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# Development of a benchmarking methodology for motion control algorithms

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**Bachelorthesis No. 1385/21**

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At the Institute of Automotive Engineering of TU Darmstadt (FZD), research on modular and service-oriented automated vehicles is being conducted in cooperation with several German universities for the UNICAR*agil* project funded by the Federal Ministry of Education and Research (BMBF). The focus is on the avoidance of technological legacy and on replacing it with new, disruptive concepts.

The vehicles to be developed possess four electric wheel hub drives that allow steering angles and drive and braking torques to be specified for individual wheels. The associated motion control system is being developed at FZD.

To evaluate the performance of implemented motion control algorithms, suitable metrics and test methods are needed. Within this thesis, such metrics and evaluation techniques shall be researched and investigated towards their applicability within the project. Afterwards, an overall benchmarking methodology for the motion control shall be developed based on the researched metrics and methods.

In detail, the following steps must be carried out:

1. Familiarization with the UNICAR*agil* project, the existing motion control system and necessary concepts of control engineering and vehicle dynamics
2. Specification of the assignment
3. Literature review of different metrics and test methods for evaluation of motion control algorithms
4. Investigation of applicability of the previously research metrics and methods, based on derived requirements
5. Development of an overall benchmarking methodology

Seite: 1/2



6. Application of the methodology to an exemplary controller within the project
7. Composing of the written report

Focus of the evaluation:

- Methodology of the procedure
- Completeness
- Comprehensibility and resilience of the argumentation
- Quality of the following results to be delivered:
  - Written report and documentation
  - Created source code
- Final presentation

The expected results of the work are the fulfillment of the individual work steps as well as a detailed, easily comprehensible documentation of the findings obtained. The submission of all measurement data and source code is required. The work remains the property of the department. Reference is made to the department's leaflet.

Prof. Dr. rer. nat. Hermann Winner

Tobias Homolla M. Sc.  
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## Abstract

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The UNICARagil project is funded by the Federal Ministry of Education and Research (Bundesministerium für Bildung und Forschung, BMBF). Along with other German universities and companies, the Institute of Automotive Engineering (Fahrzeugtechnik Darmstadt, FZD) at the Technische Universität Darmstadt is collaborating in this project whose objective is the development of fully driverless electric vehicles. The concept of the vehicle consists in a platform and add-on modules. This enables a modular approach to the development, improving the flexibility and updatability of the different components. The drivetrain structure of the project's vehicle consists of a four-wheel independently steered (4WIS) and four-wheel independently driven (4WID), that lies within the over-actuated (OA) vehicles set. This results in independent control of yaw, contrary to the traditional two-wheel steered drivetrains yielding an underactuated system.

The motion control system is currently being developed at the FZD. To evaluate the performance of the motion control algorithms, a benchmarking methodology is developed in this thesis. In the first place, a study of the UNICARagil project and investigation on the details of the project's control system is performed. Then, a revision of the state of the art of the evaluation of motion control algorithms (MCAs) is conducted. A set of requirements for MCAs based on the research made is proposed. A literature review of the different metrics and test methods is performed, and their applicability within the project is investigated. Finally, an overall benchmarking methodology that allows the evaluation of MCAs is developed based on the previously defined requirements. The methodology is then applied to an existing controller within the project.

To perform the evaluation of the different metrics and test methods within the benchmarking methodology, several models for the vehicle dynamics as well as for the different components were employed. The multiple-input multiple-output (MIMO) control system was decoupled into three single-input single-output (SISO) systems after performing Relative Gain Array (RGA) to the vehicle dynamics model. For the analytical evaluation of stability, each of the components along the control system was linearized. The analysis of the accuracy was performed with a realistic model of the vehicle. The rest of the experimental tests were simulated by realistic models developed within the UNICARagil project and using IPG CarMaker.

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## Table of Contents

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Abstract .....	I
Table of Contents .....	II
Symbols and Indices .....	IV
List of Abbreviations .....	VI
List of Figures .....	VII
List of Tables .....	VIII
1 Introduction .....	1
1.1 Motivation .....	1
1.2 Ascertainment of assignment .....	1
1.3 Methodology .....	2
1.4 UNICARagil project vehicle concept .....	3
1.4.1 System architecture .....	3
1.4.2 Controller architecture .....	4
2 Requirements of Motion Control Algorithms .....	5
2.1 Stability .....	5
2.2 Steady state accuracy .....	5
2.3 Transient response characteristics .....	5
2.4 Robustness .....	6
2.5 Control action allocation .....	6
2.6 Comfort .....	6
3 Literature review towards applicability of metrics and test methods .....	7
3.1 Stability & robustness .....	7
3.2 Steady state accuracy & transient response characteristics .....	8
3.3 Comfort .....	9
3.4 Control allocation .....	10
4 Elaboration of benchmarking methodology .....	11
4.1 Stability .....	11
4.2 Robustness .....	12
4.3 Steady state accuracy & transient response characteristics .....	13
4.4 Control allocation .....	15
5 Application to controller in the project .....	16
5.1 Stability .....	16
5.2 Robustness .....	21
5.2.1 Mass uncertainty .....	21
5.2.2 Dynamic radius uncertainty .....	22
5.3 Steady state accuracy & transient response characteristics .....	23
5.4 Control allocation .....	25

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6	Conclusions and outlook.....	27
6.1	Conclusion .....	27
6.2	Outlook.....	28
	Appendix.....	29
	Appendix 1: Mass uncertainty, transfer function parameters.....	29
	Appendix 2: Dynamic radius uncertainty, transfer function parameters.....	30
	List of References .....	31

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## Symbols and Indices

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Latin letters:

Symbol	Unit	Term
<i>a</i>	m/s <sup>2</sup>	Acceleration
<i>c</i>	N/°, N/rad	Cornering stiffness
<i>F</i>	N	Force
<i>I</i>	kg·m <sup>2</sup>	Moment of inertia
<i>l</i>	m	Length
<i>M</i>	N·m	Moment, torque
<i>m</i>	kg	Mass
<i>p</i>	m	Position
<i>r</i>	m	Radius
<i>t</i>	s	Time
<i>v, V</i>	m/s	Velocity
<i>w</i>	m	Width

Greek letters:

Symbol	Unit	Term
$\alpha$	°, rad	Slip angle
$\delta$	°, rad	Steer angle
$\zeta$	-	Damping factor
$\psi$	°, rad	Yaw angle
$\sigma$	m	Relaxation length
$\tau$	s	Time constant
$\omega$	rad/s	Angular frequency

Indices:

Symbol	Term
act	Actual
c	Course
car	Carcass
d	Desired
dyn	Dynamic
dri	Electric drive
lat	Lateral
lon	Longitudinal
n	n-direction
op	Operating point

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p	Position
s	s-direction
set	Setpoint
slip	Slip
ste	Steering
tire	Tire
tot	Total
v	Velocity
x	x-direction
y	y-direction
yaw	Yaw
z	z-direction

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## List of Abbreviations

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4WID	Four-wheel independently driven
4WIS	Four-wheel independently steered
AV	Automated vehicle
BMBF	Federal Ministry of Education and Research (Bundesministerium für Bildung und Forschung)
DOF	Degrees of freedom
ECU	Electronic Control Unit
FZD	Institute of Automotive Engineering at Technische Universität Darmstadt (Fahrzeugtechnik Darmstadt)
GCC	Global Chassis Control
GM	Gain margin
MCAs	Motion control algorithms
MIMO	Multiple-input multiple-output
OA	Over-actuated
PM	Phase margin
RGA	Relative Gain Array
SISO	Single-input single-output
TUD	Technische Universität Darmstadt

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## List of Figures

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Figure 1-1: Methodology overview. ....	3
Figure 1-2: Modular system architecture concept of the UNICARagil project. ....	3
Figure 1-3: Control loop architecture of the project. <sup>6b</sup> .....	4
Figure 4-1: Open loop general chain.....	12
Figure 4-2: Geometric relations of the position errors of the vehicle's corners. ....	14
Figure 5-1: Kinematic relations for yaw set point forces calculation. ....	17
Figure 5-2: Longitudinal open loop transfer function.....	20
Figure 5-3: Lateral open loop transfer function. ....	20
Figure 5-4: Yaw open loop transfer function. ....	20
Figure 5-5: Longitudinal open loop bode plot with PM and GM. ....	20
Figure 5-6: Longitudinal open loop bode plot for a 20% increase in mass. ....	22
Figure 5-7: Longitudinal open loop bode plot for a 50% increase in mass. ....	22
Figure 5-8: Longitudinal open loop bode plot for a 10% increase in dynamic radius.....	23
Figure 5-9: Longitudinal open loop bode plot for a 10% decrease in dynamic radius. ....	23
Figure 5-10: y-position step response at 20 km/h by 2m. <sup>32b</sup> .....	24
Figure 5-11: Lateral displacement while performing an evasive maneuver at 30 km/h.....	25
Figure 5-12: Maximum corner deviation while performing an evasive manoeuvre at 30 km/h. ....	25
Figure 5-13: Distance travelled evolution during efficiency test.....	26
Figure 5-14: Accumulated energy use during efficiency test. ....	26

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## List of Tables

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Table 2-1: List of requirements for MCAs. ....	6
Table 4-1: Parameter values for the calculation of the RGA analysis. ....	11
Table 5-1: Parameter values for the calculation of the open loop transfer functions. ....	21

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# 1 Introduction

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## 1.1 Motivation

The development of new architectures for automated vehicles (AVs) based on modular design is the objective of the UNICARagil project. The Institute of Automotive Engineering (Fahrzeugtechnik Darmstadt, FZD) at the Technische Universität Darmstadt (TUD) is working along with other German universities in this project funded by the Federal Ministry of Education and Research (Bundesministerium für Bildung und Forschung, BMBF).

The vehicle's unique powertrain consists of four independently driven and steered wheels that the motion control system developed at FZD must control.<sup>1</sup> The modular system architecture implemented in the project distributes the system in different computing units, thus improving the updatability and the possibilities for specific testing. The motion control system module's objective is to ensure the correct execution of the trajectory. To develop motion control algorithms (MCAs), a benchmarking methodology should be developed first. It is particularly necessary due to the multiple aspects that need to be considered in control scenarios, where some parameters may hide relations that would prevent two parameters from improving at the same time, making a tradeoff necessary. In that way, evaluating different metrics and implementing test methods allows a better comprehension of the general behaviour and performance of a given controller, and makes it easier to make comparisons, thus facilitating the development of the project's motion control algorithm.

The particularity of the vehicle's powertrain architecture yields a different control scheme than conventional steering systems, as the yaw angle is in this case controlled independently.<sup>2</sup> It is therefore necessary to develop a suitable benchmarking methodology considering the differences and similarities between traditional and non-traditional powertrains.

## 1.2 Ascertainment of assignment

The main goal of this thesis is to develop a methodology for evaluating the MCAs that may be implemented in the project. The control algorithms are fed a trajectory consisting in a horizontal position and the yaw angle. They include their first two derivatives with respect to time, yielding a total of six variables (as the explicit form would yield).<sup>3a</sup> Since at each wheel brake/torque and steering are controlled independently, the system's over-actuation allows more design possibilities.<sup>3b</sup>

It is then necessary to research metrics and evaluation techniques that are applicable to this scenario and to investigate how it affects the control algorithms and its evaluation possibilities. Then, a set of metrics and test methods is selected. These need to fulfill a set of requirements to be implemented in the final benchmarking methodology, which are derived from the project. Afterwards, an overall

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<sup>1</sup> Buchholz, M. et al.: Automation of the UNICARagil Vehicles (2020), p. 21.

<sup>2</sup> Huang, Y.; Chen, Y.: Estimation and Analysis of Vehicle Lateral Stability ... (2017), p. 2.

<sup>3</sup> Homolla, T.; Winner, H.: Trajectory Tracking Control ... (not published). a: p. 10, b: p. 6.

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benchmarking methodology for the motion control is developed based on the researched metrics and methods.

### **1.3 Methodology**

First, a review of the state of the art of the evaluation of MCAs is performed. The different aspects to assess the quality of MCAs are defined. This involves a general theoretic control approach to desired controller characteristics and overall behaviour, setting a starting point for the base requirements. Then, a more focused research is carried out in regards of vehicle control requirements. The main parameters involved in traditional car powertrain scheme control are discussed, benefitting from the already well-established knowledge in the field. However, it is also important to assess the different requirements over-actuated (OA) vehicles introduce, so a specific review of this type of powertrains and its control characteristics is carried out.

Based on the research done, a set of requirements for MCAs is proposed. These must allow the evaluation of the performance of a given MCA, to allow the development within the UNICARagil project. Therefore, only the effects of the trajectory control system in the project are taken into consideration, isolating them from the effects of other systems in the vehicle. Then, the requirements are analyzed to avoid possible redundancies within the requirements. A justified selection of the requirements is then performed.

Before selecting the necessary metrics and test methods to include in the benchmarking methodology, a literature review of the state of the art is performed. An investigation of the applicability to the project is done, based on the characteristics of the project's vehicle and architecture.

Finally, the benchmarking methodology is defined, containing the metrics and test methods that permit the evaluation of the requirements while being applicable to the project.

The final task consists of the application of the developed benchmarking methodology to an exemplary controller within the project. To perform the realistic simulations, IPG CarMaker with a current model of the UNICARagil vehicle within the project is used.

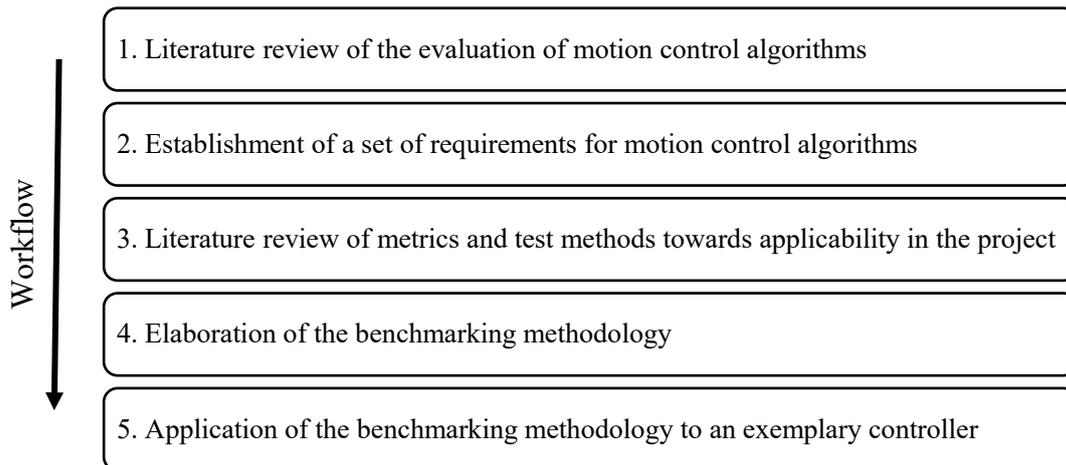


Figure 1-1: Methodology overview.

## 1.4 UNICARagil project vehicle concept

The aspects of the project's vehicle and systems relevant to the development of the benchmarking methodology for MCAs are presented briefly in this chapter.

### 1.4.1 System architecture

The UNICARagil architecture automation concept follows a modular and redundant approach.<sup>4a</sup> The three main modules are the cerebrum, the brainstem, and the set of sensor modules, depicted in Figure 1-2. There is also an underlying energy module that provides power to the actuators within the brainstem. This thesis focuses on the brainstem, the electronic control unit (ECU) that performs the control of the trajectories (among other tasks).<sup>4b</sup> The planning of said trajectories are a task of the cerebrum, a different ECU devoted to its own tasks.

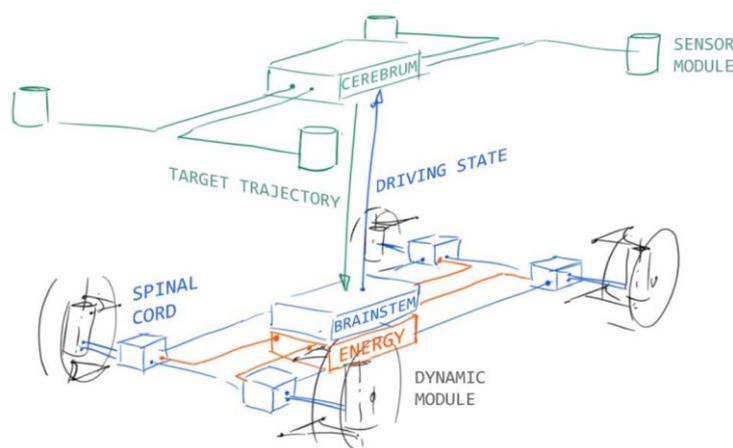


Figure 1-2: Modular system architecture concept of the UNICARagil project.<sup>5</sup>

<sup>4</sup> Buchholz, M. et al.: Automation of the UNICARagil Vehicles (2020). p. 3, p. 4.

<sup>5</sup> The UNICARagil Project: 2021-01-20-UNICARagil-Vorstellung, p. 29.



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## 2 Requirements of Motion Control Algorithms

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In this chapter, the main aspects of a motion controller are investigated and discussed. The design specifications are unique to each application, although some are of great importance across different applications. At the end of the chapter, the list of requirements is presented in Table 2-1.

### 2.1 Stability

One of the main requirements of a control system is stability. Traditionally, it was the driver who needed to stabilize the overall system driver-vehicle, and development of the vehicle's stability was oriented towards the analysis of the driver-vehicle closed system.<sup>7</sup> In the context of AVs this is of great importance, especially in SAE Level 4, as the system has complete control of the vehicle.<sup>8</sup> To keep the vehicle under control at all times, control stability must be guaranteed.

In non-holonomic vehicle drivetrains, one of the most used indicators of stability is given by sideslip together with yaw rate. They determine handling, and in a human driven vehicle this contributes to stability altogether.<sup>9</sup> In AVs however, handling is no longer a problem, so stability can be focused separately. It should be noted that the trajectory has an influence in the stability too, and hence the analysis of the controller's effect on stability has to be carefully distinguished.

### 2.2 Steady state accuracy

Driving in traffic requires precision in the movement of the vehicle, as collisions are to be avoided. The vehicle must be always kept inside a safe region, so the idea of minimizing the pose error is of main importance. In traditional two-wheel steered vehicles subjected to automated driving, there are several approaches. In this control application concretely, steady state errors are desired to be zero, at least for position, for that reason.

### 2.3 Transient response characteristics

Several aspects of the transient response are of particular interest in this control system. As a theoretical bi-level stabilization scheme (Chapter 1.4.2), critical maneuvers are avoided, as upon certain error threshold the trajectory is replanned. However, the behaviour of the response is still relevant, as during normal operation the low-level stabilization scheme is the design objective. The delay of the response must be small enough, as eliminating the error in the pose can be critical to avoid a crash. However, this may hinder other characteristics, namely comfort or stability. Overshoot of the response should be minimized, and oscillations of the response should be minimal. Again, the

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<sup>7</sup> MacAdam, C.C.; Fancher, P.S.: A study of the closed-loop ... (1986), p. 368.

<sup>8</sup> ORAD committee: Taxonomy and Definitions for Terms ... (2021), p.-.

<sup>9</sup> Furukawa, Y. et al.: A Review of Four-Wheel Steering Studies ... (1989), p. 183.

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contribution of the vehicle system should be considered, as not every desired transient response characteristic may be reachable by modifying the controller only.

## 2.4 Robustness

Robustness is defined as the resistance of the stability against parameter variations, disturbances, and noise. In the context of AVs, there are many sources of disturbances, parameter uncertainties and noise within sensor measures. As mentioned in requirement 2.1, stability must be always maintained. The analysis of the robustness of the system is then of key importance.

## 2.5 Control action allocation

One of the particularities of the project's system architecture is the capability of distributing the control action among the four tires freely. That allows the optimization towards other goals than the trajectory tracking, e.g., energy efficiency and dealing with actuator faults. The requirements regarding control allocation are then ultimately bound to the design objectives. The evaluation of control allocation in this thesis will focus on energy efficiency, since it is applicable to every final use application of the project vehicle (it is not a performance car, rather a transport vehicle).

## 2.6 Comfort

As one of the use cases of the project involves passenger inside the vehicles, a sensible aspect of the controller requirements is passenger comfort. To achieve this, the human perception and comfort conditions need to be considered. In this case the trajectory has again shared responsibility with the controller in the final motion, so a separation must be made.

Table 2-1: List of requirements for MCAs.

Number	Requirement
1	Stability
2	Steady state accuracy
3	Transient response characteristics
4	Robustness
5	Control action allocation
6	Comfort

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### 3 Literature review towards applicability of metrics and test methods

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The general specifications for MCAs have been discussed, but due to the limited extension of the present bachelor thesis, a selection among these is made. The criteria to do so is to eliminate redundancies and non-essential ones.

#### 3.1 Stability & robustness

The need for a valid stability criterion in OAs has been studied by many research teams. For non-holonomic vehicles, yaw rate has been traditionally used as a metric for lateral stability.<sup>10</sup> However, holonomic vehicles allow independent yaw control and hence stability cannot be derived from it. Most control schemes aim to achieve horizontal and yaw control, with vertical control often added with active suspensions. This results in a multiple-input multiple-output (MIMO) system which contains vehicle coupling dynamics, so a priori separated single-input single-output (SISO) control strategies are not applicable. The most common control approaches are summarized by Kissai et al. “The state of art shows that either we employ complex robust controllers based on simplified non-coupled vehicle models, or we decouple a complex vehicle model to use simplified controllers at each direction.”<sup>11a</sup>. The approach used in the UNICARagil project coincides with the second type, as the longitudinal, lateral and yaw motion are controlled separately by three linear SISO controllers. To assess stability however, couplings cannot be arbitrarily ignored. A complex model is later derived “using Newton’s laws of motion, and simplifying cross-multiplied low angles and angular velocities, e.g., multiplication of the roll velocity and the pitch velocity”<sup>11b</sup>. The model is later validated using a high-fidelity 15 degrees of freedom (DOF) model with LMS Imagine.Lab AMESim<sup>®</sup>. According to the results shown in their paper, they conclude that “the model shows good precision for all states in a coupled maneuver. [...] This model can then be chosen as a starting model for all Global Chassis Control (GCC) synthesis. It is important to start with a complex full vehicle model and then reduce it while justifying each simplification.”<sup>11c</sup>. In their case, as no active suspensions are considered, the model is simplified to the following:

$$\begin{cases} \dot{V}_x = \frac{F_{x_{tot}}}{M} + V_y \dot{\psi} & (3-1) \\ \dot{V}_y = \frac{F_{y_{tot}}}{M} - V_x \dot{\psi} & (3-2) \\ \ddot{\psi} = \frac{M_{z_{tot}}}{I_z} & (3-3) \end{cases}$$

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<sup>10</sup> Huang, Y.; Chen, Y.: Estimation and Analysis of Vehicle Lateral Stability ... (2017), p. 1.

<sup>11</sup> Kissai, M. et al.: Gain-Scheduled H<sub>∞</sub> ... (2018). a: p. 98, b: p. 98, c: pp. 98-99.

The next step to analyze the couplings is to linearize the model, which is done with a “Taylor series expansion around a nominal system trajectory representing the operating points”<sup>12</sup>. The model then becomes:

$$\begin{bmatrix} \dot{V}_x \\ \dot{V}_y \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & -\dot{\psi}_{op} & V_{y_{op}} \\ -\dot{\psi}_{op} & 0 & -V_{x_{op}} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ \psi \end{bmatrix} + \begin{bmatrix} m^{-1} & 0 & 0 \\ 0 & m^{-1} & 0 \\ 0 & 0 & I_z^{-1} \end{bmatrix} \begin{bmatrix} F_{x_{tot}} \\ F_{y_{tot}} \\ M_{z_{tot}} \end{bmatrix} \quad (3-4)$$

The representation of the model is changed into state space and then the plant transfer function matrix  $G(s)$  is obtained by linear algebra operations. The non-diagonal terms in the matrix indicate coupled behaviour. In the late 1960s, Bristol developed Relative Gain Array (RGA) method for decoupling MIMO systems into separate SISOs.<sup>13</sup> It consists in the calculation of a matrix starting from a transfer function matrix. Let  $G$  be a non-singular square complex general transfer matrix. The RGA of  $G$  is defined as:

$$RGA(G(i\omega)) = \Lambda(G(i\omega)) = G(i\omega) \circ (G(i\omega)^{-1})^t \quad (3-5)$$

The elements of the RGA matrix are close to 1 if the input and output variable pairing depicted by that element is the best selection at that frequency. If diagonal elements are equal to 1 and the off-diagonal are 0, then decentralized control is possible since the effects in the system are the following:  $F_x$  to  $V_x$ ,  $F_y$  to  $V_y$  and  $M_z$  to  $\psi$ . The project’s vehicle parameters should be introduced as well as a suitable operating point and crossover frequency to check if control decentralization is possible in the project case.

### 3.2 Steady state accuracy & transient response characteristics

In classical control analysis, the typical metric for accuracy is steady state error, and some commonly used metrics to characterize the transient behaviour are overshoot and settling time. They are easy to measure and provide useful information about the control loop analysed. However, a more experimental result is also necessary to account for hidden effects that are not assessed by the simple step response analysis. In the development of AVs, a common approach involves performing an overtake manoeuvre and keeping track of the error during it (as seen in Huang, Y et al.<sup>14</sup>, Li, p. et al.<sup>15</sup>, Soltani, A<sup>16</sup> and others). It is based in the ISO 3888-2<sup>17</sup> standard, that defines the dimensions of a test track to then perform an evasive manoeuvre at increasing speed until the vehicle exceeds the limits of the track. The simplicity of the test (it is easily performed and reproduced) and the fact that it tests the vehicle at closed loop behaviour makes it a very used standard in the automotive industry. However,

<sup>12</sup> Kissai, M. et al.: Gain-Scheduled  $H_\infty$  ... (2018), p. 99.

<sup>13</sup> Bristol, E.: On a New Measure of Interaction ... (1966), p.-.

<sup>14</sup> Huang, Y. et al.: Development and Performance Enhancement ... (2020), p. 38.

<sup>15</sup> Li, P. et al.: Velocity-Based Lateral Stability Control ... (2021), p.261.

<sup>16</sup> Soltani, A.: Low Cost Integration ... (2014), pp. 262-267.

<sup>17</sup> ISO 3888-2: Passenger cars (2011).

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the performance in this test is highly related to the vehicle's capabilities, reaching the limits of drivable trajectories for the available grip. Instead, the concept is maintained at normal driving conditions, and only the controller behaviour is analysed, as the trajectories lie within the capabilities of the vehicle.

### 3.3 Comfort

The evaluation of comfort is tied to the human perception of motion. The norm ISO 2631-1<sup>18</sup> regulates the exposure of humans to mechanical vibrations and shock. It establishes a limit for the magnitude of vibrations as a function of the frequency, as the body is more sensitive to certain frequencies than others, and the discomfort is then affected differently. In the context of AVs, research has been done regarding comfort. Strategies focus on the trajectory planner, with optimization algorithms oriented towards comfort among other objectives, as summarized in Gonzalez et al.<sup>19</sup>. So as far as control error is kept within an acceptable level, which is by itself a requirement due to the consequences of not being at the intended pose in terms of e.g., possible crashes or dangerous situations, the comfort will be determined by the trajectory fed to the controller by the trajectory planner.

The evaluation of comfort of the controller poses some challenges. Primarily, as the requirements defined by the ISO 2631-1 are related to the overall motion of the vehicle. However, this is mainly influenced by the trajectory planned at an earlier stage. There is still a contribution made by the controller, which consists in the handling of the errors. The analysis of the importance of the contribution of the controller to comfort is discussed, based on the most critical situations that may appear.

The biggest errors that may appear during operation would correspond to a safety halt operation, where the feedforward term of the controller is not acting with the correct values of future poses. From the moment of the trajectory change, the current value being fed to the actuators was calculated with the previous trajectory poses, as it is intended to compensate the vehicle dynamics. So, the error in this scenario is not just an error inherent to the control loop, that during normal operation only has the task of dealing with errors related to disturbances and uncertainties within the models. In this case it has to cope with a retarded effect, which attempted to counteract the delays in the actuators and vehicle response. However, this is clearly a case in which comfort is not important, as it is a situation where safety is at risk. As a result of this, this case is not considered.

The other concern is oscillations, which can appear when tracking a trajectory. However, an oscillating response is an indicator of marginal stability, so the existence of oscillations should not be possible if the controller is stable, which is evaluated in Chapter 2.1. Moreover, oscillations of this type should not be a concern of comfort, rather of stability.

Due to the difficulties to find a suitable scenario where the controller is entirely responsible for comfort, or a metric that clearly separates its responsibility from the other modules in the project, comfort

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<sup>18</sup> ISO 2631-1: Mechanical vibration and shock (1997-2003).

<sup>19</sup> Gonzalez, D. et al.: A Review of Motion Planning Techniques ... (2016), p. 1135.

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by itself is not evaluated in this thesis. It is considered as sufficiently fulfilled as long as stability is satisfied, and in the event of a safety concerning situation, it is considered not relevant.

### 3.4 Control allocation

The evaluation of the controller regarding control allocation is done in relation to energy only, but further tests can be done to evaluate the performance of the goals that it can be set to achieve. Energy use is mainly due to the drive actuation where the distribution of the work to each wheel can have an impact in efficiency.<sup>20</sup> During braking, energy recovery (if available) is also affected by the allocation of the forces to the wheels. As shown in Wang et al.<sup>21</sup>, torque response and energy efficiency of the electric drive are to be investigated in detail to improve operational energy efficiency and motion control performance. The evaluation of the control allocation is then realized by analyzing the efficiency at which the electric drives are operating. The evaluation continues analyzing the different sources for energy losses, as e.g., tire rolling resistance and motor friction. However, an approach that is valid among future design iterations is preferred that does not depend on the rest of the components. The solution consists in measuring the overall energy consumption per distance travelled. An analogous metric for the traditional liters per 100 km for combustion engines is already extended for electric vehicles: kWh per 100 km.<sup>22</sup>

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<sup>20</sup> Zhang, X. et al.: Energy-Efficient Torque Allocation ... (2018), pp. 292-294.

<sup>21</sup> Wang, R. et al.: Development and Performance ... (2011), p. 3962.

<sup>22</sup> Wolfram, P.; Lutsey, N.: Electric Vehicles: ... (2016), p. 3.

## 4 Elaboration of benchmarking methodology

A set of metrics and test methods is derived based on the requirements set for MCAs after the literature research and the investigation of their applicability to the UNICARagil project.

### 4.1 Stability

The control loop within the system consists of a MIMO with the three control variables: longitudinal, lateral and yaw accelerations. It is desirable to achieve a valid decoupling into separate SISO loops to evaluate stability. RGA analysis is performed for the case of the project's vehicle using the appropriate parameters. Following the criteria mentioned in the thesis by Soltani<sup>23a</sup>, the decoupling has to be met at the crossover frequencies and above, so that stability margins can be analyzed independently and thus ensuring safety although ignoring dynamic couplings. A driving operating point must be selected to perform RGA analysis, i.e., fixed values for longitudinal, lateral and yaw velocities have to be specified. To prove the validity even in demanding dynamic situations, a reasonable set of values is introduced, and the corresponding RGA result matrices are presented. The other value that has to be set to perform RGA analysis is frequency. Values in the boundaries of the theoretical crossover frequency for vehicles assuming a worst case (around 1 Hz or  $2\pi$  rad/s for the average driver, 3 Hz for competitive drivers and higher for automatic systems<sup>24</sup>) are chosen and results shown accordingly in equations 4-1 and 4-2. The operating point is chosen for a critical maneuver scenario, so  $V_{x,op} = 35$  m/s,  $V_{y,op} = 15$  m/s and  $\dot{\psi}_{op} = 0.6$  rad/s. The value for yaw rate is extracted from scenarios where typical evasive maneuvers were under test (taking as reference values found in a research article<sup>25</sup> and in a 4WID and 4WIS research paper<sup>26</sup>). The mass and inertia of the vehicle is extracted from the current values within the project ( $M = 2456.1$  kg and  $I_z = 6.7561$  kg·m<sup>2</sup>). The parameter values are summarized in Table 4-1.

Table 4-1: Parameter values for the calculation of the RGA analysis.

	Parameter	First set of values	Second set of values
<b>Vehicle</b>	$m$	2456.1 kg	2456.1 kg
	$I_z$	6.7561 kg·m <sup>2</sup>	6.7561 kg·m <sup>2</sup>
<b>Operating point</b>	$V_{x,op}$	35 m/s	35 m/s
	$V_{y,op}$	15 m/s	15 m/s
	$\dot{\psi}_{op}$	0.6 rad/s	0.6 rad/s
<b>Angular frequency</b>	$S_{op}$	$2\pi$ rad/s	$6\pi$ rad/s

<sup>23</sup> Soltani, A.: Low Cost Integration ... (2014), pp. 125-126,

<sup>24</sup> Milliken, W.F.; Milliken, D.L.: Race Car vehicle Dynamics (1995), p.-.

<sup>25</sup> Emirler, M.T. et al.: Vehicle Yaw Rate Estimation ... (2013), p. 7.

<sup>26</sup> Li, P. et al.: Velocity-Based Lateral Stability Control ... (2021), pp. 262-263.

$$\Lambda(G(i2\pi)) = \begin{bmatrix} 0.9910 & 0.0090 & 0 \\ 0.0090 & 0.9910 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix} \quad (4-1)$$

$$\Lambda(G(i6\pi)) = \begin{bmatrix} 0.9990 & 0.0010 & 0 \\ 0.0010 & 0.9990 & 0 \\ 0 & 0 & 1.0000 \end{bmatrix} \quad (4-2)$$

The higher the frequency, the better the values for system decoupling. Even for a case of low cross-over frequency the values are sufficient to justify SISO decoupling. The off-diagonal terms show that the influence of  $F_x$  on  $V_y$  is in the order of  $10^{-2}$  smaller than the effect of  $F_y$  on  $V_y$ . With these results it is concluded that SISO decoupling is possible, and stability margins are evaluated accordingly. Each of the three variables is analyzed by linearizing the different elements in the control chain and performing linear stability criteria. The project may present different elements along the development phases, so the stability analysis must keep track of the changes and adapt those changes to correctly evaluate the system. The general structure is depicted in Figure 4-1: Open loop general chain



Figure 4-1: Open loop general chain.

This evaluation method assumes linear or at least linearizable techniques are used in the system, as it is necessary to obtain the transfer functions. The actuator, tire and vehicle dynamics can be linearized with approximate models provided by the manufacturer or obtained by experimental tests. Either way will be suited for this method. The selection of the models is left open for the MCA developer, as new requirements for the controller may unfold, as well as better models for the components making up the project's vehicle.

Once the open loop transfer function is calculated, the open loop bode plot is obtained and the phase and gain margins are evaluated. The phase margin is considered as a design parameter within the project's control concept, to be maintained at  $60^\circ$  for robust control.<sup>27</sup> Gain margin (GM) is also made available for possible design objectives.

## 4.2 Robustness

The robustness of the system is dependent on many factors, so the evaluation is made on a set of parameter uncertainties that are known to cause challenges for the control of the vehicle. The method to analyze how those factors affect stability is by observing the change in stability conditions derived

<sup>27</sup> Homolla, T.; Winner, H.: Trajectory Tracking Control ... (not published), p. 14.

in Chapter 3.1. The objective is to determine whether for a determined amount of uncertainty or disturbance, the stability margins are maintained. The maximum parameter variation shall be obtained from experimental tests to ensure robust control under real conditions. In this thesis, vehicle mass and dynamic radius are chosen as uncertain parameters. Mass uncertainty is common in the use case of vehicles, where the number of passengers and the load is constantly changing by relatively high amounts compared to the nominal value. Moment of inertia is also related to this scenario, as the passengers and cargo may be positioned at unknown places. Cornering stiffness is dependent on many variables, and its value may be wrongly estimated (even with better models than the one used as an example in this thesis).

The methodology consists in employing the model developed in Chapter 4.1, and introducing different values of the parameters at the setpoint force calculation (where the models provide the values) and at the simulated vehicle dynamics, or plant (where the real vehicle dynamics take place). In this way, the controller is placed in a simulated environment where the values used for the setpoint calculation are wrong, at least to some extent.

### 4.3 Steady state accuracy & transient response characteristics

The evaluation of the accuracy and the transient response characteristics of the controller is performed by analyzing the step response of the closed loop. The simulation is carried out with a realistic model such as IPG CarMaker. In this way every of the blocks is simulated as close to a real scenario as possible, minimizing the use of simplified models. A set of values can be used to analyze the performance of the controller. The position, the velocity and the acceleration errors to a step input are analyzed. The three pose variables can be analyzed separately (longitudinal, lateral and yaw) or combined into a metric to be able to interpret the consequences of those errors. For clarification, an example is proposed: given a desired maneuver that the vehicle is intended to follow, the error in the yaw position can lead to a collision, but it is not clear a priori how much error can be manageable. Moreover, the errors of longitudinal, lateral and yaw position interact between each other. With this metric, the summed effect can be collected and the specific deviation to where the corners of the vehicle should be is obtained. The geometric derivation used to calculate this error values is presented in Figure 4-2.

To obtain the values of the deviation of the corner position, the calculations are undertaken as indicated in equations 4-3 and 4-4.

$$\mathbf{e} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} \sqrt{e_{1,s} + e_{1,n}} \\ \sqrt{e_{2,s} + e_{2,n}} \\ \sqrt{e_{3,s} + e_{3,n}} \\ \sqrt{e_{4,s} + e_{4,n}} \end{bmatrix} \quad (4-3)$$

$$\begin{bmatrix} e_{1,s} \\ e_{1,n} \\ e_{2,s} \\ e_{2,n} \\ e_{3,s} \\ e_{3,n} \\ e_{4,s} \\ e_{4,n} \end{bmatrix} = \begin{bmatrix} e_{lon} + l_{\psi} \cdot \cos(e_{yaw} + \arctan(\frac{w}{l})) - \frac{l}{2} \\ e_{lat} + l_{\psi} \cdot \sin(e_{yaw} + \arctan(\frac{w}{l})) - \frac{w}{2} \\ e_{lon} + l_{\psi} \cdot \cos(e_{yaw} + \arctan(\frac{-w}{l})) - \frac{l}{2} \\ e_{lat} + l_{\psi} \cdot \sin(e_{yaw} + \arctan(\frac{-w}{l})) + \frac{w}{2} \\ e_{lon} + l_{\psi} \cdot \cos(e_{yaw} + \arctan(\frac{w}{-l})) + \frac{l}{2} \\ e_{lat} + l_{\psi} \cdot \sin(e_{yaw} + \arctan(\frac{w}{-l})) - \frac{w}{2} \\ e_{lon} + l_{\psi} \cdot \cos(e_{yaw} + \arctan(\frac{-w}{-l})) + \frac{l}{2} \\ e_{lat} + l_{\psi} \cdot \sin(e_{yaw} + \arctan(\frac{-w}{-l})) + \frac{w}{2} \end{bmatrix} \quad (4-4)$$

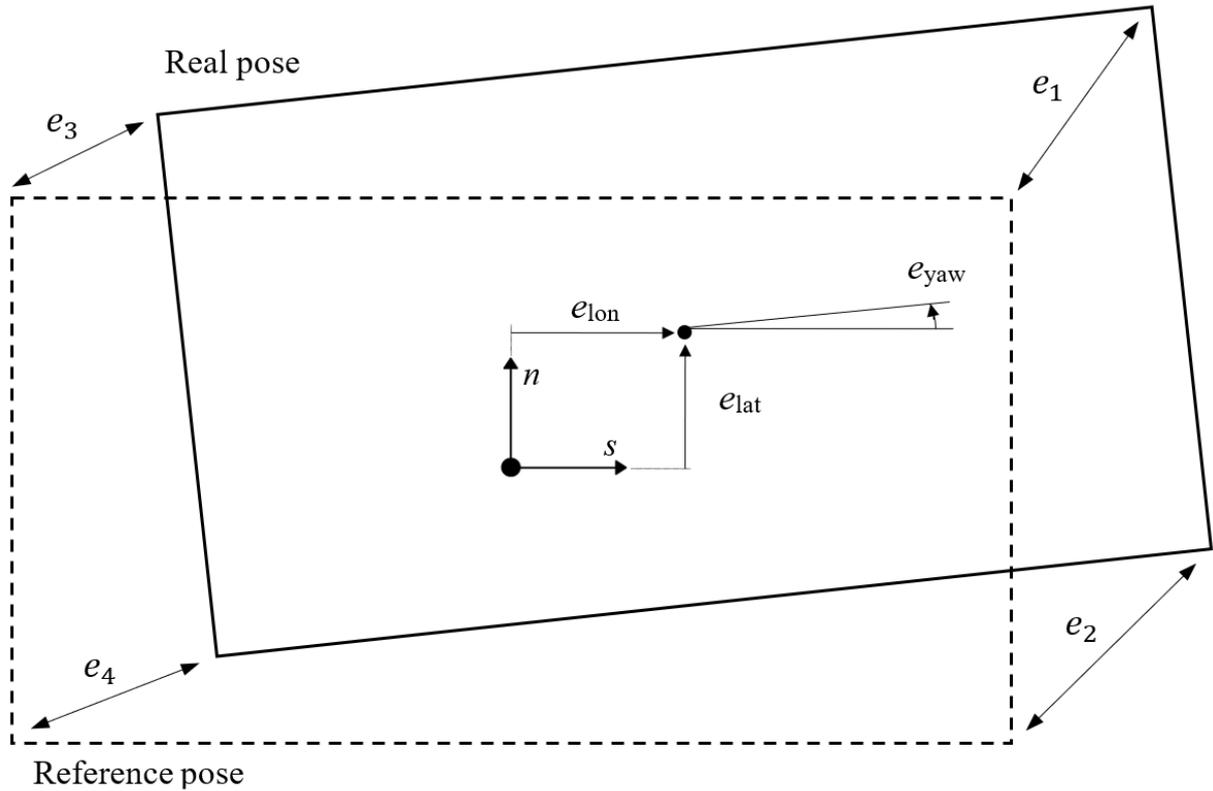


Figure 4-2: Geometric relations of the position errors of the vehicle's corners.

Several responses can be analysed: one for each of the three separate controlled variable (longitudinal, lateral and yaw) and for position, velocity, and acceleration, and combinations (lateral and yaw at the same time for a turn in heading direction). Then, a set of metrics is extracted: steady state error, overshoot and settling time (within 2% of the final value). The steady state errors can be used to determine whether the controller is of type 0, 1 or 2 and how big the errors are in relation to the input value. The overshoot and settling time metrics describe the approximate features of the transient response, showing some hints to possible underdamping and the speed of the control action.

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However, a more experimental result is also necessary to account for hidden effects that are not assessed by the simple step response analysis.

#### **4.4 Control allocation**

The evaluation of control allocation focuses on the consumption of energy. As the trajectory, mass of the vehicle and other variables play a role in the amount of energy used by the vehicle, the metric proposed is relative. It allows MCA comparisons if the rest of parameters are left unchanged. To reduce the influence of a bias due to few represented maneuvers, a long test including different driving conditions is necessary. The value for energy per kilometer, in kW/100 km is obtained, and a comparison between different controller allocation schemes can be made. The trajectory selected aims to obtain a representative value close to the real average energy efficiency. There are different standards for the energy efficiency tests, but research has shown that noticeable differences (up to 79% in some cases) in the results for an identical car appear between tests.<sup>28</sup> The selection of the trajectory is left open to the MCA developer, enabling the possibility for the evaluation of different driving scenarios (e.g., urban, inter-urban, involving maneuvers only possible for over-actuated vehicles).

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<sup>28</sup> Tzirakis, E.: Vehicle Emissions and Driving Cycles (2013), p. 288.

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## 5 Application to controller in the project

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The benchmarking methodology is now applied to an exemplary controller within the project. A detailed description of the application of each metric is presented in this chapter.

### 5.1 Stability

To assess the stability, the open loop transfer function was approximated by a linear model. The first part of the control chain is the controller. In the project, the contribution of position, velocity and acceleration errors are added and weighted by a time constant (Figure 1-3). The equivalent transfer function expressed in acceleration is the following:

$$F_{\text{controller}} = \frac{s^2 + \frac{1}{\tau_v} \cdot s + \frac{1}{\tau_v \cdot \tau_p}}{s^2} \quad (5-1)$$

Next is the control allocation, which has the task of distributing the force demand to each of the wheels and to calculate the set point values in terms of torque and steering angle. This is done by a linear operation. At the moment the distribution is uniform, with each wheel being in charge of one fourth of total longitudinal force, and the same for lateral and yaw forces. The set point value for longitudinal force is calculated as the torque demand for the electric motor of each wheel, so the mass and radius of the tire are used equation 5-2. The set point value for lateral force provides the steering angle, using the cornering stiffness and the mass (equation 5-3).

$$F_{\text{allocation lon}} = \frac{M_{\text{dri}}}{\dot{V}_x} = \frac{m \cdot r_{\text{dyn}}}{4} \quad (5-2)$$

$$F_{\text{allocation lat}} = \frac{\delta}{\dot{V}_x} = \frac{m}{4 \cdot c_{\alpha}} \quad (5-3)$$

To obtain set point values for the yaw motion, the force demand is divided into a longitudinal and a lateral force producing a maximum lever arm to the vehicle center.<sup>29a</sup> The transfer function for each of the wheels are calculated as follows<sup>29b</sup>:

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<sup>29</sup> Homolla, T.; Winner, H.: Trajectory Tracking Control ... (not published). a: p. 12, b: p. 13.

$$\begin{bmatrix} F_{d,s,z,1} \\ F_{d,n,z,1} \\ F_{d,s,z,2} \\ F_{d,n,z,2} \\ F_{d,s,z,3} \\ F_{d,n,z,3} \\ F_{d,s,z,4} \\ F_{d,n,z,4} \end{bmatrix} = \begin{bmatrix} \cos(\pi - \operatorname{atan}(l/w)) \cdot \frac{\dot{\psi} \cdot l_z}{4 \cdot l_\psi} \\ \sin(\pi - \operatorname{atan}(l/w)) \cdot \frac{\dot{\psi} \cdot l_z}{4 \cdot l_\psi} \\ \cos(\frac{\pi}{2} - \operatorname{atan}(l/w)) \cdot \frac{\dot{\psi} \cdot l_z}{4 \cdot l_\psi} \\ \sin(\frac{\pi}{2} - \operatorname{atan}(l/w)) \cdot \frac{\dot{\psi} \cdot l_z}{4 \cdot l_\psi} \\ \cos(\frac{3\pi}{2} - \operatorname{atan}(l/w)) \cdot \frac{\dot{\psi} \cdot l_z}{4 \cdot l_\psi} \\ \sin(\frac{3\pi}{2} - \operatorname{atan}(l/w)) \cdot \frac{\dot{\psi} \cdot l_z}{4 \cdot l_\psi} \\ \cos(2\pi - \operatorname{atan}(l/w)) \cdot \frac{\dot{\psi} \cdot l_z}{4 \cdot l_\psi} \\ \sin(2\pi - \operatorname{atan}(l/w)) \cdot \frac{\dot{\psi} \cdot l_z}{4 \cdot l_\psi} \end{bmatrix} \quad (5-4)$$

where  $l$  stands for the wheelbase,  $w$  for the track width,  $l_\psi$  for the distance from the vehicle center to the wheel contact point and  $\psi_{c,set}$  for the setpoint course angle. This last one is set to 0 for simplification in this example. This is graphically depicted in Figure 5-1.

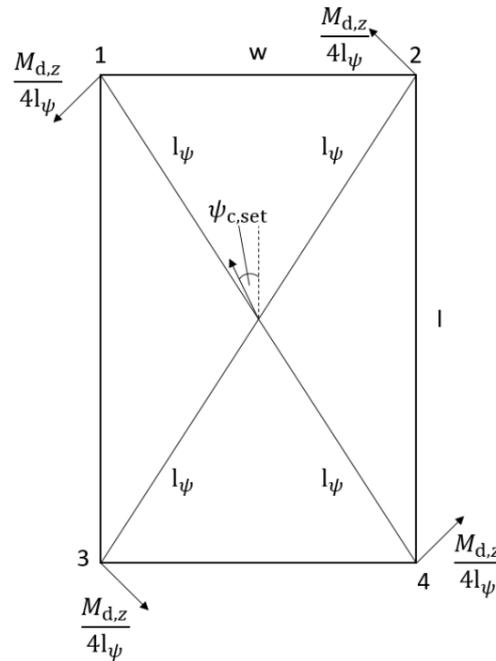


Figure 5-1: Kinematic relations for yaw set point forces calculation.<sup>30</sup>

The transfer function for the yaw motion then is separated into a steering and a torque value:

<sup>30</sup> Homolla, T.; Winner, H.: Trajectory Tracking Control ... (not published), p. 13.

$$F_{\text{allocation yaw}} = \begin{bmatrix} \frac{\delta}{\ddot{\psi}} \\ \frac{M_{\text{dri}}}{\ddot{\psi}} \end{bmatrix} = \begin{bmatrix} \frac{F_{\text{d},n,i}}{\ddot{\psi} \cdot c_{\alpha}} \\ \frac{F_{\text{d},s,i}}{\ddot{\psi}} \cdot r_{\text{dyn}} \end{bmatrix} \quad (5-5)$$

Next in the loop chain is the actuators. The actuator for the steering is currently being modeled by either a first order or a second order transfer function depending on the operating point. For now, we settle with the second order model, which has a time constant  $\tau_{\text{ste}}$  and a damping factor  $\zeta_{\text{ste}}$ . The transfer function for the actuator is modelled by a second order system:

$$F_{\text{ste}} = \frac{1}{\tau_{\text{ste}}^2 \cdot s^2 + 2 \cdot \tau_{\text{ste}} \cdot \zeta_{\text{ste}} \cdot s + 1} \quad (5-6)$$

The same approach is done for the electric drive providing torque, where depending on the operating point either a first order or a second order transfer function is used. For this testing the second order one is chosen too, with parameters  $\tau_{\text{dri}}$  and  $\zeta_{\text{dri}}$ :

$$F_{\text{dri}} = \frac{1}{\tau_{\text{dri}}^2 \cdot s^2 + 2 \cdot \tau_{\text{dri}} \cdot \zeta_{\text{dri}} \cdot s + 1} \quad (5-71)$$

The tire dynamic behaviour is modelled with the simplified transfer function (equation 5-8) for demonstration purposes. It approximates the delay caused by the time it takes for the deformation in tire to build up and producing the desired force. The parameters needed in the present model are relaxation length  $\sigma$  and velocity  $v$ . As reflected in Vantsevich and Grey<sup>31</sup>, relaxation time is dependent on the car, tire, normal load, slip angle and its rate of change. This is shown to be true for lateral and longitudinal relaxation time. In this example however, an approximate value is calculated with a model used within the project, using longitudinal tire slip stiffness  $c_{\text{lon}}$ , slip angle stiffness  $c_{\text{slip}}$ , longitudinal carcass stiffness  $c_{\text{lon,car}}$  and lateral carcass stiffness  $c_{\text{lat,car}}$  as input parameters (equations 5-9 and 5-10). Further models with more precise parameters should be implemented for more precise results.

$$F_{\text{tire}} = \frac{1}{\frac{\sigma}{|v|}s + 1} \quad (5-8)$$

$$\sigma_{\text{lat}} = \frac{c_{\text{slip}}}{c_{\text{lat,car}}} \quad (5-9)$$

$$\sigma_{\text{lon}} = \frac{c_{\text{lon}}}{c_{\text{lon,car}}} \quad (5-10)$$

To model the tire force generation and the vehicle dynamics, the inverse models with respect to the ones used to calculate the set point forces in equations 5-2, 5-3, 5-4 and 5-5. However, a more complete model accounting for other factors (e.g., self-aligning moment, combined longitudinal and lateral friction limit, models simulating dynamic behaviour) should be used for higher reliability and

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<sup>31</sup> Vantsevich, V.V.; Gray, J.P.: Relaxation Length Review ... (2015), p. 2.

studying missing effects. To enable the effect of uncertainty in the values of the parameters, the subscript veh (for vehicle) is used in these transfer functions. These values represent the parameters of the vehicle as the plant in the loop. The resulting transfer functions are:

$$F_{veh \text{ lon}} = \frac{\dot{V}_x}{M_{dri}} = \frac{4}{m_{veh} \cdot r_{dyn}} \quad (5-11)$$

$$F_{veh \text{ lat}} = \frac{\dot{V}_x}{\delta} = \frac{4 \cdot c_{\alpha,veh}}{m_{veh}} \quad (5-12)$$

$$F_{veh \text{ yaw}} = \begin{bmatrix} \frac{\ddot{\psi}}{\delta} \\ \ddot{\psi} \\ M_{dri} \end{bmatrix} = \begin{bmatrix} \frac{\ddot{\psi} \cdot c_{\alpha,veh}}{F_{d,n,i,veh}} \\ \ddot{\psi} \\ F_{d,s,i,veh} \cdot r_{dyn,veh} \end{bmatrix} \quad (5-13)$$

$$\begin{bmatrix} F_{d,s,z,1,veh} \\ F_{d,n,z,1,veh} \\ F_{d,s,z,2,veh} \\ F_{d,n,z,2,veh} \\ F_{d,s,z,3,veh} \\ F_{d,n,z,3,veh} \\ F_{d,s,z,4,veh} \\ F_{d,n,z,4,veh} \end{bmatrix} = \begin{bmatrix} \cos(\pi - \text{atan}(l/w)) \cdot \frac{\ddot{\psi} \cdot I_{z,veh}}{4 \cdot l_{\psi}} \\ \sin(\pi - \text{atan}(l/w)) \cdot \frac{\ddot{\psi} \cdot I_{z,veh}}{4 \cdot l_{\psi}} \\ \cos(\frac{\pi}{2} - \text{atan}(l/w)) \cdot \frac{\ddot{\psi} \cdot I_{z,veh}}{4 \cdot l_{\psi}} \\ \sin(\frac{\pi}{2} - \text{atan}(l/w)) \cdot \frac{\ddot{\psi} \cdot I_{z,veh}}{4 \cdot l_{\psi}} \\ \cos(\frac{3\pi}{2} - \text{atan}(l/w)) \cdot \frac{\ddot{\psi} \cdot I_{z,veh}}{4 \cdot l_{\psi}} \\ \sin(\frac{3\pi}{2} - \text{atan}(l/w)) \cdot \frac{\ddot{\psi} \cdot I_{z,veh}}{4 \cdot l_{\psi}} \\ \cos(2\pi - \text{atan}(l/w)) \cdot \frac{\ddot{\psi} \cdot I_{z,veh}}{4 \cdot l_{\psi}} \\ \sin(2\pi - \text{atan}(l/w)) \cdot \frac{\ddot{\psi} \cdot I_{z,veh}}{4 \cdot l_{\psi}} \end{bmatrix} \quad (5-14)$$

The open loop transfer functions for the three variables are shown in Figure 5-4, Figure 5-3 and Figure 5-4.



Figure 5-2: Longitudinal open loop transfer function.



Figure 5-3: Lateral open loop transfer function.

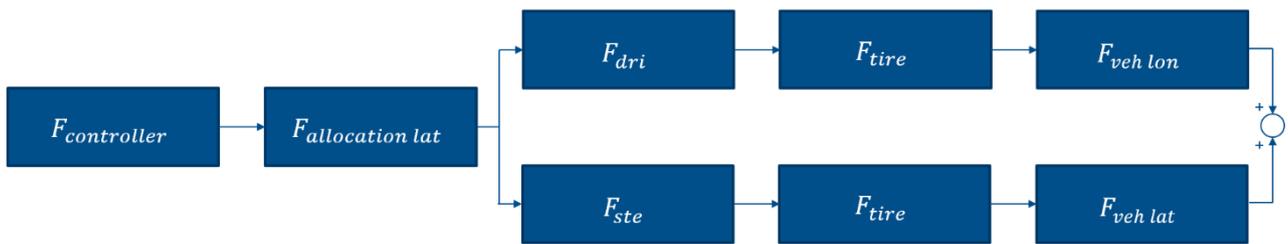


Figure 5-4: Yaw open loop transfer function.

Then the open loop bode plot can be analysed by means of the PM and GM values. The open loop bode plot of the longitudinal response is provided as an example in Figure 5-5.

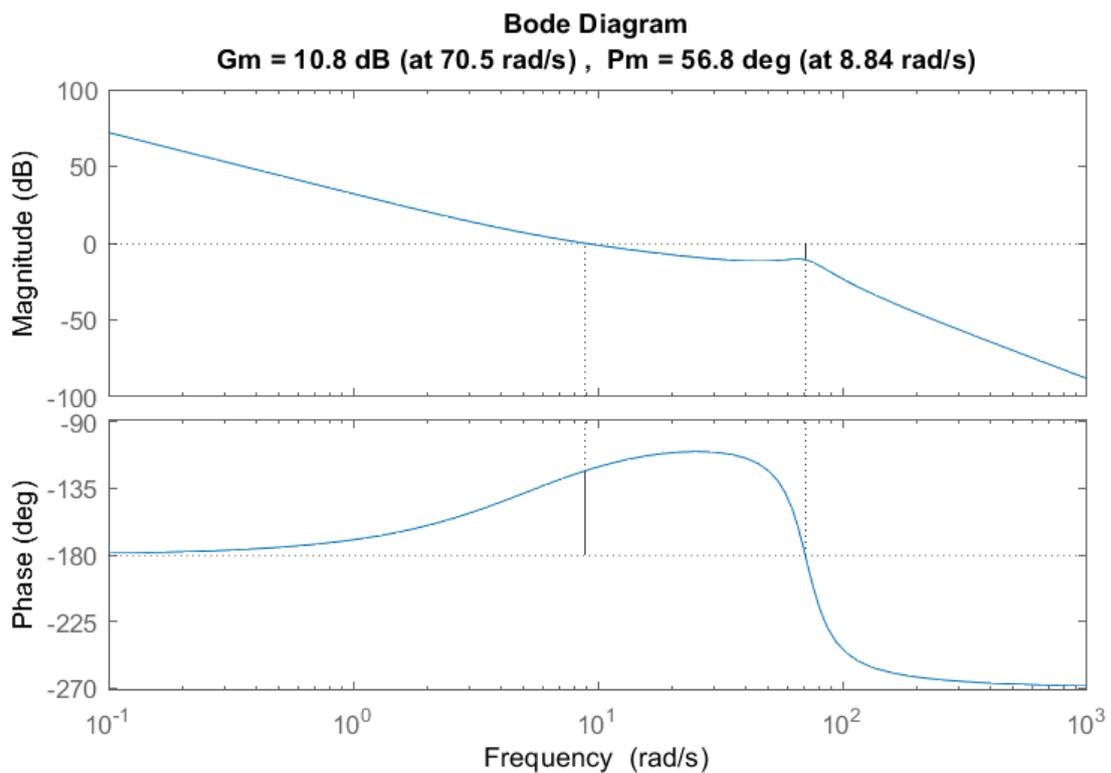


Figure 5-5: Longitudinal open loop bode plot with PM and GM.

The values for the parameters used along this example for the different models are summarized in Table 5-1. The calculations were done using MATLAB and the Control System Toolbox.

Table 5-1: Parameter values for the calculation of the open loop transfer functions.

Use	Parameter	Value
Common	$m = m_{veh}$	2456.1 kg
	$I_z = I_{z,veh}$	6.7561 kg·m <sup>2</sup>
	$r_{dyn} = r_{dyn,veh}$	0.195 m
	$c_{\alpha} = c_{\alpha,veh}$	70000 N/°
	$w$	1.78 m
	$l$	3.4 m
	$l_{\psi}$	1.9189 m
	$\tau_p$	0.316 s
	$\tau_v$	0.079 s
	Longitudinal	$\tau_{dri}$
$\zeta_{dri}$		0.189901320353302 s
$c_{lon}$		100000 N/°
$c_{lon,car}$		200000 N/m

## 5.2 Robustness

The analysis of robustness is performed by introducing erroneous values for the parameters subject to analysis. As mentioned in Chapter 4.2, the maximum variation permitted must be obtained from experimental tests to ensure robustness during real life operation. For the demonstrative purposes of this section, arbitrary approximations are selected.

### 5.2.1 Mass uncertainty

The value of mass can increase from the nominal value up to a value of around the weight of six or more passengers, for example. That corresponds to about 18% of the nominal mass, considering an average weight of 75 kg. To give a somewhat applicable scenario (again, just to provide an example), the open loop bode plot for longitudinal control is obtained for mass increase of 20% and 50% increase (Figure 5-6 and Figure 5-7).

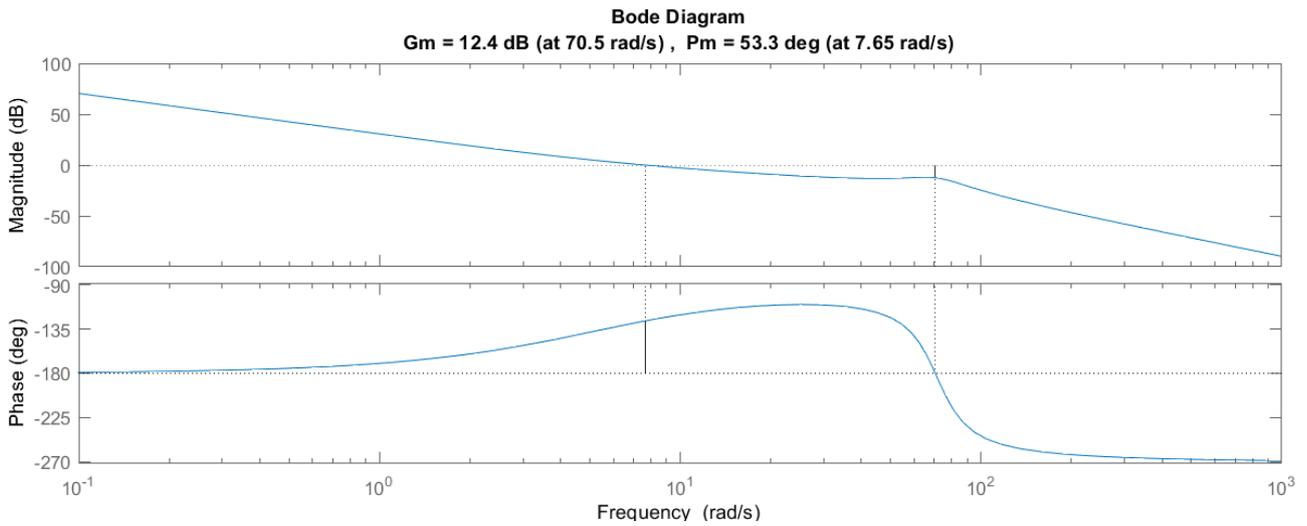


Figure 5-6: Longitudinal open loop bode plot for a 20% increase in mass.

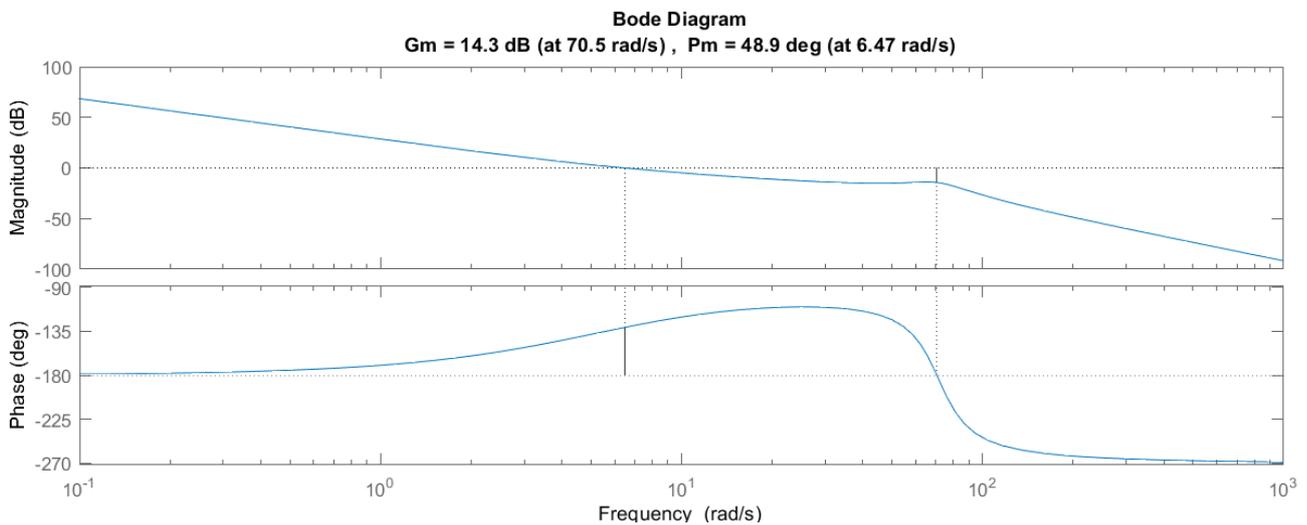


Figure 5-7: Longitudinal open loop bode plot for a 50% increase in mass.

A decrease of  $3.5^\circ$  and  $7.9^\circ$  of PM for the 20% and 50% mass increase, respectively, with respect to the perfect parameter value case in Figure 5-5 is observed. The pertinent conclusions (to low PM for the parameter variation or high enough) shall be deduced based on the PM desired by the controller developer. In this case, if a  $PM = 50^\circ$  is desired, and the 50% mass increase is expected during normal operation, some changes within the controller are needed, as the PM has fallen below the desired level. The parameters used to obtain the simulation results are contained in Appendix 1.

### 5.2.2 Dynamic radius uncertainty

Due to the direct effect of dynamic radius estimation in the setpoint force calculation (equations 5-2 and 5-5), the uncertainty in the parameter value could affect the stability. The same procedure is undertaken as for the mass uncertainty. In this case, the variation is arbitrarily chosen for a 10% increase and a 10% decrease, as a demonstration.

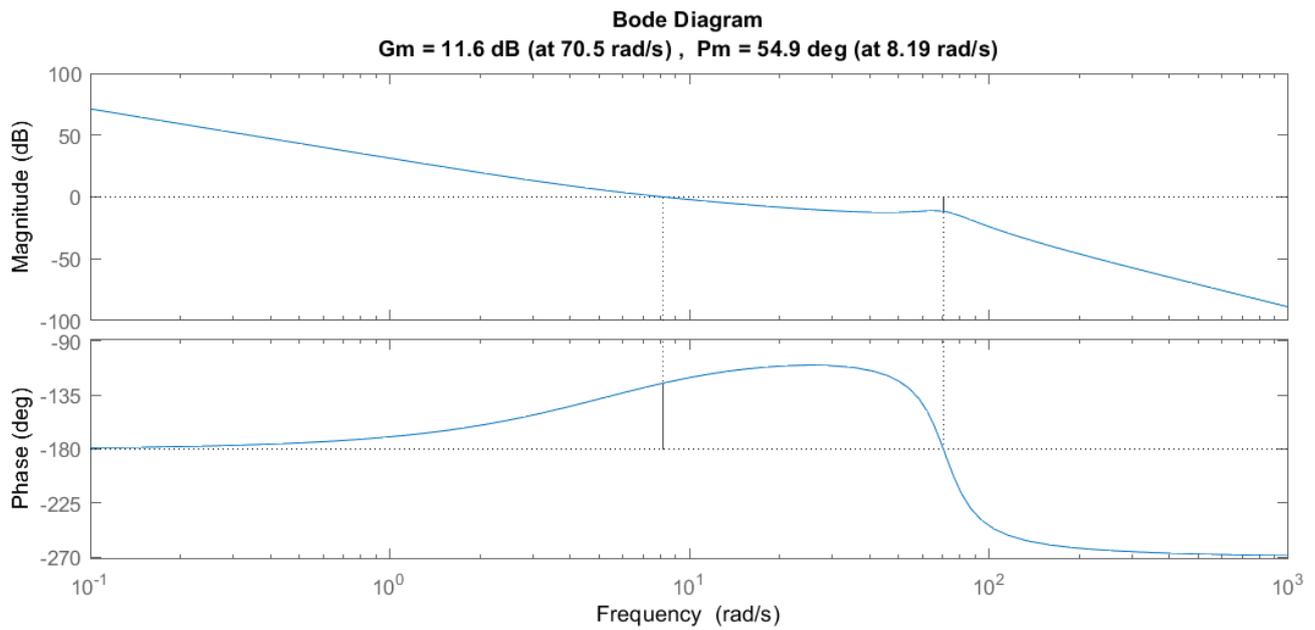


Figure 5-8: Longitudinal open loop bode plot for a 10% increase in dynamic radius.

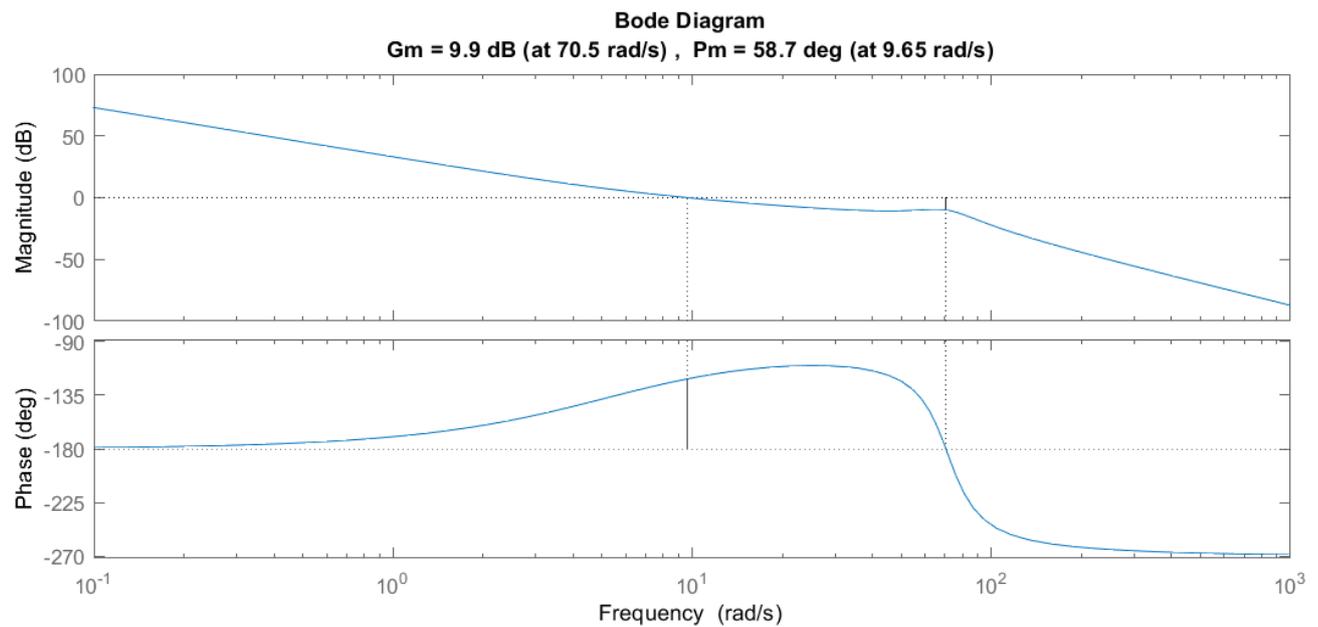


Figure 5-9: Longitudinal open loop bode plot for a 10% decrease in dynamic radius.

The PM and GM change compared to the ideal parameter value simulation seen in Figure 5-5. The same conclusions as in the mass uncertainty scenario can be reached (Chapter 5.2.1). The values of the parameters for this calculation are found in Appendix 2.

### 5.3 Steady state accuracy & transient response characteristics

The steady state and accuracy metrics are obtained for an exemplary controller with IPG CarMaker simulation environment. The first metric, consisting in the step response characteristics, is introduced

first. In order to stimulate the controller with a step input, the tool developed by Blödel, A.P.<sup>32a</sup> is used. As an example, the variable selected is a step in the position along the y-direction (corresponding to the lateral direction of the vehicle). The result can be seen in Figure 5-10.

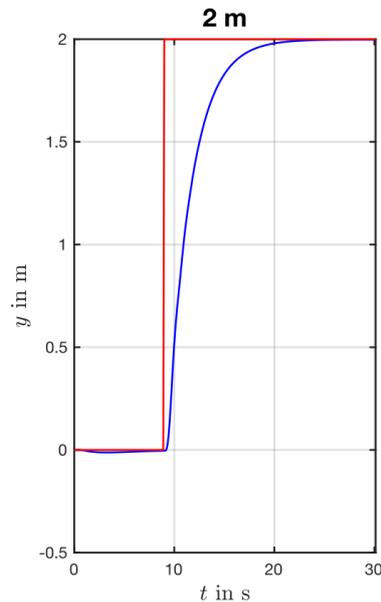


Figure 5-10: y-position step response at 20 km/h by 2m.<sup>32b</sup>

The obtained values for settling time and overshoot are then extracted. In this case, settling time is around  $t = 8.5$  s (within 2% of the final value). The response is overdamped, as no overshoot is seen. No steady state error is either observed.

The second metric involving the overtake or evasive maneuver is performed while keeping track of the error of the position of the corners as proposed in Chapter 4.3. This example is obtained by evading the imaginary object while keeping yaw rate close to zero, exploiting the features of the over-actuated vehicle. The maneuver is performed at approximately 30 km/h, and the saved distance (distance moved in lateral direction) is close to 3.5 m, enough to avoid a vehicle. The trajectory followed by the vehicle is shown in Figure 5-11. IPG CarMaker is used for this simulation, together with a current vehicle model and an existing trajectory within the project (referred to as “AAET”).

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<sup>32</sup> Blödel, A.P.: Diss., Entwicklung Und Implementierung ... (2021). a: p.-, b: p.116

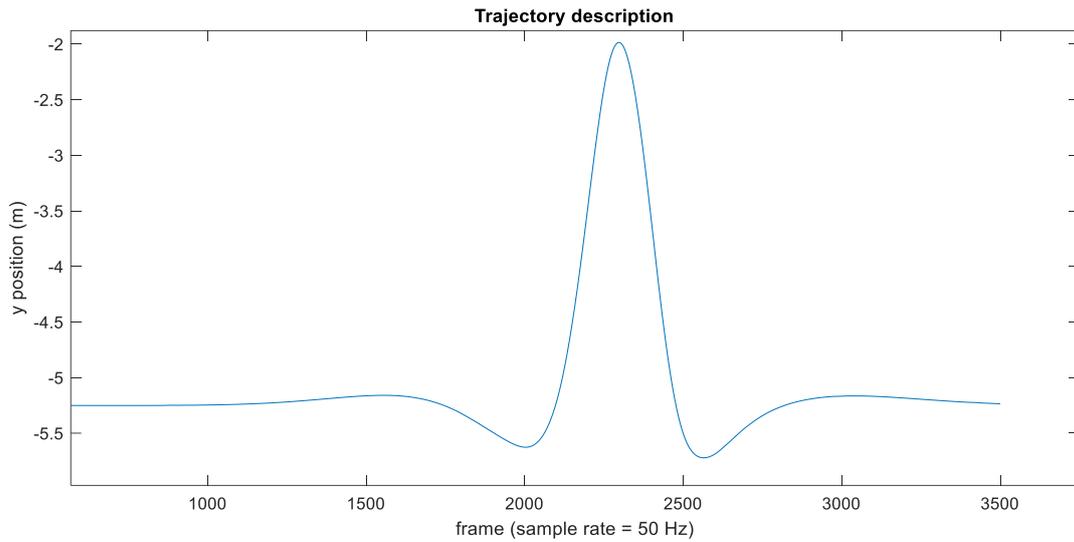


Figure 5-11: Lateral displacement while performing an evasive maneuver at 30 km/h.

The results of the performance of the controller are shown in Figure 5-12. The deviation of each of the corners of the vehicle to the desired position is calculated and the maximum among the four is plotted in each frame. A peak value of about 0.12 m is observed. With this information, we cannot guarantee a precision lower than that during driving velocities close to 30 km/h.

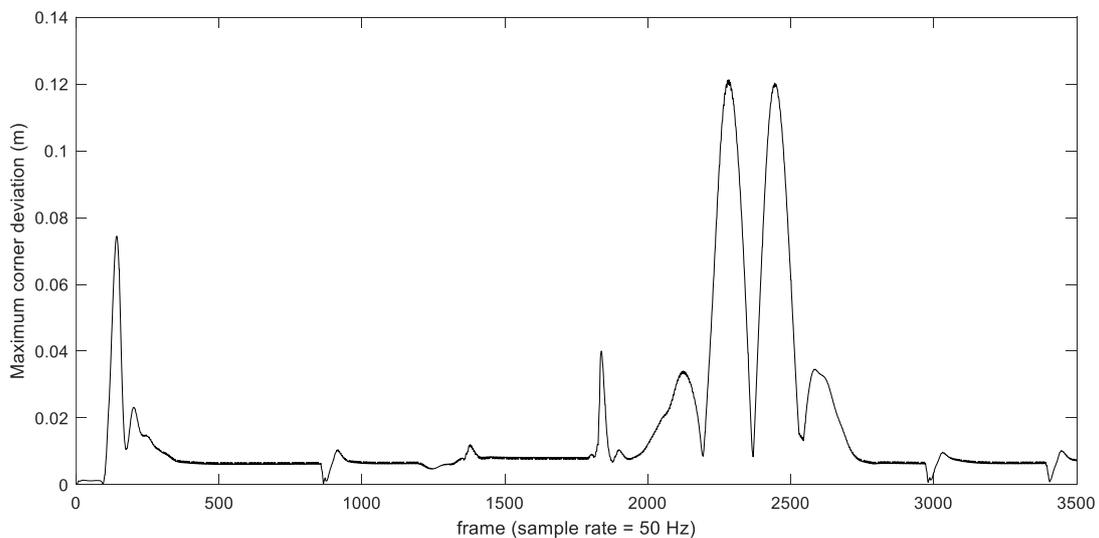


Figure 5-12: Maximum corner deviation while performing an evasive manoeuvre at 30 km/h.

## 5.4 Control allocation

The power efficiency is evaluated for an exemplary controller. The trajectory selected for the example provided simulates a city driving scenario and can be found within the project referred to as “HZE”. To obtain the energy efficiency value, the energy used is compared against the distance travelled and the value in kWh per 100 km is provided. In Figure 5-13, the accumulated distance travelled during the test is plotted. The energy use is plotted in the same way in Figure 5-14.

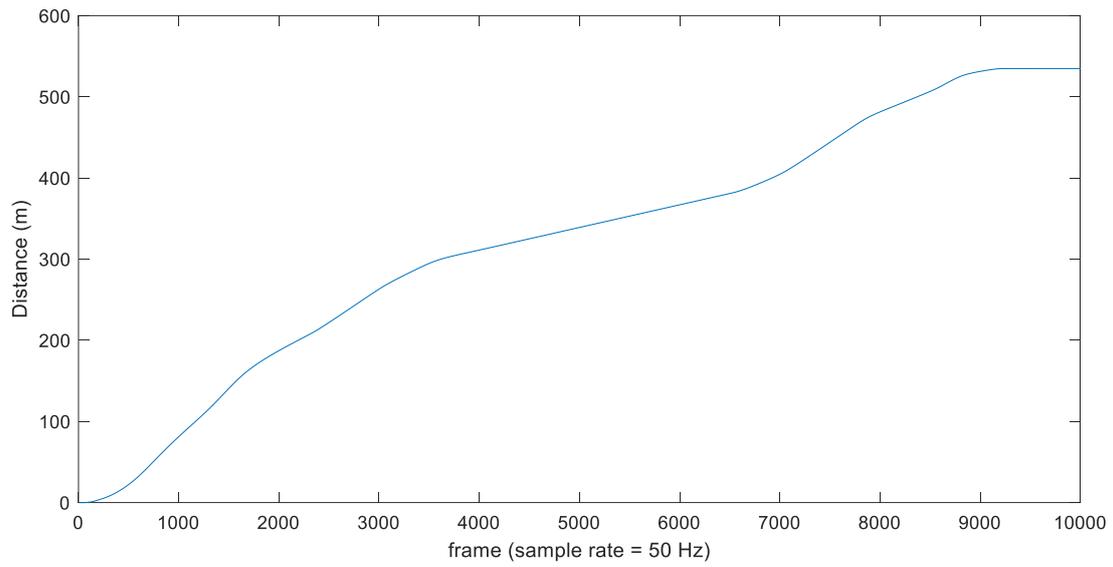


Figure 5-13: Distance travelled evolution during efficiency test.

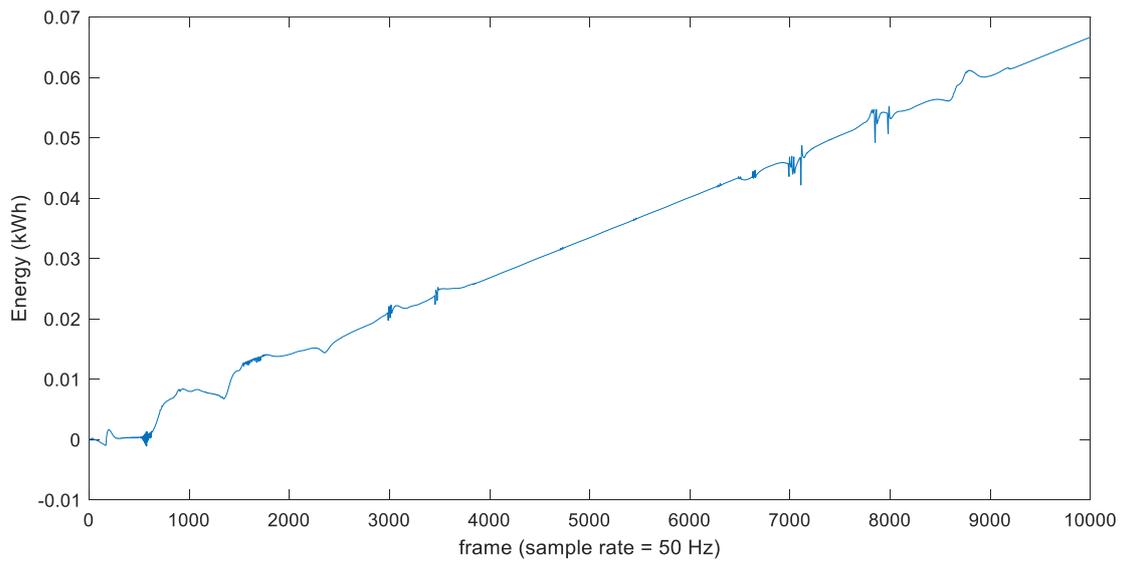


Figure 5-14: Accumulated energy use during efficiency test.

The test can be performed for longer distances for a better accuracy of the value. In this case, after 534.8051 m the energy used was 0.0667 kWh. The efficiency shown is of 12.4656 kWh/100 km.

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## 6 Conclusions and outlook

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The aim of this thesis was to develop a benchmarking methodology for motion control algorithms, to enable the evaluation of them towards their implementation in the UNICARagil project. Suitable metrics and test methods were to be researched and investigated, and a benchmarking methodology for the motion control was to be developed.

### 6.1 Conclusion

A study of the UNICARagil project was conducted, and the relevant aspects and characteristics of the vehicle, as well as the characteristics of the control scheme implemented was noted. The review of the state of the art of metrics and test methods was oriented towards the applicability towards the project. This involved studying the characteristics of four wheel steered and four wheeled driven vehicles and the challenges and possibilities that this drivetrain yields. Also important was the structure of the control system, which performs MIMO control by controlling each of the three control variables independently as if it was three different SISO systems.

The requirements for motion control algorithms were researched, based on general control criteria and then from the perspective of the application of the project, the control of over-actuated automated vehicles. The set of requirements that was selected for evaluation were stability, robustness, accuracy, transient response characteristics, control allocation and comfort.

It was found that the most important aspects of a motion control algorithm involve guarantying stability in the first place and ensuring robustness in all known scenarios. The decoupling of the MIMO system was found to be justified at frequencies of 1 Hz and above, where the stability conditions are determined. RGA analysis was performed with a sufficiently simplified model for vehicle dynamics to justify the decoupling. Then, classical stability criteria were applied to the three SISO systems, by selecting suitable models for the different components of the control loop. However, the approach relied on parametrized and linearized models, so the validation of those models is necessary to ensure the validity of the metric proposed.

The accuracy of the trajectory tracking was found to be of big importance in the context of automated vehicles. In this thesis, the analysis proposed consists in a test method inspired in the norm ISO 3888-2, that evaluates the error during an evasive maneuver. In addition to that metric, a standard step response analysis was proposed, to obtain a useful characterization of the response of the controller in terms of overshoot, settling time and steady state error.

The evaluation of comfort due to the controller actions was found to be partially implicit within the evaluation of stability and accuracy. The repercussion of the planned trajectory was found to have the most effect in the comfort of passengers, and it was determined that the analysis of the effect of the controller was not relevant enough to include it within this thesis.

Regarding control allocation, research showed that different control algorithms could be designed to target specific goals, and metrics for evaluating the success would have to be developed to adapt to

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each one. In this thesis, energy efficiency was chosen as an exemplary target, and a metric was developed to assess the performance.

The benchmarking methodology was then applied to a controller within the project, and the results were presented accordingly.

## **6.2 Outlook**

The overall development of the benchmarking methodology was found to be a large and complex task, with many secondary tasks to be researched and investigated. This resulted in the development of a general benchmarking methodology that needs further investigation in certain areas and adaptation at different levels depending on the type of controller implemented.

The use of simplified and linearized models compromises the stability and robustness analysis, as the validity of the metrics directly depend on them. In further research, the selection of the suitable and validated models at every step would be necessary.

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## Appendix

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### Appendix 1: Mass uncertainty, transfer function parameters.

Use	Parameter	Value
Common	$m$	2456.1 kg
20% mass increase	$m_{veh}$	2947.32 kg
50% mass increase	$m_{veh}$	3684.15 kg
	$I_z = I_{z,veh}$	6.7561 kg·m <sup>2</sup>
	$r_{dyn} = r_{dyn}$	0.195 m
	$c_\alpha = c_{\alpha,veh}$	70000 N/°
	$W$	1.78 m
	$L$	3.4 m
	$l_\psi$	1.9189 m
	$\tau_p$	0.316 s
	$\tau_v$	0.079 s
Longitudinal	$\tau_{dri}$	0.013999994238804 s
	$\zeta_{dri}$	0.189901320353302 s
	$c_{lon}$	100000 N/°
	$c_{lon,car}$	200000 N/m

## Appendix 2: Dynamic radius uncertainty, transfer function parameters.

Use	Parameter	Value
Common	$m = m_{veh}$	2456.1 kg
	$I_z = I_{z,veh}$	6.7561 kg·m <sup>2</sup>
	$r_{dyn}$	0.195 m
10% dynamic radius increase	$r_{dyn,veh}$	0.2145 m
10% dynamic radius decrease	$r_{dyn,veh}$	0.1755 m
	$c_\alpha = c_{\alpha,veh}$	70000 N/°
	$W$	1.78 m
	$L$	3.4 m
	$l_\psi$	1.9189 m
	$\tau_p$	0.316 s
	$\tau_v$	0.079 s
Longitudinal	$\tau_{dri}$	0.013999994238804 s
	$\zeta_{dri}$	0.189901320353302 s
	$c_{lon}$	100000 N/°
	$c_{lon,car}$	200000 N/m

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