Algorithmic Conceptual Design of Technical Systems under Uncertainty

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Vorwort des Herausgebers

Kontext — Komposition technischer Systeme unter Unsicherheit
Die ersten zwei Phasen einer technischen Entwicklung werden nach Pahl & Beitz (Konstruktionslehre, Springer, 2021) Konzept und Entwurf genannt. In der Konzeptphase wird die Funktions- und damit die Wirkstruktur der Funktionselemente festgelegt. Funktionselemente übernehmen Teilfunktionen, die sich in der Gesamtstruktur zur Gesamtfunktion aggregieren. In der auf die Konzeptphase folgende Entwurfsphase werden die cyber-physische Komponenten, die die Teilfunktionen erfüllen, „grobmaßstäblich" festgelegt. Nach Pahl & Beitz erfolgt die Bewertung von Nutzen und Aufwand (sog. Nutzwertanalyse) in der Konzeptphase. Diese in der VDI-Richtlinie Konstruktionsmethodik (Technisch- wirtschaftliches Konstruieren VDI 2225) beschriebene Nutzwertanalyse ist stark von dem Schweizer Ingenieur Fritz Kesselring beeinflusst. Mit seiner „Technischen Kompositionslehre" (Springer, 1954) gilt Kesselring als der westliche Vordenker eines Qualitätsbegriffs, den wir heute mit wirtschaftlicher Nachhaltigkeit in Verbindung bringen. Das östliche Pendant zu Kesselring ist Genichi Taguchi, der in den 1950 und 1960er Jahren erstmals die „Kosten für Gesellschaft" als Metrik betrachtet und als Begründer der Robust-Design-Methodik mit den Baukastenelementen Qualitätskostenfunktion, Qualitätsregelung, statistische Versuchsmethodik, Six-Sigma, Parameterdesign und Systemdesign gilt.
Das Problem der Konstruktionsmethode von Pahl & Beitz ist das konsekutive Vorgehen nach dem Prinzip „Teile und Herrsche". Tatsächlich kann die Qualität mit den Dimensionen a) Ressourcenverbrauch und Emissionen (Aufwand), b) Verfügbarkeit und c) Akzeptanz (Sicherheit, funktionale Qualität) sowohl in der Fertigung als auch in der Nutzung erst in der Entwurfsphase beurteilt werden. Genichi Taguchi war der Erste, der erkannte, dass dies nur gelingt, wenn phasenübergreifend gestaltet, gefertigt und betrieben wird. Die östliche Ingenieurschule von Taguchi ist der westlichen Schule von Pahl überlegen, wenn maximale Nachhaltigkeit bei minimalem Aufwand Kompositionsziel ist. Die Pahlsche Konstruktionsmethodik versteht man aus der Historie. Professor Gerhard Pahl, der in meinem Studium einer meiner Lehrer war, wirkte in den 1960 und 1970er Jahren in der Industrie. Dies war eine Zeit, abgesehen von der Ölkrise, in der Ressourcenknappheit technische Entscheidungen nur unwesentlich beeinflussten. Daher stand für Pahl und seine Lehre an erster Stelle a) Funktion, b) Verfügbarkeit und c) Sicherheit. An zweiter Stelle standen d) Montage, e) Fertigung und f) Wirtschaftlichkeit. Damit ist die Pahlsche Konstruktionsmethodik immer noch Sullivan's „form follows

function“ aus dem 19ten Jahrhundert verpflichtet.
Anders verhält es sich mit der Robust-Design-Methodik von Taguchi. In den 50er und 60er Jahren des letzten Jahrhunderts herrscht in Japan ein Mangel an qualitativ hochwertigen Halbzeugen. Kundenakzeptanz durch maximale funktionale Qualität (invers zur Differenz zwischen realisierter und erwarteter Funktion) konnte daher — so Taguchi's Einsicht - nur dann erreicht werden, wenn Beschaffung, Fertigung und Nutzung ganzheitlich gestaltet wird.
Heute ist die Erfüllung eines essentiellen Bedürfnisses, und damit die Funktion die Nebenbedingung und minimale private und soziale Kosten sowie maximale Verfügbarkeit und Sicherheit, die Zielfunktion. Trotz seines Weitblicks war Taguchi's Zeit nicht reif genug, das beschränkte Optimierungsproblem unter Unsicherheit zu lösen, da ihm weder die Modellierungsmethoden noch die mathematischen Lösungsmethoden zur Verfügung standen.
Dies ist erst heute durch Zusammenarbeit von angewandter Mathematik und Ingenieurwissenschaft möglich. Die vorliegende Dissertation von Herrn Philipp Leise ist hierfür ein leuchtendes Beispiel. Im Sonderforschungsbereich 805 erlebten wir im Jahr 2015, dass beim Getriebeentwurf entweder brutale Rechengewalt z.B. durch die Firma IAV eingesetzt wurde oder Heuristiken, wie genetische Algorithmen Anwendung fanden, wie am Institut für Mechatronische Systeme der TU Darmstadt. Brutale Rechengewalt scheitert häufig an den Berechnungskosten, genetische Algorithmen führen häufig nur zu lokalen Optima und liefern auch keine absolute Qualitätsaussage wie „besser geht's nicht“, garantieren keine globale Optimalität. Bei der Komposition haben wir Ingenieurinnen und Ingenieure mit dem Fluch der Dimension (kombinatorische Explosion) bei unklarer Sicht (Unsicherheit) zu kämpfen. Unsere Waffen sind heute mathematischer Methoden und Algorithmen. Herr Leise war also Waffenschmied.

Dargestellte und nichtdargestellte Forschung
Die Arbeit von Herrn Leise entstand im Sonderforschungsbereich (SFB) 805 der Deutschen Forschungsgemeinschaft. Der im Jahr 2021 nach 12 Jahren Forschung abgeschlossene SFB hatte Beherrschung von Unsicherheit in lasttragenden Systemen des Maschinenbaus zum Thema.
Unsicherheit ist bei der Komposition technischer Systeme immer präsent. Dies betrifft die Modelle, Strukturen (Funktions- und Bauteilstrukturen) und insbesondere die Nutzungsszenarien. Für jeden Ingenieur bzw. jede Ingenieurin ist es Aufgabe, die Unsicherheit zu quantifizieren und Strategien für die Beherrschung von Unsicherheit zu entwickeln und zu implementieren.
Herr Leise gibt mit seiner Arbeit eine Antwort auf die Frage, wie das von Taguchi angedachte Systemdesign aussieht, insbesondere wenn Unsicherheit zu berücksichtigen ist. Wie gesagt, fehlten Taguchi noch die notwendigen

Modelle und mathematischen Methoden.[1] Herr Leise entwickelt, ertüchtigt und nutzt Methoden um alle drei Unsicherheitskategorien, nämlich Daten-, Modell- und Strukturunsicherheit bei der Komposition zu berücksichtigen. Mittels Clustering-Verfahren und stochastischer Optimierung werden unsichere Nutzungsszenarien betrachtet.
Beides betrifft Daten- und Modellunsicherheit. Phänomenologische und Skalierungsmodelle technischer Komponenten wie Pumpe, Triebwerk, Propeller und Elektromotor in den Ingenieurwissenschaften sind häufig Monome (Produkte von Potenzen). Der tiefergehende Grund liegt im Bridgman-Postulat („absolute Bedeutung relativer Größen“) und dem Buckingham-Pi-Theorem. Dadurch sind Optimierungsprobleme häufig nichtlinear und nicht konvex, was exakte Optimierungen sehr erschwert.
Herr Leise zeigt wie durch „Geometric Programming“ Konvexität erreicht werden kann und damit typische technische Systeme wie ein Antriebsstrang exakt optimiert werden können. Fragt man Herrn Leise, dann würde er dies sicherlich als seine größte Leistung benennen, nämlich das Potential des „Geometric Programming“ für die Ingenieurwissenschaften entdeckt zu haben. Dadurch wird die Strukturunsicherheit erst beherrschbar, dass nämlich der vollständige Designraum durch ein exaktes Verfahren exploriert werden kann. Übliche Ingenieur-Heuristiken wie Partikelschwarm oder genetische Methoden bleiben i.d.R. weit hinter dem exakten Verfahren zurück. Interessant ist, dass die Designmethode an sich, nämlich die beschränkte Optimierung, wiederum ein Optimierungsproblem ist, bei dem minimale Rechenzeit und minimale Modellunsicherheit im Zielkonflikt stehen. Herr Leise geht darauf in seiner

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[1] Im Kontext des SFB 805 wird das Systemdesign Sustainable-Systems-Design (SSD) genannt. Sullivan’s Paradigma „form follows function“ ist dabei durch das beschränkte Optimierungsproblem „maximise quality subject to functionallity“ ersetzt. Aus der Nachhaltigkeitssicht ist dies allerdings noch nicht ausreichend. Es gibt drei komplementäre Nachhaltigkeitsstrategien, nämlich Suffizienz, Effizienz und Konsistenz. Die Suffizienz ist die Beschränkung auf wesentliche Bedürfnisse, die Effizienz und die Konsistenz wird in „maximise quality subject to functionallity“ bereits ausreichend dargestellt.

Ein sehr knappes und schönes Leitbild ist das von Dieter Rahms, nämlich „less but better“. Mit „less ... “ ist die Suffizienz dargestellt, mit „... but better“ die Effizienz und Konsistenz. Effizienz ohne Suffizienz führt nicht zur Nachhaltigkeit. Dies zeigt die Zunahme der Emissionen durch Wohnen obgleich Isolation und Heizungen immer besser bzw. effizienter werden. Grund ist die Zunahme des Wohnraums pro Person. Im Verkehr lassen sich ähnliche Beispiele finden. So werden Motoren effizienter die Suffizienz ist aber nicht erreicht, wenn Motoren immer größer werden. So wie Gestalt und Größe auseinandergehalten werden müssen und an die Stelle der Geometrie treten, muss Effizient und (notwendige) Größe bzw. Menge getrennt werden.

Arbeit ein.
Herr Leise betrachtet vier Anwendungen von denen nur zwei in der vorliegenden Arbeit diskutiert sind, nämlich erstens eine Wasserinfrastruktur (Kapitel 3) und zweitens den Antriebsstrang eines elektrischen Fahrzeuges mit mechanischem Getriebe, vgl. Kapitel 4 der Arbeit. Drittens taucht in der Arbeit immer wieder ein Beispiel auf, das er zitiert aber nicht weiter ausführt, nämlich den nachhaltigen Flugzeugentwurf eines elektrischen Flugzeugs mit einer großen Anzahl verteilter Propulsoren und viertens der Prüfstand, den Herr Leise in seiner Arbeit aufgebaut hat. An diesem Prüfstand, einem generischen Wasserversorgungssystem, hat Herr Leise die vier Resilienzfunktionen eines kybernetischen Systems, nämlich Überwachen, Reagieren, Lernen und Antizipieren realisiert und dynamische Strategien und Resilienzmetriken validiert.
Es ist notwendig den Prüfstand zu nennen, da uns allen bewusst ist, wie aufwendig und zeitraubend es ist, einen mechatronischen Prüfstand zu planen, bauen, in Betrieb zu nehmen und damit produktiv Forschung zu betreiben, die über reine Konzepte hinausgeht. Neben dynamischen Resilienz-Metriken und -Eigenschaften findet sich in der Arbeit auch statische Resilienz-Metriken wie z.B. die Nehmerqualität eines Netzwerks. Auch dies ist mir wichtig zu erwähnen. Der Begriff Resilienz ist derzeit so modern, dass er allenthalten zu hören ist, ohne dass eine Begriffsbildung stattgefunden hat. Begriffsbildung ist eine wesentliche Aufgabe von Wissenschaft.[2] Herr Leise leistet einen wesentlichen Beitrag hierzu - bei einem Wort, dass leider häufig unreflektiert verwendet wird.

Darmstadt, im Oktober 2022

Peter Pelz

[2] Die begriffsbildende Aufgabe wurde schon als nichts nützliche Scholastik z.B. von Francis Bacon kritisiert. Dies ist durchaus berechtigt, nämlich dann, wenn Wissenschaft sich nur um sich selbst kreist und zu einem geschlossenen und autistischen System mit eigner Sprache degeneriert. Die Begriffsbildung muss genutzt werden für Kommunikation, Selbstorganisation, Analyse und Synthese. Dies erfordert Offenheit und Reibung und ist damit gerade Voraussetzung für die moderne wissenschaftliche Methode, die Francis Bacon auf den Weg gebracht hat.

Vorwort

Diese Dissertation entstand während meiner Tätigkeit als wissenschaftlicher Mitarbeiter am Institut für Fluidsystemtechnik an der Technischen Universität Darmstadt. Mein ganz besonderer Dank gilt hierbei Herrn Prof. Dr.-Ing. Peter F. Pelz für die sehr gute, vertrauensvolle und konstruktive Zusammenarbeit. Ich bedanke mich für seine Offenheit gegenüber neuen interdisziplinären Forschungsansätzen, die diese Arbeit an der Schnittstelle der Ingenieurswissenschaften und der angewandten Mathematik erst ermöglicht hat. Darüber hinaus hat erst der von ihm ermöglichte tiefere Einblick in ganz unterschiedliche ingenieurswissenschaftliche Themenbereiche mit den damit verbundenen unterschiedlichen Modellierungsansätzen maßgeblich dazu beigetragen, dass der im folgenden vorgestellte methodische Ansatz erfolgreich zum Design der vorliegenden technischen Systeme genutzt werden kann.

Darüberhinaus bedanke ich mich bei Prof. Marc E. Pfetsch für die Ausfertigung des Zweitgutachtens, sowie die sehr gute Zusammenarbeit innerhalb des Teilprojekts A9 im Sonderforschungsbereich 805. Ich bedanke mich für die vielen Hinweise zur Lösung der vorliegenden mathematischen Problemstellungen, die in dieser Zeit unweigerlich auftraten. Ich möchte weiterhin noch Frau Prof. Dr.-Ing. Lena Altherr für die sehr gute Betreuung als Teilprojektleiterin und Herrn Dr. Andreas Schmitt für die ausgezeichnete interdisziplinäre Zusammenarbeit danken.

Die Zeit meiner Promotion wurde maßgeblich bereichert, durch die vielen Gespräche und Aktivitäten mit meinen Kolleginnen und Kollegen am Institut und im Sonderforschungsbereich 805. Im besonderen möchte ich hier Tim Müller, Andreas Schmitt, Imke Rehm und Marvin Meck erwähnen. Die Zusammenarbeit zwischen uns war aus meiner Sicht immer sehr konstruktiv und freundschaftlich. Unvergesslich bleiben die gemeinsamen Fahrten zu Konferenzen, wie der *OR*. Darüber hinaus möchte ich allen anderen nicht namentlich erwähnten Personen am Institut für die kollegiale Zusammenarbeit danken.

Ferner möchte ich noch allen Studierenden danken, die mich während meiner Zeit als wissenschaftlicher Mitarbeiter unterstützt haben. Im besonderen Herrn Nicolai Simon für seine langjährige Tätigkeit als studentische Hilfskraft. Zuletzt möchte ich mich noch bei meiner Frau und meiner Familie für die Unterstützung während dieser Zeit bedanken.

Hiermit erkläre ich, dass ich die vorliegende Arbeit, abgesehen von den in ihr ausdrücklich genannten Hilfen, selbständig verfasst habe.

Felsberg, im August 2022
Philipp Leise

Kurzfassung

Die vorliegende Dissertation ist im Rahmen des Sonderforschungsbereichs 805 am Institut für Fluidsystemtechnik im Fachbereich Maschinenbau der TU Darmstadt entstanden. Betrachtet wird die konzeptionelle Auslegung technischer Systeme unter Berücksichtigung von Unsicherheit mithilfe exakter Optimierungsalgorithmen. Diese eignen sich insbesondere für eine systematische Auslegung, da sie zur Auffindung global-optimaler Systemstrukturen führen und damit Strukturunsicherheit reduzieren. Die hinsichtlich einer definierten Zielfunktion optimierte Systemgestalt hängt maßgeblich von der gewählten Parametrisierung ab. Da diese Parametrisierung jedoch während der konzeptionellen Entwicklung mit Unsicherheit behaftet ist, entstehen bei Nutzung unterschiedlicher Parametrisierungen in der Regel unterschiedliche Lösungsstrukturen. Darüber hinaus findet die Lösungssuche für technische Systeme mithilfe global-optimaler Optimierungsalgorithmen in einem Spannungsfeld zwischen der Nutzung effizienter Lösungsalgorithmen, einer effizienten Modellentwicklung und einer zweckmäßigen Approximation der relevanten Wirklichkeit mithilfe des entwickelten Systemmodells statt. Oben genannte Punkte werden in der Arbeit daher adressiert. Anhand von zwei technischen Systemen wird aufgezeigt, wie eine gemischt-ganzzahlige nichtlineare Modellierung zum konzeptionellen Design unter Unsicherheit genutzt werden kann. Betrachtet werden als Anwendungsfälle die dezentrale Wasserversorgung in Hochhäusern sowie der Antriebsstrang von batterieelektrisch betriebenen Fahrzeugen mit Mehrganggetriebe. Die entwickelten gemischt-ganzzahligen Programme werden mithilfe gängiger Software gelöst. Um die Lösungszeit zur Auffindung effizienter Lösungen zu reduzieren, werden darüber hinaus für beide Anwendungsfälle neue problemspezifische Heuristiken vorgestellt. Zur Beherrschung der Unsicherheit wird die Nutzungsphase in beiden Anwendungsfällen mithilfe von Szenarien in einem deterministischen Äquivalent eines stochastischen Optimierungsprogramms abgebildet. Für den Anwendungsfall Antriebsstrangauslegung wird ein neues Verfahren zur automatisierten Erstellung von Szenarien basierend auf einem Ansatz des unüberwachten Lernens vorgestellt. Für den Anwendungsfall Wasserversorgung wird auf die algorithmische Auslegung resilienter Systeme eingegangen. Für beide Anwendungsfälle werden die Effizienzgewinne durch die beschriebene systematische Auslegung quantifiziert und am Beispiel der Antriebsstrangauslegung wird eine Verifikation des Modells vorgestellt. Darüber hinaus wird ein allgemein gültiger Modellierungs- und Lösungsansatz aufgezeigt, der auf geometrischer Programmierung beruht. In der Literatur wurde diese Modellierung bisher vorwiegend bei Modellen mit kontinuierlichen Variablen eingesetzt. Für die Antriebsstrangauslegung wird gezeigt, wie dieser Ansatz auch bei Modellen mit gemischt-ganzzahligen Variablen eingesetzt werden kann. Zur Lösung des jeweils zugrunde liegenden gemischt-ganzzahligen Programms wird ein Lösungsalgorithmus basierend auf einer Generalisierten Benders Dekomposition vorgestellt. Im Vergleich zur vollständigen Enumeration der diskreten Variablen kann gezeigt werden, dass der Ansatz die global-optimale Lösung bei deutlich geringerer Lösungszeit findet.

Abstract

This dissertation is located within the Collaborative Research Center 805 at the Chair of Fluid Systems within the Department of Mechanical Engineering at TU Darmstadt. It examines the conceptual design of technical systems under consideration of uncertainty with the help of exact optimization algorithms. These are particularly suitable for a systematic design, since they lead to the identification of globally optimal system structures and thus reduce structural uncertainty. The system design optimized with respect to a defined objective function depends significantly on the selected parameterization. However, since this parameterization is subject to uncertainty during the conceptual design, the use of a different parameterization usually results in different solution structures. Furthermore, the search for solutions for technical system designs is situated within a field of conflicting goals. These are the use of efficient solution algorithms, an efficient model development and an appropriate approximation of the relevant reality with the help of the developed system model. These above-mentioned points are addressed in the thesis. Based on two technical systems, it is shown how mixed-integer nonlinear modeling can be used for conceptual design under uncertainty. The use cases considered are a decentralized water distribution system in high-rise buildings and the powertrain of battery electric vehicles with a multi-speed transmission. The developed mixed-integer programs are solved with the help of common software. Furthermore, in order to reduce the solving time for finding efficient solutions, new problem-specific heuristics are presented for both use cases. To master uncertainty, the usage phase is modeled with multiple scenarios within a deterministic equivalent of a stochastic optimization program each in both use cases. For the powertrain design use case, a new method for an automatic generation of scenarios based on an unsupervised learning approach is presented. For the water supply use case, the algorithmic design of resilient systems is addressed. For both use cases, the efficiency gains realized by the described systematic design are quantified. Additionally, a verification of the model is presented for the powertrain design. Furthermore, a generally applicable modeling and solving approach is shown, which is based on geometric programming. In the literature, this modeling approach has so far mainly been used for models with continuous variables. For the powertrain design, it is shown how this approach can also be used for models with mixed integer variables. For the solution of the respective underlying mixed-integer program, a solution algorithm based on a Generalized Benders Decomposition is presented. In comparison to the complete enumeration of the discrete variables, it can be shown that the approach finds the global-optimal solution with significantly less solution time.

In fact the great watershed in optimization isn't between linearity and nonlinearity, but convexity and nonconvexity.

R. T. Rockafellar

Contents

Nomenclature

The following comprises a summary of the symbols used in mathematical and physical expressions in this thesis. The first column contains the symbols themselves, the second column gives a verbal description of its meaning. In the third column the dimensions of each property are given as a product of the fundamental dimensions length (L), mass (M), time (T), and currency (C). If no dimension is mentioned, the dimension varies, depending on the selected use case.

Dimensional quantities:

Symbol	**Description**	**Dimension**
a	matrix entry of A	
A	matrix	
$A^{\dagger}$	pseudo-inverse of matrix A	
A_{d}	hyperbole domain restriction parameter	1
A_{c}	cross-sectional area	L^{2}
b	binary variable	1
B	big-M constant	
c^{B}	battery capacity	$\mathrm{M}\ \mathrm{L}^{2}\ \mathrm{T}^{-2}$
c_{w}	drag coefficient	1
C	cost	C
d	diameter	L
d_{+}	diameter ratio	1
e	generic function	
f	generic function	
f^{m}	monomial function	
f^{p}	posynomial function	
g	gravitational acceleration	$\mathrm{L}^{1}\ \mathrm{T}^{-2}$
h	pressure head	L
Δh	pressure increase by a pump	L
ΔH	pressure loss	L
H	heuristic	
i	transmission ratio	1
I	transmission ratio bound parameter	1
k	scaling factor	1

K	wall roughness parameter	L
L	distance	L
$\mathcal{L}$	Lagrange function	
m	mass	M
M	mass	M
n	motor speed	T^{-1}
N	motor speed	T^{-1}
p	power	$M\ L^{2}\ T^{-3}$
P	power	$M\ L^{2}\ T^{-3}$
P	problem	
q	volume flow	$L^{3}\ T^{-1}$
Q	volume flow	$L^{3}\ T^{-1}$
r	residual	
r^{W}	wheel radius	L
R	range	L
s	slope	1
S	Stirling number	1
t	torque	$M\ L^{2}\ T^{-2}$
T	torque	$M\ L^{2}\ T^{-2}$
v	speed	$L\ T^{-1}$
$\dot{v}$	acceleration	$L\ T^{-2}$
x	generic variable	
y	generic variable	
Y	generic variable / parameter	
z	generic variable / parameter	
Z	number of parallel pumps	1

Greek Symbol	**Description**	**Dimension**
α	regression parameter	
β	regression parameter	
γ	regression parameter	
δ	regression parameter	
ζ	generic variable	
η	efficiency	1
λ	generic coefficient	
λ_{h}	hydraulic resistance coefficient	1
λ_{i}	inertia coefficient	1
λ_{r}	rolling resistance coefficient	1
$\lambda^{\mathrm{La},(k)}$	Lagrange multiplier	

Λ	load values	1
μ_{B}	lower bound variable	
μ	mean value	
ν	kinematic viscosity	L^2 T^{-1}
$\nu^{\mathrm{La},(k)}$	Lagrange multiplier	
π	probability value	1
ρ^{e}	electric density	L^2 T^{-2}
ϱ	air density	M L^{-3}
$\dot{\varphi}$	normalized angular speed	1
ϕ	basis function	
ψ	normalized torque	1
ω	rotational speed	T^{-1}
Ω	rotational speed	T^{-1}

Subscripts	**Description**
B	Benders
c	cross-sectional
const	constant
d	domain
i	inertia
In	input
L	loss
M	model
main	main
max	maximum
post	posterior
pre	previous
r	rolling
ref	reference
sub	sub
w	wind
+	normalized

Superscripts	**Description**
A	additional
avg	average
B	battery
e	electric
energy	energy
G	transmission

h	pressure head
in	input
k	iterator
L	loss
La	Lagrange
load	load
M	motor
max	maximum
out	output
p	power
P	passenger
part	partial
pipe	pipe
pump	pump
R	recuperation
W	wheel

Symbols	**Description**
$\tilde{\square}$	approximation
$\underline{\square}$	lower bound
$\hat{\square}$	log-space
$\square^*$	optimum
$\overline{\square}$	upper bound

Set	**Description**
$\mathcal{B}$	pump types
$\mathcal{C}$	convex set
$\mathcal{D}$	diameters
$\mathcal{E}$	edges
$\mathcal{F}$	desired fractional loads
$\mathcal{G}$	graph
$\mathcal{H}$	clusters
$\mathcal{J}$	optimality cuts
$\mathcal{K}$	EM domains
$\mathcal{M}$	efficiency map
$\mathcal{N}$	grid points in EM domain
$\mathcal{P}$	polyhedron
$\mathcal{R}$	operating points on convex hull
$\mathcal{S}$	scenarios
$\mathcal{T}$	transmissions

$\mathcal{U}$	active scenarios
$\mathcal{V}$	vertices
$\mathcal{X}$	generic set
$\mathcal{Y}$	binary variables
$\mathcal{Z}$	selected fractional loads

Abbreviations

AAO	all-at-once optimization
BEV	battery electric vehicle
CC	convex combination
CRC	collaborative research center
CVT	continuous variable transmission
EM	electric machine
FTP	federal test procedure
GA	genetic algorithm
GBD	generalized Benders decomposition
GP	geometric program
ILP	integer linear program
LB	lower bound
LP	linear program
MAX	maximum
MDO	multi-disciplinary design optimization
MIN	minimum
MINLP	mixed-integer nonlinear program
MIP	mixed-integer program
MILP	mixed-integer linear program
MIT	Massachusetts Institute of Technology
NLP	nonlinear program
NP	non-deterministic polynomial-time
NYCC	New York city cycle
PMSM	permanent-magnet synchronous motor
PWL	piecewise linearization
SA	sensitivity analysis
SAA	sample average approximation
SAND	simultaneous analysis and design
SCIP	solving constraint integer programs
SOS	special ordered sets
STD	standard deviation
TOR	technical operations research
UB	upper bound
WLTC	worldwide harmonized light-duty vehicles test cycles

Chapter 1

Introduction and Motivation

Technology has ever been a key driver for societal improvements. Today we are surrounded by products of industrial processes, which all required engineers to a certain extent for a design, verification and validation. Hence, it can be summarized with the words of Günther Ropohl: "We have made the world we inhabit ourselves: our biotope has become a technotope"[1]. On the one hand, the technological improvements over time and especially starting with the first Industrial Revolution[2], lead to great societal prosperity. For instance the improvements in agricultural, medical and information technology since then lead to a widespread societal growth in most economies on the world. On the other hand, this technological progress fostered the occurrence of global challenges, like the human-induced climate change. Hence, it is an obligation for current engineers to design sustainable systems, which support societal improvements and reduce societal challenges.
A guideline for sustainable systems design is given by the United Nations Sustainable Development Goals[3], but the implications for an engineering design of sustainable technical systems is still a young research field with many open research questions.
This thesis therefore focuses on the development of algorithmic design approaches to support engineers within the process of designing more sustainable systems.
Within this work, the focus lies on two different key technological areas. First, the municipal water supply. Already in 2018, more people lived in cities than

[1] Günter Ropohl, *Allgemeine Technologie : eine Systemtheorie der Technik*, ([70], 2009, p. 15)

[2] This describes the time between approximately 1760 and 1840, with a high per-capita economic growth caused by new technological changes and an increasing industrialization.

[3] United Nations Development Program, *Sustainable Development Goals*, ([187], 2015)

in rural environments[4]. Furthermore, the unstopped trend of urbanization will potentially lead to an increase of megacities and simultaneously to an increase of high-rise buildings as already shown by the Council on Tall Buildings and Urban Habitat[5]. These buildings require a significant amount of energy to enable a fresh water supply on each floor, which can be improved up to 50% with an optimized system design[6].
Second, this work addresses the development of battery-electric vehicles. To reduce the environmental impact of transport, it is beneficial to have a technological shift from traditional combustion engines to a battery-electric powertrain design[7]. To ensure a highly efficient system design all components must be sized together based on realistic requirements from the usage phase[8].

1.1 Sustainable Systems Design

At the core of this sustainable systems design[9] lies the engineering design. Within the following, the term *engineering design* is used as defined by Pahl/Beitz et al.[10]. As noted there, *design* "is used synonymously (...)"[10] for design and development. A technical system fulfills a predefined set of functions. "What belongs to a system is determined by the system boundary."[11] All already mentioned systems that are considered within this thesis show different characteristics, but can all be evaluated in a coherent manner and show all a specific function that must be fulfilled within their usage phase. Besides this definition of a *technical system* the definitions of a *mechatronic system*, shown by Isermann[12], are also applicable in this context.

[4] United Nations Secretariat, Population Division, *2018 Revision of World Urbanization Prospects*, ([188], 2018)

[5] CTBUH, *2018 Tall Building Year in Review*, ([36], 2018)

[6] Altherr, Leise, Pfetsch, and Schmitt, "Resilient layout, design and operation of energy-efficient water distribution networks for high-rise buildings using MINLP", ([6], 2019)

[7] Esser, "Realfahrtbasierte Bewertung des ökologischen Potentials von Fahrzeugantriebskonzepten", ([50], 2021)

[8] Silvas, Hofman, Murgovski, Etman, and Steinbuch, "Review of Optimization Strategies for System-Level Design in Hybrid Electric Vehicles", ([172], 2016)

[9] Pelz, Groche, Pfetsch, and Schäffner, *Mastering Uncertainty in Mechanical Engineering*, ([148], 2021, pp. 16)

[10] Pahl, Beitz, Feldhusen, and Grote, *Engineering Design — A Systematic Approach*, ([141], 2007, pp. 1)

[11] Pahl, Beitz, Feldhusen, and Grote, *Engineering Design — A Systematic Approach*, ([141], 2007, p. 27)

[12] Isermann, *Mechatronische Systeme: Grundlagen*, ([85], 2007, pp. 3)

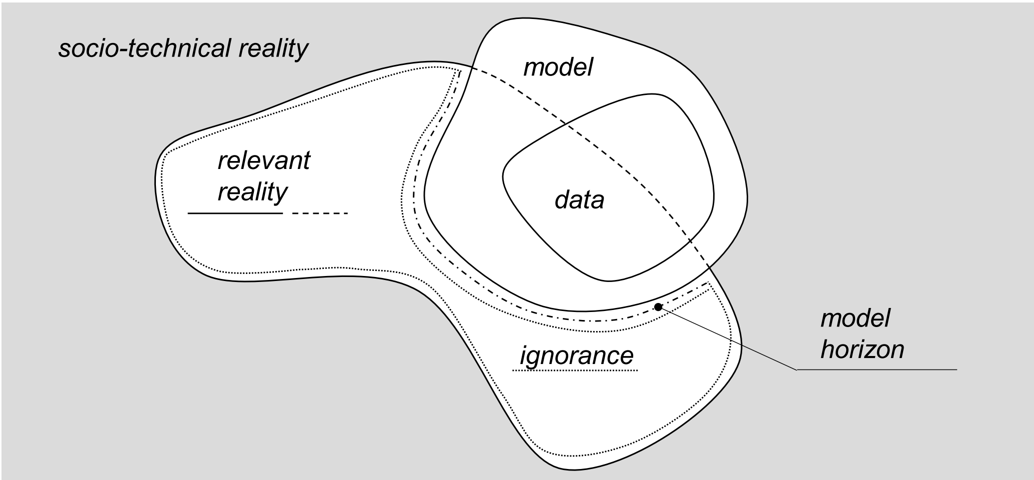

Figure 1.1 – Euler diagram for depiction of the model, model horizon, and the relevant reality, given by Pelz et al. [148, p. 9]

The entire process of work during the design of a technical system can be subdivided in[13]:

1. Planning and Task Clarification,
2. Conceptual Design,
3. Embodiment Design, and
4. Detail Design.

The focus of this thesis lies on the conceptual design, since this part is a crucial step towards more sustainable system designs. In this phase the principal solution is determined[13]. "This is achieved by abstracting the essential problems (...)."[13] Within this phase a *technical artifact*[14] is conceptually designed and evaluated. Since multiple possible solutions for the technical system at hand should be evaluated and compared on a given objective until the artifact is finalized, it is mandatory to use a systematic approach.
Mathematical modelling of the given system is therefore a promising approach within the conceptual design phase.

[13] Pahl, Beitz, Feldhusen, and Grote, *Engineering Design — A Systematic Approach*, ([141], 2007, pp. 131)

[14] Pahl, Beitz, Feldhusen, and Grote, *Engineering Design — A Systematic Approach*, ([141], 2007, p. 27)

As shown in Fig. 1.1, a derived model can only represent a specific part of the *relevant reality*, where the omitted part is called *ignorance*[15]. "The boundary between the model and the relevant reality is named *model horizon*."[15] A detailed introduction about mathematical modeling is given by Pelz et al.[16]. Returning to the conceptual design of technical systems one key statement is: "In systematic respects, designing is the optimisation of given objectives within partly conflicting constraints"[17]. Mathematical optimization methods are therefore a suitable approach to derive technical system designs that maximize or minimize given objectives[18,19].
The interdisciplinary approach of *Technical Operations Research*[20] (TOR) combines exact mathematical optimization with the engineering sciences to systematize the conceptual design. The major goal of this approach is "to derive sustainable systems designs that follow the statement"[20]:

> "Maximize quality, subject to functionality."[21]

The combination of multiple components that are only optimized individually often leads to system designs with a lower performance than if all components and subsystems are designed concurrently. Hence, the combined system-level based design can provide artifacts with a better performance, measured by their respective objective(s). As described by Fügenschuh, Lorenz and Pelz[20] the four major steps in a conceptual engineering design process are

1. "the formulation of the desired task and quality,
2. defining the design space,
3. deriving the composition, and

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15 Pelz, Groche, Pfetsch, and Schäffner, *Mastering Uncertainty in Mechanical Engineering*, ([148], 2021, p. 9)

16 Pelz, Groche, Pfetsch, and Schäffner, *Mastering Uncertainty in Mechanical Engineering*, ([148], 2021, pp. 8)

17 Pahl, Beitz, Feldhusen, and Grote, *Engineering Design — A Systematic Approach*, ([141], 2007, p. 2)

18 Suhl and Mellouli, *Optimierungssysteme: Modelle, Verfahren, Software, Anwendungen*, ([178], 2009)

19 Kallrath, *Gemischt-ganzzahlige Optimierung: Modellierung in der Praxis: Mit Fallstudien aus Chemie, Energiewirtschaft, Papierindustrie, Metallgewerbe, Produktion und Logistik*, ([89], 2013)

20 Fügenschuh, Lorenz, and Pelz, "OPTE Special Issue on Technical Operations Research (TOR)", ([59], 2021)

21 Pelz, Groche, Pfetsch, and Schäffner, *Mastering Uncertainty in Mechanical Engineering*, ([148], 2021, p. 215)

4. the evaluation of realized task, quality and acceptance."[22]

Each step can be used as a leading principle to derive a structured system design[23]. This general approach is described briefly within the following. In the first step, the needs for the technical system result in a quality description based on *effort*, *availability*, and *acceptability*[24]. Here, effort should be minimized, while the availability and acceptability should be maximized to derive a Pareto optimal solution of these partly conflicting objectives. "Effort is measured, for example, by the total cost of ownership"[23]. It can be approximated by only considering the material costs and/or energy costs in specific use cases. The availability is measured for instance by the *mean operating time between failures*[25] or the anticipated service life. The acceptability is given by the compliance of the derived design solution with regulations, but also with a positive user experience. It is the fuzziest objective, which is hard to be cast in a mathematical depiction.
Within the second step "defining the design space", available resources like material, components or technologies are specified to be available for the system design process. They are bounded by technical requirements of the desired system design. Within this step the selection of discrete components is often required to be able to describe the design space more accurately.
In the third step, the most promising solution is derived by means of mathematical optimization to ensure a feasible and efficient system design.
This proposed design is then evaluated in the final step. Here, detailed system simulation models or experiments are required for instance to ensure a proper system design.
Since the required models for description of technical systems, as the two already introduced example systems, usually require a nonlinear and partly discrete or integral description of the design space, this approach automatically results in a *Mixed-Integer Nonlinear Program*[26] (MINLP), which has to be evaluated efficiently to derive the best possible system design.
Within the design process there is one important circumstance to consider: *uncertainty*. Uncertainty is ubiquitous within the design process, as shown in

[22] Fügenschuh, Lorenz, and Pelz, "OPTE Special Issue on Technical Operations Research (TOR)", ([59], 2021)

[23] Pelz, Groche, Pfetsch, and Schäffner, *Mastering Uncertainty in Mechanical Engineering*, ([148], 2021, p. 17)

[24] Pelz, Groche, Pfetsch, and Schäffner, *Mastering Uncertainty in Mechanical Engineering*, ([148], 2021, p. 16)

[25] Birolini, *Reliability Engineering: Theory and Practice*, ([17], 2017, p. 6)

[26] Lee and Leyffer, *Mixed Integer Nonlinear Programming*, ([97], 2011)

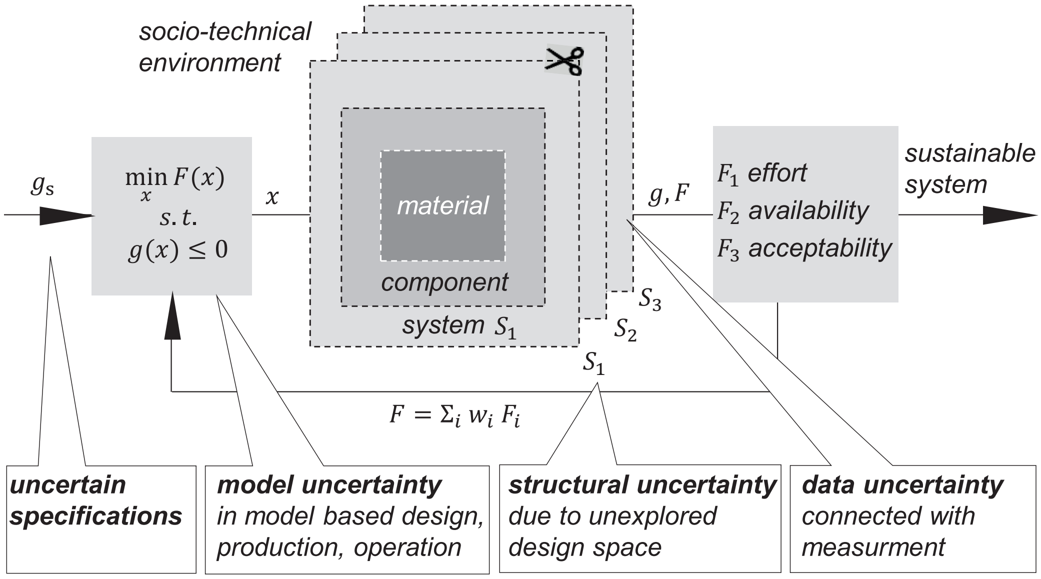

Figure 1.2 – Sustainable systems design as a closed loop with uncertainty localization, given by Pelz et al. [148, p. 18]

Fig. 1.2 by Pelz et al.[27].
It can be distinguished between "*model uncertainty*, *structural uncertainty*, and *data uncertainty*"[27] for technical systems. Furthermore, there are also uncertain specifications given by the different stakeholders.
"Data uncertainty is present, if the amount, type and distribution of required data, such as model parameters, is incomplete, unknown or insufficient (...)."[28]
Model uncertainty[29] focuses on the uncertainty that arises in the model selection process. As noted earlier, the models are usually only approximations of the reality. Two types of *ignorance* can be distinguished within this scope. First a *lack of knowledge*, where some objects or processes are unknown. Second, *disregard of knowledge*, where simplified models are chosen based on other benefits within the model evaluation phase, like for instance a more rapid model evaluation.
Structural uncertainty[30] describes the uncertainty that arises by only con-

[27] Pelz, Groche, Pfetsch, and Schäffner, *Mastering Uncertainty in Mechanical Engineering*, ([148], 2021, p. 18)

[28] Pelz, Groche, Pfetsch, and Schäffner, *Mastering Uncertainty in Mechanical Engineering*, ([148], 2021, p. 29)

[29] Pelz, Groche, Pfetsch, and Schäffner, *Mastering Uncertainty in Mechanical Engineering*, ([148], 2021, pp. 33)

[30] Pelz, Groche, Pfetsch, and Schäffner, *Mastering Uncertainty in Mechanical Engineering*, ([148], 2021, p. 38)

sidering a limited number of design variants. Then only a subspace of the design space is evaluated explicitly, which leads to a structural uncertainty by neglecting further design variants and their respective uncertainty. The comparison of multiple design variants with different system configurations leads on a system level rapidly to a "combinatorial explosion" of variants, which is not completely evaluable by humans by hand[31]. The usage of mathematical optimization for the system design can be used to reduce the structural uncertainty and consider broader parts of the design space analytically instead of manually.
One approach to master uncertainty beyond structural uncertainty is the consideration of *resilience*[32,33]. Within the collaborative research center (CRC) 805, where the main part of this thesis research was conducted in, the following definition arose for technical systems[32] within numerous discussions and meetings of a multidisciplinary team:

> "A resilient technical system guarantees a predetermined minimum of functional performance even in the event of disturbances and failures of system components, and a subsequent possibility of recovering."[32]

If the resilience of a technical system is already considered within the conceptual design, its performance within the usage phase can benefit especially within severe failure scenarios. This goes beyond the approach to use safety factors for dimensioning system components. The consideration of resilience within the design phase also leads to a more holistic design approach. A more detailed description of the derived integration of resilience will be shown throughout the thesis.

1.2 Research Questions

The design of technical systems by using mathematical optimization methods enables a faster and broader evaluation of different design decisions in comparison to manual evaluations. Nevertheless, this benefit is only valid, if a solution can be found in a fast, reliable and understandable way. Since

[31] Miller, "The Magic Number Seven Plus or Minus Two: Some Limits on our Capacity for Processing Information", ([125], 1956)

[32] Altherr et al., "Resilience in Mechanical Engineering - A Concept for Controlling Uncertainty during Design, Production and Usage Phase of Load-Carrying Structures", ([3], 2018)

[33] Leise et al., "Potentials and Challenges of Resilience as a Paradigm for Designing Technical Systems", ([107], 2021)

the domain-specific knowledge of an engineer can lead in the manual design process to a fast avoidance of unfavorable solutions in the overall design space, it is not assured that it will lead to a global optimal solution, and it is usually unknown how good this solution can be, in comparison to unconsidered solutions. On contrary, the usage of mathematical optimization methods considers the whole design space which is modelled with no preference of specific solutions, but it is usually unaware of domain-specific knowledge. This leads in case of binary decisions, where the *curse of dimensionality* leads to a fast growth of design decisions often to a long evaluation time, if a global optimal solution should be found. To reduce the computational burden the number of binary decisions is therefore often limited to a low number and long computational evaluations on high-performance hardware are required. Therefore, most approaches rely on the usage of (meta-)heuristics, like for instance genetic algorithms or particle swarm optimization to enable a faster solution without an explicit evaluation of its global optimality[34].
When a global optimal solution approach is considered, the relevant algorithmic requirements and system model requirements can not be neglected. Even more, they have to be balanced against each other to derive a fast and technically reliable system design. Therefore, within this thesis, the following research questions are answered and exemplified on the already identified reference systems of a residential water supply and the powertrain design of battery electric vehicles:

> *How does the technical system model granularity - solution procedure - interaction affect the efficiency to derive a global optimal solution?*
>
> *How should a conceptual design model be built to fulfill the technical requirements and additionally allow for an efficient solving procedure?*
>
> *How can we master the uncertainty within the conceptual system design when using a global-optimal optimization procedure?*

1.3 Contributions and Thesis Structure

This thesis is located in the interdisciplinary research field of technical operations research (TOR) with a clear view from an engineering perspective.

[34] Pelz, Leise, and Meck, "Sustainable Aircraft Design – A Review on Optimization Methods for Electric Propulsion with derived Optimal Number of Propulsors", ([147], 2021)

The focus lies on a trade-off between the model granularity for conceptual design models of technical systems and the applicability of global-optimal mathematical mixed-integer nonlinear programming.
To answer the introduced research questions, the two already mentioned reference use cases of a water distribution system design and a powertrain design have been selected. For both use cases, newly developed conceptual design models are presented that can be solved by using common MINLP solvers and which explicitly consider uncertainty. Furthermore, newly developed algorithmic approaches are shown to derive global optimal solutions or near-optimal solutions. For the former, specific modeling approaches are presented and compared. For the latter, problem-specific heuristics that still rely on solving underlying mathematical optimization programs are presented within the following chapters.
To derive successfully conceptual design models, it is also required to integrate a concept of operation[35] for the usage phase. Hence, newly developed heuristic approaches that can be embedded in the global-optimal optimization procedure are shown for each use case. For the powertrain use case a modeling approach that results in efficiently solvable MINLPs is also introduced in detail. This modeling approach has the great advantage to be not only problem-specific, but also more generally applicable. A further use case could be the design of aircraft under uncertainty.
Parts of the newly developed models and solution procedures that are shown in this thesis were already published in (peer-reviewed) conference and journal publications. A summary of all publications that show some specific aspects of the research in this thesis is given in the Appendix. Additionally, in each chapter references to the own publications are included.
The thesis is structured as follows: The required fundamentals will be shown in Chapter 2. It is followed by the first use case of a water distribution system design in Chapter 3. In Chapter 4, the modeling and solving approach for the second use case of a conceptual design for a powertrain of battery electric vehicles with multi-speed transmission is presented. The thesis is rounded off by a discussion in Chapter 5 and a summary and outlook in Chapter 6. Further relevant information is given in the Appendix. Each relevant Figure is labeled with an identifier, which links it to a digital representation. Further information about this procedure is also given in the Appendix.

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[35] National Aeronautics and Space Administration, *NASA Systems Engineering Handbook*, ([137], 2016, pp. 4)

Chapter 2

Fundamentals

2.1 Engineering Design Optimization

The solving of engineering design problems, especially within the conceptual design phase, can be supported by using mathematical optimization approaches. A general overview about the engineering design optimization is shown in Fig. 2.1. In this process, it is required to either maximize or minimize a given objective $f_0(x^{\mathrm{d}}, x^{\mathrm{h}}, U)$, which depends on *design variables*[1] $x^{\mathrm{d}} = [x_1^{\mathrm{d}}, x_2^{\mathrm{d}}, \ldots, x_n^{\mathrm{d}}]$, *hidden state variables* $x^{\mathrm{h}} = [x_1^{\mathrm{h}}, x_2^{\mathrm{h}}, \ldots, x_m^{\mathrm{h}}]$, and *parameters* $U = [U_1, U_2, \ldots, U_r]$. These parameters U are usually subject to uncertainty, which is represented in Fig. 2.1 by using multiple cumulative distribution functions. These distributions can be arbitrary, for simplicity they are all shown as Gaussian in this figure. A specification of these parameters results in a concrete instance of the optimization program, which can be solved by using a mathematical optimization solver. The hidden state variables x^{h} affect the objective f_0 and are given by the underlying physical model.

The system modeling and optimization program generation is not unique for a specific design problem. Multiple approaches are possible to derive a system model description, with for instance a diverging model fidelity and different computational requirements. Therefore, the interaction between mathematical optimization and engineering abstraction is important to derive models that are capable to predict the system behavior within the desired domain and be able to be optimized as efficiently as possible. For models with solely continuous variables, a multitude of approaches exist to derive

[1] Design variables are also known as decision variables.

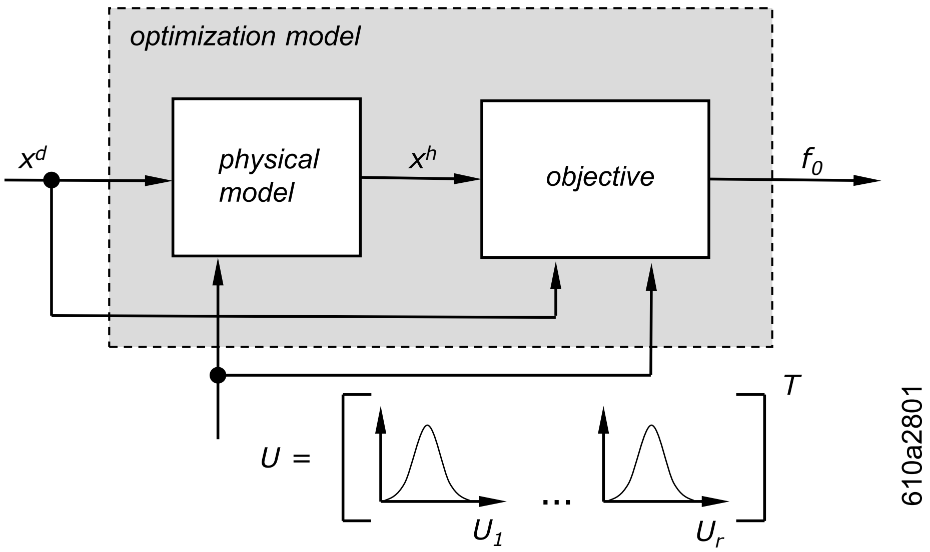

Figure 2.1 – Description of an engineering design optimization problem. Adapted from [147], which was based on [121].

optimized results[2,3].
But in engineering, especially mechanical engineering, models on a system scale often require the definition of further binary or integer decisions or hidden state variables[4]. This then results in a *Mixed-Integer Programming* model, which is usually nonlinear and nonconvex for engineering design problems[5]. In terms of complexity, a general nonconvex Mixed-Integer Nonlinear Program (MINLP) is non-deterministically polynomial-time hard[6] (*NP-hard*), which means that its solution is easily verifiable, but finding it is hard (as long as $P \neq NP$). This worst-case computational complexity therefore often leads to the usage of heuristics that are often based on meta-heuristics[7] like the *genetic*

[2] Pelz, Leise, and Meck, "Sustainable Aircraft Design – A Review on Optimization Methods for Electric Propulsion with derived Optimal Number of Propulsors", ([147], 2021)

[3] Martins and Ning, *Engineering Design Optimization*, ([123], 2021)

[4] Kallrath, *Gemischt-ganzzahlige Optimierung: Modellierung in der Praxis: Mit Fallstudien aus Chemie, Energiewirtschaft, Papierindustrie, Metallgewerbe, Produktion und Logistik*, ([89], 2013)

[5] This is then described by the term *Mixed-Integer Nonlinear Program* (MINLP), Belotti et al. [13]

[6] Floudas and Pardalos, *Encyclopedia of Optimization*, ([57], 2009, p. 2237)

[7] A heuristic is a procedure that finds a sufficient, but not necessarily optimal solution. A meta-heuristic is a generally applicable schema or principle to derive a specific heuristic, see e.g. Suhl and Mellouli [178, p. 13].

algorithm[8] or *simulated annealing*[9] for having a more time-efficient solution procedure. One drawback of this approach is the loss of information about the goodness of the derived (intermediate) solution(s), since the distance to the (unknown) global optimum is unknown. Here, duality theory and modern exact solvers like SCIP[10] or MOSEK[11] play their highest benefit, since their usage allows an estimation of the distance to the global optimum. A more detailed introduction in duality theory will follow briefly in this chapter.

2.1.1 Mixed-Integer Nonlinear Programming

A *Mixed-Integer Nonlinear Program* (MINLP) is given in general by

$$\begin{aligned} \min_{x,y} \quad & f_0(x,y) & \\ \text{s.t.} \quad & f_i(x,y) \leq 0 & \forall i = 1,\dots,b, \\ & e_j(x,y) = 0 & \forall j = 1,\dots,u, \\ & x, y \geq 0, & \\ & x \in \mathbb{R}^k, y \in \mathbb{Z}^l. & \end{aligned} \tag{2.1}$$

The objective is given by $f_0 : \mathbb{R}^k \times \mathbb{Z}^l \to \mathbb{R}$. It represents the goal that should be achieved. Each maximization objective can be transformed to an equivalent minimization objective by using a multiplication with (-1). The inequality constraints $f_i : \mathbb{R}^k \times \mathbb{Z}^l \to \mathbb{R}$ are used to model for instance logical conditions or other technical specifications. The equality constraints $e_j : \mathbb{R}^k \times \mathbb{Z}^l \to \mathbb{R}$ represent among others physical system properties, like for instance the mass and/or energy balance.

If the MINLP is additionally *nonconvex*, multiple local optima may exist and the complexity class of solving the nonconvex MINLP to global optimality is *NP-hard*, as already introduced[12]. Despite this worst-case complexity computational and algorithmic achievements lead to a high variety of use cases in which this approach is applicable. Further details on this problem class and solution strategies are summarized by Lee and Leyffer[13].

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8 Holland, *Adaptation in Natural and Artificial Systems: An Introductory Analysis with Applications to Biology, Control, and Artificial Intelligence*, ([76], 1992)

9 Kirkpatrick, Gelatt Jr, and Vecchi, "Optimization by Simulated Annealing", ([90], 1983)

10 For details see: https://www.scipopt.org/ (accessed August 18th 2022)

11 For details see: https://www.mosek.com/ (accessed August 18th 2022)

12 Floudas and Pardalos, *Encyclopedia of Optimization*, ([57], 2009, p. 2237)

13 Lee and Leyffer, *Mixed Integer Nonlinear Programming*, ([97], 2011)

2.1.2 Stochastic Optimization

In engineering design, some or all parameters that are required to define an instance (e.g. shown in Fig. 2.1) of an abstract optimization problem are uncertain. This uncertainty can be mastered within the optimization process by using stochastic optimization[14], if it is quantifiable. Within a stochastic optimization program the constraints and the objective depend on the already introduced design variables and further random variables ξ which model the uncertain parameters. Its values are unknown, but their probability distributions are known. One specific abstract definition of a stochastic optimization program is then given by:

$$\begin{aligned}
\min_{x,y} \quad & \mathbb{E} f_0(x,y,\xi) \\
\text{s.t.} \quad & \mathbb{E} f_i(x,y,\xi) \leq 0 \quad \forall i = 1,\ldots,b, \\
& \mathbb{E} e_j(x,y,\xi) = 0 \quad \forall j = 1,\ldots,u, \\
& x,y \geq 0, \\
& x \in \mathbb{R}^k, y \in \mathbb{Z}^l.
\end{aligned} \tag{2.2}$$

One approach to solve (2.2) is via the definition of *scenarios*[15]. Here (2.2) is approximated by using N_s scenarios $\{\pi_n, \xi_n\}$[16]:

$$\begin{aligned}
\min_{x,y} \quad & \sum_{n=1}^{N_s} \pi_n f_0(x,y,\xi_n) \\
\text{s.t.} \quad & \sum_{n=1}^{N_s} \pi_n f_i(x,y,\xi_n) \leq 0 \quad \forall i = 1,\ldots,b, \\
& \sum_{n=1}^{N_s} \pi_n e_j(x,y,\xi_n) = 0 \quad \forall j = 1,\ldots,u, \\
& x,y \geq 0, \\
& x \in \mathbb{R}^k, y \in \mathbb{Z}^l.
\end{aligned} \tag{2.3}$$

The probability of occurrence π_n of the n-th scenario and the according value ξ_n are used to derive a *deterministic equivalent*. This procedure is also known as a *sample average approximation* (SAA)[17]. Problem (2.3) can be solved similarly to problem (2.1).

[14] Shapiro, Dentcheva, and Ruszczynski, *Lectures on Stochastic Programming: Modeling and Theory*, ([170], 2014)

[15] Scenarios are also known as a *finite event set*.

[16] Williams, *Model Building in Mathematical Programming*, ([199], 2013, pp. 53)

[17] Nemirovski, Juditsky, Lan, and Shapiro, "Robust Stochastic Approximation Approach to Stochastic Programming", ([138], 2009)

2.1.3 Convex Optimization

Given two points $x_1, x_2 \in \mathbb{R}^n$, a *convex combination* is any point x_3 of the form[18]:

$$x_3 = \lambda x_1 + (1-\lambda)x_2, \lambda \in \mathbb{R}, 0 \leq \lambda \leq 1. \tag{2.4}$$

A set $\mathcal{C} \subseteq \mathbb{R}^n$ is *convex*, if it contains all convex combinations of pairs of points $x_1, x_2 \in \mathcal{C}$. In general, a function $f : \mathcal{C} \to \mathbb{R}$ is *convex* in $\mathcal{C}$, if the set $\mathcal{C} \subseteq \mathbb{R}^n$ is convex and for any two points $x_1, x_2 \in \mathcal{C}$[19]:

$$f(\lambda x_1 + (1-\lambda)x_2) \leq \lambda f(x_1) + (1-\lambda)f(x_2), \lambda \in \mathbb{R}, 0 \leq \lambda \leq 1 \tag{2.5}$$

holds. The convex hull conv($\mathcal{C}$) of a set $\mathcal{C}$ is given by[20]:

$$\text{conv}(\mathcal{C}) = \{\Theta_1 x_1 + \cdots + \Theta_k x_k | x_i \in \mathcal{C}, \Theta_i \geq 0, i = 1, \ldots, k, \Theta_1 + \cdots + \Theta_k = 1\}. \tag{2.6}$$

A continuous (nonlinear) optimization problem is given by[21]

$$\begin{aligned} \min_x \quad & f_0(x) \\ \text{s.t.} \quad & f_i(x) \leq 0 \quad \forall i = 1, \ldots, b, \\ & e_j(x) = 0 \quad \forall j = 1, \ldots, u, \\ & x \geq 0, \\ & x \in \mathbb{R}^k. \end{aligned} \tag{2.7}$$

Problem (2.7) can be derived for instance from (2.1) by fixing the integral variables y to a given setting. Furthermore, problem (2.7) is *convex*, if $f_0(x)$ and $f_i(x)$ are convex functions and $e_j(x) = a_j^T x - b_j$ are affine functions[21]. A convex optimization problem has the advantage that each local optimum is a global optimum. On contrary, nonconvex nonlinear programs can also have multiple local optima. The *epigraph form* of the standard problem in (2.7)

---

[18] Papadimitriou and Steiglitz, *Combinatorial Optimization: Algorithms and Complexity*, ([143], 1998, pp. 12)

[19] Boyd and Vandenberghe, *Convex Optimization*, ([20], 2004, p. 67)

[20] Boyd and Vandenberghe, *Convex Optimization*, ([20], 2004, p. 24)

[21] Boyd and Vandenberghe, *Convex Optimization*, ([20], 2004, pp. 127)

is[22]

$$
\begin{aligned}
\min_{x} \quad & t \\
\text{s.t.} \quad & f_0(x) - t \leq 0 \\
& f_i(x) \leq 0 \quad \forall i = 1, \ldots, b, \\
& e_j(x) = 0 \quad \forall j = 1, \ldots, u, \\
& x \geq 0, \\
& x \in \mathbb{R}^k.
\end{aligned} \tag{2.8}
$$

This form is equivalent to (2.7), since (2.8) is only optimal, if and only if x is optimal for (2.7)[22].
Per definition, a mixed-integer program is always non-convex, since the discrete variables lead to a discontinuous solution space. Nevertheless, throughout this thesis a MINLP is understood as "convex", if the relaxation, where the integrality requirements for the discrete variables are dropped, is convex. Hence, a MILP is called to be "convex". This imprecise wording enables an easier discussion of the derived models within this thesis.

2.1.4 Duality

The following description is based on Boyd and Vandenberghe[23]. If we consider the NLP given in (2.7), we can derive a valid *upper bound* by using any heuristic or meta-heuristic that returns a solution that doesn't violate any constraint. If multiple evaluations are performed, the solution with the lowest objective value is the current lowest upper bound. It is also possible to derive a valid *lower bound*. This can be done in this case by solving a related optimization problem; the so-called *Lagrange dual problem*. This problem is based on the evaluation of the according *Lagrange dual function*[23]

$$
g(\nu^{\mathrm{La},(k)}, \lambda^{\mathrm{La},(k)}) = \inf_{x \in \mathcal{X}} \mathcal{L}(x, \nu^{\mathrm{La},(k)}, \lambda^{\mathrm{La},(k)}) = \inf_{x \in \mathcal{X}} \left(f_0(x) + \sum_{i=1}^{b} \nu_i^{\mathrm{La},(k)} f_i(x) + \sum_{j=1}^{u} \lambda_j^{\mathrm{La},(k)} e_j(x) \right) \tag{2.9}
$$

with the non-empty valid domain $\mathcal{X}$ of the variables x and the *dual variables* (or *Lagrange multiplier vectors*) $\nu^{\mathrm{La},(k)} \in \mathbb{R}^b$ and $\lambda^{\mathrm{La},(k)} \in \mathbb{R}^u$. This dual function yields valid lower bounds on the optimal value x^*, since for any $\nu^{\mathrm{La},(k)} \geq 0$ and any $\lambda^{\mathrm{La},(k)}$

$$
g(\nu^{\mathrm{La},(k)}, \lambda^{\mathrm{La},(k)}) \leq x^* \tag{2.10}
$$

[22] Boyd and Vandenberghe, *Convex Optimization*, ([20], 2004, p. 134)
[23] Boyd and Vandenberghe, *Convex Optimization*, ([20], 2004, pp. 215)

holds. This is known as *weak duality*. The highest lower bound is found by solving[24]:

$$\begin{aligned} \max_{\nu^{\mathrm{La},(k)},\lambda^{\mathrm{La},(k)}} \quad & g(\nu^{\mathrm{La},(k)}, \lambda^{\mathrm{La},(k)}) \\ & \nu^{\mathrm{La},(k)} \geq 0. \end{aligned} \tag{2.11}$$

Problem (2.11) is convex, even if problem (2.7) would be nonconvex[24]. The consideration of duality theory can be exploited within the solution process and be used to derive bounds for the given design problem. Therefore, its explicit usage is beneficial to be able to estimate further possible performance gains, if primal solutions are found. For a detailed description of duality for linear systems, it is referred to Papadimitriou and Steiglitz[25].

2.1.5 Convexification Strategies

Since the global optimization of Mixed-Integer Nonlinear Programs is a challenging task, it is required to derive modeling approaches that on the one hand represent a technical system design as detailed as required to derive solutions that can predict the real-world behavior accurately, and on the other hand models that are solvable in an efficient way. This trade-off results in the need of interdisciplinary research to develop suitable models. Multiple approaches that proved their applicability within the research of this thesis in the research group of Technical Operations Research at the Chair of Fluid Systems from an engineering point of view, will be presented in the following.

Piecewise Linear Approximation

Frequently, engineering design optimization problems contain multiple binary decision variables to select specific components from a pool of components or are used to specify the activity of specific components within specific use cases. Further linear and nonlinear constraints are then used to model physical relationships like conservation laws. Since for engineering design problems functional relationships with two independent variables and one dependent variable have to be often integrated in the optimization program, the following will only focus on this type of requirements. If only some nonlinear functional relationships exist in the derived model, besides multiple linear and binary relationships, it is often computationally efficient to use linearized approximations of these nonlinear constraints to facilitate a rapid

[24] Boyd and Vandenberghe, *Convex Optimization*, ([20], 2004, pp. 215)

[25] Papadimitriou and Steiglitz, *Combinatorial Optimization: Algorithms and Complexity*, ([143], 1998, pp. 67)

solution procedure by state-of-the-art MILP solvers. The domain space is partitioned in small boxes that are then represented by two simplexes each in three dimensions. In the following only a 1-4 direction is used as introduced by Misener and Floudas[26].
A *polyhedron* $\mathcal{P}$, as stated by Boyd and Vandenberghe[27], is a "solution set of a finite number of linear equalities and inequalities"[27]:

$$\mathcal{P} = \{x | a_j^T x \leq b_j, j = 1 \ldots, b, d_j^T x = e_j, j = 1, \ldots, u\}. \tag{2.12}$$

Hence, it represents an intersection of a finite number of halfspaces and hyperplanes.
A *simplex* is a special polyhedron that is given by[28]

$$\text{conv}(v_0, \ldots, v_k) = \{\Theta_0 v_0 + \cdots + \Theta_k v_k | \Theta \geq 0, \mathbb{1}^T \Theta = 1\}. \tag{2.13}$$

Here, $\mathbb{1}$ denotes a vector with all entries one[29]. For instance, a simplex in one dimension is a line segment, and in two dimensions it is a triangle.
Given is a functional relationship $f(x_1, x_2)$, where $f : \mathbb{R}^2 \to \mathbb{R}$ is an arbitrary twice-differentiable relationship. Within technical systems this bivariate function represents characteristic curves of specific components, like for instance the efficiency map of a permanent magnet synchronous motor[30] or the power-flow-pressure characteristic of a pump[31]. An approximation $\tilde{f} : \mathbb{R}^2 \to \mathbb{R}$ can be created by using a lookup-table $L = \{f(x_1, x_2) \in \mathbb{R} | x_1 = X_{1,i,j} \in \mathbb{R}, x_2 = X_{2,i,j} \in \mathbb{R}\}$ of specific function evaluations $Y_{i,j}$ at given domain points $X_{1,i,j}$ and $X_{2,i,j}$ and an affine approximation in between[32].
This piecewise linear approximation can be integrated in a given MILP by

26 Misener and Floudas, "Piecewise-Linear Approximations of Multidimensional Functions", ([126], 2010, p. 9)

27 Boyd and Vandenberghe, *Convex Optimization*, ([20], 2004, pp. 31)

28 Boyd and Vandenberghe, *Convex Optimization*, ([20], 2004, p. 32)

29 Boyd and Vandenberghe, *Convex Optimization*, ([20], 2004, p. 33)

30 Leise, Simon, and Altherr, "Comparison of Piecewise Linearization Techniques to Model Electric Motor Efficiency Maps: A Computational Study", ([109], 2020)

31 Altherr, Leise, Pfetsch, and Schmitt, "Resilient layout, design and operation of energy-efficient water distribution networks for high-rise buildings using MINLP", ([6], 2019)

32 Misener and Floudas, "Piecewise-Linear Approximations of Multidimensional Functions", ([126], 2010)

multiple approaches[33,34,35]. In the following, only the method called *convex combination*[34] (CC) and a method based on *special ordered sets* (SOS)[33] are introduced in more detail. The following constraints are required to represent the two-dimensional simplexes in between three adjacent domain points each within the optimization program:

$$x_{1,i,j} = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \lambda_{i,j} X_{1,i,j}, \tag{2.14a}$$

$$x_{2,i,j} = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \lambda_{i,j} X_{2,i,j}, \tag{2.14b}$$

$$\tilde{f}_{i,j} = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \lambda_{i,j} Y_{i,j}, \tag{2.14c}$$

$$\sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \lambda_{i,j} = 1. \tag{2.14d}$$

Here $\lambda_{i,j} \in [0, 1]$ is an additional continuous set of variables to enable an affine approximation in between the according adjacent domain points. Additionally, further method specific constraints are required besides these general constraints. This then results in the addition of further binary variables. For the CC method the additional constraints:

$$\sum_{k \in \mathcal{K}} a_k = 1, \tag{2.15a}$$

$$\lambda_{i,j} \leq \sum_{k \in \mathcal{K}(i,j)} a_k \tag{2.15b}$$

with $a_k \in \{0, 1\}$ and the set of all simplexes $\mathcal{K}$ and the subset $\mathcal{K}(i, j)$ of all simplexes that are adjacent to the domain point $(x_{1,i,j}, x_{2,i,j})$ are required. This approach introduces as many binary variables as simplexes are used to approximate the functional relationship.
The SOS approximation requires no definition of specific binary variables. Instead, it uses the availability to process constraints with special ordered sets in modern MILP/MINLP solvers. Special ordered sets were first introduced by Beale and Tomlin[36]. A *special ordered set of type one (SOS1)* is a set

[33] Misener and Floudas, "Piecewise-Linear Approximations of Multidimensional Functions", ([126], 2010)

[34] Vielma, Ahmed, and Nemhauser, "Mixed-integer Models for Nonseparable Piecewise-linear Optimization: Unifying Framework and Extensions", ([191], 2010)

[35] Geissler, Martin, Morsi, and Schewe, "Using Piecewise Linear Functions for Solving MINLPs", ([61], 2012)

[36] Beale and Tomlin, "Special facilities in a general mathematical programming system for non-convex problems using ordered sets of variables", ([11], 1970)

where only one element is nonzero[37]. A *special ordered set of type 2* (SOS2) is a set where at maximum two adjacent elements are nonzero[37]. The selection of an active simplex is accomplished in this method by introducing additional SOS1 and SOS2 constraints, as shown by Misener and Floudas[37].
A further approximation approach that is closely related to PWL is the usage of a polyhedral approximation without the definition of any binary variable. If a resulting point in the optimal solution lies on the convex hull of a three-dimensional polyhedron, this domain can be integrated in the program, by adding a constraint for each intersecting hyperplane. This approach avoids the usage of additional binary variables, but is only valid if the condition for the solution is met by the optimization program.

Geometric Programming

The geometric programming approach first appeared in a publication by Zener[38] in 1961. Besides this first occurence the complete theory was first published by Duffin, Peterson, and Zener[39] in 1967. It was since its beginnings related to the solution of engineering design optimization problems, based on its possibility to consider nonconvex functional relationships efficiently without the requirement of a (piecewise) linear approximation[40]. A historical overview is given by Peterson[41]. A general introduction is given by Boyd, Kim, Vandenberghe, and Hassibi[42] and a recent review in the aerospace domain is given by Pelz, Leise, and Meck[43]. It is a modeling approach in which a log-transformation is used to convert a nonconvex nonlinear program with only continuous variables in a convex program. This transformation is possible[42], if the objective and constraints either consist of *monomials*

$$f^{\mathrm{m}}(x) = \alpha_0 x_1^{\alpha_1} x_2^{\alpha_2} \dots x_n^{\alpha_n}, \tag{2.16}$$

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[37] Misener and Floudas, "Piecewise-Linear Approximations of Multidimensional Functions", ([126], 2010)

[38] Zener, "A Mathematical Aid in optimizing Engineering Designs", ([208], 1961)

[39] Duffin, Peterson, and Zener, *Geometric Programming – Theory and Application*, ([44], 1967)

[40] Beightler and Phillips, *Applied Geometric Programming*, ([12], 1976)

[41] Peterson, "The Origins of Geometric Programming", ([150], 2001)

[42] Boyd, Kim, Vandenberghe, and Hassibi, "A Tutorial on Geometric Programming", ([19], 2007)

[43] Pelz, Leise, and Meck, "Sustainable Aircraft Design – A Review on Optimization Methods for Electric Propulsion with derived Optimal Number of Propulsors", ([147], 2021)

with $\alpha_0 > 0$ or *posynomials*

$$f^{\mathrm{p}}(x) = \sum_{k=1}^{K} \alpha_{0,k} x_{1,k}^{\alpha_{1,k}} x_{2,k}^{\alpha_{2,k}} \dots x_{n,k}^{\alpha_{n,k}} \tag{2.17}$$

with $\alpha_{0,k} > 0$. In this example the n variables are given by x, while α represents specific parameters for each constraint and K the total number of terms of the sum. The underlying optimization program can be transformed in a convex program by using a log-transformation if it has a posynomial as an objective and monomial or posynomial constraints of the following form[44]:

$$\begin{aligned} \min_{x} \quad & f_0^{\mathrm{p}}(x) \\ \text{s.t.} \quad & f_j^{\mathrm{m}}(x) = 1 \quad \forall j = 1, \dots, u \\ & f_i^{\mathrm{p}}(x) \leq 1 \quad \forall i = 1, \dots, m \\ & x > 0. \end{aligned} \tag{2.18}$$

The resulting log-transformed problem is convex. To derive this equivalent convex problem description, we use $\xi_i = \log(x_i)$ ($x_i = e^{\xi_i}$) for a variable transformation. The following is based on Boyd and Vandenberghe[45]. With these new variables, the monomial (2.16) transforms into the exponential of an affine function:

$$f^{\mathrm{m}}(x) = c(e^{\xi_1})^{\alpha_1} \dots (e^{\xi_n})^{\alpha_n} \tag{2.19a}$$

$$f^{\mathrm{m}}(x) = e^{\alpha^T \xi + \log(c)} \tag{2.19b}$$

A posynomial can be transformed analogously to

$$f^{\mathrm{p}}(x) = \sum_{k=1}^{K} e^{\alpha_k^T \xi + \beta_k} \tag{2.20}$$

The program (2.18) is then given with the new variables and a logarithmic transformation by

$$\begin{aligned} \min_{x} \quad & \log\left(\sum_{k=1}^{K_0} e^{\alpha_{0,k}^T \xi + \beta_{0,k}}\right) \\ \text{s.t.} \quad & \log\left(\sum_{k=1}^{K_i} e^{\alpha_{i,k}^T \xi + \beta_{i,k}}\right) \leq 0 \quad \forall i = 1, \dots, m \\ & \alpha_j^T \xi + \beta_j = 0 \quad \forall j = 1, \dots, u. \end{aligned} \tag{2.21}$$

44 Boyd, Kim, Vandenberghe, and Hassibi, "A Tutorial on Geometric Programming", ([19], 2007)

45 Boyd and Vandenberghe, *Convex Optimization*, ([20], 2004, pp. 162)

In general, the function $\hat{f}(\xi) = \log(e^{\xi_1} + \dots + e^{\xi_n})$ is called a *log-sum-exp* function, and it is convex on $\mathbb{R}^n$. A composition of this function with an affine function is again convex, so the transformed GP in the log-domain is convex[46]. The transformed GP problem can also be further transformed into an *exponential cone program*, which can be solved very efficiently by recently developed conic optimization software.

2.1.6 Generalized Benders Decomposition

The *Generalized Benders Decomposition*[47,48] (GBD) is a method to derive global optimal solutions of MINLPs efficiently, if they fulfill specific structural requirements[49]. The following introduction of this solution algorithm is based on Floudas and Pardalos[49]. The general MINLP description was already introduced in (2.1). In this subsection the following (nonconvex) MINLP with continuous (x) and only binary variables (y) is considered[49]:

$$\begin{aligned} \min_{x,y} \quad & f_0(x,y) \\ \text{s.t.} \quad & f_i(x,y) \le 0 \quad \forall i = 1,\dots,m, \\ & e_j(x,y) = 0 \quad \forall j = 1,\dots,u, \\ & x \ge 0, \\ & x \in \mathbb{X} \subseteq \mathbb{R}^k, y \in \{0,1\}^q. \end{aligned} \tag{2.22}$$

Here, the set $\mathbb{X}$ is nonempty and convex. The GBD is applicable, if the binary variables y can be seen as "complicating" variables. Shown by Floudas and Pardalos[49], if these variables y are fixed ($y \in \mathbb{Y} = \{0,1\}^q$), the remaining problem

(i) can be decomposed in multiple subproblems,

(ii) becomes convex in x,

(iii) takes a specific structure that is exploitable with specific known algorithms.

All three cases result in a significantly more efficient solution procedure of the given subproblem with fixed complicating variables y. The GBD approach is

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46 Martins and Ning, *Engineering Design Optimization*, ([123], 2021, p. 435)

47 Geoffrion, "Generalized Benders Decomposition", ([62], 1972)

48 Geoffrion and Graves, "Multicommodity Distribution System Design by Benders Decomposition", ([63], 1974)

49 Floudas and Pardalos, *Encyclopedia of Optimization*, ([57], 2009, pp. 1162)

based on the *Benders decomposition*, which was developed by Benders[50] for (mixed-integer) linear programs in 1962. Geoffrion extended this approach for the solution of MINLPs in 1972[51]. GBD is also applicable for a MINLP with continuous or discrete/continuous variables y, which is beyond the considered use case here.
The GBD is based on a decomposition in a *main*[52] problem and a subproblem with fixed complicating variables y. In an iterative solution procedure a main and subproblem are solved consecutively. The main problem is used to derive an optimized setting of the complicating variables in each iteration and the subproblem is used to derive an optimized solution of the remaining variables based on the fixed complicating variables in each iteration. This procedure is then executed alternating until a global optimal solution is found, or another stopping criterion is met.
Now, it is assumed that the solution procedure of the subproblem always leads to a feasible solution. Then, in each iteration k, the following subproblem with fixed y variables $Y_k \in \mathbb{Y}$ is solved:

$$\begin{aligned} \min_{x} \quad & f_0(x, Y_k) \\ \text{s.t.} \quad & f_i(x, Y_k) \leq 0 \quad \forall\, i = 1, \dots, m, \\ & e_j(x, Y_k) = 0 \quad \forall\, j = 1, \dots, u, \\ & x \geq 0, \\ & x \in \mathbb{X} \subseteq \mathbb{R}^k. \end{aligned} \tag{2.23}$$

The solution of the subproblem (2.23), also known in this context as the *primal problem*, leads to an upper bound on (2.22) and provides Lagrange multipliers, cf. Sec. 2.1.4, in each iteration. These Lagrange multipliers are used in the main problem to derive an additional *cut* (constraint) in each iteration. The solution of the main problem then leads to a lower bound and the next iterations variable assignments for y. The main problem is independent of the variables x and is given by[53]:

$$\begin{aligned} \min_{y \in \mathbb{Y}, \mu_B} \quad & \mu_B \\ \text{s.t.} \quad & \mu_B \geq \zeta(y, \nu^{\mathrm{La},(k)}, \lambda^{\mathrm{La},(k)}) \quad \forall\, \nu^{\mathrm{La},(k)}, \lambda^{\mathrm{La},(k)} \geq 0, \forall k \end{aligned} \tag{2.24}$$

with $\zeta(y, \nu^{\mathrm{La},(k)}, \lambda^{\mathrm{La},(k)})$ being the resulting cut of iteration k that relies on

50 Benders, “Partitioning procedures for solving mixed-variables programming problems”, ([15], 1962)

51 Geoffrion, “Generalized Benders Decomposition”, ([62], 1972)

52 Geoffrion and Benders use the term *master* problem.

53 Floudas and Pardalos, *Encyclopedia of Optimization*, ([57], 2009, p. 1167)

the Lagrange multipliers $\nu^{\mathrm{La},(k)}$ and $\lambda^{\mathrm{La},(k)}$. This cut is derived from the subproblem with a fixed set of complicating variables Y_k.
The shown main problem (2.24) is only valid, if the subproblem (2.23) always results in feasible solutions. If this is not the case, it is required to add further constraints based on an additional Lagrange function specific for the solution of an infeasible subproblem[54]. Furthermore, the GBD can converge to only a local optimum or even a non-stationary point, if the considered MINLP is nonconvex in x and/or y[55]. Therefore, it only leads to global optimal solutions for a subset of general nonconvex MINLPs. But on this subset, it can lead to a significant improvement of the solving performance and can even provide good estimates of the dual bound with a low number of iterations.

2.2 Modeling of Technical Systems

We differentiate between three model types: "*white-box*, *gray-box*, and *black-box*"[56] models. The selection of models has a significant impact on the final result of an optimized technical system[57]. Therefore, it is essential to derive engineering models that fit their purpose well.
White box models rely solely on axioms. The physical laws and parameters are known[58]. The opposite, a black-box model[59], only returns an evaluation based on given inputs, but its internal structure is completely unknown. Gray-box models are in an intermediate position, since they integrate an axiomatic modeling with a parameter or functional relationship identification.
In the following, important modeling approaches that are used in the considered engineering design optimization programs in this thesis are presented.

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[54] Floudas and Pardalos, *Encyclopedia of Optimization*, ([57], 2009, pp. 1162)

[55] Sahinidis and Grossmann, "Convergence Properties of Generalized Benders Decomposition", ([160], 1991)

[56] Pelz, Leise, and Meck, "Sustainable Aircraft Design – A Review on Optimization Methods for Electric Propulsion with derived Optimal Number of Propulsors", ([147], 2021)

[57] Martins and Lambe, "Multidisciplinary Design Optimization: A Survey of Architectures", ([122], 2013)

[58] Isermann and Münchhof, *Identification of Dynamic Systems*, ([86], 2011, p. 6)

[59] Ashby, *An Introduction to Cybernetics*, ([9], 1956, pp. 86)

2.2.1 White-box Modeling

Mathematical optimization algorithms can exploit the structure of a given physical model, if it is modeled as a whitebox[60]. Hence, within modeling of technical systems fundamental physical relationships from continuum mechanics, like conservation laws for energy, mass, and momentum play an important role[61]. All white-box models shown in this thesis rely on appropriate subsets of these conservation laws.

Besides these, further problem specific subsystem descriptions are often mandatory to describe the whole technical system properly. Here, problem specific subsystem models are derived by using physical-based parameterized functional relationships. The appropriate parametrizations are derived either from experiments and/or component specifications provided by the components' manufacturer. The derivation of these parameterized gray-box models is shown in the following subsection 2.2.2.

Furthermore, it is often required to determine the size of given components accordingly to the system. The optimized result then consists of optimally sized subsystems. Here, one approach is to model component series explicitly, which results in a high number of binary variables to model the selection of specific subsets for the derived system solutions. The advantage of this method is a detailed modeling. The drawback is an increase in time in the optimization program evaluation, since the higher number of binary variables increases the complexity. From a computational point of view, this approach is therefore limited within an optimization program to a relatively low number of components that can be considered explicitly. Within Chapter 3 a problem-specific preprocessing heuristic is shown, which reduces the number of components that are modeled within the optimization to improve the evaluation performance significantly and on the same time reduces omitting required subsystems from an engineering point of view.

Another approach is the usage of a reference system model and an according continuous *scaling* to derive a model representation for a subsystem. Here, frequently a change in the components size and the according effects on the system properties are considered in a functional relationship. This is also known as *allometric scaling*[62]. One early allometric scaling law is the famous Kleiber's law[63], which states that a mammals metabolic rate is proportional

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60 Pelz, Leise, and Meck, "Sustainable Aircraft Design – A Review on Optimization Methods for Electric Propulsion with derived Optimal Number of Propulsors", ([147], 2021)

61 Spurk and Aksel, *Strömungslehre*, ([177], 2010, pp. 37)

62 Huxley and Teissier, "Terminology of Relative Growth", ([80], 1936)

63 Kleiber, "Body Size and Metabolism", ([91], 1932)

to its mass by the power of 3/4. Besides biology, allometric scaling is also applicable for technical systems. Here, the usage of scaling laws requires that the scaled and reference system have a geometric and physical similarity[64]. The term "similarity" is used, if the relationship of at least one physical quantity in the reference and scaled design is constant[65]. Hence, geometric similarity is achieved, if the ratio of all the lengths of any scaled design and the according lengths in the reference model is constant.
The basis to derive scaling laws for technical systems is a dimensional analysis[66], which is based on the *Buckingham Pi-theorem*[67] and the *Bridgman postulate*[68]. One important physical similarity is a *dynamic similarity*, which describes a constant force relationship in combination with a geometric and temporal similarity[69]. Here, specific similarity measures, like the *Froude*- or *Reynolds*-number are often used in engineering.
In reality the requirements of similarity are often not met completely, which results in an *incomplete similarity*[70]. Nevertheless, scaling is a useful approach within conceptual design to derive subsystem models of components that have either not yet been developed or to reduce restrictions on the component selections that are enforced by the usage of a component series solely. An example of the usage of scaling within the conceptual system design is for instance given by Pelz, Leise, and Meck who derived an allometric scaling law for aircraft which was then used in an optimization program[71].

2.2.2 Gray-Box and Black-box Modeling

To construct a derived subsystem model from given data two regression approaches are used within this work, depending on the underlying estimated

64 Weber, "Das allgemeine Ähnlichkeitsprinzip der Physik und sein Zusammenhang mit der Dimensionslehre und der Modellwissenschaft", ([197], 1930)

65 Pahl, Beitz, Feldhusen, and Grote, *Engineering Design — A Systematic Approach*, ([141], 2007, pp. 466)

66 Spurk, *Dimensionsanalyse in der Strömungslehre*, ([176], 1992)

67 Buckingham, "On Physically Similar Systems; Illustrations of the Use of Dimensional Equations", ([27], 1914)

68 Bridgman, *Dimensional Analysis*, ([23], 1922)

69 Pahl, Beitz, Feldhusen, and Grote, *Engineering Design — A Systematic Approach*, ([141], 2007, pp. 466)

70 Spurk, *Dimensionsanalyse in der Strömungslehre*, ([176], 1992, pp. 62)

71 Pelz, Leise, and Meck, "Sustainable Aircraft Design – A Review on Optimization Methods for Electric Propulsion with derived Optimal Number of Propulsors", ([147], 2021)

physical relationships. These are first, the *linear least-squares*[72], and second the *nonlinear least-squares*[73]. The linear and nonlinear least-squares approaches are usable, if it is required to derive a functional relationship $\tilde{f} : \mathbb{R}^n \to \mathbb{R}$ based on k discrete observations $x_i \in \mathbb{R}^n$, for all $i = 1, \ldots, k$ and k according function evaluations $y \in \mathbb{R}$ from the unknown true functional relation $f : \mathbb{R}^n \to \mathbb{R}$, so $y_i = f(x_i)$ for all $i = 1, \ldots, k$. Linear least-squares is applicable, if the derived model

$$\tilde{f}(x) = \gamma_1\phi_1(x) + \cdots + \gamma_p\phi_p(x) \tag{2.25}$$

is a combination of p *basis functions* $\phi_i(x) : \mathbb{R}^n \to \mathbb{R}$ and model parameters γ_i. The resulting approximation $\tilde{f}$ is linear in its parameters γ_i. The approximation $\tilde{f}$ should minimize the prediction errors $r_i \, \forall i = 1, \ldots, k$ given by:

$$r_i = y_i - \tilde{y}_i = f(x_i) - \tilde{f}(x_i) \tag{2.26}$$

for all given data samples x_i. Here, the root-mean-square prediction error $\|r\|^2$ is used to derive a least-squares representation. With this the given problem is transformed in the convex optimization problem

$$\min_{\gamma} ||A\gamma - y_i||^2, \tag{2.27}$$

with the $k \times p$ matrix A being the matrix containing the given basis function evaluations at the known data samples x_i. This problem can be solved analytically by using the *pseudo-inverse* $A^\dagger$ of the matrix A[74]:

$$\tilde{\gamma} = A^\dagger y_i. \tag{2.28}$$

If the parameters γ_i are not linear in the derived model this results in:

$$\tilde{f}(x) = \mathrm{fn}(x, \gamma) \tag{2.29}$$

and the nonlinear least-squares approach must be used. Here, it is common practice to either use the Levenberg-Marquardt algorithm[73] or to use a generally applicable optimization algorithm for unrestricted nonlinear optimization problems[75] which mostly only finds a local optimum due to the possible non-convexity of Equation (2.29).

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[72] Boyd and Vandenberghe, *Introduction to Applied Linear Algebra: Vectors, Matrices, and Least Squares*, ([21], 2018, pp. 245)

[73] Boyd and Vandenberghe, *Introduction to Applied Linear Algebra: Vectors, Matrices, and Least Squares*, ([21], 2018, pp. 381)

[74] Boyd and Vandenberghe, *Introduction to Applied Linear Algebra: Vectors, Matrices, and Least Squares*, ([21], 2018, p. 229)

[75] Harzheim, *Strukturoptimierung*, ([73], 2008, Chapter 2)

2.2.3 Scenario Generation

For the design of technical systems it is required to derive a *concept of operations*[76] which is used to estimate the conditions within the usage period. If a stochastic optimization approach is used to derive system designs that are more robust against variations in their inputs, unsupervised learning algorithms like the *k-means*[77] clustering algorithm can be used to approximate a sample distribution by a smaller discrete distribution[78]. This procedure is also known as *scenario generation*. The clustering based on the k-means algorithm is used explicitly within the modeling of the powertrain system design[79]. For the water supply system design the scenario generation is based implicitly on a clustering[80,81].

2.2.4 Uncertainty Quantification

When comparing different solution strategies, it is required to derive point estimates and their according confidence intervals. One useful approach to derive these confidence intervals is the *bootstrap* method, which was first introduced by Efron[82] in 1979. Within this method resampling with replacement is used to create multiple sets of data samples. "The statistical accuracy of parameter estimates can then be evaluated by looking at the variability of predictions between the different bootstrap data samples."[83] This procedure is repeated a high number of times to derive more accurate estimates. Its advantage is the ability to derive confidence intervals for arbitrary distributions without further required information besides the already given drawn samples.

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[76] National Aeronautics and Space Administration, *NASA Systems Engineering Handbook*, ([137], 2016, p. 4)

[77] MacQueen et al., "Some Methods for Classification and Analysis of Multivariate Observations", ([115], 1967)

[78] Löhndorf, "An Empirical Analysis of Scenario Generation Methods for Stochastic Optimization", ([112], 2016)

[79] Leise et al., "Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods", ([106], 2021)

[80] Altherr, Leise, Pfetsch, and Schmitt, "Resilient layout, design and operation of energy-efficient water distribution networks for high-rise buildings using MINLP", ([6], 2019)

[81] Müller, Leise, Lorenz, Altherr, and Pelz, "Optimization and Validation of Pumping System Design and Operation for Water Supply in High-Rise Buildings", ([131], 2020)

[82] Efron, "Bootstrap Methods: Another Look at the Jackknife", ([47], 1979)

[83] Bishop, *Pattern Recognition and Machine Learning*, ([18], 2006, p. 23)

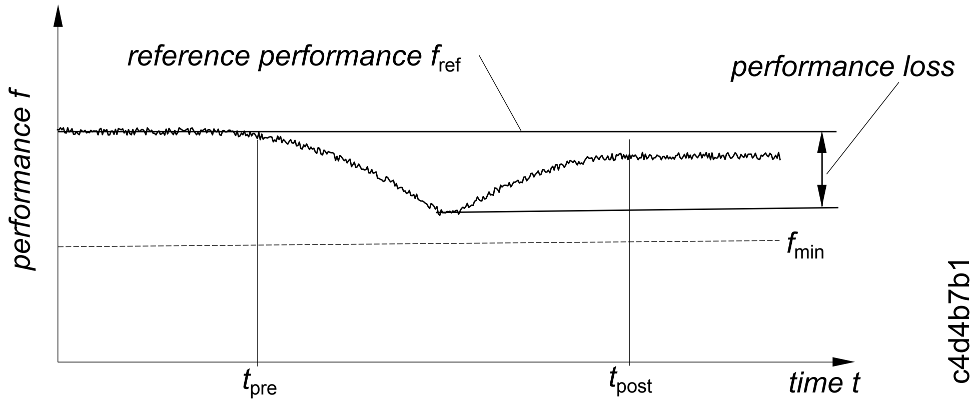

Figure 2.2 – Performance of an example resilient technical system over time, cf. [107]

2.3 Resilience

Resilience describes the intrinsic ability of a system to still ensure a minimal functional performance in situations where a failure or a severe disturbance occurs and additionally learn from these situations. Hence, a resilient system is “safe-to-fail”[84]. Resilience is subject of research in many scientific fields[85]. Nevertheless, the integration of this ability in technical systems is a challenging task and subject to current research, but it offers the opportunity to derive more sustainable systems[86].

Multiple different facets of the resilience paradigm within the engineering domain have been discovered so far. One early approach was given with the term *bounce back*, see e.g. Wink[87]. This approach describes the desired system behavior after a severe disturbance. Following this, the system should regain its pre-disturbance performance level after the occurrence of a disturbance. This describes one key property of a resilient system, but omits further important properties, like a meaningful learning and anticipation approach to avoid similar performance losses in the future.

Figure 2.2 visualizes a generalization of this dynamic behavior for an ideal technical system[88]. A severe disturbance occurs in the time step t_{pre}. After-

[84] Ahern, “From fail-safe to safe-to-fail: Sustainability and Resilience in the New Urban World”, ([1], 2011)

[85] Wink, *Multidisziplinäre Perspektiven der Resilienzforschung*, ([200], 2016)

[86] Fiksel, “Designing Resilient, Sustainable Systems”, ([54], 2003)

[87] Wink, *Multidisziplinäre Perspektiven der Resilienzforschung*, ([200], 2016, p. 127)

[88] Leise et al., “Potentials and Challenges of Resilience as a Paradigm for Designing Technical Systems”, ([107], 2021)

wards, the performance declines, but remains over a predefined minimum required performance f_{min}. Due to either topological changes or other adaptation mechanisms the performance increases. At time t_{post} the best (mean) post-disturbance performance is reached. For technical systems a complete restoration of the pre-disturbance performance is not always feasible.

In a technical context, the term resilience often overlaps with the term robustness[89]. A robust system survives a disturbance from a predefined range, because it is designed to withstand stresses within this predefined range. Even though robustness increases the resilience of a technical system, the resilience paradigm augments this behavior by enabling a further adaptation, learning and anticipation[90,91].

The research focus within the community to design resilient technical systems lies on the development of definitions of resilience and according metrics to measure the resilience property[92,93,94].

One major research direction of designing resilient technical systems within the engineering domain is, besides definitions and metrics, the consideration of network-like structures. They are modelled with the help of a mathematical graph and are improved by using optimization approaches[95,96].

Besides this, as shown by Scharte and Thoma[97], there exists currently a lack of a detailed evaluation of approaches to design resilient technical systems.

To address the resilient design of technical systems, within the CRC 805 a definition and multiple metrics that are suitable for technical systems have

[89] Pelz, Groche, Pfetsch, and Schäffner, *Mastering Uncertainty in Mechanical Engineering*, ([148], 2021, pp. 78)

[90] Leise et al., "Potentials and Challenges of Resilience as a Paradigm for Designing Technical Systems", ([107], 2021)

[91] Schulte, Kirchner, and Kloberdanz, "Analysis and Synthesis of Resilient Load-Carrying Systems", ([168], 2019)

[92] Linkov et al., "Changing the Resilience Paradigm", ([111], 2014)

[93] Righi, Saurin, and Wachs, "A Systematic Literature Review of Resilience Engineering: Research Areas and a Research Agenda Proposal", ([158], 2015)

[94] Hosseini, Barker, and Ramirez-Marquez, "A Review of Definitions and Measures of System Resilience", ([77], 2016)

[95] Altherr, Leise, Pfetsch, and Schmitt, "Resilient layout, design and operation of energy-efficient water distribution networks for high-rise buildings using MINLP", ([6], 2019)

[96] Ulusoy, Pecci, and Stoianov, "An MINLP-Based Approach for the Design-for-Control of Resilient Water Supply Systems", ([185], 2020)

[97] Scharte and Thoma, "Resilienz – Ingenieurwissenschaftliche Perspektive", ([164], 2016)

been introduced[98,99].
Figure 2.3 shows a quasi-static evaluation of the performance f of a given technical system. This approach is not limited to conceptual design, but can be used in it as well to quantify the resilience. Figure 2.3 (a) shows the *performance range*[100] which represents the range for influencing factors for which the minimum performance can be reached. The *radius of performance* is the “minimum distance between the design point and a realization of an influencing factor”[101].
Figure 2.3 (b) shows the *margin* and *gracefulness*. The first is defined as the difference of functional performance between the performance at the design point and the minimum required performance[102]. The second describes the system’s behavior “at the boundary of its performance range”[103]. Here a graceful degradation is preferred for a more resilient system[104]. Since the focus of this thesis is the conceptual design of technical systems by employing Mixed-Integer Nonlinear Optimization, the selected approach for designing a resilient system design is also presented in the next chapter.

[98] Altherr et al., “Resilience in Mechanical Engineering - A Concept for Controlling Uncertainty during Design, Production and Usage Phase of Load-Carrying Structures”, ([3], 2018)

[99] Altherr and Leise, “Resilience as a Concept to Master Uncertainty”, ([4], 2021)

[100] Altherr et al., “Resilience in Mechanical Engineering - A Concept for Controlling Uncertainty during Design, Production and Usage Phase of Load-Carrying Structures”, ([3], 2018)

[101] Altherr and Leise, “Resilience as a Concept to Master Uncertainty”, ([4], 2021, p. 414)

[102] Altherr and Leise, “Resilience as a Concept to Master Uncertainty”, ([4], 2021, p. 415)

[103] Altherr et al., “Resilience in Mechanical Engineering - A Concept for Controlling Uncertainty during Design, Production and Usage Phase of Load-Carrying Structures”, ([3], 2018, p. 190)

[104] Woods, “Essential Characteristics of Resilience”, ([202], 2017)

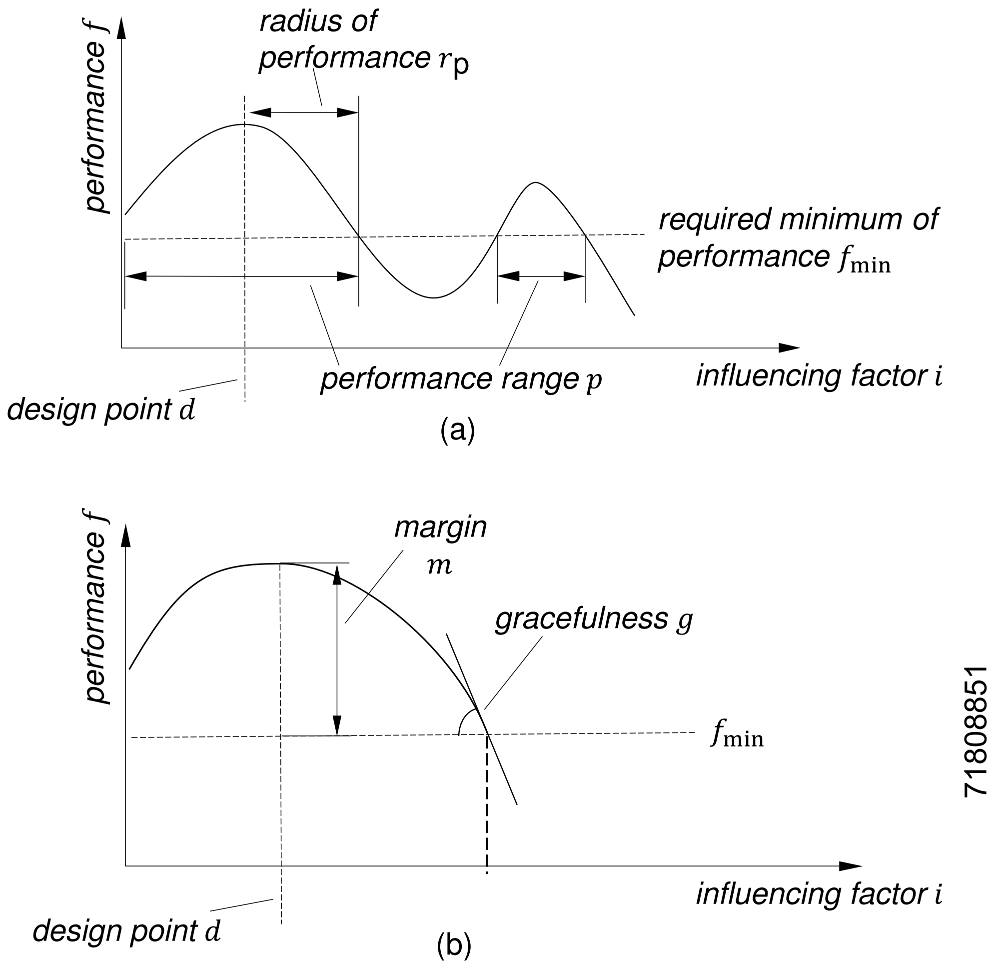

Figure 2.3 – Metrics for technical systems to quantify the resilience, cf. [3]. This figure was first published in [4, p. 415]

Chapter 3

Water Distribution System Design

In 2018 about 55% of the world's population lived in urban areas, and it is estimated that 2.5 billion people will join the urban population by 2050[1]. This trend enforces also an increase in the number of mega-cities with more than 10 million inhabitants and also rises the trend of a growing number of high-rise buildings[2]. Since the urban water distribution system is an important infrastructure, research on more sustainable system designs is crucial[3].
Therefore, the first use case which is described in details within this Chapter focuses on the development of a sustainable water distribution system (WDS) for high-rise buildings, since the efficiency of these systems can be significantly improved, as identified within the research conducted within this thesis in the CRC 805.
A prerequisite of an efficient design of these systems is the usage of efficient components. But even a system design with the most efficient components can result in a suboptimal system when considering the usage period and investment decisions together. Therefore, it is essential to consider within the conceptual design besides investment and the energy efficiency of single components also their interplay within the derived system design[4]. Within this chapter a newly derived MINLP model with specific extensions will be shown. Furthermore, the effects of different modeling approaches on

[1] United Nations Secretariat, Population Division, *2018 Revision of World Urbanization Prospects*, ([188], 2018)

[2] CTBUH, *2018 Tall Building Year in Review*, ([36], 2018)

[3] Coelho and Andrade-Campos, "Efficiency Achievement in Water Supply Systems – A Review", ([35], 2014)

[4] Tolvanen, "Life Cycle Energy Cost Savings through careful System Design and Pump Selection", ([181], 2007)

the system design and the effects of the selected solution approach will be evaluated. This chapter focuses on the underlying optimization model and the physical and computational properties. In Subsection 3.3.3, this approach is extended and discussed while considering explicitly the resilience.
This Chapter is based on multiple already published contributions[5,6,7,8,9], which specify specific parts while using this approach for the development of a sustainable water distribution system for high-rise buildings. The first model for designing a water distribution system for high-rise buildings was developed in the preceding master thesis[10], of which specific results were published by Rausch, Leise, Ederer, Altherr, and Pelz[11].
The models shown in the following extend this model significantly with an improved model accuracy and enhanced solving performance. The research within the subproject A9 within the CRC 805 mainly focused on this reference use case to broaden the understanding of efficient modeling approaches, while also considering the technical system's resilience next to the system's lifecycle performance.
A MILP approximation of this non-convex MINLP for a WDS in highrise buildings is also selected in a reference library that is used for the evaluation of general purpose Mixed-Integer Linear Solvers. More details can be found within the library MIPLIB 2017[12] and the corresponding publication[13].

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[5] Leise, Altherr, and Pelz, "Energy-Efficient Design of a Water Supply System for Skyscrapers by Mixed-Integer Nonlinear Programming", ([102], 2018)

[6] Leise and Altherr, "Optimizing the Design and Control of Decentralized Water Supply Systems – A Case-Study of a Hotel Building", ([101], 2018)

[7] Altherr, Leise, Pfetsch, and Schmitt, "Algorithmic Design and Resilience Assessment of Energy Efficient High-Rise Water Supply Systems", ([5], 2018)

[8] Altherr, Leise, Pfetsch, and Schmitt, "Resilient layout, design and operation of energy-efficient water distribution networks for high-rise buildings using MINLP", ([6], 2019)

[9] Müller, Leise, Lorenz, Altherr, and Pelz, "Optimization and Validation of Pumping System Design and Operation for Water Supply in High-Rise Buildings", ([131], 2020)

[10] Leise, "Algorithmic Design of a Decentralized Fluid System", ([98], 2016)

[11] Rausch, Leise, Ederer, Altherr, and Pelz, "A Comparison of MILP and MINLP Solver Performance on the Example of a Drinking Water Supply System Design Problem", ([157], 2016)

[12] Zuse Institute Berlin, *MIPLIB 2017 – The Mixed Integer Programming Library*, ([211], 2017)

[13] Gleixner et al., "MIPLIB 2017: Data-Driven Compilation of the 6th Mixed-Integer Programming Library", ([65], 2021)

3.1 Related Work

The optimization of water distribution systems in general has been in the interest of research for several decades. Detailed reviews are given by different authors[14,15,16]. Three types of specific optimization problems exist in the literature. First, optimization programs, in which specific components like pipes, networks, valves, or pumps are sized[17,18,19]. The problem of pipe diameter sizing in this problem class is already NP-hard[20] and is therefore often solved by using heuristics. Second, the optimization of the operation of components like pumps or valves in a given network[21,22,23]. And third, the combined optimization of sizing, layout and operation increases the solution complexity even further. It was therefore also mostly solved by heuristics[24,25,26].
Within this thesis an approach based on an exact optimization of a MINLP is presented for the combined solution of a network layout, a pump sizing and pump operation for the fresh-water supply of high-rise buildings. This approach is based on the modeling as a stochastic optimization program as already introduced in Section 2.1.2 to ensure the integration of a usage

[14] De Corte and Sörensen, "Optimisation of Gravity-fed Water Distribution Network Design: A Critical Review", ([39], 2013)

[15] D'Ambrosio, Lodi, Wiese, and Bragalli, "Mathematical Programming Techniques in Water Network Optimization", ([37], 2015)

[16] Mala-Jetmarova, Sultanova, and Savic, "Lost in Optimisation of Water Distribution Systems? A Literature Review of System Operation", ([119], 2017)

[17] Fujiwara and Khang, "A two-phase Decomposition Method for Optimal Design of Looped Water Distribution Networks", ([60], 1990)

[18] Varma, Narasimhan, and Bhallamudi, "Optimal Design of Water Distribution Systems using an NLP Method", ([189], 1997)

[19] Pecci, Abraham, and Stoianov, "Global Optimality Bounds for the Placement of Control Valves in Water Supply Networks", ([144], 2019)

[20] Yates, Templeman, and Boffey, "The Computational Complexity of the Problem of determining Least Capital Cost Designs for Water Supply Networks", ([206], 1984)

[21] Jowitt and Germanopoulos, "Optimal Pump Scheduling in Water-supply Networks", ([88], 1992)

[22] Yu, Powell, and Sterling, "Optimized Pump Scheduling in Water Distribution Systems", ([207], 1994)

[23] Gleixner, Held, Huang, and Vigerske, "Towards Globally Optimal Operation of Water Supply Networks", ([66], 2012)

[24] Dandy, Simpson, and Murphy, "Optimum Design and Operation of Pumped Water Distribution Systems", ([38], 1994)

[25] Ostfeld and Tubaltzev, "Ant Colony Optimization for Least-Cost Design and Operation of Pumping Water Distribution Systems", ([140], 2008)

[26] Prasad, "Design of Pumped Water Distribution Networks with Storage", ([153], 2009)

period that is affected by uncertainty. Furthermore, it allows to estimate the efficiency increase of an optimized system design in comparison to a reference design. A related research approach that is solely based on a MILP approach instead of a MINLP approach, and which focuses more on the optimization of booster station designs and thermo-fluid systems is given by Weber et al[27,28,29]. The standards DIN 1988-500 (2011) and DIN 1988-300 (2014) present codes of practice for drinking water installations in high-rise buildings[30,31]. They reflect current engineering best-practices to design water distribution systems for fresh-water. Within DIN 1988-500 multiple generic approaches of multi-branch system designs, where different pressure zones within the building are supplied individually, are shown. DIN 1988-300 presents relevant sizing and load estimation techniques to design the water distribution system according to the needs of the considered building. Both standards are applicable for high-rise buildings, but do not present a specific approach to derive a sustainable system design. Instead, they only provide generic approaches. Besides this, a concept of operation for the fresh water supply in buildings is given by Hirschberg[32]. Since the considered pumps within the system design have a significant impact on the energy efficiency within the usage phase, standards like the EN16480:2016 (2016)[33] or the ISO/ASME 14414 (2019)[34] are also relevant within the system design.

[27] Weber and Lorenz, "Optimizing Booster Stations", ([196], 2017)

[28] Weber and Lorenz, "Algorithmic System Design of Thermofluid Systems", ([195], 2018)

[29] Weber, Hartisch, Herbst, and Lorenz, "Towards an Algorithmic Synthesis of Thermofluid Systems", ([194], 2021)

[30] DIN 1988-300, *Codes of practice for drinking water installations – Part 300: Pipe sizing; DVGW code of practice*, ([41], 2012)

[31] DIN 1988-500, *Codes of practice for drinking water installations – Part 500: Pressure boosting stations with RPM-regulated pumps; DVGW code of practice*, ([42], 2011)

[32] Hirschberg, "Lastprofil und Regelkurve zur energetischen Bewertung von Druckerhöhungsanlagen", ([74], 2014)

[33] EN 16480:2016, *Pumps — Minimum required Efficiency of Rotordynamic Water Pumps*, ([49], 2016)

[34] ISO/ASME 14414, *Pump System Energy Assessment*, ([87], 2019)

3.2 MINLP Model

The remaining part of this chapter is based on three publications that were published in the context of the CRC 805[35,36,37]. The focus lies on the interplay between model accuracy and the applicability of an exact optimization approach.

The concept of operation for the conceptual design of a water distribution system is represented with a set of s scenarios, which can be written as:

$$\Lambda = \begin{bmatrix} \mathbf{q}^T \\ \mathbf{h}^T \\ \boldsymbol{\pi}^T \end{bmatrix} \in \mathbb{R}^{3\times s}. \tag{3.1}$$

With this abstract representation it is possible to derive a deterministic equivalent of a stochastic program, in which the load of the water distribution system is uncertain. This load is approximated with a discrete set of scenarios. Each scenario is given by a specific volume flow q_i, pressure h_i and probability π_i. The set of scenarios is given by $\mathcal{S}$ and consists of the s load scenarios given by Λ.

In the following, a nonlinear nonconvex reference model is presented. Then, multiple advancements are shown of this basic model, where each model is specialized for answering a specific research question. Two approaches for integrating multiple speed-controlled pumps in the optimization model are presented. Here, one approach is based on a scaling approach, while the second one is based on a preselection of a subset of pumps from a given series of pumps. Afterwards, we also present a model adaption that transforms the given nonconvex nonlinear model in a mixed-integer linear program. The performance of this convexification strategy is shown by a systematic evaluation, cf. [131].

For modeling of the given technical system a gray-box approach is selected. The gray-box model for deriving a conceptual design for the WDS of high-rise buildings is based on a graph-theoretical approach with the graph $\mathcal{G}$, in which the sets given in Table 3.1 are used to describe the vertices and edges. It is required that the underlying graph is directed and acyclic. The

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[35] Leise and Altherr, "Optimizing the Design and Control of Decentralized Water Supply Systems – A Case-Study of a Hotel Building", ([101], 2018)

[36] Altherr, Leise, Pfetsch, and Schmitt, "Resilient layout, design and operation of energy-efficient water distribution networks for high-rise buildings using MINLP", ([6], 2019)

[37] Müller, Leise, Lorenz, Altherr, and Pelz, "Optimization and Validation of Pumping System Design and Operation for Water Supply in High-Rise Buildings", ([131], 2020)

Table 3.1 – Index Sets, cf. [101]

Index Sets	Description
$\mathcal{V}$	set of vertices of system graph $\mathcal{G}$
$\mathcal{E}$	set of edges of system graph $\mathcal{G}$
$\mathcal{S}$	set of load scenarios s
$\mathcal{B}$	set of pump types b

models consider only the fresh water supply. As an objective the simplified lifetime costs are used, which consist of the investment and energy costs within the lifetime. The most important components in the system model are the speed-controlled centrifugal pumps that provide the volume flow and pressure to supply each floor in the given building with fresh water. Since the water pressure of municipal water supplier is only sufficient to provide water in buildings with a few floors and is regionally different, pumps are usually required for high-rise buildings. In the following, we use capital letters for parameters and lower case letters for variables.

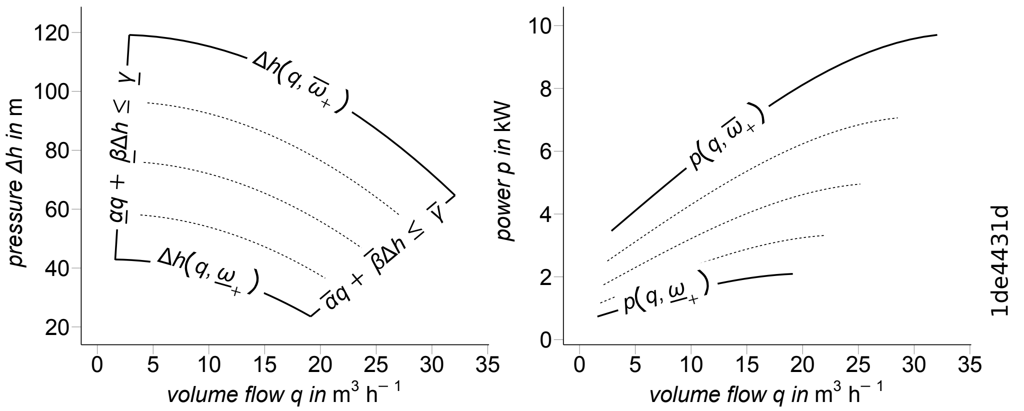

Figure 3.1 – Characteristic diagram with isolines for different speed values ω for an exemplified pump and its feasible set, cf. [6]

The behavior of the centrifugal pumps can be described by characteristic diagrams, as shown in Fig. 3.1. The pressure head Δh is measured in meter. A centrifugal pump can be used in a specified subdomain within the Δh-q domain. Here, q represents the volume flow. A characteristic diagram of a given centrifugal pump can be approximated with polynomials[38]. In the following, we use for the representation of the pumps pressure head Δh and

[38] Ulanicki, Kahler, and Coulbeck, "Modeling the Efficiency and Power Characteristics of a Pump Group", ([184], 2008)

the power p[39]:

$$\begin{aligned} \Delta h(q,\omega_+) &= \alpha^h\, q^2 + \beta^h\, q\,\omega_+ + \gamma^h\,\omega_+^2 \quad \text{and} \\ p(q,\omega_+) &= \alpha^p\, q^3 + \beta^p\, q^2\,\omega_+ + \gamma^p\, q\,\omega_+^2 + \delta^p\,\omega_+^3. \end{aligned} \tag{3.2}$$

The functional relationship of the pressure head is approximated by a second-order polynomial with the volume flow q and normalized rotational speed $\omega_+ = \omega/\omega^{\max}$. This normalized rotational speed reaches its upper bound $\overline{\omega}_+ = 1$, if the maximum speed is reached. The lower bound $\underline{\omega}_+$ depends on the considered pump and is usually above zero. The feasible set of a given pump is then described additionally to Eq. (3.2) by[39]:

$$\begin{aligned} \underline{\omega}_+ \leq \omega_+ &\leq \overline{\omega}_+, \\ \overline{\alpha} q + \overline{\beta}\Delta h &\leq \overline{\gamma}, \\ \underline{\alpha} q + \underline{\beta}\Delta h &\leq \underline{\gamma}. \end{aligned} \tag{3.3}$$

The parameters that are required within this centrifugal pump model (3.2) and (3.3) are derived by using a linear least-square approximation of measured characteristics given by the given manufacturer, cf. Section 2.2.2. Since the conceptual design task requires the appropriate sizing of pumps to supply the building at hand most sustainable, a scaling approach, as already presented in Section 2.2.1, helps to define pump characteristics that can be used as a reference construction kit from which the optimized system can be build of. If the flow conditions in two given pumps are geometrically and dynamically similar, cf. Section 2.2.1, it is possible to use a scaling law to represent a scaled pump by using the description of a reference pump ($\square_{\mathrm{M}}$) and an according scaling. For centrifugal pumps the following scaling approach is given by Gülich[40]:

$$\begin{aligned} q &= q_{\mathrm{M}} \left(\frac{d}{d_{\mathrm{M}}}\right)^3, \\ \Delta h &= \Delta h_{\mathrm{M}} \left(\frac{d}{d_{\mathrm{M}}}\right)^2 \frac{z_{st}}{z_{st,\mathrm{M}}}, \\ p &= p_{\mathrm{M}} \left(\frac{d}{d_{\mathrm{M}}}\right)^5 \frac{z_{st}}{z_{st,M}} \frac{\eta_{\mathrm{M}}}{\eta}. \end{aligned} \tag{3.4}$$

It assumes that “the volumetric and hydraulic efficiency, as well as the

[39] Altherr, Leise, Pfetsch, and Schmitt, “Resilient layout, design and operation of energy-efficient water distribution networks for high-rise buildings using MINLP”, ([6], 2019)

[40] Gülich, *Centrifugal Pumps*, ([69], 2008, p. 147)

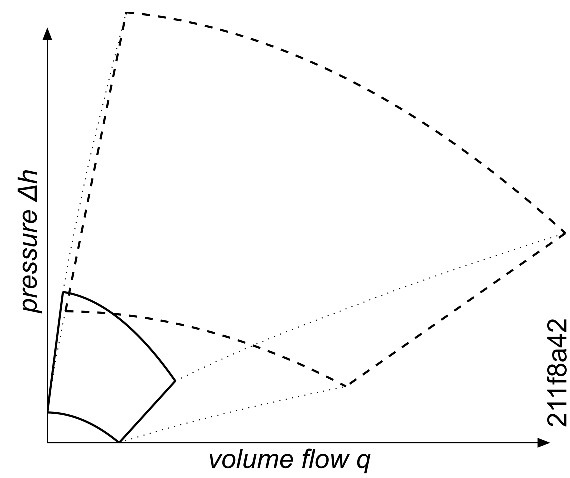

Figure 3.2 – Example for the scaling of characteristic pump diagrams based on [6]. The solid curve represents the feasible set of the reference pump. The dashed lines depict the feasible set of a scaled pump. The dotted lines depict the scaling of the corner points.

rotational speed and density of the fluid are equal for the reference pump"[41] and the derived pump. Here, the impeller diameter $d \in \mathbb{R}^+$ and the number of stages $z_{\mathrm{st}} \in \mathbb{Z}^+$ are used as scaling parameters. A scaling based on the impeller diameter is shown in Figure 3.2.
The parameter η represents the hydraulic efficiency, which increases with the size of the pump. This increase is modelled following Gülich[42] with:

$$\eta = \eta_{\mathrm{M}} + 0.4\,(1 - \eta_{\mathrm{M}}) \left(1 - \frac{Re_{\mathrm{M}}}{Re}^{0.2}\right). \tag{3.5}$$

With the constant kinematic viscosity ν and the Reynolds number $Re = \frac{\omega \pi d^2}{\nu}$ for pumps, Equation (3.5) can be written as:

$$\eta = \eta_{\mathrm{M}} + 0.4\,(1 - \eta_{\mathrm{M}}) \left(1 - \frac{d_{\mathrm{M}}}{d}^{0.4}\right). \tag{3.6}$$

With this approach, we are able to derive an arbitrary number of pumps that can be sized according to the requirements. This benefit comes also with a computational drawback, since the integration of this approach directly

41 Altherr, Leise, Pfetsch, and Schmitt, "Resilient layout, design and operation of energy-efficient water distribution networks for high-rise buildings using MINLP", ([6], 2019)

42 Gülich, *Centrifugal Pumps*, ([69], 2008, p. 155)

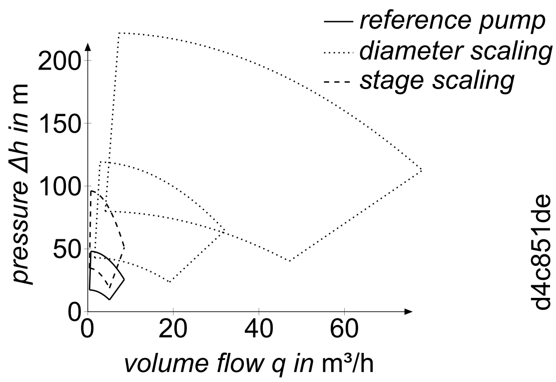

Figure 3.3 – Characteristic q–Δh diagrams of the used construction kit in [6].

in the model reduces the performance of exact optimization algorithms due to its high nonlinearity. Therefore, the scaling approach is used to define multiple construction kits of specific scaled pumps, cf. Leise and Altherr[43]. The additional degree of freedom is then evaluated by *Latin Hypercube Sampling* (LHS) – an approach known from Design of Experiments[44]. Since the number of stages and the diameter of the impeller represent two parameters to scale the pumps, the construction kit can be build of pumps that have different properties. In Fig. 3.3, a reference construction kit with differently sized pumps is shown[45].

The presented newly derived reference MINLP for the conceptual design of the fresh water distribution systems in high-rise buildings is based on a two-stage stochastic optimization program. In the first stage binary and integer decisions represent investment decisions. Here, the complete network design and pumps are selected. This reference network design, and an resulting example selection after fixing the first stage variables is shown in Figure 3.4. The black dots represent vertices of the underlying graph-representation and from an engineering point of view junctions of the water distribution network in the building to a corresponding pressure zone. Here,

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43 Leise and Altherr, "Optimizing the Design and Control of Decentralized Water Supply Systems – A Case-Study of a Hotel Building", ([101], 2018)

44 Pelz, Leise, and Meck, "Sustainable Aircraft Design – A Review on Optimization Methods for Electric Propulsion with derived Optimal Number of Propulsors", ([147], 2021)

45 Altherr, Leise, Pfetsch, and Schmitt, "Resilient layout, design and operation of energy-efficient water distribution networks for high-rise buildings using MINLP", ([6], 2019)

pressure zones[46] aggregate multiple adjacent floors. The according pressure and flow requirements in these pressure zones can either be estimated by standard procedures[47] for a newly designed building or are based on reference measurements from an existing building.

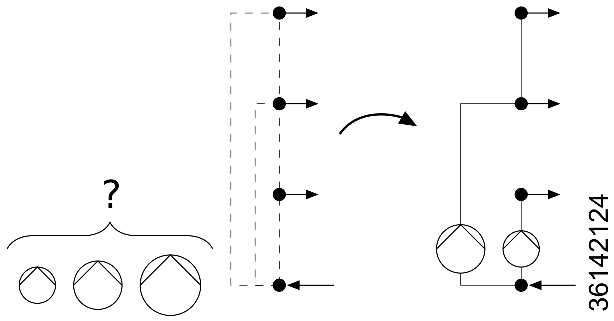

Figure 3.4 – First stage decisions – Define the topology of the network, cf. [101]

In the second stage integer and continuous variables lead to an optimal control of the selected pumps[48]. The usage phase is represented by different scenarios s and a corresponding weighting π_s, cf. Section 2.1.2. It is therefore possible to use specific pumps that were integrated in the optimized solution only in specific scenarios, while they are deactivated in other scenarios. This activation and deactivation is modeled with binary variables $y^{\text{load}}_{i,j,b,s}$ and integer variables $z^{\text{load}}_{i,j,b,s}$.
All used parameters and variables are shown in Table 3.2, cf. [101]. The corresponding two-stage stochastic optimization program is given in Equation 3.7 to Equation 3.35. Within each load scenario up to Z_p pumps of the same type can be used in a parallel setting. This restriction is valid since parallel pumps are only the most energy efficient, if they are of the same size and are controlled simultaneously[49,50].
The result of the solution of the given optimization program ((3.7) – (3.35)) with an according parametrization that is suitable for the high-rise building

[46] DIN 1988-500, *Codes of practice for drinking water installations – Part 500: Pressure boosting stations with RPM-regulated pumps; DVGW code of practice*, ([42], 2011)

[47] DIN 1988-300, *Codes of practice for drinking water installations – Part 300: Pipe sizing; DVGW code of practice*, ([41], 2012)

[48] Leise and Altherr, "Optimizing the Design and Control of Decentralized Water Supply Systems – A Case-Study of a Hotel Building", ([101], 2018)

[49] Pedersen and Yang, "Efficiency Optimization of a Multi-pump Booster system", ([145], 2008)

[50] Gross, Pöttgen, and Pelz, "Analytical Approach for the Optimal Operation of Pumps in Booster Systems", ([67], 2017)

that is under consideration, are the investment decisions for the selected pumps and pipes $\{z^{\text{pump}}_{i,j,b}, y^{\text{pipe}}_{i,j}\}$ and the control decisions of the selected pumps $\{z^{\text{load}}_{i,j,b,s}, \omega_{+,i,j,b,s}\}$.

The constraints of the resulting nonconvex MINLP are presented now in further detail. Constraint (3.8) restricts the solution to only networks, in which one pipe supplies each pressure zone. This leads to a tree-structured network, which is representable for fresh water distribution systems in high-rise buildings. Besides a single pipe, it is possible to select one of the given pump types from the construction kit (constraint (3.9)). From this selected pump type multiple parallel pumps can be used in the optimized solution. A selection of pumps is only valid, if a pipe is selected to connect the according vertices of the graph (constraint (3.10)). In a given load scenario, a pump can only be used if it is installed (constraint (3.11)). The logical behavior for each parallel pump group is modeled with Constraints (3.12) to (3.16). Constraints (3.17) and (3.18) describe the domain boundaries on the normalized rotating speed $\omega_{+,i,j,b,s}$. The boundary conditions of the considered high-rise building are represented by Constraints (3.19) to (3.21). Constraint (3.19) sets the pressure at the entry of the building based on a specified value by the municipal water supplier. Constraints (3.20) and (3.21) integrate the minimum pressure head in each pressure zone. The continuity equations for the volume flow are modeled with (3.22) to (3.28). And finally, the characteristics of each considered pump in the underlying construction kit is integrated in the MINLP with Constraints (3.29) to (3.35).

Instances of this reference model can be created by selecting a dedicated construction kit of pumps, a pressure zone description and water demand as well as environmental parameters like the water pressure at the intake of the building.

3.2.1 Solution Approach

The introduced abstract model is now evaluated on an existing reference hotel building. With this example, the effect of a retrofitting of the water supply system can be estimated. Since the selection of the pumps has an influence on the optimized solution, an approach that is based on a Latin Hypercube Sampling[51] is presented to evaluate multiple solutions within the complete design space given by the variability of the pumps. The performance of each construction kit $\mathcal{B}$ is evaluated based on the given MINLP. The parameters

[51] Pelz, Leise, and Meck, "Sustainable Aircraft Design – A Review on Optimization Methods for Electric Propulsion with derived Optimal Number of Propulsors", ([147], 2021)

Table 3.2 – Parameters and variables for the MINLP, cf. [101]

Parameter	Description	Domain
$\overline{Q}$	maximum inflow	$\mathbb{R}^+$
$Q^{\text{load}}_{v,s}$	outflow on level v in scenario s	$\mathbb{R}^+$
H^0	pressure head by water supplier	$\mathbb{R}^+$
$\underline{H}$	minimal pressure head in each level, expect the first one	$\mathbb{R}^+$
$\underline{H}_1$	minimal pressure head in first level	$\mathbb{R}^+$
$\overline{H}$	maximum water height of the building	$\mathbb{R}^+$
B	big-M constant for the pressure increase	$\mathbb{R}^+$
$\Delta H_{i,j}$	pressure loss between level i and level j	$\mathbb{R}^+$
$\overline{P}$	maximum power consumption	$\mathbb{R}^+$
Z	maximum number of parallel pumps	$\mathbb{R}^+$
$\underline{\Omega}_+$	minimum normalized speed of all pumps	$[0,1]$
C^{energy}	energy costs per Watt	$\mathbb{R}^+$
C^{pump}_b	investment cost for each pump	$\mathbb{R}^+$
$C^{\text{pipe}}_{i,j}$	investment costs per pipe	$\mathbb{R}^+$
π_s	usage probability of each scenario	$\mathbb{R}^+$
τ	time of usage	$\mathbb{R}^+$

Variable	Description	Domain
$q_{i,j,s}$	volume flow in each edge (i,j) in scenario s	$[0,\overline{Q}]$
$q^{\text{part}}_{i,j,s}$	volume flow in each pump on edge (i,j) in scenario s	$[0,\overline{Q}]$
$h_{v,s}$	pressure head on vertex v in scenario s	$[0,\overline{H}]$
$\omega_{+,i,j,b,s}$	normalized speed on edge (i,j) for pump b in scenario s	$[0,1]$
$p_{i,j,b,s}$	total power of pumps of type b on edge (i,j) in scenario s	$[0,\overline{P}]$
$p^{\text{part}}_{i,j,b,s}$	power of each pump of type b on edge (i,j) in scenario s	$[0,\overline{P}]$
$y^{\text{pipe}}_{i,j}$	indicator whether pipe (i,j) is used	$\{0,1\}$
$y^{\text{pump}}_{i,j,b}$	indicator whether pumps of type b is used in pipe (i,j)	$\{0,1\}$
$y^{\text{load}}_{i,j,b,s}$	indicator if pump b is used in pipe (i,j) in scenario s	$\{0,1\}$
$z^{\text{pump}}_{i,j,b}$	number of parallel pumps of type b on edge (i,j)	$\{0,Z\}$
$z^{\text{load}}_{i,j,b,s}$	number of active pumps of type b on edge (i,j) in scenario s	$\{0,Z\}$

Table 3.3 – Parameters for all used model pumps, cf. [101].

$b \in \mathcal{B}$	Power $p^{\text{part}}_{i,j,b,s}$				Head $\Delta h_{i,j,b,s}$			Flow $q^{\text{part}}_{i,j,s}$			
	α^{p}_b	β^{p}_b	γ^{p}_b	δ^{p}_b	α^{h}_b	β^{h}_b	γ^{h}_b	$\alpha^{\text{q},1}_b$	$\beta^{\text{q},1}_b$	$\alpha^{\text{q},2}_b$	$\beta^{\text{q},2}_b$
$b = 1$	-11.14	41.52	54.32	191.21	-3.42	2.76	45.19	1.07	0.085	0.077	0.0027
$b = 2$	-1.77	5.90	135.81	245.28	-0.92	1.18	52.30	2.34	0.15	0.098	0.0029
$b = 3$	-0.66	1.15	125.73	276.78	-0.34	0.37	47.97	3.26	0.21	0.25	0.0094
$b = 4$	-1.21	-4.67	205.12	343.73	-1.18	1.04	84.64	6.80	0.0	0.00	0.00

$$\min \sum_{b\in\mathcal{B}} \sum_{(i,j)\in\mathcal{E}} C_b^{\text{pump}} z_{i,j,b}^{\text{pump}} + \sum_{(i,j)\in\mathcal{E}} C_{i,j}^{\text{pipe}} y_{i,j}^{\text{pipe}} + C^{\text{energy}} \tau \sum_{s\in\mathcal{S}} \sum_{b\in\mathcal{B}} \sum_{(i,j)\in\mathcal{E}} \pi_s p_{i,j,b,s} \tag{3.7}$$

subject to

$$\sum_{(i,j)\in\mathcal{E}:j=v} y_{i,j}^{\text{pipe}} \leq 1 \qquad \forall (i,j)\in\mathcal{E}, \forall v\in\mathcal{V} \tag{3.8}$$

$$\sum_{\forall b\in\mathcal{B}} y_{i,j,b}^{\text{pump}} \leq 1 \qquad \forall (i,j)\in\mathcal{E} \tag{3.9}$$

$$y_{i,j,b}^{\text{pump}} \leq y_{i,j}^{\text{pipe}} \qquad \forall (i,j)\in\mathcal{E}, \forall b\in\mathcal{B} \tag{3.10}$$

$$y_{i,j,b,s}^{\text{load}} \leq y_{i,j,b}^{\text{pump}} \qquad \forall (i,j)\in\mathcal{E}, \forall b\in\mathcal{B}, \forall s\in\mathcal{S} \tag{3.11}$$

$$y_{i,j,b}^{\text{pump}} \leq z_{i,j,b}^{\text{pump}} \qquad \forall (i,j)\in\mathcal{E}, \forall b\in\mathcal{B} \tag{3.12}$$

$$y_{i,j,b}^{\text{pump}} Z \geq z_{i,j,b}^{\text{pump}} \qquad \forall (i,j)\in\mathcal{E}, \forall b\in\mathcal{B} \tag{3.13}$$

$$y_{i,j,b,s}^{\text{load}} \leq z_{i,j,b,s}^{\text{load}} \qquad \forall (i,j)\in\mathcal{E}, \forall b\in\mathcal{B}, \forall s\in\mathcal{S} \tag{3.14}$$

$$y_{i,j,b,s}^{\text{load}} Z \geq z_{i,j,b,s}^{\text{load}} \qquad \forall (i,j)\in\mathcal{E}, \forall b\in\mathcal{B}, \forall s\in\mathcal{S} \tag{3.15}$$

$$z_{i,j,b,s}^{\text{load}} \leq z_{i,j,b}^{\text{pump}} \qquad \forall (i,j)\in\mathcal{E}, \forall b\in\mathcal{B}, \forall s\in\mathcal{S} \tag{3.16}$$

$$\omega_{+,i,j,b,s} \leq y_{i,j,b,s}^{\text{load}} \qquad \forall (i,j)\in\mathcal{E}, \forall b\in\mathcal{B}, \forall s\in\mathcal{S} \tag{3.17}$$

$$\omega_{+,i,j,b,s} \geq \underline{\Omega}_{+} y_{i,j,b,s}^{\text{load}} \qquad \forall (i,j)\in\mathcal{E}, \forall b\in\mathcal{B}, \forall s\in\mathcal{S} \tag{3.18}$$

$$h_{\text{In},s} = H^0 \qquad \forall s\in\mathcal{S} \tag{3.19}$$

$$h_{1,s} \geq \underline{H}_1 \qquad \forall s\in\mathcal{S} \tag{3.20}$$

$$h_{v,s} \geq \underline{H} \qquad \forall v\in\mathcal{V}\setminus 1, \forall s\in\mathcal{S} \tag{3.21}$$

$$q_{i,j,s} \leq \overline{Q} y^{\text{pipe}}_{i,j} \qquad \forall (i,j) \in \mathcal{E}, \forall s \in \mathcal{S} \tag{3.22}$$

$$q^{\text{part}}_{i,j,s} \leq q_{i,j,s} \qquad \forall (i,j) \in \mathcal{E}, \forall b \in \mathcal{B}, \forall s \in \mathcal{S} \tag{3.23}$$

$$\sum_{(i,v)\in\mathcal{E}} q_{i,v,l} = \sum_{(v,j)\in\mathcal{E}} q_{v,j,s} + q^{\text{load}}_{v,l} \qquad \forall v \in \mathcal{V}, \forall s \in \mathcal{S} \tag{3.24}$$

$$q_{i,j,s} \leq z^{\text{load}}_{i,j,b,s} q^{\text{part}}_{i,j,s} + \overline{Q}(1 - y^{\text{load}}_{i,j,b,s}) \qquad \forall (i,j) \in \mathcal{E}, \forall b \in \mathcal{B}, \forall s \in \mathcal{S} \tag{3.25}$$

$$q_{i,j,s} \geq z^{\text{load}}_{i,j,b,s} q^{\text{part}}_{i,j,s} - \overline{Q}(1 - y^{\text{load}}_{i,j,b,s}) \qquad \forall (i,j) \in \mathcal{E}, \forall b \in \mathcal{B}, \forall s \in \mathcal{S} \tag{3.26}$$

$$q^{\text{part}}_{i,j,s} - q_{i,j,s} \leq \overline{Q} \sum_{b\in\mathcal{B}} y^{\text{load}}_{i,j,b,s} \qquad \forall (i,j) \in \mathcal{E}, \forall s \in \mathcal{S} \tag{3.27}$$

$$q^{\text{part}}_{i,j,s} - q_{i,j,s} \geq -\overline{Q} \sum_{b\in\mathcal{B}} y^{\text{load}}_{i,j,b,s} \qquad \forall (i,j) \in \mathcal{E}, \forall s \in \mathcal{S} \tag{3.28}$$

$$q^{\text{part}}_{i,j,s} \leq \alpha^{\text{q},1}_b + \beta^{\text{q},1}_b \Delta h_{i,j,b,s} + (1 - y^{\text{load}}_{i,j,b,s})\overline{Q} \qquad \forall (i,j) \in \mathcal{E}, \forall b \in \mathcal{B}, \forall s \in \mathcal{S} \tag{3.29}$$

$$q^{\text{part}}_{i,j,s} \geq \alpha^{\text{q},2}_b + \beta^{\text{q},2}_b \Delta h_{i,j,b,s} - (1 - y^{\text{load}}_{i,j,b,s})\overline{Q} \qquad \forall (i,j) \in \mathcal{E}, \forall b \in \mathcal{B}, \forall s \in \mathcal{S} \tag{3.30}$$

$$\begin{aligned}(h_{j,s} - \sum_{b\in\mathcal{B}} \Delta h_{i,j,b,s} + \Delta H_{i,j} - h_{i,s})& \\ +B(1 - y^{\text{pipe}}_{i,j}) \geq 0& \qquad \forall (i,j) \in \mathcal{E}, \forall s \in \mathcal{S}\end{aligned} \tag{3.31}$$

$$\begin{aligned}(h_{j,s} - \sum_{b\in\mathcal{B}} \Delta h_{i,j,b,s} + \Delta H_{i,j} - h_{i,s})& \\ -B(1 - y^{\text{pipe}}_{i,j}) \leq 0& \qquad \forall (i,j) \in \mathcal{E}, \forall s \in \mathcal{S}\end{aligned} \tag{3.32}$$

$$\begin{aligned}p^{\text{part}}_{i,j,b,s} \geq \alpha^{\text{P}}_b (q^{\text{part}}_{i,j,s})^3 + \beta^{\text{P}}_b (q^{\text{part}}_{i,j,s})^2 \omega_{+,i,j,b,s}& \\ +\gamma^{\text{P}}_b q^{\text{part}}_{i,j,s} \omega^2_{+,i,j,b,s} + \delta^{\text{P}}_b \omega^3_{+,i,j,b,s}& \qquad \forall (i,j) \in \mathcal{E}, \forall b \in \mathcal{B}, \forall s \in \mathcal{S}\end{aligned} \tag{3.33}$$

$$p_{i,j,b,s} \geq z^{\text{load}}_{i,j,b,s} p^{\text{part}}_{i,j,b,s} \qquad \forall (i,j) \in \mathcal{E}, \forall b \in \mathcal{B}, \forall s \in \mathcal{S} \tag{3.34}$$

$$\begin{aligned}\Delta h_{i,j,b,s} = y^{\text{load}}_{i,j,b,s} \alpha^{\text{H}}_b (q^{\text{part}}_{i,j,s})^2& \\ +\beta^{\text{H}}_b q^{\text{part}}_{i,j,s} \omega_{+,i,j,b,s}& \\ +\gamma^{\text{H}}_b \omega^2_{+,i,j,b,s}& \qquad \forall (i,j) \in \mathcal{E}, \forall b \in \mathcal{B}, \forall s \in \mathcal{S}\end{aligned} \tag{3.35}$$

Table 3.4 – Cost model parameters, cf. [6, 101]

α_1	α_2	α_3	α_4	α_5	α_6
-0.952	-0.00853	1.135	84.699	5.542	225.387

Table 3.5 – Statistical indicators based on 40 different evaluations, cf. [101].

	mean	std	min	25%	50%	75%	max
objective	41056.39	24110.50	23391.09	26060.80	30905.98	40178.98	112464.52
time	461.48	836.86	0.31	2.84	118.60	510.55	4311.16

std — standard deviation; min — minimum; max — maximum

in Table 3.3 are used as a reference for the pump construction kit.
Within Equation 3.7 the given parameter C_b^{pump} describes the investment cost of a specific pump. Since this cost is only available for pumps already available on the market, it is required to derive a cost estimation model for the applicability of the scaling approach. The cost model

$$C_b^{\text{pump}} = \alpha_1(\overline{q}^{\text{pump}})^2 + \alpha_2(\Delta\overline{h}^{\text{pump}})^2 + \alpha_3\overline{q}^{\text{pump}}\Delta\overline{h}^{\text{pump}} + \alpha_4\overline{q}^{\text{pump}} + \alpha_5\Delta\overline{h}^{\text{pump}} + \alpha_6 \tag{3.36}$$

was derived from a set of 33 available pumps of a pump series for a highrise water supply with given impeller diameters and number of stages.
The according parameters that were derived by using a linear least-square approach, cf. Section 2.2.2, are given in Table 3.4. As features the maximum flow $\overline{q}^{\text{pump}}$ and the maximum pressure increase $\Delta\overline{h}^{\text{pump}}$ were used, since they can describe the scaling by impeller size and stage number. The data fitting resulted in an adjusted R-squared value of 0.988 for the given data set. Further information is given in the corresponding publications[52,53].

3.2.2 Results

A computational study was conducted with the newly developed model given in Equations (3.7) to (3.35). For this study we use $d_{+,b} = d_b/d_{\mathrm{M},b}$ to describe the ratio between the derived diameter and the original diameter given by the reference pump in Table 3.3. A detailed description of the used parameters can be found in the Appendix A.

[52] Leise and Altherr, "Optimizing the Design and Control of Decentralized Water Supply Systems – A Case-Study of a Hotel Building", ([101], 2018)

[53] Altherr, Leise, Pfetsch, and Schmitt, "Resilient layout, design and operation of energy-efficient water distribution networks for high-rise buildings using MINLP", ([6], 2019)

Table 3.6 – Three best computational results, cf. [101]

#	$d_{+,1}$	$d_{+,2}$	$d_{+,3}$	$d_{+,4}$	objective value	solution time in s
1	0.89	1.27	1.08	1.01	23391.09	257.34
2	0.78	0.93	0.89	1.04	23931.28	974.47
3	0.67	0.86	1.68	0.89	25607.67	3.16

Since the scaling uses the relations between the diameters, the true diameters of the reference systems are not relevant in an absolute manner. Furthermore, we select only a varying impeller diameter while taking the relation between the number of stages constant. Table 3.5 shows a summary derived from 40 evaluations with differently scaled pumps. It can be seen that the objective varies significantly, depending on the underlying set of selected pumps $\mathcal{B}$. Hence, the selection of the construction kit plays an important role, when designing efficient systems. Next to the objective value, Table 3.5 also shows a summary of the solving time for all 40 instances. Here, it can be seen that the solving time also differs significantly within this problem class, despite the fact that all problems only differ in their selected set of pumps. The topological freedom by choosing different pipe layouts can compensate the effects of sets with inefficiently sized pumps for a specific part. But due to the high variability of the objective it is essential to evaluate multiple set of pumps for the construction kit properly to enable an efficient system design. This is exemplified by the difference between the maximum and minimum value in Table 3.5. The worst system design results in more than four times higher lifetime costs than the best system.
The three best sets by its objective value for $\mathcal{B}$ are shown in Table 3.6. The objective value between the best and second solution is almost equal, which exemplifies that multiple almost equally good solutions exist in the design space. Nevertheless, the computational time varies significantly, as can be seen for the three best solutions as well as within the summary data in Table 3.5. Even if a specified construction kit $\mathcal{B}$ is defined for each computational result with four differently sized pumps the optimally computed solution does not necessarily make use of all available pumps. In fact, it usually consists of only a few different pumps.
For the given evaluations the best solution in Table 3.6 uses only four times the pump $b = 4$ with an almost identical size as the reference pump ($d_{+,4} = 1.01$). As a topology two parallel pumps are combined in one booster station for to adjacent pressure zones, which consist of three floors each. This system design is shown in Figure 3.5.

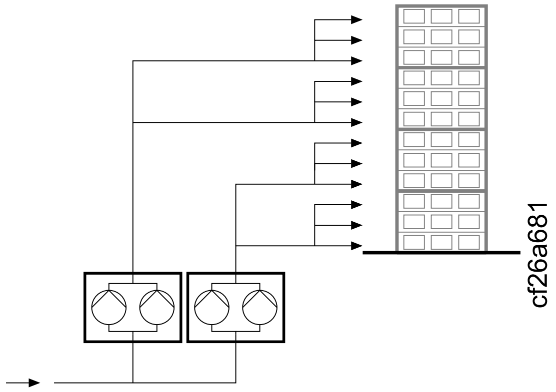

Figure 3.5 – Outlined topology result for the best found solution, cf. [101]

3.3 Extensions

3.3.1 Detailed Modeling

Based on this nonlinear and nonconvex MINLP multiple extended models with specific solution approaches were developed within the CRC 805 and at the Chair of Fluid Systems. Two specific extensions[54,55] will be presented in the following.
To increase the computational performance of this MINLP, a partially improved model was derived by Altherr, Leise, Pfetsch, and Schmitt[56]. Besides computational improvements through a problem-specific algorithm development, the model was extended from an engineering point of view by a more detailed pipe modeling. Here, different discrete pipe diameters were introduced, which are commonly used as shown by DIN 1988-300[57]. The additional set $\mathcal{D} = \{10, 13, 16, 19.6, 25.6, 32, 39, 51, 60, 72.1, 84.9, 104\}$ measured in mm is introduced[54]. One of these discrete diameters has to be chosen for each arc in the network. The investment costs in the objective for pipes are then

54 Altherr, Leise, Pfetsch, and Schmitt, "Resilient layout, design and operation of energy-efficient water distribution networks for high-rise buildings using MINLP", ([6], 2019)

55 Müller, Leise, Lorenz, Altherr, and Pelz, "Optimization and Validation of Pumping System Design and Operation for Water Supply in High-Rise Buildings", ([131], 2020)

56 An alphabetic ordering of authors was chosen for this publication, cf. [6]

57 DIN 1988-300, *Codes of practice for drinking water installations – Part 300: Pipe sizing; DVGW code of practice*, ([41], 2012)

estimated by using the generally used nonlinear approach[58,59,60] $C^{\text{pipe}}_{i,j} = \gamma L_a d^\delta$ with the problem specific individually derived coefficients $\gamma = 3593$ €/m and $\delta = 1.6975$, cf. [6]. Here, a nonlinear least-squares approach is chosen, see Section 2.2.2. Furthermore, the pressure loss in pipes is modelled with the help of the Darcy-Weisbach equation[61]

$$\Delta F(d, q, L) = \lambda_{\text{h}} \frac{1}{d^5} \frac{8}{\pi^2} \frac{q^2}{g} L.$$

Here, λ is the friction coefficient and g the standard gravity acceleration. In the considered use case for a water distribution system in high-rise buildings by Altherr, Leise, Pfetsch, and Schmitt[62], we have maximum volume flows between $25\,\text{m}^3/\text{h}$ and $35\,\text{m}^3/\text{h}$ and pipe diameters between 0.01 m and 0.1 m. These values result in Reynolds numbers that indicate a turbulent flow. Hence, we use the assumption that the flow conditions are turbulent.
For the modelling of the friction coefficient λ_{h} in pipes multiple modelling approaches exist[63]. For an integration in the optimization model it is beneficial, if the friction coefficient is independent of the volume flow on the considered arc, since the volume flow is set within optimization. If this holds, the friction coefficient does not have to be modeled as an additional constraint explicitly, but it can be integrated implicitly. One friction coefficient model that is independent of the volume flow is the friction law of Nikuradse, Prandtl and von Kármán, cf. Brkić[63], for a hydraulically rough pipe[62]:

$$\lambda_{\text{h}} = \frac{1}{(2\log_{10}(3.71\frac{d}{K}))^2}. \tag{3.37}$$

For the pipe wall roughness the parameter $K = 0.0015$ mm is used[54], which represents stainless steel pipes[64]. Besides this more detailed modelling of pipes

58 Fujiwara and Khang, "A two-phase Decomposition Method for Optimal Design of Looped Water Distribution Networks", ([60], 1990)

59 Savic and Walters, "Genetic Algorithms for Least-Cost Design of Water Distribution Networks", ([162], 1997)

60 Bieupoude, Azoumah, and Neveu, "Optimization of Drinking Water Distribution Networks: Computer-based Methods and Constructal Design", ([16], 2012)

61 Brown, "The History of the Darcy-Weisbach Equation for Pipe Flow Resistance", ([26], 2003)

62 Altherr, Leise, Pfetsch, and Schmitt, "Resilient layout, design and operation of energy-efficient water distribution networks for high-rise buildings using MINLP", ([6], 2019)

63 Brkić, "Review of Explicit Approximations to the Colebrook Relation for Flow Friction", ([24], 2011)

64 DIN 1988-300, *Codes of practice for drinking water installations – Part 300: Pipe sizing; DVGW code of practice*, ([41], 2012)

a focus is set on the concurrent solution of the given optimization problem and the explicit consideration to derive resilient system designs. For more details on the solving algorithm, please refer to [6]. More details on the resilience consideration in the conceptual design are given in Section 3.3.3.

3.3.2 Selection of Pump Sets

In the MINLP model in Eq. (3.7) – (3.35), a scaling of multiple reference pumps is used to create a specific set of pumps $\mathcal{B}$ as a construction kit. This approach is suitable, if no further knowledge about the pumps besides a reference design is known.

Since the usage of a scaling approach for the pump size leads to pump designs that are not necessarily available on the market, another approach is the integration of a specific set of pumps as shown by Müller, Leise, Lorenz, Altherr, and Pelz[65].

In this work a computational study is performed. It is based on a discrete approach with a dedicated problem-specific heuristic to select a suitable subset of 15 pumps from a given series with 200 market-available pumps[66] that are used explicitly for the design of a water supply of high-rise buildings. Since the integration of the complete set of pumps would result in a significant increase in computational time, the heuristic shown in Algorithm 1 has been developed to enable a selection of a subset. The developed MINLP in the publication [131] differs slightly from the already introduced MINLP in the last subsection, since it focuses on the design of a booster station.

Nevertheless, the heuristic approach to select a subset of pumps can be integrated as a preprocessing step in multiple conceptual design processes for high-rise buildings which are based on a MINLP modeling.

The preselection heuristic considers domain-specific knowledge to derive the subset of pumps in the given series that are most suitable for the water supply. It is a new algorithmic approach that extends common manual problem-specific expert selection processes[67,68].

The heuristic is shown in Algorithm 1. It derives a subset $\mathcal{B}$ of all available pumps $\mathcal{B}^{\text{total}}$. As an input it uses multiple scenarios in the set $\mathcal{S}$ in their respective $\Delta h - q$ domain. As already introduced in general in Section 2.2.3 the concept of operation in the usage period is given by scenarios $s \in \mathcal{S}$ with

[65] Müller, Leise, Lorenz, Altherr, and Pelz, “Optimization and Validation of Pumping System Design and Operation for Water Supply in High-Rise Buildings”, ([131], 2020)

[66] Further information on the used dataset is given in the Appendix.

[67] Larralde and Ocampo, “Centrifugal Pump Selection Process”, ([95], 2010)

[68] Larralde and Ocampo, “Pump Selection: A Real Example”, ([96], 2010)

Algorithm 1: Heuristic Selection of Pumps, cf. [131]

input : original load scenarios in Δh–q domain $\mathcal{S}$
set of all available pumps $\mathcal{B}^{\text{total}}$
distance parameter $\overline{L}^{\text{pre}}$
set of desired fractional positions $\mathcal{F}$
output : selected pumps $\mathcal{B}$

```
Z ← ∅ // selected fractional loads in Δh–q domain
B ← ∅ // selected pumps
// step 1: select fractional loads
for i ← 1 to |S| do
    generate fractional loads for parallel pump usage based on F and
     add them to Z
for i ← 1 to |Z| do
    generate fractional loads for serial pump usage based on F and
     add them to Z
sort Z by power
while true do
    compute distance matrix value a_{i,j} between every point in Z
    if at least one a_{i,j} < L̄^pre ∀ i, j ∈ Z without i = j then
        remove point with a_{i,j} < L̄^pre and lowest possible power from Z
    else
        quit loop
// step 2: get best suitable pumps for Z
for i ← 1 to |Z| do
    for j ← 1 to |B^total| do
        select best suitable pump b_j ∈ B^total for z_i ∈ Z
        save best suitable pump in B if not already selected
```

specific values for the pressure Δh_s, the volume flow q_s and a probability of occurrence π_s.

Since these scenarios must be fulfilled within the optimized solution, it is required that the corresponding pumps must either fulfill these scenarios individually or in a set where pumps are connected in parallel or in series.

As already introduced, it is most efficient to consider only parallel connection of equivalent pumps with the same speed, the volume flow of each pump in this connection can be approximated with q/N if N is the number of pumps in this subgroup. The pressure increase Δh of a parallel connection is equivalent for each pump. Furthermore, if a subgroup consists of a series connection the volume flow q is equivalent in this subgroup, but the required pressure Δh can be divided by the number of considered pumps N.

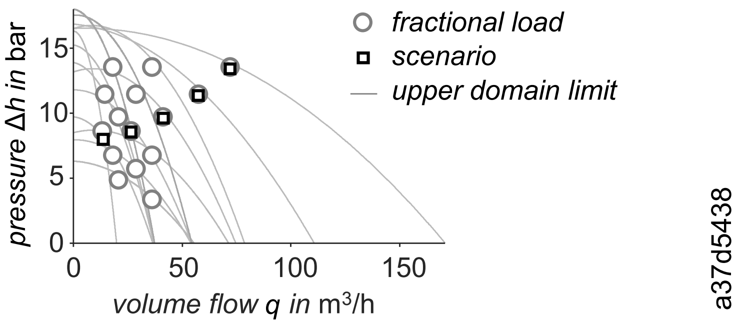

Figure 3.6 – Exemplified result of the newly derived heuristic for a preselection of pumps, cf. [131].

The heuristic is based on these two physical properties of a booster station. It uses a two-step process. In the first step a set of fractional loads is derived where each load represents the load of one pump in a parallel or series connection of pumps. Since multiple fractional loads can be placed in the $\Delta h - q$ domain close to each other, a distance parameter $\overline{L}^{\mathrm{pre}}$ is introduced. Only the load with the highest power within a given distance from each other load in the domain is stored for a later pump selection. The other loads are removed iteratively from the set of fractional loads $\mathcal{Z}$.
In the second step, the finally used pumps are selected based on their according characteristics: "A pump is selected, if it can fulfill the desired demand at these fractional loads and has the lowest possible maximum hydraulic power. In this manner we derive a final pump catalog that consists of a predefined number of pumps which are suitable to supply each original demand by single pumps or a combination of serial/parallel connected pumps. Additionally, each selected pump has the lowest power demand to supply the partial load it was selected for."[69]
This procedure reduces the given set of pumps considerably to the number that is set manually in advance of the optimization and which can be solved in an appropriate time. As shown in the corresponding publication, the performance of this heuristic is high for the considered use case, since it leads to solutions that are close to a hypothetically optimal performance[69].

[69] Müller, Leise, Lorenz, Altherr, and Pelz, "Optimization and Validation of Pumping System Design and Operation for Water Supply in High-Rise Buildings", ([131], 2020)

3.3.3 Resilience

Besides the already introduced conceptual approaches towards more resilient technical system designs in Sec. 2.3, another important metric that can be used to measure the resilience of a technical system is the *buffering capacity*[70] of type k, which we also call *k-resilience*[71]. This metric describes the ability of a system to still fulfill a predefined minimum functionality $f_{\min}$, if k components have a failure and cannot maintain their functionality they would provide under “normal” conditions. The benefit of this metric is the applicability within MINLP programs. For the WDS use case, we assume that only pumps are affected by failures and that the flow through an affected pump can be bypassed. With these assumptions, we are able to extend a reference MINLP model which is presented by Altherr, Leise, Pfetsch, and Schmitt[71] so that the resilience can also be considered while optimizing the conceptual system design. A detailed view on this metric and its application for designing resilient and efficient water distribution systems can be found in the joint publication[71] and in the dissertation of A. Schmitt[72].

3.3.4 Convexification

The already introduced MINLP is nonconvex due to the pump characteristics. One approach to derive a convex optimization program based on the given MINLP is the usage of a piecewise linearization for the nonlinear and nonconvex constraints, cf. Section 2.1.5. This approach then leads to a Mixed-Integer Linear Program (MILP) which can be solved with common state-of-the-art MILP solvers very efficiently.
Hence, it is an often used approach to derive optimized system designs in general and for water distribution system in particular[73,74]. This approach was also implemented and tested by Müller, Leise, Lorenz, Altherr, and Pelz for a high-rise WDS.

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70 Woods, “Essential Characteristics of Resilience”, ([202], 2017, p. 23)

71 Altherr, Leise, Pfetsch, and Schmitt, “Resilient layout, design and operation of energy-efficient water distribution networks for high-rise buildings using MINLP”, ([6], 2019)

72 Schmitt, “Mixed-Integer Nonlinear Programming for Resilient Water Network Design”, ([166], 2022)

73 Morsi, Geissler, and Martin, “Mixed Integer Optimization of Water Supply Networks”, ([129], 2012)

74 Rausch, Leise, Ederer, Altherr, and Pelz, “A Comparison of MILP and MINLP Solver Performance on the Example of a Drinking Water Supply System Design Problem”, ([157], 2016)

We were able to show that the solving performance is significantly improved comparable to a solution of the MINLP directly. Since the number of simplices and therefore the number of newly introduced variables significantly affects the performance, we selected the number of simplices by the ability of the optimized solution to approximate the MINLP result. As a threshold, we set a limit of 2% of the objective value which corresponds to the uncertainty that is measurable in physical experiments to validate the performance of the optimization approach.
The only possibility to improve the performance even more is the development of problem specific algorithms that exploit the problem structure and use domain-specific knowledge. Further details on the MILP approach and a specific solving algorithm as well as a systematic evaluation are shown in the respective publication by Müller, Leise, Lorenz, Altherr, and Pelz[75].
Besides this more common approach of PWL integration to convexify a nonconvex MINLP, a further generalizable approach will be shown in the next Chapter.

[75] Müller, Leise, Lorenz, Altherr, and Pelz, "Optimization and Validation of Pumping System Design and Operation for Water Supply in High-Rise Buildings", ([131], 2020)

Chapter 4

Powertrain Design

The transport sector is responsible for approximately 37% of global CO_2 emissions in the end-use sector[1]. Despite a decline of 10% in the CO_2 emissions within the transportation sector in 2020 caused by the Covid-19 pandemic, the emissions are rebounding again in 2021[2] and still remain on a high level.
This contradicts the required savings in CO_2 emissions to reduce the effects of climate change. Energy efficient system designs and an electrification of transport are fundamental to reach the required ambitious reductions in CO_2 emissions to mitigate climate change[2].
The worldwide share of battery electric vehicles (BEVs) increased rapidly within the last years[3,4]. Especially in Europe, which already became in 2021 the world's largest electric car market followed by China and the United States of America[3]. Besides passenger cars, other road transport modes, like busses or trucks were also continuously electrified[3].
As shown by the Intergovernmental Panel on Climate Change[5] (IPCC), one approach to derive a more "climate resilient development pathway"[5] is the development of environmentally sustainable and resilient technologies and infrastructures. One suitable approach here, is the fulfillment of the sustainable development goals that were already introduced by the United

[1] IEA, *IEA Transport*, ([82], 2021)

[2] IEA, *Tracking Transport 2021*, ([83], 2021)

[3] IEA, *Electric Vehicles*, ([81], 2021)

[4] Palmer, Tate, Wadud, and Nellthorp, "Total Cost of Ownership and Market Share for Hybrid and Electric Vehicles in the UK, US and Japan", ([142], 2018)

[5] Pörtner et al., "Climate Change 2022: Impacts, Adaptation, and Vulnerability", ([152], 2022)

Nations in 2015[6].
Besides the already presented approach in the previous Chapter to derive a sustainable water supply infrastructure, a second example use case is shown in this Chapter. The technological development in this Chapter strengthens the sustainable development goal *climate action*[7]. The focus lies on a conceptual design approach for an efficient powertrain design in BEVs, and it is applicable not only for road passenger vehicles, but also for road freight vehicles and can therefore have a significant impact.
The representation of the usage phase in an optimization program for conceptual design is always affected by uncertainty. This uncertainty is currently mastered within this problem class, by considering a specific driving cycle as a concept of operation. Examples are legislative standard driving cycles like for instance, the *Worldwide harmonized Light-duty vehicles Test Cycles*[8] (WLTC). Additional approaches to derive driving cycles specific for BEVs also exist in the literature[9].
Besides this common approach, the focus is set on a new approach, where the driving behavior is modeled as a stochastic variable, which is then approximated by scenarios, cf. Section 2.2.3. With this approach, it is possible to create models that are comparable in its structure to the model presented in the previous Chapter.
Some modeling approaches and results presented in this chapter have already been published in a peer-reviewed journal article[7] and in peer-reviewed proceedings[10,11].
The optimization program considers a powertrain that consists of a battery, a single electric machine (EM) with power electronics and a transmission which are all optimized towards energy efficiency together[7].
A first Mixed-Integer Nonlinear Program was presented by Leise, Altherr, Simon, and Pelz[10]. This program was then extended, and the results were verified by comparison with the solution of a comparable physical model that

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6 United Nations Development Program, *Sustainable Development Goals*, ([187], 2015)

7 Leise et al., "Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods", ([106], 2021)

8 Tutuianu et al., *Development of a World-wide Worldwide harmonized Light duty driving Test Cycle (WLTC)*, ([183], 2013)

9 Pfriem and Gauterin, "Development of Real-World Driving Cycles for Battery Electric Vehicles", ([151], 2016)

10 Leise, Altherr, Simon, and Pelz, "Finding Global-Optimal Gearbox Designs for Battery Electric Vehicles", ([103], 2019)

11 Leise, Simon, and Altherr, "Comparison of Piecewise Linearization Techniques to Model Electric Motor Efficiency Maps: A Computational Study", ([109], 2020)

was solved with a genetic algorithm[7]. As shown previously, the application of a piecewise linear approximation of the characteristic diagrams of the pumps leads to an efficient solving approach for the water distribution system design problem, cf. Chapter 3. This approach was transferred to the powertrain MINLP, where the highly nonlinear EM efficiency map was approximated in a piecewise linear manner[12]. This approach leads to a reduction in the computational time to derive solutions compared to solving the original MINLP, but the usage of an outer polyhedral approximation that does not require the integration of binary variables[13] in the modeling resulted in the considered (nonconvex) use case in a better scalable approach.
An extension of the solution approach for the nonconvex MINLP, shown in these publications, is presented in this Chapter as well. It exploits the specific structure of the MINLP by transforming the problem in a MINLP with convex subproblems and applying a generalized Benders decomposition. This solution approach also enables the derivation of a specific class of heuristics that can lead to near-optimal solutions in a fraction of the time that is required to find and prove the global optimality. The efficiency and correctness of the newly developed modeling and solution approach is shown by a comparison to an efficient brute force approach that omits the computation of mathematically symmetric solutions. The shown modeling and solution approach is not only applicable to the powertrain conceptual design problem, but is also suitable for a high number of conceptual design problems. An outlook on the transferability of the derived approach will be shown in Chapter 6.

4.1 Related Work

The optimization of a powertrain for hybrid-electric vehicles and battery electric vehicles is a major research field in mechanical engineering[14,15]. As a foundation for the modeling and optimization of BEVs, common approaches from literature for the vehicle system modeling can be used to model the

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[12] Leise, Simon, and Altherr, "Comparison of Piecewise Linearization Techniques to Model Electric Motor Efficiency Maps: A Computational Study", ([109], 2020)

[13] Leise et al., "Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods", ([106], 2021)

[14] Chan, Bouscayrol, and Chen, "Electric, Hybrid, and Fuel-cell Vehicles: Architectures and Modeling", ([31], 2009)

[15] Tran et al., "Thorough State-of-the-Art Analysis of Electric and Hybrid Vehicle Powertrains: Topologies and integrated Energy Management Strategies", ([182], 2020)

fundamental physical properties[16,17].
As already shown in Chapter 3 for the water distribution system, the combined consideration of the system design (by selecting specific components either by scaling or from a discrete set) and the usage phase (by having a concept of operation) leads to more sustainable system designs. This is also true for the powertrain design[18]. Hence, it results in an optimization program that represents a combination of a component design and a corresponding control problem.
The research focus in the literature was set primarily on powertrains for hybrid electric vehicles, as shown by numerous publications[19,20,21,22,23]. Besides passenger cars, also hybrid-electric busses were considered[24]. In recent years, with an increasing importance of BEVs, further research publications were presented for the powertrain design of battery electric vehicles as well[25,26].
As shown previously, a common basis to derive a concept of operation for the usage-phase is the consideration of representative driving cycles. These cycles approximate a specific real driving dataset and are commonly derived

[16] Mitschke and Wallentowitz, *Dynamik der Kraftfahrzeuge*, ([127], 2014)

[17] Guzzella, Sciarretta, et al., *Vehicle Propulsion Systems*, ([71], 2007)

[18] Silvas, Hofman, Murgovski, Etman, and Steinbuch, “Review of Optimization Strategies for System-Level Design in Hybrid Electric Vehicles”, ([172], 2016)

[19] Salmasi, “Control Strategies for Hybrid Electric Vehicles: Evolution, Classification, Comparison, and Future Trends”, ([161], 2007)

[20] Sinoquet, Rousseau, and Milhau, “Design Optimization and Optimal Control for Hybrid Vehicles”, ([173], 2011)

[21] Wu, Cao, Li, Xu, and Ren, “Component Sizing Optimization of Plug-in Hybrid Electric Vehicles”, ([204], 2011)

[22] Hu, Zou, and Yang, “Greener Plug-in Hybrid Electric Vehicles incorporating Renewable Energy and Rapid System Optimization”, ([78], 2016)

[23] Qin et al., “Simultaneous Optimization of Topology, Control and Size for Multi-mode Hybrid Tracked Vehicles”, ([155], 2018)

[24] Xu, Li, Hua, Li, and Ouyang, “Optimal Vehicle Control Strategy of a Fuel Cell/Battery Hybrid City Bus”, ([205], 2009)

[25] Grunditz and Thiringer, “Characterizing BEV Powertrain Energy Consumption, Efficiency, and Range during Official and Drive Cycles from Gothenburg, Sweden”, ([68], 2016)

[26] Schönknecht, Babik, and Rill, “Electric Powertrain System Design of BEV and HEV applying a Multi Objective Optimization Methodology”, ([167], 2016)

by using a stochastic synthesis process[27,28,29,30].
In the following, instead of using a given driving cycle directly, a new approach that is based on a scenario generation in an additional preprocessing step, as published by Leise et al.[31], is introduced. Here, it could be shown that the scenario-based approach can lead to results comparable to a classic optimization approach, where the driving cycles are used directly within the optimization program, if the same underlying datasets are used as an input. This was evaluated for a variety of legislative standard driving cycles.
Besides this, the advantage of this new approach is the ability to integrate arbitrary datasets. One related approach that also considers scenarios for a powertrain design is given by Caillard, Gillon, Hecquet, Randi, and Janiaud[32]. In comparison to this publication, a universally applicable pre-processing heuristic to derive relevant scenarios automatically is presented in this thesis, cf. [103, 106]. This heuristic can be used for any optimization approach for the conceptual design of powertrains in a preprocessing step and is not only specific to an exact optimization. It will be described in more detail in the following sections. For the optimal calibration of combustion engines, a scenario-based approach was also shown by Wasserburger, Hametner, and Didcock[33].
The usage of a multi-speed transmission increases the energy-efficiency of the whole powertrain[34]. Commonly the transmission is often, due to its complexity increase in solving, not modeled explicitly in the optimization programs.
In contrast within this thesis, the design of the multi-speed transmission is explicitly integrated in each optimization program. This explicit consideration leads to additional binary decisions in the model representation of the usage

[27] Souffran, Miègeville, and Guérin, "Simulation of Real-World Vehicle Missions using a Stochastic Markov Model for Optimal Powertrain Sizing", ([175], 2012)

[28] Silvas, Hereijgers, Peng, Hofman, and Steinbuch, "Synthesis of Realistic Driving Cycles With High Accuracy and Computational Speed, Including Slope Information", ([171], 2016)

[29] Esser, Zeller, Foulard, and Rinderknecht, "Stochastic Synthesis of Representative and Multidimensional Driving Cycles", ([52], 2018)

[30] Leise et al., "Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods", ([106], 2021)

[31] Leise et al., "Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods", ([106], 2021)

[32] Caillard, Gillon, Hecquet, Randi, and Janiaud, "An Optimization Methodology to Pre Design an Electric Vehicle Powertrain", ([29], 2014)

[33] Wasserburger, Hametner, and Didcock, "Risk-averse Real Driving Emissions Optimization considering Stochastic Influences", ([193], 2020)

[34] Rinderknecht and Meier, "Electric Power Train Configurations and their Transmission Systems", ([159], 2010)

phase. Here, the program has to evaluate, which discrete transmission should be used. This leads to the appropriate control strategy. Additionally, the powertrain components also have to be sized, which is done by continuous decision variables. These binary decisions for the control strategy result in a higher complexity compared to only sizing the powertrain based on continuous variables without an explicit consideration of a transmission or by simplifying the transmission design. Within the literature the explicit consideration of multi-speed transmissions were shown in some publications[35,36,37].
The resulting optimization program is highly nonlinear and results in a MINLP, when considering a multi-speed transmission. If no transmission is considered explicitly in the program, it results in a NLP.
Due to the nonlinearity (and usually nonconvexity) of the underlying optimization program, the NLP or MINLP are mostly solved by one or multiple primal heuristics. The most used approaches are a genetic algorithm[38,39,40,41], an evolutionary algorithm[42] or a particle swarm algorithm[43,44,45]. The disadvantage of these methods compared to exact optimization approaches are the unavailability of dual bounds and the possibility to find only a local optimum. On the other hand they usually result in faster solution times compared to exact solvers which compute until they find the provable global optimal

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[35] Wu, Zhang, and Dong, *Impacts of Two-Speed Gearbox on Electric Vehicle's Fuel Economy and Performance*, ([203], 2013)

[36] Morozov, Humphries, Zou, Martins, and Angeles, "Design and Optimization of a Drivetrain with Two-Speed Transmission for Electric Delivery Step Van", ([128], 2014)

[37] Tan, Yang, Zhao, Hai, and Zhang, "Gear Ratio Optimization of a Multi-Speed Transmission for Electric Dump Truck operating on the Structure Route", ([179], 2018)

[38] Li, Chang, Wang, and Wei, "Multi-objective Optimization Design of Gear Reducer based on Adaptive Genetic Algorithm", ([110], 2008)

[39] Fang, Qin, Xu, Li, and Zhu, "Simultaneous Optimization for Hybrid Electric Vehicle Parameters based on Multi-objective Genetic Algorithms", ([53], 2011)

[40] Schleiffer and Rinderknecht, *BEREIT: Schlussbericht Verbundvorhaben Bezahlbare Elektrische REIchweite durch ModularitäT: TP4 Entwurf und Simulation (Betriebsstrategie)*, ([165], 2017)

[41] Esser, Schleiffer, Eichenlaub, and Rinderknecht, "Development of an Optimization Framework for the Comparative Evaluation of the Ecoimpact of Powertrain Concepts", ([51], 2019)

[42] Deb and Jain, "Multi-speed Gearbox Design using Multi-objective Evolutionary Algorithms", ([40], 2003)

[43] Savsani, Rao, and Vakharia, "Optimal Weight Design of a Gear Train using Particle Swarm Optimization and Simulated Annealing Algorithms", ([163], 2010)

[44] Chen, Hung, Wu, and Huang, "Optimal Energy Management of a Hybrid Electric Powertrain System using Improved Particle Swarm Optimization", ([32], 2015)

[45] Zhou et al., "Intelligent Sizing of a Series Hybrid Electric Power-Train System based on Chaos-enhanced Accelerated Particle Swarm Optimization", ([209], 2017)

solution.
The usage of exact optimization approaches for the given powertrain design problem is currently still an open research field. One recent MINLP approach only focussed on the explicit design of the multi-speed gearboxes solely[46], but did not consider the complete powertrain.
Another recent research direction within powertrain conceptual design is the development of convex optimization programs, which can then be solved very efficiently. Tate and Boyd[47] presented an early approach to derive limits for the performance of hybrid electric powertrains by utilizing a piecewise linearization of the complete NLP without considering the design of a multi-speed transmission explicitly. Further approaches, primarily for hybrid-electric vehicles without sizing a multi-speed transmission were shown by Murgovski, Johannesson, Sjöberg, and Egardt[48], Murgovski, Johannesson, and Sjöberg[49], Egardt, Murgovski, Pourabdollah, and Mardh[50] and Hu, Zou, and Yang[51].
Another more recent convex optimization approach was presented by Verbruggen, Salazar, Pavone, and Hofman[52], where the resulting NLP to derive a joint sizing and control problem for the powertrain conceptual design of BEVs was cast as a second-order conic program. It focussed only on a fixed-gear transmission (FGT) and a continuous variable transmission (CVT) but did not consider a multi-speed transmission. In following publications by members of this research group, this approach was evaluated for different use cases and extended to multi-speed transmissions with the help of heuristics[53,54].
In the following, a new extended approach for the joint optimization of the

[46] Dörig, Ederer, Pelz, Pfetsch, and Wolf, "Gearbox Design via Mixed-Integer Programming", ([43], 2016)

[47] Tate and Boyd, "Finding Ultimate Limits of Performance for Hybrid Electric Vehicles", ([180], 2000)

[48] Murgovski, Johannesson, Sjöberg, and Egardt, "Component Sizing of a Plug-in Hybrid Electric Powertrain via Convex Optimization", ([135], 2012)

[49] Murgovski, Johannesson, and Sjöberg, "Engine On/Off Control for Dimensioning Hybrid Electric Powertrains via Convex Optimization", ([136], 2013)

[50] Egardt, Murgovski, Pourabdollah, and Mardh, "Electromobility Studies based on Convex Optimization: Design and Control Issues egarding Vehicle Electrification", ([48], 2014)

[51] Hu, Zou, and Yang, "Greener Plug-in Hybrid Electric Vehicles incorporating Renewable Energy and Rapid System Optimization", ([78], 2016)

[52] Verbruggen, Salazar, Pavone, and Hofman, "Joint Design and Control of Electric Vehicle Propulsion Systems", ([190], 2020)

[53] Hurk, "Optimal Design and Control of Electric Vehicle Transmissions", ([79], 2021)

[54] Duhr et al., "Time-optimal Gearshift and Energy Management Strategies for a Hybrid Electric Race Car", ([45], 2021)

sizing and control of a powertrain for BEVs is presented[55]. It decomposes the resulting nonconvex MINLP that also explicitly considers multi-speed transmissions in a generalized Benders decomposition in a MIP and a convex NLP that are solved iteratively. The resulting decomposed programs can be solved very efficiently and lead to the global optimal solution.
The modeling and solution approaches shown in this Chapter extend the literature within the field of research. Furthermore, the developed modeling and solution approach is a generally applicable optimization approach that is suitable for a multitude of engineering conceptual design problems. The transferability of the developed approach will be shown in Chapter 5. In the following section, we present the general modeling approach and its implication on the solution strategy.

4.2 MINLP Model

The following model description is based on a peer-reviewed journal publication by Leise et al.[56]. Furthermore, parts of this section were published in a preprint[57]. The concept of operation used to describe the usage-period is given by the following representation that is based on three discrete time-series which are aggregated in the usage phase representation Λ:

$$\Lambda = \begin{bmatrix} \mathbf{v}^T \\ \mathbf{s}^T \\ \boldsymbol{\pi}^T \end{bmatrix} \in \mathbb{R}^{3\times l}. \tag{4.1}$$

The first time-series $\mathbf{v}$ describes velocity values and the second time-series $\mathbf{s}$ represents the slope value for each discrete time-step t. These two time-series are usually used within an optimization framework, as already introduced in the previous section, to derive optimized powertrain designs[58]. This common approach is extended by a third time-series of weights $\pi_s,\ s \in (1,\dots,l)$ for each time step. If the uniform weights $\pi_s = 1/l,\ s \in (1,\dots,l)$ are used, this would result in a description that is comparable to the common procedure when using solely a complete driving cycle. This representation of the usage

55 Leise and Pelz, *Efficient Powertrain Design – A Mixed-Integer Geometric Programming Approach*, ([108], 2021)

56 Leise et al., "Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods", ([106], 2021)

57 Leise and Pelz, *Efficient Powertrain Design – A Mixed-Integer Geometric Programming Approach*, ([108], 2021)

58 Silvas, Hofman, Murgovski, Etman, and Steinbuch, "Review of Optimization Strategies for System-Level Design in Hybrid Electric Vehicles", ([172], 2016)

phase is comparable to the approach already introduced in Eq. (3.1) for the water distribution system.
The explicit consideration of weights π_s allows to master the uncertainty present in modeling the usage-period in an optimization program. The general driving conditions that a vehicle is facing within its usage-period are highly uncertain and depend on multiple influencing factors, like geographic conditions and/or individual preferences. Therefore, a driving cycle, as it is commonly used in optimization programs for the conceptual design of powertrains, is only an estimate that approximates the underlying probability density function which describes the driving behavior.
In Section 2.1.2, this stochastic view on optimization problems was already introduced and in Section 2.2.3 it was shown why and how to derive meaningful scenarios to generate a finite event set approximation of a stochastic program with recourse. The previously shown representation in Eq. (4.1) can also be seen as a set of scenarios with the according weights π_s. With this approach, we are able to model the drive conditions a BEV is experiencing in its usage-period by a set of scenarios that approximate the underlying probability density function of the uncertain driving condition. This approach is useful due to multiple reasons:

- Most time steps in the driving cycle do not affect the optimized solution.
- It avoids overfitting due to a specific driving cycle.
- It enables a faster solution of the optimization program due to its reduced complexity.

Nevertheless, the results obtained by this approach should be comparable to results that are obtained when using a complete driving cycle as the representation of the underlying probability density function. This comparability is shown in the corresponding publication[59] by verifying this new approach with a common optimization approach, which is based on the driving cycle representation solely and which uses a genetic algorithm.

4.2.1 Vehicle

A quasi-static modeling approach is used, based on a backwards dynamic longitudinal vehicle model[60]. It is used to derive the torque and speed

[59] Leise et al., "Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods", ([106], 2021)

[60] Tran et al., "Thorough State-of-the-Art Analysis of Electric and Hybrid Vehicle Powertrains: Topologies and integrated Energy Management Strategies", ([182], 2020)

Table 4.1 – Vehicle parameters, cf. [106]

PARAMETER	VALUE	UNIT	DESCRIPTION
A_c	2.2	m^2	vehicle reference area
c_w	0.3	–	drag coefficient
g	9.81	m/s^2	specific gravitational constant
M^A	70	kg	additional component mass
M^P	75	kg	mass of the considered driver
M^0	1150	kg	vehicle mass
r^W	0.3	m	wheel radius
v	based on dataset	m/s	vehicle speed
$\dot{v}$	based on dataset	m/s^2	vehicle acceleration
s	0	–	terrain slope
η^B	0.97	–	efficiency of battery (dis)charging
η^G	$\in [0.98, 0.975]$	–	efficiency of transmission; first value for one-speed; second value for two-speed
λ_i	1.0	–	inertia consideration factor
λ_r	0.008	–	rolling resistance coefficient
ϱ	1.2041	kg/m^3	air density at 20 °C and sea level
$\overline{\Omega}^M$	10000	1/min	maximum speed of the selected EM

Note: EM – electric machine.

requirements at the wheel. With these values, it is possible to set up the combined sizing and control problem to find an optimal assembly of efficiently sized components. The set $\mathcal{S}$ is used to describe the discrete scenarios which are used in the concept of operation. In the scenario-based approach this set is significantly smaller than a complete representation of a driving cycle. All parameter definitions and values for the longitudinal vehicle model are given in Table 4.1.
The longitudinal vehicle dynamics is modeled as[61]

$$t^{W} = \left(\lambda_{i}\, m\dot{v} + mg\left(\sin\left(s\right) + \lambda_{r}\right) + \frac{1}{2}\varrho c_{w} A_{c} v^{2}\right) r^{W}. \tag{4.2}$$

Equation 4.2 describes the torque at the wheels t^W. The effect of rotational inertias in Equation 4.2 was omitted by setting $\lambda_i = 1$, since they only result in second-order effects. Additionally, they would increase the computational effort significantly and are usually not considered in a conceptual design study[62].
The mass of the BEV is modeled as a sum of the battery mass m^B, the predefined net mass M^0, the additional mass for powertrain components M^A, and the passenger mass M^P[62]:

$$m = m^{B} + M^{0} + M^{A} + M^{P}. \tag{4.3}$$

[61] Mitschke and Wallentowitz, *Dynamik der Kraftfahrzeuge*, ([127], 2014, pp. 83)

[62] Leise et al., “Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods”, ([106], 2021)

For a verification of the scenario-based modeling approach, an exemplary value for a long-range capable vehicle with a range $R = 500\,\mathrm{km}$ is used. Besides this, it is required that the optimized system is able to fulfill a minimum required speed of $160\,\mathrm{km/h}$. Additionally, the vehicle should also fulfill a specific launch torque at low speed, which was integrated with an exemplified gradeability on a slope of 30°. These two restrictions are added in the optimization program as additional scenarios with a weight $\pi_i = 0$. As a result, they are added as constraints in the optimization program, but do not affect the objective of the optimized solution.

4.2.2 Battery

The battery capacity c^{B} and mass m^{B} are modelled with the energy density ρ^{e} based on an approach developed by Esser[63] and Zimmerling[64]:

$$c^{\mathrm{B}} = m^{\mathrm{B}} \rho^{\mathrm{e}}, \tag{4.4}$$

$$\rho^{\mathrm{e}} = 0.0008/\mathrm{kg}\, c^{\mathrm{B}} + 0.0788\,\mathrm{kWh/kg}. \tag{4.5}$$

Within the model, we also consider a (dis-)charging efficiency η^{B} of 97% and an useable depth of discharge of 95%[65].

4.2.3 Electric Machine

We use as a reference electric machine (EM) a permanent-magnet synchronous motor, which was developed by An and Binder[66], to demonstrate the practical applicability of the derived methodology. The characteristics of the motor are given by its efficiency map $\mathcal{M}$, which describes the relation of the efficiency η^{M} and the motor torque t^{M} and speed ω^{M}, $\eta^{\mathrm{M}} = \eta^{\mathrm{M}}\left(t^{\mathrm{M}}, \omega^{\mathrm{M}}\right) : \mathbb{R}^2 \to \mathbb{R}$. It is shown in Figure 4.1. The sub-figure (a) shows the characteristics for the first quadrant (driving mode) and (b) shows the characteristics for the fourth quadrant (recuperation).
Within the optimization program it is required to scale the motor based on the concept of operation and the selection of the other components in the

63 Esser, “Realfahrtbasierte Bewertung des ökologischen Potentials von Fahrzeugantriebskonzepten”, ([50], 2021)

64 Zimmerling, *Erweiterung einer Optimierungsumgebung zur vergleichenden Bewertung von Antriebskonzepten*, ([210], 2020)

65 Leise et al., “Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods”, ([106], 2021)

66 An and Binder, “Operation Strategy with Thermal Management of E-Machines in Pure Electric Driving Mode for Twin-Drive-Transmission (DE-REX)”, ([7], 2017)

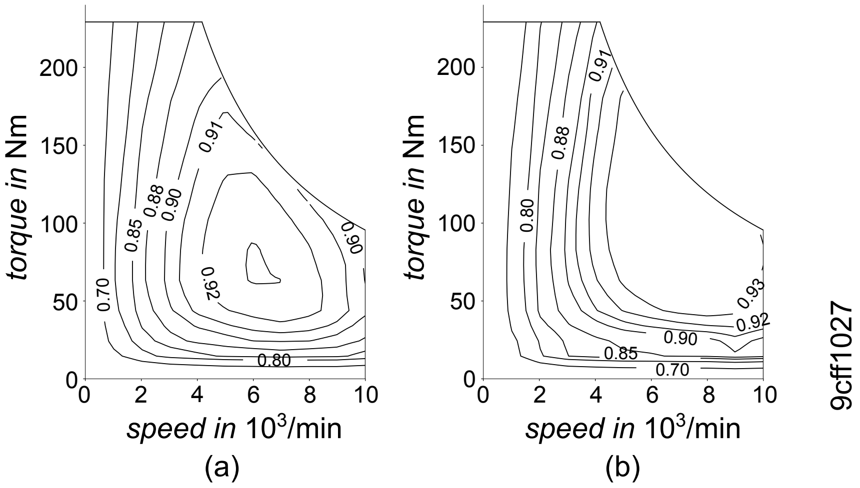

Figure 4.1 – Efficiency map of the used reference electric machine. Given by [7, 8]. (a) first quadrant / driving mode, (b) fourth quadrant / recuperation mode

powertrain. This scaling is done according to the literature[67,68] by scaling the maximum torque $\overline{t}^{\mathrm{M}}$ and the reference efficiency map $\mathcal{M}$. The upper speed limit of the motor $\overline{\Omega}^{\mathrm{M}}$ remains the same for all scaled versions. The scaling of the mass of the motor is omitted due to its small fraction in comparison to the overall system mass.
The electrical power p^{M} required in each scenario s is computed in driving with:

$$p^{\mathrm{M}} = \frac{t^{\mathrm{M}}\omega^{\mathrm{M}}}{\eta^{\mathrm{M}}} \tag{4.6}$$

and in recuperation with

$$p^{\mathrm{M}} = t^{\mathrm{M}}\omega^{\mathrm{M}}\eta^{\mathrm{M}}. \tag{4.7}$$

4.2.4 Transmission

In a vehicle a transmission with a ratio $i \neq 1$ is required to transform the torque t^{M} and speed ω^{M} that an EM can provide to the torque t^{W} and speed ω^{W} at the wheel that is required to accomplish the required driving situation. Furthermore, it is required to map all driving conditions in the feasible domain of the EM which is limited by an upper speed limit $\overline{\Omega}^{\mathrm{M}}$, an upper torque limit $\overline{t}^{\mathrm{M}}$ and an upper power limit $\overline{p}^{\mathrm{M}}$.

[67] Balazs, "Optimierte Auslegung von Hybridantriebsträngen unter realen Fahrbedingungen", ([10], 2015)

[68] Lange, "Optimierung modularer Elektro- und Hybridantriebe", ([94], 2018)

We model the transmission with:

$$t^{\mathrm{M}} i \eta^{\mathrm{G}} = t^{\mathrm{W}} \tag{4.8}$$

or

$$t^{\mathrm{M}} i = \eta^{\mathrm{G}} t^{\mathrm{W}} \tag{4.9}$$

for recuperation and

$$\omega^{\mathrm{M}} = i \omega^{\mathrm{W}}. \tag{4.10}$$

The reference value in the following computational study is $\eta^{\mathrm{G}} = 0.975$ for a transmission with two speeds and $\eta^{\mathrm{G}} = 0.98$ for a transmission with one speed[69,70].

4.2.5 Scenario Generation

A scenario generation approach is used to derive the deterministic equivalent of a stochastic optimization program that represents the conceptual design problem with uncertain driving conditions. Here, we assume that we can estimate the distribution of driving conditions (speed, slope), but do not know exactly the driving behavior or the concrete driving cycle.
To transform this stochastic MINLP in a deterministic equivalent, which can be solved by employing exact optimization algorithms, we derive scenarios with corresponding weights.
For the scenario generation, we use instead of a manual approach, a newly developed automatic preprocessing routine, which is based on an unsupervised learning algorithm to extract meaningful scenarios. The *k-means* algorithm[71] is employed, since Löhndorf[72] showed that it is suitable to derive the deterministic equivalent problem description. With the help of this algorithm, it is possible to derive a predefined set of scenarios that represents the underlying complete dataset.
The algorithm is used for partitioning a given set of vectors in different classes. This problem is already NP-hard[73] for two-dimensional vectors and therefore

69 Esser, Zeller, Foulard, and Rinderknecht, "Stochastic Synthesis of Representative and Multidimensional Driving Cycles", ([52], 2018)

70 Leise et al., "Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods", ([106], 2021)

71 MacQueen et al., "Some Methods for Classification and Analysis of Multivariate Observations", ([115], 1967)

72 Löhndorf, "An Empirical Analysis of Scenario Generation Methods for Stochastic Optimization", ([112], 2016)

73 Mahajan, Nimbhorkar, and Varadarajan, "The Planar K-Means Problem is NP-hard", ([116], 2012)

solved by employing a heuristic. Here, the implementation of the k-means algorithm, which is available in the machine learning library *scikit-learn*[74] is used.
The k-means algorithm usually converges, even for large sample dataset, but most often only to a local optimum. Therefore, a multi-start approach is beneficial, where different initial points for the cluster centers for the iterative clustering are selected and afterwards the solution with the lowest objective value is selected. This leads to a more deterministic clustering. Nevertheless, it is not guaranteed that the same clustering solution is found after rerunning the algorithm. From a practical point of view this drawback is negligible, since it only results in minor differences between different runs of the clustering algorithm. Only some discrete points from the sample distribution at the border between adjacent clusters are then assigned in one run the one cluster and in the other run to the other cluster. This then only results in minor changes in the cluster center positions, which are used as scenarios in the optimization program.
The selected scenarios for the approximation of the underlying sample distribution are based on the speed at the wheel ω^{w} and the torque at the wheel t^{w}. We use the longitudinal vehicle model in Eq. (4.2) and the given parameters in Table 4.1 to derive the torque values t^{w} for a given reference mass of 1500 kg[75]. If the mass of the vehicle is predefined, t^{w} can be computed directly by employing Eq. (4.2) for each sample point in the sample distribution. In the developed model by Leise et al.[75], we extended this approach with a variable mass that depends on the capacity of the battery as a further degree of freedom. Therefore, the mass, the torque at the wheel and the power are not fixed in advance to the optimization. This leads to the task to enable a scenario generation with an integrated mass scaling and therefore also torque scaling within the optimization program.
The scenario generation is done by employing the aforementioned reference mass. Then, the sample distribution is scaled to the unit cube to avoid a skewed cluster generation due to high differences in the scales of both axis. As a further step, the clustering is employed and the center points of the clusters in the $(t^{\mathrm{w}}, \omega^{\mathrm{w}}) \in \mathbb{R}^2$ domain are used as scenarios. These cluster centers are then rescaled to the velocity and slope domain by using an inverse model of Eq. (4.2) and added in the optimization program.
Within the optimization, we use a further linear constraint that maps the

[74] Pedregosa et al., "Scikit-learn: Machine Learning in Python", ([146], 2011)
[75] Leise et al., "Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods", ([106], 2021)

relation between the mass m and the torque at the wheel t^{W}:

$$t_s^{\mathrm{W}} = m \underbrace{\left(r^{\mathrm{W}} \lambda_{\mathrm{i}}\, \dot{v}_s + r^{\mathrm{W}} g \left(\sin\left(s\right) + \lambda_{\mathrm{r}}\right)\right)}_{\alpha_{1,s}} + \underbrace{\frac{1}{2} \varrho c_{\mathrm{w}} A_{\mathrm{c}} v_s^2 r^{\mathrm{W}}}_{\alpha_{2,s}} . \tag{4.11}$$

This function is affine in m with the parameters $\alpha_{1,s}$ and $\alpha_{2,s}$:

$$t_s^{\mathrm{W}} = m\alpha_{1,s} + \alpha_{2,s}. \tag{4.12}$$

The parameters are precomputed based on the longitudinal vehicle model in Eq. (4.2) with the derived acceleration and speed values given by the already computed scenarios $(t_s^{\mathrm{W}}, \omega_s^{\mathrm{W}}), s \in \mathcal{S}$, cf. [106]. An example use case is shown in Fig. 4.2. This heuristic approach enables an integration of a variable mass in the exact optimization program, but omits corner cases, like sign-changes caused by a changing mass. For each scenario, we also derive weights π_s based on the total number of measurements and the average power in each cluster. Additionally, to the center points of the clusters, which are used as scenarios, also the corner points are used on the convex hulls of each cluster as further constraints. These points are also scaled comparable to Eq. (4.11) and must be located in the feasible domain of the electric machine. With this approach, we are able to ensure a robust solution, which fulfills all demanded driving situations, but generates optimized solutions, which are the most energy-efficient.

4.2.6 Efficiency Map Approximation

The efficiency map $\mathcal{M}$[76] represents the efficiency of an electric machine over its feasible domain $(t^{\mathrm{M}}, \omega^{\mathrm{M}})$. It is commonly used to represent the performance characteristics of an electric machine[77]. The integration of this functional relationship is possible in different forms, but affects the solution time of the optimization solving algorithm. For instance, an approximation by polynomials was already introduced in the literature[78]. Besides these, further approaches like a piecewise linearization are suitable, if only a primal heuristic, like a genetic algorithm is used for solving the (MI)NLP, see e.g. [106]. The usage of a piecewise linearization (PWL) with an exact solution approach

[76] Leise et al., “Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods”, ([106], 2021)

[77] Lukic and Emado, “Modeling of Electric Machines for Automotive Applications using Efficiency Maps”, ([113], 2003)

[78] McDonald, “Electric Motor Modeling for Conceptual Aircraft Design”, ([124], 2013)

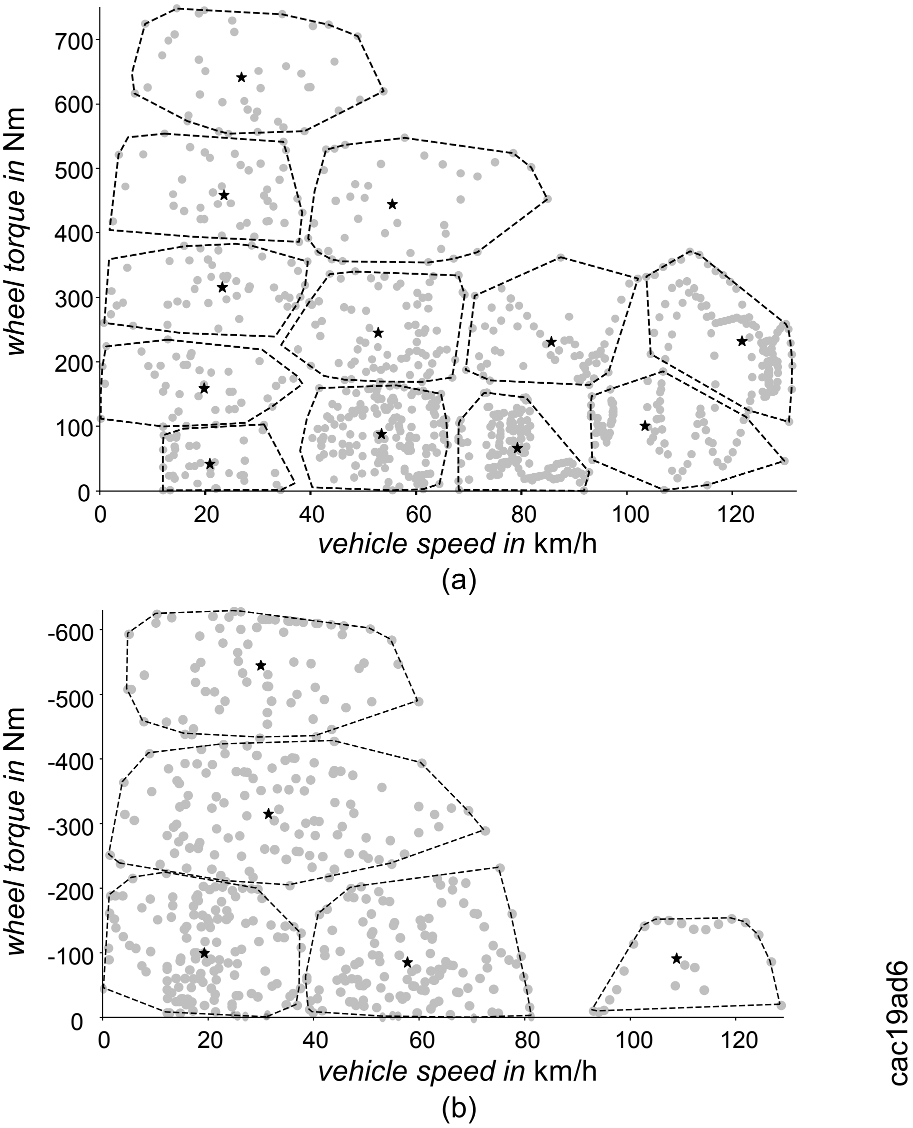

Figure 4.2 – Result of the clustering preprocessing for a reference vehicle with a mass of 1500 kg. First published by Leise et al. in [106].

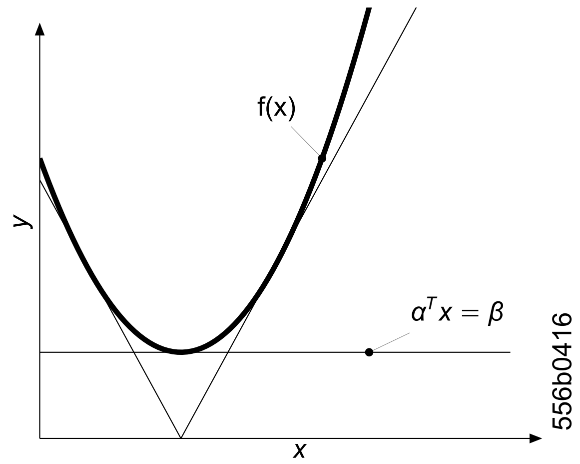

Figure 4.3 – Example of a polyhedral outer approximation with three hyperplanes in two dimensions

was also evaluated by Leise, Altherr, Simon, and Pelz[79]. In a follow-up study, three different PWL approaches were evaluated computationally[80].
For an integration in a model that is solved by employing an exact solving approach, the PWL approach resulted in a suitable modeling, but its solving efficiency is lower than the approaches presented and discussed in the following, due to the nonlinear constraints besides the efficiency map description. For the WDS problem, a PWL of the pump characteristics resulted in a MILP formulation, which was then solvable in an efficient manner. For the powertrain conceptual design problem, the resulting optimization program is still a (nonconvex) MINLP. Therefore, other approaches had to be evaluated. One approach that results in an increase in the solution performance is the usage of an outer polyhedral approximation of the efficiency map $\mathcal{M}$, where each quadrant is considered individually.
An example of this approach is shown in Figure 4.3, where the inner space of the exemplified one-dimensional convex function $f(x)$ is approximated by the intersection of three half-spaces that are based on three affine functions. Since the efficiency map can be approximated as being concave, this approach can be used to derive multiple affine half-spaces which are then added in the optimization program.
The efficiency map $\mathcal{M}$ is almost concave, if the first and fourth quadrant is considered individually, cf. Figure 4.1. In total, 226 points were used in each

[79] Leise, Altherr, Simon, and Pelz, "Finding Global-Optimal Gearbox Designs for Battery Electric Vehicles", ([103], 2019)

[80] Leise, Simon, and Altherr, "Comparison of Piecewise Linearization Techniques to Model Electric Motor Efficiency Maps: A Computational Study", ([109], 2020)

quadrant at which hyperplanes are calculated. The set of these hyperplanes is then used to represent the efficiency map within the optimization program. Since the efficiency map can be understood as the maximum value that the optimized solution can achieve, the relaxation of the equality constraints in an inequality constraints will not affect the optimized solution. The optimized system will result in a maximum efficiency, which is then given by the border of the approximation.
The polyhedral outer approximation of the efficiency map is shown in Figure 4.4 for the first quadrant (a) and fourth quadrant (c). Additionally, the approximation errors are shown. The approach leads to a good approximation in general. Only near the axis, it leads to higher differences between the original measured data and the approximation, which is used in the optimization program.

4.2.7 Complexity Evaluation

The MINLP for the powertrain conceptual design has two stages. This is comparable to the already introduced MINLP for the water distribution system design, cf. Chapter 3. In the first stage, we derive a solution for the scaling of the EM and for the sizing of the transmission. In the second stage, we retrieve the optimized control strategy for each scenario. Here, the assignment of the derived first-stage decisions for the transmission ratios i_1 and i_2 to each considered scenario is conducted.
We now focus on the combinatorial complexity of the considered use case with a discrete assignment of transmission ratios to scenarios. The optimal solution of the MINLP has an optimal binary vectorized mapping $b_{s,t}$ for each scenario $s \in \mathcal{S}$ and each transmission ratio $t \in \mathcal{T}$. For the two-speed transmission the set $\mathcal{T}$ is given by $\mathcal{T} = \{1, 2\}$. This binary decision indicates if either the first or second transmission ratio is chosen within the considered scenario. The second stage decision to assign transmission ratios to each scenario is a combinatorial subproblem, that depends on the results of the first stage. This coupling between both stages in combination with the non-convex property within the original design space are the major challenges within the solution process of the underlying MINLP.
The assignment of a single transmission ratio to each scenario, which represents a single-speed solution is trivial. This would result in a reduction of both stages to only one stage and therefore in an NLP in general. We will only consider transmissions that have at least 2 transmission ratios ($|\mathcal{T}| \geq 2$).
Furthermore, $|\mathcal{T}| < |\mathcal{S}|$ must hold, since if $|\mathcal{T}| = |\mathcal{S}|$, we compute a CVT solution, where each scenario has its own specific transmission ratio. This results again in a NLP and avoids the combinatorial second stage decisions.

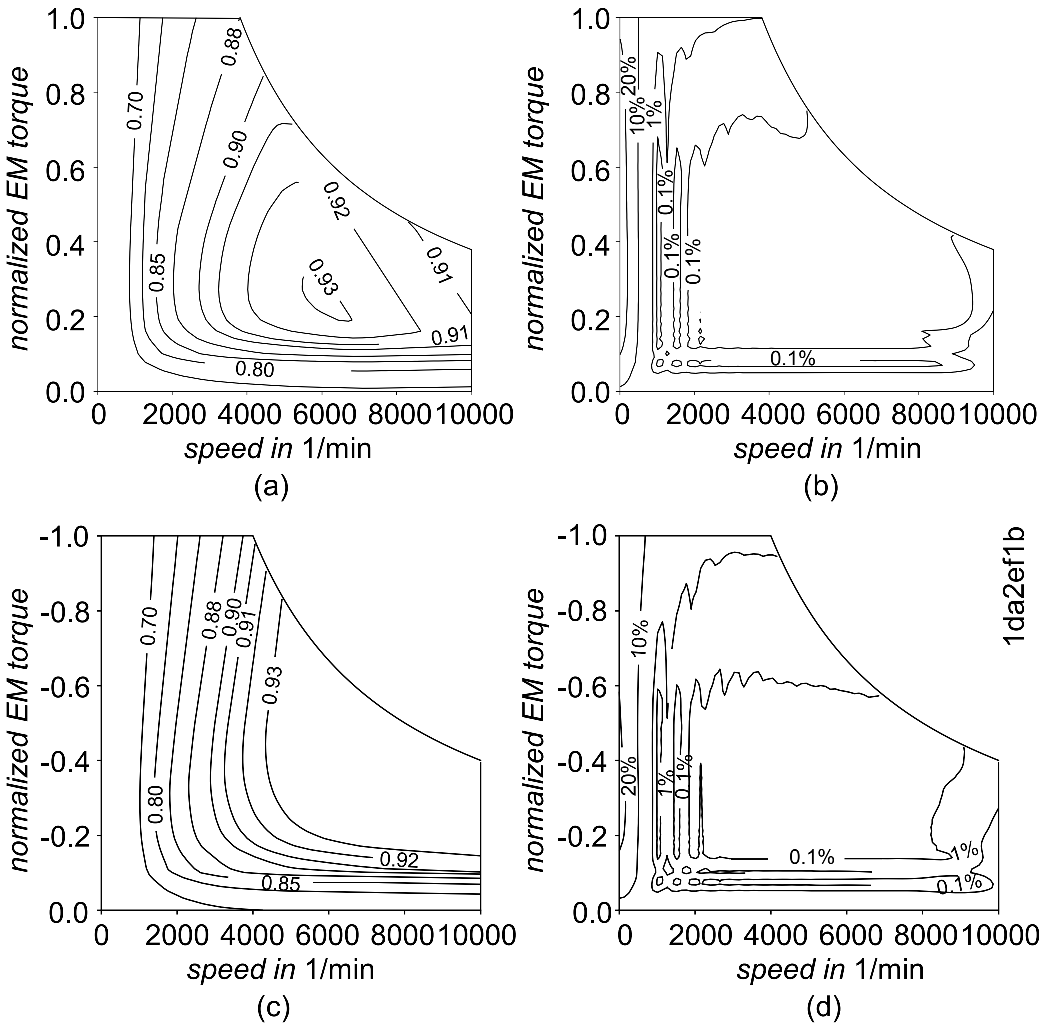

Figure 4.4 – Result of the polyhedral outer approximation. (a) first quadrant (b) error of the approximation in the first quadrant (c) fourth quadrant (d) error of the approximation in the fourth quadrant. First published by Leise et al. in [106].

If $|\mathcal{T}| < |\mathcal{S}|$, all possible assignments of transmission ratios $t \in \mathcal{T}$ to scenarios are given by the cartesian product:

$$N_{t \to s} = |\mathcal{T}|^{|\mathcal{S}|}. \tag{4.13}$$

Even for a low number of transmission ratios and scenarios, this is a rapidly growing number.
From a mathematical point of view, the underlying optimization program has a number of equivalent solutions. These solutions all have in common that they result in equivalent system designs, but the variable assignments in the optimized system design are different. They are known in combinatorics as symmetric solutions. The reduction of symmetries to increase the solver performance is a current research topic in mathematics[81].
The given conceptual design problem for powertrains has a high number of symmetric solutions, since a permutation of the transmission ratios and an according permutation of the binary assignments will result in an equivalent solution. The number of combinatorial second-stage solutions without symmetric solutions is represented by *Stirling numbers of the second kind*[82] $\left\{ {n \atop k} \right\}$. These are defined as following:

$$S_{n,k} = \left\{ {n \atop k} \right\} := \frac{1}{k!} \sum_{i=0}^{k} (-1)^i \binom{k}{i} (k-i)^n .$$

They describe the number of possibilities to assign a set of n distinguishable objects into k nonempty indistinguishable subsets. For the powertrain conceptual design problem n relates to the scenarios, while the transmission ratios are represented by k. For a two-speed transmission this reduces further to:

$$S_{n,2} = \left\{ {n \atop 2} \right\} = 2^{n-1} - 1. \tag{4.14}$$

This evaluation shows that the number of solutions that have to be evaluated is less than half of the solutions that are derived by Eq. (4.13).

81 Margot, “Symmetry in Integer Linear Programming”, ([120], 2010)
82 Knuth, *Art of Computer Programming*, ([93], 1997, p. 66ff.)

Additionally, to the combinatorial complexity of the control strategy assignment in the second stage of the MINLP comes additionally the computational complexity to solve the assignment-depending NLP of first-stage decisions. This depends on the problem formulation. Usually a convex problem formulation results in a better solving performance than a nonconvex NLP.

4.2.8 Model Description

The complete model[83] is given by objective (4.15) subject to the Constraints (4.16) to (4.38). The sets, parameters and variables for the program are shown in Table 4.2.

The goal of the conceptual design is to derive energy-efficient system designs for powertrains with multi-speed transmissions. As previously shown, we use a set range of 500 km that must be achieved. As an objective, we then minimize the brutto battery capacity c^{B}, which results in the best possible powertrain system design to achieve this goal. Since the total mass m depends on the battery capacity c^{B}, it has a significant impact on the system design and the optimized solution.

To facilitate the scaling of the EM and to enable multiple approximation approaches of the EM efficiency map $\mathcal{M}$ in a coherent manner, Constraints (4.16) and (4.17) create a normalized EM torque $\psi^{\mathrm{M}}_{s,k}$ and a normalized EM speed $\dot{\varphi}^{\mathrm{M}}_{s,k}$. These variables are initiated for each scenario $s \in \mathcal{S}$ and EM domain $k \in \mathcal{K}$.

Constraint (4.18) models the relaxation of the EM efficiency map as an affine approximation with the set $\mathcal{N}$ of considered grid points in the domain of the EM efficiency map. This constraint is active in an optimized solution and then results in an equality constraint, since the efficiency is maximized as much as possible to enable an efficient system design. Constraint (4.19) models the maximum power in the normalized domain. Constraint (4.20) is used to order the transmission ratios. The logic to select only one transmission ratio in each scenario and motor domain is modeled with Constraint (4.21). If the binary variable is active ($b_{t,s,k} = 1$), the motor and wheel speed are linked by the given relation. This linking is implemented with the help of big-M constraints.

Constraints (4.23) and (4.24) model the relation between the motor torque and wheel torque. Constraint (4.25) and (4.26) are used to model the mass changes due to capacity changes of the traction battery. Here, the energy density ρ^{e} is measured in Wh/kg.

[83] Leise et al., *Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods (Supplemental)*, ([105], 2021)

Table 4.2 – Notation for the powertrain conceptual design program

SET		DESCRIPTION
$\mathcal{T}$		set of transmissions
$\mathcal{K}$		set of EM domains
$\mathcal{N}$		set of grid points of polyhedral approximation
$\mathcal{R}$		set of operating points on the convex hull
$\mathcal{S}$		set of scenarios
ADDITIONAL PARAMETER	**VALUE**	**DESCRIPTION**
A_d	0.4	EM hyperbole domain restriction parameter
L	based on cycle	cycle length in km
R	500	defined range in km
$\overline{T}^\mathrm{M}$	500/300	max. EM torque (single-speed/two-speed) in Nm
$\alpha_{1,s,k}, \alpha_{2,s,k}$	based on $\mathcal{S}$	coefficients for torque-mass relation
$\alpha_{1,s,k,r}, \alpha_{2,s,k,r}$	based on $\mathcal{R}$	coefficients for torque-mass relation on convex hulls
$\beta_{0,k,n}, \beta_{1,k,n}, \beta_{2,k,n}$	based on $\mathcal{N}$	parameters for polyhedral approximation
$\pi_{s,k}$	based on $\mathcal{S}$	weight of each scenario s in domain k
$\overline{\Omega}^\mathrm{M}$	1047.198	max. EM velocity in 1/s
$\Omega^\mathrm{W}_{s,k}$	based on $\mathcal{S}$	angular velocity input of scenario s and domain k
$\Omega^\mathrm{W,R}_{s,k,r}$	based on $\mathcal{R}$	angular velocity input based on convex hull
VARIABLE	**DOMAIN**	**DESCRIPTION**
$b_{t,s,k}$	$\{0,1\}$	binary variable to choose a transmission
c^B	$[0, 150]$	battery capacity in kWh
i_t	$[2, 25]$	transmission ratio for transmission t
m	$[1150, 2800]$	total mass in kg
m^B	$[0, 1500]$	battery mass in kg
$p^\mathrm{C}_{s,k}$	$[0, 1500]$	power in each cluster in kW
$t^\mathrm{M}_{s,k}$	$[0, \overline{T}^\mathrm{M}]$	EM torque in Nm
$\bar{t}^\mathrm{M}$	$[0, \overline{T}^\mathrm{M}]$	max. EM torque in Nm
$t^\mathrm{W}_{s,k}$	$[0, \overline{T}^\mathrm{M}]$	wheel torque in Nm
$t^\mathrm{M,R}_{s,k,r}$	$[0, \overline{T}^\mathrm{M}]$	EM torque for op. points on convex hulls in Nm
$t^\mathrm{W,R}_{s,k,r}$	$[0, \overline{T}^\mathrm{M}]$	wheel torque for op. points on convex hulls in Nm
$\eta^\mathrm{M}_{s,k}$	$[0, 1]$	EM efficiency
ρ^e	$[0, 1000]$	energy density in Wh/kg
$\dot{\varphi}^\mathrm{M}_{s,k}$	$[0, 1]$	normalized angular velocity
$\dot{\varphi}^\mathrm{M,R}_{s,k,r}$	$[0, 1]$	norm. angular velocity for op. points on convex hulls
$\psi^\mathrm{M}_{s,k}$	$[0, 1]$	normalized torque
$\psi^\mathrm{M,R}_{s,k,r}$	$[0, 1]$	norm. torque for domain k for op. points on convex hulls
$\omega^\mathrm{M}_{s,k}$	$[0, \overline{\Omega}^\mathrm{M}]$	EM angular velocity in 1/s
$\omega^\mathrm{M,R}_{s,k,r}$	$[0, \overline{\Omega}^\mathrm{M}]$	EM angular velocity for op. points on convex hulls in 1/s

EM — electric machine

$$
\begin{aligned}
\min \quad & c^{\mathrm{B}} && && (4.15)\\
\text{s.t.} \quad & && && \\
& t^{\mathrm{M}}_{s,k} = \bar{t}^{\mathrm{M}} \psi^{\mathrm{M}}_{s,k} && \forall s \in \mathcal{S},\, \forall k \in \mathcal{K} && (4.16)\\
& \omega^{\mathrm{M}}_{s,k} = \overline{\Omega}^{\mathrm{M}} \dot{\varphi}^{\mathrm{M}}_{s,k} && \forall s \in \mathcal{S},\, \forall k \in \mathcal{K} && (4.17)\\
& \eta^{\mathrm{M}}_{s,k} \le \beta_{0,k,n} \psi^{\mathrm{M}}_{s,k} + \beta_{1,k,n} \dot{\varphi}^{\mathrm{M}}_{s,k} + \beta_{2,k,n} && \forall s \in \mathcal{S},\, \forall k \in \mathcal{K},\, \forall n \in \mathcal{N} && (4.18)\\
& \psi^{\mathrm{M}}_{s,k}\, \dot{\varphi}^{\mathrm{M}}_{s,k} \le A_{\mathrm{d}} && \forall s \in \mathcal{S},\, \forall k \in \mathcal{K} && (4.19)\\
& i_t \le i_{t-1} && \forall t > 1 && (4.20)\\
& \sum_{t \in \mathcal{T}} b_{t,s,k} = 1 && \forall s \in \mathcal{S},\, \forall k \in \mathcal{K} && (4.21)\\
& b_{t,s,k} = 1 \implies \omega^{\mathrm{M}}_{s,k} = \Omega^{\mathrm{W}}_{s,k} i_t && \forall s \in \mathcal{S},\, \forall k \in \mathcal{K},\, \forall t \in \mathcal{T} && (4.22)\\
& t^{\mathrm{M}}_{s,k} \left(\sum_{t \in \mathcal{T}} i_t b_{t,s,k} \right) \eta^{\mathrm{G}} = t^{\mathrm{W}}_{s,k} && \forall s \in \mathcal{S},\, \text{if } k = 0 && (4.23)\\
& t^{\mathrm{M}}_{s,k} \left(\sum_{t \in \mathcal{T}} i_t b_{t,s,k} \right) = \eta^{\mathrm{G}} t^{\mathrm{W}}_{s,k} && \forall s \in \mathcal{S},\, \text{if } k = 1 && (4.24)\\
& \rho^{\mathrm{e}}/1000 = 0.8 c^{\mathrm{B}}/1000 + 78.8/1000 && && (4.25)\\
& m^{\mathrm{B}} \rho^{\mathrm{e}} = 1000 c^{\mathrm{B}} && && (4.26)\\
& m = m^{\mathrm{B}} + M^{0} + M^{\mathrm{P}} + M^{\mathrm{A}} && && (4.27)\\
& t^{\mathrm{W}}_{s,k} = \alpha_{1,s,k}\, m + \alpha_{2,s,k} && \forall s \in \mathcal{S},\, \forall k \in \mathcal{K} && (4.28)\\
& 1000\, p^{\mathrm{C}}_{s,k} \eta^{\mathrm{G}} \eta^{\mathrm{M}}_{s,k} = t^{\mathrm{W}}_{s,k} \Omega^{\mathrm{W}}_{s,k} && \forall s \in \mathcal{S},\, \text{if } k = 0 && (4.29)\\
& 1000\, p^{\mathrm{C}}_{s,k} = \eta^{\mathrm{G}} \eta^{\mathrm{M}}_{s,k} t^{\mathrm{W}}_{s,k} \Omega^{\mathrm{W}}_{s,k} && \forall s \in \mathcal{S},\, \text{if } k = 1 && (4.30)\\
& 3600\, c^{\mathrm{B}}\, 0.95 - \left(\sum_{s \in \mathcal{S}} \pi_{s,0}\, p^{\mathrm{C}}_{s,0} \right) \frac{R}{L \eta^{\mathrm{B}}} \dots && && \\
& \dots + \eta^{\mathrm{B}} \left(\sum_{s \in \mathcal{S}} \pi_{s,1}\, p^{\mathrm{C}}_{s,1} \right) \frac{R}{L} = 0 && && (4.31)
\end{aligned}
$$

$$t^{\mathrm{M,R}}_{s,k,r} = \overline{t}^{\mathrm{M}} \psi^{\mathrm{M,R}}_{s,k,r} \qquad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \tag{4.32}$$

$$\omega^{\mathrm{M,R}}_{s,k,r} = \overline{\Omega}^{\mathrm{M}} \dot{\varphi}^{\mathrm{M,R}}_{s,k,r} \qquad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \tag{4.33}$$

$$\dot{\varphi}^{\mathrm{M,R}}_{s,k,r} \, \psi^{\mathrm{M,R}}_{s,k,r} \leq A_{\mathrm{d}} \qquad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \tag{4.34}$$

$$t^{\mathrm{W,R}}_{s,k,r} \geq \alpha_{1,s,k,r} \, m + \alpha_{2,s,k,r} \qquad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \tag{4.35}$$

$$b_{t,s,k} = 1 \implies \omega^{\mathrm{M,R}}_{s,k,r} = \Omega^{\mathrm{W,R}}_{s,k,r} i_t \qquad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R}, \forall t \in \mathcal{T} \tag{4.36}$$

$$t^{\mathrm{M,R}}_{s,k,r} \left(\sum_{t \in \mathcal{T}} i_t b_{t,s,k} \right) \eta^{\mathrm{G}} = t^{\mathrm{W,R}}_{s,k,r} \qquad \forall s \in \mathcal{S}, \forall r \in \mathcal{R}, \text{ if } k = 0 \tag{4.37}$$

$$t^{\mathrm{M,R}}_{s,k,r} \left(\sum_{t \in \mathcal{T}} i_t b_{t,s,k} \right) = \eta^{\mathrm{G}} t^{\mathrm{W,R}}_{s,k,r} \qquad \forall s \in \mathcal{S}, \forall r \in \mathcal{R}, \text{ if } k = 1 \tag{4.38}$$

We model the mass according to Equation (4.3) with Constraint (4.27). The (heuristic) relation between the vehicle mass and the resulting wheel torque is given by Constraint (4.28).

The power demand is modeled with Constraints (4.29) and (4.30). The required battery capacity is modelled with Constraint (4.31). The remaining constraints model comparable relations as the already introduced constraint for the points on the convex hull of the cluster, where the center represents the scenarios. These values affect the objective only indirectly, since they must be fulfilled in the optimized system design but are not considered directly in the objective.

4.2.9 Results

The newly developed optimization program in Objective (4.15) to Constraint (4.38) was implemented using PyScipOpt[84]. The solvers SCIP[85] 6.0.2 with SoPlex v. 4.0.2 and Ipopt v. 3.12. were used[86]. As termination criteria (i) a time limit of 30 minutes, (ii) RAM limit of 12 GB, and (iii) a duality gap limit of 0.25 % were used[86].

For single speed transmission, 25 scenarios plus one scenario each for the speed and gradeability requirements were used. For the two-speed transmission, 19 scenarios were used in total.

[84] Maher et al., “PySCIPOpt: Mathematical Programming in Python with the SCIP Optimization Suite”, ([117], 2016)

[85] Gleixner et al., *The SCIP Optimization Suite 6.0*, ([64], 2018)

[86] Leise et al., “Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods”, ([106], 2021)

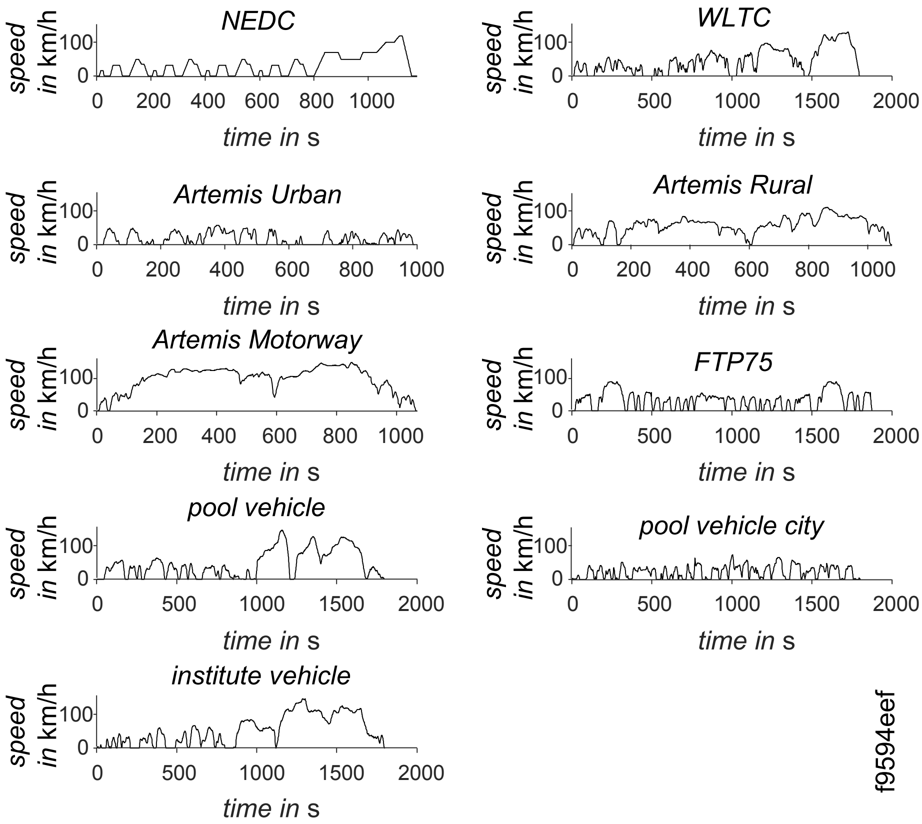

Figure 4.5 – Used driving cycles for the verification, cf. [106]

To verify the optimization results returned by the nonconvex optimization program, it was compared with an already implemented optimization approach developed at the Institute of Mechatronic Systems at TU Darmstadt that is based on the complete driving cycle and which is solved with a genetic algorithm. Furthermore, the reference approach uses a piecewise linearization of the efficiency map of the EM. Further details can be found in [50, 106]. For the clustering in the preprocessing the same driving cycles were used, as for the reference model that was solved with a GA. As a basis of comparison, nine different driving cycles are used to represent the sample distributions each. Based on these, a model and solving procedure verification is performed. These driving cycles are shown in Figure 4.5. The set of driving cycles consists of six commonly used legislative driving cycles. Additionally, it contains three driving cycles recorded at the Institute of Mechatronic Systems at TU Darmstadt while using two hybrid-electric vehicles[87]. The cycles were

[87] Leise et al., "Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods", ([106], 2021)

Table 4.3 – Rounded computational results for a single-speed transmission, cf. [106]. The first sub-column in each column group shows the result for a reference solved with a GA, while the second shows the minimum results of 10 evaluations each for the objective of the given MINLP. All deviation values are absolute. First published by Leise et al. in [106].

	$\overline{p}^{\mathrm{M}}$ in kW		i		m in kg		c^{B} in kWh		c_{mean} in kWh/100km	
WLTC	157	160	7.07	7.07	1841	1847	76.5	78.0	14.5	14.8
NEDC	155	157	7.07	7.07	1818	1814	70.9	70.0	13.5	13.3
Art. Urban	162	161	7.07	7.07	1859	1856	81.0	80.3	15.3	15.3
Art. Rural	153	157	7.07	7.07	1799	1807	66.5	68.3	12.6	13.0
Art. Motorway	168	170	7.00	7.07	1946	1954	107.0	109.9	20.3	20.9
FTP75	153	156	7.07	7.07	1795	1798	65.7	66.4	12.5	12.6
pool vehicle	158	162	7.07	7.07	1863	1871	82.0	84.1	15.6	16.0
pool veh. city	156	156	7.04	7.07	1814	1798	69.8	66.3	13.0	12.6
inst. vehicle	160	163	7.07	7.07	1874	1878	85.1	86.2	16.2	16.4
mean dev.	2.44		0.01		6.67		1.69		0.29	
median dev.	3.00		0.00		6.00		1.50		0.30	
max. dev.	4.00		0.07		16.00		3.50		0.60	

then synthesized by the method described by Esser, Zeller, Foulard, and Rinderknecht[88]. Even though the approach allows integrating slope changes within the scenario generation, all considered driving cycles do not contain any slope information. This preserves comparability between all considered cycles. The results for a single-speed transmission are shown in Table 4.3. The comparison between both approaches shows that the newly developed modeling and solution approach results in comparable results with only minor differences. It can therefore be verified that the optimization approach is suitable to develop energy-efficient powertrain system designs.
The results for a two-speed transmission are shown in Table 4.4. Here, again, the newly developed approach leads to comparable results.
In comparison to the solving approach that is based on a GA, the exact approach allows to quantify the optimality of each found solution based on duality theory. This additional certificate of performance can then lead to a more informed decision-making process within the conceptual design phase.

[88] Esser, Zeller, Foulard, and Rinderknecht, "Stochastic Synthesis of Representative and Multidimensional Driving Cycles", ([52], 2018)

Table 4.4 – Rounded computational results for a two-speed transmission. The first sub-column in each column group shows the result for a reference solved with a GA, while the second shows the minimum results of 10 evaluations each for the objective of the given MINLP. All deviation values are absolute. First published by Leise et al. in [106].

	$\overline{p}^{\mathrm{M}}$ in kW		i_1		i_2		m in kg		c^{B} in kWh		c_{mean} in kWh/100km	
WLTC	70	67	16.61	16.61	5.99	5.76	1804	1808	67.7	68.5	12.9	13.0
NEDC	69	67	20.30	16.16	6.34	5.70	1759	1764	58.2	59.2	11.0	11.2
Art. Urban	69	66	20.09	19.49	6.60	7.07	1745	1758	55.4	58.0	10.5	11.0
Art. Rural	80	67	14.21	16.23	6.18	5.88	1773	1768	61.0	60.0	11.6	11.4
Art. Motorw.	111	82	13.19	14.56	5.48	5.14	1931	1928	102.1	101.2	19.4	19.2
FTP75	68	65	18.57	18.93	6.30	5.70	1733	1737	53.1	53.9	10.1	10.2
pool vehicle	76	75	18.27	17.28	5.88	5.56	1829	1832	73.4	74.2	13.9	14.1
pool veh. city	68	67	19.23	21.31	6.68	7.07	1718	1721	50.3	50.9	9.5	9.7
inst. vehicle	70	67	18.61	18.38	5.78	5.79	1843	1846	76.8	77.6	14.6	14.7
mean dev.	6.44		1.31		0.37		4.78		1.03		0.20	
median dev.	3.00		0.99		0.34		4.00		0.80		0.20	
max. dev.	29.00		4.14		0.64		13.00		2.60		0.50	

4.3 MIGP Model

The model presented in Section 4.2 is suitable for a powertrain conceptual design. This was shown by the verification in Section 4.2. When solving the MINLP usually relaxations are generated in which binary and integer variables are partially replaced by continuous variables. For the given model this results in nonconvex relaxations. We employed the usage of the branch-cut-and-reduce framework SCIP[89] to solve the MINLP, because of its efficiency when solving nonconvex MINLPs.
Nevertheless, a MINLP with nonconvex relaxations is computationally difficult to solve, due to the nonconvexity and the existence of multiple local optima. Therefore, it is beneficial, if we can create a MINLP model, which can be relaxed to a convex optimization program.
For the water distribution system design in Chapter 3, we were able to generate a convex relaxation by using a piecewise linearization approach. This was possible due to the already large quantity of affine constraints in the model. The powertrain conceptual design problem, on the other hand, has a high quantity of nonlinear constraints, which would result in a high number of additional binary variables, when employing a piecewise linearization.
Within this Section, we present a newly developed modeling approach that is based on the already introduced model for a powertrain design, but exploits the specific structure to generate a convex relaxation of the MINLP by

[89] Gleixner et al., *The SCIP Optimization Suite 6.0*, ([64], 2018)

modeling it as a *Mixed-Integer Geometric Program*[90] (MIGP).
If we reconsider the nonconvex MINLP with its objective (4.15) and the constraints (4.16) to (4.38), we can already identify a high quantity of constraints that are aligned with the requirements of a geometric program, cf. Sec. 2.1.5. Within a MIGP the resulting relaxation, when fixing the binary and/or integer variables, becomes a geometric program (GP), cf. Section 2.1.5. The constraints that do not fulfill the MIGP requirements are adapted to finally derive a MIGP model. This is shown in the following.

4.3.1 Efficiency Map Approximation

To be able to model the conceptual design problem as a MIGP, we have to approximate the efficiency map $\mathcal{M}$ of the EM with a set of constraints that are GP-compliant. As shown in the literature[91,92], the efficiency map modeling of a permanent-magnet synchronous motor (PMSM) can be accomplished in comparison to the already applied PWL or outer polyhedral approximation, by using a physical based modeling approach that models specific losses following a power law. This modeling has the advantage to be GP-compliant due to the monomial/posynomial structure. If we transfer this modeling approach, losses like core losses, armature losses or stray load losses can be modelled in each scenario $s \in \mathcal{S}$ and EM domain $k \in \mathcal{K}$ as:

$$p_{s,k}^{\mathrm{M,L,j}}(t_{s,k}^{\mathrm{M}}, \omega_{s,k}^{\mathrm{M}}) = P_{\mathrm{L},ref,j,k} \left(\frac{t_{s,k}^{\mathrm{M}}}{\overline{t}^{\mathrm{M}}}\right)^{\alpha_{j,k}} \left(\frac{\omega_{s,k}^{\mathrm{M}}}{\overline{\Omega}^{\mathrm{M}}}\right)^{\beta_{j,k}}, \tag{4.39}$$

each with the parameters $P_{\mathrm{L},ref,j,k}$, $\alpha_{j,k}$ and $\beta_{j,k}$. The resulting total power loss $p_{s,k}^{\mathrm{M,L}}$ of the EM is then given in each scenario $s \in \mathcal{S}$ and EM domain $k \in \mathcal{K}$ by the sum of N loss terms:

$$p_{s,k}^{\mathrm{M,L}}(t_{s,k}^{\mathrm{M}}, \omega_{s,k}^{\mathrm{M}}) = \sum_{j=1}^{N} p_{s,k}^{\mathrm{M,L,j}}(t_{s,k}^{\mathrm{M}}, \omega_{s,k}^{\mathrm{M}}). \tag{4.40}$$

The resulting efficiency map approximation in the domain $k \in \mathcal{K}$ can then be calculated with a suitable set of parameters $\{P_{\mathrm{L},ref,j,k}, \alpha_{j,k}, \beta_{j,k}, \forall j \in 1, \dots, N\}$ for this semi-analytical model by

$$\eta_k(t^{\mathrm{M}}, \omega^{\mathrm{M}}) = \frac{t^{\mathrm{M}}\omega^{\mathrm{M}}}{t^{\mathrm{M}}\omega^{\mathrm{M}} + p_{\star,k}^{\mathrm{M,L}}(t^{\mathrm{M}}, \omega^{\mathrm{M}})}. \tag{4.41}$$

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90 Leise and Pelz, *Efficient Powertrain Design – A Mixed-Integer Geometric Programming Approach*, ([108], 2021)

91 Mahmoudi, Soong, Pellegrino, and Armando, "Efficiency Maps of Electrical Machines", ([118], 2015)

92 Vratny, "Conceptual Design Methods of Electric Power Architectures for Hybrid Energy Aircraft", ([192], 2019)

The advantage of this modeling approach is the applicability to integrate this EM model as constraints in a MIGP, which is shown in this thesis. Equation (4.39) is a monomial and therefore GP compliant. Equation (4.40) can be cast as a posynomial, if the equality constraint is relaxed into an inequality constraint:

$$p_{s,k}^{\mathrm{M,L}}(t^{\mathrm{M}},\omega^{\mathrm{M}}) \geq \sum_{j=1}^{N} p_{s}^{\mathrm{M,L,j}}(t^{\mathrm{M}},\omega^{\mathrm{M}}). \tag{4.42}$$

This relaxation is comparable to the relaxation used within the already introduced MINLP in the previous section for the polyhedral approximation, cf. Section 4.2.6. Within the optimized solution the overall efficiency is maximized. This is only possible, if the best possible efficiency in each point $(t^{\mathrm{M}},\omega^{\mathrm{M}})$ is selected. Hence, an optimized solution lies on the upper boundary of the relaxation, which is given by the efficiency map.

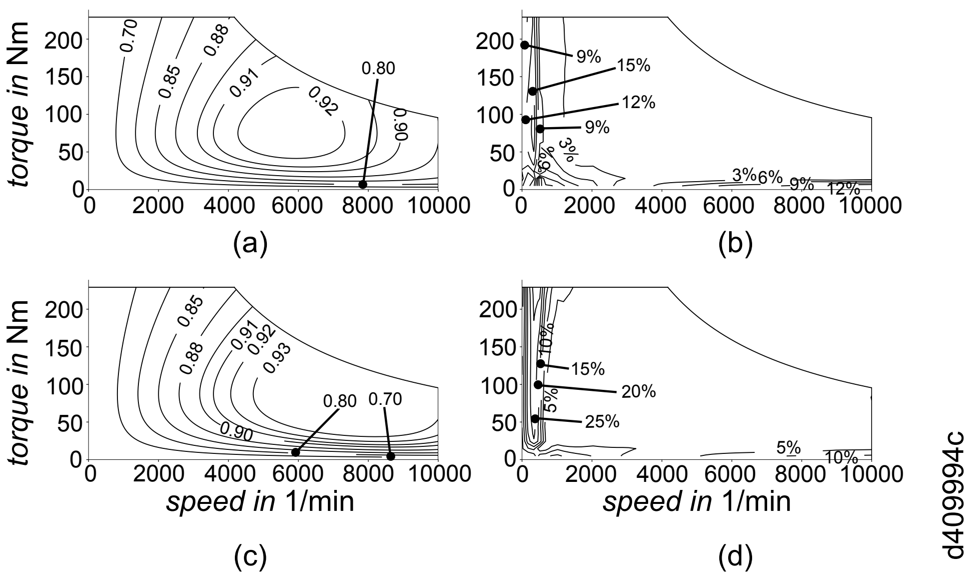

Figure 4.6 – (a) GP-compatible efficiency map model in the first quadrant for a motor with 100 kW (b) error of the approximation in the first quadrant (c) GP-compatible efficiency map model (absolute torque) in the fourth quadrant for a motor with 100 kW (d) error of the approximation in the fourth quadrant

To approximate the given reference efficiency map $\mathcal{M}$, a nonlinear least-square fitting, cf. Section 2.2.2, is conducted by employing a genetic algorithm to derive the set of fitting parameters by minimizing the mean squared error between the approximation and the given dataset $\mathcal{M}$. In total four loss terms $(N = 4)$ were considered for the driving domain $(k = 0)$ and five loss terms

$(N = 5)$ for the recuperation domain $(k = 1)$. The result is shown in Fig. 4.6 for an EM with a maximum power of 100 kW.
The integration of a scaling of the EM model is accomplished by adding a scaling factor $k^{\mathrm{M,p}}$ to Eq. (4.39).

4.3.2 Model

Based on the already introduced model in Section 4.2, the already introduced modeling approach of a geometric program in Sec. 2.1.5 and the GP-compliant EM modeling in Sec. 4.3.1, it is possible to derive a new model for the conceptual design of a powertrain that is a MIGP. To accomplish this, we slightly adapt the already introduced modeling approach in the previous section for the nonconvex MINLP. The used parameters, sets and variables are given in Tab. 4.5.
The complete model is shown in objective (4.43) to constraint (4.87). As an objective, we use a comparable approach to the previously shown objectives for the water distribution system design and the nonconvex powertrain conceptual design. To be compliant with the GP requirements, we move the objective to the first constraint (4.44) as a posynomial. This approach is already known from Section 2.1.5 as the epigraph form of the problem. The new objective (4.43) also has a physical meaning, as it represents the power required to fulfill all scenarios.
Constraint (4.45) limits the choosable transmission ratios between a preset upper $(\overline{I})$ and lower $(\underline{I})$ bound. The selection of a discrete transmission ratio is modelled with big-M constraints in Constraints (4.46) and (4.47). Only one transmission ratio can be selected within each scenario. This is given by the logic constraint (4.48).
The transformation between the torque and speed within the multi-speed transmission is given by Constraints (4.49), (4.50) and (4.51). Instead of the angular speed in s^{-1} we use as a unit for the speed in the MIGP approach min^{-1}. This results in a better scaling of the variable ranges within the optimization program, but can be changed to the angular speed $\omega^{\mathrm{M}} = 2\pi n^{\mathrm{M}}/60$, if desired. We enable a scaling of the EM besides the selection of a suitable multi-speed transmission to derive energy-efficient system designs. This EM scaling is also conducted, as previously introduced, for the general MINLP approach. A scaled EM results in a higher maximum power $\overline{p}^{\mathrm{M}}$ and a higher maximum torque $\overline{t}^{\mathrm{M}}$. The reference power that is used for scaling is added to the program by using Constraint (4.52). Constraint (4.53) is used to model the hyperbola that restricts the feasible domain of the EM to a power below the maximum power $\overline{p}^{\mathrm{M}}$. The upper limits of the speed and torque are added to the program with Constraints (4.54) and (4.55).

Table 4.5 – MIGP model notation, cf. [108]

SET			DESCRIPTION
$\mathcal{S}$			set of all considered scenarios
$\mathcal{S}^+$			set of all steps based on the given cycle

PARAMETER	VALUE	UNIT	DESCRIPTION
A_d	0.4161	–	EM hyperbola constant
$\underline{I}$	1.0	–	minimum transmission ratio
$\overline{I}$	18.0	–	maximum transmission ratio
N^W	based on cycle	/min	wheel rotational speed
$\overline{N}^\mathrm{M}$	10000	/min	EM maximum rotational speed
$\overline{P}^{\mathrm{M,\ in}}$	8000	kW	upper limit of EM power
P_ref	100	kW	power of reference EM
$P_{\mathrm{L,\ ref,\ 1,\ 0}}$	787.35	W	power reference loss 1
$P_{\mathrm{L,\ ref,\ 2,\ 0}}$	1566.67	W	power reference loss 2
$P_{\mathrm{L,\ ref,\ 3,\ 0}}$	9904.85	W	power reference loss 3
$P_{\mathrm{L,\ ref,\ const,\ 0}}$	1059.34	W	power reference constant loss
$P_{\mathrm{L,\ ref,\ 1,\ 1}}$	2450	W	recuperation power reference loss 1
$P_{\mathrm{L,\ ref,\ 2,\ 1}}$	904.58	W	recuperation power reference loss 2
$P_{\mathrm{L,\ ref,\ 3,\ 1}}$	13.8	W	recuperation power reference loss 3
$P_{\mathrm{L,\ ref,\ 4,\ 1}}$	9425	W	recuperation power reference loss 4
$P_{\mathrm{L,\ ref,\ const,\ 1}}$	1212	W	recuperation power reference constant loss
T^W	based on cycle	N m	wheel torque
η^G	0.98	–	transmission efficiency of gearbox
π	$1/\sum_{k=1}^{l} 1$	–	probability of occurrence for all $\lvert\mathcal{S}^+\rvert$ scenarios

VARIABLE	DOMAIN	UNIT	DESCRIPTION
i	$[\underline{I}, \overline{I}]$	–	transmission ratio
$k^{\mathrm{M,p}}$	$\mathbb{R}$	–	EM power scaling factor
n^M	$[0, \overline{N}^\mathrm{M}]$	/min	EM speed
$n^{\mathrm{M,R}}$	$[0, \overline{N}^\mathrm{M}]$	/min	EM speed on convex hulls
p^M	$\mathbb{R}$	W	EM design power
$p^{\mathrm{M,in,avg}}$	$\mathbb{R}$	W	average used power in cycle
$p^{\mathrm{M,in}}$	$\mathbb{R}$	W	EM power requirement drawn from battery
$p^{\mathrm{M,L}}$	$\mathbb{R}$	W	EM power loss
$\overline{p}^{\mathrm{M,out}}$	$\mathbb{R}$	W	EM maximum design power
$p^{\mathrm{M,L,R}}$	$\mathbb{R}$	W	EM power loss on convex hull
t^M	$[0, \overline{t}^\mathrm{M}]$	N m	EM torque
$t^{\mathrm{M,R}}$	$[0, \overline{t}^\mathrm{M}]$	N m	EM torque on convex hulls
$\overline{t}^\mathrm{M}$	$\mathbb{R}$	N m	EM maximum torque

EM — electric machine

$$\begin{aligned}
\min \quad & p^{\mathrm{M,in,avg}} && && (4.43)\\
\text{s.t.} \quad & && && \\
& \sum_{s\in\mathcal{S}}\sum_{k\in\mathcal{K}} \pi_{s,k} p^{\mathrm{M,in}}_{s,k} \leq p^{\mathrm{M,in,avg}} && && (4.44)\\
& \underline{I} \leq i_t \leq \overline{I} && \forall t \in \mathcal{T} && (4.45)\\
& i_{s,k} \leq i_t + B\left(1 - b_{t,s,k}\right) && \forall t \in \mathcal{T}, \forall s \in \mathcal{S}, \forall k \in \mathcal{K} && (4.46)\\
& i_{s,k} \geq i_t - B\left(1 - b_{t,s,k}\right) && \forall t \in \mathcal{T}, \forall s \in \mathcal{S}, \forall k \in \mathcal{K} && (4.47)\\
& \sum_{t\in\mathcal{T}} b_{t,s,k} = 1 && \forall s \in \mathcal{S}, \forall k \in \mathcal{K} && (4.48)\\
& t^{\mathrm{M}}_{s,k} i_{s,k} \eta^{\mathrm{G}} = T^{\mathrm{W}}_{s,k} && \forall s \in \mathcal{S}, \text{ if } k = 0 && (4.49)\\
& t^{\mathrm{M}}_{s,k} i_{s,k} = \eta^{\mathrm{G}} T^{\mathrm{W}}_{s,k} && \forall s \in \mathcal{S}, \text{ if } k = 1 && (4.50)\\
& n^{\mathrm{M}}_{s,k} = i_{s,k} N^{\mathrm{W}}_{s,k} && \forall s \in \mathcal{S}, \forall k \in \mathcal{K} && (4.51)\\
& \overline{p}^{\mathrm{M,out}} = \overline{N}^{\mathrm{M}} \frac{2\pi}{60} A_{\mathrm{d}} \overline{t}^{\mathrm{M}} && && (4.52)\\
& n^{\mathrm{M}}_{s,k} \frac{2\pi}{60} t^{\mathrm{M}}_{s,k} \leq \overline{p}^{\mathrm{M,out}} && \forall s \in \mathcal{S}, \forall k \in \mathcal{K} && (4.53)\\
& n^{\mathrm{M}}_{s,k} \leq \overline{N}^{\mathrm{M}} && \forall s \in \mathcal{S}, \forall k \in \mathcal{K} && (4.54)\\
& t^{\mathrm{M}}_{s,k} \leq \overline{t}^{\mathrm{M}} && \forall s \in \mathcal{S}, \forall k \in \mathcal{K} && (4.55)\\
& k^{\mathrm{M,p}} = \frac{\overline{p}^{\mathrm{M,out}}}{P_{\mathrm{ref}}} && && (4.56)\\
& p^{\mathrm{M}}_{s,k} = t^{\mathrm{M}}_{s,k} \frac{2\pi}{60} n^{\mathrm{M}}_{s,k} && \forall s \in \mathcal{S}, \forall k \in \mathcal{K} && (4.57)\\
& p^{\mathrm{M,L,1}}_{s,k} = k^{\mathrm{M,p}} P_{\mathrm{L,\ ref,\ 1,k}} \left(\frac{t^{\mathrm{M}}_{s,k}}{\overline{t}^{\mathrm{M}}}\right) \left(\frac{n^{\mathrm{M}}_{s,k}}{\overline{N}^{\mathrm{M}} A_{\mathrm{d}}}\right)^{3.93} && \forall s \in \mathcal{S}, \text{ if } k = 0 && (4.58)\\
& p^{\mathrm{M,L,2}}_{s,k} = k^{\mathrm{M,p}} P_{\mathrm{L,\ ref,\ 2,k}} \left(\frac{t^{\mathrm{M}}_{s,k}}{\overline{t}^{\mathrm{M}}}\right) && \forall s \in \mathcal{S}, \text{ if } k = 0 && (4.59)\\
& p^{\mathrm{M,L,3}}_{s,k} = k^{\mathrm{M,p}} P_{\mathrm{L,\ ref,\ 3,k}} \left(\frac{t^{\mathrm{M}}_{s,k}}{\overline{t}^{\mathrm{M}}}\right)^{2} && \forall s \in \mathcal{S}, \text{ if } k = 0 && (4.60)\\
& p^{\mathrm{M,L,const}}_{s,k} = k^{\mathrm{M,p}} P_{\mathrm{L,ref,const,k}} && \forall s \in \mathcal{S}, \text{ if } k = 0 && (4.61)
\end{aligned}$$

$$p_{s,k}^{\mathrm{M,L,1}} = k^{\mathrm{M,p}} P_{\mathrm{L,\ ref,\ 1,k}} \left(\frac{t_{s,k}^{\mathrm{M}}}{\overline{t}^{\mathrm{M}}} \right) \qquad \forall s \in \mathcal{S}, \text{ if } k = 1 \tag{4.62}$$

$$p_{s,k}^{\mathrm{M,L,2}} = k^{\mathrm{M,p}} P_{\mathrm{L,\ ref,\ 2,k}} \left(\frac{t_{s,k}^{\mathrm{M}}}{\overline{t}^{\mathrm{M}}} \right) \qquad \forall s \in \mathcal{S}, \text{ if } k = 1 \tag{4.63}$$

$$p_{s,k}^{\mathrm{M,L,3}} = k^{\mathrm{M,p}} P_{\mathrm{L,\ ref,\ 3,k}} \left(\frac{t_{s,k}^{\mathrm{M}}}{\overline{t}^{\mathrm{M}}} \right) \left(\frac{n_{s,k}^{\mathrm{M}}}{\overline{N}^{\mathrm{M}} A_{\mathrm{d}}} \right)^{7.36} \qquad \forall s \in \mathcal{S}, \text{ if } k = 1 \tag{4.64}$$

$$p_{s,k}^{\mathrm{M,L,4}} = k^{\mathrm{M,p}} P_{\mathrm{L,\ ref,\ 4,k}} \left(\frac{t_{s,k}^{\mathrm{M}}}{\overline{t}^{\mathrm{M}}} \right)^{2.64} \qquad \forall s \in \mathcal{S}, \text{ if } k = 1 \tag{4.65}$$

$$p_{s,k}^{\mathrm{M,L,const}} = k^{\mathrm{M,p}} P_{\mathrm{L,ref,const,k}} \qquad \forall s \in \mathcal{S}, \text{ if } k = 1 \tag{4.66}$$

$$p_{s,k}^{\mathrm{M,in}} \geq p_{s,k}^{\mathrm{M}} + \sum_{j=1}^{3} p_{s,k}^{\mathrm{M,L},j} + p_{s,k}^{\mathrm{M,L,const}} \qquad \forall s \in \mathcal{S}, \text{ if } k = 0 \tag{4.67}$$

$$p_{s,k}^{\mathrm{M,in}} \geq p_{s,k}^{\mathrm{M}} + \sum_{j=1}^{4} p_{s,k}^{\mathrm{M,L},j} + p_{s,k}^{\mathrm{M,L,const}} \qquad \forall s \in \mathcal{S}, \text{ if } k = 1 \tag{4.68}$$

$$p_{s,k}^{\mathrm{M,in}} \leq \overline{P}^{\mathrm{M,\ in}} \qquad \forall s \in \mathcal{S}, \forall k \in \mathcal{K} \tag{4.69}$$

$$t_{s,k,r}^{\mathrm{M,R}} i_{s,k} \eta^{\mathrm{G}} = T_{s,k,r}^{\mathrm{W,R}} \qquad \forall s \in \mathcal{S}, \text{ if } k = 0, \forall r \in \mathcal{R} \tag{4.70}$$

$$t_{s,k,r}^{\mathrm{M,R}} i_{s,k} = \eta^{\mathrm{G}} T_{s,k,r}^{\mathrm{W,R}} \qquad \forall s \in \mathcal{S}, \text{ if } k = 1, \forall r \in \mathcal{R} \tag{4.71}$$

$$n_{s,k,r}^{\mathrm{M,R}} = i_{s,k} N_{s,k,r}^{\mathrm{W,R}} \qquad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \tag{4.72}$$

$$n_{s,k,r}^{\mathrm{M,R}} \frac{2\pi}{60} t_{s,k,r}^{\mathrm{M,R}} \leq \overline{p}^{\mathrm{M,out}} \qquad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \tag{4.73}$$

$$n_{s,k,r}^{\mathrm{M,R}} \leq \overline{N}^{\mathrm{M}} \qquad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \tag{4.74}$$

$$t_{s,k,r}^{\mathrm{M,R}} \leq \overline{t}^{\mathrm{M}} \qquad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \tag{4.75}$$

$$p^{\mathrm{M,L,R,1}}_{s,k,r} = k^{\mathrm{M,p}} P_{\mathrm{L,\ ref,\ 1,K}} \left(\frac{t^{\mathrm{M,R}}_{s,k,r}}{\overline{t}^{\mathrm{M}}}\right) \left(\frac{n^{\mathrm{M,R}}_{s,k,r}}{\overline{N}^{\mathrm{M}} A_{\mathrm{d}}}\right)^{3.93} \quad \forall s \in \mathcal{S}, \text{ if } k = 0, \forall r \in \mathcal{R} \tag{4.76}$$

$$p^{\mathrm{M,L,R,2}}_{s,k,r} = k^{\mathrm{M,p}} P_{\mathrm{L,\ ref,\ 2,K}} \left(\frac{t^{\mathrm{M,R}}_{s,k,r}}{\overline{t}^{\mathrm{M}}}\right) \quad \forall s \in \mathcal{S}, \text{ if } k = 0, \forall r \in \mathcal{R} \tag{4.77}$$

$$p^{\mathrm{M,L,R,3}}_{s,k,r} = k^{\mathrm{M,p}} P_{\mathrm{L,\ ref,\ 3,K}} \left(\frac{t^{\mathrm{M,R}}_{s,k,r}}{\overline{t}^{\mathrm{M}}}\right)^{2} \quad \forall s \in \mathcal{S}, \text{ if } k = 0, \forall r \in \mathcal{R} \tag{4.78}$$

$$p^{\mathrm{M,L,R,const}}_{s,k,r} = k^{\mathrm{M,p}} P_{\mathrm{L,ref,const,k}} \quad \forall s \in \mathcal{S}, \text{ if } k = 0, \forall r \in \mathcal{R} \tag{4.79}$$

$$p^{\mathrm{M,L,R,1}}_{s,k} = k^{\mathrm{M,p}} P_{\mathrm{L,\ ref,\ 1,k}} \left(\frac{t^{\mathrm{M,R}}_{s,k}}{\overline{t}^{\mathrm{M}}}\right) \quad \forall s \in \mathcal{S}, \text{ if } k = 1, \forall r \in \mathcal{R} \tag{4.80}$$

$$p^{\mathrm{M,L,R,2}}_{s,k} = k^{\mathrm{M,p}} P_{\mathrm{L,\ ref,\ 2,k}} \left(\frac{t^{\mathrm{M,R}}_{s,k}}{\overline{t}^{\mathrm{M}}}\right) \quad \forall s \in \mathcal{S}, \text{ if } k = 1, \forall r \in \mathcal{R} \tag{4.81}$$

$$p^{\mathrm{M,L,R,3}}_{s,k} = k^{\mathrm{M,p}} P_{\mathrm{L,\ ref,\ 3,k}} \left(\frac{t^{\mathrm{M,R}}_{s,k}}{\overline{t}^{\mathrm{M}}}\right) \left(\frac{n^{\mathrm{M,R}}_{s,k}}{\overline{N}^{\mathrm{M}} A_{\mathrm{d}}}\right)^{7.36} \quad \forall s \in \mathcal{S}, \text{ if } k = 1, \forall r \in \mathcal{R} \tag{4.82}$$

$$p^{\mathrm{M,L,R,4}}_{s,k} = k^{\mathrm{M,p}} P_{\mathrm{L,\ ref,\ 4,k}} \left(\frac{t^{\mathrm{M,R}}_{s,k}}{\overline{t}^{\mathrm{M}}}\right)^{2.64} \quad \forall s \in \mathcal{S}, \text{ if } k = 1, \forall r \in \mathcal{R} \tag{4.83}$$

$$p^{\mathrm{M,L,R,const}}_{s,k} = k^{\mathrm{M,p}} P_{\mathrm{L,ref,const,k}} \quad \forall s \in \mathcal{S}, \text{ if } k = 1, \forall r \in \mathcal{R} \tag{4.84}$$

$$p^{\mathrm{M,R}}_{s,k,r} = t^{\mathrm{M,R}}_{s,k,r} \frac{2\pi}{60} n^{\mathrm{M,R}}_{s,k,r} \quad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \tag{4.85}$$

$$p^{\mathrm{M,R,in}}_{s,k,r} \geq p^{\mathrm{M,R}}_{s,k,r} + p^{\mathrm{M,L,R,1}}_{s,k,r} + p^{\mathrm{M,L,R,2}}_{s,k,r} + p^{\mathrm{M,L,R,3}}_{s,k,r} + p^{\mathrm{M,L,R,const}}_{s,k,r} \quad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \tag{4.86}$$

$$p^{\mathrm{M,R,in}}_{s,k,r} \leq \overline{P}^{\mathrm{M,\ in}} \quad \forall s \in \mathcal{S}, \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \tag{4.87}$$

Constraint (4.56) is used to add the scaling with the scaling factor $k^{\mathrm{M,p}}$ of the EM based on the already introduced maximum power variable $\overline{p}^{\mathrm{M}}$. The parameter P_{ref} is used as a reference for the motor power. Constraint (4.57) models the shaft power.
As already presented in Eq. (4.39), we add further constraints for the modeling of the efficiency map $\mathcal{M}$. Here, we use Constraints (4.58) – (4.61) to model specific losses for the driving domain. To be able to scale the efficiency map and accordingly the losses, we add the scaling factor $k^{\mathrm{M,p}}$ in each constraint. The Constraints (4.62) to (4.66) are used to model the EM efficiency map in the fourth quadrant.
Constraint (4.67) and (4.68) model the required power that must be supplied by the battery in driving mode and is recuperated. To be compliant with the GP requirements, we also model an upper bound of the total power $p^{\mathrm{M,in}}$ in (4.69). Within the optimized solution this bound constraint should be inactive.
The remaining Constraints (4.70) to (4.87) model the relevant parts for the selected corner points of the convex hulls to be able to derive a more robust solution, as already introduced in Section 4.2.5.
The mass is not modeled as a variable within the MIGP, as it was done in the previously shown MINLP. The focus of the following sections lies on the efficient modeling and solving, exemplified by the powertrain conceptual design. Instead, we predefine the battery size in advance to the powertrain sizing and control strategy computations. Either with a bisection solving approach or with an explicit modeling it could also be added in a later step.

4.3.3 Solution Approaches

In the following, three different approaches to solve the given MIGP are shown. To solve the given MIGP, the basic idea is a separation of first and second-stage decisions which can improve the solving performance. The second stage decisions are in this use case only combinatorial decisions for the control strategy within the usage phase. The first-stage decisions are only continuous decisions for scaling the EM and selecting the transmission ratios. As seen previously in objective (4.43) to constraint (4.87), this results in a geometric program for the continuous decisions, which can be solved efficiently[93], if the assignments for the control strategy are known. In general, these are only known for corner cases like a continuous variable transmission (CVT), where each scenario has its own transmission ratio variable, or for a

[93] Boyd, Kim, Vandenberghe, and Hassibi, "A Tutorial on Geometric Programming", ([19], 2007)

single-speed transmission, where each scenario has the same transmission ratio assigned. For multi-speed transmissions this assignment of a transmission ratio and scenario is not trivial, since it also depends on the first stage decisions. Therefore, it is not suitable to divide the optimization program in two independent optimization programs to receive the optimized system design.

Brute Force

As a first approach, we iterate over all possible combinatorial control-strategy solutions. As a result, in each assignment only a GP has to be solved to dimension the components of the powertrain, since the second-stage decisions are fixed in each iteration. This approach reduces the MIGP to a series of GP evaluations. This brute force approach is only useable for a small quantity of scenarios, since the number of possible solutions increases rapidly, as shown in Section 4.2.7. To increase the performance of this approach, it is beneficial to evaluate only one solution of multiple symmetric solutions, cf. Section 4.2.7. This is achieved with a lexicographic ordering of all binary assignments before optimization of each instance. After using a lexicographic ordering of the binary variable assignment vectors, we consider only the first half. With this approach the solution time is reduced by approximately 50 % in comparison to evaluate all possible assignment vectors. It is important to mention that the used approach with a lexicographic ordering is only valid for two-speed transmissions. For three- or more-speed transmission the subset with all removed symmetric binary assignments has to be chosen differently.
The brute force approach is used as a comparison to check the global optimality and performance of the two remaining approaches that are shown in the following.

Generalized Benders Decomposition

A decomposition approach based on a generalized Benders decomposition (GBD), cf. Section 2.1.6, is shown as a second solution approach. As stated in this Section, a generalized Benders decomposition is suitable to compute global optimal solutions of a MINLP, if the resulting subproblem becomes convex when fixing specific complicating variables. In the given case, the complicating variables are the binary control variables $b_{t,s,k}$. When fixing these variables, the remaining NLP has a special structure and becomes a GP, which can be transformed in a convex optimization program by using a variable transformation in the log-domain. This convexification of the subproblem of the general nonconvex MINLP has from a computational perspective to be

considered as very beneficial.
The usage of a generalized Benders decomposition for solving the conceptual design problem in conjunction with a GP-compatible modeling has until now only been shown for very few more abstract examples[94]. Many open research questions for the applicability and modeling of MIGPs still remain. Therefore, the approach shown here extends the examples in the literature considerably by presenting a practical example use case and a modern implementation with a sophisticated computational evaluation.

Algorithm 2: Decomposition Approach for MIGP

input : Set of Scenarios $\mathcal{S}$; Main program $\mathrm{P}_{\mathrm{main}}$; Subproblem $\mathrm{P}_{\mathrm{sub}}$
$N_{\mathrm{iter,max}} = 100$; $\epsilon = 0.000005$;
call_heuristic $\in \{0, 1\}$; heuristic H
output : Optimized powertrain design with control strategy

```
j = 0;
Set UB = ∞ and LB = −∞;
if call_heuristic = 1 then
    Call heuristic H to get a first estimate for a good assignment of all
      binary control variables b_{t,s,k};
    Retrieve current objective value as UB^j;
    if UB^j < UB then
        UB = UB^0
else
    Solve main program P_main to get a first estimate for all binary
      values b_{t,s,k} and the current lower bound LB^j;
    if LB^j > LB then
        LB = LB^j
while UB − LB > ϵ AND j < N_{iter,max} do
    j = j + 1;
    Solve subproblem P_sub with given binary assignments for b_{t,s,k};
    Retrieve dual variables λ from the solution;
    if UB^j < UB then
        UB = UB^j
    Add optimality cut based on dual variables to the main problem;
    Solve main problem P_main; Retrieve LB^j and binary assignment;
    if LB^j > LB then
        LB = LB^j
    Set binary assignments (b_{t,s,k}) based on result in P_main;
```

94 Choi and Bricker, "Geometric Programming with Several Discrete Variables: Algorithms employing Generalized Benders' Decomposition", ([34], 1995)

$$\min \quad \mu^{\mathrm{B}} \tag{4.88}$$

$$\text{s.t.} \quad \sum_{t\in\mathcal{T}} b_{t,s,k} = 1 \qquad \forall s \in \mathcal{S}, k \in \mathcal{K} \tag{4.89}$$

$$\sum_{s\in\mathcal{S}} b_{t,s,k} \leq |\mathcal{S}| - 1 \qquad \forall t \in \mathcal{T} \tag{4.90}$$

$$\mu^{\mathrm{B}} \geq \tilde{f}^{(j)}_{0,k} - \tilde{B} \sum_{s\in\mathcal{S}} \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{K}} \left(\tilde{\lambda}^{\mathrm{La},(j)}_{s,g,k,<} + \tilde{\lambda}^{\mathrm{La},(j)}_{s,g,k,>}\right) (1 - b_{s,g,k}) \qquad \forall j \in \mathcal{J} \tag{4.91}$$

$$\mu^{\mathrm{B}} \in \mathbb{R}; 0 \leq \mu^{\mathrm{B}} \leq \overline{\mu}^{\mathrm{B}} \tag{4.92}$$

Figure 4.7 – Main program within the Generalized Benders Decomposition approach

The used decomposition approach is shown in Algorithm 2 in more detail. The algorithm solves iteratively a main problem $\mathrm{P}_{\mathrm{main}}$ and a sub-problem $\mathrm{P}_{\mathrm{sub}}$. The main problem aggregates the complicating variables, in this case the binary control variables. It results for the powertrain conceptual design problem in a Mixed-Integer Linear Problem, which can be solved with any MILP-Solver. As a result, we get in each iteration j a new control strategy for all considered scenarios. Furthermore, the objective of $\mathrm{P}_{\mathrm{main}}$ refers to a current lower bound LB^j. With this, we can derive the best lower bound $\mathrm{LB}(j)$ in each iteration j by selecting the lowest value found so far.

With this given control strategy, the MIGP is reduced to a GP, which can be solved very efficiently with a state-of-the-art GP solver. The objective value of the solved sub-problem $\mathrm{P}_{\mathrm{sub}}$ yields a current estimate for the upper bound UB^j.

The gap $\mathrm{UB}(j) - \mathrm{LB}(j)$ between the highest lower bound and the lowest upper bound is used to terminate the algorithm, when it reaches a predefined value ϵ. To avoid infinitely long computations, as a second termination criterion a predefined maximum number of iterations has also been added.

While running the algorithm, the relaxation of the MIGP is continuously reduced to close the gap between the upper and lower bound. This is done by adding in each evaluation of the main problem $\mathrm{P}_{\mathrm{main}}$ an additional optimality cut.

The main problem is shown in Figure 4.7. The objective is the newly introduced continuous variable μ^{B}. Additionally, the logical relations between the binary decision variables for the control strategy are given by Constraint (4.89) and Constraint (4.90). The first Constraint ensures that only one transmission ratio is selected for each scenario. The second Constraint ensures that only solutions are selected, in which both transmission ratios are used. This forbids the single-speed solution.

The linking between the newly introduced variable μ^{B} and the binary decision variables $b_{t,s,k}$ is achieved by the optimality cuts (4.91). A detailed explanation about the optimality cut generation is given in the following.

When conducting the optimization, we also introduced a rounding scheme for binary variables comparable to the approach shown by Fischetti, Ljubić, and Sinnl[95] to ensure a correct binary variable setting in the subproblem. This reduces the effects of a transfer of computational imprecision of the solution of the main problem on the solution of the following subproblem. Without this rounding of binary variables the generalized Benders decomposition would sometimes lose its ability to close the gap due to a definition of redundant optimality cuts.

The adding of feasibility cuts, cf. Sec. 2.1.6, in the main problem is not required in the given use case, since the control strategy derived by the main problem is always feasible due to the combinatorial nature of the given problem. The optimality cuts were derived by using the approach shown in Sec. 2.1.6 based on the Lagrangian and the dual variables from the solution of the subproblem.

In the original problem formulation, we couple the binary decision variables for the control strategy $b_{t,s,k}$ with the continuous transmission ratios i_t by the following big-M constraints:

$$i_s \leq i_t + B\,(1 - b_{t,s,k}) \qquad \forall t \in \mathcal{T}, \tag{4.93a}$$

$$i_s \geq i_t - B\,(1 - b_{t,s,k}) \qquad \forall t \in \mathcal{T} \tag{4.93b}$$

These constraints are the only constraints which contain both types of variables: complicating variables for the main problem and continuous variables from the subproblem. These constraints are not GP-compliant. But if we only consider the projection on the variable space of the subproblem $\mathrm{P}_{\mathrm{sub}}$, we derive the constraints:

$$i_s \leq i_t, \tag{4.94a}$$

$$i_s \geq i_t, \tag{4.94b}$$

for the active subset of control assignments. These constraints are monomials and therefore GP-compliant. They are also invariant to the log-transformation.

[95] Fischetti, Ljubić, and Sinnl, "Redesigning Benders Decomposition for Large-scale Facility Location", ([55], 2017)

Since the logic of the assignment must also hold in the log-space after conducting a GP transformation, we can also add the big-M constraints in the log-space with the transformed big-M constant $\tilde{B} = \log(B)$:

$$\tilde{i}_s \leq \tilde{i}_t + \tilde{B}\left(1 - b_{t,s,k}\right) \qquad \forall t \in \mathcal{T}, \tag{4.95a}$$
$$\tilde{i}_s \geq \tilde{i}_t - \tilde{B}\left(1 - b_{t,s,k}\right) \qquad \forall t \in \mathcal{T}. \tag{4.95b}$$

This then results in linear mixed-integer constraints in the log-space.
If the binary variables are set to zero, the big-M approach leads to two additional constraints per scenario that per definition do not affect the given optimization program:

$$\tilde{i}_s \leq \tilde{i}_t + \tilde{B} \qquad \forall t \in \mathcal{T}, \tag{4.96a}$$
$$\tilde{i}_s \geq \tilde{i}_t - \tilde{B} \qquad \forall t \in \mathcal{T}. \tag{4.96b}$$

Since the log-transfomed GP in the subproblem $\mathrm{P_{sub}}$ is a convex optimization program, we can also employ duality theory for convex optimization programs. When the GP is solved to optimality, the primal and dual of the GP are equal. Then, the *complementary slackness* states that "(...) roughly speaking, (...) the ith optimal Lagrange multiplier is zero unless the ith constraint is active at the optimum."[96] Since, per definition of the big-M constraints, only the projections on the GP space of the selected binary assignments are active, we also only have to consider the dual variables for these constraints in the definition of the optimality cuts for the main problem $\mathrm{P_{main}}$. Since the two monomial constraints in (4.94) are used to model the selection of the transmission ratio in a given scenario, we also have to consider both dual variables $\tilde{\lambda}^{\mathrm{La},(j)}_{s,t,k,<}$ and $\tilde{\lambda}^{\mathrm{La},(j)}_{s,t,k,>}$ within the cut generation. The used optimality cuts in Eq. (4.91) use these values, besides the current objective value $\tilde{f}^{(j)}_{0,k}$ of the solved GP in iteration j and the log-transformed big-M parameter B. The solving performance is reduced with a selection of a larger big-M parameter than necessary. Hence, the smallest possible selection is the best choice, as commonly done for big-M constraints in linear programs.

Primal Heuristic

The third approach to solve the given MIGP is a newly developed primal heuristic. This approach results in fast solution times, but does not guarantee a global-optimal solution. It is based on an iterative two-step process and is only valid for a two-speed transmission. The algorithm is shown in detail in Alg. 3. It can also be extended to a higher number of transmission ratios.

[96] Boyd and Vandenberghe, *Convex Optimization*, ([20], 2004, p. 243)

Algorithm 3: Pseudocode for the primal heuristic to solve the powertrain conceptual design MIGP.

input : MIGP model
empty set of assigned binary control variables in driving and recuperation mode $\mathcal{Y} = \varnothing$; $\mathcal{Y}^{\mathrm{R}} = \varnothing$
percentage value $\gamma = 0.1$; Loop decision variables $q = 0$; $q^{\mathrm{R}} = 0$

output : Optimized powertrain design with according control strategy

while $q = 0$ *or* $q^{R} = 0$ **do**
  solve relaxed MIGP model with partial assignments $\mathcal{Y}$ and $\mathcal{Y}^{\mathrm{R}}$;
  get set of unassigned scenarios in driving mode $\mathcal{U}$;
  get set of unassigned scenarios in recuperation mode $\mathcal{U}^{\mathrm{R}}$;
  if $\mathcal{U} = \varnothing$ **then**
    $q = 1$;
  else
    Get current maximum transmission value $\bar{i}$ in $\mathcal{U}$;
    Get current minimum transmission value $\underline{i}$ in $\mathcal{U}$;
    $\Delta i = \bar{i} - \underline{i}$;
    Get upper rounding limit $\bar{l} = \bar{i} - \gamma \Delta i$;
    Get lower rounding limit $\underline{l} = \underline{i} + \gamma \Delta i$;
    for $k \in \mathcal{U}$ **do**
      if $i_k > \bar{l}$ **then**
        $b_{k,0} = 1$; $b_{k,1} = 0$
      if $i_k < \underline{l}$ **then**
        $b_{k,0} = 0$; $b_{k,1} = 1$
  if $\mathcal{U}^{R} = \varnothing$ **then**
    $q^{\mathrm{R}} = 1$;
  else
    Get current maximum transmission value $\bar{i}^{\mathrm{R}}$ in $\mathcal{U}^{\mathrm{R}}$;
    Get current minimum transmission value $\underline{i}^{\mathrm{R}}$ in $\mathcal{U}^{\mathrm{R}}$;
    $\Delta i^{\mathrm{R}} = \bar{i}^{\mathrm{R}} - \underline{i}^{\mathrm{R}}$;
    Get upper rounding limit $\bar{l}^{\mathrm{R}} = \bar{i}^{\mathrm{R}} - \gamma \Delta i^{\mathrm{R}}$;
    Get lower rounding limit $\underline{l}^{\mathrm{R}} = \underline{i}^{\mathrm{R}} + \gamma \Delta i^{\mathrm{R}}$;
    for $k \in \mathcal{U}$ **do**
      if $i_k^{R} > \bar{l}^{R}$ **then**
        $b_{k,0}^{\mathrm{R}} = 1$; $b_{k,1}^{\mathrm{R}} = 0$
      if $i_k^{R} < \underline{l}^{R}$ **then**
        $b_{k,0}^{\mathrm{R}} = 0$; $b_{k,1}^{\mathrm{R}} = 1$

Starting from a complete relaxation of all binary variables, which is equivalent to the CVT solution, the binary control variables are iteratively assigned. Therefore, further constraints, as shown in (4.94), are added to the CVT solution, which restrict the transmission ratio variable i_s in specific scenarios to the global decision variables i_t which are used in multiple scenarios. This is equivalent to setting specific binary variables $b_{t,s,k}$ to one and the opposite to zero. Different to the decomposition approach not all binary variables are set in this approach in each iteration, but the number of assigned scenarios to the variables i_t is continuously growing. The algorithm terminates when all scenarios $s \in \mathcal{S}$ are assigned to one of the two transmission ratios $i_t; t \in \mathcal{T}$.
A rounding scheme is employed to select scenarios where a specific transmission ratio is used. The CVT solution results in as many values of transmission ratios as there are scenarios. But some of these values are close to each other. This closeness between assigned transmission ratios in different scenarios is used to assign close to each other located scenarios in the CVT solution to discrete transmission ratios. In Alg. 3, we use a closeness factor of $\gamma = 0.1$, which relates to assign the 10 % closest values to the given reference values. As a reference, we use the highest and lowest transmission ratio within the CVT solution as a starting point. With this approach we support the manual assignment heuristic to assign scenarios with a high torque and low speed to one transmission ratio and one with low torque and high speed to another.
Since the CVT solution is in general not feasible anymore if specific binary variables are assigned, a recomputation of this extended NLP is done. With this newly sized powertrain components from the first stage, a further iteration is started to assign further binary variables to either the first or second transmission ratio. After this assignment, a recomputation of the powertrain components is done again with the given new additional assignments of binary variables. This procedure is iteratively executed until all available scenarios are assigned.
With the given approach, we are able to compute in a low number of iterations a near-optimal solution by using the binary assignment of the given heuristic and the consecutive solving of GPs even for a high number of scenarios. The performance of this heuristic, next to the decomposition approach, is evaluated with a high number of computations in comparison to the brute force approach within the following.

Table 4.6 – Influencing variables for the computational datasets to evaluate the MIGP solving approaches

	dataset 1
number of clusters	$\{6, 7, 8, 9, 10, 11, 12\}$
cycles	WLTC, NYCC, Artemis Motorway 150, FTP 75, Institute vehicle
mass	$\{1800, 2000\}$
recuperation status	{True, False}
convex hull status	{True, False}
maximum iterations	400

	dataset 2
number of clusters	$\{13, 14, 15, 16\}$
cycles	WLTC, NYCC, Artemis Motorway 150, FTP 75, Institute vehicle
mass	$\{1800, 2000\}$
recuperation status	{True, False}
convex hull status	{True, False}
maximum iterations	600

4.3.4 Results

The performance and correctness of the decomposition approach and the primal heuristic are evaluated in comparison to the brute force approach. Since the computational time required to solve the underlying MIGP depends on the number of binary control decisions, multiple instances with different scenarios were evaluated in a computational study. To allow a comparison between all instances, we conducted the computations on specialized server hardware that was only used for the computations. The scenarios are generated with the method shown in Section 4.2.5.

Based on the complexity of the control strategy in the second stage, we divided the performed evaluation runs in two datasets. The complete set of combinations is shown in Table 4.6. For the first dataset, in total 280 MIGPs were solved. For the second dataset, 160 MIGP evaluations were performed.

Besides four commonly used legislative driving cycles for which scenarios were generated, we also used one driving cycle developed at the Institute of Mechatronic Systems at TU Darmstadt that was already used in the computational study for the (nonconvex) MINLP approach in Sec. 4.2.9.

Table 4.7 – Comparison of solution approaches for 280 instances.

	Decomposition	Decomposition + Heuristic	Heuristic	Brute Force
global identified	279	277	90	280
gap closed	264	262	–	280

Over both datasets, we considered between 6 to 16 scenarios in each instance. For the low number of scenarios in dataset 1, we were able to also compute the global optimal solution by employing the brute force approach. Since the complexity grows exponential, cf. Sec. 4.2.7, we were only able to evaluate the brute force approach on the first dataset. On the second dataset, we then only compared the decomposition with and without a pre-evaluation based on the developed heuristic.
All computations were conducted on a server with an AMD Ryzen 5 5600X and 32 GB RAM. The code was implemented in Python 3[97]. The GP models were implemented by using the Python library GPKit[98]. As a solver for the GP we used MOSEK 9.0[99]. For the implementation of the main problem of the Generalized Benders approach, we used SCIP 7.0.2 with the LP-Solver SoPlex 5.0.2 and as an interface PyScipOpt[100].

Comparison of Solution Approaches

For dataset 1 in total 280 MIGPs were solved. Table 4.7 shows a comparison of the different approaches and the possibility to find the global solution.
The brute force approach was used as a reference for each MIGP. It was executed iteratively until all possible control strategies were evaluated, and the best solution was stored as the global optimal solution. As shown previously, we omitted the computation of symmetric solutions, which improved the solution performance considerably, cf. Sec. 4.3.3. Within each iteration of the brute force approach, we computed a GP with the given binary assignments for the specific control strategy. Due to the convexity of the subproblems, these could be solved very efficiently. Hence, we were able to find the global-optimal solution for all 280 instances.

97 Python, *Python Programming Language*, ([154], 2022)

98 Burnell, Damen, and Hoburg, “GPkit: A Human-centered Approach to Convex Optimization in Engineering Design”, ([28], 2020)

99 Mosek SPA, *Mosek*, ([130], 2022)

100 Maher et al., “PySCIPOpt: Mathematical Programming in Python with the SCIP Optimization Suite”, ([117], 2016)

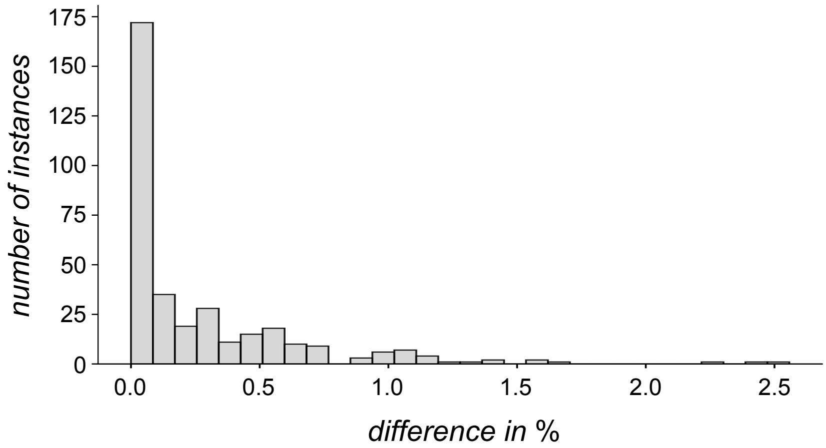

Figure 4.8 – Difference in percent between the best found solution with the heuristic and the found global solution by employing a Benders decomposition approach.

The usage of the decomposition approach with and without a pre-evaluation based on the heuristic can find for most instances the global solution within the given limit of maximum iterations (cf. Table 4.6). At the maximum number of iterations only for one instance we were unable to identify the global optimal solution, when using the decomposition. Within Table 4.7, we also show the number of instances, where the duality gap between the upper and lower bound was closed at the end of computation. From the difference to the first row, it can be seen that the global solution was identified in more instances, even, if the gap could not be closed completely until the end of the maximum iterations per MIGP evaluation.
As a third approach, we also evaluated the heuristic to find the global-optimal solutions. In approximately one third of the MIGPs, we were able to find the global-optimal solution in the first dataset by employing the heuristic. The heuristic does not provide any information about the dual. Hence, no duality gap can be estimated. Since the possibilities for a control-strategy increase with the number of scenarios, it is in general not possible to find the global optimal solution by employing the heuristic solely. Nevertheless, it provides near-optimal solutions. This can be seen in the histogram in Figure 4.8 which shows the error between the global solutions and the best found solutions when using the heuristic for all instances that could be solved to global optimality.
Figure 4.9 shows a comparison of the mean solution times for the brute force approach and the decomposition approach. Besides a run of the decomposition, we also evaluated the performance, if the heuristic was employed before solving

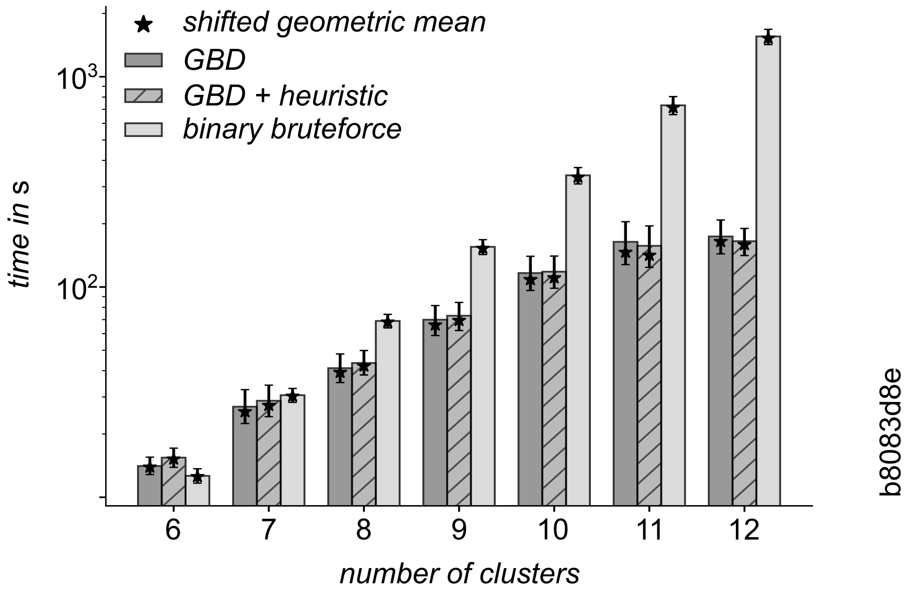

Figure 4.9 – Comparison of the mean required solution time (on a logarithmic scale) for each solving approach for the derived MIGPs in dataset 1. The uncertainty is quantified by using bootstrapping and the 95 %-confidence interval is presented by the error bars. Additionally, the shifted geometric mean with a shift value of 10 is shown as a star ($\star$).

the MIGP with the generalized Benders decomposition algorithm. The idea in using the heuristic in advance was to generate a near-optimal starting solution relatively fast with the heuristic and then using this solution to start the decomposition algorithm.

The results in Figure 4.9 show that the effects of the heuristic on the overall solution time are negligible. The complete time for finding and proving the global optimal solution is not reduced by adding the heuristic in advance to the decomposition. Nevertheless, the comparison with the brute-force approach shows that the solution time with the decomposition approach can be reduced significantly.

For 6 scenarios, the brute-force approach is faster than the decomposition approach. This is caused by the overhead that is required to model the optimization programs and call the main programs' MILP solver. As shown by the logarithmic scale, all approaches are rather fast to solve the given optimization program for this size. But starting from 8 scenarios (which is equivalent to $2^{8-1} - 1 = 127$ different possible control strategies) the overhead of the decomposition approach outperforms the brute force approach. For

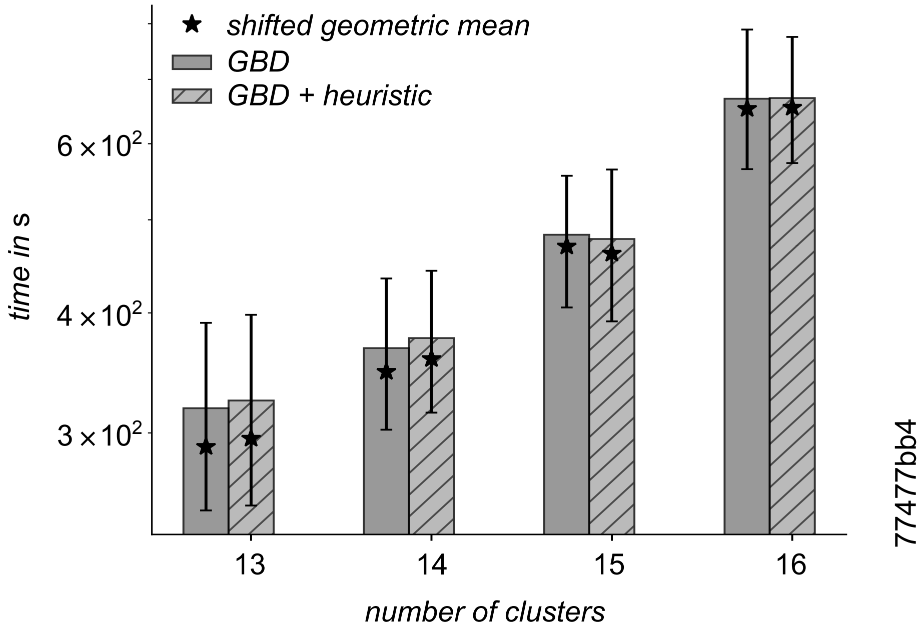

Figure 4.10 – Comparison of the mean required solution time for each solving approach for the derived MIGPs in dataset 2. The uncertainty is quantified by using bootstrapping and the 95 %-confidence interval is presented by the error bars. Additionally, the shifted geometric mean with a shift value of 10 is shown as a star ($\star$).

12 scenarios (2047 possible control strategies) the difference becomes even more significant, since it is almost one order of magnitude faster in the given implementation.

We also quantify the uncertainty of this point estimate by employing bootstrapping. This approach to quantify the uncertainty of arbitrary point estimates was already introduced in Sec. 2.2.4. The 95 %-confidence interval of the arithmetic mean is shown as error bars for each subset in Figure 4.9. Besides the arithmetic mean of the solution time, which is commonly used in engineering, we also present the shifted geometric mean as stars ($\star$) with a shift value 10. The shifted geometric mean is usually used for performance evaluations in the optimization community. The advantage of the shifted geometric mean is its robustness against small and large outliers. But as shown by the arithmetic mean and the uncertainty quantification, the difference between both values is not very high and the shifted geometric mean lies within the uncertainty set.

Dataset 2 was evaluated solely by using the decomposition approach with and without a pre-evaluation with the developed heuristic. The results for 13 to

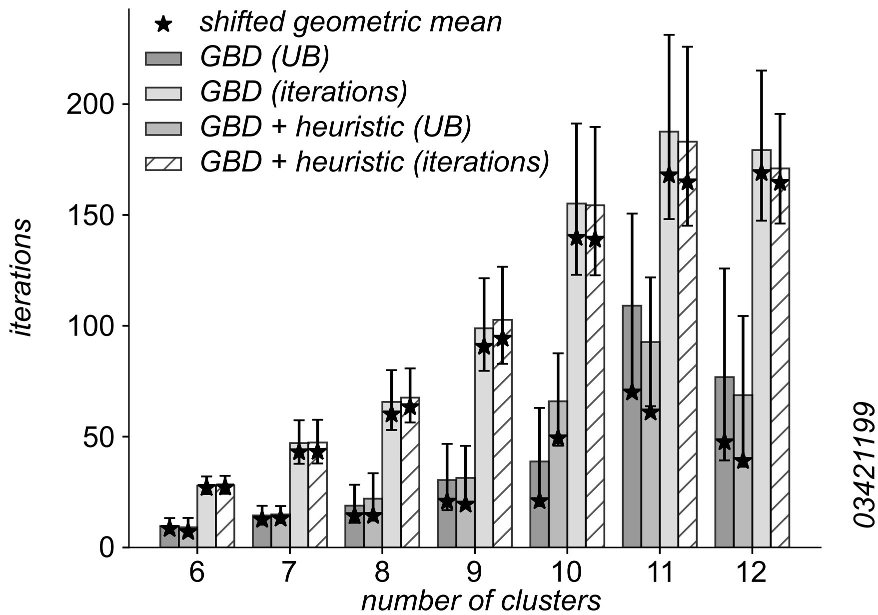

Figure 4.11 – Comparison of the number of iterations required to identify the global optimal solution (marked with UB) and to prove these found solutions to be global optimal (marked with iterations).

16 scenarios are shown in Figure 4.10. The benefit of the heuristic as a first step to estimate a good primal solution is not given for each scenario group. The solving time increases with the considered scenarios, but remains on an acceptable level for evaluations of specific instances to derive an optimized conceptual powertrain design. Furthermore, we only considered instances in the results, which were solved to global-optimality.

The decomposition approach is based on an iterative solving procedure. In Figure 4.11, we evaluate the number of iterations that are required to (a) find the global optimal solution and (b) to prove this global optimality by closing the gap between the primal and dual bound. From an evaluation on dataset 1, we can see that the decomposition approach usually finds the global optimal solution in a much earlier iteration than is required to prove the found solution. It is already found by the algorithm after approximately $30-50\%$ of the total number of iterations that are required for proving the global optimal solution. This underlines the usefulness of this developed approach for solving a given MIGP.

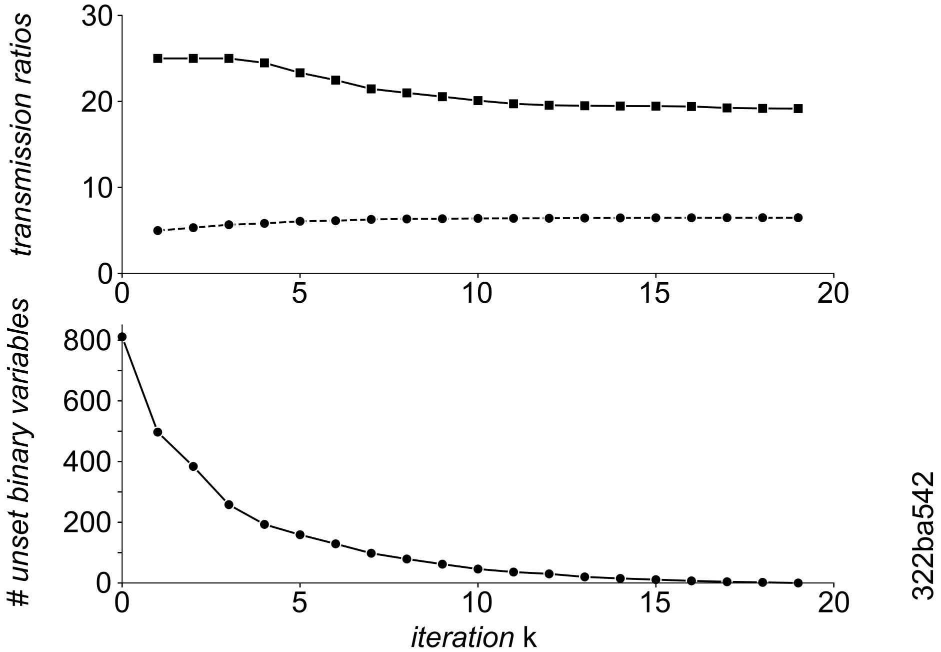

Figure 4.12 – (a) Heuristic transmission ratio assignment over required iterations for WLTC data with 811 scenarios. (b) assigned binary variables by iteration.

Primal Heuristic

The developed primal heuristic that iteratively solves GP models was used until now only for generating a good primal starting solution for the decomposition approach. Unfortunately, it did not accelerate the performance of the decomposition. But its advantage is clearly to find a good solution for a given MIGP rapidly. This performance is shown here with a use case, where not only a few scenarios are used.

The heuristic can be used to compute a control strategy for a complete driving cycle within a few iterations. For the shown use case, we consider exemplarily the WLTC legislative driving cycle. We consider a vehicle mass of $M = 1800\,\mathrm{kg}$ and select only loads in the first quadrant. This results in 811 scenarios (time steps) with torque and speed requirements.

If we reconsider the combinatorial complexity associated with this problem we have $2^{811-1} - 1 \approx 6.8 \times 10^{243}$ possible control strategies.

With the developed heuristic, we are able to derive a good solution that is near global-optimal with only 20 iterations with one GP solving each. For the given problem this was solved in approximately 168 s.

The usage of the considered two transmission ratios and the number of

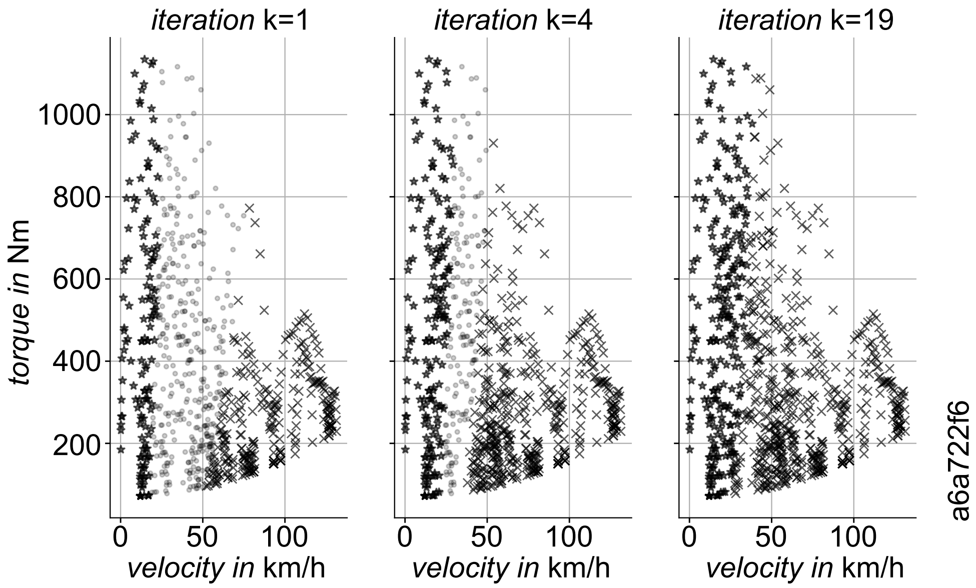

Figure 4.13 – Exemplary iterations of the developed heuristic to derive near-global-optimal solutions for the WLTC. The iterations count from $k = 0$ to $k = 19$. A star ($\star$) marks the usage of the first transmission ratio, while a cross ($\times$) shows the second transmission ratio. Unset binaries are marked with a circle ($\circ$). In iteration $k = 19$ all binary decision variables are set.

unassigned binary variables in each iteration are shown in Figure 4.12. The heuristic iteratively switches between the computation of the first and second-stage variables. Within the first iterations, the number of unset binary variables for the control strategy is reduced the most. In the later iterations only a few scenarios are still unset and assigned to the given transmission ratios. Three exemplified assignment results for the given use case are shown in Figure 4.13: within the first, fourth and 20[st] iteration. The already set binary variables are shown in stars ($\star$) and crosses ($\times$). The unset binary variables in the given iteration are shown as circles ($\circ$).

Engineering Results

Within the last sections, we presented a computational study to evaluate the applicability of the decomposition approach and the usage of the developed heuristic. Within this section, we will show briefly the results obtained by using the MIGP approach from an engineering point of view.

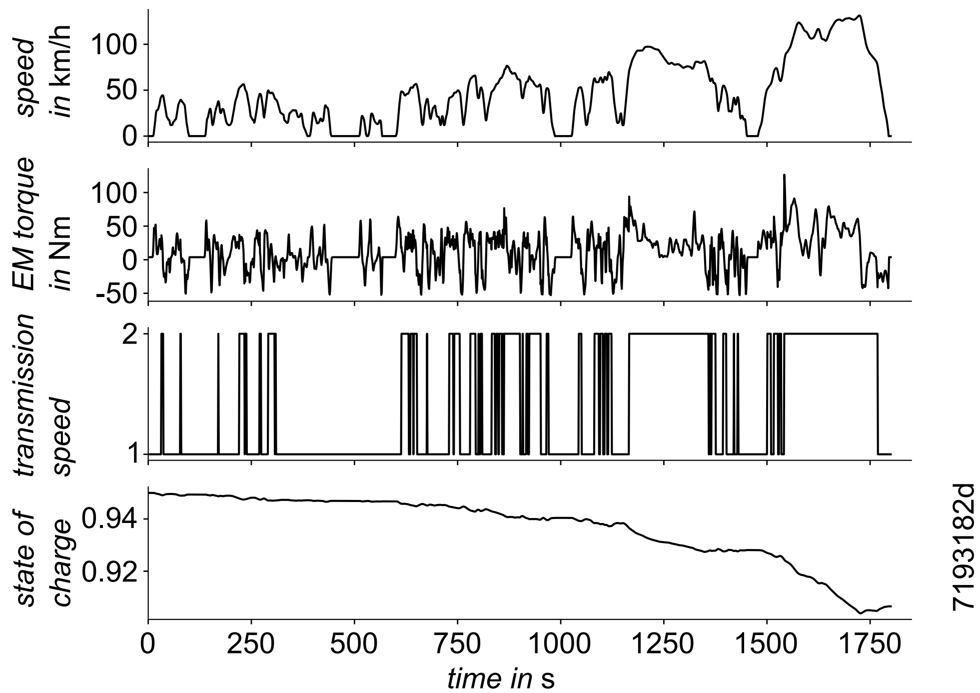

Figure 4.14 – Optimized powertrain system design for the WLTC and a vehicle mass of 1800 kg.

Figure 4.14 shows an exemplified powertrain design that is based on the WLTC driving cycle. The resulting powertrain design is presented according

to the optimized results shown within the supplementary material for the MINLP model presented above[101].
It shows a simultaneous view on the underling driving cycle, the used EM torque, the switching behavior between the two speeds of the optimized powertrain design and the state of charge (SoC) of the used battery. The resulting system design that is based on the MIGP approach behaves similar to the system design optimized by using a genetic algorithm with a higher-fidelity modeling, cf. [105]. This shows that the developed optimization and design approach can be useful to derive first conceptual designs that are based on binary control decisions and a nonlinear system modeling. The MIGP modeling approach leads to a system design that avoids a high number of switching between both transmission ratios within the considered example use case. This behavior is beneficial for a later embodiment design and transfer on a real-world test environment. This underlines the usefulness of this modeling approach, both from an engineering point of view and from an algorithmic point of view.

[101] Leise et al., *Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods (Supplemental)*, ([105], 2021)

Chapter 5

Discussion

At the beginning of this thesis the question was raised how a good trade-off between model accuracy and solving performance for a conceptual design of a technical system can be achieved. To answer this research question, two different use cases, the water distribution system design and the powertrain design for battery electric vehicles (BEVs), were modeled and evaluated.
As known from optimization research[1], there is always a trade-off, if the goal is to derive a very efficient solving algorithm for a specific problem class.
Additionally, when developing and solving optimization-based conceptual design models, engineers always have to trade off the effort required to model and to solve the given system at hand.
The number of model evaluations after a successful modeling in conceptual design is rather low, since the most important result is the system design itself. This is derived with only one evaluation. Even if parameter studies are conducted with a given conceptual design model, the number of evaluations is at least one order of magnitude smaller than in other domains where optimization based approaches are used. In these domains a given model is repeatedly solved multiple times with a different parametrization in each run. Furthermore, solving-performance requirements exist to solve these models up to real-time[2]. Examples for this are demand-planning or trajectory-planning. Consequently, the importance of the efficiency of the solution algorithm shifts to the applicability of the solution procedure for a given problem in conceptual design. Furthermore, the transferability of the given solving procedure on new systems also becomes more important. The applicability in engineering, the algorithmic expedience and the suitability for mastering uncertainty are

[1] Wolpert and Macready, “No Free Lunch Theorems for Optimization”, ([201], 1997)
[2] Chachuat, Srinivasan, and Bonvin, “Adaptation Strategies for Real-Time Optimization”, ([30], 2009)

taken up in this chapter to discuss the developed conceptual design approach against this background.

5.1 Applicability in Engineering

For conventional aircraft the design has "converged to an overriding design scheme"[3] since "the shape of the body and wings remains almost identical for all designs"[3], currently manufactured.
On contrary, if new powertrain concepts with (hybrid-)electric propulsion are considered for aircraft, the optimal design is still an open research question[3]. This example shows that it is required in the conceptual design of a technical system to be able to compare a high number of possible design decisions, even when changes in a sub-system are introduced and solutions for related systems are already identified. The optimal new system can then differ significantly from the already found old solution of the related system. For instance, when introducing (hybrid-)electric powertrains in aircraft, the optimal number of propulsors significantly increases in comparison to a conventional aircraft, when employing an optimization-based approach[3].
The benefits of an optimization-based approach in conceptual design are obvious. This approach is able to provide (i) a quantification of the performance gains, (ii) evaluate and compare a high variety of possible system designs, and (iii) allows a comprehensible preference of solutions due to the predefined objective.
When using optimization-based approaches in the conceptual design, engineers always have the possibility to choose from multiple approaches to derive an optimized system design. This thesis focussed on the usage of an exact optimization approach by modeling the given problem in a domain-specific modeling language for optimization programs, like PyScipOpt[4], JuMP[5] or GPKit[6]. Afterwards, an exact optimization solver or specifically developed heuristics were used to solve the problems. Even though this approach is widespread in the operations research community, it is not within engineering.

[3] Pelz, Leise, and Meck, "Sustainable Aircraft Design – A Review on Optimization Methods for Electric Propulsion with derived Optimal Number of Propulsors", ([147], 2021)

[4] Maher et al., "PySCIPOpt: Mathematical Programming in Python with the SCIP Optimization Suite", ([117], 2016)

[5] Dunning, Huchette, and Lubin, "JuMP: A Modeling Language for Mathematical Optimization", ([46], 2017)

[6] Burnell, Damen, and Hoburg, "GPkit: A Human-centered Approach to Convex Optimization in Engineering Design", ([28], 2020)

For this reason, this thesis is embedded in the research area *technical operations research*[7]. Besides concrete modeling approaches it presents further algorithmic developments and an engineering applicability.
The first impression of a higher effort in modeling of the approach in this thesis is outweighed by a better traceability and extensibility. With this approach different open-source and commercially available solvers can easily be replaced by each other to improve the solving performance with no extra costs in modeling and only minor costs in maintenance. Furthermore, this explicit modeling allows a superior traceability due to the already described optimization problem with the help of the given domain-specific modeling language.
The developed approaches for designing more sustainable systems designs that were presented in the last two chapters can also be discussed within the domain of *multi-disciplinary design optimization* (MDO), which gained attention in the engineering sciences starting in the 1990s[8].
In general, the system design is modeled in an object-oriented manner, where each object can represent a subsystem or discipline. The variables in all objects can then be either optimized together, or only within an object. In the latter approach, only the optimized results are passed-on to the objects which require these values[9].
Following the classification of Martins and Lambe[10], the developed approach for a water distribution system in Chapter 3 and for the powertrain in Chapter 4 can be seen as all-at-once (AAO) approaches, or more precisely as *simultaneous analysis and design* (SAND) approaches. This architecture naming convention was coined by Haftka[11].
Usually local optima are searched within MDO. Furthermore, usually submodels that are only given as black-boxes are integrated in the optimization architecture.
On contrary, since the derived models in this thesis allow for a white-box optimization, the developed solving algorithms can exploit the given model structure and additionally lead to the global optimal solution. This global-optimality is usually not searched for in common MDO approaches. Examples

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[7] Fügenschuh, Lorenz, and Pelz, "OPTE Special Issue on Technical Operations Research (TOR)", ([59], 2021)

[8] Sobieszczanski-Sobieski, *Multidisciplinary Design Optimization: An Emerging New Engineering Discipline*, ([174], 1993)

[9] Perez, Liu, and Behdinan, "Evaluation of Multidisciplinary Optimization Approaches for Aircraft Conceptual Design", ([149], 2004)

[10] Martins and Lambe, "Multidisciplinary Design Optimization: A Survey of Architectures", ([122], 2013)

[11] Haftka, "Simultaneous Analysis and Design", ([72], 1985)

as given by Chernukhin and Zingg[12] show that a search for global optima can have a benefit in engineering design and can lead to better designs.
The associated dual solution that is evaluated within the solution process of the exact optimization program has the great benefit to create a certificate that proves the global optimality, when the gap between the primal and dual is closed. Furthermore, it enables the estimation of the difference between the best solution already found and the best possible solution (currently unfound) within the solution process. With this additional knowledge an engineer can estimate, if an additional investment in further solving time is worth the performance gains of the complete system. Therefore, even if the solving until its global optimum is not always required from an engineering point of view, the shown approach has the benefit to quantify the possible performance gains until the global optimum is found.
The water distribution system in Chapter 3 was first modelled as a (nonconvex) MINLP, which can then be linearized and transformed in a (convex) MILP. By employing the epigraph form given in Eq. (2.8), without loss of generality, the objective can always be modelled as a linear function. But for the constraints, this is not true anymore. In conceptual design, most system models require the integration of at least partly nonlinear constraints. As seen within Chapter 3, one approach that is useful is a linearization, if the total number of nonlinear constraints is reasonable.
A second approach to derive a convex optimization program, is shown in Chapter 4. Here, a new modeling as a (mixed-integer) geometric program is shown. In the engineering sciences, the applicability of this modeling approach is significant, since a lot of physical constraints and scaling laws are of an exponential nature. Already the initial developments of this modeling approach by Zener were motivated by engineering design[13]. Within recent years, cf. Sec. 2.1.5, multiple models have been discovered within conceptual design, i.e. for electronics[14], aircraft[15,16], and water distribution systems[17]. So far, the focus in engineering design optimization with geometric programming

[12] Chernukhin and Zingg, “Multimodality and Global Optimization in Aerodynamic Design”, ([33], 2013)

[13] Zener, “A Mathematical Aid in optimizing Engineering Designs”, ([208], 1961)

[14] Boyd, Kim, Patil, and Horowitz, “Digital Circuit Optimization via Geometric Programming”, ([22], 2005)

[15] Hoburg and Abbeel, “Geometric Programming for Aircraft Design Optimization”, ([75], 2014)

[16] Brown and Harris, “Vehicle Design and Optimization Model for Urban Air Mobility”, ([25], 2020)

[17] Sela Perelman and Amin, “Control of Tree Water Networks: A Geometric Programming Approach”, ([169], 2015)

was focussed solely on continuous GP models, while the approach shown in Chapter 4 extends this to partly discrete MIGP models. This has not been done in this detail within the literature to the knowledge of the author for a conceptual design of a technical system.

Within engineering design, two further results are important besides the optimized system design. First, the change of the objective and topology within the design space. Second, the sensitivity due to changes in the parametrization.

In Chapter 3, different sets of pumps were created by employing a latin-hypercube sampling (LHS). Each subset was then integrated in the derived MINLP formulation and was then optimized. The comparison of the found solutions in Table 3.5 shows the significance of the selected set of pumps on the objective value. This meta-evaluation can also be seen as a discrete sampling within the design space of the pump sets. Due to this sampling, different solutions in this design space can be compared with each other. Besides this meta-level due to the LHS, the developed solution procedure in Chapter 4 for the MIGP allows an equivalent view on the powertrain use case.

The decomposition in a main problem, which specifies a new set of discrete control variables, and a subproblem, which evaluates the performance, results in a similar view. The iterative solution process allows to derive additional information about the performance of local optimal solutions within the design space of the control strategy. Additionally, the modeling as a MIGP has the great benefit to automatically derive sensitivity values for each of the evaluated solutions without further computational costs.

For a general overview about sensitivity analysis within engineering design it is referred to [147][18]. In general, within sensitivity analysis the effects of model inputs on the output are quantified[19]. The developed MIGP modeling and solution approach enables the examination of local sensitivities for each subproblem evaluation without any further computation besides the model solving. Due to the sampling within the main problem a more global sensitivity quantification due to the *sensitivity tracking*[20] within the design space is also possible without any extra computational costs. Therefore, the developed approach supports additionally very effectively the evaluation of sensitivities.

Within the introduction in Sec. 1.1 the research domain *technical operations*

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[18] Pelz, Leise, and Meck, "Sustainable Aircraft Design – A Review on Optimization Methods for Electric Propulsion with derived Optimal Number of Propulsors", ([147], 2021)

[19] Iooss and Saltelli, "Introduction to Sensitivity Analysis", ([84], 2017)

[20] Harzheim, *Strukturoptimierung*, ([73], 2008, p. 382)

research was introduced. To ensure that the derived optimized engineering designs are applicable in a real usage, it is mandatory to implement further steps following the conceptual design. One crucial step is the verification and validation of the given system topologies. The focus of this thesis lies on algorithmic approaches to derive conceptual designs by employing exact optimization approaches. Nevertheless, within the research at the Chair of Fluid Systems and within the CRC 805 multiple experimental evaluations were conducted for the water distribution system design[21,22,23]. Here, it could be shown that the developed modeling can lead to system designs which result in comparable solutions, when building these system topologies on a test rig. For the powertrain conceptual design the conducted verification also showed that the developed optimization approach leads to comparable results when considering an equivalent driving cycle as within a heuristic-based solution procedure[24,25].
The last two chapters focussed more on the algorithmic development and mathematical implications, when building and solving conceptual design models for two example use cases in engineering. Nevertheless, the optimized results of the system design can significantly improve the energy-efficiency of the considered systems. For instance improvements of up to 50 % can be achieved within the water distribution system design[26].
Due to the scalability of the developed optimization models by using a different parametrization and the expandability due to the modeling with a domain-specific modeling language, both models can be used within the conceptual design for each use case also if the underlying building design or vehicle differ from the shown examples. This allows to use these models and their implementations as a “tool” for engineers to design these systems properly within conceptual design.

[21] Müller, Leise, Lorenz, Altherr, and Pelz, “Optimization and Validation of Pumping System Design and Operation for Water Supply in High-Rise Buildings”, ([131], 2020)

[22] Müller, Leise, Meck, Altherr, and Pelz, “Systemic Optimization of Booster Stations - From Data Collection to Validation”, ([132], 2019)

[23] Müller et al., “Validation of an Optimized Resilient Water Supply System”, ([133], 2021)

[24] Leise et al., “Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods”, ([106], 2021)

[25] Esser, Schleiffer, Eichenlaub, and Rinderknecht, “Development of an Optimization Framework for the Comparative Evaluation of the Ecoimpact of Powertrain Concepts”, ([51], 2019)

[26] Altherr, Leise, Pfetsch, and Schmitt, “Resilient layout, design and operation of energy-efficient water distribution networks for high-rise buildings using MINLP”, ([6], 2019)

The term *sustainable development* was defined in 1987 by the United Nations as the ability to meet "the needs of the present without compromising the ability of future generations to meet their own needs"[27]. With the methodological research conducted in this thesis, a further step in this direction is shown, since a higher energy-efficiency allows lower consumption of resources within the lifetime. As already presented in the introduction, the environmental effect when applying both optimization approaches can be significant.

5.2 Algorithmic Expedience

Besides an applicability in engineering, this section discusses the applicability from an algorithmic point of view.
The water distribution system design model in Chapter 3 and the powertrain design model in Chapter 4 contain both nonlinear constraints, which make a modeling of the given systems solely with linear constraints without an employing of a PWL approximation impossible. As shown by Fügenschuh, Hayn, and Michaels[28], a modeling of technical systems with a MINLP can increase the solving performance in comparison to using a MILP approach. Hence, the start of each system modeling was a MINLP. For both use cases these (nonconvex) MINLPs were solved with common state-of-the-art MINLP solvers. In case of the water distribution system design a piecewise linearization as shown in the accompanying publication[29] can be introduced to derive a convex MILP formulation. Its benefit is the convexity of the relaxations which can be solved very efficiently. Its drawback is the requirement to add additional binary variables and sets of constraints. The performance of this PWL approximation to derive a MILP has been compared within the accompanying publication.
Within the powertrain use case next to the nonconvex MINLP a convex MINLP model has been identified, developed and presented. The core of the developed convex MINLP within the powertrain use case is a geometric program. Its specific model structure allows solving high dimensional problems very efficiently[30].

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[27] United Nations Commission on Environment and Development, *Our Common Future*, ([186], 1987)

[28] Fügenschuh, Hayn, and Michaels, "Mixed-integer Linear Methods for Layout-optimization of Screening Systems in Recovered Paper Production", ([58], 2014)

[29] Müller, Leise, Lorenz, Altherr, and Pelz, "Optimization and Validation of Pumping System Design and Operation for Water Supply in High-Rise Buildings", ([131], 2020)

[30] Boyd, Kim, Vandenberghe, and Hassibi, "A Tutorial on Geometric Programming", ([19], 2007)

The consideration of the computational solution time depends on multiple influencing factors. On the one hand, the hardware used to perform the computations has a direct influence on the performance. A further influencing factor is given by the used solvers to solve either the main or subproblem within each iteration. Within the main problem, we used Scip with the LP solver SoPlex. An additional computational evaluation with a linking to another LP solver was not done within this computational study, but can increase the solving performance even further.
Furthermore, within the developed Generalized Benders Decomposition the main problem and subproblems have always been solved to global optimality to receive optimality cuts or binary assignments. Especially within the main problem this leads to increasing solving times with increasing iterations, due to the addition of further optimality cuts. Different methods to increase the performance were already discovered but have not yet been implemented in the current algorithm[31].
One feasible approach is given by Geoffrion and Graves[32], in which the main problem was only solved to ϵ-optimality, instead of to global optimality, to derive a binary assignment.
Additionally, as can be shown in Fig. 4.11, the decomposition approach leads in average to the global optimal solution far earlier than this global optimality can be proven. Therefore, besides its ability to derive global optimal solutions, it is also applicable as a very good heuristic approach to find the global optimal solution, even though a duality gap still exists.

5.3 Mastering Uncertainty

As already introduced in Chapter 1, the consideration of uncertainty is an important aspect when designing technical systems. Here, it was distinguished between *model uncertainty*, *structural uncertainty*, *data uncertainty* and *uncertain specifications*[33]. It is referred to Figure 1.2 for further details. Within this thesis, the first three mentioned sources of uncertainty were considered explicitly.
First, the structural uncertainty that is given by the underlying design space and potentially unexplored domains within has been addressed. The

[31] Rahmaniani, Crainic, Gendreau, and Rei, "The Benders decomposition algorithm: A literature review", ([156], 2017)

[32] Geoffrion and Graves, "Multicommodity Distribution System Design by Benders Decomposition", ([63], 1974)

[33] Pelz, Groche, Pfetsch, and Schäffner, *Mastering Uncertainty in Mechanical Engineering*, ([148], 2021, pp. 25)

introduction and usage of an optimization-based approach, which derives global optimal solutions, instead of local-optimal solutions, enables to review the total modelled design space and gives with the dual bound a certificate on the optimality of the found solutions throughout this design-space.
Second, the usage of a stochastic optimization as the basis of the modeling leads to the possibility to consider explicitly data uncertainty, as introduced in Fig. 2.1. And third, thorough verification and validation tasks are performed to estimate model uncertainty.
Furthermore, the developed decomposition approach for the powertrain use case allows the application of a sensitivity analysis, as shown in Sec. 5.1, which can be used to enable a further quantification of the model uncertainty. Within the following, all three approaches to address each source of uncertainty are discussed in more detail.
When modeling a real-world technical system, usually some input parameters are not known exactly in advance to the optimization. Hence, they are affected by data uncertainty. This uncertain parametrization results in different objective values and system designs, if even slightly changed parameter values are used in the optimization program. To be able to find an optimized solution with an uncertain parametrization, two main approaches exist in the literature: *robust optimization*[34] and *stochastic optimization*[35].
The robust optimization approach only considers bounds of the uncertain parameters, which can be estimated given the specific system at hand. It is usually more easy to solve than a stochastic optimization problem, where the parameters are modeled as random variables with a given probability density function. Additionally, a robust optimization program can lead to very conservative solutions. This reduces its applicability for practical problems[36]. Contrary, a stochastic optimization approach is very flexible, but also more complex to solve. All models in this thesis followed the stochastic optimization approach to be able to exploit this flexibility in modeling.
To transform the given stochastic problem, a scenario generation was used for both use cases. While the water distribution system design use case is based on a manual scenario generation, the powertrain use case integrates a newly developed algorithm within the preprocessing to derive a predefined number of scenarios that represent the given input data. This enabled the transformation in the deterministic equivalent of the stochastic two-stage optimization problem, cf. Sec. 2.2.3.

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[34] Ben-Tal, El Ghaoui, and Nemirovski, *Robust optimization*, ([14], 2009)

[35] Shapiro, Dentcheva, and Ruszczynski, *Lectures on Stochastic Programming: Modeling and Theory*, ([170], 2014)

[36] Fischetti and Monaci, “Light robustness”, ([56], 2009)

As shown in Chapter 4 and published in [106][37] a significantly lower number of (weighted) scenarios can represent a given driving cycle accurately to be able to derive results that are comparable to the results of an approach that uses a complete driving cycle for the optimization. This down sampling of the number of considered loads in combination with a weighting of the specific scenarios can also reduce over-fitting to specific driving cycles and is therefore applicable to master data uncertainty.

The focus of this thesis is set on the modeling and computation of more sustainable system designs for two use cases, the water distribution system for high-rise buildings and the powertrain design for battery electric vehicles. For each use case, the basis is a MINLP formulation which considers the relevant physics as detailed as required. Every model is subject to model uncertainty. Within the introduction in Section 1.1 the Euler diagram of a model with its model horizon was introduced in Figure 1.1. The consideration of this model uncertainty is mandatory in engineering sciences to derive a model that fulfills its intended use. In general, the model uncertainty is reduced by increasing the models' level of detail. But this richness of detail granularity is often not required to derive the relevant outcomes for the considered questions[38].

Furthermore, when employing global optimal optimization approaches, in which integral decisions are modelled as well by using binary variables, the curse of dimensionality for this combinatorial problem requires the integration of models that follow a principle of simplicity: As detailed as required, but as simple as possible. This approach to modeling is also known as the *Occam's razzor*[39] and is a common guiding principle for axiomatic and data-driven models[40]. The developed MINLP models in this thesis follow as well this guiding principle of simplicity.

To be able to assess the quality of the developed models, in engineering sciences as well as computational sciences *verification and validation*[41] play an important role. Within systems engineering verification is defined as following:

[37] Leise et al., "Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods", ([106], 2021)

[38] Pelz, Leise, and Meck, "Sustainable Aircraft Design – A Review on Optimization Methods for Electric Propulsion with derived Optimal Number of Propulsors", ([147], 2021)

[39] MacKay, *Information Theory, Inference and Learning Algorithms*, ([114], 2003, pp. 343)

[40] One important example in this context is the *Akaike Information Criterion*, cf. [2], for the selection of statistical models, which provides a formal performance indicator that considers a trade-off between the goodness-of-fit and the model simplicity by penalizes the complexity of the considered models.

[41] National Aeronautics and Space Administration, *NASA Systems Engineering Handbook*, ([137], 2016)

"Verification of a product shows proof of compliance with requirements."[42] In contrast, validation is defined as the following: "Validation of a product shows that the product accomplishes the intended purpose in the intended environment."[42] This general definitions can be specified for a usage in computational engineering. Following Oberkampf, Trucano, and Hirsch[43], a verification is given by a comparison of the models results to known solutions, or by comparison to other implementations of the same problem. A validation is derived by comparing the results of the newly developed model with the results of experiments, which represent the intended usage and intended environment. With both approaches the confidence in a newly developed model can be increased, and the model uncertainty can be quantified. Nevertheless, the latter is more difficult, due to the requirements to derive experiments, which imply a proper design of a test-rig for the given model.
For the water distribution system design, a test-rig was developed at the Chair of Fluid Systems in a joint work and presented in one of the accompanying publications of this thesis[44]. This test-rig represents a scaled version of a high-rise building. On this test-rig, it is possible to evaluate the performance of a derived optimized solution in comparison to the true experimental data by rebuilding manually in multiple experiments the topology and pump placement, as well as the according control strategy. It could be shown that the approach for designing high-rise buildings by employing a global-optimal optimization scheme with a comparable model formulation to the one in Chapter 3 leads to a good compliance between the optimized solution and the real-world system. Further details on the experimental design and results can be found in the relevant publications[45,46,47].
For the powertrain conceptual design use case a verification approach was chosen to evaluate the performance and applicability of the developed non-

[42] National Aeronautics and Space Administration, *NASA Systems Engineering Handbook*, ([137], 2016, p. 11)

[43] Oberkampf, Trucano, and Hirsch, "Verification, Validation, and Predictive Capability in Computational Engineering and Physics", ([139], 2004)

[44] Müller, Leise, Lorenz, Altherr, and Pelz, "Optimization and Validation of Pumping System Design and Operation for Water Supply in High-Rise Buildings", ([131], 2020)

[45] Müller, Leise, Lorenz, Altherr, and Pelz, "Optimization and Validation of Pumping System Design and Operation for Water Supply in High-Rise Buildings", ([131], 2020)

[46] Müller, Leise, Meck, Altherr, and Pelz, "Systemic Optimization of Booster Stations - From Data Collection to Validation", ([132], 2019)

[47] Müller et al., "Validation of an Optimized Resilient Water Supply System", ([133], 2021)

convex MINLP modeling approach[48]. The results given in Table 4.3 and Tab. 4.4 show the results of the developed model for multiple commonly used legislative driving cycles in this thesis compared to the results of a comparable model that was based on a different modeling and optimization approach. Instead of a simultaneous analysis and design approach it uses a different structural setup. All constraints were moved to the objective by adding penalty-factors for each constraint. It was then solved by a genetic algorithm (GA). Furthermore, the approximation for the efficiency map was not modeled as in the presented approach in this thesis, but instead was modeled as piecewise linear approximation in the given domain. This results in a nonconvex efficiency map due to the partly nonconvex nature of the original dataset. The shown polyhedral outer approximation in the nonconvex MINLP and the convex approximation in the newly developed MIGP do not consider the efficiency map in that detail but enable a better solvability in the global-optimal framework. The computed results show the applicability of the shown modeling approach by resulting in only minor differences to the second approach which was based on a GA. A detailed discussion on the comparability can be found in the accompanying publication[48].

One approach important in computational engineering is the consideration of sensitivities, as introduced in Sec. 5.1. Sensitivity analysis can be understood as a methodological approach to master data and/or model uncertainty and results after a successful application in the conceptual design process in either a model refinement or model acceptance.

The newly developed MIGP approach for the powertrain design in Sec. 4.3 has a further positive side effect besides its efficient solving for providing estimates of the model uncertainty. Each run of a sub-problem (which is a GP) results besides the objective and sizing in local sensitivity values at this point in the design space with no extra cost. Even though it is not a global sensitivity analysis it still provides multiple sensitivity values scattered within the design space at local optima that can provide an input for a computation of an estimate of the global sensitivities.

A further approach to sensitivity analysis has been introduced in Chapter 3 with Latin Hypercube Sampling in the domain of the construction kit of pumps that are used as an input to the optimization by the developed MINLP. By employing this approach known from Design of Experiments, it could be shown how the optimized solution changes with an additional degree of freedom. Further details can be found in Table 3.5. The usage of Design of Experiments can also be seen as a type of sensitivity analysis as shown

[48] Leise et al., "Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods", ([106], 2021)

by Kleijnen[49]. The benefit of the selected approach for the WDS conceptual design is the explicit consideration of the complete design space spanned by the scaled pumps and still be able to evaluate the performance of a given solution with the developed MINLP model.
The concept of resilience for technical systems has already been introduced in Section 2.3. Here, we discuss the applicability of this general concept based on the two developed conceptual design problems given in the previous chapters. When considering resilience, the focus in this thesis was set on a static evaluation of the resilience. First steps in the direction of dynamic resilience have also been shown by specific publications within the research in the subproject A9 within the CRC 805[50,51]. This dynamic view on the concept of resilience "is based on the four functions monitoring, responding, learning and anticipating"[51] and was evaluated on a newly designed test-rig as well as by algorithmic evaluations.
As a static view on resilience the buffering capacity, first mentioned by Woods[52], has been introduced for the water distribution system design in Chapter 3. Here, resilience as a strategy to master uncertainty has been exemplified for an integration in a MINLP to derive solutions that are resilient against arbitrary failures of pumps. With the developed approach in the accompanying publication[53], it is possible to derive optimized solutions for a conceptual design of a water distribution system for a high-rise building that is efficient and more resilient than a given reference system. The potential energy-efficiency improvements can still be acquired, and additionally the system can cope with a predefined number of pump failures, which is given by the k-value within the k-resilience approach.
A second static view on resilience was already shown in the introduction by presenting the *performance range* and the *radius of performance*. A more resilient technical system is then characterized by a broader performance range, in which a predefined minimum performance can be reached. Furthermore, it is characterized by a graceful degradation when the system reaches its performance limit.
The developed model and solution process for the powertrain conceptual

[49] Kleijnen, "Verification and Validation of Simulation Models", ([92], 1995)

[50] Leise, Breuer, Altherr, and Pelz, "Development, Validation and Assessment of a Resilient Pumping System", ([104], 2020)

[51] Leise and Altherr, "Experimental Evaluation of Resilience Metrics in a Fluid System", ([100], 2021)

[52] Woods, "Essential Characteristics of Resilience", ([202], 2017)

[53] Altherr, Leise, Pfetsch, and Schmitt, "Resilient layout, design and operation of energy-efficient water distribution networks for high-rise buildings using MINLP", ([6], 2019)

design approach can also be extended to derive further insights about the resilience of the given system design. Due to the integration of multiple scenarios with the developed pre-processing heuristic in which it is possible to integrate multiple legislative driving cycles as a reference for sizing the powertrain, the radius of performance can be calculated for influencing factors that are important for the designer. Hence, the developed modeling and optimization approach is also applicable to derive resilience metrics based on the result of the optimization. Due to the white-box optimization approach, where each constraint is editable, it is also able to be used in later research, where explicit resilience considerations are implemented as done for the water distribution system conceptual design.

Chapter 6

Summary and Outlook

The IPCC[1] showed in its recent report that the development of environmentally sustainable and resilient technologies and infrastructures is mandatory to reduce the impact of climate change. Within this thesis both paths have been considered already before this report was published.
The focus of this thesis is the conceptual design of sustainable technical systems by employing a rigorous mathematical modeling under consideration of uncertainty. The current scientific knowledge is extended with this thesis twofold:
First, from an engineering point of view, specific approaches for two use cases, namely a water distribution system design for high-rise buildings and the powertrain design of battery electric vehicles have been introduced. In this context, the potential system efficiency improvements have been outlined.
Second, from an algorithmic point of view, a widely applicable general system modelling approach for technical systems under uncertainty has been presented.
The usage of exact optimization approaches by engineers for solving real-world engineering problems in comparison to other approaches is still in its infancy. To enable a broader usage of these very promising set of tools in engineering, it is required to reduce the modelling effort, but still be able to solve the models efficiently. Between these poles, this thesis gives answers how to jointly fulfill both requirements.
Therefore, the considered path of modeling was consequently followed. This leads to a modeling as a Mixed-Integer Geometric Program (MIGP), which is being solved very efficiently by using a Generalized Benders Decomposition. The attention of modelling conceptual design problems as Geometric Programs

[1] Pörtner et al., "Climate Change 2022: Impacts, Adaptation, and Vulnerability", ([152], 2022)

has increased within recent years considerably, due to its modelling and solving performance. This thesis extends this continuous GP-modelling approach to programs with additional discrete variables. Up to date, MIGPs have not been frequently in the focus of engineering research, but the usage of integer and/or binary variables is important for modeling a multitude of technical systems more accurately. In this thesis it could be shown that this field of research is very promising. Furthermore, the recent developments in conic and integer optimization result in a very reliable solving of this problem class. As a MIGP the underlying model can be modeled efficiently and solved to global optimality, while still having the great advantage of allowing to master uncertainty. This thesis additionally extends the knowledge in system modeling as a Mixed-Integer Geometric Program and gives answers to the transfer of problem-specific modeling approaches on this problem class.
Methodologically, this approach is not only restricted to the considered use case of the conceptual design of a powertrain, but it can be employed for numerous technical systems. Hence, the developed systematic modeling and solving can be employed for further technical systems beyond the presented use cases.

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Appendix A

Additional Data

A.1 Water Distribution System Design

The presented conceptual design model in Section 3.2 models an existing hotel building in Germany. It has 9 stories (each 3 m high) that must be supplied by the fresh water distribution system. The first three represent the lobby. From the 4th to the 12th story, the hotel rooms are located, which must be supplied by the fresh water distribution system. On top of the hotel are further individual flats and a restaurant located. Since these are supplied by another water supply, they are excluded from the consideration in this example calculation. The used parameters within the presented computational study are given in Table A.1.

In total, 5 parallel pumps can be placed of each type. Four different load scenarios have been selected. Furthermore, 4 pressure zones must be supplied by the fresh water distribution system, each consisting of three stories. The modelled graph consists of one vertex for the modeling of the water transfer inside the building from the water supplier and four vertices for the pressure

Table A.1 – Values of the parameters in Chapter 3, cf. [101]

Parameter	Description	Value
$\overline{Q}$	maximum inflow	$24\,\mathrm{m}^3/\mathrm{h}$
H^0	pressure head by water supplier	28.63 m
$\underline{H}$	minimal pressure head in each level, expect the first one	15.25 m
$\underline{H}_1$	minimal pressure head in first level	20 m
$\overline{H}$	maximum water height of the building	43.5 m
B	big-M constant for the pressure increase	84 m
$\overline{P}$	maximum power consumption	6000 W
Z	maximum number of parallel pumps	5
$\underline{\Omega}_+$	minimum normalized speed of all pumps	0
C^{energy}	energy costs per Watt	0.30 €/(kWh)
τ	time of usage	3.75 a

Table A.2 – Used scenarios within the MINLP to model the usage-period

	scenario 1	scenario 2	scenario 3	scenario 4
volume flow $Q_{v,s}$	$24\,\mathrm{m^3/h}$	$18\,\mathrm{m^3/h}$	$12\,\mathrm{m^3/h}$	$6\,\mathrm{m^3/h}$
pressure $\underline{H}$	as stated in Table A.1			
probability π_s	0.06	0.15	0.35	0.44

zones of adjacent floors, cf. DIN 1988-500 ([42]). The investment costs for each pump are calculated based on the cost model and scaling parameter d_+. The required pressure in each pressure zone is estimated by using the standard DIN 1988-300, cf. [41].
The investment costs per pipe $C_{i,j}^{\text{pipe}}$ are computed based on the selected length within the solution process and a fixed cost of 50 €/m.
Since the building considered in Chapter 3 is based on an existing hotel building, which has already been equipped with a pump, the volume flow demand and the scenarios are also based on the recorded data. The used scenarios are given in Table A.2.
The pressure difference between the different vertices within the model in Section 3.2 has been computed by considering the height difference between each vertex, since it causes the main pressure loss within the system, cf. [101]. As shown in Section 3.3.1, this can be further detailed by also considering the pressure losses within each pipe due to pipe friction. A detailed description of this approach can be found in the accompanying publication by Altherr, Leise, Pfetsch, and Schmitt ([6]). All computations were performed by using a Linux-based machine with an Intel i7-6600U processor and 16 GB of RAM with SCIP 4.0.0 and CPLEX as an LP-solver. All raw computational results can be found in the provided supplementary dataset.

A.2 Powertrain Design

The computations of the MINLP were conducted on a Linux-based machine with an Intel Core i7-10510U with 16 GB RAM and Scip 6.0.2. The termination criteria were (i) a time limit of 30 minutes, (ii) a RAM limit of 12 GB and a duality gap limit of 0.25%.
For the computations of the MIGP, shown in Section 4.3, a Linux-based server with an AMD Ryzen 5 5600X and 32 GB RAM was used. The computations were executed in a Docker image on a Gitlab runner. A maximum time limit of 6 days was set for all computations on the Gitlab runner. Scip 7.0.2 with SoPlex was used for the main problem and Mosek 9.0 was used for solving the GP sub-problems. No further solver-specific settings were selected to improve the computational performance.

Appendix B

Research Data Management

The computational evaluation of the developed algorithms leads to multiple datasets, which are then evaluated to derive the shown figures and tables. This thesis follows the *FAIR*[1] principles (Findability, Accessibility, Interoperability and Reusability) of research data management.
Since the thesis focus is the conceptual design of technical systems by employing exact optimization approaches, the presented approach already allows a direct reimplementation based on the shown models.
To enhance the reusability of the algorithms and the computed results even further, all corresponding datasets are stored within a repository as supplementary data[2]. This repository is identifiable by using the following uniform resource identifier (URI):

`https://tudatalib.ulb.tu-darmstadt.de/handle/tudatalib/3550.2`

Each relevant figure is labeled with an identifier that is unique within the thesis. It links the shown figure with an accompanying `.json`-file that contains further metadata. This metadata provides additional information about the relevant data source(s). Additionally, it contains for figures that present evaluated data the underlying data in a machine-readable format. This facilitates a re-use of the shown results.
Besides these additional files with metadata for the presented figures, further `Read-Me` files are provided, which contain additional information about the structure and usage of the provided datasets and figures. To enable a reusability of the accompanying files, open file formats, like `.csv`, `.zip` and `.txt` were used for the raw datasets.

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1 Wilkinson et al., "The FAIR Guiding Principles for scientific data management and stewardship", ([198], 2016)

2 Leise, *Supplementary Material: Algorithmic Design of Technical Systems under Uncertainty*, ([99], 2022)

The supplementary research data can be accessed over open protocols like `https` on the institutional data repository `tudatalib`[3] and is provided upon request. It is provided with a `3-Clause BSD` license.
As explained in Chapter 4, an algorithmic preprocessing was conducted to derive relevant scenarios for the conceptual design of the powertrain use case. This preprocessing is not only required within the shown MINLP/MIGP modeling, but can be reused in other optimization- and/or simulation-based approaches for the conceptual design of powertrains for battery electric vehicles. Therefore, the latest development version of this Python-based toolbox is provided also within the supplementary data repository. Important information about the toolbox and examples are located in a `Read-Me` file. The toolbox is installable via the common Python package manager `pip` from the provided local copy of the toolbox. It also contains examples for the usage and `pytest` test cases. It is also licensed under the `3-Clause BSD` license.
Additionally, to the previously mentioned supplementary data, two further additional datasets are relevant for the content of this thesis. The first one contains additional information for the MINLP model in the powertrain use case.[4] The second one contains additional information[5] about a relevant dataset of pump characteristics that were used within publications relevant to the conceptual design of the water distribution system, cf. [131].

[3] The institutional repository of TU Darmstadt can be found under the following address: https://tudatalib.ulb.tu-darmstadt.de/ (accessed 18. August 2022)

[4] Leise et al., *Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods (Supplemental)*, ([105], 2021)

[5] Müller et al., *Pump System Test Instances and Electronical Pump Catalogues*, ([134], 2022)

Own Publications

Publications

[1] L. C. Altherr, P. Leise, M. E. Pfetsch, and A. Schmitt. “Optimization of Wear Related Material Costs of a Hydrostatic Transmission System via MINLP”. In: (2021). submitted for publication.

[2] L. C. Altherr, N. Brötz, I. Dietrich, T. Gally, F. Geßner, H. Kloberdanz, P. Leise, P. F. Pelz, P. D. Schlemmer, and A. Schmitt. “Resilience in Mechanical Engineering - A Concept for Controlling Uncertainty during Design, Production and Usage Phase of Load-Carrying Structures”. In: *Uncertainty in Mechanical Engineering III*. Ed. by P. F. Pelz and P. Groche. 885. Trans Tech Publications, Switzerland, Nov. 2018, pp. 187–198. URL: http://tubiblio.ulb.tu-darmstadt.de/109766/.

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Supplementary Material

[1] P. Leise. *Supplementary Material: Algorithmic Design of Technical Systems under Uncertainty*. 2022. URL: `https://tudatalib.ulb.tu-darmstadt.de/handle/tudatalib/3550`.

[2] P. Leise, A. Eßer, T. Eichenlaub, J.-E. Schleiffer, L. C. Altherr, S. Rinderknecht, and P. F. Pelz. *Sustainable System Design of Electric Powertrains – Comparison of Optimization Methods (Supplemental)*. May 2021. DOI: `10.6084/m9.figshare.14664937.v1`.

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