COMPUTER AIDED CONTROLLER DESIGN FOR A MULTIAXIAL SERVOHYDRAULIC VIBRATION TEST BENCH WITH LARGE PARAMETER UNCERTAINTIES

A. Gräser, W. Neddermeyer and H. Tolle

Institute of Control Engineering, Technical University of Darmstadt, Federal Republic of Germany

Abstract. In 1973 I. Horowitz and M. Sidi have given a method for quantitative design by cascaded control of processes with large parameter uncertainties. This paper indicates the necessary modifications in case of bandwidth limitations and the differences in the results, which can be reached in such a case. As an example the design of a car test bench - which motivated the investigations - is discussed.

Keywords. Horowitz/Sidi-design; parameter uncertainty; cascade control; car test bench.

I. INTRODUCTION

For life span tests of wheel suspensions and complete cars servohydraulic test benches are used. In one of the possible test cases the car body is fixed and horizontal and vertical loads as measured during road tests are generated by servohydraulic actuators. Figure 1 shows such a configuration schematically. One problem of such a simulation of reality is that the dynamics of the car on the road are to a certain extent different from the dynamics of the laboratory system car plus actuators. Since test benches are only useful, if one gets fairly exact results on life span and on the location of eventual material crashes, one needs either a lot of experience in the compensation of unwanted effects or a detailed modelling, which enables one to design control loops, guaranteeing that the input signals brought onto the car axles and/ or suspensions in the tests are nearly identical to the respective input signals on the road. A modelling of this kind has been performed by A. Gräser (1982) for the problem discussed here and for some other car test bench problems.

In the situation of figure 1 one can concentrate on the design of one horizontal displacement in combination with the vertical displacement. This is due to the fact that in first approximation there is a unilateral coupling from the vertical input onto the horizontal input only, since the horizontal wheel suspension is much stiffer than the vertical one. Figure 2 clarifies the basis of such a coupling: The horizontal actuator must follow movements of the vertical actuator simulating the jumping of the wheel on the road. The kinematics give in general a quadratic relation that means we have a nonlinear coupling.

Since in the vertical direction the actuator simulates the wheel displacement one usually runs the horizontal actuator in force control due to the small displacements in this direction, which can be measured, however. As a second measurable value for the vertical direction one may use the oil pressure of the actuator. If one separates furtheron in the actuators between the conversion of the electrical input signal into the mechanical control of the oil flow and the connection between this input and the oil pressure and/or the actuator displacement, one gets the process description given by figure 3.

The detailed analysis (Gräser 1982) shows that S_1 can be modelled by a linear transfer function, but that S_2 and S_3 are heavily dependent on the working point due to valve nonlinearities and on the vehicle to be tested. However, the overall behaviour can be described by a linear transfer function with poles and zeros very near to the imaginary axis but still in the left half of the complex plane with a relatively high uncertainty on the frequency values characterizing these singularities.

Table 1 gives the transfer functions for the model of figure 2 and representative parameter variations for the problem looked at.

The parameter uncertainties can be handled by appropriate feedback (cascaded system). Here a quantitative design procedure developed by I. Horowitz and M. Sidi 1972, which has been extended by the same authors to cascaded systems in 1973 has been applied. The method guarantees that the design demands are fulfilled with controller amplifications as small as possible, which is important with respect to disturbance and sensitivity amplification unavoidable in the high frequency range for real plants. However, in Horowitz/Sidi 1973 the design restrictions are assumed to be others than being important here. Therefore we shall discuss in section two some modifications of Horowitz/Sidi 1973. Section III will deal with the design of the car test bench control and section IV concludes the paper with some summarizing remarks.

II. THE DESIGN OF CASCADED LOOPS FOR PROCESSES WITH LARGE PARAMETER UNCERTAINTIES

The design method of Horowitz/Sidi (1972) makes use of the connection between open and closed loop frequency response in the Nichols chart. Parameter uncertainties lead to an enlargement of the single point characterizing phase and gain of the process for a certain frequency in this chart to a certain area in the chart. The controller shifts this area without changing it into another position and one can check which controller gain is just necessary to guarantee that the process variations do not lead to a closed looped amplitude variation higher than specified. In addition to these frequency dependent lower boundaries, above which the controller has to raise the plant frequency response for some selected nominal process parameter values, stability has to be secured. This leads to a so-called "high frequency boundary" which guarantees the avoidance of a certain area around the critical point OdB, -180° and which itself is representative for the maximum amplification of parameter uncertainty effects on closed loop response, which are unavoidable for real plants in the high frequency range if one suppresses them in the low frequency range by high controller gain. (Bode-Westcott-theorem

$$\int_0^{\infty} \log |1+q(j\omega)| d\omega = 0 \quad ;$$

q(s) open loop transfer-function - see e.g. Krebs 1973 -). Details of the method and its semiautomatic handling via an interactive computer program can be found in a companion paper (A. Gräser, W. Neddermeyer, H. Tolle, 1982). Here we are going to deal with the extension of this method to cascaded control loops by Horowitz and Sidi (1973). They start with the design of the outermost control loop, neglecting the parameter variations of the inner ones in detail, but taking account of them in some overdesign - higher controller gain than necessary -. With the next loop going to the inside of the cascaded system the closed loop transfer function variation due to parameter variations has to stay inside the overdesign margin, will have some overdesign again as far as further inner loops are still to be closed, but is free in all other aspects. By this one gets stepwise the necessary controllers and guarantees that the overall design requirements are fulfilled. However, due to the Bode-Westcott-theorem one has to pay for the parameter uncertainty effect suppression in the closed loop in the low frequency area by some amplification in the high frequency area, so that in general the necessary bandwidth is growing for the inner loops steadily. Especially, one doesn't know in advance what bandwidth requirement one will have for the actuator acting on the process before one has designed the whole cascaded system. Now in a number of control tasks like the car test bench control bandwidth limitations for the innermost loop exist: Either because the high frequency behaviour of the process is not well known or because high frequency resonances are possible or because the actuators have a limited bandwidth only. Furtheron, coupled parameter variations of the different process blocks can complicate the boundary computations for the Nichols charts for inner loop design, if one starts from the outermost loop. Out of these reasons the design starting from the innermost loop and going stepwise to the outer loops is proposed here as an alternative to the Horowitz/Sidi method. By this, one can easily handle bandwidth limitations. But now, the required sensitivity suppression to a defined frequency may not be reached. However, one gets a clear picture, which results may be reached with a certain process model and/or selected actuator and what cannot be reached.

Besides of this change in design direction the car test bench control loop design indicated that the very simple unique high frequency boundary of Horowitz/Sidi 1972 has to be modified, if the phase variation is not negligible up to relatively high frequencies and the phase variations are not only positive compared with the phases of the process with the nominal parameter set. In this case one is only sure to avoid the specified region around the critical point for all parameter variations, if one calculates for a high enough number of selected discrete frequencies forbidden regions directly with the parameter variation areas - see figure 4 -.

III. CAR TEST BENCH DESIGN

The process description by a block diagram has been given in figure 3, the transfer functions and respective parameter values in table 1. Due to possible resonances in the high frequency range an open loop bandwidth limitation of $\omega \leq 2\,000\ \mathrm{sec}^{-1}$ has been chosen and to get the best design possible under such conditions all measurements available have been used, which leads to the cascaded closed loop structure of figure 5. The nonlinear coupling from the vertical loop onto the horizontal loop has been compensated by modelling the process nonlinearity. If the inner loop for control of the horizontal movement would be able to suppress the effects of $S_1^h \cdot S_2^h$ - parameter variations fully, the compensation would need the nonlinear term only. However, the bandwidth limitation doesn't allow this up to the necessary extent, as will be shown later on and this makes the additional lead term necessary.

We shall discuss here the design of the horizontal loop only. The design steps are the same for the vertical loop and will not be shown therefore. Figure 6 is a Bode plot of phase and gain for the nominal values and their variations for $S_1^h \cdot S_2^h$ according to table 1. The command frequency region is for both the vertical and the horizontal loop $0 \le \omega \le \omega_w = 80\ \sec^{-1}$. However, the quadratic nonlinearity leads to inputs up to $2\omega_w$ as inputs from the vertical loop. The open loop bandwidth limitation doesn't allow to specify the inner loop variations as tight as one would like to do. Figure 7 shows the possible closed loop gain variation specification as chosen after some preliminary test investigations and figure 8 a number of open loop gain variation boundaries calculated from this closed loop requirements and the unique high frequency boundary of Horowitz/Sidi applicable here (see fig. 6) for a disturbance amplification smaller than 10 dB as a maximum. The frequency plot shown is already the process with the nominal parameter set plus a satisfactory controller: The high frequency boundary is avoided and for example for the frequency $\omega_{11} = 349\ \sec^{-1}$ the frequency plot point lies exactly on the respective boundary 11, with a good approximation to this situation for the other frequencies. The controller $R_1^h(s)$ leading to this result is of the fourth order in the nominator and the denominator:

$$R_1^h = \frac{27{,}7\left(1+\frac{1{,}75}{31}s+\frac{1}{31^2}s^2\right)\left(1+\frac{1{,}68}{933}s+\frac{1}{933^2}s^2\right)}{s\left(1+0{,}84\cdot 10^{-3}s\right)\left(1+\frac{0{,}45}{1653}s+\frac{1}{1653^2}s^2\right)}$$

The open loop crosses the -180° phase line at $\omega = 1693\ \sec^{-1}$, that means the bandwidth requirement is fulfilled with no undue margin. Figure 9 shows the gain and phase variations of the loop closed with R_1^h and figure 10 the gain and phase variations which one gets by combining these not suppressed inner loop variations with the variations of S_3^h. We have now the situation that for the nominal parameter set we have also negative phase variations. This makes it necessary to use the more complicated closed boundaries of figure 4 instead of the unique high frequency boundary of Horowitz/Sidi. Since this leads to software difficulties, the real variation areas have been enclosed in rectangles and the boundaries were calculated with these rectangles which means a certain amount of overdesign. Figure 11 gives the Bode plot of the closed loop after completion of the outer loop design. The result is acceptable. The controller R_2^h has a denominator of the third order and a nominator of the fifth order:

$$R_2^h = \frac{8{,}98\left(1+1{,}4\cdot 10^{-2}s\right)\left(1+\frac{1{,}18}{113}s+\frac{1}{113^2}s^2\right)}{s\left(1+\frac{8{,}2}{10^3}s\right)\left(1+\frac{6{,}25}{10^4}s\right)\left(1+\frac{0{,}37}{91}s+\frac{1}{91^2}s^2\right)}$$

Figure 12 shows the step response of the horizontal loop with a number of parameter variations. However, the step input is not the main problem. More important are the answers to sinusoidal inputs and due to the doubling of frequency through the quadratic nonlinearity the critical case is the input:

$$w^v \sim \sin 80\ t \quad ; \quad w^h = 0 \ .$$

Figure 13 gives the answers for $w^v = 0{,}28 \sin 80\ t$, $w^h = 0$ for the case without compensation of the nonlinearity (a), with compensation of the nonlinearity

$$-\ x_1^h = -\ 0{,}12(x_1^v)^2\ -$$

but without the additional lead (PD)-term

$$-\ \left(1 + \frac{1}{180}s\right)\ -$$

(b) and finally with the design shown in figure 5. Actually, the compensation without the lead term doesn't improve the situation vis-a-vis the case without compensation: One has in both cases an unacceptable system answer. The reason is that one has at the frequency $\omega = 2\cdot 80 = 160\ \sec^{-1}$ already a phase variation of 52° for the inner horizontal loop - see figure 10 -. The lead term - which would have to be replaced by a lead-lag element with a pole far to the left in case of realisation - doesn't bring down the phase variation, but the minimum and maximum values of the phase are shifted from $-60^\circ \le \phi \le -8^\circ$ to $-32^\circ \le \phi \le 20^\circ$, which gives the improvement shown in figure 13c.

IV. CONCLUDING REMARKS

The design method of Horowitz/Sidi 1972 is a very useful tool for processes with large parameter uncertainties, as car test benches are due to linearization and different test vehicles. However, bandwidth limitations as may be existent for other problems, too, make some modifications necessary of the extension of this method to cascaded systems as given by Horowitz/Sidi 1973. The therefore used design from the inner cascade loops to the outer cascade loops does not guarantee, however, any longer the fulfillment of the design requirements. But it shows clearly what can be reached with the limitations given.

REFERENCES

Gräser, A. (1982). Zur Modellbildung und Regelung mehraxialer Kraftfahrzeugprüfstände Dissertation Technische Hochschule Darmstadt.

Gräser, A., Neddermeyer, W., Tolle, H. (1982) CAD of the Horowitz/Sidi-design for feedback systems with large plant parameter uncertainty. IFAC-Symposium on CAD of Multivariable Technological Systems, West-Lafayette, USA.

Horowitz, I.M. and Sidi, M. (1972). Synthesis of feedback systems with large plant ignorance for prescribed time-domain tolerances. Int. J. Control, vol. 16, pp. 287-309.

Horowitz, I.M. and Sidi, M. (1973). Synthesis of cascaded multiple-loop feedback systems with large parameter ignorance. Automatica, vol. 9, pp. 589-600, Pergamon Press.

Krebs, V. (1973). Das Gleichgewichtstheorem, eine grundsätzliche Aussage über das Verhalten von Regelkreisen. Regelungstechnik, 21 (1973), S. 25-27, 56-59.

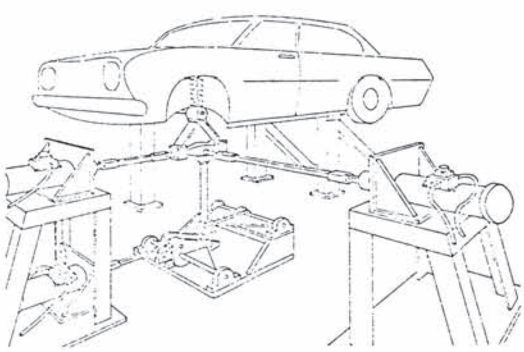

Fig.1. Sketch of a servohydraulic test bench with three actuators coupled to one wheel suspension

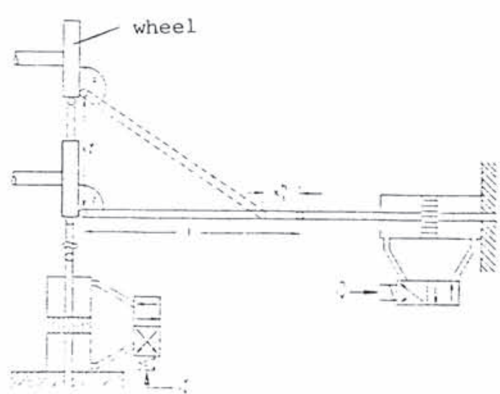

Fig. 2. Mechanical coupling of the actuators due to the wheel suspension

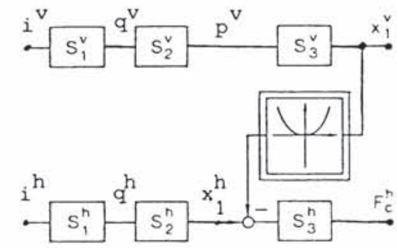

Fig.3. Process model:
- index v = vertikal
- index h = horizontal
- i : elektrical input
- q : oil flow
- p : oil pressure
- x_1: movement
- F_c: force applied by the actuator

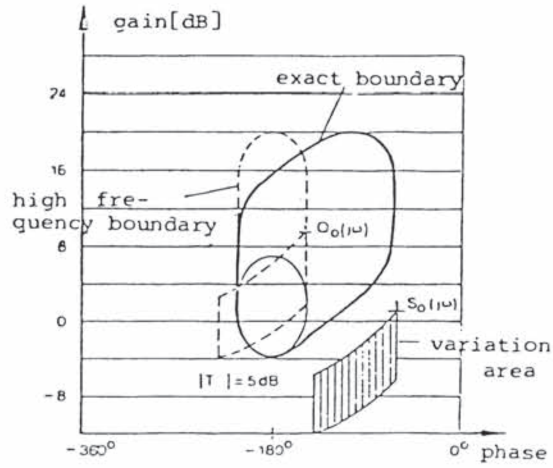

Fig.4. High frequency boundaries calculated with two different methods

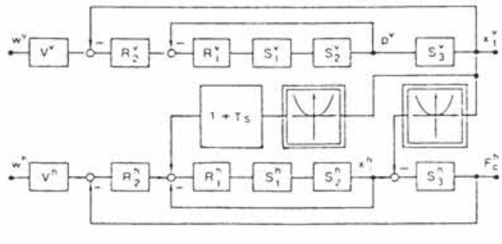

Fig.5. Structure of the cascade control system with disturbance feedforward decoupling

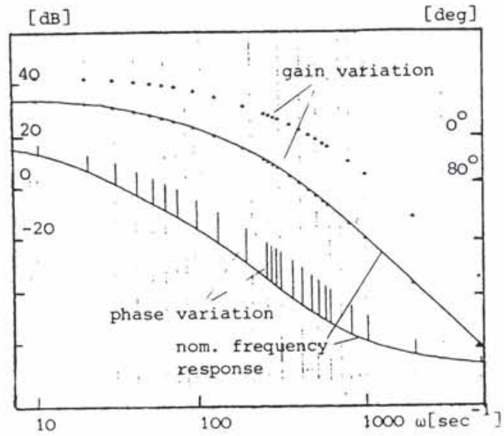

Fig. 6. Gain and phase variations of $S_1^h(j\omega)\cdot S_2^h(j\omega)$ - Bode plot

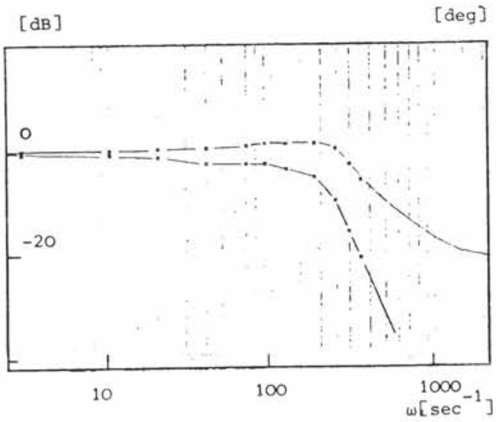

Fig. 7. Design demands for the inner control loop $T_1^h(j\omega)$ - Bode plot

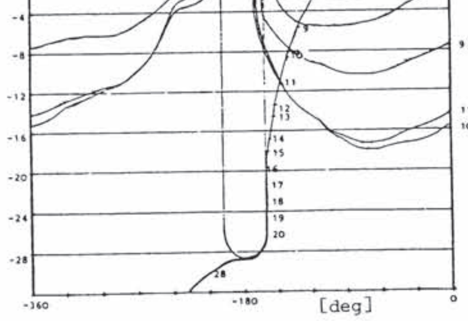

Fig. 8.
Nichols chart with the nominal open loop frequency response and the corresponding boundaries

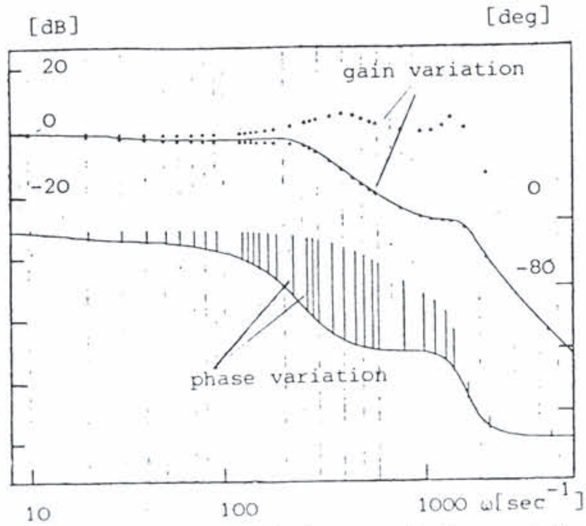

Fig. 9. Magnitude and phase variations of the closed loop $T_1^h(j\omega)$ - Bode plot

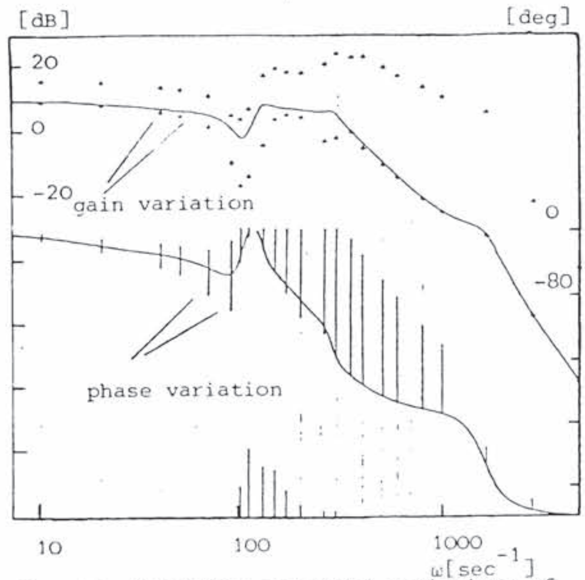

Fig. 1o. Magnitude and phase variations of $T_1^h(j\omega)\cdot S_3^h(j\omega)$ - Bode plot

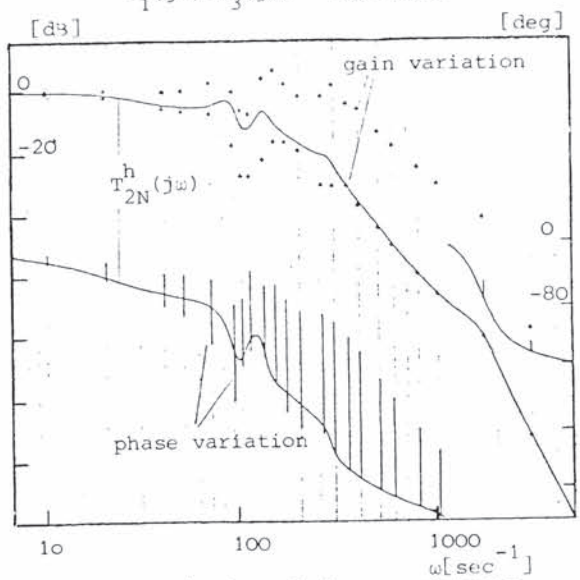

Fig. 11. Magnitude and phase variations of the closed loop $T_2^h(j\omega)$-Bode plot

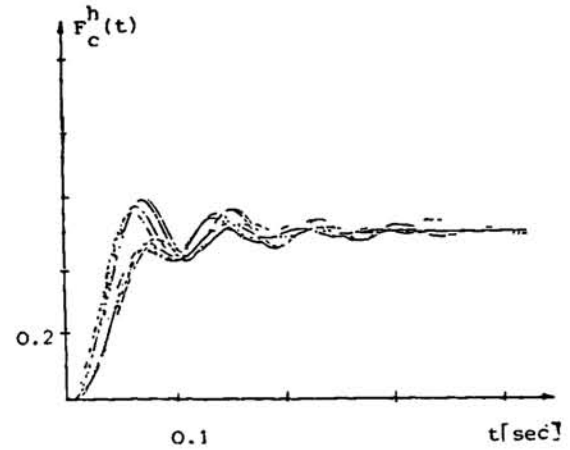

Fig. 12. Step responses of the horizontal loop

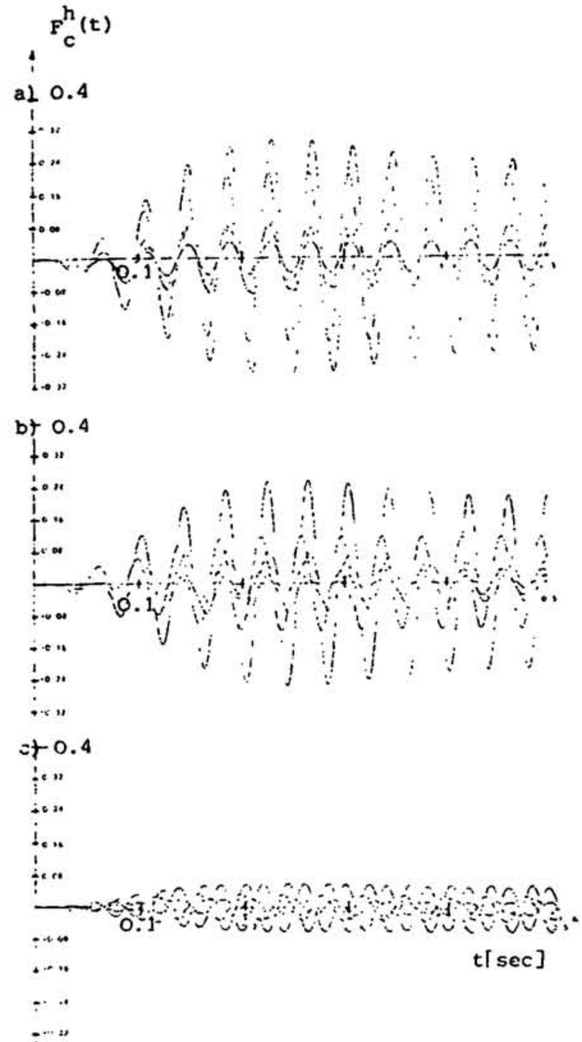

Fig. 13a). Output signal of the horizontal control loop without disturbance feedforward system

b). Output signal of the horizontal control loop with disturbance feedforward system

c). Output signal of the horizontal control loop with disturbance feedforward system and additional lead-lag system

transfer functions	nominal values	parameter variations
$F_1^v = \frac{1}{(1+T_v^v s)^2}$	$T_v^v = 1.6\cdot10^{-3}$	$1.6\cdot10^{-3} \le T_v^v \le 3.2\cdot10^{-3}$
$F_2^v = \frac{K_2(d_1^v,\omega_1^v)}{s(d_2^v,\omega_2^v)}$	$K_2 = 260$ $d_1 = 0.5$ (?) $d_2 = 0.15$ $\omega_1 = 60$ $\omega_2 = 98$	$110 \le K_2 \le 340$ $0.4 \le d_1 \le 0.6$ $0.06 \le d_2 \le 0.5$ $54 \le \omega_1 \le 69$ $90 \le \omega_2 \le 116$
$F_3^v = \frac{K_3}{s(d_2^v,\omega_2^v)}$	$K_3 = 2.4$ $d_2 = 0.1$ (?) $\omega_2 = 98$	$1.8 \le K_3 \le 2.6$ $0.06 \le d_2 \le 0.1$ (?) $98 \le \omega_2 \le 111$
$F_1^h = \frac{1}{(1+T_v^h s)^2}$	$T_v^h = 1.6\cdot10^{-3}$	$1.6\cdot10^{-3} \le T_v^h \le 3.2\cdot10^{-3}$
$F_2^h = \frac{K_2^h}{1+T_2^h s}$	$K_2^h = 60$ (?) $T_2^h = 2.24\cdot10^{-2}$	$50 \le K_2^h \le 130$ (?) $1.6\cdot10^{-2} \le T_2^h \le 3.2\cdot10^{-2}$
$F_4^h = \frac{K_4^h(d_1^h,\omega_1^h)(d_2^h,\omega_2^h)(d_3^h,\omega_3^h)}{(1+T_2 s)(d_4^h,\omega_4^h)(d_5^h,\omega_5^h)(d_6^h,\omega_6^h)}$ $(d,\omega) = 1 + \frac{2d}{\omega}s + \frac{1}{\omega^2}s^2$ nominal values $d_1 = $ [illegible] $\omega_1 = $ [illegible] $d_2 = $ [illegible] $\omega_2 = $ [illegible] $d_3 = $ [illegible] $\omega_3 = $ [illegible] $d_4 = $ [illegible] $\omega_4 = $ [illegible] $d_5 = $ [illegible] $\omega_5 = $ [illegible] $d_6 = $ [illegible] $\omega_6 = $ [illegible] $T_2 = $ [illegible] $K_4^h = $ [illegible] [illegible]		[illegible] $\le K_4^h \le 600$ $0.04 \le d_1^h \le$ [illegible] $\le \omega_1 \le$ [illegible] $0.07 \le d_2^h \le$ [illegible] $\le \omega_2 \le$ [illegible] $0.07 \le d_3 \le$ [illegible] $\le \omega_3 \le$ [illegible] [illegible] $\le d_4 \le$ [illegible] $\le \omega_4 \le$ [illegible] [illegible] $\le d_5 \le 0.15$ [illegible] $\le \omega_5 \le$ [illegible] $0.1 \le d_6 \le 0.2$ [illegible] $\le \omega_6 \le$ [illegible]

Tab. 1.