Complicated Dynamics
in a
Flow Model of the Labor Market

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Abstract

We develop a worker flow model with a nonlinear and endogenous outflow rate from unemployment. Inconsistent claims on the output lead to changing inflation rates which feedback on job offers through the real money supply. Via simulations one can show that the nonlinear outflow rate causes asymmetric adjustment of unemployment to the 'equilibrium rate of unemployment'. In addition, and depending on the parameters, the 'equilibrium rate of unemployment' may also become locally unstable. Then, there is a downward sloping Phillips curve but no trade-off between unemployment and inflation in the short run, as there is none in the long run.

Keywords: Natural rate theory, nonlinearity, asymmetric adjustment, endogenous cycles, chaos
JEL-Classification: E24, J64.

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1 Introduction

The paper develops a flow model of the labor market with wage bargaining that can generate asymmetric persistence of unemployment rates and, depending on the parameters, endogenous cycles. We understand asymmetric persistence in the following way: if the economy is hit by shocks of the same size but different signs, unemployment rates fall rather slowly after a negative (adverse) shock that raised unemployment, but return comparably fast to the long run level after a positive (favorable) shock. If we change parameters, we get a second case, where the 'equilibrium rate of unemployment' is locally unstable. Unemployment permanently overshoots the long run rate. Adjustment processes never work themselves out and may even become very complicated. In both cases, dynamics are driven by a nonlinear and convex outflow rate from unemployment into employment.

In one of the first papers on asymmetric economic time series Neftci (1984) claims that U.S. time series on unemployment are characterized by sudden jumps of unemployment rates but slower drops. In the following years, it has been argued by various authors (cf. Rothman (1998), Koop and Potter (1999), Coakley et al. (2000)) that nonlinear time series models cope better with asymmetric behavior than their linear counterparts. Forecasts on the grounds of nonlinear models would yield, at least in some cases, better results than linear models do.

Support for a nonlinear deterministic core in time series data for labor markets comes from Brock and Sayers (1988), Alogoskoufis and Stengos (1991), and Neugart (1999). However, while it is known that once nonlinearities enter economic models a whole range of dynamic behavior can emerge (i.e. Day (1994), Lorenz (1997), or Day (2000)), these econometric studies cannot reveal the type of nonlinearity that is responsible for the dynamic behavior. With respect to labor market flows, it was Mortensen (1999) who only recently brought attention to the possibility of endogenous cycles in flow models of the labor market.

We will make a rather simple argument that might explain asymmetric adjustment paths of unemployment rates to exogenous shocks or even endogenous cycles. After an adverse shock, unemployment may only gradually converge to the 'equilibrium rate of unemployment,' as comparably many workers compete for jobs. Hence, the outflow rate is smaller than in equilibrium and it is less likely that an unemployed person finds a new job. Outflows, the outflow rate times unemployment, exceed inflows into unemployment, but only to a small extent. Thus, unemployment drops only slowly to the long run level. Contrary, unemployment rates return to the long run equilibrium within rather short periods after favorable shocks. Now, even though the outflow rate is higher, inflows exceed outflows as employment is higher than in equilibrium.

The next section sets out the model. It starts with a basic flow equation for the labor market where the outflow rate from unemployment is endogenous. The denominator of the outflow rate consists of unemployed workers who all search for a job and a fraction of employed workers who search on-the-job. In a next step we derive the numerator of the outflow rate that is the number of jobs that comes to the market every period. We do not model the matching process. However, jobs are not fixed either. There is a cyclical component to job creation. When the economy is not at its equilibrium rate of unemployment, claims on the output are inconsistent. That leads to changing prices which feedback on job offers through the real money supply. At the end of section 2 we arrive at a two dimensional, nonlinear, first order difference equation in unemployment and inflation. The dynamics of that model are analyzed in section 3. The last section closes and gives policy implications.

2 The model

2.1 Worker flows

We start from the following basic identity: changes in unemployment occur if inflows into unemployment do not match outflows from unemployment into employment, or

$$U_{t+1} - U_t = i \cdot (L - U_t) - o_t \cdot U_t, \tag{1}$$

where U denotes unemployment, i is the inflow rate, $L = 1$ stands for the labor force, and o_t is the outflow rate from unemployment. Inflows into unemployment shall be due to some kind of reallocation. Jobs go sour because of structural shifts and people lose their jobs. Workers may also decide to quit. The rate at which inflows occur is assumed to be exogenous at this stage (with $i > 0$). This is not to state that the inflow rate is constant over the cycle. One may easily think of a feedback mechanism from wages on the inflow rate. In a tighter labor market wages may rise making jobs less profitable. Also, workers may be more inclined to quit jobs when reemployment prospects are good as unemployment is low. However, holding the inflow rate constant eases the analytical treatment of the model. In addition, when analyzing the dynamics of the model we can derive some results by varying the inflow rate.[1] We put most of our efforts into modelling the outflow rate from unemployment. That is defined as the fraction of jobs that comes to the market at time t and the people who want to get one of these jobs. Job offers consist of a constant number that roots in the structural characteristics of the economy and a cyclical component. The cyclical component shall be driven by changes in the real money supply, that is the difference in the money growth rate and the inflation rate. The inflation rate out of equilibrium is determined by inconsistent claims on aggregate income of workers and firms and expected inflation rates.

2.2 The outflow rate from unemployment

There is a considerable body of evidence for on-the-job search. Rosenfeld (1977) finds that 4.2% of the employed workers search. In Black (1981) and Pissarides and Wadsworth (1994) that ratio is 5.5% and 5.2% respectively. Hartog and van Ophem (1996) report on-the-job search ratios that go up to 25%. Rather high on-the-job search rates can also be found in Parsons (1991) for young men (18.9%) and young women (15.9%). Additional evidence for on-the-job search comes from measures of job-to-job mobility. A comparative study of van Ours (1990) finds job-to-job mobility rates at 5% of employees in the Netherlands, 9% in the U.K., 10.4% in France, 12.2% in Sweden, and 6% in Japan for data from 1985. Yearly job-to-job flows as a percentage of employment are between 6.2% and 18.4% for the 13 countries Boeri (1999) presents estimates for. Hence, we model an outflow rate from unemployment that takes on-the-job searchers into account. This is in line with a number of other flow models of the labor market (see i.e. Burdett (1978), Pissarides (1994), or Burgess (1994)).

We depart from the standard approach (cf. Pissarides (1990)) by not modelling the matching process of vacancies and job searchers explicitly. Skipping vacancies, we can keep the system down to two dimensions, allowing some analytical treatment of a model in inflation and unemployment. Still,

[1] In the appendix we make the inflow rate endogenous. There, it is a linear function of the unemployment rate. We present simulations for two cases, $\partial i_t/\partial U_t > 0$ and $\partial i_t/\partial U_t < 0$, to compare in some numerical examples the properties with the case that is elaborated more thoroughly throughout the paper.

rather than setting job creation as fixed, we allow for a cyclical component to job creation. It follows that the shape of the outflow rate does not differ from other flow models. As in those cases, the outflow rate written as the fraction of jobs that come to the market at time t to jobs searchers

$$o_t = \frac{J_t}{U_t + d \cdot (1 - U_t)} \tag{2}$$

is convex to the origin. The parameter $0 < d < 1$ gives on-the-job searchers as a constant fraction of employed workers.[2] J_t denotes job creations. With such an outflow rate from unemployment there are two congestion effects. Given a certain number of jobs in the economy it is less likely that an unemployed worker will leave unemployment if there is an increasing number of other unemployed workers who want to have one of these jobs or the unemployed worker has to compete with an increasing number of on-the-job searchers. Thus, on-the-job searchers can crowd out unemployed job searchers from finding jobs. Next we model job creations J_t of firms.

There shall be a constant number of jobs J_s that are filled by firms. It is the number of job creations that originate from the structural characteristics of the economy. In addition to the structural component there is a feedback on jobs from the real money supply that drives the cyclical component $J_{c,t}$. Job creations J_t shall be the sum of the structural J_s and the cyclical component

$$J_t = J_s + J_{c,t}. \tag{3}$$

From the quantity relationship we find that aggregate income changes if the money growth rate does not equal the inflation rate. With a constant marginal labor productivity y there is a proportional relationship between output and employment, so that we can write the cyclical component of job creations as $J_{c,t} = \gamma \cdot (m - \pi_t)$ where $\gamma > 0$ is a parameter, m the exogenous money growth rate, and π_t the inflation rate at time t. Hence, if workers' wage claims and expected inflation cause an inflation rate that is higher than the money growth rate, real money supply declines and will show up in a reduction of jobs. Vice versa, more jobs will be created if the inflation

[2] In Equation (2) all the unemployed and a constant fraction d of the employed workers are looking for a job. In Neugart (2000) we analyze a model with an outflow rate from unemployment where the parameter d itself is a function of the outflow rate. For ease of analytical treatment this feature, for which some evidence exists (cf. Pissarides and Wadsworth (1994) and Broersma and van Ours (1999)), is omitted here.

rate falls short of the money growth rate, increasing the real money supply. Finally, we get the outflow rate from unemployment

$$o_t = \frac{J_s + \gamma \cdot (m - \pi_t)}{U_t + d \cdot (1 - U_t)}. \tag{4}$$

2.3 Prices and expectations

The inflation rate π_t shall follow from the assumptions that nominal wage increases are driven by the wage gap $\frac{w_b - w_p}{w_p}$, where w_b is the bargained real wage and w_p is the price determined real wage, and the expected inflation rate. Furthermore firms can only change prices by a fraction of nominal wage changes at time t.

The part of the inflation rate that is driven by the wage gap can be explained as follows. With relatively low unemployment rates, workers find themselves in a good bargaining position. The bargained real wage will exceed the price determined real wage, and consequently firms will try to capture their share of the output by raising prices. On the other hand, if unemployment is rather high, workers face a bad bargaining position. The price determined real wage will be higher than the bargained real wage. As claims on the output fall short of the latter, prices will decline.

A price determined real wage function can be derived from an imperfect competition model where firms maximize profits $G = P(h(E)) \cdot h(E) - W \cdot E$ (cf. Carlin and Soskice (1990)) where P is the price level, E denotes labor input, h is a production function, and W stands for the nominal wage. This yields the price determined real wage w_p as

$$w_p = (1 - \mu) \cdot y \tag{5}$$

where $\mu \geq 0$ is the inverse of the demand elasticity, or the mark-up, and y is the constant marginal labor productivity that will be normalized to one in the following.

Usually, the bargained real wage is derived from value functions for workers having a job or alternatively being unemployed, and the value for a firm from having a job filled. A bargained real wage divides that surplus between the two parties (see Pissarides (1990)). Generally, the resulting wage bargaining curve is a decreasing function of the unemployment rate as higher unemployment rates ease the filling of jobs for firms and make it more costly for workers to become unemployed as their re-employment chances are lower.

Hence, the bargaining power shifts towards firms as the unemployment rate grows higher. We will use the following simplified function for the bargained real wage

$$w_{b,t} = 1 - (1 - b) \cdot U_t. \tag{6}$$

If there is no unemployment workers bargain for a real wage $w_{b,t}$ that equals the marginal labor productivity ($y = 1$). When all workers are unemployed the bargained real wage will equal the reservation wage $0 < b < 1$, that can be thought of as being determined by the unemployment benefits or unemployment assistance system of a country, or income from black market activity.

As a positive wage gap will result in rising prices, workers have to think about how much to bargain for in nominal terms to get 'their' share of the output. Thus, they have to predict the inflation rate of the following period. Expected inflation rates π_e of the following period shall be a weighted average of the actual inflation rate and the predicted inflation rate

$$\pi_{e,t+1} = a \cdot \pi_t + (1 - a) \cdot \pi_{e,t}. \tag{7}$$

We are aware that such a model of our agents' behavior can be questioned on the grounds of rational behavior. Workers may systematically over- or underestimate inflation rates. However, it has been shown recently (by e.g. Hommes (1998)) that if models with backward looking expectations become chaotic, even agents who only use past information may act rationally.

Finally, we assume that firms cannot raise prices to the same extent as nominal wages increase at time t. That is why $\delta > 1$ enters. Then, the inflation rate can be written as

$$\pi_t = \frac{1}{\delta} \cdot (\pi_{e,t} + \frac{w_{b,t} - w_p}{w_p}), \tag{8}$$

where the sum of the expected inflation rate $\pi_{e,t}$ and the wage gap stand for the nominal wage increase that translates into the inflation rate. Substituting out from Equation (8) the price determined real wage (Equation 6) and the bargained real wage (Equation 5), one reaches

$$\pi_t = \frac{1}{\delta} \cdot (\pi_{e,t} + \frac{\mu - (1 - b) \cdot U_t}{1 - \mu}). \tag{9}$$

Now, we are in a position to put all the pieces together. The identity in Equation (1) determines unemployment dynamics with the outflow rate o_t

as given by Equation (4). Thus, one finds the unemployment rate U_{t+1} as a function of the current unemployment rate and the current inflation rate

$$U_{t+1} = f(U_t, \pi_t). \tag{10}$$

The second dynamic equation is derived in the following way: time is shifted forward in Equation (9) so that π_{t+1} becomes a function of $\pi_{e,t+1}$ and U_{t+1}. For the latter we use expression (10) and for $\pi_{e,t+1}$ the equation on expectations formation (7) where $\pi_{e,t}$ comes from Equation (9). That gives inflation π_{t+1} as a function of current inflation and current unemployment

$$\pi_{t+1} = g(U_t, \pi_t). \tag{11}$$

We assume that if all adjustment processes have worked themselves out ($\pi_{t+1} = \pi_t$ and $U_{t+1} = U_t$), actual inflation will equal the money growth rate $\pi^* = m$. From that we develop an expression for J_s.

3 Analysis of the model

The full model writes

$$f(U_t, \pi_t) = U_t + i \cdot (1 - U_t) - U_t \cdot \frac{J_s + \gamma \cdot (m - \pi_t)}{U_t + d \cdot (1 - U_t)} \tag{12}$$

and

$$\begin{aligned} g(U_t, \pi_t) = &\frac{1}{\delta} \cdot (\frac{\mu}{1-\mu} + a \cdot \pi_t + (1-a) \cdot (\delta \cdot \pi_t - \frac{\mu - (1-b) \cdot U_t}{1-\mu}) \\ &- \frac{1-b}{1-\mu} \cdot (U_t + i \cdot (1 - U_t) - U_t \cdot \frac{J_s + \gamma \cdot (m - \pi_t)}{U_t + d \cdot (1 - U_t)})). \end{aligned} \tag{13}$$

J_s follows from the assumption that in steady state inflation π equals the money growth rate m. Thus, the structural component of job creations becomes $J_s = i \cdot \frac{(1-U^*) \cdot (U^* + d \cdot (1-U^*))}{U^*}$. Our simulations of the model, made by using suitable values of the parameters and presented in more detail later on, show that feasible trajectories exist. Therefore, let us now elaborate existence and uniqueness of the steady state, as well as the stability properties of the model before we turn to the numerical examples.

As it is imposed that $\pi^* = m$ when all adjustment processes have worked themselves out, a steady state p

$$[U^*, \pi^*] = [\frac{\mu - m \cdot (\delta - 1) \cdot (1 - \mu)}{1 - b}, m], \tag{14}$$

exists, provided that $0 \leq \frac{\mu - m \cdot (\delta - 1) \cdot (1 - \mu)}{1 - b} \leq 1$.[3] A sufficient condition for the second inequality to hold, given that the money growth rate is positive, is that $b \leq 1 - \mu = w_p$. This means that the reservation wage is not higher than the price-determined real wage.

For showing that the steady state is unique we refer to the formulation of the dynamic system in *expected* inflation and unemployment, $\overline{F}(\overline{f}, \overline{g})$. Rearranging the steady state conditions $U_t = \overline{f}$ and $\pi_{e,t} = \overline{g}$, and differentiating those conditions with respect to U yields $\frac{\partial \pi_e}{\partial U} = \frac{1-b}{1-\mu} + i \cdot \frac{\delta}{\gamma} \cdot (1 - d \cdot (1 - \frac{1}{U^2}) > 0$ and $\frac{\partial \pi_e}{\partial U} = -\frac{1}{\delta - 1} \cdot \frac{1-b}{1-\mu} < 0$, respectively. Hence, the steady state p is also unique.

Differentiating the 'equilibrium rate of unemployment' U^* after the mark-up μ and the reservation wage b shows the usual properties. The 'equilibrium rate of unemployment' is increasing in the mark-up ($\frac{dU^*}{d\mu} > 0$) and in the reservation wage ($\frac{dU^*}{db} > 0$). The 'equilibrium rate of unemployment' is also a function of the money growth rate m. Hence, in this model money has an effect on real variables ($\frac{dU^*}{dm} < 0$). But we will see in the section on endogenous cycles that this does not necessarily establish a stable Phillips curve with a menu of choice between inflation and unemployment.

3.1 Stability analysis: analytically and numerically

The stability of the steady state is determined by the eigenvalues of the Jacobian matrix evaluated at the steady state. The Jacobian matrix J is given by

$$J = \begin{pmatrix} \frac{\partial f}{\partial U}|_p & \frac{\partial f}{\partial \pi}|_p \\ \frac{\partial g}{\partial U}|_p & \frac{\partial g}{\partial \pi}|_p \end{pmatrix}$$

[3] Note that for deriving the steady state it is more comfortable to start from the dynamic system written in unemployment and *expected* inflation: $\overline{f}(U_t, \pi_{e,t}) = U_t + i \cdot (L - U_t) - U_t \cdot \frac{J_s + \gamma \cdot (m - \frac{1}{\delta} \cdot (\pi_{e,t} + \frac{\mu - (1-b) \cdot U_t}{1-\mu})}{U_t + d \cdot (1 - U_t)}$ and $\overline{g}(U_t, \pi_{e,t}) = (1 - a + \frac{a}{\delta}) \cdot \pi_{e,t} + \frac{a}{\delta} \cdot \frac{\mu - (1-b) \cdot U_t}{1-\mu}$. We also use $\overline{F}(\overline{f}, \overline{g})$ to show uniqueness of the steady state. However, for the local bifurcation analysis and the simulations, we refer to the system in unemployment and inflation $F(f, g)$.

where

$$\begin{aligned}
\frac{\partial f}{\partial U}|_p &= j_{11} = 1 - i \cdot (1 + d \cdot \frac{1-U^*}{U^* \cdot (U^* + d \cdot (1-U^*))}) \\
\frac{\partial f}{\partial \pi}|_p &= j_{12} = \gamma \cdot \frac{U^*}{U^* + d \cdot (1-U^*)} \\
\frac{\partial g}{\partial U}|_p &= j_{21} = \frac{1-b}{\delta \cdot (1-\mu)} \cdot (1 - a - j_{11}) \\
\frac{\partial g}{\partial \pi}|_p &= j_{22} = \frac{1}{\delta} \cdot (a + (1-a) \cdot \delta - \frac{1-b}{1-\mu} \cdot j_{12})
\end{aligned}$$

The corresponding eigenvalues of J are

$$\lambda_{1,2} = \frac{j_{11} + j_{22}}{2} \pm \frac{1}{2}\sqrt{(j_{11} - j_{22})^2 + 4 \cdot j_{21} \cdot j_{12}}.$$

The steady state p is locally stable for eigenvalues $|\lambda_{1,2}| < 1$ and locally unstable otherwise.[4]

We will numerically show that the steady state loses stability for reasonable parameters, and unemployment rates stay bounded in $[0, 1]$. Nickell (1997) reports replacement rates for OECD countries at around 60%. Given that the equilibrium real wage is at about 90% of the constant marginal labor productivity y, a reservation wage of $b = 0.5$ might be a good choice. Firms can translate wage changes into price changes by half ($\delta = 2$) at time t. We set $\gamma = 0.5$, which is in the range of what Davis et al. (1997) report on the cyclical component of job creation. The mark-up for what follows in the stability analysis is $\mu = 0.04$, and workers weigh past inflation and expected past inflation equally ($a = 0.5$). Taking the eigenvalue with the negative root and setting it equal to -1 yields with those parameters an expression for the inflow rate of $i = (1/56) \cdot (258048 \cdot d \cdot m^2 + 112896 \cdot d \cdot m - 5152 \cdot d - 243648 \cdot m^2 + 20304 \cdot m - 423)/(-4 + 192 \cdot m - 621 \cdot d - 192 \cdot d \cdot m - 2304 \cdot m^2 + 2304 \cdot d \cdot m^2)$. Then, for an on-the-job search ratio of $d = 0.01$ and a money growth rate of 3%, a period doubling bifurcation occurs at $i \approx 0.13199$. If one solves for the on-the-job search ratio under the assumption that $i = 0.14$ (cf. Pissarides

[4]In case of a locally unstable system, it may happen that unemployment hits the 'capacity constraints' of a fully utilized labor force or all workers being unemployed. Then, the underlying dynamic system F would have to be written as $H(h, g) = (min\{1, max\{f, 0\}\})$. However, the simulations suggest that there exist cases where the unemployment rates stay bounded in $[0, 1]$.

(1986) or Nickell (1998)), the bifurcation value becomes $d^* \approx 9.125 \cdot 10^{-3}$. Solving for the money growth rate at which a period doubling bifurcation occurs yields $m^* \approx 2.956 \cdot 10^{-2}$ (the other solution is of no interest as it coincides with a negative steady state unemployment rate). Figures 1 and 2 show examples of bifurcation curves in $(i, d) - space$ and $(d, m) - space$, respectively. Along each of those curves a period-doubling bifurcation occurs.

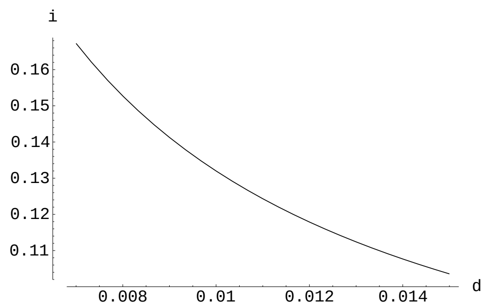

Figure 1: Bifurcation curve in $(d, i)-space$. $b = 0.5, \mu = 0.04, \gamma = 0.5, \delta = 2, a = 0.5, m = 0.03$

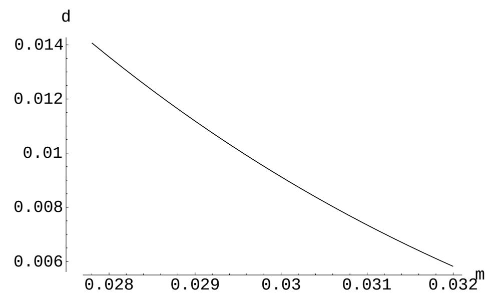

Figure 2: Bifurcation curve in $(m, d) - space$. $b = 0.5, \mu = 0.04, \gamma = 0.5, \delta = 2, a = 0.5, i = 0.14$

Extending the analysis to the eigenvalue with a positive root by equalizing it to 1 yields an expression $i = (25/24) \cdot (24 \cdot m - 1)^2/(192 \cdot (1 - d) \cdot m - 2304 \cdot (1 - d) \cdot m^2 - 621 \cdot d - 4)$. As the numerator is always positive and the denominator negative, the inflow rate would have to be negative, which is ruled out by assumption. Therefore the eigenvalue with a positive root never crosses 1.

The panel of Figures 3 to 6 shows bifurcation diagrams. Those plot the variables u and π for the long run for various parameter values. They give some insight on whether the trajectories stay bounded in reasonable ranges. The upper two relate the unemployment and inflation rate to various levels of the inflow rate, respectively. Both figures show that there exists a flip-bifurcation. As the inflow rate increases from 0.12 the system loses its stable steady state. At $i \approx 0.13199$, as calculated before, a period-two cycle emerges. Increasing the inflow rate even further leads to additional bifurcations. The period two cycle looses stability and becomes a period four cycle and so on. The corresponding bifurcation diagram for the other variable, the inflation rate, is plotted in Figure 4, where flip bifurcations occur at exactly the same values as in Figure 3. Figures 5 and 6 show the long run behavior of the unemployment rates over varying money growth rates (m) and different on-the-job search ratios (d), respectively. Compared to the other two bifurcation diagrams, the inflow rate is fixed to 0.14, now, but all other parameters are the same. Here, it can be seen that the steady state looses stability as the money growth rate is raised. At $m \approx 0.029558$ the system bifurcates and a stable period two cycle emerges. As the money growth rate becomes larger further period doubling bifurcations occur. In terms of the on-the-job search ratio the same dynamic properties can be observed. For sufficiently low values of d the steady state is stable. It looses stability via a flip bifurcation at $d \approx 9.125 \cdot 10^{-3}$, and as d increases the system bifurcates again, increasing the number of long run states for the unemployment rate to four, eight and so on. Observe also, that for the parameters chosen, the unemployment rates do not hit the lower and upper bounds of the labor force in any of the bifurcation diagrams shown.

In the following we will show numerically that the stable and unstable manifolds of F intersect. Observing such homoclinic points can imply very complicated dynamics (see for example Lorenz (1997), Guckenheimer and Holmes (1997), or Kuznetsov (1998)). This has been shown in an addiction model by Feichtinger et al. (1997), a cobweb model by Brock and Hommes (1997), an overlapping generations model by de Vilder (1996), and for a Cournot competition model by Droste et al. (2002). However, as far as we know, homoclinic points have not been detected in a flow model of the labor market.

Consider our map F and recall that the steady state is denoted with p. It has been argued before that the Jacobian of F evaluated at the steady state p may have two real eigenvalues λ_1 and λ_2 such that $0 < |\lambda_2| < 1 < |\lambda_1|$, implying that p is a fixed saddle point. The stable and unstable manifolds of F are defined as

$$W^s(p) = \{x; F^n(x) \to p \;\; as \;\; n \to +\infty\}$$
$$W^u(p) = \{x; F^n(x) \to p \;\; as \;\; n \to -\infty\},$$

respectively. The properties of the unstable and stable manifold are such that they are tangent to the eigenvectors at the steady state p that correspond to the eigenvalues λ_1 and λ_2, respectively. Whereas in the linear case, the manifolds are given by the eigenvectors, they can have a very complicated structure for a nonlinear map. That is why the manifolds are calculated

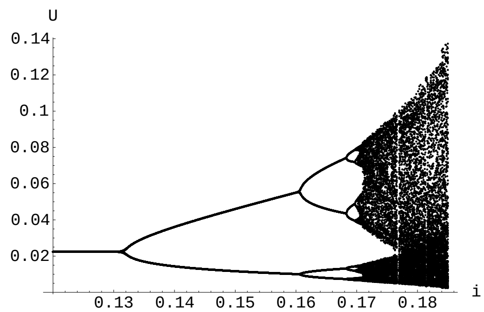

Figure 3: Bifurcation diagram U over i. $d = 0.01, b = 0.5, \mu = 0.04, \gamma = 0.5, \delta = 2, a = 0.5, m = 0.03$

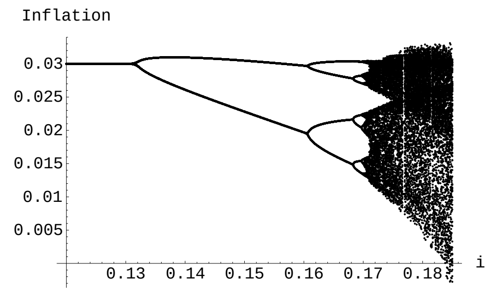

Figure 4: Bifurcation diagram π over i. $d = 0.01, b = 0.5, \mu = 0.04, \gamma = 0.5, \delta = 2, a = 0.5, m = 0.03$

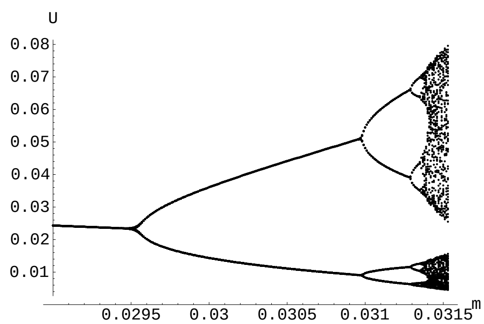

Figure 5: Bifurcation diagram U over m. $d = 0.01, b = 0.5, \mu = 0.04, \gamma = 0.5, \delta = 2, a = 0.5, i = 0.14$

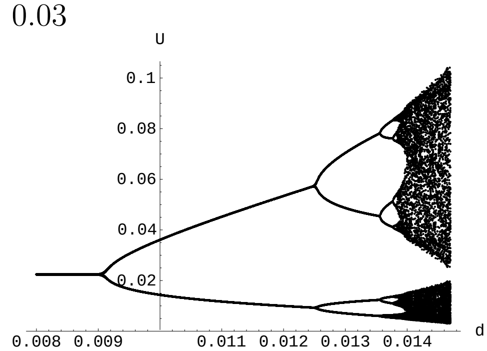

Figure 6: Bifurcation diagram U over d. $b = 0.5, \mu = 0.04, \gamma = 0.5, \delta = 2, a = 0.5, i = 0.14, m = 0.03$

numerically. The following parameters have been chosen: $i = 0.185, d = 0.01, m = 0.03, \delta = 2, \mu = 0.04, b = 0.5, a = 0.5$ and $\gamma = 0.5$. In this case, the eigenvalues are $\lambda_1 \approx -1.78$ and $\lambda_2 \approx 0.74$. The eigenvectors v_u and v_s at the steady state p follow from

$$\begin{pmatrix} j_{11} - \lambda_1 & j_{12} \\ j_{21} & j_{22} - \lambda_1 \end{pmatrix} v_u = 0$$

and

$$\begin{pmatrix} j_{11} - \lambda_2 & j_{12} \\ j_{21} & j_{22} - \lambda_2 \end{pmatrix} v_s = 0,$$

respectively. In our numerical example, the unstable eigenvector is $v_u \approx \{-0.97, 0.23\}$ and stable eigenvector is $v_s \approx \{0.14, 0.99\}$. Figure 7 shows numerically computed points on the two branches of the stable manifold in a neighborhood of the steady state, and on one branch of the unstable manifold.[5] It can be seen that one branch of the stable manifold intersects the unstable manifold. Given such homoclinic points, one may expect complicated dynamics in our model. We elaborate this with further numerical examples in the following section.

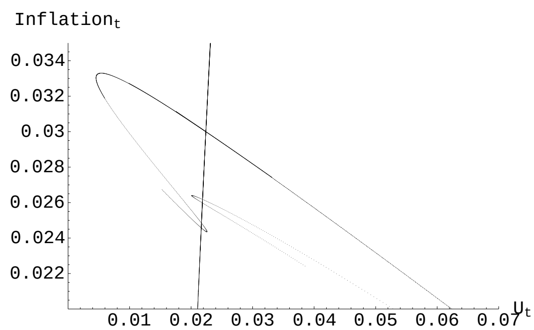

Figure 7: Numerically computed points on two branches of the stable manifold and on one branch of the unstable manifold: $d = 0.01, b = 0.5, \mu = 0.04, \gamma = 0.5, \delta = 2, a = 0.5, i = 0.185, m = 0.03$

3.2 Numerical examples

In Figures 8 and 9 we show the adjustment paths for the unemployment and inflation rates after adding positive (favorable) and negative (adverse) shocks of 0.05 percentage points of unemployment to Equation (12). The parameters are chosen such that the equilibrium rate of unemployment is 10.5% ($d = 0.05, b = 0.5, \mu = 0.08, \gamma = 0.5, \delta = 2, a = 0.5, i = 0.15, m = 0.03$). The structural component of jobs that come to the market every year amounts to approximately 19% of the labor force in this example. It can be seen that the adjustment path of the unemployment rate differs for positive and negative signs of the exogenous shock. While the unemployment rates increase

[5]For computing the manifolds small segments on the eigenvectors very close to the steady state consisting of 500 points were iterated a number of times. Note also that our map F is not invertible. For computing some points on the stable manifold, we took that part of the solution that contains the steady state.

rapidly after a favorable shock, it takes longer until the unemployment rate has reached its long run level after an adverse shock of the same size. Hence, for this set of parameters there is asymmetric persistence of unemployment. Asymmetry stems from the congestion effect that the nonlinear outflow rate from unemployment captures. If the actual unemployment rate is higher than the 'equilibrium rate of unemployment', unemployment rates converge to the steady state only gradually. As many workers compete for jobs, the outflow rate is comparably small. Even though unemployment is rather high, outflows from unemployment are only slightly larger than inflows into unemployment. During booms, when unemployment is below the 'equilibrium rate of unemployment', inflows into unemployment exceed outflows from unemployment. Now, there are relatively few workers who compete for one of the jobs. The outflow rate is high, but unemployment is low, meaning high employment rates and, at a fixed inflow rate, comparably large inflows. Unemployment rises, and due to the rather sharp drop of outflows when unemployment approaches zero, unemployment increases quickly. The corresponding time series on inflation rates shows that inflation rates increase with a favorable shock to the market (where unemployment drops) as claims on the aggregate output are inconsistent while inflation rates fall for a shock that temporarily raises the unemployment rate. In the long run, inflation rates converge to the money growth rate.

For the rest of the paper we set $d = 0.01$ and lower the mark-up to $\mu = 0.04$ (as in the numerical stability analysis), but leave all parameters unchanged. We will analyze the dynamics of the model by varying the inflow rate i. From Equation 14 one can see that a lower mark-up reduces the equilibrium rate of unemployment. For $\mu = 0.04$ it is $U^* \approx 0.022$. Figures 10 and 11 show time series for period two cycles of the unemployment rate and the inflation rate. The 'equilibrium rate of unemployment' is never reached. There is a steady state but actual unemployment varies endogenously. Unemployment cycles and changes in the inflation rate are persistent without exogenous shocks. In other words, market participants permanently fail to clearing the market. The oscillations of the unemployment rate are not symmetric to the locally unstable 'equilibrium rate of unemployment'. Therefore, the downward pressure on prices differs from the upward pressure on prices over the cycle. Average inflation rates are downward biased with respect to the money growth rate ($m = 0.03$). Such an outcome can be observed as, with a lower mark-up, firms lose market power and the price determined real wage declines. As shown before, to get consistent claims on the output, the equilibrium rate of unemployment has to be lower. This shifts the long run equilibrium into the regime where the outflow rate drops rather sharply. Any deviation from the steady state changes outflows from unemployment severely. At an unemployment rate lower than the steady state, workers bargain for a wage that is not consistent with the price determined real wage. This drives prices up, and firms will offer fewer jobs as the real money supply declines. Hence, unemployment rises in the next period. Now, confronted with an unemployment rate that exceeds the steady state rate of unemployment, the bargained real wage will fall short of the price determined real wage. Prices decline, and firms offer more jobs. The chances for an unemployed worker to find a job increase. However, the market overshoots again. Unemployment falls below the equilibrium rate, the only unemployment rate that would ensure compatible claims on the output in the long run.

In Figures 12 and 13 the inflow rate is increased to 0.18. Now, the unemployment rate never settles to the 'equilibrium rate of unemployment,' nor does the inflation rate converge to a single value or a regular cycle. The cycles are irregular even though there is no exogenous component added.

An attractor of F, plotted in Figure 14, shows the inflation rate over the unemployment rate for 500 iterations (where the initial 100 values are dropped). One may interpret the attractor as a Phillips curve (note, that it is downward sloping here), but a researcher confronted with this data set, not knowing about the underlying system, would probably conclude that there is a stable trade-off. However, there is none, as the system shifts up and down a negatively sloped Phillips curve erratically. A pair of inflation-unemployment rates today cannot tell where the economy will be in the long-run. The Phillips curve consists of 'unstable disequilibria.' Only by luck would one run into the steady state, which is a very unlikely long-run equilibrium, as a small perturbation will move the system away from it, again.

4 Conclusion

The model that we developed generates a range of adjustment paths. For the stable case we get asymmetric persistence. While unemployment rates quickly converge to the steady state after a favorable shock, it can take longer until the unemployment rate has reached the steady state after an adverse shock. Given the number of job searchers, there are too few job offers to establish an immediate return to the 'equilibrium rate of unemployment.'

Outflows from unemployment compensate inflows into unemployment, but are too low to reduce unemployment quickly. Besides asymmetric persistence, and depending on the calibration, the model can generate endogenous cycles; in that case, the labor market will never return to its long run equilibrium. Unemployment rates and expected inflation rates may even become irregular. The attractor generated by our model implies a negatively sloped Phillips curve. However, this Phillips curve consists of 'unstable disequilibria' so that there is no orthodox inflation-unemployment trade-off.

With respect to the asymmetric adjustment of unemployment rates to symmetric shocks, policy makers might underestimate the self regulating forces of labor markets. When unemployment falls below the 'equilibrium rate of unemployment' in our model, it may return to its long run rate rather quickly. Long periods of high inflation rates do not occur. Policy makers who are not aware of this asymmetric behavior might choose measures to beat inflation and do harm in the end. Irregular endogenous cycles can have important policy implications, too. Monetary authorities often take the 'natural rate of unemployment' as the reference point for upcoming rising or falling inflation rates. However, comparing the actual rate of unemployment with a locally unstable 'equilibrium rate of unemployment' might be a policy trap as the causality that runs from a gap between the 'equilibrium rate of unemployment' and the actual rate of unemployment and the inflation rate is blurred if both variables change irregularly and the long run behavior is not predictable. In addition, there is no menu from which policy makers can choose although there appears to be a negative trade-off. While people order, the menu will change, and it does so because of the orders. It is the cook who decides what is on the table, and customers will find themselves happy or sour with what they get. There is definitely no room for the management to accept or refuse orders of customers, such as in the old-fashioned orthodox policy-matters-approaches of Phillips curves.[6]

Appendix 1 *Thus far we have discussed a flow model of the labor market where the outflow rate from unemployment was endogenous and the inflow rate i was exogenous. The purpose of this section is to make the inflow rate endogenous i_t, also dependent on the state of the labor market. As outlined before there may be countervailing forces on the inflow rate from the state of the labor market. Let us take unemployment as an indicator for economic*

[6] We owe this metaphor to Michael Bolle.

activity. Then, if unemployment is rather high, firms' lay off rate may be higher too. On the other hand, workers will be less inclined to quit into unemployment, as getting a new job is more difficult when unemployment is rather high. If the latter effect is stronger than the former, $\partial i_t/\partial U_t < 0$ *(first case). If the former effect is stronger than the latter,* $\partial i_t/\partial U_t > 0$ *(second case). For the simulation of the flow model when the inflow rate is endogenous we choose a linear specification of the following form:*

$$i_t = i_e + \frac{i - i_e}{U^*} \cdot U_t.$$

Hence, at the steady state the endogenous inflow rate equals the exogenous inflow rate, $i(U^*) = i$. *The choice of* i_e *determines the elasticity of the inflow rate with respect to unemployment which is* $\varepsilon = 1 - i_e/i$. *If* $i_e = i$ *then the elasticity is zero, for* $i_e > i$ *the elasticity is negative, and positive otherwise.*

Figures 15 and 16 show simulations for $\varepsilon < 0$, *and Figures 17 and 18 for the second case where the inflow rate increases with higher unemployment. In both cases asymmetric adjustment to the steady state after equally sized shocks with different signs remains as a qualitative feature of the model. Note that for the simulations of asymmetric adjustment, parameter values equal the case with an exogenous inflow rate. In Figure 16 the evolution of unemployment is irregular. Here, all parameters equal the case with an exogenous inflow rate, except for the choice of* $i = 0.17$. *Endogenous unemployment cycles can only be observed at a positive elasticity if the steady state inflow rate is increased to* $i = 0.22$*; see Figure 18. Hence,* $\partial i_t/\partial U_t > 0$ *seems to exert a stabilizing force. Once unemployment deviates from the steady state, for example to a lower level of unemployment, the inflow rate becomes smaller so that inflows are smaller, too, and the market does not overshoot. Summing up, these simulations suggest that, at least for the elasticities of the inflow rate chosen here, in small neighborhoods to the steady state the qualitative dynamic properties remain if the inflow rate is also endogenous.*

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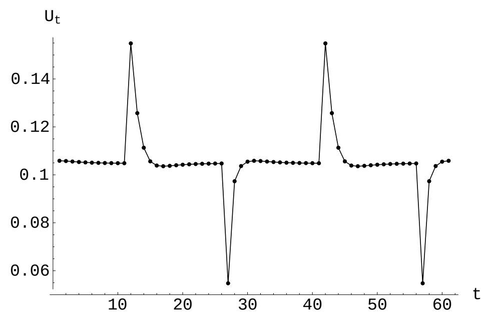

Figure 8: Unemployment rate following impulses to Equation 12 of $\epsilon = \pm 0.05$ after every 15th period. $d = 0.05, b = 0.5, \mu = 0.08, \gamma = 0.5, \delta = 2, a = 0.5, i = 0.15, m = 0.03$

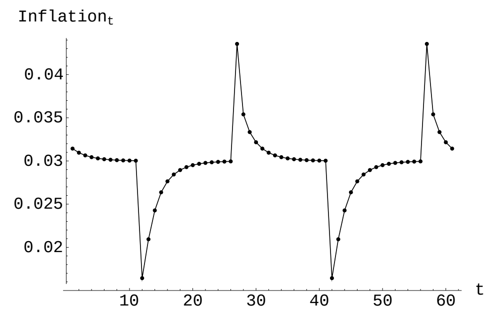

Figure 9: Inflation rate following impulses to Equation 12 of $\epsilon = \pm 0.05$ after every 15th period. $d = 0.05, b = 0.5, \mu = 0.08, \gamma = 0.08, \delta = 2, a = 0.5, i = 0.15, m = 0.03$

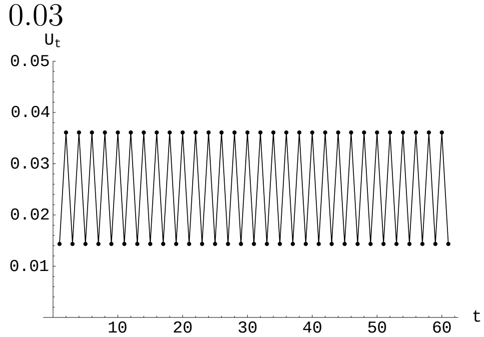

Figure 10: Unemployment rate. $d = 0.01, b = 0.5, \mu = 0.04, \gamma = 0.5, \delta = 2, a = 0.5, i = 0.14, m = 0.03$

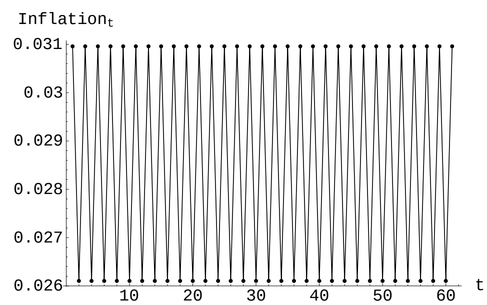

Figure 11: Inflation rate. $d = 0.01, b = 0.5, \mu = 0.04, \gamma = 0.5, \delta = 2, a = 0.5, i = 0.14, m = 0.03$

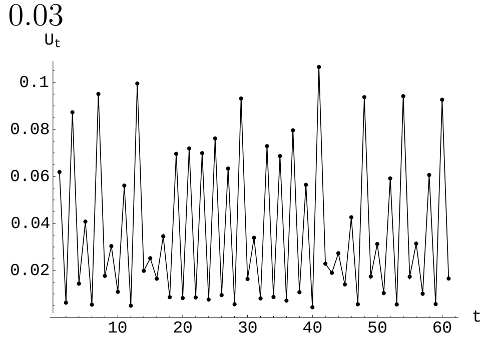

Figure 12: Unemployment rate. $d = 0.01, b = 0.5, \mu = 0.04, \gamma = 0.5, \delta = 2, a = 0.5, i = 0.18, m = 0.03$

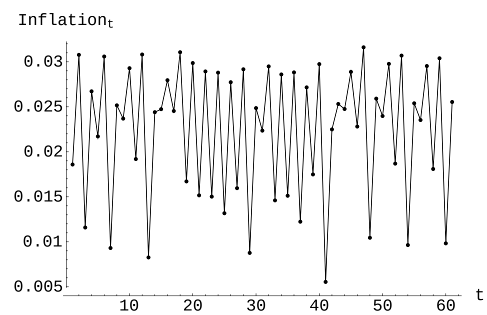

Figure 13: Inflation rate. $d = 0.01, b = 0.5, \mu = 0.04, \gamma = 0.5, \delta = 2, a = 0.5, i = 0.18, m = 0.03$

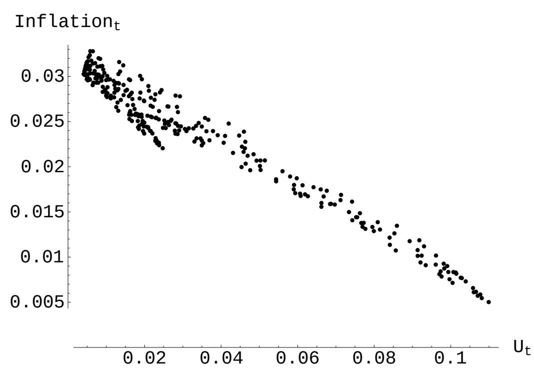

Figure 14: Attractor: 'Phillips curve'. $d = 0.01, b = 0.5, \mu = 0.04, \gamma = 0.5, \delta = 2, a = 0.5, i = 0.18, m = 0.03$

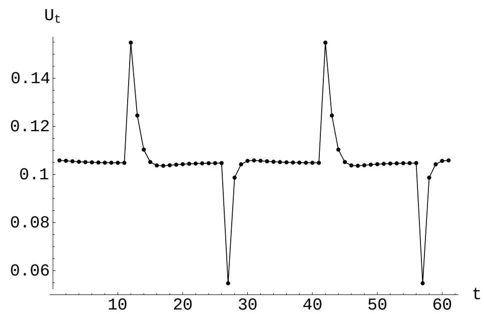

Figure 15: Unemployment rate following impulses of $\epsilon = \pm 0.05$ after every 15th period, inflow rate is endogenous with $\varepsilon = -0.02$. $d = 0.05, b = 0.5, \mu = 0.08, \gamma = 0.5, \delta = 2, a = 0.5, i = 0.15, m = 0.03$

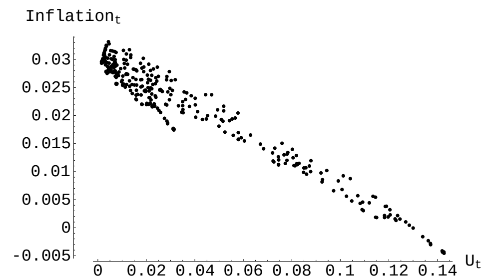

Figure 16: Attractor, 'Phillips Curve', inflow rate is endogenous with $\varepsilon = -0.02$. $d = 0.01, b = 0.5, \mu = 0.04, \gamma = 0.5, \delta = 2, a = 0.5, i = 0.17, \epsilon = -0.02, m = 0.03$

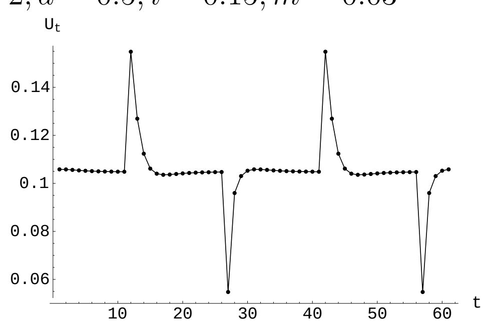

Figure 17: Unemployment rate following impulses of $\epsilon = \pm 0.05$ after every 15th period, inflow rate is endogenous with $\varepsilon = 0.02$. $d = 0.05, b = 0.5, \mu = 0.08, \gamma = 0.5, \delta = 2, a = 0.5, i = 0.15, \epsilon = 0.02, m = 0.03$

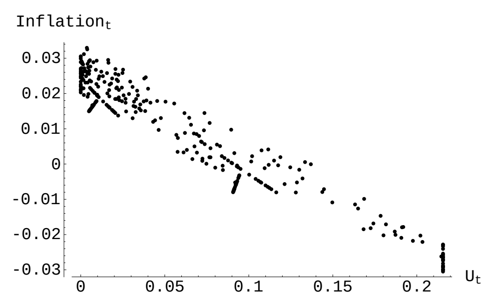

Figure 18: Attractor, 'Phillips Curve', inflow rate is endogenous with $\varepsilon = 0.02$. $d = 0.01, b = 0.5, \mu = 0.04, \gamma = 0.5, \delta = 2, a = 0.5, i = 0.22, \epsilon = 0.02, m = 0.03$