CAD OF THE HOROWITZ/SIDI-DESIGN FOR FEEDBACK SYSTEMS WITH LARGE PLANT PARAMETER UNCERTAINTY

A. Gräser, W. Neddermeyer and H. Tolle

Institute of Control Engineering, Technical University of Darmstadt, Federal Republic of Germany

Abstract. I. M. Horowitz and M. Sidi (1972) presented a design procedure which guarantees quantitative demands on disturbance rejection and suppression of plant variation using the minimum controller gain just necessary for this effect.

This paper describes an interactive, computer-aided implementation of this design procedure, which has proved to be very effective. The plant variations are handled by some expansions of a method from L. Longdon and D. J. East (1979), the controller design by a parameter optimization method using a vectorial performance criterion in an interactive manner.

Keywords. CAD; Horowitz/Sidi-design; vector performance index.

INTRODUCTION

Feedback allows to reduce effects of disturbance signals and of plant parameter variations by a suitable dynamic correction in the closed loop, the controller. However, the command response is likewise influenced by the controller. I. M. Horowitz (1963) introduced therefore the distinction between control loops with one and two degrees of freedom and proposed to use in general two degrees of freedom - two dynamic corrections - the first one being the controller and the second one a dynamic prefilter for command response shaping - see fig. 1 -.

Concentrating on the problem of plant parameter variations one may describe the plant by the parameter dependent mathematical model $P(s,\underline{a})$, where $\underline{a}$ is a vector describing parameters varying between specific bounds:

$$\underline{\varepsilon}_1 \le \underline{a} \le \underline{\varepsilon}_2 \tag{1}$$

The design objective for the two degrees of freedom control loop is to keep step function responses of the closed loop in a permissible domain:

$$\alpha(t) \le y(t,\underline{a}) \le \beta(t) \tag{2}$$

which may be reformulated at least for minimum phase plants as a requirement on a domain for the gain variation of the closed loop frequency response:

$$\alpha(\omega) \le |T_w(\omega,\underline{a})| \le \beta(\omega) \tag{3}$$

(see Krishnan and Cruickshanks 1977). Due to possible amplification of the unavoidable measurement noise and design economy the design objective should be reached with minimum loop bandwidth and/or controller gain.

THE DESIGN METHOD OF HOROWITZ/SIDI

The controller design method devised by I.M. Horowitz and M. Sidi (1972) works according to the following principle: For a number of selected frequencies ω_i the plant gain and phase variation is determined giving a certain phase-gain area in the Nichols chart, which can be shifted in the phase and the gain direction by selection of phase and gain of the controller. With the Nichols chart - see fig. 2 - there is a graphical connection between phase and magnitude of the open and the closed loop. Choosing a nominal parameter set according to (1) with the plant template one can look at distinct phase values of the open loop to the minimal magnitude of the nominal open loop to fulfill the required maximum magnitude variation of the closed loop.

Connecting points for different phase shifts using a nominal plant parameter set one gets boundaries above which the controller has to bring the nominal open loop plot in the Nichols chart for this frequency ω_i. Since one guarantees by this procedure the closed loop gain variation only, one has to adjust later on with the prefilter that the command response has not only the required small variation but is actually really inside of the given domain for the closed loop response.

However, one has to consider besides these boundaries for each ω_i the stability problem in the Nichols chart, too. In most cases this can be handled by a so-called high frequency boundary, which is based on the following situation:

For a stable closed loop and plants with at least two more poles than zeros - which is unavoidable, if one takes together process plus actuator as the plant to be considered - one can diminish the closed loop variation vis-a-vis the plant variation only up to a certain frequency, which is unfortunately smaller than the cross over frequency in the Bode-diagram (see e.g. Gräser, Dickmann and Neddermeyer 1982). Therefore the requirement:

$$|\Delta T_w(j\omega)| \leq \frac{|\beta(\omega)|}{|\alpha(\omega)|} \tag{4}$$

makes sense only in the frequency command band $0 \leq \omega \leq \omega_c$. For higher frequency one limits the unavoidable amplification of the variations, which can be done for example by a requirement:

$$|T(j\omega)| \leq \gamma, \qquad \gamma > 1 \tag{5}$$

which leads to a closed curve in the Nichols chart with the critical point 0dB, -180° in its interior. Now the templates must not penetrate into this area. Normally the phase variation is already small for ω-values with $\omega > \omega_c$, so that one gets only some gain variation now, which can be approximated by the high frequency gain variation, which is the plant gain variation for $\omega \to \infty$. One finds in this way in addition to the phase and gain dependent boundaries for selected ω_i one further boundary - see forbidden area in fig. 7 - for all frequencies, the socalled high frequency boundary.

CALCULATION OF THE ω_i-BOUNDARY

The high frequency boundary follows directly from (5) and $\lim_{s\to\infty} \Delta P(s, \underline{a})$.

However, the ω_i boundaries are explained up to now in a way, which can be handled very easily by hand and vision in a trial and error procedure, but not very easy by a computer. The basis for an automation has been given by L. Longdon and D.J. East (1979). It starts from the relative plant variation vis-a-vis some suitably selected plant description P^N out of the possible $P(s, \underline{a})$ and the easily verifyable equation:

$$\left|\frac{T^N}{T}\right| = \left|\frac{P^N/P + L^N}{1 + L^N}\right| . \tag{6}$$

With upper and lower boundaries on T^N/T and the calculable relative plant variation P^N/P one can now get the ω_i-boundaries in the Nichols-chart by the computer (for details see Longdon and East 1979). An upper and lower boundary on T^N/T can be found in the following way: From (3) we have:

$$|\alpha(\omega)| = |T_w^{min}| \leq |T_w^N| \leq |T_w^{max}| = |\beta(\omega)| \tag{7}$$

and with $T_w = T\cdot F$ we can write:

$$\left|\frac{T^N}{T}\right| = \left|\frac{T_w^N}{T_w}\right| \geq \left|\frac{T_w^N}{T_w^{max}}\right| = \tilde{\alpha}(\omega),$$

$$\left|\frac{T^N}{T}\right| = \left|\frac{T_w^N}{T_w}\right| \leq \left|\frac{T_w^N}{T_w^{min}}\right| = \tilde{\beta}(\omega),$$

so that we would have lower and upper boundaries $\tilde{\alpha}(\omega)$, $\tilde{\beta}(\omega)$, if we would know T_w^N.

Longdon and East choose now the nominal closed loop transfer function in such an manner, that it has the same percentage deviation from the upper and lower boundary, which gives some $\tilde{\alpha}(\omega)$, $\tilde{\beta}(\omega)$ and allows by that to calculate with (4) some controller R, which meets all requirements set forward in this way. However, the controller has minimum gain only if the actual value of T_w^N is chosen with respect to the value T_w^N used to calculate $\tilde{\alpha}$ and $\tilde{\beta}$. Since this is not always the case with the assumption of Longdon and East, one may get by their method a certain overdesign (higher controller gain than necessary). The simple, but very effective modification is to take a certain number of different T_w^N in the region allowed by (5), getting by this different boundaries in the Nichols-chart, which may cross each other for different phases. If one takes the lower envelope, one gets now for all phases a boundary of minimum gain and avoids by this an overdesign.

To elucidate the above connections, an example is given below.

Example: For the plant model

$$P(s,\underline{a}) = \frac{K\cdot b}{s(s+b)}; \quad \underline{a} = \begin{pmatrix} K \\ b \end{pmatrix}; \quad \underline{a}_N = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \leq \begin{pmatrix} K \\ b \end{pmatrix} \leq \begin{pmatrix} 10 \\ 10 \end{pmatrix}$$

a boundary for L^N is to be calculated at frequency point $\omega_1 = 2$.

The admissible gain variation is ΔT

$$\left|T_w^{max}\right|_{dB} - \left|T_w^{min}\right|_{dB} = \left|\Delta\, T(\omega_1)\right|_{dB} = 6{,}5 \text{ dB}.$$

Fig. 2 shows boundaries for seven allowed values of T_w^N, giving seven paired values of $\tilde{\alpha}$ and $\tilde{\beta}$. The resulting envelope is line B_1.

CONTROLLER DESIGN

The controller design uses the ω_i-boundaries and the high frequency boundary in the Nichols chart. Horowitz and Sidi (1972) are choosing for each of the selected frequencies ω_i and some frequencies in the high frequency range a phase and gain for $L^N = P^N \cdot R$ by inspection and calculate from this gain and phase requirements an R in certain frequencies. They select then a certain denominator and nominator degree for the controller and approximate by appropriate parameter selection the requested controller behaviour. Since this cannot be met exactly in general, they have to control by some additional calculation, how far the selected controller fulfills the requirements. Horowitz/Gera (1980) used a Bode-Integral to automate the evaluation of the frequency response of the open loop. The determination of the controller then needs some optimization steps, too. Here another way is used. The denominator and nominator degree of the controller is chosen at first and the parameters of the controller are optimized directly in such a way, that the open loop plot $L^N(j\omega)$ fulfills as far as possible the requirements set by the boundaries in the Nichols-chart. The optimization uses after exploring some other possibilities now a procedure, which was successfully applied by G. Kreisselmeier and R. Steinhauser (1979) for some other problems:
The constraints for $L^N(j\omega)$ and/or the controller are formulated as a vectorial performance index $\underline{G}(\underline{r})$.
The latter is normally composed of the following six elements:

- $g_1(\underline{r})$: Sum of the deviations of the frequency response points $L^N(j\omega_i)$ from the boundaries B_i to higher gains,
- $g_2(\underline{r})$: Sum of the deviations of the frequency response points $L^N(j\omega_i)$ from the boundaries B_i to lower gains,
- $g_3(\underline{r})$: Component resulting from the violation of the single high-frequency boundary by $L^N(j\omega, \underline{a}_N)$,
- $g_4(\underline{r})$: Gain of the controller $(j\omega \to \phi)$,
- $g_5(\underline{r})$: High frequency gain k^∞ of the controller,
- $g_6(\underline{r})$: Stability of the controller.

The number of the vector components g_i is easy to vary, so that additional constraints can be introduced into the design. For this purpose the performance index components have to be formulated such that their values become the smaller the better the design requirements are met.

Then the design of the controller $R(\underline{r})$ has to be transformed into the task of determining the controller parameters $\underline{r}$ in such a manner that each component of the performance index $\underline{G}(\underline{r})$ becomes sufficiently small.

This goal is achieved by iterative selection of a target vector $\underline{c}^{\gamma+1}$ with $\underline{G}(\underline{r}^{\gamma}) < \underline{c}^{\gamma+1} < \underline{c}^{\gamma}$ and by solution of one optimization task in every iterative step.

The optimization task is formulated:

$$\text{find} \quad x = \min_{\underline{r}} x(\underline{r}) \quad \text{for which}$$

$$\underline{G}(\underline{r}^{\gamma+1}) < x\, \underline{C}^{\gamma+1} \quad ; \quad x<1$$

$$\text{with} \quad x(\underline{r}) = \max_{1\le i\le n} (G_i(\underline{r})/C_i)$$

is fulfilled.

In the program package described here this problem is solved numerically using a zeroth order optimization procedure and a linear one-dimensional search strategy.

INTERACTIVE DESIGN

In practice a design based on the method described in the foregoing starts with the fixing of the vectorial performance index to be used in the optimization procedure. Generally the standard six dimensional vector agreed for the program package serves this purpose well. Upon establishment of the performance criterion, the development engineer selects a controller order as well as controller start values $\underline{r}^o$.

In doing so, the program enables him to separate individual controller parameters - for example, an integarting pole $S_p = 0$ - from the optimization process and to predetermine them exactly.

For the numerical optimization it is necessary to choose the target vector $\underline{C}^I$, for which $\underline{G}(\underline{r}^o) < \underline{C}^I$ holds.

Beginning the optimization with these start values, one obtains the parameter vector $\underline{r}^I$ and the performance vector $\underline{G}(\underline{r}^I) < x_I \underline{C}^I$ after the first iterative step. By selecting the following target vector, the designer is in the position to control the optimization process interactively. If a specific criterion is intended to be improved the corresponding vector component $C_i^{\gamma+1}$ chosen has to be as small as possible - however, always larger than $g_i(r^{\gamma})$. When a criterion is already fulfilled satisfactorily it suffices to select $C_i^{\gamma+1} = C_i^{\gamma}$.

As an aid to select the target vector, the designer can have the actual frequency response of the nominal open loop plotted into a Nichols diagram, together with the boundaries to be kept. By means of his geometrical conception of the optimum shape of the frequency response of L^N (Horowitz 1973) he then recognizes which criterion in the performance vector is still to be improved.

With the aid of the above-mentioned plots the designer can observe the progress of the design and take appropriate measures, if required.

If it turns out, for instance, that no substantial improvement of the locus can be achieved by a change of the target vector there is the possibility of continuing the optimization process by means of new start values for the controller parameters or with a new controller.

After a sufficient approximation of the optimum frequency response of L^N has been reached, the designer can stop the optimization process and arrange for the output of the controller parameters.

In addition, the described interactive program package offers a variety of possibilities of checking the control result obtained - for example, the calculation of step function responses and frequency responses.

Hence, to show the performance of the design support, the synthesis of a controller regulating the pressure of a servo-hydraulic actuator is considered in the following example (Drechsler 1982).

EXAMPLE

The plant investigated, $P(s,\underline{a})$, was identified on a motor-vehicle test bench as

$$P(s,\underline{a}) = \frac{K_1\left(1 + \frac{2d_A}{\omega_A} s + \frac{1}{\omega_A^2} s^2\right)}{s(1 + Ts)^2\left(1 + \frac{2d}{\omega_o} s + \frac{1}{\omega_o^2} s^2\right)}$$

where the parameter set $\underline{a}$ varied within the bounds

$$\underline{110} \le K_1 \le 340$$

$$\underline{98} \le \omega_o \le 116 \quad ; \quad \underline{0.08} \le d \le 0.5$$

$$56 \le \omega_A \le \underline{69} \quad ; \quad 0.4 \le d_A \le \underline{0.6}$$

The underlined magnitudes were chosen as nominal parameter values.

Considering the above-mentioned plant in the Bode diagram (Fig. 3), one finds that the maximum gain variation is approximately 35 dB.

The requirements in the closed control loop are given in the frequency domain and represented in Fig. 4.

The transformation of these requirements into boundaries took place at 11 frequency points. Fig. 5 shows these boundaries $B_1 \ldots\ldots B_{11}$ with the nominal plant frequency response P^N the frequency points belonging to the boundaries are marked in the curve. Fig. 6 illustrates the progress of the design. The frequency response of the open loop meets to the requirements imposed by the boundaries B_i. Some of the frequency response points coincide with the boundaries, for example, ω_4, ω_{10}, ω_{11} whereas the remaining ones lie above them, which corresponds to an overdesign.

Frequency response L^N, however, still violates the single high-frequency boundary, which was determined here from the high-frequency gain variation $\Delta k_\infty = 26.24$ dB and the constraint $|T| \le 10$ dB.

Fig. 7 shows the frequency response of L^N after five further iterative steps. It can be seen that the wanted optimum curve according to Horowitz is reached in good approximation by L^N.

The controller parameters obtained are:

$$P(s) = \frac{K(s - n_1)(s - n_2)}{1 + \frac{2D_1}{\omega_1} s + \frac{1}{\omega_1^2} s^2}$$

$$K = 2.3 \cdot 10^{-6}$$

$$n_1 = -\ 820.88$$

$$n_2 = -\ 303.12$$

$$D_1 = 0.28$$

$$\omega_1 = 1353.2$$

To verify the design the magnitude variation of the closed loop is shown in Fig. 8. The design requirements are marked with (x) and the actual variation of the closed loop with (+). It can be seen that the pre-determined gain variation was met at all points. The wanted absolute position of the frequency response is easy to attain by use of a preceding filter.

Note:

In this example the unusually loose constraint $|T| \le 10$ dB was selected because of a bandwidth limitation.

CONCLUSION

The method of Horowitz and Sidi for a quantitatively prescribed suppression of plant parameter variation consequences in the command frequency area has been implemented into an interactive computer program, using the ideas of Horowitz and Sidi and Longdon and East with some modifications. The program runs on a small HP 9825A desk computer. An improved version using a light pencil input on the relative plant variation areas for a DEC-PDP 11/34 and/or a DEC-MINC is foreseen to be available at the end of the year. The method of Horowitz/Sidi and the program has been proved to be very helpful for design considerations on car test benches, since it gives clear indications, what closed loop step response boundaries may be reached with actuators with a given bandwidth and/or, what actuators are necessary if a certain behaviour shall be guaranteed for a specified plant uncertainty.

REFERENCES

Drechsler, A. (1982). Analyse und quantitativer Entwurf von nichtphasenminimalen Mehrgrößensystemen mit großen Parameterunsicherheiten.
Studienarbeit am Institut für Regelungstechnik der Technischen Hochschule Darmstadt (directed by A. Gräser and W. Neddermeyer).

Gera, A. and Horowitz, I.M. (1980). Optimization of the loop transfer function.
Int. J. Control, vol. 31, pp. 389-398.

Gräser, A., Dickmann, M., Neddermeyer, W., (1982). Robuster Regelkreisentwurf: Das Verfahren von Horowitz und seine Automatisierung.
Regelungstechnik 1982.

Horowitz, I.M. (1963). Synthesis of Feedback Systems.
Academic Press

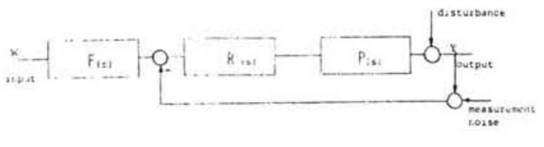
Horowitz, I.M. and Sidi, M. (1972). Synthesis of feedback systems with large plant ignorance for prescribed time-domain tolerances.
Int. J. Control, vol. 16, pp. 287-309.

Horowitz, I.M. (1973). Optimum loop transfer function in single-loop minimum-phase feedback systems.
Int. J. Control, vol. 18, pp. 97-113.

Krishnan, K.R. and Cruickshanks, A. (1977). Frequency Domain Design of Feedback Systems for Specified Insensitivity of Time Domain Response to Parameter Variation.
Int. J. Control, vol. 25, pp. 709-620.

Kreiselmeier, G. and Steinhauser, R. (1979). Systematic Control Design by Optimizing a Vector Performance Index.
IFAC-Symposium on CAD of Control Systems, Zürich, Switzerland.

Longdon, L. and East, D.J. (1979). A Simple Geometrical Technique for Determining Loops Frequency Response Bounds which achieve Prescribed Sensitivity Specifications.
Int. J. Control, vol. 30, pp. 153-158.



$L = RP \quad , \quad T = \frac{L}{1+L} \quad , \quad T_W = \frac{FL}{1+L}$

Figure 1. Two degrees-of-freedom structure

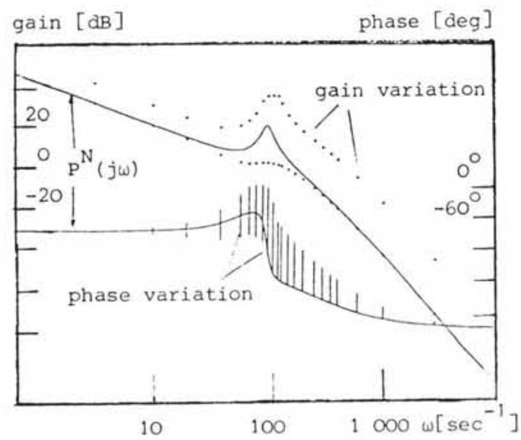


Figure 3. Frequency response of $P(j\omega)$ - Bode plot

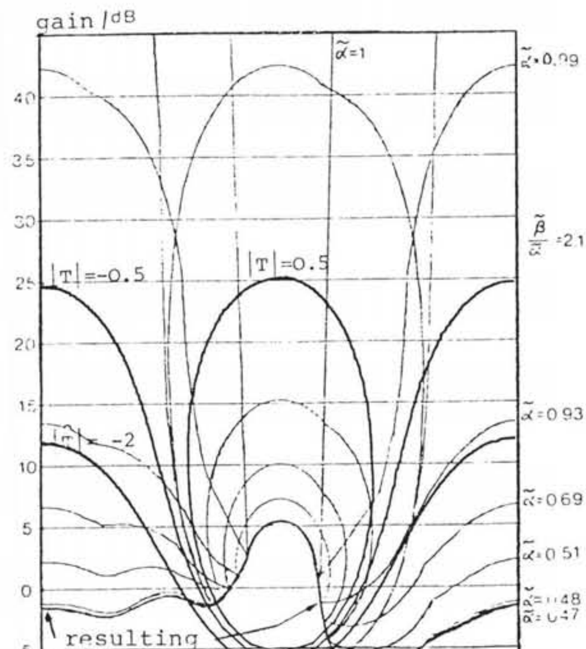


Figure 2. Boundaries for seven paired values of $\tilde{\alpha}$, $\tilde{\beta}$ - Nichols chart

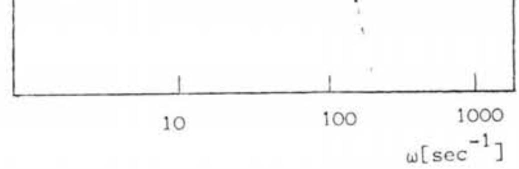


Figure 4. Tolerances on the frequency response $T_W(j\omega)$ - Bode plot

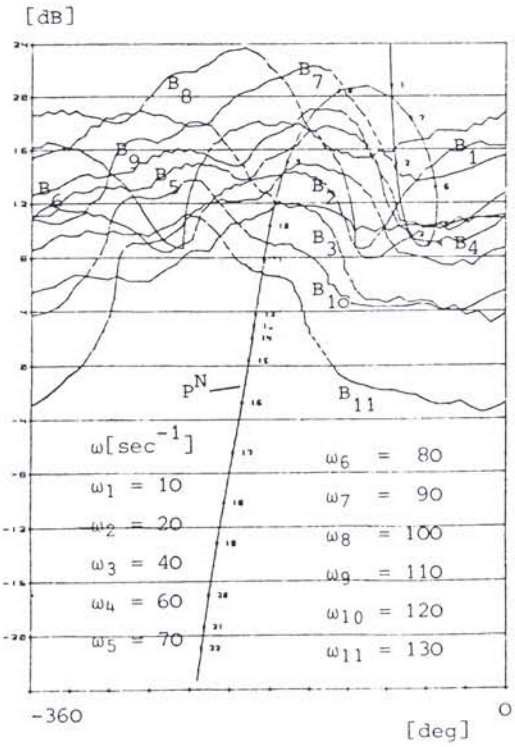


Figure 5. Resulting boundaries and nom. plant frequency response $P^N(j\omega)$ - Nichols chart

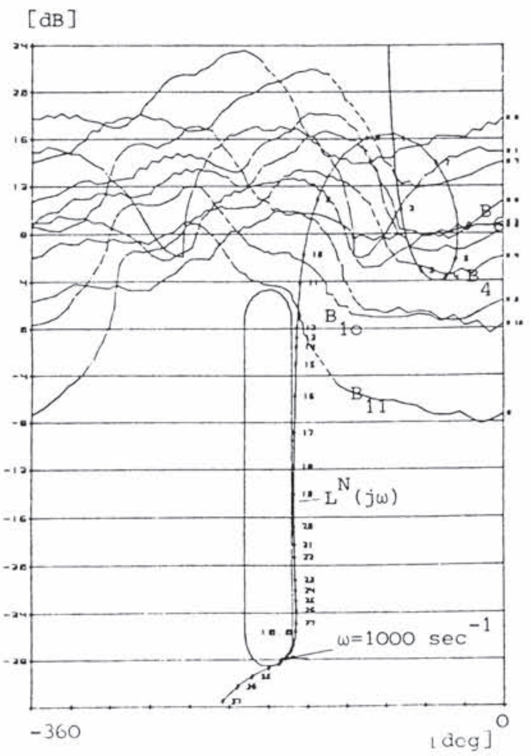


Figure 7. Frequency response of $L^N(j\omega)$ after the last iteration step - Nichols chart

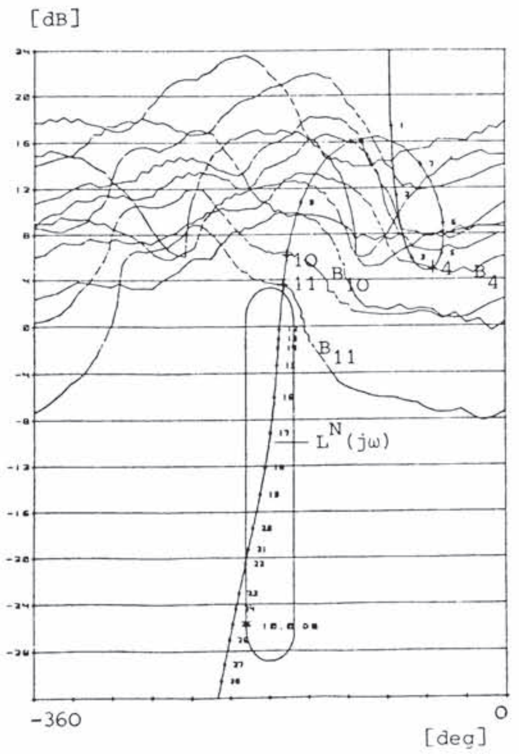


Figure 6. Frequency response of $L^N(j\omega)$ - Nichols chart

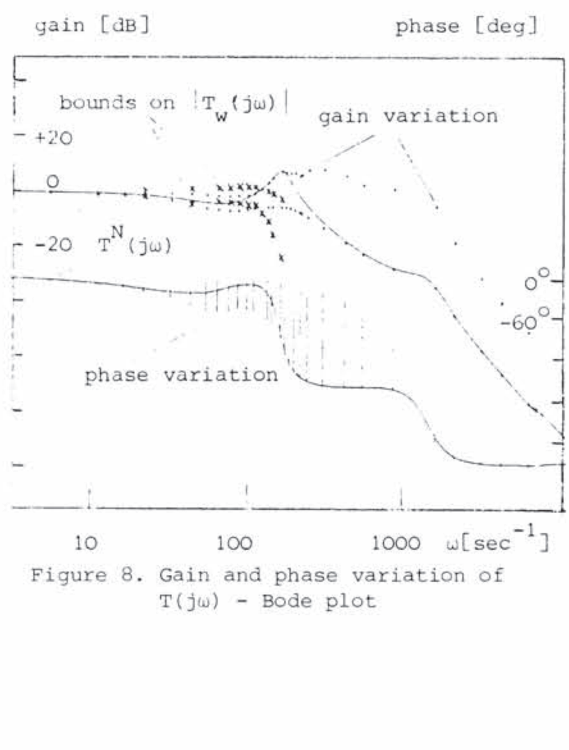


Figure 8. Gain and phase variation of $T(j\omega)$ - Bode plot