Essays on Options and Portfolio Management

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Abstract

This dissertation investigates several research topics on options. The first article explores the properties of the dividend variance and skewness risk premium and whether dividend futures excess returns are predictable by them. The second article examines the usefulness of model-free option implied upside and downside volatilities to enhance the performance of portfolios consisting of US large cap stocks. The third article shows how an investor should use options and the forward given that she either thinks the level, slope or convexity of the respective implied variance curve should be higher or lower than currently priced.

Kurzzusammenfassung

In dieser Dissertation werden mehrere Forschungsthemen im Bereich Optionen untersucht. Der erste Artikel erforscht die Eigenschaften der Dividenden-Varianz- und Schiefe-Risikoprämie und ob sie Futures-Überrenditen vorhersagen können. Der zweite Aufsatz untersucht die Nützlichkeit von modellfreien options-impliziten Aufwärts- und Abwärtsvolatilitäten zur Verbesserung der Performance von Portfolios, die aus US-amerikanischen Large-Cap-Aktien bestehen. Der dritte Artikel zeigt, wie eine Investorin Optionen und den Forward nutzen sollte, wenn sie entweder das Niveau, die Steigung oder die Konvexität der jeweiligen impliziten Varianzkurve höher oder niedriger einschätzt als aktuell eingepreist.

To my beloved wife and son.

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Chapter 1

Synopsis

1.1 Motivation

This dissertation contains three chapters, each of which contains independent research papers that examine various topics related to option markets. More specifically, the unifying element of all three studies is the use of forward-looking option-implied information, either for quantifying time-varying risk compensations to predict movements in the options' underlying, for portfolio optimization or option trading purposes.

Undoubtedly, the Black Merton Scholes (BMS) implied volatility is the most popular forward-looking measure derived from option prices among practitioners and academic researchers. Its very concept builds on the BMS European option pricing model of Black and Scholes (1973) and Merton (1973): it is the unique volatility parameter that can be obtained inverting the BMS option pricing formula using the observed price of a European option. The model assumes that the underlying of the option follows a geometric Brownian motion with constant volatility. This implies that for every (European) option written on the same underlying, regardless of the strike and maturity of the respective option, the BMS implied volatility should be the same. Rubinstein (1994) finds that this assumption was approximately true before the crash in October 1987 but largely changed afterwards with S&P 500 implied volatilities for options with different strikes and fixed maturities nowadays forming a smirk pattern with a negative slope[1], where out-of-the-money (in-the-money) put (call) options have higher implied volatilities than in-the-money (out-of-the-money) put (call) options.

Option prices also offer the possibility to obtain the entire (forward-looking) risk-neutral density (RND) of the respective underlying in a model-free way. Building on the findings of Ross (1976), Breeden and Litzenberger (1978) famously show that one can ob-

[1] The same shape is also documented by Foresi and Wu (2005) for implied volatilities of options on other major world equity indices.

tain the full RND of the underlying by taking the second partial derivative of the European call option price function with respect to the strike. To generate a smooth density, their methodology requires a continuum of options prices. However, since only a limited range of discrete strikes are available in the real world, authors such as Jackwerth and Rubinstein (1996), Aït-Sahalia and Lo (1998), Bliss and Panigirtzoglou (2002) each suggest different approaches to obtain smooth RNDs nevertheless.

Carr and Madan (1998) use the approach of Breeden and Litzenberger (1978) and demonstrate how to quantify the risk-neutral expected value of future realized variance, i.e. the fixed variance swap rate, with option prices. In contrast to the BMS implied variance, they suggest to obtain this implied variance measure in a model-free manner. Furthermore, the measure is derived from the entire set of option prices with the same maturity, rather than from the price of a single option. The Chicago Board of Options Exchange (CBOE) adopted the methodology of Carr and Madan (1998) and changed the calculation of the CBOE Volatility index (VIX index) accordingly in 2003. From its introduction in 1993 until 2003, the VIX index was computed on the basis of BMS implied volatilities of four put and four call options with near-the-money strikes[2]. After the new methodology was implemented, the CBOE launched futures and options on the new VIX index. Building upon the work of Carr and Madan (1998) and Bakshi and Madan (2000), Bakshi et al. (2003) show how to measure the risk-neutral moments of the RND while Neuberger (2012) and Kozhan et al. (2013) utilize their results to estimate the risk-neutral expected value of the future realized third moment, i.e. the fixed skew swap rate.

I use the results of the mentioned research articles on option-implied information to formulate the following main research questions relevant for this dissertation:

- Is dividend variance and skewness risk priced and do they predict returns of dividend futures?
- Can implied upside and downside volatility estimates help to improve the out-of-sample performance of stock portfolios?
- With just a portfolio of options with three different strikes, how can one trade the level, slope and convexity of a BMS implied variance curve?

[2]The CBOE also switched from using options on the S&P 100 to options on the broader S&P 500 index. For a more detailed discussion about the differences of both methodologies, see Carr and Wu (2006).

1.2 Dissertation structure

The main part of my dissertation comprises of three articles. Each article aims to answer one research question formulated in Section 1.1. Two are published articles (Chapter 2 and 4) and one is an unpublished article (Chapter 3).

In Chapter 2, titled *Dividend Predictability and Higher Moment Risk Premia*, I study the dividend variance and skewness risk premium, defined as the difference between the realized variance of returns and the model-free implied variance of returns, and defined as the difference between the realized third moment returns and the implied third moment of returns, respectively. To quantify them, I utilize EURO STOXX 50 Dividend Points index futures and option data and report that both are statistically significant and therefore priced risk factors. To examine whether they are also unique, I similarly construct the variables for the EURO STOXX 50 index and find that the dividend variance and skewness risk premium are only weakly related to them or the Fama-French 5 (Fama and French, 2015) factors (market, size, value, profitability and investment) and the momentum factor. I also evaluate the prediction quality of both moment risk premia to predict movements in dividend futures with constant maturities of one to four years in an out-of-sample setup. Three regression models are used for this purpose. The first two models each rely on one of the two variables while the third model relies on both variables to create forecasts of future dividend excess returns. The results demonstrate that these models mostly outperform a benchmark model that uses only past dividend excess returns data for contracts with maturities of three and four years. The out-of-sample performance is almost always improved when both moment risk premia are used jointly.

In Chapter 3, titled *Portfolio Optimization with Implied Good and Bad Volatility*, I estimate option-implied semivolatilities to construct portfolios consisting of S&P 500 stocks. The two main portfolio strategies proposed there are build by minimizing the risk based on a semicovariance matrix that is computed with implied upside (downside) volatilities and historical upside (downside) correlations. The performances of both strategies are compared to others that instead calculate the respective semicovariance matrix based on historical data only, strategies that use the symmetric covariance matrix estimated with historical data, implied data or a combination of both, and the naïve $1/N$ strategy. The main out-of-sample results indicate that portfolio optimization with implied downside volatilities and historical downside correlations beats the competing strategies by generating the highest Sharpe and Sortino ratio and lowest upside, downside and symmetric portfolio volatility. I then change the estimation window for the historical data from one year, used for the main results, to two years, and half a year, let each portfolio strategy select from smaller stock universes consisting of either 100, 200, 300, and

400 randomly picked stocks, and check whether the results materially differ in crisis and non-crisis periods and conclude that the portfolio strategy is still successful generating lower downside and symmetric volatility overall. Can some risk factors help to explain the excess returns of the portfolio strategy? I use multifactor regressions to examine that by using innovations in the implied volatility, skewness or kurtosis of the market, as well as the Fama-French 5 (Fama and French, 2015) plus momentum and betting against beta (Frazzini and Pedersen, 2014) factors as explanatory variables and find that in each setup, a sizable portion of the excess returns remains unexplained.

In Chapter 4, titled *Vol, Skew and Smile Trading*, we investigate how a portfolio of options can be utilized to trade the (BMS) implied variance curve level, slope and convexity. We assume that the forward and the implied variance curve are risk-neutral stochastic processes and show that the mean gain rate of a portfolio consisting of out-of-the-money put options with strike K_p, at-the-money put and call options with strike K_a and out-of-the-money call options with strike K_c is driven by the risk-neutral dynamics of the instantaneous variance of the log forward, the instantaneous variance of the log implied volatility curve and the instantaneous covariation (between the log forward and the log implied volatility curve) process. We require that a vol, skew and smile trade isolates the dependence on two of the three latter process to be considered as such. We demonstrate how certain positions in long (short) at-the-money puts and calls correspond to a long (short) vol trade, positions in out-of-the-money short (long) puts and long (short) calls correspond to a long (short) skew trade and positions in long (short) out-of-the-money puts and calls combined with short (long) at-the-money puts and calls correspond to a long (short) smile trade. We show how the positions need to be modified since the instantaneous gain of the respective option portfolios would still be exposed to delta and cash vega risks. Our empirical exercise using S&P 500 index and option data reveals that the average returns of the long vol and smile trade are negative and whereas the average returns of the long skew trade are positive. We calculate the returns of each trade with options held for one exchange day and constant maturities ranging from one to twelve months at the time when an option position is closed and report, among other findings, that the term structure of Sharpe ratios for short vol, long skew and short smile trades is downward sloping.

Chapter 2

Dividend Predictability and Higher Moment Risk Premia

Chapter 2 has been published as a journal article:

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Chapter 3

Portfolio Optimization with Implied Good and Bad Volatility

Chapter 3 is a working paper and has not been published yet.

Abstract

This paper investigates the performance of portfolios constructed using forward-looking option-implied semivolatilities. Using data from 1996 to 2022, I show that portfolio optimization with implied bad volatilities generates significantly lower out-of-sample bad and symmetric portfolio volatilities than comparable portfolio strategies, including $1/N$. The results survive a series of robustness checks. I find that the excess returns of portfolios constructed with either implied good or bad volatility yield significantly positive Fama-French 5 plus momentum factor model alphas.

3.1 Introduction

Even though the mean-variance portfolio selection framework of Markowitz (1952) is generally recognized as the cornerstone of modern portfolio theory, determining its two input parameters, the vector of expected returns and the covariance matrix of returns, remains a challenging task. However, since covariance estimates are usually more predictable than expected return estimates (Merton, 1980; Jorion, 1985) and mean-variance efficient portfolio weights are known to be highly sensitive to changes in the expected return estimates (Best and Grauer, 1991), growing attention has been paid in the last decades to the only portfolio on the mean-variance efficient frontier that does not depend on the expected returns as an input parameter to determine its weights: the minimum variance portfolio.

Instead of following the typical approach and using historical volatilities and correlations to minimize the portfolio variance, in this article, I use implied upside (good) and downside (bad) volatilities and historical semicorrelations to construct what I label as minimum good and bad variance portfolios and examine their out-of-sample performance against a set of benchmark strategies. To calculate the weights of these portfolios, I apply the results of Bollerslev et al. (2020) who decompose the covariance matrix into three semicovariance components: a positive, a negative and a mixed semicovariance matrix. I show how to integrate good and bad implied volatilities, which are obtained by computing the risk-neutral price of a contract that pays the realized good and bad variance, into the positive and negative semicovariance matrix, respectively. The corresponding portfolio performances are compared against the portfolio performances of a set of other portfolio strategies: $1/N$, minimum variance portfolios based on the sample covariance matrix, a covariance matrix that uses shrinkage on the sample covariance matrix, a covariance matrix consisting of implied volatilities and historical correlations, and a covariance matrix consisting of implied volatilities and correlations as well as the minimum good and bad variance portfolios constructed with historical return data only.

For the empirical exercise I use S&P 500 stock and option data and present evidence that the minimum bad variance portfolio constructed with implied bad volatilities and historical semicorrelations produces the lowest out-of-sample portfolio volatility, good volatility, bad volatility and highest out-of-sample Sharpe and Sortino ratio than the other strategies. To check for robustness, I vary the default estimation window, produce random portfolios and conduct a subperiod analysis. In all exercises, the most consistent results are obtained for the performance in terms of the out-of-sample portfolio's bad volatility. The strategy is only outperformed by a tiny margin for the case when 100 (but not when 200, 300 or 400) stocks are randomly selected every month. Even though the proposed strategy shows satisfactory out-of-sample properties, outperformance comes at a price since it is also among the portfolio strategies that generate the highest portfolio turnover and concentration. However, I show that with reasonable transaction costs, the Sharpe and Sortino ratio remain comparatively high. To analyze the source of the outperformance, I examine each portfolio strategy's excess return sensitivity towards common equity risk factors and market moment risk factors and find that the alpha of the strategy is high and statistically significant. It is also the portfolio strategy that produces the lowest market beta and the lowest absolute sensitivity towards changes in the markets' implied volatility.

This paper extends the sparse literature on portfolio optimization with option-implied data. DeMiguel et al. (2013) find that using a combination of historical correlations and option-implied volatilities as inputs, helps to significantly reduce the out-of-sample portfolio volatility of a minimum variance portfolio significantly. The empirical results I provide are similar in most cases. Furthermore, they document worse results for a minimum variance portfolio constructed with historical volatilities and implied correlations. Inline with their results, I also find that both, using option-implied rather than historical estimators of volatility and correlation increases portfolio turnover. Kempf et al. (2014) report that a minimum variance portfolio constructed with option-implied data beats other minimum variance portfolios with other estimators, $1/N$ and a capital weighted benchmark strategy out-of-sample. Analogous to the findings of DeMiguel et al. (2013), they show that a minimum variance portfolio based on a combination of implied correlations and historical volatilities underperforms but using implied rather than historical volatilities decreases out-of-sample portfolio volatilities significantly. In contrast to these studies, however, I additionally examine the performance of portfolios that are optimized with implied semivolatility estimates and historical semicorrelations to minimize the out-of-sample risk on a portfolio level. In this regard, the findings I report indicate that portfolios that are constructed with implied bad volatilities and historical semicorrelations deliver better and more robust results than portfolios that are build with implied and historical symmetric volatility and correlation estimates.

This paper also extends the literature on semivolatility and semicovariance. Feunou et al. (2018) evaluate the predictive power of the difference between the implied and realized variance (the variance risk premium) and their two components, i.e., the difference between good and bad implied and realized variance (the good and bad variance risk premium) of the S&P 500 to predict subsequent excess returns of the index. Their results suggest that the reported statistically significant predictive power of the variance risk premium to changes in S&P 500 excess returns by Bollerslev et al. (2009) mainly results from the bad component of the variance risk premium. In a similar exercise, Kilic and Shaliastovich (2019) find that using the S&P 500 good and bad variance risk premium jointly as predictor variables in a multivariate setting for S&P 500 excess returns produces better prediction results than using each variable or the total variance risk premium individually in a univariate setting. Bollerslev et al. (2022) decompose the standard market beta into four semibetas utilizing the results of Bollerslev et al. (2020) and document that the risk premium associated with the semibeta computed with covariances of negative stock and negative market returns is higher than the risk premium associated with the standard market beta. I contribute to this strand of literature by showing that semivolatilities and semicovariances can also be used beneficially for portfolio optimization purposes.

The remainder of this paper proceeds as follows. In Section 3.2 I discuss the data used in the empirical analysis. In Section 3.3, I describe how model-free implied good and bad volatilities are estimated and used to construct portfolios. In Section 3.4, I assess the empirical performance of the portfolio strategies and Section 3.5 concludes.

3.2 Data

For the empirical part of the study, I focus on the S&P 500 Index and its constituents. The main sample period of interest is January 1996 to October 2022. To calculate the implied measures of the index and the stocks with a 30-day fixed maturity, I obtain end-of-day option, forward and discount rate data from the IvyDB OptionMetrics database.

The Volatility_Surface file of the database includes kernel regression interpolated implied volatilities of put and call options for fixed maturities from 10 to 730 days and fixed absolute deltas from 0.1 to 0.9. While S&P 500 Index options are European style, stock options are American style. OptionMetrics uses a proprietary model that is based on the Cox-Ross-Rubinstein binomial tree model to account for the early exercise premia of American style options to create implied volatility surfaces. The implied moment estimates in this study are derived using implied volatilities of out-of-the-money (OTM) call options with strikes above the forward and of OTM put options with strikes below or equal to the forward with a fixed maturity of 30 days. For the OTM stock options, the impact of this premia on the implied volatility estimate is known to be negligible, as reported by Bakshi et al. (2003).

For each stock and the index, I use linear interpolation to estimate the 30-day forward prices based on data from the Forward_Price file. These forward prices are calculated using projected dividends. I also use the Zero_Curve file that contains discount rates that are derived from LIBOR rates and Eurodollar futures prices, which are used for this study to compute the target 30-day discount rate through linear interpolation too.

I also rely on constituents and end-of-day underlying price data which are derived from Bloomberg to compute historical covariances. The beginning of the main sample aligns with the beginning of the OptionMetrics data. In order to compute the optimal weights of the portfolio strategies using the historical covariance with estimation windows of up to 2 years directly at sample start in January 1996, the price data start in January 1994. On average, the number of S&P 500 constituents for which option and price data are available is 488.

3.3 Methodology

In this section, I first describe how implied good and bad volatilities are derived and then show how they are used to form minimum variance portfolios. Finally, I introduce the benchmark strategies against with which they are compared in the empirical part.

3.3.1 Implied Good and Bad Volatility

This study uses the concept of model-free implied (semi-) variance estimates to form portfolios. Based on the seminal work of Breeden and Litzenberger (1978), Carr and Madan (1998) show that an arbitrary twice differentiable payoff function $g(F_T)$ written on the underlying forward price process F that pays at a future time T can be statically replicated using the underlying F, a continuum of OTM European put P and call options C that expire at T, and unit face value bonds B that expire at T. They and Bakshi and Madan (2000) show that under the risk-neutral measure $\mathbb{Q}$, the expectation at current time $t < T$ for the payoff at maturity T is then given by:

$$\mathbb{E}_t^{\mathbb{Q}}[g(F_{T,T})] = g(F_{t,T}) + \frac{1}{B_{t,T}} \left[\int_0^{F_{t,T}} g''(K) P_{t,T}(K) dK + \int_{F_{t,T}}^{\infty} g''(K) C_{t,T}(K) dK \right], \quad (3.1)$$

where K denotes the strike price. The model-free implied variance IV is derived based on a contract that pays the realized variance $RV_{t,T}$ observed from time t to maturity T and is defined as:

$$RV_{t,T} = \int_t^T \sigma_s^2 ds. \quad (3.2)$$

Based on the relationship in (3.1), Carr and Madan (1998) and Britten-Jones and Neuberger (2000) show that at time t, the risk-neutral expected value of such a contract, i.e. IV, can be derived by:

$$IV_{t,T} = \mathbb{E}_t^{\mathbb{Q}}[RV_{t,T}] = \frac{2}{B_{t,T}} \left[\int_0^{F_{t,T}} \frac{P_{t,T}(K)}{K^2} dK + \int_{F_{t,T}}^{\infty} \frac{C_{t,T}(K)}{K^2} dK \right]. \quad (3.3)$$

The calculation of the widely followed VIX Index is based on an annualized, truncated, and discretized formulation of (3.3).

Using the same logic as shown before, to obtain implied good and bad variance estimates, it is necessary to first define their realized counterparts. Similar to Barndorff-Nielsen et al. (2010), I decompose $RV_{t,T}$ as follows:

$$RV_{t,T}^b = \int_t^T \sigma_s^2 \mathbb{I}_{(F_{s,T} \leq F_{t,T})} ds, \quad (3.4)$$

$$RV_{t,T}^g = \int_t^T \sigma_s^2 \mathbb{I}_{(F_{s,T} > F_{t,T})} ds, \quad (3.5)$$

where $RV^b_{t,T}$ ($RV^g_{t,T}$) is the realized variance of negative (positive) returns, denoted as realized bad (good) variance[1] and $\mathbb{I}$ is the indicator function taking the value of 1 if the argument in the subscript is true. By construction, the sum of both variance measures yields $RV_{t,T}$:

$$RV_{t,T} = RV^b_{t,T} + RV^g_{t,T}. \tag{3.6}$$

Following Andersen and Bondarenko (2010) and Feunou et al. (2018), the risk-neutral estimate of $RV^b_{t,T}$ is calculated with OTM put option prices whereas the risk-neutral estimate $RV^g_{t,T}$ is calculated using OTM call option prices[2]:

$$IV^b_{t,T} = \mathbb{E}^{\mathbb{Q}}_t[RV^b_{t,T}] = \frac{2}{B_{t,T}} \int_0^{F_{t,T}} \frac{P_{t,T}(K)}{K^2} dK, \tag{3.7}$$

$$IV^g_{t,T} = \mathbb{E}^{\mathbb{Q}}_t[RV^g_{t,T}] = \frac{2}{B_{t,T}} \int_{F_{t,T}}^{\infty} \frac{C_{t,T}(K)}{K^2} dK, \tag{3.8}$$

where analogously to (3.6):

$$IV_{t,T} = IV^b_{t,T} + IV^g_{t,T}. \tag{3.9}$$

To evaluate the integrals (3.7) and (3.8), I proceed as follows: First, I obtain, for the range $F_\tau \pm 8$ times the average implied volatility, 2,000 equidistant strikes by interpolating linearly the implied volatilities estimates of the Volatility_Surface file of OptionMetrics as a function of strike. Second, for strikes outside of the range of the file, I extrapolate using the respective left and right boundaries. Third, a total of 2,000 implied volatilities are then used to calculate a fine grid Black-Scholes-Merton prices for OTM put and call options. Lastly, the integrals are calculated using the trapezoidal rule. A similar procedure to calculate the integrals is implemented by, among others, Jiang and Tian (2005) and Carr and Wu (2009).

3.3.2 Portfolio Strategies

Main Strategy

I consider the minimum variance portfolio optimization strategy to allocate wealth across risky assets with implied good and bad volatilities. The optimization problem does not

[1]Some studies refer to both measures as realized downside and upside variance.

[2]The concept of implied good and bad variance relies on the theoretical foundation of the corridor variance contract developed by Carr and Madan (1998), which is a generalized version of the variance contract.

rely on often noisy expected return estimates. Given a population covariance matrix of asset returns C with dimension $N \times N$, the short-sale constrained[3] minimum variance portfolio optimization problem is given by:

$$\min_{\mathbf{w}} \quad \mathbf{w}^\top \mathbf{C} \mathbf{w} \tag{3.10}$$

$$s.t. \quad \mathbf{w} \geq 0, \quad \mathbf{w}^\top \mathbf{1} = 1, \tag{3.11}$$

where $\mathbf{w} = [w_1, w_2, ..., w_N]^\top$ denotes the portfolio weights vector and $\mathbf{1}$ the vector of ones. To optimize portfolios according to (3.10) subject to (3.11) with good and bad (implied) volatilities, consider the Bollerslev et al. (2020) decomposition of the realized covariance matrix $\hat{\mathbf{C}}$ to three semicovariance matrices. Let $\mathbf{r_t} = [r_{t,1}, r_{t,2}, ..., r_{t,N}]^\top$ denote the return vector of N assets, then the corresponding bad and good return vectors, $\mathbf{r_t}^b$ and $\mathbf{r_t}^g$, are given as

$$\mathbf{r_t}^b \equiv \mathbf{r_t} \odot \mathbf{I}_t^b, \quad \mathbf{r_t}^g \quad \equiv \mathbf{r_t} \odot \mathbf{I}_t^g \tag{3.12}$$

where $\mathbf{I}_t^b = [\mathbb{I}_{(r_{t,1} \leq 0)}, \mathbb{I}_{(r_{t,2} \leq 0)}, ..., \mathbb{I}_{(r_{t,N} \leq 0)}]^\top$ and $\mathbf{I}_t^g = [\mathbb{I}_{(r_{t,1} > 0)}, \mathbb{I}_{(r_{t,2} > 0)}, ..., \mathbb{I}_{(r_{t,N} > 0)}]^\top$. Then, the authors define the three additive components of $\hat{\mathbf{C}}$ as:

$$\begin{aligned}
\hat{\mathbf{N}}_T &\equiv \sum_{t=1}^{T} \mathbf{r_t}^b \mathbf{r_t}^{b\top}, \\
\hat{\mathbf{P}}_T &\equiv \sum_{t=1}^{T} \mathbf{r_t}^g \mathbf{r_t}^{g\top}, \\
\hat{\mathbf{M}}_T &\equiv \sum_{t=1}^{T} \mathbf{r_t}^b \mathbf{r_t}^{g\top} + \mathbf{r_t}^g \mathbf{r_t}^{b\top}, \\
\hat{\mathbf{C}}_T &= \hat{\mathbf{N}}_T + \hat{\mathbf{P}}_T + \hat{\mathbf{M}}_T.
\end{aligned} \tag{3.13}$$

While $\hat{\mathbf{C}}$, $\hat{\mathbf{N}}$ and $\hat{\mathbf{P}}$ are positive semidefinite[4], $\hat{\mathbf{M}}$ is indefinite (Bollerslev et al., 2020). The two main strategies tested in this paper take advantage of the fact that the two matrices $\hat{\mathbf{N}}$ and $\hat{\mathbf{P}}$ can be decomposed into the following components:

$$\begin{aligned}
\hat{\mathbf{N}}_T &\equiv \mathrm{Diag}(\hat{\mathbf{RV}}^{\mathbf{b}}_{\mathbf{T}})^{1/2} \hat{\mathbf{R}}^{\mathbf{b}}{}_T \, \mathrm{Diag}(\hat{\mathbf{RV}}^{\mathbf{b}}_{\mathbf{T}})^{1/2}, \\
\hat{\mathbf{P}}_T &\equiv \mathrm{Diag}(\hat{\mathbf{RV}}^{\mathbf{g}}_{\mathbf{T}})^{1/2} \hat{\mathbf{R}}^{\mathbf{g}}{}_T \, \mathrm{Diag}(\hat{\mathbf{RV}}^{\mathbf{g}}_{\mathbf{T}})^{1/2}
\end{aligned} \tag{3.14}$$

where $\mathrm{Diag}(\hat{\mathbf{RV}}^{\mathbf{b}}_{\mathbf{T}})^{1/2}$ and $\mathrm{Diag}(\hat{\mathbf{RV}}^{\mathbf{g}}_{\mathbf{T}})^{1/2}$ are the diagonal matrices with realized bad

[3]Authors such as Lamont (2012) and Engelberg et al. (2018) document that short selling produces additional risks such as recall risk and increasing stock loan costs. Frost and Savarino (1988), Jagannathan and Ma (2003) and Kempf et al. (2014) report that imposing the short-sale constraint on the minimum variance portfolio enhances the out-of-sample performance.

[4]This guarantees that $\mathbf{w}^\top \mathbf{C} \mathbf{w} \geq 0$, $\mathbf{w}^\top \mathbf{N} \mathbf{w} \geq 0$ and $\mathbf{w}^\top \mathbf{P} \mathbf{w} \geq 0$ for all $\mathbf{w}$.

and good volatilities on the diagonal and $\hat{\mathbf{R}^{\mathbf{b}}}_T$ and $\hat{\mathbf{R}^{\mathbf{b}}}_T$ are the realized bad and good semicorrelation matrices, respectively.

Using these insights, the main portfolio strategies for the empirical investigation are formed as follows. At the last trading day of each month in the sample, the minimum variance portfolios with S&P 500 stocks are constructed using implied good (bad) volatilities from that day replacing $\mathbf{C}$ with $\hat{\mathbf{N}}$ ($\hat{\mathbf{P}}$) in (3.10) and replacing the realized good (bad) volatilities on the diagonal matrices in (3.14) with implied good (bad) volatilities. The respective semicorrelations are based on historical daily returns with an estimation window of 1 year. At each rebalancing date, stocks with no option data at that date or limited underlying price data in the corresponding estimation window are ignored. I henceforth denote the portfolio strategy that utilizes good (bad) implied volatility data as **GV-Hybr** (**BV-Hybr**).

Undoubtedly, a strategy that is successful in minimizing out-of-sample bad instead of good portfolio volatility is more desirable. However, there are two reasons to include **GV-Hybr** for the empirical part of this study nonetheless. First, it is unclear which of those two strategies outperforms the other out-of-sample. Second, the out-of-sample results for **GV-Hybr** serve as an additional check with regard to the empirical properties of implied semivolatilities.

Benchmark Strategies

To evaluate the out-of-sample performance of the main strategies, six benchmark strategies are considered. Apart from the naive $1/N$ strategy, the other benchmark strategies obtain their optimal weights as **GV-Hybr** and **BV-Hybr** and diverge merely on which covariance estimator is used to solve (3.10) subject to (3.11).

The first four benchmark strategies use only historical data to build minimum variance portfolios. I denote **V-Hist** as the strategy that uses the sample covariance matrix as an input, while **GV-Hist** and **BV-Hist** denote the strategies that use the matrices $\hat{\mathbf{N}}$ and $\hat{\mathbf{P}}$ as an input, respectively. Furthermore, I consider the shrinkage estimator proposed by Ledoit and Wolf (2003) which sets the single-factor model of Sharpe (1963) as the shrinkage target to reduce the estimation error of the sample covariance matrix[5]. The strategy that uses the resulting covariance matrix is denoted by **V-Shri**.

Similar to the main strategies, the next competitor, denoted as **V-Hybr**, uses historical data to obtain the sample correlation matrix but uses implied volatilities to obtain the final (hybrid) covariance estimator.

Finally, I also construct fully-implied covariance matrices based on the methodology introduced by Chang et al. (2012). Under the assumption of a single-factor model and

[5]For an excellent review on shrinkage estimators, see Ledoit and Wolf (2020).

zero skewness of idiosyncratic shocks, they suggest to compute the implied market beta estimate of a stock as follows:

$$\beta_i^* = \left(\frac{IS_i^*}{IS_m^*}\right)^{1/3} \left(\frac{IV_i^*}{IV_m^*}\right)^{1/2}, \tag{3.15}$$

where IS_i^* (IV_i^*) is the implied skewness (variance) of the stock formed according to the methodology of Bakshi et al. (2003). The subscript m denotes the market, which is approximated by the S&P 500 in this study. At each rebalancing date I calculate β^* for all relevant stocks and fill each off-diagonal element of the fully-implied covariance matrix by $\beta_i^* \beta_j^* IV_m^*$ and fill each diagonal element with $\sqrt{IV_i^*}$. I denote the respective strategy that uses this matrix as an input as **V-Impl**. Table 3.1 lists all defined strategies including their abbrevations used in the following tables and figures.

[Table 3.1 about here.]

3.4 Empirical Analysis

This section examines whether implied good or bad volatility help to improve out-of-sample performance of the portfolio strategies. I begin with reporting the performance of each portfolio strategy introduced in the previous section. Then, I run a series of robustness checks to validate the findings. Lastly, I check whether the out-of-sample performance of the portfolios can be attributed to their exposure to certain risk factors.

3.4.1 Main Results

Table 3.2 reports the out-of-sample annualized sample/symmetric volatility ($\hat{\sigma}$), annualized realized good volatility ($\hat{\sigma}^g$), annualized realized bad volatility ($\hat{\sigma}^b$), Sharpe and Sortino ratio of each portfolio strategy. The volatility measure $\hat{\sigma}^g$ ($\hat{\sigma}^b$) is calculated as the annualized sum of absolute positive (negative) daily returns. To test the one-sided null hypothesis that a given strategy underperforms the benchmark, I generate 10,000 bootstrapped pairs of daily strategy and benchmark returns via resampling with replacement. The one-sided p-value is reported for the null that the difference between the portfolio and benchmark volatility (risk-adjusted return) measure is larger (smaller) than or equal to zero. Each portfolio strategy is tested against the three benchmarks $1/N$, **V-Hist** and **V-Impl**.

[Table 3.2 about here.]

The lowest out-of-sample symmetric (10.87%), good (8.07%) and bad volatility (7.32%) is produced by **BV-Hybr**. All p-values calculated for these measures are smaller than 1%, except for the comparison relative to **V-Hist** where the p-value is below 10%. How much does option-implied (downside) information contribute to this outperformance? This question can be answered by comparing the strategies that use historical data only (**V-Hist**, **GV-Hist** and **BV-Hist**) to their hybrid counterparts (**V-Hybr**, **GV-Hybr** and **BV-Hybr**). While the out-of-sample volatility measures are lower for **V-Hybr** and **BV-Hybr**, the opposite is true for **GV-Hybr**. Except for **V-Impl**, all minimum variance portfolio strategies are able to beat $1/N$ significantly in each volatility metric[6]. In particular, the three out-of-sample volatility measures for the **BV-Hybr** portfolio are about 50% lower than the corresponding numbers for the $1/N$ portfolio.

The highest annualized Sharpe ratio (0.93) and Sortino ratio (0.10), the latter being calculated with a minimum acceptable return of 0%, are both generated by **BV-Hybr**. Both are statistically significant at the 1% significance level relative to the three benchmarks. Similar to the findings of DeMiguel et al. (2009), for example, **V-Hist** does not produce significantly higher Sharpe ratios than $1/N$. In fact, among all tested strategies only **GV-Hybr** and **BV-Hybr** produce Sharpe and Sortino ratios that are significantly higher than each benchmark strategy at the 5% level. While high Sharpe and Sortino ratios are generally desirable, I consider the three out-of-sample volatility measures as the main performance metrics due to the fact that the respective objective function of each minimum variance portfolio strategies is to minimize the corresponding portfolio volatility/variance measure.

3.4.2 Robustness Checks

Estimation Windows

In order to assess the robustness of the main conclusions drawn from Table 3.2, I consider alternative historical estimation windows of 6- and 24-months, which corresponds to halving and doubling the default historical estimation window size of 12-month. Table 3.3 reports the results with a rolling 6-month estimation window. While the lowest sample and good volatility are now produced by **GV-Hist**, the lowest bad volatility is still generated by **BV-Hybr**, all significant at the 1% confidence level. In terms of highest Sharpe and Sortino ratio, **BV-Hybr** remains at the top. Both are significant at the 5% level.

[6] Studies such as DeMiguel et al. (2013) and Kempf et al. (2014) also confirm that minimum variance portfolios formed with various types of covariance estimators beat the $1/N$ portfolio in terms of out-of-sample volatility most of the time.

[Table 3.3 about here.]

Table 3.4 displays the results with a rolling 24-month estimation window. Overall, the results are comparable to the base case results shown in Table 3.2 in both absolute and relative terms.

[Table 3.4 about here.]

Random Portfolio Universes

As a next robustness check, instead of taking the entire set of S&P 500 stocks, I randomly select 100, 200, 300 and 400 stocks from the S&P 500 universe each month from which the portfolio strategies can allocate wealth to. Table 3.5 reports the results with 100 randomly selected stocks. Contrary to the results shown earlier, **BV-Hybr** does not outperform the others in this setting. Instead, **V-Shri** produces the lowest out-of-sample sample, good and bad portfolio volatilities and, with one exception, significantly lower than the three benchmarks at the 1% level. However, no strategy is able to produce significantly higher Sharpe and Sortino ratios against any benchmark at the 5% level.

[Table 3.5 about here.]

Table 3.6, 3.7 and 3.8 report the results for 200, 300 and 400 randomly selected stocks, respectively. With regard to the three portfolio volatility measures, **BV-Hybr** produces the lowest numbers, mostly significant at the 1% level. While no strategy is able to provide the highest Sharpe and Sortino ratios in all three settings, both metrics increase for **BV-Hybr** and the corresponding p-values drop as the number of randomly selected stocks increases.

[Table 3.6 about here.]

[Table 3.7 about here.]

[Table 3.8 about here.]

Good and bad times

To conduct a further robustness check, I divide the main sample into two subsamples: good and bad times. The period from April 2000 to March 2003, from May 2007 to December 2009 and from February 2020 to April 2020 is defined as bad times and the rest of the main sample as good times. Table 3.9 reports the respective results. Panel A of Table 3.9 indicates that **BV-Hybr** produces the best performance in each metric in good times, all significant at the 1% level. It is also the portfolio strategy that produces the lowest, good and bad volatility in bad times, as displayed in Panel B of Table 3.9. Furthermore, they are statistically significant at the 1% level except when compared to **V-Hist** with regard to good volatility. However, regarding Sharpe and Sortino ratios, no strategy is able to beat any benchmark in bad times significantly. All portfolio volatility measures are roughly twice as large in bad times as in good times. Nevertheless, with the exception of **V-Impl**, all minimum variance portfolio strategies manage to produce significantly lower volatilities than the $1/N$ benchmark.

[Table 3.9 about here.]

To shed more light on the consistency of the portfolio volatility results in each subsample, Figure 3.1 depicts the evolution of the three volatility measures with a rolling window of 12-month over time for the five strategies $1/N$, **V-Hist**, **V-Hybr**, **GV-Hybr** and **BV-Hybr**. Visual inspection shows that the three rolling volatility metrics of **BV-Hybr** are most of the time lower than those of the other strategies in good and in bad times and that the respective findings in the two subsamples reported in Table 3.9 are not the result of particular outliers. To sum up, the plots indicate that the minimum variance portfolio strategies, and in particular **BV-Hybr**, produce consistently lower sample, good and bad volatilities than $1/N$ in good times and in bad times.

[Figure 3.1 about here.]

Transaction costs and weights analysis

In the final robustness check I analyze the portfolio weights and evaluate the impact of transaction costs on the Sharpe and Sortino ratios reported in Table 3.2. I compute the turnover of each portfolio strategy as the average sum of absolute changes in weights of the portfolio from one month to the next to assess the amount of trading needed for the strategy to be fully implemented. Furthermore, I calculate the transaction cost that equates the Sharpe or Sortino ratio of the strategy to that of the benchmark and label it the equivalent transaction cost ETC, following DeMiguel et al. (2009). The degree of portfolio weight concentration is measured by the Herfindahl index HI and is computed as the sum of squared portfolio weights for each portfolio strategy. The results are shown in Table 3.10. The outperformance of the **BV-Hybr** strategy seems to come at the cost of high turnover: 114.89% compared to 52.72% turnover produced by **V-Hist**, for example. As one would expect, the strategy with the lowest turnover is $1/N$ with 1.57%. Overall, it seems that the type of data used to form the portfolios is the main determinant of the resulting turnover level: among the minimum variance strategies, those who use historical data only have the lowest turnover, followed by the hybrid strategies and with **V-Impl** at the top, which solely uses implied data. Turnover appears to also increase by using implied good or bad volatilities instead of implied volatilities to form minimum variance portfolios, which is observable by comparing the turnover of **V-Hybr** (94.18%) with **GV-Hybr** (120.57%) and **BV-Hybr** (114.89%). The corresponding semicorrelations that are used by **GV-Hybr** and **BV-Hybr** do not seem to be the reason behind the relatively high turnover as their counterparts **GV-Hist** and **BV-Hist** that use them as well generate significantly lower turnover (54.61% and 51.16%, respectively).

[Table 3.10 about here.]

Despite the high turnover of **BV-Hybr**, outperformance in terms of the Sharpe ratio (Sortino ratio) vs. $1/N$ and **V-Hist** is sustained with transaction costs below 32 (35) and 44 (47) bps and vs. **V-Impl** above 100 (100) bps, respectively. The results are similar for **GV-Hybr** and roughly 10 bps smaller for **V-Hybr** and **BV-Hist**. These numbers are higher than the transaction costs reported by Frazzini et al. (2018) who study a large trade execution database of an institutional investor. From August 1998 to June 2016, they estimate average transaction costs of 9 bps for US large cap stocks.

The degree of portfolio concentration is also highly dependent on which covariance estimator is used for minimum variance optimization. While it is well known that minimum variance portfolios tend to be relatively concentrated (see, e.g., Clarke et al. (2013)), the concentration is especially higher for the strategies using semivolatilities and semicorrelations only as the HI ranges for them from 53.23 to 80.43 compared to the other minimum variance portfolios where the HI ranges from 24.84 to 35.39. Again, the only exception is **V-Impl** which produces the highest HI of 92.32.

3.4.3 Risk Factor Sensitivities

Equity Risk Factors

Next, I investigate to which degree the returns of the portfolio strategies are related to popular equity risk factors. To do so, the excess returns of the strategies are regressed on the Fama-French 5 factors (Fama and French, 2015) - market MKT, size SMB, value HML, profitability RMW and investment CMA - extended by the momentum factor MOM[7] and the betting against beta factor BAB of Frazzini and Pedersen (2014)[8]. The t-statistics are computed with Newey and West (1987) standard errors. The results of the

[7]The Fama-French 5 factor and the momentum factor data are obtained from `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`. The latter is similar to the momentum factor described in Carhart (1997).

[8]The data can be found at `https://www.aqr.com/Insights/Datasets/`.

standard multifactor regressions are based on monthly data and are presented in Table 3.11. The regressions in Panel A of Table 3.11 exclude the BAB factor and show that the adjusted R^2s range between 0.36 and 0.59 for the minimum variance portfolio strategies and 0.96 for the $1/N$ strategy. Interestingly, strategies that utilize implied good, bad or symmetric volatility produce lower adjusted R^2s than their counterparts that rely on historical data only. Given that all strategies examined in this study engage in long-only investments, it is not surprising that the MKT factor is positively and significantly related with the (excess) returns of each strategy. The corresponding coefficient is close to unity for $1/N$, and ranges from 0.41 to 0.70 for the minimum variance strategies[9]. No portfolio except for $1/N$ has significant SMB or HML exposure. RMW is significantly positively related with most strategies while the CMA and UMD coefficients are only in some cases significant.

[Table 3.11 about here.]

Panel B of Table 3.11 augments the regression by the BAB factor. The results indicate that except for $1/N$ and **V-Impl**, all portfolio strategies have a significant positive exposure to the BAB factor and that the addition of this factor mostly increases the adjusted R^2s to a small degree.

Overall, the regression results for the excess returns of **GV-Hybr** and **BV-Hybr** stand out in several ways. Only these strategies generate positive, highly significant and economically meaningful alphas of 0.465% and 0.357% per month (0.514% and 0.396% without the BAB factor), the lowest market betas of 0.47 and 0.41 and the lowest adjusted R^2s of 0.40 and 0.38 (0.39 and 0.36 without the BAB factor), respectively.

Market Moment Risk Factors

Given the limited success of common equity risk factors to explain the excess returns of all considered portfolio strategies, I now turn to analyze whether market moment risk factors are better able to do so. Chang et al. (2013) find that excess market returns are significantly related with innovations in implied market volatility ($\Delta\sqrt{IV_m^*}$), skewness (ΔIS_m^*) and kurtosis (ΔIK_m^*). As in their original paper, I use S&P 500 index options data and apply the Bakshi et al. (2003) moment estimation methodology to calculate the three implied measures $\sqrt{IV_m^*}$, IS_m^* and IK_m^*. The corresponding put and call integrals are evaluated applying the methodology described in 3.3.1. Since all three implied market moments are highly serially correlated, I follow Chang et al. (2013) and obtain

[9] Chow et al. (2014) investigate the returns of minimum variance, inverse volatility and inverse beta portfolios and find that among these risk-based portfolio strategies the former produces the lowest market betas in each of the three datasets (U.S., global and emerging market stocks) considered.

estimates of $\Delta\sqrt{IV_m^*}$ by calculating first differences in $\sqrt{IV_m^*}$, and obtain estimates of ΔIS_m^* and ΔIK_m^* by using the residuals of a fitted ARMA(1,1) model to IS_m^* and IK_m^*, respectively[10]. The monthly excess returns of each portfolio strategy are regressed on the monthly estimates of the innovations in the three implied market moments. Panel A and B of Table 3.12 and Panel A of Table 3.13 show the standard regression results obtained when the excess returns are regressed on $\Delta\sqrt{IV_m^*}$, ΔIS_m^* and ΔIK_m^*, respectively. The coefficients of $\Delta\sqrt{IV_m^*}$ (ΔIK_m^*) are all negative (positive) and statistically significant. The coefficients of ΔIS_m^* are mostly significant and always negative. These coefficient signs match with those presented in Chang et al. (2013). While the ΔIS_m^* and ΔIK_m^* regressions reported here produce relative low adjusted adjusted R^2s ranging from 0.01 to 0.12, the corresponding numbers for the $\Delta\sqrt{IV_m^*}$ regressions are significantly higher but vary strongly depending on the portfolio strategy: at the lower end between 0.16 and 0.18 for **GV-Hist**, **GV-Hybr** and **BV-Hybr** and at the upper end between 0.31 and 0.56 for **V-Hist**, **V-Shri** and $1/N$. The intercept coefficient is always positive and highly statistically significant irrespective of the explanatory variable used.

[Table 3.12 about here.]

[Table 3.13 about here.]

Panel B of Table 3.13 reports the regression results where the excess returns are regressed on all three implied market moment risk factors simultaneously. The results emphasize that the only relevant factor overall seems to be $\Delta\sqrt{IV_m^*}$, since the coefficients remain negative and statistically significant as in Panel A of Table 3.12 and both, the intercept coefficients and the adjusted R^2s remain largely unchanged with the other two factors included. This is also reflected by the coefficient estimates of ΔIS_m^* and ΔIK_m^*, since the former is in no setting significant anymore and the latter is only significant for the $1/N$ strategy. Compared to the results shown in 3.11, it is fair to conclude that the (implied) market moment risk factors seem to provide less explanatory power for the portfolio strategy excess returns.

3.5 Conclusion

In this study I show how forward-looking implied good and bad volatilities can be used to form minimum good and bad variance portfolios and compare their out-of-sample performance with several other strategies including the $1/N$ strategy and minimum variance portfolio strategies that use historical and/or implied data.

[10] In line with Chang et al. (2013), I find that taking first differences of $\sqrt{IV_m^*}$ is enough to eliminate autocorrelation in the data, but not for IS_m^* and IK_m^* and that fitted ARMA(1,1) models are able to remove autocorrelation in both time series.

I present evidence that a minimum (bad) variance strategy that constructs portfolios with implied bad volatilities and historical semicorrelations provides the highest out-of-sample Sharpe and the highest Sortino ratio. Although the use of implied semivolatilities in particular seems to significantly increase portfolio turnover, the Sharpe and Sortino ratio remain sufficiently high even after considering transaction costs.

Even more important and more robust is the evidence provided by this article that the strategy produces consistently lower out-of-sample bad and in most cases symmetric portfolio volatilities than the other strategies tested.

Only the minimum good and bad variance portfolio strategies that use implied good and bad volatilities, respectively, produce highly statistically and economically significant Fama-French 5 plus momentum factor alphas and the lowest market betas. Adding the betting against beta factor to the factor model do not materially affect this finding.

Figures

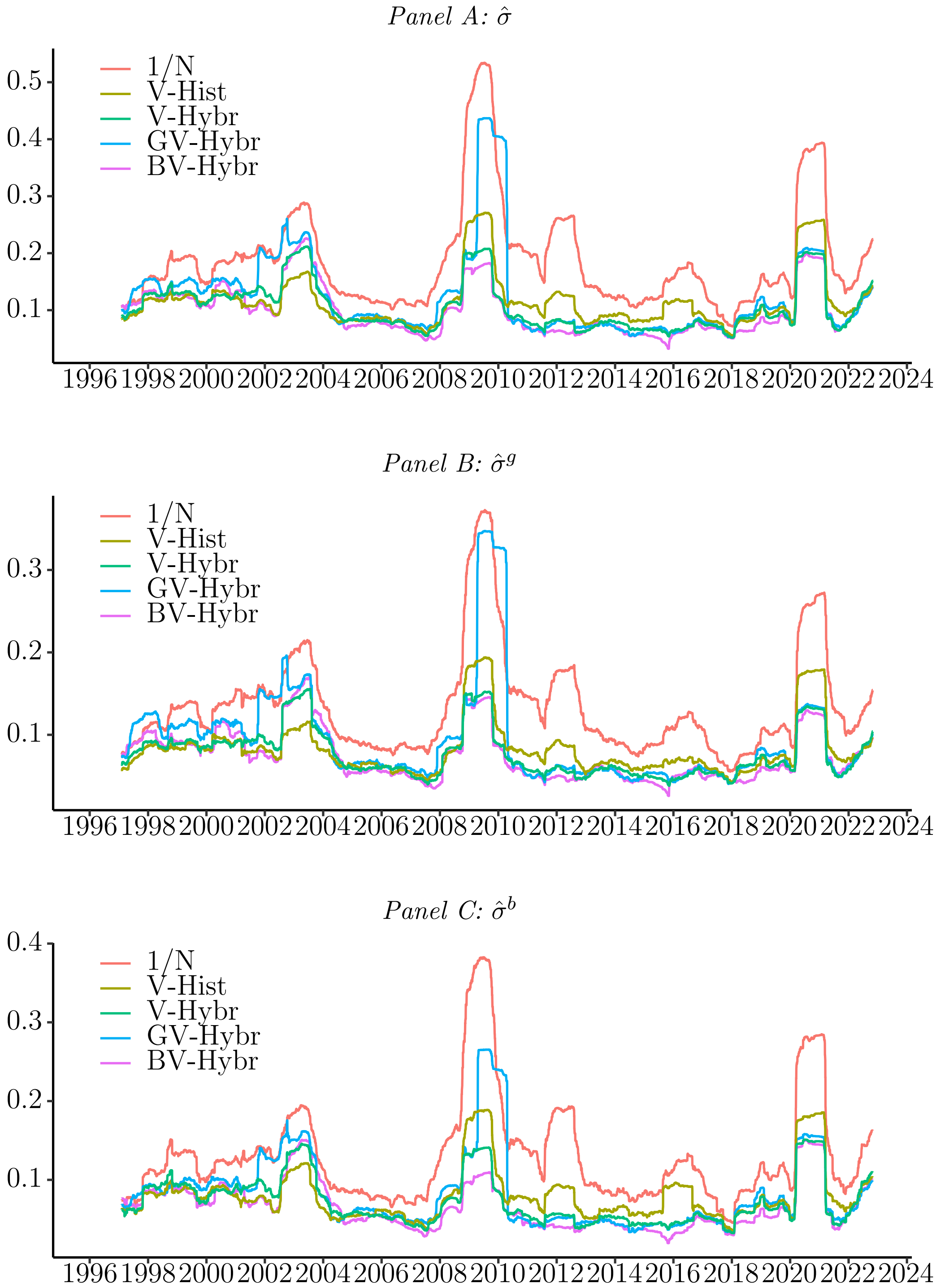

Figure 3.1: Rolling portfolio volatilities.
This figure displays the rolling 12-month out-of-sample annualized volatility ($\hat{\sigma}$), good volatility ($\hat{\sigma}^g$) and bad volatility ($\hat{\sigma}^b$) of five portfolio strategies. The sample period goes from January 1996 to October 2022.

Tables

Table 3.1: List of evaluated portfolio strategies.

This table lists the various portfolio strategies considered for the empirical analysis.

No.	Portfolio Strategy	Abbreviation
Naive		
1	Rebalanced $1/N$ strategy	$1/N$
Minimum Variance Portfolio		
2	Minimum variance portfolio based on historical data	V-Hist
3	Minimum variance portfolio based on shrinkage	V-Shri
4	Minimum variance portfolio based on implied and historical data	V-Hybr
5	Minimum variance portfolio based on implied data	V-Impl
Minimum Good Variance Portfolio		
6	Minimum (good) variance portfolio based on historical data	GV-Hist
7	Minimum (good) variance portfolio based on implied and historical data	GV-Hybr
Minimum Bad Variance Portfolio		
8	Minimum (bad) variance portfolio based on historical data	BV-Hist
9	Minimum (bad) variance portfolio based on implied and historical data	BV-Hybr

Table 3.2: Portfolio performance.

This table provides the out-of-sample performance of the portfolio strategies. $\hat{\sigma}$ is the annualized sample volatility, $\hat{\sigma}^g$ is the annualized good volatility, $\hat{\sigma}^b$ is the annualized bad volatility, 'Sharpe' is the Sharpe ratio and 'Sortino' is the Sortino ratio. All volatility measures are displayed in percentage terms. In each of the first three (last two) columns, the lowest (highest) number is shown in bold. The parentheses include the bootstrapped p-values for the null hypothesis that the portfolio strategy performs equally as good as the respective benchmark ($1/N$, **V-Hist** or **V-Impl**, presented in this order). The sample period goes from January 1996 to October 2022.

	$\hat{\sigma}$	$\hat{\sigma}^g$	$\hat{\sigma}^b$	Sharpe	Sortino
$1/N$	20.74	14.74	14.61	0.48	0.06
	(N.A./1.00/0.00)	(N.A./1.00/0.00)	(N.A./1.00/0.08)	(N.A./0.61/0.30)	(N.A./0.60/0.36)
V-Hist	12.39	8.81	8.73	0.56	0.06
	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.39/N.A./0.26)	(0.40/N.A./0.31)
V-Shri	12.38	8.80	8.72	0.56	0.07
	(0.00/0.13/0.00)	(0.00/0.18/0.00)	(0.00/0.30/0.00)	(0.36/0.21/0.24)	(0.38/0.23/0.30)
V-Hybr	11.38	8.20	7.91	0.72	0.08
	(0.00/0.00/0.00)	(0.00/0.01/0.00)	(0.00/0.00/0.00)	(0.07/0.07/0.06)	(0.08/0.08/0.09)
V-Impl	22.60	16.58	15.38	0.39	0.05
	(1.00/1.00/N.A.)	(1.00/1.00/N.A.)	(0.92/1.00/N.A.)	(0.70/0.74/N.A.)	(0.64/0.69/N.A.)
GV-Hist	12.76	9.16	8.90	0.56	0.07
	(0.00/0.95/0.00)	(0.00/0.95/0.00)	(0.00/0.76/0.00)	(0.40/0.48/0.27)	(0.40/0.46/0.31)
GV-Hybr	14.51	11.00	9.50	0.84	0.09
	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.00/0.76/0.00)	(0.04/0.05/0.01)	(0.04/0.05/0.02)
BV-Hist	13.37	9.61	9.32	0.59	0.07
	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.30/0.33/0.20)	(0.30/0.31/0.24)
BV-Hybr	**10.87**	**8.07**	**7.32**	**0.93**	**0.10**
	(0.00/0.00/0.00)	(0.00/0.01/0.00)	(0.00/0.00/0.00)	(0.01/0.01/0.01)	(0.01/0.01/0.01)

Table 3.3: Portfolio performance: 6-month window.

This table provides the out-of-sample performance of the portfolio strategies. Compared to Table 3.2, historical volatilities and correlations are estimated with a 6-month window. $\hat{\sigma}$ is the annualized sample volatility, $\hat{\sigma}^g$ is the annualized good volatility, $\hat{\sigma}^b$ is the annualized bad volatility, 'Sharpe' is the Sharpe ratio and 'Sortino' is the Sortino ratio. All volatility measures are displayed in percentage terms. In each of the first three (last two) columns, the lowest (highest) number is shown in bold. The parentheses include the bootstrapped p-values for the null hypothesis that the portfolio strategy performs equally as good as the respective benchmark ($1/N$, **V-Hist** or **V-Impl**, presented in this order). The sample period goes from January 1996 to October 2022.

	$\hat{\sigma}$	$\hat{\sigma}^g$	$\hat{\sigma}^b$	Sharpe	Sortino
$1/N$	20.73	14.74	14.60	0.48	0.06
	(N.A./1.00/0.00)	(N.A./1.00/0.00)	(N.A./1.00/0.03)	(N.A./0.34/0.54)	(N.A./0.33/0.59)
V-Hist	12.02	8.47	8.55	0.46	0.06
	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.66/N.A./0.64)	(0.67/N.A./0.69)
V-Shri	11.89	8.38	8.45	0.50	0.06
	(0.00/0.00/0.00)	(0.00/0.00/0.00)	(0.00/0.00/0.00)	(0.56/0.02/0.57)	(0.58/0.03/0.63)
V-Hybr	11.84	8.48	8.28	0.65	0.07
	(0.00/0.18/0.00)	(0.00/0.52/0.00)	(0.00/0.05/0.00)	(0.17/0.05/0.27)	(0.18/0.05/0.32)
V-Impl	23.08	17.03	15.61	0.49	0.06
	(1.00/1.00/N.A.)	(1.00/1.00/N.A.)	(0.97/1.00/N.A.)	(0.46/0.36/N.A.)	(0.41/0.31/N.A.)
GV-Hist	**11.00**	**7.81**	7.77	0.57	0.07
	(0.00/0.00/0.00)	(0.00/0.01/0.00)	(0.00/0.00/0.00)	(0.40/0.20/0.44)	(0.41/0.20/0.49)
GV-Hybr	14.56	11.14	9.42	0.92	**0.10**
	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.00/0.81/0.00)	(0.02/0.01/0.02)	(0.02/0.01/0.02)
BV-Hist	12.70	9.14	8.85	0.61	0.07
	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.00/0.97/0.00)	(0.29/0.09/0.36)	(0.29/0.08/0.40)
BV-Hybr	11.09	8.25	**7.45**	**0.94**	0.10
	(0.00/0.00/0.00)	(0.00/0.27/0.00)	(0.00/0.00/0.00)	(0.01/0.00/0.02)	(0.01/0.00/0.04)

Table 3.4: Portfolio performance: 24-month window.

This table provides the out-of-sample performance of the portfolio strategies. Compared to Table 3.2, historical volatilities and correlations are estimated with a 24-month window. $\hat{\sigma}$ is the annualized sample volatility, $\hat{\sigma}^g$ is the annualized good volatility, $\hat{\sigma}^b$ is the annualized bad volatility, 'Sharpe' is the Sharpe ratio and 'Sortino' is the Sortino ratio. All volatility measures are displayed in percentage terms. In each of the first three (last two) columns, the lowest (highest) number is shown in bold. The parentheses include the bootstrapped p-values for the null hypothesis that the portfolio strategy performs equally as good as the respective benchmark ($1/N$, **V-Hist** or **V-Impl**, presented in this order). The sample period goes from January 1996 to October 2022.

	$\hat{\sigma}$	$\hat{\sigma}^g$	$\hat{\sigma}^b$	Sharpe	Sortino
$1/N$	20.73	14.74	14.60	0.48	0.06
	(N.A./1.00/0.00)	(N.A./1.00/0.00)	(N.A./1.00/0.03)	(N.A./0.70/0.28)	(N.A./0.69/0.32)
V-Hist	12.86	9.17	9.04	0.59	0.07
	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.30/N.A./0.18)	(0.31/N.A./0.22)
V-Shri	12.86	9.17	9.03	0.60	0.07
	(0.00/0.30/0.00)	(0.00/0.51/0.00)	(0.00/0.24/0.00)	(0.26/0.02/0.16)	(0.28/0.03/0.20)
V-Hybr	11.49	8.33	7.95	0.81	0.09
	(0.00/0.00/0.00)	(0.00/0.00/0.00)	(0.00/0.00/0.00)	(0.02/0.03/0.02)	(0.03/0.03/0.04)
V-Impl	22.52	16.39	15.45	0.37	0.05
	(1.00/1.00/N.A.)	(1.00/1.00/N.A.)	(0.97/1.00/N.A.)	(0.72/0.82/N.A.)	(0.68/0.78/N.A.)
GV-Hist	13.12	9.40	9.18	0.55	0.06
	(0.00/0.97/0.00)	(0.00/0.95/0.00)	(0.00/0.85/0.00)	(0.43/0.67/0.27)	(0.42/0.65/0.30)
GV-Hybr	14.63	11.11	9.56	0.81	0.09
	(0.00/0.99/0.00)	(0.00/0.99/0.00)	(0.00/0.66/0.00)	(0.05/0.09/0.02)	(0.05/0.08/0.02)
BV-Hist	13.76	9.91	9.55	0.58	0.07
	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.33/0.51/0.19)	(0.32/0.47/0.22)
BV-Hybr	**10.82**	**7.99**	**7.33**	**0.93**	**0.10**
	(0.00/0.00/0.00)	(0.00/0.00/0.00)	(0.00/0.00/0.00)	(0.01/0.01/0.01)	(0.01/0.01/0.01)

Table 3.5: Portfolio performance: 100 random stocks.

This table provides the out-of-sample performance of the portfolio strategies. Compared to Table 3.2, the investment universe consists of 100 S&P 500 stocks selected at random each month. $\hat{\sigma}$ is the annualized sample volatility, $\hat{\sigma}^g$ is the annualized good volatility, $\hat{\sigma}^b$ is the annualized bad volatility, 'Sharpe' is the Sharpe ratio and 'Sortino' is the Sortino ratio. All volatility measures are displayed in percentage terms. In each of the first three (last two) columns, the lowest (highest) number is shown in bold. The parentheses include the bootstrapped p-values for the null hypothesis that the portfolio strategy performs equally as good as the respective benchmark ($1/N$, **V-Hist** or **V-Impl**, presented in this order). The sample period goes from January 1996 to October 2022.

	$\hat{\sigma}$	$\hat{\sigma}^g$	$\hat{\sigma}^b$	Sharpe	Sortino
$1/N$	21.02	15.04	14.72	0.51	0.06
	(N.A./1.00/1.00)	(N.A./1.00/0.98)	(N.A./1.00/1.00)	(N.A./0.91/0.71)	(N.A./0.90/0.73)
V-Hist	13.73	9.86	9.59	0.73	**0.08**
	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.09/N.A./0.26)	(0.10/N.A./0.31)
V-Shri	**13.71**	**9.84**	**9.58**	0.72	0.08
	(0.00/0.01/0.00)	(0.00/0.01/0.00)	(0.00/0.22/0.00)	(0.10/0.81/0.27)	(0.11/0.82/0.32)
V-Hybr	14.08	10.20	9.73	0.68	0.08
	(0.00/0.94/0.00)	(0.00/0.88/0.00)	(0.00/0.80/0.00)	(0.16/0.68/0.35)	(0.16/0.64/0.39)
V-Impl	19.24	14.07	13.15	0.60	0.07
	(0.00/1.00/N.A.)	(0.02/1.00/N.A.)	(0.00/1.00/N.A.)	(0.29/0.74/N.A.)	(0.27/0.69/N.A.)
GV-Hist	14.42	10.38	10.04	0.69	0.08
	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.19/0.65/0.35)	(0.20/0.63/0.38)
GV-Hybr	14.62	10.67	10.02	0.65	0.07
	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.00/0.99/0.00)	(0.25/0.72/0.42)	(0.24/0.67/0.44)
BV-Hist	14.69	10.64	10.16	**0.77**	0.08
	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.07/0.30/0.19)	(0.08/0.27/0.22)
BV-Hybr	14.46	10.82	9.62	0.65	0.08
	(0.00/0.97/0.00)	(0.00/0.98/0.00)	(0.00/0.57/0.00)	(0.25/0.72/0.43)	(0.21/0.61/0.40)

Table 3.6: Portfolio performance: 200 random stocks.

This table provides the out-of-sample performance of the portfolio strategies. Compared to Table 3.2, the investment universe consists of 200 S&P 500 stocks selected at random each month. $\hat{\sigma}$ is the annualized sample volatility, $\hat{\sigma}^g$ is the annualized good volatility, $\hat{\sigma}^b$ is the annualized bad volatility, 'Sharpe' is the Sharpe ratio and 'Sortino' is the Sortino ratio. All volatility measures are displayed in percentage terms. In each of the first three (last two) columns, the lowest (highest) number is shown in bold. The parentheses include the bootstrapped p-values for the null hypothesis that the portfolio strategy performs equally as good as the respective benchmark ($1/N$, **V-Hist** or **V-Impl**, presented in this order). The sample period goes from January 1996 to October 2022.

	$\hat{\sigma}$	$\hat{\sigma}^g$	$\hat{\sigma}^b$	Sharpe	Sortino
$1/N$	20.90	14.89	14.68	0.50	0.06
	(N.A./1.00/0.99)	(N.A./1.00/0.82)	(N.A./1.00/1.00)	(N.A./0.45/0.38)	(N.A./0.44/0.42)
V-Hist	13.31	9.48	9.36	0.51	0.06
	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.55/N.A./0.43)	(0.56/N.A./0.47)
V-Shri	13.28	9.44	9.35	0.52	0.06
	(0.00/0.01/0.00)	(0.00/0.01/0.00)	(0.00/0.27/0.00)	(0.53/0.12/0.40)	(0.53/0.15/0.45)
V-Hybr	13.02	9.32	9.11	0.57	**0.07**
	(0.00/0.11/0.00)	(0.00/0.28/0.00)	(0.00/0.07/0.00)	(0.39/0.30/0.30)	(0.40/0.30/0.34)
V-Impl	19.87	14.48	13.62	0.45	0.06
	(0.01/1.00/N.A.)	(0.18/1.00/N.A.)	(0.00/1.00/N.A.)	(0.62/0.57/N.A.)	(0.58/0.53/N.A.)
GV-Hist	13.75	9.81	9.65	0.44	0.05
	(0.00/0.99/0.00)	(0.00/0.95/0.00)	(0.00/0.96/0.00)	(0.70/0.78/0.59)	(0.69/0.76/0.61)
GV-Hybr	13.90	10.17	9.50	**0.65**	0.07
	(0.00/0.96/0.00)	(0.00/0.95/0.00)	(0.00/0.71/0.00)	(0.23/0.16/0.17)	(0.22/0.15/0.18)
BV-Hist	14.13	10.16	9.84	0.64	0.07
	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.22/0.07/0.18)	(0.23/0.07/0.21)
BV-Hybr	**12.62**	**9.15**	**8.71**	0.56	0.07
	(0.00/0.01/0.00)	(0.00/0.17/0.00)	(0.00/0.00/0.00)	(0.43/0.37/0.33)	(0.41/0.35/0.35)

Table 3.7: Portfolio performance: 300 random stocks.

This table provides the out-of-sample performance of the portfolio strategies. Compared to Table 3.2, the investment universe consists of 300 S&P 500 stocks selected at random each month. $\hat{\sigma}$ is the annualized sample volatility, $\hat{\sigma}^g$ is the annualized good volatility, $\hat{\sigma}^b$ is the annualized bad volatility, 'Sharpe' is the Sharpe ratio and 'Sortino' is the Sortino ratio. All volatility measures are displayed in percentage terms. In each of the first three (last two) columns, the lowest (highest) number is shown in bold. The parentheses include the bootstrapped p-values for the null hypothesis that the portfolio strategy performs equally as good as the respective benchmark ($1/N$, **V-Hist** or **V-Impl**, presented in this order). The sample period goes from January 1996 to October 2022.

	$\hat{\sigma}$	$\hat{\sigma}^g$	$\hat{\sigma}^b$	Sharpe	Sortino
$1/N$	20.82	14.81	14.65	0.48	0.06
	(N.A./1.00/0.70)	(N.A./1.00/0.34)	(N.A./1.00/0.88)	(N.A./0.44/0.63)	(N.A./0.44/0.66)
V-Hist	12.94	9.20	9.11	0.49	0.06
	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.56/N.A./0.66)	(0.56/N.A./0.68)
V-Shri	12.90	9.17	9.09	0.51	0.06
	(0.00/0.00/0.00)	(0.00/0.04/0.00)	(0.00/0.00/0.00)	(0.53/0.06/0.64)	(0.53/0.07/0.66)
V-Hybr	12.32	8.86	8.59	0.62	0.07
	(0.00/0.01/0.00)	(0.00/0.13/0.00)	(0.00/0.00/0.00)	(0.23/0.13/0.38)	(0.23/0.13/0.42)
V-Impl	20.55	15.03	14.04	0.54	0.06
	(0.30/1.00/N.A.)	(0.66/1.00/N.A.)	(0.12/1.00/N.A.)	(0.37/0.34/N.A.)	(0.34/0.32/N.A.)
GV-Hist	13.29	9.55	9.25	0.52	0.06
	(0.00/0.92/0.00)	(0.00/0.90/0.00)	(0.00/0.73/0.00)	(0.48/0.39/0.59)	(0.47/0.37/0.61)
GV-Hybr	15.13	11.49	9.89	**0.82**	**0.09**
	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.00/0.79/0.00)	(0.05/0.03/0.07)	(0.04/0.03/0.06)
BV-Hist	13.81	9.95	9.59	0.67	0.07
	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.15/0.02/0.28)	(0.15/0.02/0.32)
BV-Hybr	**11.93**	**8.77**	**8.11**	0.71	0.08
	(0.00/0.00/0.00)	(0.00/0.11/0.00)	(0.00/0.00/0.00)	(0.13/0.08/0.23)	(0.12/0.07/0.24)

Table 3.8: Portfolio performance: 400 random stocks.

This table provides the out-of-sample performance of the portfolio strategies. Compared to Table 3.2, the investment universe consists of 400 S&P 500 stocks selected at random each month. $\hat{\sigma}$ is the annualized sample volatility, $\hat{\sigma}^g$ is the annualized good volatility, $\hat{\sigma}^b$ is the annualized bad volatility, 'Sharpe' is the Sharpe ratio and 'Sortino' is the Sortino ratio. All volatility measures are displayed in percentage terms. In each of the first three (last two) columns, the lowest (highest) number is shown in bold. The parentheses include the bootstrapped p-values for the null hypothesis that the portfolio strategy performs equally as good as the respective benchmark ($1/N$, **V-Hist** or **V-Impl**, presented in this order). The sample period goes from January 1996 to October 2022.

	$\hat{\sigma}$	$\hat{\sigma}^g$	$\hat{\sigma}^b$	Sharpe	Sortino
$1/N$	20.75	14.76	14.61	0.48	0.06
	(N.A./1.00/0.00)	(N.A./1.00/0.00)	(N.A./1.00/0.08)	(N.A./0.45/0.22)	(N.A./0.45/0.29)
V-Hist	12.56	8.91	8.87	0.50	0.06
	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.55/N.A./0.26)	(0.55/N.A./0.33)
V-Shri	12.54	8.90	8.85	0.51	0.06
	(0.00/0.05/0.00)	(0.00/0.21/0.00)	(0.00/0.05/0.00)	(0.51/0.10/0.24)	(0.52/0.10/0.31)
V-Hybr	11.94	8.56	8.35	0.65	0.07
	(0.00/0.00/0.00)	(0.00/0.09/0.00)	(0.00/0.00/0.00)	(0.17/0.09/0.08)	(0.18/0.10/0.13)
V-Impl	23.54	17.47	15.78	0.33	0.05
	(1.00/1.00/N.A.)	(1.00/1.00/N.A.)	(0.92/1.00/N.A.)	(0.78/0.74/N.A.)	(0.71/0.67/N.A.)
GV-Hist	12.94	9.34	8.97	0.53	0.06
	(0.00/0.95/0.00)	(0.00/0.96/0.00)	(0.00/0.69/0.00)	(0.47/0.38/0.24)	(0.45/0.35/0.28)
GV-Hybr	15.03	11.34	9.89	0.76	**0.09**
	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.00/0.91/0.00)	(0.08/0.06/0.02)	(0.08/0.06/0.03)
BV-Hist	13.55	9.76	9.42	0.58	0.07
	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.33/0.18/0.15)	(0.33/0.17/0.21)
BV-Hybr	**11.48**	**8.43**	**7.82**	**0.79**	0.09
	(0.00/0.00/0.00)	(0.00/0.06/0.00)	(0.00/0.00/0.00)	(0.06/0.03/0.02)	(0.06/0.03/0.04)

Table 3.9: Portfolio performance: good and bad times.

This table provides the out-of-sample performance of the portfolio strategies divided into two subsamples: good and bad times. $\hat{\sigma}$ is the annualized sample volatility, $\hat{\sigma}^g$ is the annualized good volatility, $\hat{\sigma}^b$ is the annualized bad volatility, 'Sharpe' is the Sharpe ratio and 'Sortino' is the Sortino ratio. All volatility measures are displayed in percentage terms. In each of the first three (last two) columns, the lowest (highest) number is shown in bold. The parentheses include the bootstrapped p-values for the null hypothesis that the portfolio strategy performs equally as good as the respective benchmark ($1/N$, **V-Hist** or **V-Impl**, presented in this order). Bad times are defined from April 2000 to March 2003, from May 2007 to December 2009 and from February 2020 to April 2020. The rest of the main sample, January 1996 to October 2022, is defined as good times.

	$\hat{\sigma}$	$\hat{\sigma}^g$	$\hat{\sigma}^b$	Sharpe	Sortino
Panel A: Good Times					
1/N	15.86	11.44	11.04	0.97	0.10
	(N.A./1.00/0.00)	(N.A./1.00/0.00)	(N.A./1.00/0.00)	(N.A./0.29/0.03)	(N.A./0.27/0.05)
V-Hist	10.13	7.23	7.13	0.88	0.09
	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.71/N.A./0.11)	(0.73/N.A./0.15)
V-Shri	10.10	7.22	7.10	0.90	0.09
	(0.00/0.00/0.00)	(0.00/0.04/0.00)	(0.00/0.01/0.00)	(0.69/0.16/0.10)	(0.71/0.15/0.13)
V-Hybr	9.29	6.79	6.39	1.27	0.13
	(0.00/0.00/0.00)	(0.00/0.00/0.00)	(0.00/0.00/0.00)	(0.04/0.00/0.00)	(0.05/0.00/0.00)
V-Impl	17.70	12.82	12.23	0.59	0.07
	(1.00/1.00/N.A.)	(1.00/1.00/N.A.)	(1.00/1.00/N.A.)	(0.97/0.89/N.A.)	(0.95/0.85/N.A.)
GV-Hist	10.11	7.33	6.99	0.86	0.09
	(0.00/0.46/0.00)	(0.00/0.84/0.00)	(0.00/0.22/0.00)	(0.71/0.56/0.16)	(0.69/0.51/0.18)
GV-Hybr	10.00	7.55	6.63	1.37	0.14
	(0.00/0.23/0.00)	(0.00/0.98/0.00)	(0.00/0.00/0.00)	(0.03/0.00/0.00)	(0.02/0.00/0.00)
BV-Hist	10.67	7.80	7.31	0.88	0.09
	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.00/0.95/0.00)	(0.71/0.51/0.12)	(0.67/0.43/0.14)
BV-Hybr	**8.88**	**6.67**	**5.93**	**1.46**	**0.15**
	(0.00/0.00/0.00)	(0.00/0.00/0.00)	(0.00/0.00/0.00)	(0.01/0.00/0.00)	(0.01/0.00/0.00)
Panel B: Bad Times					
1/N	32.54	22.86	23.15	-0.22	0.00
	(N.A./1.00/0.07)	(N.A./1.00/0.03)	(N.A./1.00/0.49)	(N.A./0.72/0.85)	(N.A./0.72/0.85)
V-Hist	18.23	12.91	12.87	0.00	0.02
	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.00/N.A./0.00)	(0.28/N.A./0.69)	(0.28/N.A./0.69)
V-Shri	18.25	12.89	12.91	0.00	0.02
	(0.00/0.69/0.00)	(0.00/0.42/0.00)	(0.00/0.98/0.00)	(0.27/0.40/0.68)	(0.27/0.41/0.69)
V-Hybr	16.76	11.92	11.78	-0.20	0.00
	(0.00/0.01/0.00)	(0.00/0.09/0.00)	(0.00/0.01/0.00)	(0.59/0.83/0.85)	(0.59/0.83/0.85)
V-Impl	34.74	25.78	23.28	0.09	0.03
	(0.93/1.00/N.A.)	(0.97/1.00/N.A.)	(0.51/1.00/N.A.)	(0.15/0.31/N.A.)	(0.15/0.31/N.A.)
GV-Hist	19.39	13.78	13.63	0.09	0.03
	(0.00/0.97/0.00)	(0.00/0.93/0.00)	(0.00/0.89/0.00)	(0.23/0.30/0.57)	(0.23/0.30/0.57)
GV-Hybr	24.48	18.61	15.91	**0.29**	**0.05**
	(0.00/1.00/0.00)	(0.06/1.00/0.00)	(0.00/0.93/0.00)	(0.12/0.20/0.34)	(0.11/0.20/0.32)
BV-Hist	20.19	14.24	14.31	0.14	0.03
	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.00/1.00/0.00)	(0.14/0.15/0.52)	(0.14/0.15/0.53)
BV-Hybr	**16.02**	**11.74**	**10.89**	0.03	0.02
	(0.00/0.00/0.00)	(0.00/0.09/0.00)	(0.00/0.00/0.00)	(0.31/0.47/0.66)	(0.31/0.47/0.65)

Table 3.10: Turnover, transaction costs and concentration.

This table provides transaction costs, turnover and concentration statistics for the portfolio strategies. ETC is the equivalent transaction cost and is calculated as the transaction cost in bps that equates the Sharpe or Sortino ratio of the portfolio strategy with that of the respective benchmark ($1/N$, **V-Hist** or **V-Impl**). If the transaction cost adjusted performance of the portfolio strategy is better (worse) than that of the benchmark for each transaction cost level from 1 to 100 bps, it is displayed as '+' ('-'). Turnover is the average sum of absolute changes in portfolio weights from each monthly rebalancing date to the next and is displayed in percentage terms. HI is the Herfindahl index and is calculated as the sum of squared weights. The sample period goes from January 1996 to October 2022.

		ETC_{Sharpe}			$ETC_{Sortino}$			
	Turnover	vs. $1/N$	vs. V-Hist	vs. V-Impl	vs. $1/N$	vs. V-Hist	vs. V-Impl	HI
1/N	1.57	N.A.	-	+	N.A.	-	+	0.67
V-Hist	52.72	14	N.A.	+	17	N.A.	+	27.35
V-Shri	50.39	16	+	+	19	+	+	24.84
V-Hybr	94.18	23	32	+	25	34	+	35.39
V-Impl	178.57	-	-	N.A.	-	-	N.A.	92.32
GV-Hist	54.61	15	+	+	18	+	+	70.07
GV-Hybr	120.57	32	51	+	37	55	+	64.93
BV-Hist	51.16	22	+	+	26	+	+	53.23
BV-Hybr	114.89	32	44	+	35	47	+	80.43

Table 3.11: Equity factor risk loadings.

This table provides standard regression results (coefficient estimates, t-statistics and adjusted R^2s) of out-of-sample returns of each portfolio strategy on the Fama-French 5 (Fama and French, 2015) plus momentum factors and the betting against beta factor of Frazzini and Pedersen (2014). Panel A excludes the latter factor while Panel B includes it in the regressions. MKT is the excess return of the market, SMB is the size factor, HML is the value factor, RMW is the profitability factor, CMA is the investment factor, UMD is the momentum factor and BAB is the betting against beta factor. The intercept coefficients α are displayed in percentage terms. The t-statistics are calculated with Newey and West (1987) standard errors and are reported in parentheses. The sample period goes from January 1996 to October 2022.

	α	MKT	SMB	HML	RMW	CMA	UMD	BAB	Adj. R^2
Panel A: FF5 + UMD									
$1/N$	0.105	1.015	0.086	0.147	0.172	0.134	-0.161		0.96
	(1.75)	(54.30)	(2.43)	(4.12)	(3.88)	(3.02)	(-6.21)		
V-Hist	-0.051	0.572	0.015	-0.047	0.343	0.311	0.025		0.58
	(-0.45)	(18.32)	(0.25)	(-0.75)	(5.20)	(3.84)	(0.55)		
V-Shri	-0.046	0.576	0.015	-0.044	0.344	0.311	0.022		0.59
	(-0.42)	(19.06)	(0.25)	(-0.72)	(5.24)	(3.82)	(0.51)		
V-Hybr	0.205	0.501	-0.030	-0.012	0.149	0.166	0.028		0.49
	(1.85)	(13.75)	(-0.63)	(-0.18)	(1.95)	(1.74)	(0.70)		
V-Impl	0.094	0.700	0.183	0.057	0.469	0.203	-0.188		0.50
	(0.46)	(11.36)	(1.64)	(0.55)	(4.11)	(1.37)	(-1.94)		
GV-Hist	-0.001	0.479	-0.024	0.012	0.294	0.371	0.113		0.41
	(0.00)	(10.77)	(-0.41)	(0.18)	(4.55)	(4.71)	(2.42)		
GV-Hybr	0.514	0.466	0.082	-0.042	0.328	0.252	-0.152		0.39
	(3.69)	(9.82)	(0.78)	(-0.43)	(2.74)	(1.58)	(-1.18)		
BV-Hist	0.062	0.559	-0.042	0.003	0.344	0.321	-0.015		0.48
	(0.45)	(15.05)	(-0.56)	(0.04)	(4.98)	(3.67)	(-0.31)		
BV-Hybr	0.396	0.410	-0.094	0.021	0.146	0.171	0.056		0.36
	(3.42)	(8.57)	(-1.80)	(0.29)	(1.87)	(1.61)	(1.59)		
Panel B: FF5 + UMD + BAB									
$1/N$	0.086	1.016	0.078	0.134	0.133	0.128	-0.175	0.061	0.96
	(1.47)	(58.04)	(2.23)	(3.93)	(3.16)	(2.67)	(-7.59)	(3.11)	
V-Hist	-0.114	0.575	-0.008	-0.088	0.217	0.290	-0.021	0.197	0.62
	(-0.93)	(19.69)	(-0.16)	(-1.32)	(3.61)	(3.76)	(-0.47)	(4.37)	
V-Shri	-0.110	0.578	-0.009	-0.086	0.217	0.290	-0.023	0.198	0.63
	(-0.93)	(20.77)	(-0.18)	(-1.30)	(3.70)	(3.76)	(-0.54)	(4.48)	
V-Hybr	0.153	0.502	-0.050	-0.046	0.045	0.149	-0.009	0.163	0.52
	(1.39)	(13.95)	(-1.00)	(-0.72)	(0.55)	(1.51)	(-0.24)	(3.66)	
V-Impl	0.059	0.701	0.170	0.034	0.399	0.192	-0.214	0.109	0.50
	(0.28)	(11.56)	(1.51)	(0.33)	(3.08)	(1.22)	(-2.38)	(1.50)	
GV-Hist	-0.067	0.481	-0.049	-0.032	0.161	0.349	0.065	0.208	0.46
	(-0.45)	(11.47)	(-0.86)	(-0.47)	(2.55)	(4.65)	(1.67)	(3.93)	
GV-Hybr	0.465	0.468	0.064	-0.075	0.231	0.236	-0.188	0.152	0.40
	(3.25)	(9.83)	(0.59)	(-0.90)	(1.83)	(1.56)	(-1.55)	(2.69)	
BV-Hist	-0.002	0.562	-0.065	-0.040	0.216	0.300	-0.061	0.200	0.52
	(-0.01)	(16.15)	(-0.95)	(-0.63)	(2.79)	(3.76)	(-1.34)	(3.57)	
BV-Hybr	0.357	0.412	-0.109	-0.005	0.066	0.158	0.027	0.124	0.38
	(2.81)	(8.16)	(-2.21)	(-0.07)	(0.84)	(1.56)	(0.80)	(2.31)	

Table 3.12: Market moment factor loadings (1).

This table provides standard regression results (coefficient estimates, t-statistics and adjusted R^2s) of out-of-sample excess returns of each portfolio strategy on innovations in implied market volatility $\Delta\sqrt{IV_m^*}$ (Panel A), and on innovations in implied market skewness ΔIS_m^* (Panel B). These market moments are constructed as suggested by Chang et al. (2013). The intercept coefficients α are displayed in percentage terms. The t-statistics are calculated with Newey and West (1987) standard errors and are reported in parentheses. The sample period goes from January 1996 to October 2022.

	α	$\Delta\sqrt{IV_m^*}$	ΔIS_m^*	Adj. R^2
Panel A: $\Delta\sqrt{IV_m^*}$				
$1/N$	0.891	-0.751		0.55
	(4.54)	(-18.17)		
V-Hist	0.556	-0.348		0.31
	(3.91)	(-10.70)		
V-Shri	0.562	-0.352		0.32
	(3.98)	(-10.89)		
V-Hybr	0.653	-0.305		0.26
	(5.21)	(-10.13)		
V-Impl	0.776	-0.474		0.22
	(3.37)	(-7.60)		
GV-Hist	0.571	-0.247		0.16
	(3.75)	(-5.79)		
GV-Hybr	0.972	-0.299		0.16
	(5.23)	(-5.81)		
BV-Hist	0.647	-0.375		0.30
	(4.19)	(-11.72)		
BV-Hybr	0.785	-0.233		0.18
	(6.28)	(-6.28)		
Panel B: ΔIS_m^*				
$1/N$	0.842		-0.043	0.03
	(3.20)		(-3.37)	
V-Hist	0.534		-0.018	0.01
	(3.41)		(-2.07)	
V-Shri	0.541		-0.018	0.01
	(3.44)		(-2.13)	
V-Hybr	0.634		-0.017	0.01
	(4.59)		(-1.80)	
V-Impl	0.750		-0.016	0.00
	(2.79)		(-1.28)	
GV-Hist	0.555		-0.014	0.01
	(3.58)		(-1.66)	
GV-Hybr	0.953		-0.017	0.01
	(4.75)		(-1.44)	
BV-Hist	0.624		-0.018	0.01
	(3.69)		(-2.05)	
BV-Hybr	0.769		-0.018	0.01
	(6.58)		(-2.14)	

Table 3.13: Market moment factor loadings (2).

This table provides standard regression results (coefficient estimates, t-statistics and adjusted R^2s) of out-of-sample excess returns of each portfolio strategy on innovations in implied market kurtosis ΔIK_m^* (Panel A), and on innovations in implied market volatility $\Delta\sqrt{IV_m^*}$, implied market skewness ΔIS_m^* and ΔIK_m^* together (Panel B). These market moments are constructed as suggested by Chang et al. (2013). The intercept coefficients α are displayed in percentage terms. The t-statistics are calculated with Newey and West (1987) standard errors and are reported in parentheses. The sample period goes from January 1996 to October 2022.

	α	$\Delta\sqrt{IV_m^*}$	ΔIS_m^*	ΔIK_m^*	Adj. R^2
Panel A: ΔIK_m^*					
$1/N$	0.820			0.025	0.12
	(3.26)			(5.61)	
V-Hist	0.524			0.011	0.06
	(3.44)			(4.21)	
V-Shri	0.530			0.011	0.06
	(3.48)			(4.36)	
V-Hybr	0.624			0.010	0.06
	(4.63)			(3.60)	
V-Impl	0.733			0.014	0.04
	(2.82)			(4.38)	
GV-Hist	0.547			0.009	0.04
	(3.58)			(4.15)	
GV-Hybr	0.943			0.010	0.04
	(4.86)			(3.70)	
BV-Hist	0.613			0.012	0.05
	(4.00)			(5.25)	
BV-Hybr	0.761			0.009	0.05
	(6.63)			(3.90)	
Panel B: $\Delta\sqrt{IV_m^*} + \Delta IS_m^* + \Delta IK_m^*$					
$1/N$	0.873	-0.690	0.025	0.016	0.56
	(4.49)	(-15.25)	(0.95)	(2.04)	
V-Hist	0.548	-0.320	0.014	0.007	0.31
	(3.77)	(-8.22)	(0.65)	(1.23)	
V-Shri	0.555	-0.324	0.014	0.008	0.32
	(3.84)	(-8.50)	(0.66)	(1.26)	
V-Hybr	0.645	-0.269	0.021	0.010	0.27
	(5.04)	(-8.16)	(0.95)	(1.62)	
V-Impl	0.764	-0.407	0.059	0.020	0.23
	(3.20)	(-4.43)	(1.68)	(1.90)	
GV-Hist	0.564	-0.220	0.015	0.007	0.16
	(3.78)	(-4.89)	(0.70)	(1.29)	
GV-Hybr	0.964	-0.266	0.018	0.009	0.16
	(5.32)	(-4.05)	(0.45)	(0.85)	
BV-Hist	0.640	-0.345	0.018	0.008	0.30
	(4.30)	(-9.24)	(0.76)	(1.30)	
BV-Hybr	0.778	-0.209	0.006	0.006	0.18
	(6.78)	(-4.74)	(0.28)	(0.97)	

Chapter 4

Vol, Skew, and Smile Trading

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