Received: 5 October 2022 | Accepted: 10 October 2022

DOI: 10.1002/pamm.202200077

Analysis of strut-based lattice cores in sandwich panels using homogenization and dehomogenization methods

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The design freedom provided by additive manufacturing offers new opportunities to fabricate novel structures with a high lightweight potential, such as strut-based lattice structures. These lattice structures consist of periodically repeated unit cells and can be used in several applications due to their outstanding mechanical performance. One of the possible applications are cores of sandwich panels since the strut-based lattices offer comparable mechanical properties to conventional honeycomb structures. Moreover, multifunctional use of the sandwich core is enabled by allowing the heat and fluid transfer through the sandwich due to the open-celled lattice structure. However, strut-based lattices are rarely utilized as cores in sandwich panels in engineering practice. One of the main reasons for that is the unknown mechanical behavior of lattice cores. In particular, when the sandwich is subjected to concentrated loads, localized stresses and deformations occur in the sandwich core, leading to core damage. In this work, we present a novel analytical model to determine stresses and deformations in the struts of lattice cores of sandwich panels using homogenization and dehomogenization methods. The local core compression caused by localized transverse forces can also be determined by the derived model.

1 Introduction

The high bending stiffness and the simultaneously low weight provided by sandwich panels make sandwich structures attractive to design lightweight constructions. Typically, honeycomb and foams are employed as cores in sandwich panels. Recently, novel light-weight strut-based lattices have been investigated in several studies, revealing that these lattice structures can provide an alternative for typical sandwich cores since they offer comparable mechanical properties to conventional honeycomb cores [1]. This trend of considering strut-based lattice as cores in sandwich panels was mainly motivated due to advances in additive manufacturing, which enables the fabrication of these complex structures [2]. In case the face sheets and the core are made of the same material, the sandwich structure may be manufactured merely in one print job and no assembly using adhesive layers is required [3].

However, strut-based lattices are rarely employed as cores in sandwich panels in industrial applications. The use of lattices as cores is complicated by the lack of knowledge about the lattice behavior under different loads. Existing approaches to lattice modeling are based on replacing the lattice with an equivalent homogeneous material, called homogenization [4]. Making use of the effective properties of lattices, simple sandwich theories can be used to calculate the displacements and the effective stresses in the core. However, most theories do not take into account the core local deformations caused by concentrated loads and the stiffness mismatch between the core and face sheet material [5]. Advanced theories enable the determination of local core compressions [6], but do not provide information about the stresses in the lattice struts since the lattice struts are neglected due to lattice homogenization. Therefore, a dehomogenization method to determine the strut stresses in the lattice core is presented in this study. Furthermore, a higher-order sandwich model is introduced to determine sandwich deformations and in particular, the local core compression induced by concentrated transverse loads in sandwich panels with strut-based lattice cores.

2 Modelling approach

In this work, we consider a 2D symmetric sandwich model with two isotropic face sheets ($E^{(f)}, \nu^{(f)}$), as illustrated in Fig. 1(a). The quantity n describes the face sheet number, where the bottom face sheet is indicated by $n = 1$ and the top face sheet by $n = 2$. The face sheet thickness and the core thickness are described by $h^{(f)}$ and $h^{(c)}$, respectively. A periodic lattice with the unit cell shown in Fig. 1(b) represents the sandwich core. Furthermore, we assume that the face sheets and the lattice struts are made of the same material and show a linear elastic material behavior. A Cartesian coordinate system xz with $-l/2 \leq x \leq l/2$ and $-h/2 \leq z \leq h/2$ is placed in the sandwich center where l and h denote the sandwich total length and sandwich total thickness, respectively. All geometric parameters of the sandwich are presented in Fig. 1.

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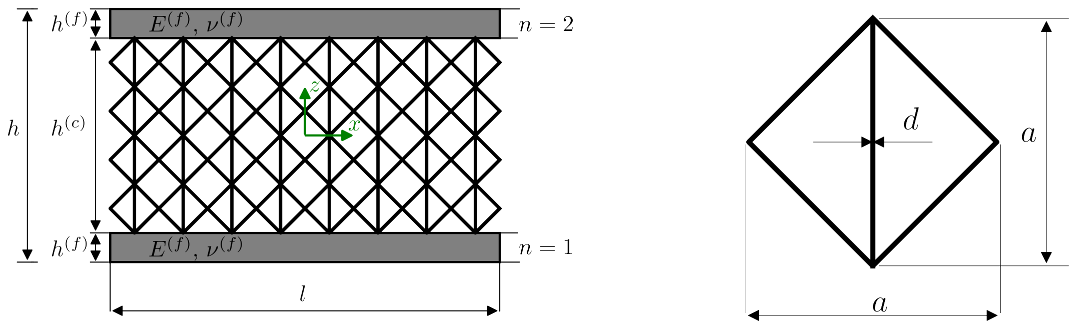


(a) Sandwich structure with a lattice core

(b) Lattice unit cell

Fig. 1: The considered sandwich model and the corresponding unit cell

2.1 Lattice homogenization

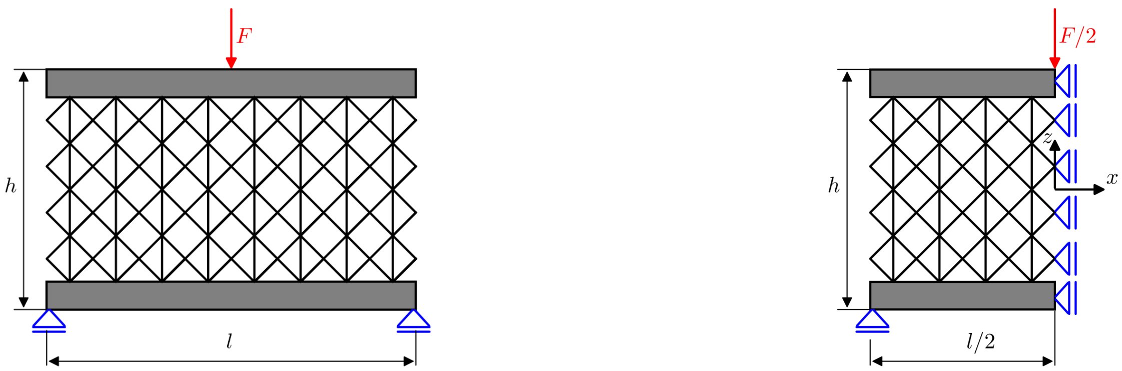
To enable the calculation of the lattice strut stresses, the core will be first modeled as a homogeneous anisotropic material which shows the same mechanical response as the lattice. For this reason, the effective mechanical properties of the considered lattice unit cell are required. The considered unit cell is assumed to behave in an orthotropic manner. The effective orthotropic material behavior in the considered plane can be described by four elastic constants: the effective elastic modulus in the horizontal direction E^*_{xx}, the effective elastic modulus in the vertical direction E^*_{zz}, the effective shear modulus in the xz-plane G^*_{xz} and the effective Poisson's ratio ν^*_{xz}. These elastic constants are obtained using FE simulations of basic load cases of the lattice unit cell. In the FE simulations, the unit cell is subjected to an uniaxial load case in the horizontal and in the vertical direction and a shear load in the xz-plane. The simulations are conducted for several aspect ratios where only the strut diameter of the unit cell d is changed and the cell size a remains constant (a=5 mm). The results obtained by these numerical simulation show that the relative elastic constants depend on the aspect ratio of the unit cell. This dependence relation for the different moduli can be given as

$$\frac{E^*_{xx}}{E_s} = 1.6 \left(\frac{a}{d}\right)^{-2}, \qquad \frac{E^*_{zz}}{E_s} = 3.9 \left(\frac{a}{d}\right)^{-2}, \qquad \frac{G^*_{xz}}{G_s} = 7.4 \left(\frac{a}{d}\right)^{-2}, \tag{1}$$

where E_s is the elastic modulus of the lattice solid isotropic material.

2.2 Higher-order sandwich theory

In the current study, we consider a sandwich structure that is subjected to a single transverse load on the mid of the top face sheet upper surface and simply supported at the ends of the bottom face sheet, reassembling the set-up of a three-point bending test, as shown in Fig. 2(a). Since the considered load and geometry are symmetric, merely a half model is taken into account (Fig. 2(b)). To determine the sandwich displacements, the principle of minimum potential energy is used.



(a) Boundary conditions of a 3-point bending test

(b) Sandwich half model with the corresponding boundary conditions

Fig. 2: The considered sandwich model and the boundary conditions

First, the employed displacement approaches of the face sheets are presented. Since the face sheets are assumed to be slender, a beam-like behavior is expected. Therefore, Timoshenko beam deformation approaches are employed to model the face sheets deflections

$$u^{(n)}(x,z) = u_0^{(n)}(x) + z\psi^{(n)}(x), \qquad w^{(n)}(x,z) = w_0^{(n)}(x), \tag{2}$$

where $w_0^{(n)}$ and $u_0^{(n)}$ are the vertical and horizontal displacement of the face sheet mid-axis and $\psi^{(n)}$ is the angular deformation of the face sheet. Strains of the face sheets are obtained by the derivatives of displacement functions concerning the horizontal coordinate x or vertical coordinate z

$$\varepsilon_{xx}^{(n)} = \frac{\partial u^{(n)}}{\partial x}, \qquad \varepsilon_{zz}^{(n)} = \frac{\partial w^{(n)}}{\partial z}, \qquad \gamma_{xz}^{(n)} = \frac{\partial u^{(n)}}{\partial z} + \frac{\partial w^{(n)}}{\partial x}. \tag{3}$$

Assuming a plane-stress state in the xz-plane, the face sheet stresses can be given as

$$\sigma_{xx}^{(n)} = \frac{E^{(f)}}{1-{\nu^{(f)}}^2}\,\varepsilon_{xx}^{(n)}, \qquad \tau_{xz}^{(n)} = G^{(f)}\gamma_{xz}^{(n)}. \tag{4}$$

Since the core is expected to show a more complex deformation behavior than the face sheets, using simple linear approaches would not reveal the core compression induced by concentrated loads. Thus, higher-order approaches are introduced to extend the linear interpolation between the face sheet displacements. Adding quadratic to 4th-order terms ($\hat{w}, \tilde{w}, \breve{w}, \hat{u}, \tilde{u}, \breve{u}$) enables the calculation of linear, quadratic, and cubic strain distributions. The core displacement approaches are given as

$$\begin{aligned} u^{(c)}(x,z) =& \frac{u_0^{(1)}(x) + u_0^{(2)}(x) + \frac{h^{(f)}}{2}\psi^{(1)}(x) - \frac{h^{(f)}}{2}\psi^{(2)}(x)}{2} \\ &+ \frac{-u_0^{(1)}(x) + u_0^{(2)}(x) - \frac{h^{(f)}}{2}\psi^{(1)}(x) - \frac{h^{(f)}}{2}\psi^{(2)}(x)}{h^{(c)}} z + +\tilde{u}(x)\tilde{f}(z) + \hat{u}(x)\hat{f}(z) + \breve{u}(x)\breve{f}(z), \end{aligned} \tag{5}$$

$$w^{(c)}(x,z) = \frac{w_0^{(1)}(x) + w_0^{(2)}(x)}{2} + \frac{w_0^{(2)}(x) - w_0^{(1)}(x)}{h^{(c)}} z + \tilde{w}(x)\tilde{f}(z) + \hat{w}(x)\hat{f}(z) + \breve{w}(x)\breve{f}(z), \tag{6}$$

where the distribution functions $\tilde{f}(z), \hat{f}(z)$, and $\breve{f}(z)$ are quadratic, cubic and 4th-order functions of the coordinate z and satisfy the displacement continuity on the interfaces between the core and the face sheets

$$\tilde{f}(z) = \left(1 - \frac{4z^2}{{h^{(c)}}^2}\right), \qquad \hat{f}(z) = \left(\frac{z}{h^{(c)}} - \frac{4z^3}{{h^{(c)}}^3}\right), \qquad \breve{f}(z) = \frac{1}{2}\left(\frac{z^2}{{h^{(c)}}^2} - \frac{4z^4}{{h^{(c)}}^4}\right). \tag{7}$$

The core strains are calculated in the same manner as in the face sheets. In contrast to the face sheets, the lattice core is assumed to show an effective orthotropic material behavior. Therefore, four elastic parameters are required to determine the core stresses using the following material laws

$$\sigma_{xx}^{(c)} = \frac{E_{xx}^{(c)}}{(1-\nu_{xz}^{(c)}\nu_{zx}^{(c)})}\left[\varepsilon_{xx}^{(c)} + \nu_{xz}^{(c)}\varepsilon_{zz}^{(c)}\right], \quad \sigma_{zz}^{(c)} = \frac{E_{zz}^{(c)}}{(1-\nu_{xz}^{(c)}\nu_{zx}^{(c)})}\left[\varepsilon_{zz}^{(c)} + \nu_{zx}^{(c)}\varepsilon_{xx}^{(c)}\right], \quad \tau_{xz}^{(c)} = G_{xz}^{(c)}\gamma_{xz}^{(c)}. \tag{8}$$

Regarding all introduced degrees of freedom, the considered sandwich has a total of 12 degrees of freedom which are functions of the coordinate x. To determine these unknown functions, the method of minimum potential energy is employed. Considering the half model presented in Fig. 2(b), the external energy can be determined depending on the applied load F by $\Pi_a = -\frac{F}{2}w_0^{(2)}(x=0)$. The sandwich strain energy is composed of the sandwich layers energies. Since the transverse normal stresses in the face sheets are neglected, the face sheet strain energy can be given as

$$\Pi_i^{(n)} = \frac{1}{2}\int_{-l/2}^{0}\int_{-h^{(f)}/2}^{h^{(f)}/2}\left(\sigma_{xx}^{(n)}\varepsilon_{xx}^{(n)} + \tau_{xz}^{(n)}\gamma_{xz}^{(n)}\right)\mathrm{d}z\,\mathrm{d}x, \tag{9}$$

In contrast, the core strain energy involves all stress components in the xz-plane

$$\Pi_i^{(c)} = \frac{1}{2}\int_{-l/2}^{0}\int_{-h^{(c)}/2}^{h^{(c)}/2}(\sigma_{zz}^{(c)}\varepsilon_{zz}^{(c)} + \tau_{xz}^{(c)}\gamma_{xz}^{(c)} + \sigma_{xx}^{(c)}\varepsilon_{xx}^{(c)})\,\mathrm{d}z\,\mathrm{d}x. \tag{10}$$

The sum of all energies yields the total sandwich potential energy Π. Considering the rules of the calculus of variation, the condition $\delta\Pi = 0$ yields 12 coupled second-order differential equations of the sandwich degrees of freedom

$$\underline{\underline{A}}\,\ddot{\underline{\Psi}} + \underline{\underline{B}}\,\dot{\underline{\Psi}} + \underline{\underline{C}}\,\underline{\Psi} = \underline{0}, \tag{11}$$

with $\underline{\Psi}$ the vector of the 12 unknown deflections of the sandwich layers

$$\underline{\Psi} = \begin{bmatrix} u_0^{(1)} & u_0^{(2)} & \psi^{(1)} & \psi^{(2)} & w^{(1)} & w^{(2)} & \tilde{w} & \tilde{u} & \hat{w} & \hat{u} & \breve{w} & \breve{u} \end{bmatrix}^T. \tag{12}$$

By converting this system to a first-order equation system, the first-order system can be solved using an exponential ansatz function of the eigenvalues of the first-order system matrix. Further information about solving and converting the differential equation system is discussed in detail in [7].

2.3 Lattice dehomogenization

After calculating the displacement functions, these functions are evaluated on the lattice nodes so that the vertical and horizontal displacements ($u_1, u_2, u_3, u_4, w_1, w_2, w_3$ and w_4) of each unit cell node are obtained (Fig. 3). These displacements will be transformed to the respective strut local coordinate system to enable the calculation of the corresponding strut extension or compression Δl_s

$$\begin{aligned} \Delta l_1 &= (u_2 - u_1)\cos(\pi/4) + (w_2 - w_1)\sin(\pi/4), \\ \Delta l_2 &= (u_3 - u_2)\cos(3\pi/4) + (w_3 - w_2)\sin(3\pi/4), \\ \Delta l_3 &= (u_4 - u_3)\cos(5\pi/4) + (w_4 - w_3)\sin(5\pi/4), \\ \Delta l_4 &= (u_1 - u_4)\cos(-\pi/4) + (w_1 - w_4)\sin(-\pi/4), \\ \Delta l_5 &= (w_3 - w_1). \end{aligned} \tag{13}$$

Finally, the lattice strut stresses are determined using Hooke's law $\sigma_s = E_s\,\frac{\Delta l_s}{l_s}$.

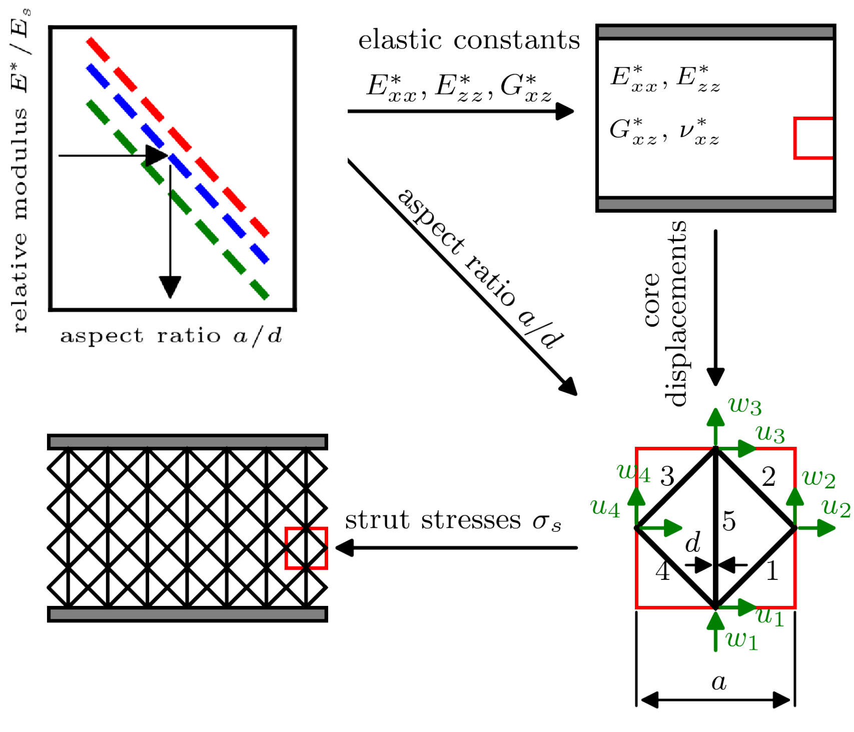
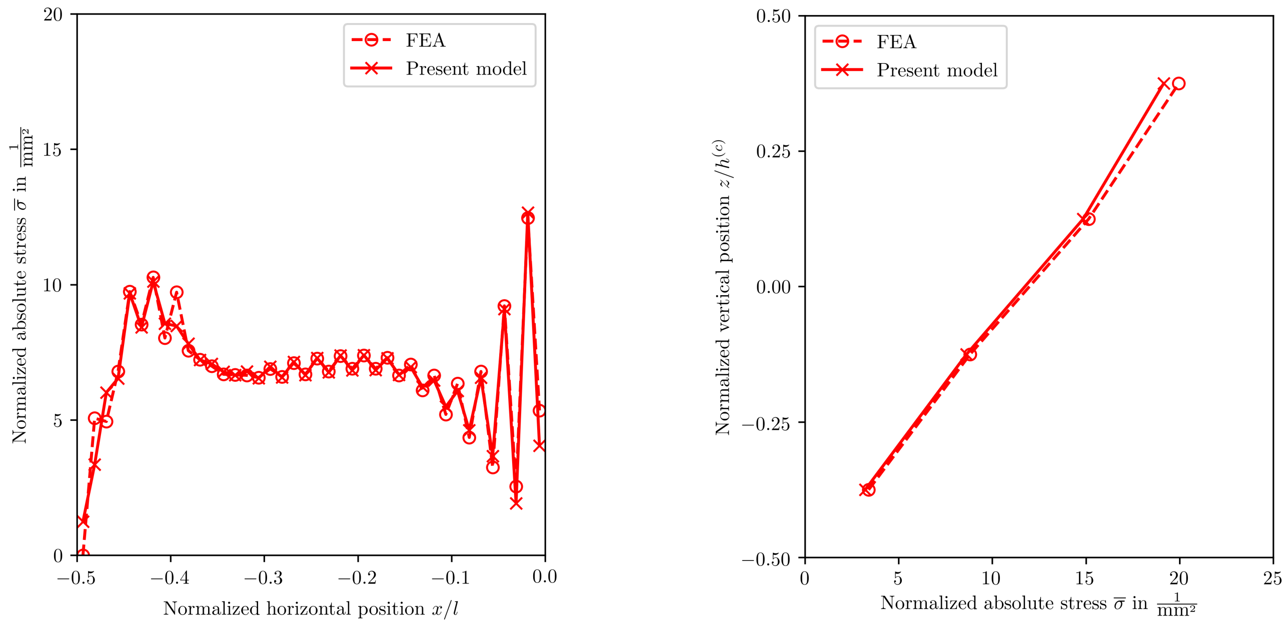


Fig. 3: Dehomogenization of the homogenized core in sandwich panels

3 Results and discussion

In this section, the results of the calculation of the lattice strut stresses in sandwich panels subjected to a transverse force using the dehomogenization method are presented. The considered sandwich involves two face sheets made of the same material and a lattice core with a unit cell aspect ratio $a/d = 40$. The selected geometric parameters yield a lattice core with 4 layers through the core thickness. The face sheet material is used to model the lattice solid material ($E^{(f)} = E_s = 70000$ MPa and $\nu^{(f)} = \nu_s = 0.35$). The face sheets have a thickness of 1 mm and the thickness ratio $h^{(c)}/h^{(f)}$ and the slenderness ratio

$l/h^{(c)}$ are assumed to be 20 and 10, respectively. An equivalent FE model is used to verify the results obtained by the present model. In the FE model, the lattice struts are simply joined with no moment transfer using truss elements, and the face sheets are modeled as solid layers using plane stress elements. The absolute normalized stress ($\overline{\sigma} = |\sigma/F|$) in the lattice core struts along the sandwich length in the core layer under the top face sheet is illustrated in Fig. 4(a). Since the vertical struts are partially not loaded outside the model boundary areas, the stress distribution through the core thickness in the vertical struts is only illustrated in the struts near the load application area (Fig. 4(b)). The stress in the vertical struts exhibits high values near the load introduction area and decreases with increasing distance from the applied load. The inclined struts show lower stresses in the load application area. In contrast to the vertical struts, the inclined struts are higher stressed outside the load application area. Comparing the results presented by the derived model to the FE results, the stresses in the lattice struts are well captured by the present model and show an excellent agreement with the FE results.



(a) Stress distribution in the inclined core struts at $(x, (h^{(c)} - a)/2)$

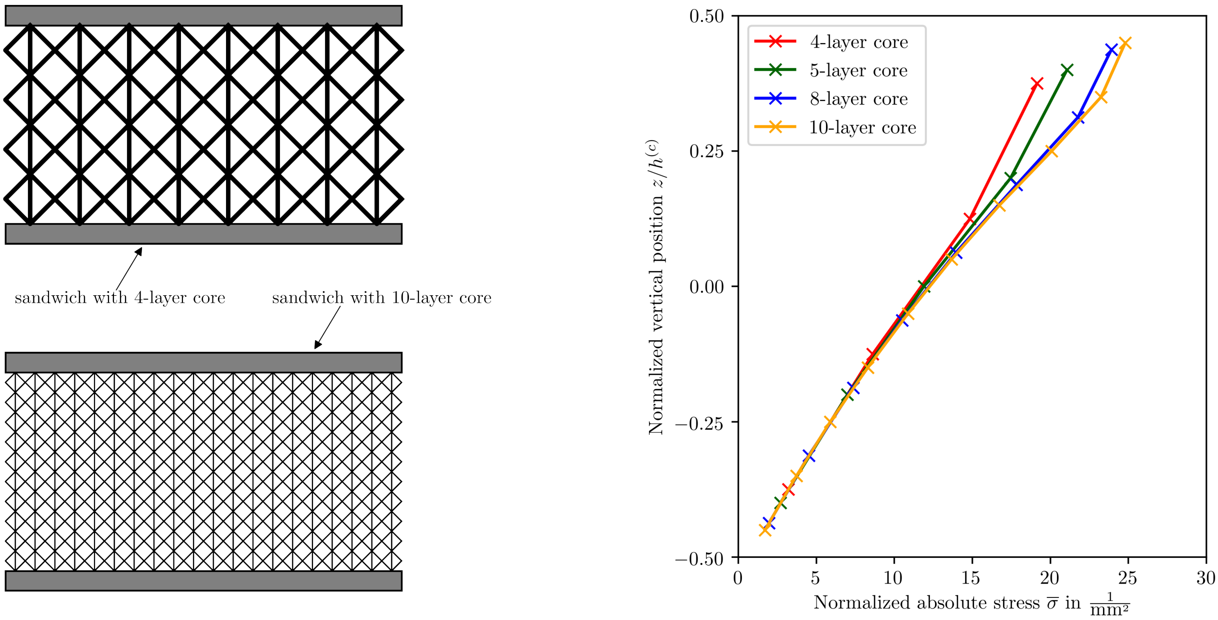
(b) Stress distribution in the vertical core struts at $(-a/2, z)$

Fig. 4: Stress distribution in the inclined struts of the core layer under the top face sheet and the vertical struts near the load application area

The sandwich lattice core can be replaced by a lattice with the same aspect ratio but different unit cell size a and strut diameter d. In Fig. 5(a), two cores with the same aspect ratio but different unit cell size and strut diameter are shown. These two sandwiches with different cores are equivalent since the effective elastic constants of the lattice depend merely on the aspect ratio. Replacing the unit cell by a cell with a smaller cell size and a thinner strut diameter results in cores with more layers through the thickness in case the thickness remains constant. Fig. 5(b) shows the stress distribution in the vertical struts near the load application area through the core thickness in cores with different cell sizes but the same aspect ratio. The 4 investigated sandwiches consist of cores with 4, 5, 8 and 10 layers and the same thickness and the same effective stiffness. The highest stress in the load application area is observed in the core with the thinnest strut diameter, namely the 10-layer core. It can be shown that the stress in the load application area increases with increasing number of the core layers since the strut diameter decreases. Outside the load application area (approximately starting from the mid of the sandwich), the stresses in the vertical struts exhibit no deviation in the different cores since they are not affected by the load application and all lattice cores have the same effective stiffness. With the knowledge gained from Fig. 5, the relevance of the dehomogenization and the determination of the lattice strut stresses become clear. By using effective properties alone, the influence of the strut diameter and the cell size on the occurring local stresses cannot be considered since the effective core stress does not change.

4 Conclusion

In the present work, a dehomogenization method to determine the lattice strut stresses in lattice cores of sandwich panels is introduced. By using higher-order displacement approaches, the core compression and local displacements can be captured with reasonable accuracy. In contrast to the models using merely homogenized lattice cores, the presented model enables the consideration of the impact of the unit cell size and the strut diameter on the local stresses in the lattice struts. Due to the flexibility of manufacturing provided by additive manufacturing, knowledge gained by this study can be used to grade the lattice core and tailor the core properties to be adapted to the local strut stresses in the core without demanding lattice modeling using FE software programs.



(a) Two equivalent sandwiches

(b) Stress distribution in vertical struts at $(-a/2, z)$

Fig. 5: Vertical strut stress near the load application area in equivalent sandwiches with different core layer numbers

Acknowledgements Open access funding enabled and organized by Projekt DEAL.

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