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Capital depreciation allowances, redistributive taxation, and economic growth

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Abstract

Are capital depreciation allowances when coupled with capital income taxes good instruments for redistribution in the long run? In a simple two-agent-economy I find that accelerated depreciation is good for growth, but bad for redistribution. The opposite holds for capital income taxes. However, in a feedback Stackelberg equilibrium, where the government is the leader and the private sector the follower, the depreciation allowance is maximal in the long run, time-consistent optimum. This removes the accumulation distortion of capital income taxes. Furthermore, the latter, and so redistribution, is found to be generically nonzero in the time-consistent optimum, and depends on the social weight of transfers receivers, the pretax factor income distribution, the intertemporal elasticity of substitution and the time preference rate. Thus, accelerated depreciation allowances are an important indirect tool for redistribution. The tax scheme allows for a separation of "efficiency" and "equity" concerns for redistributive policies.

1 | INTRODUCTION

Important economic stimuli in normal and crisis times are capital depreciation allowances. They imply that investment outlays can be deducted from taxable income within a certain time period. The motivation for such measures is seen in the investment promoting effect of capital depreciation allowances. This is well known.

However, the long-run distributional implication of depreciation allowances is less clear. For instance, if redistributive transfers are financed with taxes, then higher capital depreciation allowances reduce tax revenues and appear to be a bad instrument for redistributive policies. The opposite is often found to hold for capital income taxes. This is the problem that is being analyzed in this paper.

In particular, in the present analysis capital income taxes are coupled with depreciation allowances in a neoclassical growth setup. Therefore, I relate to the literature on optimal capital income taxation. According to the "celebrated" result by Judd (1985) and Chamley (1986) capital income taxes are not a good instrument for pure redistribution in a neoclassical growth framework.[1]

Subsequent research considered other capital income policy packages, including consumption taxes, and found the same result as in, for instance, Judd (1999).[2]

Most contributions in this context have focussed on open-loop Stackelberg set-ups which may feature problems of time-inconsistency. On this point see, for example, Kemp et al. (1993).

I relate to these findings in a simple two-agent, neoclassical growth framework and analyze a closed-loop Stackelberg equilibrium with a focus on a particular tax scheme. Closed-loop Stackelberg equilibria are, of course, time-consistent by construction. This is the novel feature of the present contribution.

It is shown that coupling capital income taxes with accelerated depreciation allowances to finance pure redistributive transfers to the nonaccumulated factor of production ("workers") may also imply a nondistortionary policy package, similar to a consumption tax on "capitalists."[3]

In this context, I build on Sinn (1987), ch. 3, and use a simple way to analyze the granting of accelerated depreciation allowances. The latter are usually granted above the true depreciation rate of a capital good. Depreciation allowances are similar in nature to granting investment subsidies. See, for example, Atkinson and Stiglitz (1989), ch. 5.3.[4] I relate to this and analyze a (simple) two-agent, closed economy framework with a capital-income-cum-depreciation-allowance (CICDA) tax scheme.

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[1]The intuition for the result is astounding. Even workers who may not own capital and may, therefore, not accumulate resources might benefit more from higher steady state wages resulting from nondistorted accumulation with zero taxes than having redistributive transfers now at the expense of a lower steady state capital stock and so wages in the long run. Sargent and Ljungqvist (2004) call this a "celebrated result." Guo and Lansing (1999), fn. 1, point out that a similar result had already been discussed by Arrow and Kurz (1970), p. 191-203, in the context of a neoclassical growth model with inelastic labor supply and productive public expenditures.

[2]But the result that capital income taxes are not good instruments for redistribution need not always hold. This has been shown by many contributions and is well-known. See, for example, Kemp et al. (1993), Aiyagari (1995), Uhlig and Yanagawa (1996), Lansing (1999) Grüner and Heer (2000), Chamley (2001), Erosa and Gervais (2002), Domeij and Heathcote (2004), Abel (2005), Mathieu-Bolh (2006), Werning (2007), Spataro and de Bonis (2008), Conesa et al. (2009), Zhang et al. (2008), Selim (2010), Saez (2013), Reinhorn (2018), Straub and Werning (2020) and others.

[3]Most governments redistribute resources but also grant depreciation allowances to be deducted from collectable tax revenues. This appears to be a pervasive phenomenon in most countries. Hence, these realistic features may justify the policy package under consideration.

[4]The present paper also relates to, for example, Jones et al. (1997) who show for a *representative agent* framework that an investment subsidy can offset the growth distortion associated with a capital income tax and that a consumption tax is the optimal second best policy. For a related argument see also Guo and Lansing (1999). A similar point was made by

The following results then emerge under that tax scheme: Depreciation allowances and capital income taxes appear to have opposite effects on redistribution and capital accumulation in the model. However, when a benevolent government represents the weighted interests of the workers and the capitalists and the latter act optimally things are different. In particular, I analyze a dynamic, closed-loop (feedback) Stackelberg game between the government and the private sector. The government is the Stackelberg leader and the private sector is the Stackelberg follower. The feedback structure of the game implies that the optima derived in this paper are time-consistent.[5]

In this environment the government finds it optimal for the long run to grant maximal depreciation allowances. The reason is that it would remove the distorting effect policy has on capital accumulation. Consequently the paper shows that full expensing of investment outlays is optimal for the long run.[6]

The present paper derives that optimality result in a simple dynamic, heterogeneous agent model with potential distributional conflicts. Having maximal capital depreciation allowances in the long-run, time-consistent optimum does not depend on the social weights attached to the interests of different factor owners or any other things in this model. Importantly, even an entirely pro-labor government would choose maximal depreciation allowances, even though this could mean less tax revenues and so less redistributive transfers. Thus, in the model accelerated capital depreciation allowances work like a synthetic consumption tax and serve as an important *indirect* redistribution device, because transfers ultimately depend on the capital income tax rate chosen.

As regards the latter, it turns out that optimal capital income tax rates are *generically nonzero* in the paper's time-consistent optima. That result complements, for example, Long and Shimomura (2002) and holds under quite plausible conditions.

As one might expect from actual taxation by governments the optimal choice of capital income taxes and so redistribution in the long run depends on the social weight of those who receive redistributive transfers, the physical wear and tear of capital, the distribution of pretax income among individuals, the intertemporal elasticity of substitution and the rate of time preference.

Interestingly, it is found the optimal long-run capital income tax can be negative. The reason appears to be that labor is supplied inelastically in the model. Thus, given certain parameter constellations it may be better from a welfare point of view to have maximal depreciation allowances with no distortion to accumulation so that the combined income tax scheme is tantamount to a tax on consumption, which may also be levied on the workers', instead on the capital owners' consumption.

The intuition for these results is similar to the "celebrated result" in the following sense. No matter whether the government is more pro capital or labor, it chooses not to distort accumulation. That can be taken to represent the essence of the "celebrated result." However, the redistribution implied in this paper is different from the long-run zero-tax results. Here social preferences are

Kaldor (1955) and Fisher (1937) as well who basically proposed that taxable "income" should be "income after savings are taken out." See Fisher (1937), p. 54.

[5]It is well-known that open-loop Stackelberg tax policy games such as the ones analyzed by, for instance, Judd and Chamley may yield time-inconsistent solutions. See, for example, Kemp et al. (1993), Xie (1997) and others. A previous version of the present project also analyzed an open-loop setup of the problem under study. See, Rehme (2007).

[6]That finding has, for example, also been obtained in partial equilibrium analyses by Samuelson (1964), Hall and Jorgenson (1967) and Hall and Jorgenson (1971). In a representative agent, dynamic general equilibrium framework Abel (2007) established the same result. In that sense the assumption of full expensing of investment outlays made in Rehme (1995), and Rehme (1995), which provided verbal arguments why this may be optimal in a general equilibrium, endogenous growth framework, is endogenized and found to be optimal in the present neoclassical growth framework. For a recent related result see, for example, Davies et al. (2009).

important for how the nondistorted accumulation proceeds are distributed among the agents. In this paper that implies that long-run, time-consistent capital income tax rate is optimally nonzero and (pure) redistribution can go either way for the workers and the capital owners.

In summary, the main message of the paper is that capital depreciation allowances may well serve as a redistributive device, especially in the long run and when the private sector and the government would act optimally and in a time-consistent way. Complementing previous results it is corroborated that the long-run, optimal time-consistent capital income tax rate is generically *not* equal to zero.

The paper is organized as follows: Section 2 presents the model. Section 3 analyzes the optimality for depreciation allowances and tax rates in long-run equilibrium. Section 5 provides concluding remarks.

2 | THE MODEL

The model is set in continuous time and the following conventions are used. A variable m functionally depending on another variable z is denoted by $m = m[z]$, that is, square brackets $[\cdot]$ denote a functional dependence. For all variables that are continuous functions of time the subscript t denotes their dependence on time. Thus, we define $h_t \equiv h[t]$ for some variable h depending on time t. As is common, the change of a variable depending on time, that is, $\partial h_t/\partial t$, is denoted by $\dot{h}_t$. In contrast, a change in a variable z with its effect on m is interchangeably denoted by m_z or $\partial m/\partial z$. For a simple derivative I use the convention $m' \equiv m'[z] \equiv dm/dz$.

In terms of the description of the economy the model is set in the following environment. The economy consists of a government, identical competitive firms and two types of infinitely-lived, equally patient and price taking individuals called workers and capitalists. All agents derive utility from the consumption of a homogenous, malleable good. For simplicity, we normalize the population so that the group of capitalists and workers can be treated as one individual each.

The model abstracts from population growth, uncertainty, and technological progress. We assume the workers supply one unit of labor inelastically and do not save or invest.[7] Thus, all the wealth is concentrated in the hands of the capitalists who do not work.

2.1 | Capitalists

In each period the *capital owners* choose how much of their income to consume or invest, and they take prices and policy as given. The instantaneous budget constraint of a representative capitalist is given by

$$c_t + i_t = r_t k_t - T_t \text{ and } i_t = \dot{k}_t + \delta k_t, \tag{1}$$

where c_t denotes consumption of the representative capitalist, i_t his/her (gross) investment, and T_t taxes to be paid to the government. Thus, the capitalists derive income from renting their capital, k_t, to competitive firms at the rate r_t.

[7]The assumption may be rationalized by imposing transaction costs on the workers when borrowing small amounts. Thus, the model uses the commonly used framework of Kaldor (1956) and Pasinetti (1962), which is also employed by Judd (1985) and Lansing (1999).

The capitalist's investment must cover the change in net assets, $\dot{k}_t$, and the depreciation of the capital stock, δk_t. The latter is assumed to happen in a linear way and is determined by technological wear and tear of capital that is not under the control of agents. For simplicity we assume the depreciation rate δ is constant over time. In this paper δk_t captures the *true* (technological) depreciation of capital. Finally, the capital owners have to pay taxes T_t to the government which is taken into account when they make their decisions about consumption and investment.

The representative capital owner derives the following intertemporal utility stream

$$\int_0^\infty u[c_t]e^{-\rho t}dt,$$

where ρ is the constant rate of time preference, common to all agents, that is, common to the capitalists and workers.

The instantaneous utility function $u[c_t]$ satisfies the properties $u' > 0, u'' < 0$, as well as $\lim_{c_t \to \infty} u' = 0$ and $\lim_{c_t \to 0} u' = \infty$ where $u' \equiv du[c_t]/dc_t$ and $u'' \equiv d^2u[c_t]/dc_t^2$.

2.2 | Workers

The *workers* do not invest and are not taxed by assumption. Each worker supplies one unit of labor inelastically at each date and derives utility from consuming his/her entire wage and transfer income. A worker's consumption equals income, x_t, which depends on wage income, w_t, and lump-sum transfers, TR_t, granted by the government,

$$x_t = w_t + TR_t.$$

The intertemporal utility of the worker is given by $\int_0^\infty v[x_t]e^{-\rho t}dt$ where the instantaneous utility $v[x_t]$ function need not be the same as that of the capitalists, but it is also assumed to satisfy $v' > 0, v'' < 0$ and the conditions $\lim_{x_t \to \infty} v' = 0$ and $\lim_{x_t \to 0} v' = \infty$ where $v' \equiv dv[x_t]/dx_t$ and $v'' \equiv d^2v[x_t]/dx_t^2$.

2.3 | Firms

The *firms* operate in a perfectly competitive environment and maximize profits. The capital owners rent capital to and demand shares of the firms, which are collateralized one-to-one by capital. The markets for assets, capital and labor clear at each point in time so that the firms face a path of uniform, market clearing rental rates for capital and labor, r_t and w_t. Given perfect competition the firms rent capital and hire labor in spot markets in each period. Output serves as numéraire and its price is set equal to 1 at each date, implying that the price of capital, k_t, in terms of overall consumption stays at unity.

Aggregate production is constant returns to scale in capital and labor inputs. The (total) labor input equals 1. Thus, k_t can also be interpreted as the capital labor-ratio. Furthermore, the paper's normalization implies that k_t corresponds to the capital stock held by a representative capital owner.

The production function $f[k_t]$ for the representative firm is assumed to be increasing and strictly concave in k_t with $\lim_{k_t \to \infty} f'[k_t] = 0$ and $\lim_{k_t \to 0} f'[k_t] = \infty$. Perfect competition and profit maximization imply

$$r_t = f'[k_t] \text{ and } w_t = f[k_t] - r_t \cdot k_t \tag{2}$$

and free entry and exit of firms means that profits, $f[k_t] - r_t k_t - w_t$, are zero. By implication the share of capital, called α, is then given by $\alpha = (r_t \cdot k_t)/f[k_t] = (f'[k] \cdot k)/f[k_t]$ where $0 < \alpha < 1$. For a Cobb-Douglas production α would be a constant when $f[k_t] = Ak_t^{\alpha}$ denotes per-capita output.

2.4 | Government

As in Judd (1985) and Lansing (1999), I rule out a market for government bonds. A missing bond market may be a justified for an analysis that focuses on the long-run and where Ricardian Equivalence holds. Thus, the paper concentrates on pure redistribution financed by real resources in a two-class model, and abstracts from the intricate issues associated with an analysis of the link between public debt, pure redistribution and capital accumulation.

By assumption the government taxes capital income. But taxable income is such that the government allows for a depreciation allowance on capital as is the case in most countries, that is, it allows for a deduction of taxable income related to the depreciation of capital. As shown above, we will consider *true* economic capital depreciation, δk_t, and the possibility of *accelerated* tax depreciation.

For analyzing the latter I relate it to capital investment as in Sinn (1987), ch. 3. In particular, assume that a proportion $p_t \in [0, 1]$, of an investment expenditure is depreciated immediately and the remainder $1 - p_t$ gradually over time by keeping the tax depreciation at a level of $1 - p_t$ times the true economic depreciation. At each point in time gross investment of the capital owner is $i_t = \dot{k}_t + \delta k_t$.

As the true economic depreciation is δk_t, the flow of immediate depreciation on new investment is $p_t i_t$ and the flow of depreciation on existing assets is $(1 - p_t)\delta k_t$. Thus, the current flow of tax depreciation (see Sinn, 1987, p. 59, eq. 3.16, and fn. 26) is

$$D_t \equiv p_t i_t + (1 - p_t)\delta k_t = p_t \dot{k}_t + \delta k_t. \tag{3}$$

The government taxes capital income net of the depreciation allowance and uses the tax revenues for transfers to the workers. The latter are not taxed by assumption.

The total tax revenues are denoted by T_t and the total transfers to the workers by TR_t. The government runs a balanced budget by assumption and, thus, we have

$$T_t = \theta_t \cdot (r_t k_t - p_t \dot{k}_t - \delta k_t) = TR_t \tag{4}$$

for the government where θ_t denotes the tax rate on (net) capital income.

The tax rate θ is not restricted at this stage of the analysis. Thus, we leave it an open question now whether and how θ_t is bounded from below. In fact, it may turn out in this paper that the optimal solution implies negative θ. On the other hand, θ cannot be greater than one in the model's optima.[8]

2.5 | The private sector

Inserting the tax T_t to be paid by the capital owner in Equation (4) into Equation (1) and rearrangement of the resulting expression yields the capital owner's budget constraint

$$\dot{k}_t = \frac{(1-\theta_t)(r_t-\delta)k_t - c_t}{(1-\theta_t p_t)}. \tag{5}$$

From that one can then state the *capital owner's* problem as

$$\max_{c_t} \int_0^\infty u[c_t]e^{-\rho t}dt$$

subject to the budget constraint in (5) and a given initial capital stock $k(0) = k_0$.

We solve that intertemporal problem by dynamic programming, which always yields time-consistent solutions. Thus, the capitalist's problem is then

$$V_c[k_0] \equiv \max_{c_t} \int_0^\infty u[c_t]e^{-\rho t}dt \text{ s.t. } \dot{k}_t = \frac{(1-\theta_t)(r_t-\delta)k_t - c_t}{(1-\theta_t p_t)}, k(0) = k_0,$$

where the capital owner takes policy as given.

For this autonomous problem the value function of the capital owners, indexed by subscript c, is simply denoted by $V_c(k_t)$, with the understanding that it depends on time as well, and must satisfy the Hamilton–Jacobi–Bellman (HJB) equation

$$\rho V_c[k_t] = \max_{c_t}\left\{u[c_t] + V'_c[k_t]\left(\frac{(1-\theta_t)(r_t-\delta)k_t - c_t}{(1-\theta_t p_t)}\right)\right\},$$

where the value function is assumed to be twice continuously differentiable, and the usual initial as well terminal boundary point conditions are imposed. For convenience let $V_c^{'}[k_t] \equiv \partial V_c[k_t]/\partial k_t$ and $V_c''[k_t] \equiv \partial^2 V_c[k_t]/\partial k_t^2$ and note these expressions are also functions of time t.[9]

As is well-known, the first order necessary conditions for this problem involve the following equations to be met,

$$u'[c_t] = \frac{V'_c[k_t]}{1-\theta_t p_t} \tag{6}$$

[8]Thus, the support of θ is $\theta \in (-\infty, 1)$ which is simply assumed to hold from now on. An earlier version of this project restricted the choice of θ to be nonnegative. See Rehme (2011). Thus, allowing for a larger potential solution space offers interesting new insights in the present paper.

[9]This form of the HJB equation for our autonomous problem with exponential discounting follows, for example, Kamien and Schwartz (1991), sec. 21, or Acemoglu (2009), ch. 7.

$$\rho V'_c[k_t] = V'_c[k_t]\left(\frac{(1-\theta_t)(r_t-\delta)}{(1-\theta_t p_t)}\right) + V''_c[k_t]\left(\frac{(1-\theta_t)(r_t-\delta)k_t - c_t}{(1-\theta_t p_t)}\right) \tag{7}$$

plus an initial and a transversality condition.

Important here is Equation (6), because it implicitly defines the capital owners' optimal decision rule. The latter is of the feedback type, because it depends only on the current state k, and the current policy variables θ and p. Thus, in the optimum for the capitalist we have $c_t = c[k_t; \theta_t, p_t]$. It is important to note that the capitalist follows a rule for the state of the problem, that is, a rule following the state variable k_t. The policy variables θ_t and p_t are parameters and beyond the control of the capital owner. Then[10]

$$\frac{dV'_c[k_t]}{dt} = V''_c[k_t]\cdot\dot{k}_t = V''_c[k_t]\left(\frac{(1-\theta_t)(r_t-\delta)k_t - c_t}{(1-\theta_t p_t)}\right).$$

Thus, we get

$$\frac{\dot{V}'_c[k_t]}{V'_c[k_t]} = \rho - \left(\frac{(1-\theta_t)(r_t-\delta)}{(1-\theta_t p_t)}\right) \tag{8}$$

which is commonly referred to as the *Euler* equation, showing how agents evaluate the evolution of their capital stock in terms of their welfare.

Equation (6) implies $u'[c_t](1-\theta_t p_t) = V'[k_t]$. Taking time derivatives yields

$$u''[c_t]\cdot\dot{c}_t\cdot(1-\theta_t p_t) - u'[c_t]\cdot(\dot{\theta}_t p_t + \theta_t\dot{p}_t) = \dot{V}'_c[k_t].$$

Dividing by $u'[c_t](1-\theta_t p_t) = V'[k_t]$ and using Equation (8) establishes

$$\frac{u''[c_t]}{u'[c_t]\cdot(1-\theta_t p_t)}\cdot\dot{c}_t\cdot(1-\theta_t p_t) - \frac{u'[c_t]\cdot(\dot{\theta}_t p_t + \theta_t\dot{p}_t)}{u'[c_t]\cdot(1-\theta_t p_t)} = \frac{\dot{V}'_c[k_t]}{V'_c[k_t]} = \rho - \left(\frac{(1-\theta_t)(r_t-\delta)}{(1-\theta_t p_t)}\right)$$

$$\begin{aligned}
\dot{c}_t\cdot\left(\frac{u''[c_t]}{u'[c_t]}\cdot\frac{c_t}{c_t}\right) - \frac{(\dot{\theta}_t p_t + \theta_t\dot{p}_t)}{1-\theta_t p_t} &= \rho - \left(\frac{(1-\theta_t)(r_t-\delta)}{(1-\theta_t p_t)}\right)\\
\frac{\dot{c}_t}{c_t}\cdot\left(\frac{u''[c_t]}{u'[c_t]}\cdot c_t\right) &= \rho - \left(\frac{(1-\theta_t)(r_t-\delta)}{(1-\theta_t p_t)}\right) + \frac{(\dot{\theta}_t p_t + \theta_t\dot{p}_t)}{1-\theta_t p_t}\\
\frac{\dot{c}_t}{c_t} &= \eta\left\{\left(\frac{(1-\theta_t)(r_t-\delta)}{(1-\theta_t p_t)}\right) - \frac{(\dot{\theta}_t p_t + \theta_t\dot{p}_t)}{1-\theta_t p_t} - \rho\right\},
\end{aligned}$$

[10]Notice that under the optimal choice and state variable solutions there is a link between a Hamiltonian and the HJB. Furthermore, in the general case $V_c'[k_t] = \lambda_t$, where λ_t is commonly interpreted as the costate variable, representing the *shadow* value of more capital. Of course, the same interpretation applies to $V_c'(k_t)$ for the optimal k_t in the dynamic programming problem. Thus, Equation (8) represents the evolution the shadow value of more capital.

where $\eta[c_t] \equiv -\frac{u'[c_t]}{c_t \cdot u''[c_t]} < 0$ denotes the intertemporal elasticity of substitution of the capital owners.

Lastly, one has to check for the terminal, that is, the transversality condition of the optimum. To that end and for our case we simply require

$$\lim_{t\to\infty} V'[k_t] \cdot k_t \cdot e^{-\rho t} = 0$$

which is satisfied because k_t approaches a positive constant in the steady state.[11]

Furthermore, Equation (6) implies $V'[k_t] = u'[c_t](1 - \theta_t p_t)$ which, under the paper's assumptions, is finite and positive for $c_t > 0$, $p \in [0, 1]$ and $\theta_t < 1$. But then the transversality condition is indeed satisfied.

Thus, the time-consistent household optimum is characterized by the following equations:

$$\dot{k}_t = \frac{(1-\theta_t)(r_t - \delta)k_t - c}{(1-\theta p)}, \tag{9}$$

$$\dot{c}_t = c_t \cdot \eta[c_t] \cdot \left[\frac{(1-\theta_t)(r_t-\delta)}{1-\theta_t p_t} - \frac{\dot{\theta}_t p_t + \dot{p}_t \theta_t}{1-\theta_t p_t} - \rho\right]. \tag{10}$$

In general the consumption growth rate is not necessarily constant, when policy changes, that is, when p_t and θ_t move over time. That is interesting, but most private sector agents expect the tax code and policy not to change systematically over time. For realism's sake that is what is assumed below.

When the factor and goods markets clear the representative worker's income and so consumption is

$$x_t = w_t + TR_t = f[k_t] - r_t k_t + \theta_t \cdot (r_t k_t - p_t \dot{k}_t - \delta k_t),$$

where Equations (2) and (4) have been used. In equilibrium the overall resource constraint is such that the agents satisfy their budget constraints. By substitution of Equation (5) into the expression for x_t above one then obtains[12]

$$\begin{aligned} x_t &= f[k_t] - \left(\frac{1-\theta_t}{1-\theta_t p_t}\right) r_t k_t - \frac{\theta_t(1-p_t)\delta k_t}{1-\theta_t p_t} + \frac{\theta_t p_t c_t}{1-\theta_t p_t}, \\ &= f[k_t] - \left(\frac{1-\theta_t}{1-\theta_t p_t}\right)(r_t - \delta)k_t - \delta k_t + \frac{\theta_t p_t c_t}{1-\theta_t p_t}. \end{aligned} \tag{11}$$

As a consequence in equilibrium the income of the representative worker is increasing in the consumption of the capital owner, for given θ_t and p_t.

[11]That the particular transversality condition used above is applicable, when k approaches a constant value in the steady state, is based on, for example, Acemoglu (2009), theorem 7.12 and 7.13.

[12]The second line of this equation follows when one adds and subtracts $(1-\theta p)/(1-\theta p) \cdot \delta k$ in the first line, collects terms and rearranges.

One verifies from Equations (9) and (11) that on impact (a) an increase in investment subsidies does not appear to raise after-tax wages and so workers' consumption. So p_t looks as if it is a noneffective redistribution tool; (b) an increase in capital income taxes seems to be positive for redistribution as it may raise after-tax wages; (c) higher capital income taxes seem to imply lower investment; (c) an increase in investment subsidies may imply more investment and so more capital accumulation.[13]

2.6 | Nondistortion of accumulation

One important consequence of the Judd (1985) and Chamley (1986) result that capital income taxes be optimally zero in the long run is that the capital accumulation process will not be disturbed by political interference in that case.

In the model the impact of any accumulation distortion can be inferred from the Euler equation in Equation (8). We will rerender it here where the shadow value of more capital is denoted by $V_c'[k_t]$,

$$-V'_c[k_t]\left(\frac{(1-\theta_t)(r_t-\delta)}{1-\theta_t p_t}\right)+\rho V'_c[k_t]=\dot{V}'_c[k_t].$$

This equation shows how agents evaluate the evolution of the state variable k_t in terms of their welfare, measured by the evolution of the shadow price $V_c'[k_t]$, which then leads them to pursue a particular accumulation programme. Policy would in general distort this evaluation which is captured by the term $\frac{1-\theta_t}{1-\theta_t p_t}$.

The government does *not* distort this evaluation in a long-run equilibrium with $\dot{V}_c'[k_t]=0$ when $\theta_t=0$ or $p_t=1, \forall\, t$, or both. In this paper all these solutions are in principle possible and I analyze that in detail below.[14]

If $p_t=1$, then the tax arrangement with nonzero θ_t reduces to a tax on the capital owner's consumption. As is well known, consumption taxes are not distorting accumulation. To see this consider Equation (5) where $\dot{k}_t=(r_t-\delta)k_t-\frac{c_t}{1-\theta_t}$ when $p_t=1$. The taxes are then tantamount to taxing c_t. In that sense, a policy with $p_t=1$ and $\theta>0$ is equivalent to synthetic, nondistorting consumption tax on the capital owners.

3 | THE OPTIMAL LONG-RUN CAPITAL DEPRECIATION ALLOWANCE AND CAPITAL INCOME TAX

Consider a benevolent government that respects the private sector's problem and/or its optimality conditions and represents the agents' interests by attaching weights to their welfare. By assumption the government is as impatient as the private sector and so has the same rate of time preference as the agents.

[13]See, for example, Goode (1955) for similar arguments, which correspond to what one usually expects.
[14]Notice that in the case of $p_t=1$ all values of θ_t in its domain are possible, not just $\theta_t=0$.

Let $\gamma \in (0, \infty)$ represent the social weight attached to the welfare of the representative worker, $v[x_t]$, relative to that of the capitalist, $u[c_t]$.[15] If $\gamma \to 0$, the government is only concerned about the representative capitalist, whereas it only cares about the representative worker when $\gamma \to \infty$. By assumption the government inherits a capital income tax rate $\theta(0)$ that is less than one and takes this as given at time zero. This makes the tax problem nontrivial.[16]

The government keeps the agents on their respective supply and demand curves, and chooses a policy that can be realized as a competitive equilibrium *in quantities*. Thus, the government is taken to choose its policy instruments, but lets the market determine the path of the (pretax) return on capital. The solution of the government's problem is then compatible with a private ownership competitive equilibrium *in quantities*.[17] Hence, I relate to the dual approach for solving Ramsey tax problems as described in, for example, Sargent and Ljungqvist (2004), ch. 15.3. Similar approaches are used by Judd (1985), Judd (1999), and Lansing (1999).

Furthermore, from now on time subscripts are dropped for convenience whenever it is clear that a particular variable depends on time, and variables that attain a long-run equilibrium position such as a steady state are indexed by an asteristk (*).

Unless stated otherwise the government's problem is to choose a policy pair (θ, p), where the choice of θ is in principle unrestricted. Furthermore, in line with most tax provisions and as the focus of the analysis is on the long run, I only concentrate on optimal long-run policies that are constant over time. Thus, the focus is on situations where $\dot{\theta} = \dot{p} = 0$, because that is what most agents expect to be the case for the long-run.

Consider now a dynamic game between the private sector and the government. The latter moves first by announcing a policy which the follower, that is, the private sector takes into account when making its decision. Thus, the government is the Stackelberg leader, and the private sector is the Stackelberg follower. For an introduction into these kinds of games see, for example, Dockner et al. (2000) and Long (2010).

Judd and Chamley focussed on open-loop-Stackelberg equilibria in their setups. The equilibria of such games do require the assumption that the government can commit itself to a policy announced at the outset of the game. As is known, that may yield time inconsistency issues.

In turn and building on Kemp et al. (1993), consider now a game in which the government plays a feedback strategy $\theta = \theta(k)$ and $p = p(k)$, which is time-consistent by construction. The representative capitalist treats θ, p and r as known and given functions of time. The equilibria of such games do not require the assumption that the government can commit itself to a policy announced at the outset of the game.

Following Kemp et al. (1993), p. 421, assume that the private sector optimum in Equations (5) and (10), that is,

[15]The model's normalization implies that we consider a representative capital owner and worker each. As a stronger microfoundation of the political process is beyond the scope of the paper, I follow the common procedure to attach fixed (exogenous) weights on the representative agents' welfare. For a similar setup see, for example, Lansing (1999), p. 432.

[16]The assumption rules out taxing the initial capital stock via a so-called capital levy that would constitute a lump sum tax, since initial capital is in fixed supply. See Judd (1985), and Chamley (1986) or, for example, Sargent and Ljungqvist (2004), ch. 15.3.

[17]This builds on Jones (1965), Atkinson and Stiglitz (1989), lec. 6, and Turnovsky (2000), ch. 12.6. Here I follow Turnovsky's setup.

$$\dot{k} = \frac{(1-\theta)(r-\delta)k - c}{(1-\theta p)}$$
$$\dot{c} = c\cdot\eta[c]\cdot\left[\frac{(1-\theta)(r-\delta)}{1-\theta p} - \frac{\dot{\theta}p + \dot{p}\theta}{1-\theta p} - \rho\right]$$

forms a system that has a unique stationary saddle point. The stable path through that point, say $c[k]$, is the equilibrium consumption path of the capitalist. Thus, the optimal (feedback) decision rule of the capital owner satisfies $c = c[k;\theta, p]$ where policy is taken parametrically by the household and policy is constant, that is, $\dot{\theta} = \dot{p} = 0$.

The dynamic problem facing the government is then

$$V_g(k_0) \equiv \max_{\theta,p}\int_0^{\infty}\{\gamma v[x] + u[c[k;\theta,p]]\}e^{-\rho t}dt, \tag{12a}$$

$$\text{s. t.}\quad v[x] = v\left[f(k) - \left(\frac{1-\theta}{1-\theta p}\right)(r-\delta)k - \delta k + \frac{\theta\cdot p\cdot c[k;\theta,p])}{1-\theta p}\right], \tag{12b}$$

$$\dot{k} = \frac{(1-\theta)(r-\delta)k - c[k;\theta,p]}{(1-\theta p)} \quad \text{and} \quad k(0) = k_0. \tag{12c}$$

For this autonomous problem it is known that the value function of the government, denoted by $V_g(k)$, satisfies the HJB equation[18]

$$\begin{aligned}\rho V_g(k) = \max_{\theta,p}\Bigg\{&\gamma v\left[f(k) - \left(\tfrac{1-\theta}{1-\theta p}\right)(r-\delta)k - \delta k + \tfrac{\theta\cdot p\cdot c[k;\theta,p])}{1-\theta p}\right]\\ &+u[c[k;\theta,p]] + V'_g(k)\left(\tfrac{(1-\theta)(r-\delta)k - c[k;\theta,p]}{(1-\theta p)}\right)\Bigg\}.\end{aligned} \tag{13}$$

The solution to (12) then satisfies

$$\theta[k] \equiv \arg\max_{\theta}\left\{\gamma v[x] + u[c[k;\theta,p]] + V'_g(k)\left(\tfrac{(1-\theta)(r-\delta)k - c[k;\theta,p]}{(1-\theta p)}\right)\Big| p\right\},$$
$$p[k] \equiv \arg\max_{p}\left\{\gamma v[x] + u[c[k;\theta,p]] + V'_g(k)\left(\tfrac{(1-\theta)(r-\delta)k - c[k;\theta,p]}{(1-\theta p)}\right)\Big| \theta\right\},$$

where $v[x]$ is given by Equation (12b). Thus, recalling Basar and Olsder (1995), p. 227/8, it is acceptable to consider $(c[k], \theta[k], p[k])$ as a feedback equilibrium with the government as the leader. As $\theta[k], p[k]$ are derived from the HJB Equation (13), the equilibrium is time-consistent.

I now simplify the analysis by assuming that $\eta[c]$ is a *negative constant* η. Again we look for constant optimal policies. A tilde over a variable will denote that we look for a solution that depends on the feedback rule $\tilde{c} = \tilde{c}[k,\theta,p]$ or, expressed in reduced form, simply $\tilde{c} = \tilde{c}[k]$.

[18]Again, as in Kamien and Schwartz (1991), section 21, assume that $V_g(k)$ is twice continuously differentiable, or that it satisfies the conditions for the infinite horizon leading to Theorem 3.4 in Dockner et al. (2000).

Thus, the capitalists' consumption follows a rule and is not a state variable anymore in the subsequent problem.

Then the current-value Hamiltonian associated with the problem formulated in Equation (12), called $\mathcal{H}^F$, is then[19]

$$\mathcal{H}^F = \gamma v[\tilde{x}] + u[\tilde{c}] + \nu \cdot \tilde{\Delta} \text{ where } \tilde{\Delta} \equiv \left[\frac{(1-\theta)(r-\delta)k - \tilde{c}[k,\theta,p]}{(1-\theta p)}\right],$$

where $\dot{k} = \tilde{\Delta}$, and $\tilde{c} = \tilde{c}[k,\theta,p]$, as well as $\tilde{x} = f[k] - \frac{1-\theta}{1-\theta p} \cdot (r-\delta)k - \delta k + \frac{\theta p \tilde{c}[\cdot]}{1-\theta p}$.

Here the control variables are θ and p. The single state variable is k.[20] The first order conditions for θ, p and k are

$$\gamma v'[\cdot] \cdot \tilde{x}_\theta + u'[\cdot] \cdot \tilde{c}_\theta + \nu \cdot \tilde{\Delta}_\theta = 0, \tag{14a}$$

$$\gamma v'[\cdot] \cdot \tilde{x}_p + u'[\cdot] \cdot \tilde{c}_p + \nu \cdot \tilde{\Delta}_p = 0, \tag{14b}$$

$$\gamma v'[\cdot] \cdot \tilde{x}_k + u'[\cdot] \cdot \tilde{c}_k + \nu \cdot \tilde{\Delta}_k = \rho\nu - \dot{\nu}. \tag{14c}$$

Plus the requirements that

$$\dot{k}_t = \frac{(1-\theta)(r-\delta)k - \tilde{c}[k,\theta,p]}{(1-\theta p)} \text{ and } \lim_{t\to\infty} \nu k e^{-\rho t} = 0.$$

The co-state variable ν denotes the shadow price of capital for the government. The requirement for $\dot{k}$ is, of course, that the budget constraint be obeyed, and the second one captures the transversality condition, ruling out asymptotic left-overs or running infinite debt. The partial derivatives of $\tilde{x}, \tilde{c}$ and $\tilde{\Delta}$ are derived in Appendix A.

As we are interested in finding constant optimal policies for the long run we evaluate the first order conditions in a steady state. In such a state we must then have $\dot{k} = \dot{\nu} = 0$. For a long-run equilibrium we also require that $\dot{c} = 0$ in Equation (10).

In Appendix A.2 the partial derivatives are derived that are necessary to evaluate the first order conditions in a steady state. From that it is not difficult to verify that the Equations (14a) to (14c) evaluated in a steady state, indexed by an asterisk, become

$$\gamma v'[\cdot]\left[\frac{(r-\delta)k^* + \theta p \cdot \tilde{c}^*_\theta}{1-\theta p}\right] + u'[\cdot] \cdot \tilde{c}^*_\theta - \nu\left[\frac{(r-\delta)k^* + \tilde{c}^*_\theta}{1-\theta p}\right] = 0, \tag{15a}$$

$$\gamma v'[\cdot]\left(\frac{\theta p \cdot \tilde{c}^*_p}{1-\theta p}\right) + u'[\cdot] \cdot \tilde{c}^*_p - \nu \cdot \left(\frac{\tilde{c}^*_p}{1-\theta p}\right) = 0, \tag{15b}$$

[19] Here the solution procedure in Kemp et al. (1993), sec. 4, is followed.

[20] The feedback setup eliminates c as a state variable. This is an important point because in open-loop formulations that is one reason one may obtain time-inconsistent solutions. Also the present problem is quite different from a command (nudge) approach of the government which may also yield a time-consistent solution. That is analyzed in a companion paper to this one.

$$\gamma v'[\cdot]\left\{f' - \frac{(1-\theta)(r-\delta)}{1-\theta p} - \delta + \frac{\theta p \cdot \tilde{c}_k^*}{1-\theta p}\right\} + u'[\cdot]\cdot \tilde{c}_k^* + \nu \cdot \left[\frac{(1-\theta)(r-\delta)}{1-\theta p} - \frac{\tilde{c}_k^*}{1-\theta p}\right] = \rho \nu. \tag{15c}$$

One also verifies that $\dot{k} = 0$ implies $\tilde{c}^* = \tilde{c}^*[k, \theta, p] = (1-\theta)(r-\delta)k^*$.

3.1 | Is $\theta = 0$ with $p \in [0, 1)$ an optimum?

If $\theta = 0$, then $\tilde{c}_p = 0$ (see Appendix A.2) so that from Equation (15b) any p is optimal. In this case, that is, when $\theta = 0$, Equation (15a) becomes

$$\gamma v'[\cdot](r-\delta)k^* + u'[\cdot]\cdot \tilde{c}_\theta^* - \nu(r-\delta)k^* - \nu\tilde{c}_\theta^* = 0.$$

From Appendix A.2 one verifies for $\theta = 0$ that

$$\tilde{c}_\theta^* = \frac{-\eta \cdot \tilde{c}^* \cdot p}{1-\theta p} = -\eta \cdot p \cdot \tilde{c}^* = -\eta\, p\,(r-\delta)k^*.$$

Substitution then yields that Equation (15a) must satisfy

$$\gamma v'[\cdot] + u'[\cdot]\cdot(-\eta)\,p = \nu\cdot(1-\beta\, p).$$

Given this and the assumption $\theta = 0$ we can substitute in Equation (15c) to obtain

$$\gamma v'[\cdot]\{f' - (r-\delta) - \delta\} + u'[\cdot]\cdot \tilde{c}_k^* = \left(\frac{\gamma v'[\cdot] - u'[\cdot]\eta p}{1-\eta p}\right)\cdot\left[\rho - (r-\delta) + \tilde{c}_k^*\right].$$

By profit maximization we have $f' = r$, and Equation (10) implies $(r-\delta) = \rho$ when $\dot{c} = 0$ and $\theta = 0$. It is then not difficult to verify that an optimum with $\theta = 0$ as a feedback solution would require

$$\gamma v'[f[k^*] - r^* k^*] = u'[\rho k^*],$$

where, again, k^* is determined by $f'[k^*] = r = \delta + \rho$. But an optimum that simultaneously satisfies $\theta = 0$ and the last equation is generically impossible.

Proposition 1. *A time-consistent, feedback solution with $\theta = 0$ and $p \in [0, 1)$ as long-run policy optima in a steady state equilibrium does generically not exist.*

The result, therefore, casts doubt on the general validity of the *celebrated* Judd–Chamley result and corroborates earlier findings such as Long and Shimomura (2000), Long and Shimomura (2002) and others.

3.2 | Is $\theta \neq 0$ with $p = 1$ an optimum?

If θ is nonzero, then so is $\tilde{c}_p^*$. In that case we can divide Equation (15b) by $\tilde{c}_p^*$ to obtain

$$\gamma v'[\cdot]\frac{\theta p}{1-\theta p} + u'[\cdot] = \frac{\nu}{1-\theta p} \text{ i. e. } \gamma v'[\cdot]\theta p + u'[\cdot](1-\theta p) = \nu. \tag{16}$$

We can substitute for ν in Equation (15a) to get

$$\gamma v'[\cdot]\left\{\frac{(r-\delta)k^* + \theta p\cdot\tilde{c}_\theta^*}{1-\theta p}\right\} + u'[\cdot]\cdot\tilde{c}_\theta^* = \{\gamma v'[\cdot]\theta p + u'[\cdot](1-\theta p)\}\left[\frac{(r-\delta)k^* + \tilde{c}_\theta^*}{1-\theta p}\right].$$

Collecting terms, rearranging and division by $(r-\delta)k^*$ yield $\gamma v'[x^*] = u'[c^*]$.[21] Thus, in the long-run optimum the social marginal utilities of the agents must be equated.

Again by profit maximization $f' = r$ so that substitution of ν from Equation (16) into the optimality condition for the capital stock in Equation (15c) implies that

$$\gamma v'[\cdot]\left\{(r-\delta)\left(1-\frac{1-\theta}{1-\theta p}\right)\right\} + \{\gamma v'[\cdot]\theta p + u'[\cdot](1-\theta p)\}\left(\frac{(1-\theta)(r-\delta)}{1-\theta p} - \rho\right) = 0 \tag{17}$$

must be satisfied.

In a steady state equilibrium $\dot{c} = 0$ so that

$$\frac{(1-\theta)(r-\delta)}{1-\theta p} = \rho.$$

But then Equation (17) is only satisfied when $\frac{1-\theta}{1-\theta p} = 1$, because $\gamma v'[\cdot]$ and $(r-\delta)$ are nonzero. Thus, the solution requires $\theta = 0$ or $p = 1$.

As was shown in the previous subsection a solution with $\theta = 0$ is generically impossible. Thus, we now concentrate on solutions with $p = 1$ and θ being nonzero. This is not difficult to see since the optimum must satisfy $\gamma v' = u'$. With $p = 1$ it amounts to the condition

$$\gamma v'[\tilde{x}^*] = u'[\tilde{c}^*] \text{ where } \tilde{c}^* = (1-\theta)(r-\delta)k^* = (1-\theta)\rho k^*$$
$$\text{and } \tilde{x}^* = f(k^*) - rk^* + \frac{\theta\tilde{c}^*}{1-\theta} = f(k^*) - rk^* + \theta\rho k^*.$$

Hence, the optimal tax rate θ, when $p = 1$, is implicitly determined by

[21] The intermediate step is

$$\gamma v'[\cdot]\left\{\frac{(r-\delta)k^*}{1-\theta p} + \frac{\theta p\cdot\tilde{c}_\theta^*}{1-\theta p} - \frac{\theta p(r-\delta)k^*}{1-\theta p} - \frac{\theta p\cdot\tilde{c}_\theta^*}{1-\theta p}\right\} + u'[\cdot]\left\{\tilde{c}_\theta^* - \frac{(1-\theta p)(r-\delta)k^*}{1-\theta p} - \frac{(1-\theta p)\tilde{c}_\theta^*}{1-\theta p}\right\} = 0$$
$$\gamma v'[\cdot]\cdot(r-\delta)k^* = u'[\cdot]\cdot(r-\delta)k^*$$

$$\gamma v'[f(k^*) - rk^* + \theta\rho k^*] = u'[(1-\theta)\rho k^*] \tag{18}$$

and is denoted by $\tilde{\theta}^*$ where the tilde indicates the feedback solution and the asterisk that this would be a long-run optimum. Of course, the feedback solution is *time-consistent* by construction, and the optimal tax rate $\tilde{\theta}^*$ is implicitly determined by the model's parameters.

It remains to check whether the transversality condition is met for the conditions identified so far. Notice that the transversality condition can be written as

$$\lim_{t\to\infty} \nu k e^{-\rho t} = \lim_{t\to\infty} \nu \cdot \lim_{t\to\infty} k \cdot \lim_{t\to\infty} e^{-\rho t} = 0.$$

We know that $k \to k^*$, a positive constant in the steady state. Furthermore, $\lim_{t\to\infty} e^{-\rho t} = 0$. Thus, it remains to determine that in the optimum ν will nonnegative and finite in the long-run equilibrium The following arguments show that that is the case, when ruling out implausible policy solutions.

From Equation (14b) one obtains

$$\nu = -\frac{\gamma v'[\cdot]\cdot x_p + u'[\cdot]\cdot c_p}{\Delta_p} = -\frac{\gamma v'[\cdot]\cdot \frac{\theta p\cdot c_p}{1-\theta p} + u'[\cdot]\cdot c_p}{-c_p/(1-\theta p)} = \gamma v'[\cdot]\theta p + u'[\cdot](1-\theta p)$$

and by Equation (18) the optimum involves $p = 1$ and $\gamma v'[\cdot] = u'[\cdot]$. Thus, ν equals $u'[\cdot]$ or $\gamma v'[\cdot]$ and is positive, because $u'[\cdot]$ and $\gamma v'[\cdot]$ are positive by assumption.

In the long-run optimum we have $\gamma v'[\tilde{x}^*]$ and $u'[\tilde{c}^*]$. But then any viable optimum must rule out a tax rate such that $\tilde{c}^* = (1-\theta)\rho k^* \to 0$ implying $u'[\tilde{c}^*] \to \infty$. As a consequence for a finite ν one must have that $\tilde{\theta}^* < 1$ in any optimum.

Similarly, any solution with $\tilde{x}^* \to 0$ must be ruled out. So $\tilde{\theta}^*$ has to be such that $\tilde{x}^*$ is positive. That does not preclude a negative $\tilde{\theta}^*$, however. One verifies that $\tilde{x}^* = 0$ would require a $\theta_{\min} < 0$ such that $\theta_{\min} \equiv \frac{(\alpha-1)f(k^*)}{\rho k^*} = \frac{(\alpha-1)r}{\alpha\rho}$ which is a negative number because the capital share is less than one, $\alpha < 1$. Thus, for any optimum the lower bound on any optimal $\tilde{\theta}^*$ is $\theta_{\min} < 0$ so that $\tilde{\theta}^* > \theta_{\min}$.

But if the optimal $\tilde{\theta}^*$ is bounded in the way shown above then $0 < \nu < \infty$ as $t \to \infty$ (in the long-run steady state) so that the transversality condition is indeed satisfied.

Summarizing, in the optimum the long-run $\tilde{\theta}^* \in (\theta_{\min}, 1)$ and $\tilde{p}^* = 1$. Furthermore, total differentiation of Equation (18) reveals that $\tilde{\theta}^*$ is increasing in γ.[22]

Proposition 2. *In an interior, time-consistent feedback Stackelberg optimum the following holds for the steady state equilibrium.*

[22]The total differential implies

$$v'[\cdot]d\gamma + (\gamma v''[\cdot] + u''[\cdot])\rho k^* d\theta = 0 \quad \Leftrightarrow \quad \frac{d\gamma}{d\theta} = -\frac{(\gamma v''[\cdot] + u''[\cdot])\rho k^*}{v'[\cdot]} > 0.$$

The expression is positive because $v''[\cdot]$ and $u''[\cdot]$ are negative and the other terms positive.

1. *Generically the social marginal utilities of the worker's and capitalist's consumption are equated in a long-run equilibrium, and the optimal, long-run capital income tax* $\widetilde{\theta}^*$ *solves*

$$\gamma v'[f(k^*) - r^*k^* + \theta\rho k^*] = u'[(1-\theta)\rho k^*].$$

   Where $r^* = \rho + \delta$, *and* $k^* = k^*[\alpha, \rho, \delta]$.
2. *The long-run, optimal depreciation allowances are maximal, that is,* $\widetilde{p}^* = 1$, *and that implies nondistortion of capital accumulation.*
3. *The optimal, long-run time-consistent solution does not distort accumulation and turns the capital income tax scheme into a (synthetic) consumption tax scheme.*
4. *Depending on the strength of the reaction of the marginal utilities of consumption to taxes, the optimal, long-run capital income tax rate* $\widetilde{\theta}^*$, *can be positive or negative,.*

$$\widetilde{\theta}^* \in (\theta_{min}, 1) \text{ where } \theta_{min} = \frac{(\alpha-1)f(k^*)}{\rho k^*} = \frac{(\alpha-1)r}{\alpha\rho} < 0.$$

   And depends in an important way on the parameters characterizing the economy.
5. *The optimal tax rate* $\widetilde{\theta}^* = \widetilde{\theta}^*[\gamma, \alpha, \rho, \delta]$ *is higher when the social weight on the marginal utility of the workers' consumption* γ *is higher.*

Notice that the results do not depend on production externalities or any other things, the capital income taxes may be used for, except for using an accelerated capital depreciation allowance scheme.

Thus, high capital income depreciation allowances are good for the workers in an *indirect* way, because, with a maximal accelerated depreciation allowance, there exists the possibility of positive redistributive effects of capital income taxes, namely when the optimal capital income tax is positive. These effects become strongest in a long-run equilibrium when the accelerated depreciation allowance is maximal. In that sense depreciation allowances are an indirect redistributive device in the long run.

Clearly, the result is in contrast to the model's predictions for the short run and arbitrary behavior. In the short run p may be a bad instrument for redistribution. Hence, the effects of the policy instrument p depends on behavior and the time horizon.

The fact that the optimal tax rate is increasing in γ appears quite realistic. Thus, as the workers's welfare gets more social weight the government would choose higher capital income taxes in this tax scheme with capital depreciation allowances.[23]

An important implication of the model is that the long-run optimal capital income tax rate can be negative. This is, for example, the case if γ is low. Thus, political preferences are not important when considering the policy distortion on accumulation. Irrespective of γ it is optimal not to distort accumulation. But then the often identified redistribution-accumulation trade-off is separated out in this setup. As a consequence the benefits of a nondistorted

[23]Notice that the steady state capital stock k^* would be the same under any other capital income tax scheme for which it is shown that the long-run capital income tax should be zero. This is an important point, because overall welfare (sum of utilities) may be higher under the present tax scheme in comparison to those other capital income tax schemes.

economy are redistributed, and that redistribution can go either way, to the workers or the capital owners.[24]

In summary, the deep structural parameters of the economy determine the optimal policy mix for the long run. Maximal accelerated capital depreciation allowances and nonzero capital income taxes are found to be optimal for the long-run. Redistributive taxation depends in an important way on social preferences and other deep structural variables characterizing the economy.

4 | A PARAMETRIZATION

Proposition 2 provides a general result for the tax scheme under study. Clearly the optimal tax rate will depend on many parameters. To consider the possible effects of the latter consider an example based on the observation that it is not entirely clear why workers should evaluate a consumption good any differently than a capital owner. For that reason now assume that the two representative agents have the same form of the utility function, which is taken to be of the often-used and well-known constant intertemporal elasticity of substitution (CIES) type,

$$u[c] = \frac{c^{1-\frac{1}{\eta}} - 1}{1 - \frac{1}{\eta}} \quad \text{and} \quad v[x] = \frac{x^{1-\frac{1}{\eta}} - 1}{1 - \frac{1}{\eta}}$$

with η as the constant intertemporal elasticity of substitution. Then Equation (18) boils down to

$$\gamma[f[k] - (\rho + \delta)k^* + \theta\rho k^*]^{-\frac{1}{\eta}} = [(1 - \theta)\rho k^*]^{-\frac{1}{\eta}},$$

where $\rho + \delta = r^*$. This equation is not easily solvable, but clearly the optimal solution is a function of the form $\widehat{\theta} = \theta[\gamma, k^*, \eta, \rho, \delta]$.

Notice that k^* is a function of parameters too. To this end assume that the production function is of the standard type $f = Ak^\alpha$ where $0 < \alpha < 1$. Then $f' = \alpha A k^{\alpha-1}$ and $f' = r$ and so $k^* = \left(\frac{\alpha A}{r}\right)^{\frac{1}{1-\alpha}}$. Thus, in steady state the output-capital ratio is given by $f^*/k^* = A(k^*)^{\alpha-1} = A\frac{r^*}{\alpha A} = \frac{r^*}{\alpha}$. With this one can rearrange the equation above and divide by k^* to get

$$\gamma^\eta = \frac{1}{\rho}(f^*/k^* - r^*) + \theta(1 + \gamma^\eta).$$

Now notice that $f^*/k^* = r^*/\alpha$. Thus, the optimal θ, called $\widetilde{\theta}^*$, satisfies

[24]Of course, that result depends on the assumption of an inelastic labor supply, just as assumed in the previous research this paper relates to. Again, it is important to notice that the optimum with nondistorted capital accumulation implies that the redistribution question can be separated from accumulation considerations. Hence, more generally speaking, the optimum implicitly implies a separation of "efficiency" and "equity" concerns.

$$\tilde{\theta}^* = \tilde{\theta}[\gamma, \alpha, \eta, \rho, \delta] = \left\{\gamma^\eta + \frac{r^*}{\rho}\left(1 - \frac{1}{\alpha}\right)\right\}(1 + \gamma^\eta)^{-1}, \tag{19}$$

where $r^* = \rho + \delta$. As $\frac{1}{\alpha} > 1$ the sign of the optimal tax rate $\tilde{\theta}^*$ depends in an important way on the social weight going to the workers. As it is very unlikely that $\gamma = \left\{\frac{r^*}{\rho}\left(\frac{1}{\alpha} - 1\right)\right\}^{\frac{1}{\eta}}$, the long-run capital income tax rate $\tilde{\theta}^*$ is generically nonzero.

Corollary 1. *If the agents possess the same CIES utility functions, it is optimal to set $\tilde{p}^* = 1$ in a long-run equilibrium under a capital-income-cum-depreciation-allowance tax scheme (CICDA).*

- *The value of the optimal, long-run capital income tax rate $\tilde{\theta}^*$ is generically nonzero.*
- *The optimal, long-run capital income tax rate is positive, that is, $\tilde{\theta}^* > 0$, if $\gamma > \left\{\frac{r^*}{\rho}\left(\frac{1}{\alpha} - 1\right)\right\}^{\frac{1}{\eta}}$.*
- *Otherwise, the optimal long-run capital income tax rate is negative, that is, $\tilde{\theta}^* < 0$, if $\gamma < \left\{\frac{r^*}{\rho}\left(\frac{1}{\alpha} - 1\right)\right\}^{\frac{1}{\eta}}$.*

In Appendix C it is then shown that $\tilde{\theta}^*$ depends on the parameters as follows.

Corollary 2. *Under CICDA and when the agents all have the same CIES utility functions, the optimal, long-run capital income tax rate $\tilde{\theta}^*$ is*

- *higher the higher the share of capital (α) is. Thus, the income distribution matters.*
- *lower the higher is the physical depreciation rate (δ). Thus, technology matters.*
- *higher, the more impatient (ρ) or the more willing the agents are to exchange current for future consumption (η). Thus, private-sector-preferences matter.*
- *higher, the more social weight is attached to the marginal utility of the workers' consumption (γ). Thus, public-sector-preferences matter.*

Thus, under the capital income tax scheme under consideration (CICDA) distributional and preference parameters matter for the long-run equilibrium, and that may complement the results of Judd (1985) and Chamley (1986). Importantly, the results establish that there capital income taxes are optimally nonzero in the long run when coupled with accelerated capital depreciation allowances.

To get a feeling for the nature of the solutions I have conducted a numerical simulation exercise based on calibrations from the business cycle and other literature. They are presented in Appendix D and reveal that, unsurprisingly, the weight γ plays a crucial role for the value of the optimal tax rate.

The simulation is able to mimic income tax rate values that apply today and in history, particularly for the U.S., under the counterfactual assumption that it pursues a maximal depreciation allowance policy. Naturally the simulated numbers crucially depend on all the parameters too. In that sense the numerical exercise highlights a dependency of any nonzero capital income tax rates on social and other parameters.

5 | CONCLUSION

In this paper it is analyzed whether politically determined capital depreciation allowances are bad instruments for redistribution. When coupling the latter with capital income taxes it turns out that an increase in accelerated capital depreciation allowances is a bad tool for redistribution, but good for economic growth, when the private sector and the government act nonoptimally in the short run. In turn, capital income taxes are bad for economic growth and good for redistribution under these conditions.

However, for the long run and with optimizing behavior things are quite different, when the agents act in a time-consistent manner. To capture this a feedback Stackelberg game between the government and the private sector is analyzed, where the government is the Stackelberg leader and moves before the private sector. The coupling of capital income taxes with accelerated capital depreciation allowances for financing pure redistribution implies maximal depreciation allowances and generically nonzero capital income taxes in the long-run, time-consistent optimum.

The policy package under consideration in this paper is nondistortionary for accumulation and, importantly, time-consistent in the optimum. It is found that capital income taxes are optimally nonzero in the long run which depends on realistic conditions for time-consistent taxation policy. The most important conditions identified in this paper are: (a) the social weight of those who receive redistributive transfers, (b) the distribution and so inequality in pretax factor incomes, (c) the physical wear and tear of capital, (d) the intertemporal elasticity of substitution (especially, of the capital owners), and (e) the rate of time preference. The results imply that pure redistribution may optimally be financed by capital income taxes when using accelerated capital depreciation allowances as a complementing instrument.

The results suggest that it might be a good thing to use quite generous depreciation allowance schemes for pure redistribution in the long run and, coupled with it, that a long-run, nonzero capital income tax is optimal. The model's optimum also suggests that a separation of "efficiency" and "equity" concerns seems possible under the tax scheme analyzed in this contribution.

Several caveats apply. For instance, it may interesting to analyze the medium run properties of the solutions and what the optimal policy trajectory in the transition to the steady state is. Issues of public debt and long-run optimal policy are certainly also interesting to analyze. These and other questions are left for future research.

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APPENDIX A: REACTIONS OF c, x, AND Δ IN THE FEEDBACK STACKELBERG GAME

A.1 The general reactions

The static first order condition of the capitalists requires

$$u'[c] = \frac{V'_c[k]}{1-\theta p}, \tag{A1}$$

where $V_c[k]$ denotes the value function of the capital owner. Thus, the optimal decision rule for consumption depends on the current value of the state variable k and the policy variables. Hence, the optimal feedback rule satisfies

$$\tilde{c} = \tilde{c}[k;\theta,p],$$

where from now on feedback relationships are denoted by a tilde.

We obtain by implicit differentiation

$$\tilde{c}_k = \frac{V''_c[k]}{u''[\cdot]}\cdot\frac{1}{1-\theta p},\ \tilde{c}_\theta = \frac{V'_c[k]}{u''[\cdot]}\cdot\frac{p}{(1-\theta p)^2},\ \text{and}\quad \tilde{c}_p = \frac{V'_c[k]}{u''[\cdot]}\cdot\frac{\theta}{(1-\theta p)^2}.$$

Recall $\eta[c] \equiv -\frac{u'[\cdot]}{u''[\cdot]\cdot c}$, which denotes the intertemporal elasticity of substitution of the capital owners and is a positive number since $u''[\cdot]<$ by assumption. Using Equation (A1) we then get

$$\tilde{c}_k = \frac{V''_c[k]}{u''[\cdot]}\cdot\frac{1}{1-\theta p} > 0, \tag{A2a}$$

$$\tilde{c}_\theta = \frac{V'_c[k]}{u''[\cdot]}\cdot\frac{p}{(1-\theta p)^2} = \frac{u'[\cdot]}{u''[\cdot]}\frac{p}{(1-\theta p)} = -\eta[\tilde{c}]\cdot\tilde{c}[\cdot]\cdot\left(\frac{p}{1-\theta p}\right) < 0, \tag{A2b}$$

$$\tilde{c}_p = \frac{V'_c[k]}{u''[\cdot]}\cdot\frac{\theta}{(1-\theta p)^2} = \frac{u'[\cdot]}{u''[\cdot]}\frac{\theta}{(1-\theta p)} = -\eta[\tilde{c}]\cdot\tilde{c}[\cdot]\cdot\left(\frac{\theta}{1-\theta p}\right) < 0. \tag{A2c}$$

Second, with the rule $\tilde{c} = \tilde{c}[k,\theta,p]$ we have

$$\tilde{x} = f[k] - \frac{1-\theta}{1-\theta p}\cdot(r-\delta)k - \delta k + \frac{\theta p\cdot\tilde{c}[\cdot]}{1-\theta p}.$$

The partial derivatives of x are given by

$$\tilde{x}_k = f' - \frac{(1-\theta)(r-\delta)}{1-\theta p} - \delta + \frac{\theta p \cdot \tilde{c}_k}{1-\theta p}, \tag{A3a}$$

$$\tilde{x}_\theta = \frac{(1-p)(r-\delta)k + p \cdot \tilde{c}[k,p,\theta]}{(1-\theta p)^2} + \frac{\theta p \cdot \tilde{c}_\theta}{1-\theta p}, \tag{A3b}$$

$$\tilde{x}_p = \frac{\theta \cdot \{\tilde{c}[k,p,\theta] - (1-\theta)(r-\delta)k\}}{(1-\theta p)^2} + \frac{\theta p \cdot \tilde{c}_p}{1-\theta p}. \tag{A3c}$$

The signs of these derivatives are not immediately clear.

Third, $\tilde{\Delta}$ is given by

$$\tilde{\Delta} = \frac{(1-\theta)(r-\delta)k - \tilde{c}[k,\theta,p]}{1-\theta p}.$$

So that its partial derivatives amount to

$$\tilde{\Delta}_k = \frac{(1-\theta)(r-\delta)}{1-\theta p} - \frac{\tilde{c}_k}{1-\theta p}, \tag{A4a}$$

$$\tilde{\Delta}_\theta = \frac{\{-(r-\delta)k - \tilde{c}_\theta\}(1-\theta p) + p\{(1-\theta)(r-\delta)k - \tilde{c}[k,p,\theta]\}}{(1-\theta p)^2}, \tag{A4b}$$

$$\tilde{\Delta}_p = \frac{-\tilde{c}_p \cdot (1-\theta p) + \theta\{(1-\theta)(r-\delta)k - \tilde{c}[k,p,\theta]\}}{(1-\theta p)^2}. \tag{A4c}$$

Again, the signs of these derivatives are not clear.

A.2 Reactions in the steady state

In the steady state $\dot{v} = \dot{k} = 0$. Form that latter condition it follows that $\dot{k} = 0 \Rightarrow \tilde{c}^* = (1-\theta)(r^* - \delta)k^*$ where steady state variables are again marked by an asterisk. Notice that $\tilde{c}_k$ must obey (A2a), and must hold in a steady state too. Furthermore, recall that η is constant in the steady state.

Thus, from the Equations (A1), and (A2a)–(A2c), noting that $-\eta < 0$ and $V_c'', u'' < 0$ and for $\theta, p \leq 1$, the reactions of $\tilde{c}$, $\tilde{x}$ and $\tilde{\Delta}$ in the steady state are the following.

$$\tilde{c}_k^* = \frac{V''_c[k]}{u''[\tilde{c}^*]} \cdot \frac{1}{1-\theta p} > 0, \tilde{c}_\theta^* = \frac{-\eta \cdot \tilde{c}^* \cdot p}{1-\theta p} < 0, \tilde{c}_p^* = \frac{-\eta \cdot \tilde{c}^* \cdot \theta}{1-\theta p} < 0. \tag{A5}$$

With this the partial effects on $\tilde{x}$ become

$$\tilde{x}_k^* = f'[k^*] - \frac{(1-\theta)(r^*-\delta)}{1-\theta p} - \delta + \frac{\theta p \cdot \tilde{c}_k^*}{1-\theta p}, \tag{A6a}$$

$$\tilde{x}_\theta^* = \frac{(r^*-\delta)k^*}{(1-\theta p)} + \frac{\theta p \cdot \tilde{c}_\theta^*}{1-\theta p}, \tag{A6b}$$

$$\tilde{x}_p^* = \frac{\theta p \cdot \tilde{c}_p^*}{1 - \theta p}. \tag{A6c}$$

Third, the partial effects on $\tilde{\Delta}$ are then given by

$$\tilde{\Delta}_k^* = \frac{(1-\theta)(r^* - \delta)}{1 - \theta p} - \frac{\tilde{c}_k^*}{1 - \theta p}, \tag{A7a}$$

$$\tilde{\Delta}_\theta^* = -\frac{(r^* - \delta)k^* + \tilde{c}_\theta^*}{1 - \theta p}, \tag{A7b}$$

$$\tilde{\Delta}_p^* = -\frac{\tilde{c}_p^*}{1 - \theta p}. \tag{A7c}$$

APPENDIX B: THE SECOND ORDER SUFFICIENT CONDITIONS

To check the sufficiency conditions of the solutions use Arrow's theorem. That requires that the Hamiltonian evaluated at the optimum solution for the choice variable (as a function of the state and costate variables) be concave in the state variable for a given co-state variable. If the concavity is strict, the solution (for the control variable) is then also the unique optimizer. Notice that in our case the optimum is restricted to the long-run, steady state. Thus, the Hamiltonian is also evaluated at the steady state optimum. For Arrows's theorem and a similar steady state problem see, for example, Weitzman (2003), ch. 3, especially pp. 85–93.

For our problem the Hamiltonian for the feedback solution, $\mathcal{H}^F$, is given by

$$\mathcal{H}^F = \gamma v[\tilde{x}] + u[\tilde{c}] + \nu \cdot \tilde{\Delta} \quad \text{where} \quad \tilde{\Delta} \equiv \left[\frac{(1-\theta)(r-\delta)k - \tilde{c}[k, \theta, p]}{(1-\theta p)}\right].$$

The choice variables for $\mathcal{H}^F$ are θ and p, the state variable is k and the costate is ν. In the long-run optimum $p = 1$ and θ is implicitly determined from

$$\gamma v'[\tilde{x}^*] = u'[\tilde{c}^*] \quad \text{where} \quad \tilde{c}^* = (1-\theta)(r^* - \delta)k^* = (1-\theta)\rho k^*$$
$$\text{and } \tilde{x}^* = f[k^*] - rk^* + \tfrac{\theta \tilde{c}^*}{1-\theta} = f[k^*] - r^*k^* + \theta\rho k^*.$$

From that one verifies that in the optimum the Hamiltonian $\mathcal{H}^F$ under the optimal long-run solution is given by

$$\mathcal{H}^{F*} \equiv \mathcal{H}^F[k^*, \theta[k^*], p[k^*]] = \gamma v[f[k^*] - r^*k^* + \theta[k^*] \cdot \rho k^*] + u[(1-\theta[k^*])\rho k^*] + \nu \cdot 0,$$

where the optimal p satisfies $p[k^*] = 1$, the optimal tax rate $\theta[k^*]$ is, again, implicity determined by

$$\gamma v'[f[k^*] - r^*k^* + \theta\rho k^*] = u'[(1-\theta)\rho k^*]$$

and $\tilde{\Delta} = 0$ in the long-run optimum, that is, in the steady state.

Arrow's theorem is satisfied if $\mathcal{H}^{F*}$ is concave in k^*. To that end we check the sign of the second derivative of $\mathcal{H}^{F*}$. For simplicity let $f' = f'[k^*]$ and $f'' = f''[k^*]$. From

$$\mathcal{H}_k^{F*} = \gamma v'[\cdot]\left(f' - r^* + \frac{\partial\theta}{\partial k^*}\cdot\rho k^* + \theta\rho\right) + u'[\cdot]\left(-\frac{\partial\theta}{\partial k^*}\cdot\rho k^* + (1-\theta)\rho\right)$$

one obtains the expression for the second derivative as

$$(\mathcal{H}^{F*})_{kk} = \gamma v''[\cdot]\left(f' - r^* + \tfrac{\partial\theta}{\partial k^*}\cdot\rho k^* + \theta\rho\right)^2 + \gamma v'[\cdot]\left(f'' + \tfrac{\partial^2\theta}{\partial(k^*)^2}\cdot\rho k^* + \tfrac{\partial\theta}{\partial k^*}\cdot\rho + \tfrac{\partial\theta}{\partial k^*}\cdot\rho\right)$$
$$+ u''[\cdot]\left(-\tfrac{\partial\theta}{\partial k^*}\cdot\rho k^* + (1-\theta)\rho\right)^2 + u'[\cdot]\left(-\tfrac{\partial^2\theta}{\partial(k^*)^2}\cdot\rho k^* - \tfrac{\partial\theta}{\partial k^*}\cdot\rho - \tfrac{\partial\theta}{\partial k^*}\cdot\rho\right).$$

This expression can be simplified to

$$(\mathcal{H}^{F*})_{kk} = \gamma v''[\cdot]\cdot D_1 + u''[\cdot]\cdot D_2 + \gamma v'[\cdot]\cdot f'' + (\gamma v'[\cdot] - u'[\cdot])\cdot D_3,$$

where $D_1 \equiv \left(f^{'}[k^*] - r^* + \frac{\partial\theta}{\partial k^*}\cdot\rho k^* + \theta\rho\right)^2 > 0$, $D_2 \equiv \left(-\frac{\partial\theta}{\partial k^*}\cdot\rho k^* + (1-\theta)\rho\right)^2 > 0$, because they are squared expressions, and $D_3 \equiv \left(\frac{\partial^2\theta}{\partial(k^*)^2}\cdot\rho k^* + 2\frac{\partial\theta}{\partial k^*}\rho\right) \lesseqgtr 0$. Because $\gamma v'[\cdot] = u'[\cdot]$, $\gamma v'[\cdot] > 0$, and $f''[k^*] < 0$, it follows that $(\mathcal{H}^{F*})_{kk} < 0$. Hence, the Hamiltonian $\mathcal{H}^{F*}$ is concave in k^*.

APPENDIX C: COMPARATIVE STATICS OF THE OPTIMAL INCOME TAX RATE

The optimal capital income tax rate has to satisfy the following equation:

$$\gamma^\eta = \frac{1}{\rho}(f^*/k^* - r^*) + \theta(1+\gamma^\eta).$$

Where $f^*/k^* = r^*/\alpha$ and $r^* = \rho + \delta$. Thus, $\widehat{\theta}$ has to satisfy

$$\gamma^\eta = \left(1 + \frac{\delta}{\rho}\right)\cdot\left(\frac{1}{\alpha} - 1\right) + \theta(1+\gamma^\eta).$$

Taking total differentials implies the following.

$$\gamma: \quad \eta\gamma^{\eta-1}d\gamma = (1+\gamma^\eta)d\theta + \theta\eta\gamma^{\eta-1}d\gamma$$
$$(1-\theta)\eta\gamma^{\eta-1}d\gamma = (1+\gamma^\eta)d\theta \quad \Rightarrow \quad d\theta/d\gamma > 0$$
$$\alpha: \quad 0 = \left(1+\frac{\delta}{\rho}\right)\cdot\left(-\frac{1}{\alpha^2}\right)d\alpha + (1+\gamma^\eta)d\theta \quad \Rightarrow \quad d\theta/d\alpha > 0$$
$$\delta: \quad 0 = (1/\rho)\cdot\left(\frac{1}{\alpha}-1\right)d\delta + (1+\gamma^\eta)d\theta \quad \Rightarrow \quad d\theta/d\delta < 0$$
$$\rho: \quad 0 = -(\delta/\rho^2)\cdot\left(\frac{1}{\alpha}-1\right)d\rho + (1+\gamma^\eta)d\theta \quad \Rightarrow \quad d\theta/d\rho > 0$$
$$\eta: \quad \ln\gamma \cdot e^{\eta\ln\gamma}d\eta = \theta\ln\gamma\cdot e^{\eta\ln\gamma}d\eta + (1+\gamma^\eta)d\theta$$
$$(1-\theta)\ln\gamma\cdot e^{\eta\ln\gamma}d\eta = (1+\gamma^\eta)d\theta \quad \Rightarrow \quad d\theta/d\eta > 0, \forall\gamma > 1.$$

These results are summarized in the following table.

Table C1

APPENDIX D: NUMERICAL SIMULATION

For the simulation of Equation (19) I use the calibration values in Walsh (2010), ch. 2. He bases his calibration on quarterly data for the United States and roughly sets $\alpha = 0.36$, $\delta = 0.02$ and $1/\eta = 2$. Jordá et al. (2019) report a long-run estimate of the return on wealth of $r_w \approx 0.06$ for a weighted panel of countries and the long-run period 1870–2015. In terms of the model I set the ρ equal to $r^* = \rho + \delta = \rho + 0.02 = 0.06$ so that $\rho = 0.04$.

It is certainly true that more people receive the major part of their income in the form of wages than in the form of capital income. For a population-based justification for values of γ I, therefore, relate to Lansing (2015) who sets the number of workers relative to capitalists equal to nine.

Table D1

With these values $\gamma > 7.11$ satisfies the condition for positive optimal capital income tax rates when there is maximal depreciation, that is, $\widetilde{p}^* = 1$. Table D2 reports the results of a numerical simulation for the calibrated economy varying γ, bearing in mind Lansing's calibration where the relative ratio of workers to capitalists, called n_w here, is nine. Thus, when political representation would be based on that ratio, γ should take a value of nine. If $\gamma = 9$ then the optimal tax rate $\widetilde{\theta}^*$ is positive.

The numbers suggest that the social weight γ is an important determinant of the optimal tax rate. That corresponds to common intuition. Governments that give more weight to the interests of the workers seem to choose higher capital income tax rates. However, according to this model the social weight must be sufficiently high for the government to choose positive capital income tax rates. For example, the (highest marginal) capital income tax rate in the United States is currently around 35 percent. To obtain such a number would require a γ of around 20 in this model where the depreciation allowance were maximal. For Germany a γ of roughly 30 would correspond to the general (top) marginal income tax rate of 42%. The value of $\gamma = 3600$ basically corresponds to the historically most progressive income tax rate in US

TABLE C1 The reaction of $\widetilde{\theta}^*$ to changes in parameters

α	δ	ρ	η	γ
+	−	+	+	+

TABLE D1 Baseline parameter values

α	δ	ρ	$\frac{1}{\eta}$	n_w
0.36	0.02	0.04	2	9

TABLE D2 Optimal capital income tax rates $\tilde{\theta}^*$

γ	1	5	9	15	20	30	40	100	1000	3600
θ^*	−0.83	−0.13	0.08	0.25	0.33	0.43	0.50	0.67	0.89	0.94

Note: The results are calculated for the optimal capital depreciation allowance $\hat{p} = 1$ and as a function of the social weight factor γ.

history of 95% in 1944–1945. That γ would correspond to a social weight of approximately 180 times of the US today's imputed value of the capital income tax rate. That would, of course only hold, if the fiscal depreciation of capital were maximal.[25]

That tax rate may seem high. But it is important to realize that the determination of the social weight that the government attaches to the welfare of a (representative) worker relative to that of a (representative) capital owner is outside this model. Thus, many things may lead to a particular value of γ. Furthermore, these numbers crucially depend on the other parameters too. In that sense the numerical exercise is only intended to highlight a dependency of any nonnegative capital income tax rates on social parameters.

[25]In the United States in 1944 the marginal tax rate was applicable for incomes above 200.000 $ which corresponds to roughly 2.4 million $ today, given inflation over that period.