is about 7%. On the senior management level, the frequency is 23.97% for women and 27.06% for men, giving rise to a gap of about 3%. The raw data indicates a larger gender gap on the junior level compared with the senior level. In column (1) of Table 7, we present a referral regression for junior management jobs and in column (3) for senior management jobs. Our results support Hypothesis H3. There is a significant gender gap in referral hiring at the junior management level, but the gender variable is insignificant for

TABLE 7 Logistic regressions: H3, H4, and H5.

Dependent variable	H3 incidence of referral Junior management (1)	Different employer (2)	Senior management (3)	H4 incidence of referral All jobs (4)	All jobs (5)	H5 being employed in lower management job (6)
Fem	−0.337**	−0.465***	0.033	−0.184**	0.162	−1.094***
	(0.140)	(0.156)	(0.185)	(0.080)	(0.205)	(0.132)
h (=Share of women in occupation)					−0.275	−0.080
					(0.538)	(0.217)
Fem ## h					−2.869**	2.379***
					(1.377)	(0.356)
h^2					−0.547	
					(1.183)	
Fem ## h^2					5.283*	
					(2.888)	
Part-time	−0.079	0.017	0.138	0.256***	0.262***	−0.730***
	(0.197)	(0.203)	(0.296)	(0.065)	(0.064)	(0.075)
Nationality	0.200	−0.144	0.233	0.372***	0.378***	−0.460***
	(0.247)	(0.241)	(0.252)	(0.036)	(0.035)	(0.101)
Age	−0.034	−0.003	−0.163*	−0.016	−0.017	0.113***
	(0.042)	(0.058)	(0.099)	(0.013)	(0.013)	(0.010)
Age sq.	0.000	−0.000	0.002	0.000	0.000	−0.001***
	(0.001)	(0.001)	(0.001)	(0.000)	(0.000)	(0.000)
Education	−0.607**	−0.395	−0.323	−0.361***	−0.373***	0.293***
	(0.299)	(0.350)	(0.622)	(0.047)	(0.050)	(0.077)
Education sq.	0.019*	0.012	0.009	0.012***	0.012***	−0.009***
	(0.010)	(0.012)	(0.021)	(0.002)	(0.002)	(0.003)
ISEI	0.000	−0.003	−0.003	−0.006***	−0.005**	0.025***
	(0.004)	(0.003)	(0.006)	(0.002)	(0.002)	(0.001)
Rural	0.091	0.060	0.383	0.095	0.094	0.086
	(0.250)	(0.261)	(0.239)	(0.073)	(0.072)	(0.064)
East	−0.495***	−0.862***	1.938***	0.088***	0.075***	−0.095***
	(0.061)	(0.091)	(0.213)	(0.017)	(0.019)	(0.009)
Constant	4.653*	2.436	4.565	2.120***	2.321***	−6.588***
	(2.451)	(2.812)	(3.701)	(0.468)	(0.459)	(0.466)
Observations	1373	996	634	13,231	13,231	25,445
Wald χ^2	1425.69	484.08	1027.35	982.24	31,640.86	8483.01
Pseudo-R^2	0.0621	0.0616	0.0943	0.0463	0.0471	0.0766

(Continues)

TABLE 7 (Continued)

Dependent variable	H3 incidence of referral			H4 incidence of referral		H5 being employed
	Junior management (1)	Different employer (2)	Senior management (3)	All jobs (4)	All jobs (5)	in lower management job (6)
Log likelihood	−785.9	−593.6	−327.3	−7904	−7897	−13973
Fixed effects	Y	Y	Y	Y	Y	Y

Note: All regressions include fixed effects for year, industry, state of residence, and firm size. The underlying sample includes full- and part-time workers who are not self-employed. Workers are younger than 68 years. For columns (4)–(6), the sample includes workers working in occupations for which more than 50 observations are given, and in which the share of female workers does not exceed 50%. Standard errors in parenthesis are clustered at the state level. Data source: SOEP (2020).

*$P < .1$; **$P < .05$; ***$P < .01$.

the "senior management" level. One interesting regional effect that we observe in these regressions is that referrals to junior (senior) positions are less (more) common in East Germany compared with West Germany. In addition, in column (2) we check if the gender gap in junior management is observed when workers move from one employer to another by setting "*Type of job change* = 3." Our results in Table 7 show that, again, there is a gender gap. Overall, we find support for Hypothesis H3, and our model is compatible with the evidence, except that in the data referrals are also used in senior jobs, which is not captured by our model.

Next, we continue our analysis by testing the second part of the theoretical *Result 4*. This result suggests that the gender gap in the probability of a referral ($\tilde{\alpha}_1 - \alpha_1$) is negligibly small when there are hardly any women in the occupation $h \to 0$ and when women represent a half of the occupational labor force ($h \to 0.5$), but it is substantial in the middle range when $h \in (0 \dots 0.5)$. To test for this nonmonotonic relationship, we estimate the following logistic regression:

$$Ref_{ij} = \beta_0 + \beta_1 Fem_i + \beta_2 h_j + \beta_3 h_j^2 + \beta_4 Fem_i \cdot h_j + \beta_5 Fem_i \cdot h_j^2 + \eta_0 ISEI_j + X_i' \eta_1 + \varepsilon_{ij}. \tag{2}$$

Subindex j refers to the occupation, and variable h_j is a female participation rate in the occupation. The nonmonotonic gender gap can be captured by the coefficients β_4 and β_5. We summarize these results, originating from the theoretical *Result 4*, in the following hypothesis:

> **Hypothesis H4.** The gender gap in the probability of being recommended for a job is nonmonotonic: It is small in absolute terms for $h \to 0$ and for $h \to 0.5$, but it is large in absolute terms for intermediate values of h in the range of $(0 \dots 0.5)$. For Equation (2) this implies $\beta_4 < 0, \beta_5 > 0$.

To test Hypothesis H4, we draw on variation in female participation rates across occupations. We construct a new variable h_j corresponding to the female participation rates in occupations using occupational information based on the ISCO-88 classification of occupations. Our sample includes information on 326 different occupations. However, the number of employees in some rare occupations can be small giving rise to all-men or all-women observations. Thus, in the following, we restrict the sample to occupations with more than 50 observations to obtain a meaningful estimate for the gender ratio. This holds for 223 occupations in the sample. In the final sample, women are underrepresented in occupations with occupational codes below 2000, this corresponds to high prestige occupations such as legislators and senior officials, corporate managers and general managers. The gender representation is more balanced on average in occupations with ISCO-88 codes between 2000 and 6000, including science, health, legal and teaching professionals, associate professionals, office clerks, and market sales workers. However, women are also underrepresented on average in occupations with ISCO-88 codes above 6000. These are agricultural and fishery workers, craft and trade workers, plant and machine operators.

Since Hypothesis H4 refers to occupations where women are the minority, that is, $h_j < 0.5$, we restrict the sample to satisfy this condition first. This reduces the sample to 146 occupations. The corresponding regression is presented in column (4) of Table 7. As before female workers get fewer referrals. In the next step, we add four more variables: h_j, h_j^2, $Fem_i \cdot h_j$, and $Fem_i \cdot h_j^2$. The corresponding logistic regression is presented in column (5) of the table. These

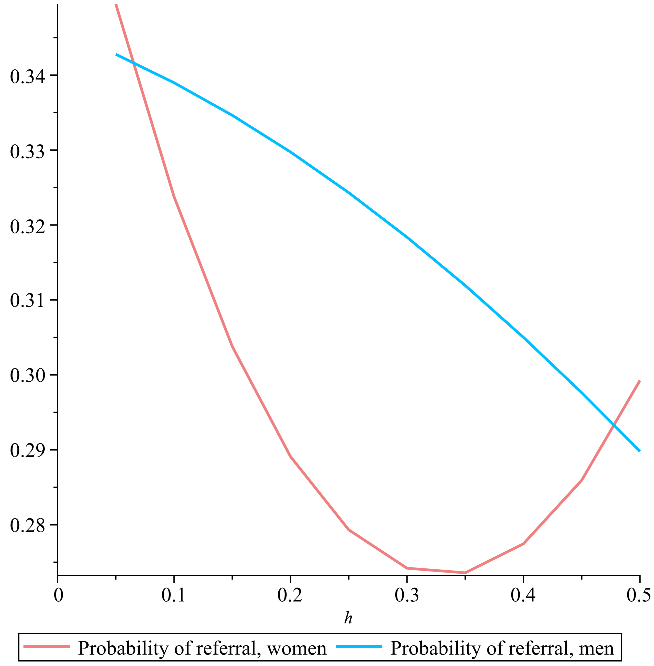

FIGURE 2 Predicted referral probability for men and women, based on estimated regression (5) in Table 7. [Color figure can be viewed at wileyonlinelibrary.com]

regressions show that $\beta_4 < 0$ and $\beta_5 > 0$, which is in line with Hypothesis H4 and *Result 4*. Intuitively, these results mean that for women the probability of a job referral is falling initially, reaches a minimum at $h_j = 0.33$ and starts increasing thereafter. Figure 2 presents an illustration of this effect and shows that the gender gap in referral hiring disappears for low values of h close to 5% and when the gender representation in the occupation is almost equal.

Our theoretical model delivers an intuitive explanation for this functional relationship. When there are only a few women in the occupation, the gender composition of their networks is not different from their male colleagues and so are the chances of being recommended for the job. Recall that workers in the model do not discriminate their contacts by gender when giving recommendations. However, as the fraction of women in the occupation is increasing, social networks of men and women start diverging leading to a substantially lower fraction of men in the networks of women and reducing their chances of getting a job by referral. The reason is that women are situated in less influential positions compared with men. However, if the fraction of women in the occupation continues to increase and reaches $h = 0.5$, both groups become equally large and their career patterns equalize.

We have presented a comprehensive testing of the theoretical *Result 4* split into several testable hypotheses (H1–H4), so far. We continue our empirical analysis by testing comparative statics predictions described in *Result 6*. *Result 6* suggests that female workers have a lower probability of reaching management jobs compared with men, but the gender gap in this probability is decreasing in absolute terms with a higher participation of women h. To test this prediction, we use the following regression:

$$Manager_{ij} = \beta_0 + \beta_1 \cdot Fem_i + \beta_2 \cdot h_j + \beta_3 Fem_i \cdot h_j + \eta_0 ISEI_j + X_i'\eta_1 + \varepsilon_{ij}. \quad (3)$$

Since we already know from H3 that the gender gap in the probability of a referral is generated at the junior management level rather than the senior level, our dependent variable $Manager_{ij}$ takes value 1 for jobs in junior management and 0 for nonmanagement jobs. Thus we consider the management entry margin. In relation to *Result 6*, we formulate Hypothesis H5.

Hypothesis H5. The probability of becoming a manager for females is lower compared with males, but this probability is increasing in the female participation rate h, so that the gender gap in the probability of becoming a manager is getting smaller with higher h. For Equation (3) this implies $\beta_1 < 0, \beta_2 + \beta_3 > 0, \beta_3 > 0$.

The initial gender gap in reaching a management job is captured by the coefficient β_1. The coefficient β_2 shows how the chances of reaching a management job for men can be affected by the higher participation rate of women h. For women the effect of higher h is captured by the compound coefficient $\beta_2 + \beta_3$. Moreover, if $\beta_3 > 0$, then the gender gap is closing with a higher participation rate h. Our estimation results are presented in column (6) of Table 7. As we can see from column (6) $\beta_1 < 0$. This shows that women face lower chances of entering a management job even if the standard worker and firm characteristics are controlled for. In addition, the data supports Hypothesis H5 since $\beta_3 > 0$ meaning that the gender gap in becoming a junior manager is decreasing in absolute terms with a higher participation of women in the occupation (h). Our theoretical model does not have a clear prediction for the sign of the coefficient β_2, which reflects the chances of becoming a manager for men. However, Figure 1 shows that the quantitative effect is likely to be small for men. In line with this expectation, we can see that the coefficient β_2 is insignificant in column (6), meaning that $\beta_2 + \beta_3 > 0$. So the gender gap is closing because the chances of reaching a management job are improving for women rather than worsening for men.

6 | DISCUSSION AND CONCLUSIONS

In this paper, we explore the impact of social networks on the career progression of men and women and the mechanism by which networks can contribute to stronger gender inequality. We address the question by setting up and analyzing an analytical search and matching model and confronting various predictions of the theoretical analysis with data from the German SOEP.

Our theoretical model is producing several new and interesting results. We show that a disproportionately low representation of women in managerial positions in firms may emerge without discrimination. Two requirements have to be fulfilled for this to occur: the participation rate of women in the labor market should be low, and network formation has to be homophilous. Network homophily leads to a situation when women are underrepresented in the networks of men. This becomes a disadvantage for women regarding career progression if men constitute a majority group and referral hiring is used to fill vacant positions. The reason is that senior managers use their social networks to suggest applicants if suitable internal candidates are unavailable to the firm. This is why women have a low speed of progression in the job ladders of firms, although senior managers do not discriminate but rather refer candidates randomly from their homophilous network.

Our theoretical apparatus finds empirical support in various directions. We can provide evidence that male workers' networks are larger than female workers' networks and that networks are gendered (homophily). As predicted by our model, female workers get fewer referrals. There is also evidence that part of the gender gap in the referral probability is because men have more other men in their networks situated in more influential positions and that the probability of getting a job in lower management for women is positively correlated with the share of women in the occupation. All this evidence supports our primary theoretical results, namely, that underrepresentation of women in top management positions can be the result of referral hiring in labor markets, where women's labor market participation is lower than men's labor market participation, and where social networks are homophilous.

Thus, there are reasons, women do not make it to the top positions in firms, which are not necessarily related to discrimination or occupational segregation. It may suffice that the participation of women is lower compared with men—which is the case in many occupations (International Labour Office, 2017)—and that workers are more inclined to form network links with others of the same gender. Whether these requirements could already be the result of discrimination needs discussion. If, for example, lower female participation in the occupation is the result of young women not investing in human capital because prevailing social norms make it difficult for them to enter higher education in general or specific fields, as is often observed in engineering studies or the natural sciences, then discrimination would already have taken place before our analysis starts.

Our work focuses on studying the role of networks in hierarchical male-dominated occupations (e.g., management and business jobs). Even though mathematically, the model can be used to obtain symmetric predictions for occupations dominated by female workers with $h_j > 0.5$, we believe that these occupations have several structural differences going beyond the gender composition. These include limited promotion possibilities (e.g., nurses) or a large share of civil servants where network hiring is restricted (e.g., teachers). Thus, we refrained from stretching the model predictions to female-dominated occupations.

Although our analysis evolves along the gender dimension, the model is not necessarily gender-specific. More broadly, our results suggest that being underrepresented in the social networks of the majority group can lead to disadvatages if social networks are used as a source of information for labor market decisions. A similar line of argument could be made with other groups in the labor market facing low chances of reaching top management positions. An analysis along the gender dimension only has, however, the advantage that labor market data on gender issues is available. This data allows us to calibrate our model meaningfully and, to some extent, test its predictions. Whether the predictions of our model also hold for other minority groups in the labor market would be an interesting empirical project for the future.

Our analysis also bears some interesting policy implications. As homophilous networks are one driver behind the disproportionate gender distribution in managerial jobs, instruments may be called for that support the gender-mixing of networks. Policies that encourage women's only networks at workplaces, as can be often observed nowadays, seem to be the wrong way to go. Our analysis also suggests that policies that successfully raise female participation to the levels of male workers will contribute to equal opportunities by strengthening female networks even when network formation remains homophilous. Thus, although their main goal is to eliminate discrimination at the hiring stage, affirmative action policies could achieve a higher representation of women in management positions. This would lead to a better quality of women's professional networks and amplify the policy's direct effect.

Our theoretical and empirical analysis of the glass-ceiling effect in the context of social networks rests on various simplifying assumptions and data availability, respectively. For example, we make simplifying assumptions with regard to the network formation or firm hierarchies. An extension to firms with more hierarchies would pave the way to explore further channels, such as whether referral hiring in conjunction with deeper firm hierarchies affects gender segregation in management positions. Furthermore, network formation in our model is random. We do not explore clustered networks, which may emerge from deliberate decisions of the actors with whom to connect or the social context in which actors are embedded. It is an interesting extension to derive the defining feature of female networks from women's condition in society or the labor market environment, that is, modeling networks as emerging from different responsibilities for childcare or different approaches to networking more generally. It is also conceivable that the formation of networks along the gender dimension is already the result of discrimination. Men might want to form a network link with another man rather than a woman because they want to keep women out of their relevant labor market. Data beyond the available sources would be needed to test for the more involved chain of causes that our theory postulates and conduct a causal analysis. In particular, information about the gender of the referrer could be helpful. Extending the theoretical analysis and testing with better data as it becomes available are interesting routes for future work in this area.

ACKNOWLEDGMENTS

We would like to thank Herbert Dawid, Roberto M. Fernandez, Laszlo Goerke, and the members of the ZIF Research Group "In Search of the Global Labor Market: Actors, Structures and Policies" for their contributions. The paper also benefited from colleagues' comments at the Workshop on Applied Economics at Philips-Universität Marburg, the research seminars at DICE, Düsseldorf, KIT, Karlsruhe, and IAAEU, Trier as well as from participants' suggestions on the 2019 EALE conference in Uppsala, the 2020 conference of the German Economic Association in Cologne, the 2021 conference of the Search and Matching network group, and the 2021 conference of the European Economic Association in Copenhagen. Anna Zaharieva acknowledges the financial support of the Center for Interdisciplinary Research at Bielefeld University (ZIF) and the German Research Foundation (DFG) under grant ZA 827/2. Open Access funding enabled and organized by Projekt DEAL.

ORCID

Anna Zaharieva http://orcid.org/0000-0002-2952-8904

NOTES

[1] Two notable exceptions include Rubineau and Fernandez (2013, 2015).

[2] In this section, we restrict attention to studies, where social networks are used to exchange information about vacancies, giving rise to referral hiring. However, we acknowledge that social networks could contribute to gender inequality even after the job is secured. For example, McDowell and Kiholm Smith (1992) and most recently Ductor et al. (2023) find that women in the economic profession have fewer possibilities to collaborate since economists tend to work with coauthors of the same gender. The former study shows additionally, that this type of gender homophily contributes to lower promotion rates of women in economics (due to a lower number of coauthored publications). Lindenlaub and Prummer (2021) capture a similar idea and develop a formal theory that links individuals' network structure to their productivity on the job.

[3] Pistaferri (1999) shows it for Italy, Addison and Portugal (2002) for Portugal, Bentolila et al. (2010) and Pellizzari (2010) for the United States and the European Union, and Rebien et al. (2020) for Germany.

[4] This means that there is a large population of constant size $N \to \infty$, but the model is size-invariant making separate notation for variable N redundant.

[5] Our assumption is also motivated by the mixed empirical evidence on gender differences in internal promotions. Whereas Blau and DeVaro (2007), Kauhanen and Napari (2015), and Cassidy et al. (2016) show that women have lower promotion probabilities within firms in the USA and in Finnland, unequal promotion chances are not supported for countries like Germany and the United Kingdom, see Chadi and Goerke (2018) and Booth et al. (2003), respectively.

[6] We make this assumption to isolate the effect of network composition (homophily) on the probability of referring a female applicant from other possible channels such as explicit preferences for recommending applicants of specific gender (taste-based discrimination). Whereas these other channels driven by gender-discrimination are also relevant and have been studied in the literature (Beaman et al., 2018; Beugnot & Peterlé, 2020), a separate impact of network homophily on referral chances of women received little attention so far. For the same reason, we also do not consider the effects of referrals after hiring, such as differences in the turnover of referred workers compared with not-referred workers (Burks et al., 2015; Friebel et al., 2022) or improved collaboration on the job of a referred worker with the referrer (Pallais & Sands, 2016).

[7] We use the job-filling rates obtained for a manager with an average network as an approximation for the average job-filling rates obtained over all possible random network combinations of the manager. We find that numerically the two approaches are indistinguishable from each other.

[8] In Table 3, we show that the total job-filling rate in the equilibrium is equal to $\bar{q}_1^{MM} + \bar{q}_1^{FM} = 0.6218$, indicating that firms with open junior positions enjoy 62% probability of filling their position per unit of time. The total job-filling rate $\bar{q}_1^{MM} + \bar{q}_1^{FM}$ consists of the formal hiring rate $q_1 = 0.4718$ and the referral hiring rate $\tilde{q}_1^{M} = 0.15$, so the average fraction of employees hired by recommendation can be evaluated at $0.15/0.6218 \approx 0.24$.

[9] A detailed summary of the coding of variables and for which years they are available is contained in Appendix A.

[10] To set parameters in the theoretical model, we restricted the sample to high-skill workers in full-time jobs, because it is this group of workers where the glass-ceiling effect is more likely to prevail. But we keep workers of all skill groups in full- or part-time employment in the sample for the purpose of the empirical analysis and include education and working time as control variables.

[11] In Appendix A, we show how β_1 is related to our theoretical results.

[12] In general the stock variable $d_{F0}(x, t)$ may depend on time t, so the total derivative is given by

$$\frac{\partial d_{F0}(x,t)}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial d_{F0}(x,t)}{\partial t} = \rho\left(d_{FM}^{N}(x) + d_{FF}^{N}(x)\right) - (\rho + q_2)d_{F0}(x).$$

Since the distribution of firms $d_{F0}(x, t)$ is stationary in the steady state, we set the time derivative $\dot{d}_{F0} = \frac{\partial d_{F0}(x,t)}{\partial t}$ equal to zero. Moreover, experience x is accumulating one to one with time because $\dot{x} = \partial x/\partial t = 1$).

REFERENCES

Addison, J. T., & Portugal, P. (2002). Job search methods and outcomes. *Oxford Economic Papers*, *54*(3), 505–533.

Alaverdyan, S., & Zaharieva, A. (2022). Immigration, social networks and occupational mismatch. *Economic Modelling*, *114*, 105936.

Albrecht, J., Björklund, A., & Vroman, S. (2003). Is there a glass ceiling in Sweden? *Journal of Labor Economics*, *21*(1), 145–177.

Arbex, M., O'Dea, D., & Wiczer, D. (2019). Network search: Climbing the job ladder faster. *International Economic Review*, *60*(2), 693–720.

Beaman, L., Keleher, N., & Magruder, J. (2018). Do job networks disadvantage women? Evidence from a recruitment experiment in Malawi. *Journal of Labor Economics*, *36*(1), 121–157.

Behtoui, A. (2008). Informal recruitment methods and disadvantages of immigrants in the Swedish labour market. *Journal of Ethnic and Migration Studies*, *34*(3), 411–430.

Bentolila, S., Michelacci, C., & Suarez, J. (2010). Social contacts and occupational choice. *Economica*, *77*(305), 20–45.

Bertrand, M., Black, S. E., Jensen, S., & Lleras-Muney, A. (2018). Breaking the glass-ceiling? The effect of board quotas on female labor market outcomes in Norway. *The Review of Economic Studies*, *86*, 191–239.

Beugnot, J., & Peterlé, E. (2020). Gender bias in job referrals: An experimental test. *Journal of Economic Psychology*, *76*, 102209.

Blau, F. D., & DeVaro, J. (2007). New evidence on gender differences in promotion rates: An empirical analysis of a sample of new hires. *Industrial Relations: A Journal of Economy and Society*, *46*(3), 511–550.

Blau, F. D., & Kahn, L. M. (2017). The gender wage gap: Extent, trends and explanations. *Journal of Economic Literature*, *55*(3), 789–865.

Boorman, S. A. (1975). A combinatorial optimization model for transmission of job information through contact networks. *Bell Journal of Economics*, *6*(1), 216–249.

Booth, A. L., Francesconi, M., & Frank, J. (2003). A sticky floors model of promotion, pay and gender. *European Economic Review*, *47*, 295–322.

Burks, S. V., Cowgill, B., Hoffman, M., & Housman, M. (2015). The value of hiring through employee referrals. *The Quarterly Journal of Economics*, *130*(2), 805–839.

Calvó-Armengol, A. (2004). Job contact networks. *Journal of Economic Theory*, *115*(1), 191–206.

Cassidy, H., DeVaro, J., & Kauhanen, A. (2016). Promotion signaling, gender, and turnover: New theory and evidence. *Journal of Economic Behavior and Organization*, *126*, 140–166.

Chadi, A., & Goerke, L. (2018). Missing at work-sickness-related absence and subsequent career events. *Journal of Economic Behavior and Organization*, *153*, 153–176.

Chetty, R., Jackson, M., Kuchler, T., Stroebel, J., Hendren, N., Fluegge, R., Gong, S., Gonzalez, F., Grondin, A., Jacob, M., Johnston, D., Koenen, M., Laguna-Muggenburg, E., Mudekereza, F., Rutter, T., Thor, N., Townsend, W., Zhang, R., Bailey, M., Barbera, P., Bhole, M., & Wernerfelt, N. (2022). Social capital ii: Determinants of economic connectedness. *Nature*, *608*, 122–149.

Cingano, F., & Rosolia, A. (2012). People I know: Job search and social networks. *Journal of Labor Economics*, *30*(2), 291–332.

Corcoran, M., Datcher, L., & Duncan, G. J. (1980). Information and influence networks in labor markets. *Five Thousand American Families: Patterns of Economic Progress*, *8*, 1–37.

Dawid, H., Mitkova, M., & Zaharieva, A. (2023). Professional careers, promotions and organizational hierarchies in a frictional labour market. *Journal of Economics and Management Strategy*, *32*(1), 100–131.

Diamond, P. A. (1982). Wage determination and efficiency in search equilibrium. *The Review of Economic Studies*, *49*(2), 217–227.

Ductor, L., Sanjeev, G., & Prummer, A. (2023). Gender and collaboration. *The Review of Economics and Statistics*, *105*(6), 1–13.

Erosa, A., Fuster, L., Kambourov, G., & Rogerson, R. (2022). Hours, occupations, and gender differences in labor market outcomes. *American Economic Journal: Macroeconomics*, *14*(3), 543–590.

Fernandez, R. (2013). Cultural change as learning: The evolution of female labor force participation over a century. *The American Economic Review*, *103*(1), 472–500.

Fernandez, R. M., & Sosa, L. M. (2005). Gendering the job: Networks and recruitment at a call center. *American Journal of Sociology*, *111*(3), 859–904.

Fontaine, F. (2008). Why are similar workers paid differently? The role of social networks. *Journal of Economic Dynamics and Control*, *32*(12), 3960–3977.

Friebel, G., Heinz, M., Hoffman, M., & Zubanov, N. (2022). What do employee referral programs do? Measuring the direct and overall effects of a management practice. *Journal of Political Economy*, *131*(3), 633–686.

Galenianos, M. (2021). Referral networks and inequality. *The Economic Journal*, *131*(633), 271–301.

Galeotti, A., & Merlino, L. P. (2014). Endogenous job contact networks. *International Economic Review*, *55*(4), 1201–1226.

Glitz, A. (2017). Coworker networks in the labour market. *Labour Economics*, *44*, 218–230.

Gneezy, U., Niederle, M., & Rustichini, A. (2003). Performance in competitive environments: Gender differences. *The Quarterly Journal of Economics*, *118*3, 1049–1074.

Goebel, J., Grabka, M. M., Liebig, S., Kroh, M., Richter, D., Schröder, C., & Schupp, J. (2019). The German Socio-Economic Panel (SOEP). *Jahrbücher für Nationalökonomie und Statistik*, *239*(2), 345–360.

Granovetter, M. (1995). *Getting a job: A study of contacts and careers* (2nd ed.). University of Chicago Press.

Guertler, M., & Guertler, O. (2019). Promotion signaling, discrimination,and positive discrimination policies. *RAND Journal of Economics*, *50*(4), 1004–1027.

Horvath, G. (2014). On-the-job search and finding a good job through social contacts. *The B.E. Journal of Theoretical Economics*, *14*(1), 93–125.

International Labour Office. (2017). *World employment social outlook: Trends for women 2017*.

Kauhanen, A., & Napari, S. (2015). Gender differences in careers. *Annals of Economics and Statistics*, *117–118*, 61–88.

Lindenlaub, I., & Prummer, A. (2021). Network structure and performance. *The Economic Journal*, *131*, 851–898.

Marsden, P. V., & Gorman, E. H. (2001). Social networks, job changes and recruitment. *Sourcebook of Labour Markets: Evolving Structures and Processes*, *19*, 467–502.

McDowell, J. M., & Kiholm Smith, J. (1992). The effect of gender-sorting on propensity to coauthor: Implications for academic promotion. *Economic Inquiry*, *30*(1), 68–82.

McKinsey&Company. (2017). *Women in the workplace*.

McPherson, M., Smith-Lovin, L., & Cook, J. M. (2001). Birds of a feather: Homophily in social networks. *Annals Review of Sociology*, *27*, 415–444.

Milgrom, P., & Oster, S. (1987). Job discrimination, market forces, and the invisibility hypothesis. *The Quarterly Journal of Economics*, *102*(3), 453–476.

Montgomery, J. D. (1991). Social networks and labor-market outcomes: Toward an economic analysis. *The American Economic Review*, *81*(5), 1408–1418.

Mortensen, D. T. (1982). The matching process as a noncooperative bargaining game. In J. McCall (Ed.), *The economics of information and uncertainty* (pp. 233–258). University of Chicago Press.

Pallais, A., & Sands, E. G. (2016). Why the referential treatment? Evidence from field experiments on referrals. *Journal of Political Economy*, *124*(6), 1793–1828.

Pellizzari, M. (2010). Do friends and relatives really help in getting a good job? *Industrial and Labor Relations Review*, *63*(3), 494–510.

Petit, P. (2007). The effects of age and family constraints on gender hiring discrimination: A field experiment in the French financial sector. *Labor Economics*, *14*, 371–391.

Petrongolo, B., & Pissarides, C. A. (2001). Looking into the black box: A survey of the matching function. *Journal of Economic Literature*, *39*(2), 390–431.
Pissarides, C. A. (1985). Short-run equilibrium dynamics of unemployment, vacancies, and real wages. *The American Economic Review*, *75*(4), 676–690.
Pistaferri, L. (1999). Informal networks in the Italian labor market. *Giornale degli Economisti e Annali di Economia*, *58*(3–4), 355–375.
Plug, E., van der Klaauw, B., & Ziegler, L. (2018). Do parental networks pay off? Linking children's labor-market outcomes to their parents' friends. *The Scandinavian Journal of Economics*, *120*(1), 268–295.
Rebien, M., Stops, M., & Zaharieva, A. (2020). Formal search and referrals from a firm's perspective. *International Economic Review*, *61*(4), 1679–1748.
Rivera, M., Soderstrom, S., & Uzzi, B. (2010). Dynamics of dyads in social networks: Assortative, relational, and proximity mechanisms. *Annual Review of Sociology*, *36*, 91–115.
Rubineau, B., & Fernandez, R. M. (2013). Missing links: Referrer behavior and job segregation. *Management Science*, *59*(11), 2470–2489.
Rubineau, B., & Fernandez, R. M. (2015). Tipping points: The gender segregating and desegregating effects of network recruitment. *Organization Science*, *26*(6), 1646–1664.
SOEP. (2020). *Socio-Economic Panel (SOEP), data for years 1984–2018* (Version 35). https://doi.org/10.5684/soep.v35
Stupnytska, Y., & Zaharieva, A. (2015). Explaining U-shape of the referral hiring pattern in a search model with heterogeneous workers. *Journal of Economic Behavior and Organization*, *119*, 211–233.
Topa, G. (2011). Labor markets and referrals. *Handbook of Social Economics*, *1*, 1193–1221.
Winter-Ebmer, R., & Zweimüller, J. (1997). Unequal assignment and unequal promotions in job ladders. *Journal of Labor Economics*, *15*(1), 43–71.
Zaharieva, A. (2015). Social contacts and referrals in a labor market with on-the-job search. *Labour Economics*, *32*, 27–43.

How to cite this article: Neugart, M., & Zaharieva, A. (2024). Social networks, promotions, and the glass-ceiling effect. *Journal of Economics & Management Strategy*, 1–33. https://doi.org/10.1111/jems.12603

APPENDIX A

A.1 | Description of variables

Table A1 documents the coding of variables used in the network analysis.

TABLE A1 Variable description.

Variable	Description
Referral (Ref)	If found out about current job with the help of another person (=1), otherwise (=0); Years: 2000–2018
Referral type	If found out about current job through internet, friends, family, or colleagues; Years: 2015–2018
Leadership position	Top management (=1); middle management (=2); lower management (=3); highly qualified specialist (=4); Years: 2007, 2009, 2011, 2013, 2015, 2017
Management	Individual supervises other in position (=1); otherwise (=0); Years: 2007, 2009, 2011, 2013, 2015, 2017
Type of job change	Type of occupational change; entered employment for the first time in my life (=1); returned to a past employer after a break in employment (=2); new position different employer (=3);

TABLE A1 (Continued)

Variable	Description
	has been taken on by the company (4); changed position within the company (=5) became self-employed (=6) Years: 2000–2018
Num. close friends	Number of close friends Years: 2003, 2008, 2011, 2013, 2015, 2017, 2018
Share fem. friends	Share of female friends among three best friends Years: 2001, 2006, 2011, 2016
Employment status	Full-time and part-time employed (=1); other employment (=0); Years: 2000–2018
Occupation	Current occupational classification (ISCO-88 COM) Years: 2000–2018
Firm size	Core category size of the company; <20 (=1); ≥20 < 200 (=2); ≥200 < 2000 (=3); ≥2000 (=4); self-employed without coworkers (=5); Years: 2000–2018
ISEI	Last reached international socioeconomic index Years: 2000–2018
Part-time	Type of employment; if full-time (=0), part-time (=1) Years: 2000–2018
Nationality	Nationality of individual; if German (=0), otherwise (=1) Years: 2000–2018
Age	Age of individual in years Years: 2000–2018
Female (Fem)	Gender of individual, male (=0), female (=1) Years: 2000–2018
Education	Number of years in education Years:

Industry	1 digit industry code of individual; agriculture (=1);
	energy (=2); mining (=3); manufacturing (=4);
	construction (=5); trade (=8); transport (=7);
	bank, insurance (=8); services (=9); other (=10)
	Years: 2000–2018

State	State of residence Years: 2000–2018
Rural	Urban density; if rural (=1), otherwise (=0); following official definition of the Federal Institute for Research on Building, Urban Affairs and Spatial Development (BBSR) urbanized areas include cities with more than 100000 inhabitants as well as

(Continues)

TABLE A1 (Continued)

Variable	Description
	counties with more than 150 inhabitants per km^2
	Years: 2000–2018
East	Residence in East or West Germany; if East (=1), otherwise (=0)
	Years: 2000–2018
Wave	Year of the wave from which observations stem
	Years: 2000–2018

Data source: SOEP (2020).

A.2 | Derivation of H1

H1 links to the theoretical result in the following way. Recall from the theoretical part that $\tilde{\alpha}_1 = P\{Fem = 0|Ref = 1\}$ and $\alpha_1 = P\{Fem = 0|Ref = 0\}$. In the equation below, we show that the difference $\tilde{\alpha}_1 - \alpha_1$, which is a male bias associated with the referral channel versus the formal channel, is closely related to the gender gap in the probability of being referred for the job captured by the regression parameter β_1:

$$\begin{aligned} P\{R=1|F=1\} - P\{R=1|F=0\} &= \frac{P\{R=1\}P\{F=1|R=1\}}{P\{F=1\}} - \frac{P\{R=1\}P\{F=0|R=1\}}{P\{F=0\}} \\ &= \frac{P\{R=1\}(1-\tilde{\alpha}_1)}{1-(\tilde{\alpha}_1 P\{R=1\} + \alpha_1(1-P\{R=1\}))} \\ &\quad - \frac{P\{R=1\}\tilde{\alpha}_1}{\tilde{\alpha}_1 P\{R=1\} + \alpha_1(1-P\{R=1\})} \\ &= \frac{P\{R=1\}(1-P\{R=1\})}{P\{F=0\}(1-P\{F=0\})}[\alpha_1 - \tilde{\alpha}_1]. \end{aligned}$$

We use this transformation to formulate H1.

A.3 | Deriving the steady-state distributions of firms

In Section 3.3 we already described the evolution of the stock of firms in state d_{00}. In the following, we derive the differential equations of all the remaining states and provide solutions.

Consider changes in the stocks of firms $d_{F0}(x)$, $d^N_{FM}(x)$, and $d^N_{FF}(x)$. Note that workers with experience $0 \le x \le \bar{x}$ are not yet searching on the job since their experience is not sufficient for managerial positions and there are no gains from changing to another junior job. This means that the inflow of firms into state $d_{F0}(x)$ is equal to $\rho(d^N_{FM}(x) + d^N_{FF}(x))$. These are the firms where the manager exits at rate ρ and they are left with only one junior worker of type F. If the manager exits firms post the open position in the second submarket for experienced workers and find a manager at rate q_2. This means that the outflow of workers from the state $d_{F0}(x)$ is equal to $(q_2 + \rho)d_{F0}(x)$ where the term $\rho d_{F0}(x)$ corresponds to the job destruction shock ρ of the junior position. So we get the following differential equation[12]:

$$\frac{\partial d_{F0}(x)}{\partial x} = \rho\left(d^N_{FM}(x) + d^N_{FF}(x)\right) - (\rho + q_2)d_{F0}(x).$$

Next, we take into account changes in the stock of firms $d^N_{FM}(x)$ and $d^N_{FF}(x)$. Each of these firms has exactly two filled positions, so the job destruction shock arrives at the increased rate 2ρ. The inflow of firms into category $d^N_{FF}(x)$ is equal to $q_2(1-\alpha_2)d_{F0}(x)$. These are the firms $d_{F0}(x)$ filling their senior position with a type F candidate. In a similar way, the inflow of firms into category $d^N_{FM}(x)$ is equal to $q_2\alpha_2 d_{F0}(x)$. These are the firms $d_{F0}(x)$ filling their senior position with a type M candidate. So we get the following two differential equations:

$$\partial d_{FF}^{N}(x)/\partial x = q_2(1-\alpha_2)d_{F0}(x) - 2\rho d_{FF}^{N}(x),$$
$$\partial d_{FM}^{N}(x)/\partial x = q_2\alpha_2 d_{F0}(x) - 2\rho d_{FM}^{N}(x).$$

The coefficient matrix of the three first-order linear differential equations for $\left\{d_{F0}(x), d_{FF}^{N}(x), d_{FM}^{N}(x)\right\}$ has three eigenvalues equal to: -2ρ, $-\rho$, and $-(2\rho+q_2)$ (see the next part of Appendix A). So the general solution is given by

$$d_{F0}(x) = k_2^F\frac{\rho^2}{q_2}\mathrm{e}^{-\rho x} - k_3^F q_2\mathrm{e}^{-(2\rho+q_2)x},$$
$$d_{FF}^{N}(x) = k_1^F\mathrm{e}^{-2\rho x} + k_2^F\rho(1-\alpha_2)\mathrm{e}^{-\rho x} + k_3^F q_2(1-\alpha_2)\mathrm{e}^{-(2\rho+q_2)x},$$
$$d_{FM}^{N}(x) = -k_1^F\mathrm{e}^{-2\rho x} + k_2^F\rho\alpha_2\mathrm{e}^{-\rho x} + k_3^F q_2\alpha_2\mathrm{e}^{-(2\rho+q_2)x}.$$

To find the constant terms k_1^F, k_2^F, and k_3^F, we use the following initial conditions: $q_1(1-\alpha_1)d_{00} = d_{F0}(0)$, $\bar{q}_1^{FF}d_{0F} = d_{FF}^{N}(0)$, and $\bar{q}_1^{FM}d_{0M} = d_{FM}^{N}(0)$. The first condition implies that the stock $d_{F0}(0)$ consists of firms d_{00} finding a junior worker of type F, that is, $q_1(1-\alpha_1)d_{00}$. The second condition implies that the stock of firms $d_{FF}^{N}(0)$ consists of firms d_{0F} who find a junior worker of type F at rate $\bar{q}_1^{FF}$. The third condition implies that the stock of firms $d_{FM}^{N}(0)$ consists of firms d_{0M} who find a junior worker of type F at rate $\bar{q}_1^{FM}$. Exact expressions for k_1^F, k_2^F, and k_3^F are provided in the next part of Appendix A.

Note that in all three states $d_{F0}(x)$, $d_{FF}^{N}(x)$, $d_{FM}^{N}(x)$ female workers employed in junior positions remain inactive and accumulate experience till it reaches the minimum level $\bar{x}$ necessary for the senior position. If the senior position is free, the junior worker is immediately promoted, so the stock of firms $d_{F0}(\bar{x})$ is one of the entries into the stock d_{0F}. However, if the senior position is not vacant, then junior workers start searching and applying for senior positions in other firms. This means that stocks of firms $d_{FM}^{N}(\bar{x})$ and $d_{FF}^{N}(\bar{x})$ are the entries into d_{FM}^{S} and d_{FF}^{S}, respectively. This mechanism allows us to obtain the total stocks of firms d_{F0}, d_{FF}^{N}, and d_{FM}^{N} be integrating from $x=0$ till $x=\bar{x}$. This yields the following:

$$d_{F0} = \frac{k_2^F\rho}{q_2}\left(1-\mathrm{e}^{-\rho\bar{x}}\right) - \frac{k_3^F q_2}{2\rho+q_2}\left(1-\mathrm{e}^{-(2\rho+q_2)\bar{x}}\right),$$
$$d_{FF}^{N} = \frac{k_1^F}{2\rho}\left(1-\mathrm{e}^{-2\rho\bar{x}}\right) + k_2^F(1-\alpha_2)\left(1-\mathrm{e}^{-\rho\bar{x}}\right) + \frac{k_3^F q_2(1-\alpha_2)}{2\rho+q_2}\left(1-\mathrm{e}^{-(2\rho+q_2)\bar{x}}\right),$$
$$d_{FM}^{N} = -\frac{k_1^F}{2\rho}\left(1-\mathrm{e}^{-2\rho\bar{x}}\right) + k_2^F\alpha_2\left(1-\mathrm{e}^{-\rho\bar{x}}\right) + \frac{k_3^F q_2\alpha_2}{2\rho+q_2}\left(1-\mathrm{e}^{-(2\rho+q_2)\bar{x}}\right).$$

Next, we repeat our analysis with the stocks of firms $d_{M0}(x)$, $d_{MM}^{N}(x)$, and $d_{MF}^{N}(x)$, where there is a male worker employed in the junior position. This yields the following system of differential equations:

$$\partial d_{M0}(x)/\partial x = \rho\left(d_{MF}^{N}(x) + d_{MM}^{N}(x)\right) - (\rho+q_2)d_{M0}(x),$$
$$\partial d_{MM}^{N}(x)/\partial x = q_2\alpha_2 d_{M0}(x) - 2\rho d_{MM}^{N}(x),$$
$$\partial d_{MF}^{N}(x)/\partial x = q_2(1-\alpha_2)d_{M0}(x) - 2\rho d_{MF}^{N}(x).$$

Firms of the type $d_{M0}(x)$ are searching for a senior manager and find one at rate q_2. With probability α_2 the chosen candidate is a male worker, so the firm makes transition into the state $d_{MM}^{N}(x)$. Here, the junior employee is also a male worker with experience $x<\bar{x}$. With the counter-probability $1-\alpha_2$ the new senior manager is a female worker, so the firm makes a transition into the state $d_{MF}^{N}(x)$. The three eigenvalues of this system of differential equations are again -2ρ, $-\rho$, and $-(2\rho+q_2)$, so the general solution is

$$d_{M0}(x) = k_2^M\frac{\rho^2}{q_2}\mathrm{e}^{-\rho x} - k_3^M q_2\mathrm{e}^{-(2\rho+q_2)x},$$
$$d_{MM}^{N}(x) = k_1^M\mathrm{e}^{-2\rho x} + k_2^M\rho\alpha_2\mathrm{e}^{-\rho x} + k_3^M q_2\alpha_2\mathrm{e}^{-(2\rho+q_2)x},$$
$$d_{MF}^{N}(x) = -k_1^M\mathrm{e}^{-2\rho x} + k_2^M\rho(1-\alpha_2)\mathrm{e}^{-\rho x} + k_3^M q_2(1-\alpha_2)\mathrm{e}^{-(2\rho+q_2)x}.$$

To find the constant terms k_1^M, k_2^M, and k_3^M we use the following initial conditions: $q_1\alpha_1 d_{00} = d_{M0}(0)$, $\bar{q}_1^{MM}d_{0M} = d_{MM}^{N}(0)$, and $\bar{q}_1^{MF}d_{0F} = d_{MF}^{N}(0)$. The first condition implies that the stock $d_{F0}(0)$ consists of firms d_{00} finding a junior worker of type M, that is, $q_1\alpha_1 d_{00}$. The second condition implies that the stock of firms $d_{MM}^{N}(0)$ consists of firms d_{0M} who find a junior worker of type M at rate $\bar{q}_1^{MM}$. The third condition implies that the stock of firms

$d_{MF}^N(0)$ consists of firms d_{0F} who find a junior worker of type M at rate $\bar{q}_1^{MF}$. Exact expressions for k_1^M, k_2^M, and k_3^M are provided later in Appendix A. Finally, integrating variables $d_{M0}(x)$, $d_{MM}^N(x)$, and $d_{MF}^N(x)$ from $x = 0$ till $x = \bar{x}$ we get the following:

$$d_{M0} = \frac{k_2^M \rho}{q_2}\left(1 - e^{-\rho\bar{x}}\right) - \frac{k_3^M q_2}{2\rho + q_2}\left(1 - e^{-(2\rho+q_2)\bar{x}}\right),$$
$$d_{MM}^N = \frac{k_1^M}{2\rho}\left(1 - e^{-2\rho\bar{x}}\right) + k_2^M\alpha_2\left(1 - e^{-\rho\bar{x}}\right) + \frac{k_3^M q_2\alpha_2}{2\rho + q_2}\left(1 - e^{-(2\rho+q_2)\bar{x}}\right),$$
$$d_{MF}^N = -\frac{k_1^M}{2\rho}\left(1 - e^{-2\rho\bar{x}}\right) + k_2^M(1-\alpha_2)\left(1 - e^{-\rho\bar{x}}\right) + \frac{k_3^M q_2(1-\alpha_2)}{2\rho + q_2}\left(1 - e^{-(2\rho+q_2)\bar{x}}\right).$$

To close the model, consider the stocks of firms d_{FF}^S, d_{FM}^S, d_{MM}^S, and d_{MF}^S. In all these firms the junior worker has experience more than $\bar{x}$ and is already searching for a senior position. We already know that $d_{FM}^N(\bar{x})$ and $d_{FF}^N(\bar{x})$ are the only entries into d_{FM}^S and d_{FF}^S, respectively. In a similar way, variables $d_{MF}^N(\bar{x})$ and $d_{MM}^N(\bar{x})$ are the only entries into d_{MF}^S and d_{MM}^S. There are three possible events that can alter the state of these firms. Either one of the two employees is dismissed from the job at rate ρ, or the junior worker finds another employment as a senior manager and quits the firm at rate λ_2. Thus we get

$$\dot{d}_{FF}^S = d_{FF}^N(\bar{x}) - (2\rho + \lambda_2)d_{FF}^S, \quad \dot{d}_{MM}^S = d_{MM}^N(\bar{x}) - (2\rho + \lambda_2)d_{MM}^S,$$
$$\dot{d}_{FM}^S = d_{FM}^N(\bar{x}) - (2\rho + \lambda_2)d_{FM}^S, \quad \dot{d}_{MF}^S = d_{MF}^N(\bar{x}) - (2\rho + \lambda_2)d_{MF}^S.$$

Finally, consider the stock of firms d_{0F}. We already know that $d_{F0}(\bar{x})$ is one of the entries into d_{0F}, because junior workers are promoted to senior positions upon reaching experience $\bar{x}$. Also the firms d_{FM}^S and d_{FF}^S promote their female junior employees to senior positions in the event when the senior manager is dismissed, which happens at rate ρ. So the inflow of firms into d_{0F}, which is *due to immediate or delayed promotions* is given by $d_{F0}(\bar{x}) + \rho\left(d_{FM}^S + d_{FF}^S\right)$.

However, also empty firms d_{00} are searching for senior managers and find one at rate q_2. With probability α_2 the new manager is a male worker, so the firm d_{00} becomes d_{0M}, but with probability $1 - \alpha_2$ the new manager is a female worker, so the firm enters the state d_{0F}. Hence the entry of firms into state d_{0F}, which is *due to outside hiring*, is equal to $q_2(1 - \alpha_2)d_{00}$.

In addition, we know that any of the firms d_{FF}^N, d_{MF}^N, d_{FF}^S, and d_{MF}^S may lose their junior employees at rate ρ *due to the exogenous exit* and therefore enter the state d_{0F} as the only remaining worker in these firms is a senior female manager. So the next entry is $\rho\left(d_{FF}^N + d_{MF}^N + d_{FF}^S + d_{MF}^S\right)$. Moreover, it can also happen that junior employees in firms d_{FF}^S and d_{MF}^S separate from their employers *due to quitting* and taking employment in other firms, which happens at rate λ_2. This yields the last entry into the state d_{0F}, namely, $\lambda_2\left(d_{FF}^S + d_{MF}^S\right)$. Summarizing, we find that the entry of firms into the state d_{0F} is given by $d_{F0}(\bar{x}) + \rho\left(d_{FM}^S + d_{FF}^S\right) + q_2(1 - \alpha_2)d_{00} + \rho\left(d_{FF}^N + d_{MF}^N + d_{FF}^S + d_{MF}^S\right) + \lambda_2\left(d_{FF}^S + d_{MF}^S\right)$.

Next, we investigate the exits of firms from the state d_{0F}. On the one hand, senior managers may exit the market at rate ρ rendering the firm empty (d_{00}). On the other hand, firms may fill their open junior position with a female worker, which happens at rate $\bar{q}_1^{FF}$, or with a male worker, which happens at rate $\bar{q}_1^{MF}$. Note, that these rates include the possibility of formal and referral hiring to junior positions. So the exit of firms from the state d_{0F} is given by $\left(\rho + \bar{q}_1^{FF} + \bar{q}_1^{MF}\right)d_{0F}$. This yields the following differential equations for d_{0F} and d_{0M}:

$$\begin{aligned}
\dot{d}_{0F} = {} & d_{F0}(\bar{x}) + \rho\left(d_{FM}^S + d_{FF}^S\right) + q_2(1 - \alpha_2)d_{00} \\
& + \rho\left(d_{FF}^N + d_{MF}^N + d_{FF}^S + d_{MF}^S\right) + \lambda_2\left(d_{FF}^S + d_{MF}^S\right) \\
& - \rho d_{0F} - \left(\bar{q}_1^{FF} + \bar{q}_1^{MF}\right)d_{0F}, \\
\dot{d}_{0M} = {} & d_{M0}(\bar{x}) + \rho\left(d_{FM}^S + d_{MM}^S\right) + q_2\alpha_2 d_{00} + \rho\left(d_{MM}^N + d_{FM}^N + d_{MM}^S + d_{FM}^S\right) \\
& + \lambda_2\left(d_{MM}^S + d_{FM}^S\right) - \rho d_{0M} - \left(\bar{q}_1^{MM} + \bar{q}_1^{FM}\right)d_{0M}.
\end{aligned}$$

We restrict our attention to steady-state equilibria, so we set $\dot{d}_{00} = \dot{d}^{S}_{MF} = \dot{d}^{S}_{FM} = \dot{d}^{S}_{MM} = \dot{d}^{S}_{FF} = \dot{d}_{0F} = \dot{d}_{0M} = 0$.

A.4 | Solving for the differential equations

We consider the system of differential equations for female workers $\dot{d}_{F0}$, $\dot{d}^{N}_{FF}$, and $\dot{d}^{N}_{FM}$, first. The coefficient matrix and the characteristic equation for r are given by

$$\begin{pmatrix} -(\rho + q_2) & \rho & \rho \\ q_2(1-\alpha_2) & -2\rho & 0 \\ q_2\alpha_2 & 0 & -2\rho \end{pmatrix},$$

$$(-\rho - q_2 - r)(-2\rho - r)(-2\rho - r) - \rho q_2(1-\alpha_2)(-2\rho - r) - \rho q_2\alpha_2(-2\rho - r) = 0.$$

The first eigenvalue is given by $r_1 = -2\rho$. The remaining quadratic term is

$$r^2 + r(q_2 + 3\rho) + 2\rho^2 + 2\rho q_2 - \rho q_2 = 0.$$

The discriminant of this quadratic equation is $(q_2 + \rho)^2$, so the second and the third eigenvalues are given by $r_2 = -\rho$, $r_3 = -(q_2 + 2\rho)$. The corresponding three eigenvectors are given by

$$\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}\begin{pmatrix} \frac{\rho^2}{q_2} \\ \rho(1-\alpha_2) \\ \rho\alpha_2 \end{pmatrix}\begin{pmatrix} -q_2 \\ q_2(1-\alpha_2) \\ q_2\alpha_2 \end{pmatrix}.$$

The general solution is given by

$$\begin{aligned} d_{F0}(x) &= k_2^F\frac{\rho^2}{q_2}\mathrm{e}^{-\rho x} - k_3^F q_2\mathrm{e}^{-(2\rho+q_2)x}, \\ d^N_{FF}(x) &= k_1^F\mathrm{e}^{-2\rho x} + k_2^F\rho(1-\alpha_2)\mathrm{e}^{-\rho x} + k_3^F q_2(1-\alpha_2)\mathrm{e}^{-(2\rho+q_2)x}, \\ d^N_{FM}(x) &= -k_1^F\mathrm{e}^{-2\rho x} + k_2^F\rho\alpha_2\mathrm{e}^{-\rho x} + k_3^F q_2\alpha_2\mathrm{e}^{-(2\rho+q_2)x}. \end{aligned}$$

The three constant terms k_1^F, k_2^F, and k_3^F can be found from the following initial conditions: $q_1(1-\alpha_1)d_{00} = d_{F0}(0)$, $\bar{q}_1^{FF}d_{0F} = d^N_{FF}(0)$, and $\bar{q}_1^{FM}d_{0M} = d^N_{FM}(0)$:

$$\begin{aligned} d_{F0}(0) &= k_2^F\frac{\rho^2}{q_2} - k_3^F q_2 = q_1(1-\alpha_1)d_{00}, \\ d^N_{FF}(0) &= k_1^F + k_2^F\rho(1-\alpha_2) + k_3^F q_2(1-\alpha_2) = \bar{q}_1^{FF}d_{0F}, \\ d^N_{FM}(0) &= -k_1^F + k_2^F\rho\alpha_2 + k_3^F q_2\alpha_2 = \bar{q}_1^{FM}d_{0M}. \end{aligned}$$

Adding the latter two equations we can express $k_3^F q_2 = \bar{q}_1^{FF}d_{0F} + \bar{q}_1^{FM}d_{0M} - k_2^F\rho$. Then inserting it into the first equation we get

$$\begin{aligned} k_2^F &= \frac{q_2}{\rho(\rho+q_2)}\left[q_1(1-\alpha_1)d_{00} + \bar{q}_1^{FF}d_{0F} + \bar{q}_1^{FM}d_{0M}\right], \\ k_3^F &= \frac{\rho}{q_2(\rho+q_2)}\left[\bar{q}_1^{FF}d_{0F} + \bar{q}_1^{FM}d_{0M}\right] - \frac{q_1(1-\alpha_1)}{\rho+q_2}d_{00}, \\ k_1^F &= \alpha_2\bar{q}_1^{FF}d_{0F} - (1-\alpha_2)\bar{q}_1^{FM}d_{0M}. \end{aligned}$$

Integrating $d_{F0}(x)$ over x in the interval $[0\ldots\bar{x}]$ we get the total stock of firms d_{F0}:

$$d_{F0} = \int_0^{\bar{x}} \left[k_2^F \frac{\rho^2}{q_2} \mathrm{e}^{-\rho x} - k_3^F q_2 \mathrm{e}^{-(2\rho+q_2)x} \right] dx = \frac{k_2^F \rho}{q_2} \left(1 - \mathrm{e}^{-\rho \bar{x}}\right) - \frac{k_3^F q_2}{2\rho + q_2} \left(1 - \mathrm{e}^{-(2\rho+q_2)\bar{x}}\right).$$

Integrating $d_{FF}^N(x)$ over x in the interval $[0 \dots \bar{x}]$ we get the total stock of firms d_{FF}^N:

$$\begin{aligned} d_{FF}^N &= \int_0^{\bar{x}} \left[k_1^F \mathrm{e}^{-2\rho x} + k_2^F \rho (1-\alpha_2) \mathrm{e}^{-\rho x} + k_3^F q_2 (1-\alpha_2) \mathrm{e}^{-(2\rho+q_2)x} \right] dx \\ &= \frac{k_1^F}{2\rho} \left(1 - \mathrm{e}^{-2\rho\bar{x}}\right) + k_2^F (1-\alpha_2)\left(1 - \mathrm{e}^{-\rho\bar{x}}\right) + \frac{k_3^F q_2 (1-\alpha_2)}{2\rho + q_2} \left(1 - \mathrm{e}^{-(2\rho+q_2)\bar{x}}\right). \end{aligned}$$

Integrating $d_{FM}^N(x)$ over x in the interval $[0 \dots \bar{x}]$ we get the total stock of firms d_{FM}^N:

$$\begin{aligned} d_{FM}^N &= \int_0^{\bar{x}} \left[-k_1^F \mathrm{e}^{-2\rho x} + k_2^F \rho \alpha_2 \mathrm{e}^{-\rho x} + k_3^F q_2 \alpha_2 \mathrm{e}^{-(2\rho+q_2)x} \right] dx \\ &= -\frac{k_1^F}{2\rho} \left(1 - \mathrm{e}^{-2\rho\bar{x}}\right) + k_2^F \alpha_2 \left(1 - \mathrm{e}^{-\rho\bar{x}}\right) + \frac{k_3^F q_2 \alpha_2}{2\rho + q_2} \left(1 - \mathrm{e}^{-(2\rho+q_2)\bar{x}}\right). \end{aligned}$$

Next we consider the system of differential equations for male workers $\dot{d}_{M0}$, $\dot{d}_{MM}^N$, and $\dot{d}_{MF}^N$. The coefficient matrix is given by

$$\begin{pmatrix} -(\rho + q_2) & \rho & \rho \\ q_2 \alpha_2 & -2\rho & 0 \\ q_2(1-\alpha_2) & 0 & -2\rho \end{pmatrix}.$$

The eigenvalues are the same, but the eigenvectors are slightly different and given by

$$\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \begin{pmatrix} \frac{\rho^2}{q_2} \\ \rho\alpha_2 \\ \rho(1-\alpha_2) \end{pmatrix} \begin{pmatrix} -q_2 \\ q_2\alpha_2 \\ q_2(1-\alpha_2) \end{pmatrix}.$$

So the general solution becomes

$$\begin{aligned} d_{M0}(x) &= k_2^M \frac{\rho^2}{q_2} \mathrm{e}^{-\rho x} - k_3^M q_2 \mathrm{e}^{-(2\rho+q_2)x}, \\ d_{MM}^N(x) &= k_1^M \mathrm{e}^{-2\rho x} + k_2^M \rho\alpha_2 \mathrm{e}^{-\rho x} + k_3^M q_2 \alpha_2 \mathrm{e}^{-(2\rho+q_2)x}, \\ d_{MF}^N(x) &= -k_1^M \mathrm{e}^{-2\rho x} + k_2^M \rho(1-\alpha_2) \mathrm{e}^{-\rho x} + k_3^M q_2 (1-\alpha_2) \mathrm{e}^{-(2\rho+q_2)x}. \end{aligned}$$

The initial conditions are: $q_1 \alpha_1 d_{00} = d_{M0}(0)$, $\bar{q}_1^{MM} d_{0M} = d_{MM}^N(0)$, and $\bar{q}_1^{MF} d_{0F} = d_{MF}^N(0)$. So we find the three constant terms k_1^M, k_2^M, and k_3^M from the following system of equations:

$$\begin{aligned} d_{M0}(0) &= k_2^M \frac{\rho^2}{q_2} - k_3^M q_2 = q_1 \alpha_1 d_{00}, \\ d_{MM}^N(0) &= k_1^M + k_2^M \rho\alpha_2 + k_3^M q_2 \alpha_2 = \bar{q}_1^{MM} d_{0M}, \\ d_{MF}^N(0) &= -k_1^M + k_2^M \rho(1-\alpha_2) + k_3^M q_2 (1-\alpha_2) = \bar{q}_1^{MF} d_{0F}. \end{aligned}$$

Adding the latter two equations we can express $k_3^M q_2 = \bar{q}_1^{MM} d_{0M} + \bar{q}_1^{MF} d_{0F} - k_2^M \rho$. Then inserting it into the first equation we get

$$k_2^M = \frac{q_2}{\rho(\rho+q_2)}\left[q_1\alpha_1 d_{00} + \bar{q}_1^{MM} d_{0M} + \bar{q}_1^{MF} d_{0F}\right],$$
$$k_3^M = \frac{\rho}{q_2(\rho+q_2)}\left[\bar{q}_1^{MM} d_{0M} + \bar{q}_1^{MF} d_{0F}\right] - \frac{q_1\alpha_1}{\rho+q_2} d_{00},$$
$$k_1^M = (1-\alpha_2)\bar{q}_1^{MM} d_{0M} - \alpha_2 \bar{q}_1^{MF} d_{0F}.$$