

## SELF-TUNING CROSS PROFILE CONTROL FOR A PAPER MACHINE

A. Gräser\* and W. Neddermeyer\*\*

\*Lippke GmbH and Co., Wilhelm-Leuschner-Str. 12, D-5450 Neuwied 1,  
Federal Republic of Germany

\*\*Section Control Systems Theory, Institute of Control Engineering, Technical  
University of Darmstadt, Schlossgraben 1, D-6100 Darmstadt,  
Federal Republic of Germany

**Abstract.** The paper discusses a self tuning control algorithm for a multivariable plant applied to a grammage cross profile control of a paper machine. The plant model represents the steady-state couplings as well as the dynamics behaviour of the system. The control algorithm handles special situations including the case where the number of measurements is not equal to the number of actuators and the case where special actuators are locked.

The whole identification algorithm with a special filter and an estimation of the required measurement accuracy for a successful identification of the steady-state couplings will be discussed. The paper shows how to proof the stability of the control system including the cases of locked actuators and identification errors.

Finally, the whole control system is verified by in house simulations and at a real plant. The obtained control results are discussed.

**Keywords.** Computer Control, Direct Digital Control, Paper Industry, Paper Cross Profile Control, Multivariable Control Systems, Decoupling, Stability

### I. INTRODUCTION

Paper and extruded plastics are produced continuously on fast running machines with web width up to 10 m.

The quality requirements in paper and extrusion coating productions are very high, especially for low variance in cross direction. The main quality sheet parameters of interest are

- grammage
- moisture content
- caliper.

The quality parameters across the web can be influenced by several actuators as

- slice positioners at the head box (grammage)
- steam showers (moisture)
- infrared heaters (moisture)
- air showers (caliper, smoothness)
- inductive heating coils (caliper, smoothness)

The actuators consist of several equal parts, which are controlled independently and influence areas of different size of the web.

For cross profile grammage control the slice opening of the head box is changed. Due to the physical interaction between actuator, web and the flow in the head box, a single actuator element influence broad areas of the sheet. This results

in strong couplings between the single actuator responses. From the control point of view these interactions result in an multivariable plant description with strong couplings between the single control loops. The grammage control is the most demanding control loop compared with moisture and caliper ones. Therefore we will focus our interest to this quality control loop. The derived algorithm is also useful for other kinds of actuators and controlled variables (e.g. moisture).

The paper is organized in 8 sections. The second, third and fourth describes the plant behaviour, the mathematical representation and the applied control strategy. Section five is applied to the identification algorithm.

Section five to nine describe the complete multivariable control system, the stability analysis and compare some in house simulation results with commissioning results at a paper machine.

### II. PROCESS DESCRIPTION

The process has been discussed in some earlier paper. The work of Beecker and Bareiss [1] is the most complete early treatment of the subject from the cross-profile-control point of view.

Some later papers of Boyle [2,3] and Wilhelm [4,5] discusses the weight control in cross direction by actuators which are located at the wet end of the machine, too.

All these papers discusses some special aspects of cross profile control. This paper tries to give a solution for the complete problem including an identification procedure and a stability proof.

However, the major problem for the controller design is the strong interaction of the effects of adjacent cross-direction actuators at the wet end of the machine.

Fig. 1 shows the flow onto the wire. An increase of the flow within one section leads to changes in the grammage profile within a third of the web width.

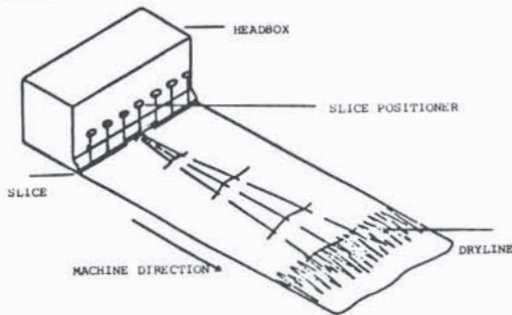


Fig. 1 Wet end of a paper machine

In order to obtain the steady-state behaviour of the systems we assume that the *i*-th actuator is moved. The result is a decreasing or increasing of the mass-flow at this point depending the sign of the slice positioner displacement. Due to cross flows in the head box we obtain at the dry-line a typical response of the grammage profile as shown and described in some papers before (see also fig. 2).

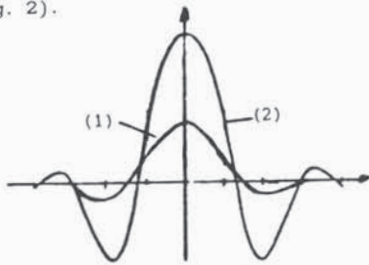


Fig. 2 Typical weight profile responses to slice displacements in one position

We call such a response "coupling curve". One can measure this effect on-line by scanning sensors mounted at several locations within the paper machine (see fig. 3).

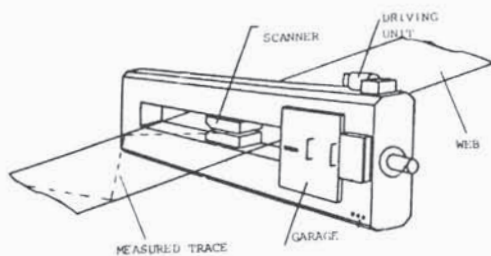


Fig. 3 Measurement unit

These units measure the web parameters (e.g. grammage, moisture, caliper) along a triangular measurement trace as shown in fig. 3. The measured values contain the information in cross-direction and in machine direction. We discuss the cross direction measurements only. These has to be calculated from the measured diagonal profil. The necessary filter algorithm is not considered here. For further information on this point see [6].

### III. PROCESS MODEL

The steady state response can be described by eq. 1, if the displacement of one slice position *j* is considered only.

$$\begin{pmatrix} y_1 \\ \vdots \\ y_i \\ \vdots \\ y_m \end{pmatrix} = \begin{pmatrix} q_{1j} \\ \vdots \\ q_{ij} \\ \vdots \\ q_{mj} \end{pmatrix} u_j \quad \text{eq. 1}$$

with  $y_i$  - The measured grammage values in cross direction.

$q_{ij}$  - The values of the coupling curve.

Considering all actuators eq. 1 expands to the matrix equation

$$y = Q u \quad \text{eq. 2}$$

Normally the number of measured points (*m*) of the web in cross direction is not equal to the number of actuator elements (*n*). Accordingly the coupling matrix has the order *m* x *n*. Due to the fact that the coupling curves influence a part of the web only the coupling matrix has a band-structure.

The dynamic part of the process model can be described by the lag behaviour of the actuators and a transport delay  $T_t$ .

These two effects are represented in eq. 3 in the frequency domain as

$$D(s) = \begin{pmatrix} \frac{1}{1+T_t s} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{1+T_t s} \end{pmatrix}_{n \times n} \quad \text{eq. 3}$$

$$T(s) = \begin{pmatrix} e^{-T_t s} & & \\ & \ddots & \\ & & e^{-T_t s} \end{pmatrix}_{m \times m}$$

$$\text{with: } T_t = \frac{l}{v_{pap}}$$

$v_{pap}$  - paper velocity

$l$  - distance between actuator and measuring unit

The diagonal matrix elements have generally all the same value.



$$T_i = T \quad i = 1, 2, \dots, n$$

Finally, the whole process model is shown in fig. 4.

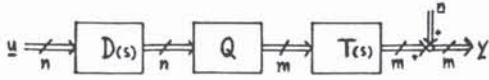


Fig. 4 Block diagram of the process model

IV. CONTROL STRATEGY

For the control task it is most important to influence the grammage on a width of one measure point, that means one has to eliminate the coupling terms. This can be done via a decoupling matrix.

From multivariable control theory it is known, that strong couplings between control loops cause stability problems. A common solution is the introduction of a decoupling matrix which yields to decoupled single control loops. In the case of  $m = n$  the coupling matrix is a quadratic one.

To calculate the required slice position  $\underline{u}$  we define eq. 4.

$$\underline{u} = R^I \underline{y}_D \quad \text{eq. 4}$$

with  $\underline{y}_D$  the required grammage profile and  $R^I$  the decoupling matrix. With the goal that  $\underline{u}$  should be equal to  $\underline{y}_D$  ( $\underline{y} = \underline{y}_D$ ) we obtain  $R^I = Q^{-1}$ . (see also fig. 5)



Fig. 5 Decoupling with the matrix  $R^I$

In the case of  $m > n$  which is the normal one, the inverse matrix is not defined. A possible way to find still a decoupling matrix is the introduction of a performance index and the minimization of this index.

A typical performance index  $J$  for such a problem is given with eq. 5

$$J = \underline{e}^T \underline{e} \quad \text{eq. 5}$$

with  $\underline{e} = (\underline{y}_D - \underline{y})$

Minimizing the index  $J$  the resulting decoupling Matrix  $R^I$  can be written as

$$R^I = (Q^T Q)^{-1} Q^T \quad \text{eq. 6}$$

This matrix is the pseudo inverse of  $Q$ . The multivariable system is decoupled by the matrix  $R^I$  and the multivariable control loop can furtheron be discussed as a system with  $n$  independent loops.

All these loops has the same time constants. This yields to identical controller parameters.

The complete control loop is described by five matrices as shown in fig. 6.

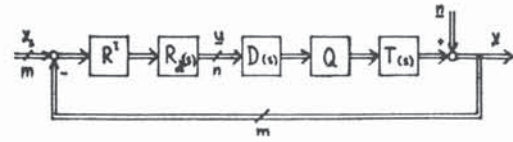


Fig. 6 Structure of the control loop

$R^I$  represents the decoupling matrix and  $R_d(s)$  the dynamic part of the controller, which is a diagonal matrix with identical elements  $r(s)$ .

$$R_d(s) = \begin{pmatrix} r(s) & & \\ & \ddots & \\ & & r(s) \end{pmatrix}_{n \times n} \quad \text{eq. 7}$$

The plant model is represented by  $D(s)$ ,  $Q$  and  $T(s)$ .

The dynamic behaviour of the cross profile with the above described strategy depends very closely on the accuracy of the decoupling matrix  $R^I$ . The matrix is calculated from the steady state plant model  $Q$  which is calculated from the coupling curves. The identification of the coupling curves is described in section five.

V. IDENTIFICATION

The identification is done with a least square method. One slice position  $u_j$  is changed at time  $k$  and all others remain in the old position. Equation 8 describes the response to be measured.

$$\underline{y}(k + d + e) - \underline{y}(k + d) = \underline{q}_j (u_j(k) - u_j(k - 1)) \quad \text{eq. 8}$$

$d$  is the transport delay

$e$  is the response time

both are rated in sampling steps.

In order to decrease the influence of measurement disturbances the measurements can be repeated  $s$  times without slice changes.

$$\sum_{l=0}^s (\underline{y}(k + d + e + 1) - \underline{y}(k + d - 1)) = \underline{q}_j (s+1) (u_j(k) - u_j(k-1)) \quad \text{eq. 9}$$

With  $N$  changes of the slice positioner one obtains eq. 10.

$$\underline{y} = u_j \underline{q}_j^T \quad \text{eq. 10}$$

$$\underline{y}^T = (\underline{y}_d(1), \underline{y}_d(2), \dots, \underline{y}_d(N))$$

$$\underline{u}_j^T = (u_{jd}(1), u_{jd}(2), \dots, u_{jd}(N))$$

with

$$\underline{u}_{jd}(i) = \sum_{l=0}^s (\underline{y}(d + 1 + l + ie + (i-1)s) - \underline{y}(d - 1 + 1 + (i-1)e + (i-1)s))$$

$$u_{jd}(i) = \sum_{l=0}^s (u_j(1 + (i-1)e + (i-1)s) - u_j((i-1)e + (i-1)s))$$

For the estimation of the  $q_j$ -terms eq. 11 can be used (see [7]).

$$q_j^T = (u_j^T u_j)^{-1} u_j^T y \tag{eq. 11}$$

With eq. 11 an identification of a single coupling curve is carried out.

The identification algorithm calculates each point of the coupling curve independent from its neighbours. Due to the physical behaviour of the flow onto the web, the coupling curve must be smooth.

This knowledge leads to a filter algorithm of the coupling curve, which is shown in fig. 7.

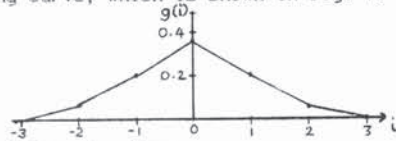


Fig. 7 Filter function

$$g(i) = \begin{cases} e^{-|i|/a} / \left( \sum_{j=-b/2}^{b/2} e^{-|j|/a} \right) & |i| \leq b/2 \\ 0 & |i| > b/2 \end{cases} \tag{eq. 12}$$

With the weighting factor  $a$  and the width of the filter  $b$  the filter response can be chosen.

The filtered values of the coupling curve  $q^F$  are calculated with eq. 13

$$q_1^F = \sum_{j=1}^n q_j g(j-1) \quad l=1,2,\dots,n \tag{eq. 13}$$

**VI. STRUCTURE OF THE MULTIVARIABLE SELF-TUNING CONTROL LOOP**

The identification and the multivariable control loop strategy are combined in the whole control structure which is shown in fig. 8.

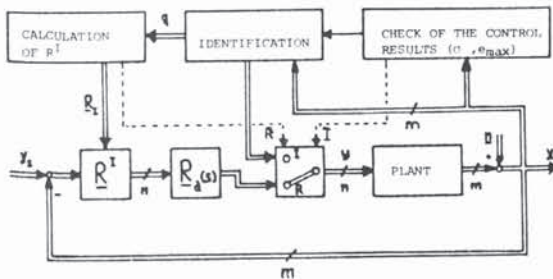


Fig. 8 Structure of the multivariable self-tuning loop

The plant level in fig. 8 describes the basic control loop with the decoupling matrix  $R^I$  and the controller matrix  $R_d(s)$ .

The supervisor level observes the control results and calculates the necessary position of the switch in fig. 8. The position of the switch indicates closed loop control or identification mode. There are several mathematic or heuristic criterions available to calculate the position of the switch. One criterion for rating the control loop result is the  $\sigma$ -value of the cross profile (see eq. 14)

$$\sigma = \sqrt{\frac{1}{m} \sum_{i=1}^m (y_i - y_{Di})^2} \tag{eq. 14}$$

$m$  - number of measured points in cross direction.

Another one is the maximum of the difference between the commanded and measured value of the cross profile

$$e_{\max} = \max(|y_i - y_{Di}|) \quad i=1\dots m \tag{eq. 15}$$

A practical way is to switch the control off and start the identification if the  $e_{\max}$ -value or the  $\sigma$  value exceeds their upper limits.

The identification results are needed to calculate a new decoupling matrix. When the identification is finished this matrix will be used and the switch is set back to the basic control mode.

**VII. STABILITY ANALYSIS**

The stability of the basic control loop has to be verified especially in the case of locked actuators and errors of the plant model which is used for decoupling.

In order to proof the stability of the multivariable control loop we break the loop between  $R_d(s)$  and  $D(s)$  in fig. 6. Then one obtains for the open loop  $L(s)$

$$L(s) = R_d(s) R^I T(s) Q D(s) \tag{eq. 16}$$

Assuming that  $T(s)$  is a diagonal matrix with identical elements one can change the position of  $R^I$  and  $T(s)$  in eq. 16 and obtain

$$L(s) = R_d(s) \tilde{T}(s) R^I Q D(s) \tag{eq. 17}$$

with  $R^I = (\hat{Q}^T \hat{Q})^{-1} Q^T$

$$L(s) = R_d(s) \tilde{T}(s) (\hat{Q}^T \hat{Q})^{-1} \hat{Q}^T Q D(s) \tag{eq. 18}$$

with  $(\hat{Q}^T \hat{Q})^{-1} \hat{Q}^T Q = S$

$$L(s) = R_d(s) \tilde{T}(s) S D(s) \tag{eq. 19}$$

In the case of  $\hat{Q} = Q$  is  $S$  equal  $I$  and the whole multivariable loop can be written as a  $n$ -single loop system since the matrices  $R_d(s)$ ,  $\tilde{T}(s)$ ,  $D(s)$  are diagonal ones. In this case the multivariable system is stable if the  $n$ -single loops are stable.

The design of the multivariable loop is now simplified to  $n$ -single loop designs which can easily be carried out by wellknown procedures.

In such a system a locked actuator effects that one single loop is not controlled but this has no influence on the other ones. Due to the fact that the uncontrolled single plants are stable the



whole multivariable system is stable, too.

The assumption  $\hat{Q} = Q$  is not a serious one for a real plant. The measurement errors and disturbances lead always to an identification error  $\hat{Q} \neq Q$  with the result that  $S$  is not equal  $I$ .

However, the above described facts are valid also for stability, if the matrix  $S$  is a diagonally dominant matrix. For a proof look at [8]. A stability theorem based on the concept of a set of "normalized" Gershgorinbands and a practical rule how to calculate the normalized radii are given in [9,10].

With this rule we try to answer the question: Which size of model errors are acceptable in order not to destroy stability?

This question can not be answered in general. For a specific number of actuators, measurement points and coupling curve one can get an empirical value by a lot of simulations. For the case of 64 measurement points, 18 actuators and a coupling curve which is shown in fig. 9b we did some simulations with different sizes of measurement disturbances. Fig. 9a/b shows a typical result.

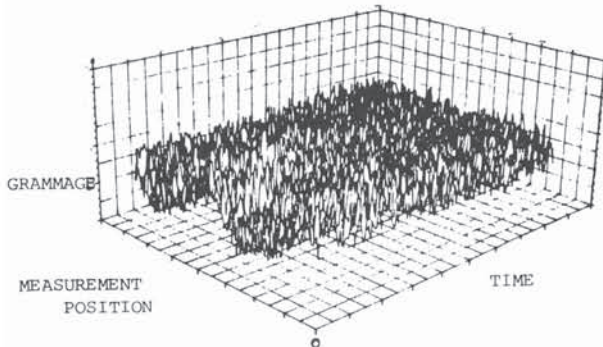


Fig. 9a Simulated cross profile

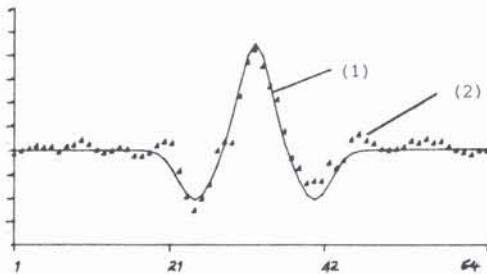


Fig. 9b Coupling curves (1) plant model (2) identified

In fig. 9a the web grammage profile (nominal value  $80 [g/m^2]$ ) with a measurement disturbance of  $1 [g/m^2]$  is shown. The identification of the coupling curve is done with an actuator stimulation of 30 counts that leads to a maximum response of approximately  $2.8 [g/m^2]$ . The identified coupling curve is marked by stars in fig. 9b, the real coupling curve is given by the line. With this identified coupling curve one obtains Gershgorinbands, which satisfy the stability conditions.

Due to a lot of simulations we found that for disturbances of the identification step response up to approximately 30 percent the matrix  $S$  remains diagonally dominant.

VIII. CONTROL RESULTS

This point figures out three control situations. The first two are simulated ones. The third is measured at a real plant controlled by a Lippke 4012 system.

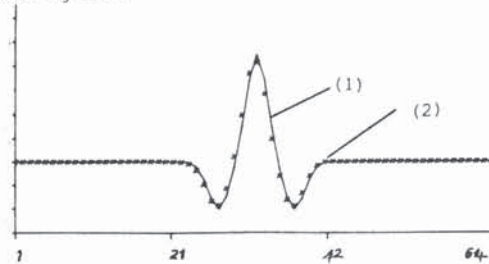


Fig. 10a Coupling curves (1) plant model (2) identified

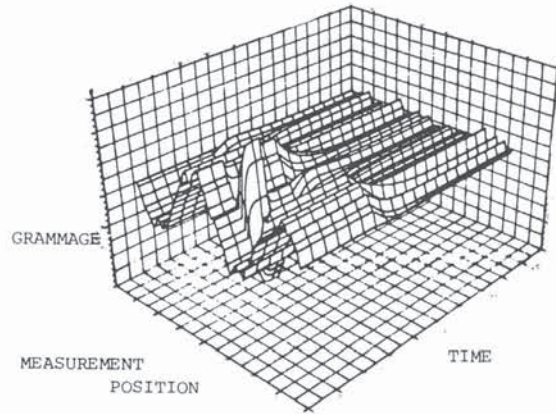


Fig. 10b Grammage cross profile

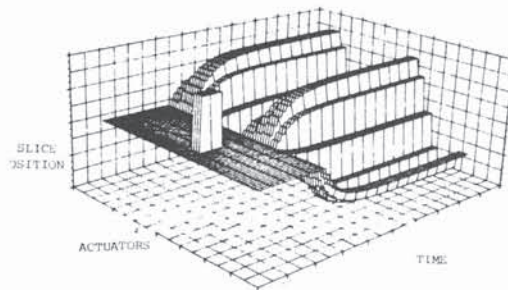


Fig. 10c Actuator positions

Fig. 10 shows the simulated coupling curve and the identified one (stars). The plant is simulated without any disturbances, the web cross profile and the actuator positions for 62 seconds are in part(b) and (c) of this figure. The  $\sigma$ -value calculated according eq.14 is shown in part (d) over the same time interval. The increasing part at the beginning results by the stimulation of one actuator during the identification cycle. The steady state behaviour is shown in fig. 10e by the start cross profile and the final one.

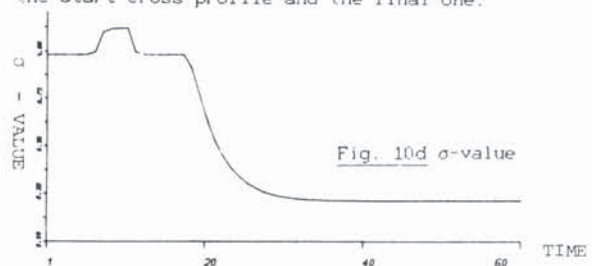


Fig. 10d  $\sigma$ -value

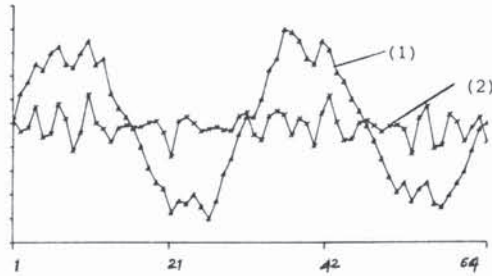


Fig. 10e cross profile (1) start profile (2) end profile

Fig. 11 sketches the second simulation run with three locked actuators (positions: 5,6,12).

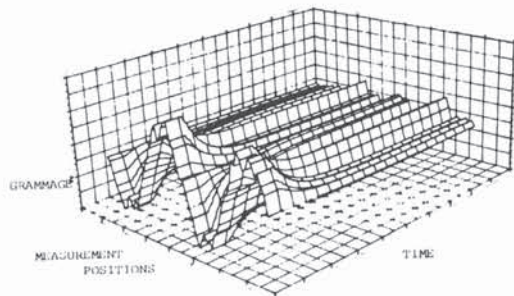


Fig. 11a Grammage cross profile

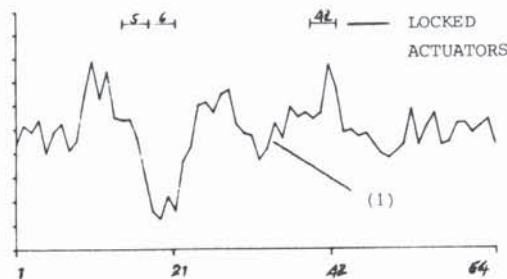


Fig. 11b Final cross profile (1)

The cross profile over the whole simulation time is in part (a) of this figure. Part (b) shows the final profile. One can see that the final profile is not so good as in the first simulation run especially at the regions where the actuators are locked. However, the whole multivariable loop is stable.

The last fig. 12 shows the  $\sigma$ -value of the grammage cross profile of a paper machine which was controlled by the above described algorithm. After an identification cycle the control loop decreased the  $\sigma$ -value very fast (compare fig. 12a with fig. 10d).



Fig. 12a  $\sigma$ -value

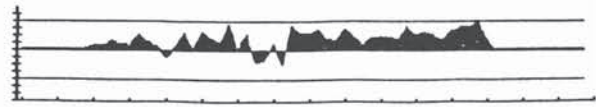


Fig. 12b Start profile

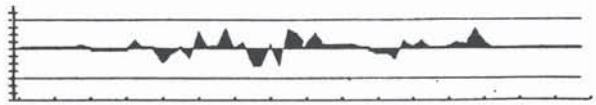


Fig. 12c End profile

The start profile and the steady state profile are given in part (b) and (c) of this figure.

**IX. CONCLUSIONS**

We did this simulation and control tests with a lot of different paper qualities. The measured results were always in a good agreement with the simulated ones as shown in the example case above. This indicates, that the control loop shown in fig. 8 is an acceptable solution for the discussed control problem.

**X. REFERENCES**

- [1] Beecker, A.E. Theory and Practice of automatic control of basis weight profiles; TAPPI, VOL.53, No.5, 1970
- [2] Boyle, T.J. Can. J. Chem. Eng. (1977)
- [3] Boyle, T.J. Practical algorithms for cross-direction control; TAPPI, VOL. 61, No.1, 1978
- [4] Wilhelm, R.G. Self tuning control strategies: Multi-faceted solutions to paper machine control problems ISA JOINT SYMPOSIUMS 1982, COLUMBUS, OHIO
- [5] Wilhelm, R.G. Norwegian Institute of Technologie, PRP 5, Automation, Sept. 1983
- [6] Richards, G.A. Cross Direction Weight Control JAPAN PULP and Paper, 1982
- [7] Isermann, R. Prozeßidentifikation Springer-Verlag
- [8] Mee, D.H. Specifications and criteria for multivariable control system design; Joint Automatic Control Conference, Purdue University, Indiana, USA, 1976
- [9] Kantor, J.C. Andres, R.P. A note on the extension of Rosenrock's Nyquist array techniques to a larger class of transfer matrices, Int. J. Control, Vol.30, 1979
- [10] Tolle, H. Mehrgroßenregelkreissynthese; Band I, Oldenbourg Verlag, 1983