

CONDITIONING OF TAIL WATER FLOW BY DRAFT TUBE DESIGN – A NEW APPROACH FOR LOW HEAD HYDROPOWER

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Abstract: The optimization of fluid systems, i.e. draft tubes in low head hydropower stations is up to now commonly carried out by CFD analysis or based on empiric data. The reasons for that is a lack of analytical optimization methods. The present paper provides a new analytic approach to optimization of draft tubes. Furthermore it names and corrects two common fallacies in hydropower literature. The optimized draft tube design satisfies the optimal operation condition for hydropower in an open channel-flow [1].

1 Optimal Operation for Low Head Hydropower

A common criterion for efficiency of low pressure hydro power plants is the hydraulic efficiency η of the facility

$$\eta = \frac{P_T}{\rho g Q (H_1 - H_2)} \quad (1)$$

It is the ratio of shaft power to the hydraulic energy at a particular operating point and it is a measure for the dissipation within the hydraulic machine and connected parts like draft tubes. The turbine head $H_T / \eta := H_1 - H_2$, i.e. the difference of total head at positions 1 and 2 and the volume flow rate Q are considered as given by the hydrologic conditions of the surface water course. Anyway, while operating low pressure power stations it can be observed, that the hydraulic machine's operating point itself has an influence on the head and volume flow rate Q . The operating point determines which fraction of the available power can be converted into shaft power P_T . The coefficient of performance is defined as that ratio

$$C_p = \frac{P_T}{\rho g Q (h_1 + \frac{u^2}{2g} + z_1 - z_2)} \quad (2)$$

Since the shaft power of the system is normalized by the available power P_{avail} , the first step is to define the available power. Figure 1 shows a schematic hydro power plant. Velocities are denoted by u , water levels by h and ground levels by z . The effective head is defined as $H_{eff} := h_1 + u^2/2g + z_1 - z_2$. The second author defines the available power, as the shaft power of a hypothetical ideal machine (without tail water – in fact there is no better machine one can think of) [1]

$$P_{avail} = \rho g Q \left(h_1 + \frac{u^2}{2g} + z_1 - z_2 \right) \quad (3)$$

The coefficient of performance C_p can be expressed as a function of volume flow rate per width unit $q_2 = Q/b_2$ and tail water level height h_2 . Those two parameters define the operating point of the installed hydraulic machine.

As a consequence of the first law of thermodynamics, the coefficient of performance is

$$C_p = \frac{\eta}{2} \left(\frac{5}{2}\right)^{5/2} q_+ \left(1 - h_+ - \frac{1}{2} \frac{q_+^2}{h_+^2}\right). \quad (4)$$

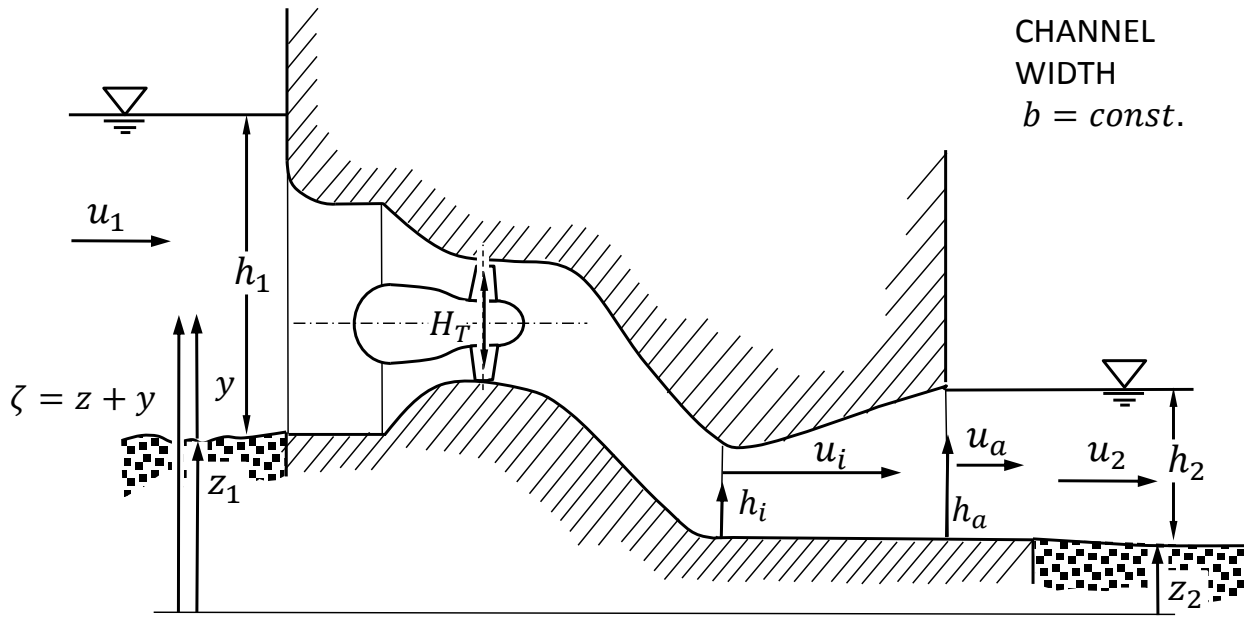


Figure 1: Schematic cross section of low head hydropower plant (Case II)

With dimensionless operating parameters q_+ and h_+ (see Figure 2 for the definitions), C_p is plotted in Figure 2.

Since equation (4) is exact, it clearly shows the difference between hydraulic efficiency η and coefficient of performance C_p once more (see Betz [2], Pelz [1]). The first is a measure for dissipation, the later a measure for system efficiency. It has to be emphasized, that considering hydraulic efficiency only can be misleading and is indeed insufficient for optimization of low head hydro power plants.

The optimum of equation (4) is $C_{p,opt}/\eta = 0.5$. In other words: half of the available power in an open channel-flow can be converted into shaft power at the optimal operation point (see Figure 2) provided

$$q_{+,opt} = \left(\frac{2}{5}\right)^{3/2} \quad \text{and} \quad h_{+,opt} = \frac{2}{5}. \quad (5)$$

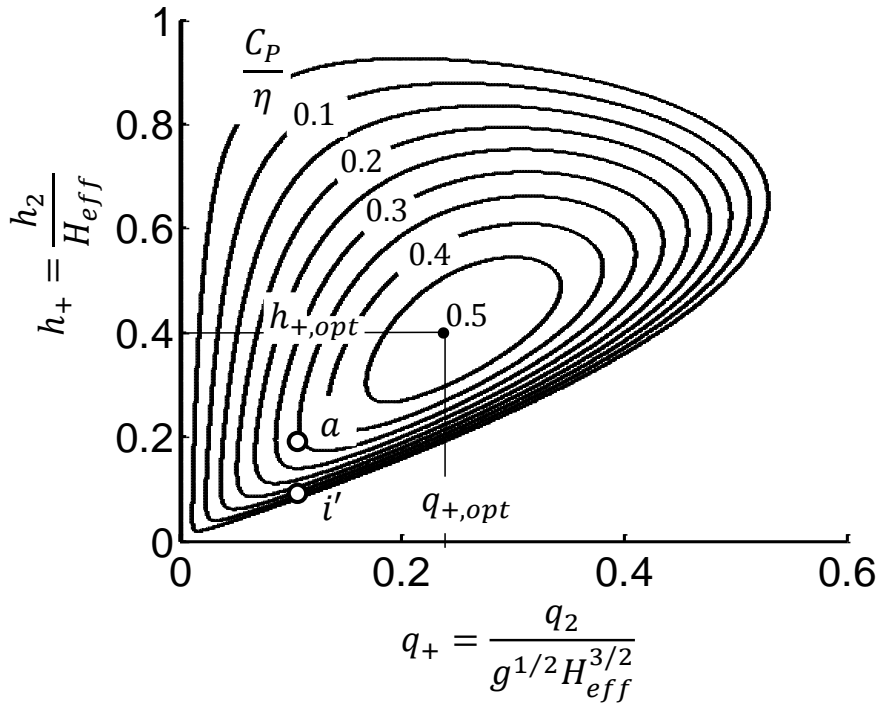


Figure 2: Coefficient of performance as function of operating point [1]

2 Terminology of Total Pressure

In hydro power literature there is a multitude of terms in the context of pressure. In some cases they are contradictory. The terms for the present paper follow the definitions of E. Becker [3] in his excellent and crystal clear monograph on fluid mechanics. The total pressure p_t is the sum of static pressure p_s potential of the specific gravitational field $\rho g \zeta$, with $\zeta = z + y$ and dynamic pressure $\rho u^2 / 2$

$$p_t := p_s + \rho g \zeta + \frac{\rho}{2} u^2. \quad (6)$$

Provided $\partial u / \partial x \approx 0$ (x = flow direction) the static pressure at $0 < y < h$ is given by $p = \rho g(h - y)$. Hence the piezometric pressure is constant across any cross sections $p^* := \rho g h + \rho g z$ and is no function of the coordinate y (see Figure 1). The same is true for the total pressure and the total head defined by

$$H = \frac{p_t}{\rho g} = h + z + \frac{u^2}{2g}. \quad (7)$$

(It is homogenous at any cross section with $\partial u / \partial x \approx 0$). The total head H has the dimension of energy per volume unit [1], [3]. h denotes pressure head, which is equivalent to the water level height in an open channel-flow.

3 State of the Art Draft Tube Theory – a Limited Approach

With technological progress in hydro power a trend from machines of large diameter at low specific speed to machines of smaller diameter at high specific speed can be observed. With the decreasing size of machines investment cost per installed power decreased as well. Otto Cordier [4] showed, based on empirical data, the correlation of specific speed and specific diameter of hydraulic machines. The correlation between specific diameter and investment cost is considered by Pelz and Metzler [5]. Due to small diameters, the meridian speed of flow along the hydraulic machine increased compared to ancient machines of low rotational speed and large diameter. Hence, the fraction of dynamic pressure in the total pressure is high (see equation (6)) at the rotor outlet. Experiments indicate that draft tubes with large diffusers enhanced the shaft power significantly, for machines of high rotational speed.

The common explanation for this effect is an assumed *recuperation of energy* within the draft tube, due to conversion of dynamic pressure into static pressure. Deniz [6], as one of various authors writes:

“To achieve high hydraulic efficiency it is necessary to recuperate the major part of the outlet energy. Velocity energy has to be converted in pressure energy at highest possible efficiency.”

In hydro power literature, kinetic energy that leaves the power plant is considered as lost. This loss is expressed by Deniz [6] and Raabe [7] as the ratio of kinetic energy at the rotor outlet and the turbine head defined as the difference of total head between head water and tail water [6].

$$k := \frac{u_2^2}{2g(H_1 - H_2)} \quad (8)$$

Due to Deniz [6] this loss has to be minimized, while it is assumed that the numerator of definition (8) is variable and the denominator is fixed. Furthermore Deniz claims, that the pressure gain leads to a pressure drop at the rotor outlet. Without a doubt this pressure drop has been measured and can be considered as proofed. Only the explanation is not adequate and hence misleading for optimization tasks.

4 Extended Approach

Ruprecht [8] already noticed that the tail water condition has an influence on the draft tubes outlet flow, but did consider the tail water height h_2 as fixed, so did Deniz [6] and Raabe [7].

In the following section it will be shown, why the a conversion of hydrodynamic pressure into hydrostatic pressure inside the draft tube does not lead to higher shaft power since the total pressure drop does not change. Surprisingly this fact is not taken into account in state of the art draft tube theory.

Unlike Deniz, Raabe and Ruprecht, within the later sections of this work, we widen the view and take the downstream flow of the draft tube into account. By doing so, we enlighten the limitations of classical approaches and furthermore open the view to the two major functions of draft tubes.

The first one is to reduce dissipation namely the Carnot shock loss and hence increases the hydraulic efficiency η of the system. This function is understood by most hydraulic engineers.

The second function is less obvious and up to now not clearly addressed: the draft tube determines the tail water Froude number and tail water level and hence, the none dissipative losses which are considered mainly by the coefficient of performance divided by the hydraulic efficiency C_p/η .

5 The Impossibility of Energy Recuperation by Pressure Conversion

The term *energy recuperation*, that is used in hydropower literature to describe the function of the diffuser ([6], [9]), is misleading, since the energy content of a flow cannot be changed by the conversion of hydrodynamic pressure into hydrostatic pressure. Neither the energy flux at the outlet of the hydro power plant within the tail water can be lowered by conversion of pressure.

$$P_T = \eta \Delta p_t Q = \eta \rho g (H_1 - H_2) Q . \quad (9)$$

Equation (9)=(1) is shaft power P_T of the turbine, expressed by the energy equation. Independently of the amount of piezometric pressure $p^* = \rho g (h + z)$ and dynamic pressure $\rho u^2/2$ the sum of both, the total pressure p_t (see equation (6)), is constant in a system without an energy source or sink, like it is the case in between rotor outlet and diffuser outlet, for a only at the moment assumed ideal diffuser (no separation, no friction). The volume flow rate Q is constant for stationary operation of hydro power plants. Hence, converting hydrodynamic pressure into hydrostatic, i.e. piezometric pressure does not have any influence on the shaft power of the turbine P_T .

In hydropower literature plants are modeled analytically under use of Bernoulli's equation, which leads to feasible results for hydropower with large head. In low head hydropower effects of the free surface must be considered. The model concept of Bernoulli's equation misleads at this point: Bernoulli's equation is valid along a streamline for frictionless flow. For reasons of simple description this streamline, in case of hydro power plants, usually starts at the surface of the head water and ends at the surface of the tail water. Now the common fallacy is, that the tail water level h_2 is considered as fixed, while the tail water velocity u_2 is considered to be adjustable by the diffuser. The definition of $k := u_2^2/2g(H_1 - H_2)$ implicit has this as a preliminary. This assumption leads to a violation of mass conservation, as it will be shown in the following section. For an adequate description of the flow, including free surface effects, the first law of thermodynamics, which is of superior generality, compared to Bernoulli's equation has to be applied. Then both equations, (i) the energy equation and (ii) the mass conservation law have to be satisfied.

6 Consideration of Free Surface

To explain the function of the diffuser in an adequate manner, the effects of the free surface have to be included in the analytic considerations. Therefore both, the first law of thermodynamics and mass conservation have to be satisfied as mentioned above. The first law of thermodynamics for the scheme in Figure 1 reads (see [1])

$$H_T = h_1 + \frac{u_1^2}{2g} + z_1 - h_2 - \frac{u_2^2}{2g} - z_2 - h_L . \quad (10)$$

All of dissipative losses in the whole device including screen, inlet, hydraulic machine and the draft tube, are summarized in the head loss h_L . The main difference to the Bernoulli's equation is, that first law of thermodynamics (equation (10)) is not only valid along a streamline. It is a conservation equation for the whole fluid body between any cross section of an open channel.

For a stationary operating hydro power plant with volume flow rate Q and rectangular channel cross section of constant width b the mass conservation reads¹

$$u_2 = \frac{Q}{h_2 b} . \quad (11)$$

As a conclusion of equation (11) it is established that the tail water flow velocity u_2 is a function of tail water level height h_2 and must not be considered as independent variable, as it is done implicit using the definition (8). Hence, equation (11) is substituted in equation (10) for satisfaction of mass conservation and first law of thermodynamics

$$H_T = H_{eff} - h_2 - \frac{Q^2}{2g b^2 h_2^2} - h_L . \quad (12)$$

Figure 3 shows an example for the possible states of an open channel-flow, i.e. the possible water level heights for each total head. As well, as equations (10), (11) and (12) it is valid for a rectangular channel of constant width b for a given volume flow rate Q . Both, total head and water level height are divided by H_{eff} for a dimensionless description of the flow. The state variables at the channel cross sections are shown in Figure 1 and Figure 4 (1, i , i' , a , 2). At cross section 1 the total head equals the effective head, provided the reference level is the tail water channel bottom. The function of the diffuser is discussed for three exemplary cases. Case I is

¹ All statements of this paper are valid for any kind of nonrectangular channels as well. The rectangular channel is only chosen for reasons of simple analytic description.

a draft tube without diffuser (see Figure 4). Case II is a draft tube with diffuser (see Figure 1). Case III is a draft tube with submerged diffuser (see Figure 5).

Case I: Draft tube without diffuser (see Figure 4)

In this case the index i' represents the position at the outlet of the draft tube without diffuser. The velocity head at the turbine outlet $u_{i'}^2/2g$ is comparatively high, while the pressure head $h_{i'}$ is low.

Point i in Figure 3 i.e. Figure 4 is the tail water state of case I. The transition from state i' to i is known as hydraulic jump. For each total head within the codomain of the total head for a given volume flow rate Q and $Fr = u/\sqrt{gH} \neq 1$ there are two possible water depths. One of them with $Fr_i > 1$ is called supercritical, the other with $Fr_i < 1$ is subcritical. The highest possible difference in total head between head water and tail water occurs when the tail water flow is critical ($Fr_a = 1$ (see Figure 3)). This statement has been established by the second author [1].

In case I, the flow is supercritical at the draft tube outlet ($Fr = u_{i'}/\sqrt{gh_{i'}} > 1$). A hydraulic jump occurs in between draft tube outlet and tail water (see Figure 4). The total head in the tail water H_i/H_{eff} is comparatively high. The shaft power is low due to an inefficient operation point and high dissipative losses.

$$\frac{H_{T,I}}{H_{eff}} = 1 - \frac{h_i}{H_{eff}} - \frac{Q^2}{2g b^2 h_i^2 H_{eff}} - \frac{h_{L,I}}{H_{eff}}. \quad (13)$$

The dissipative losses related to the hydraulic jump from state i' to state i , $h_{L,j}$ are included in $h_{L,I}$. For all of the cases considered in the present paper, the width of the tail water is assumed to be the same ($b = const.$).

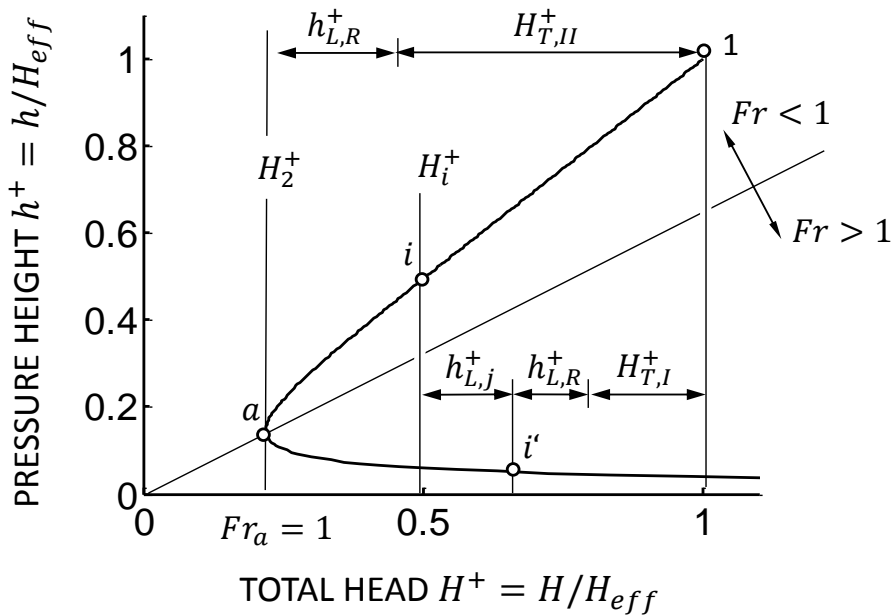


Figure 3: State diagram for open channel-flow

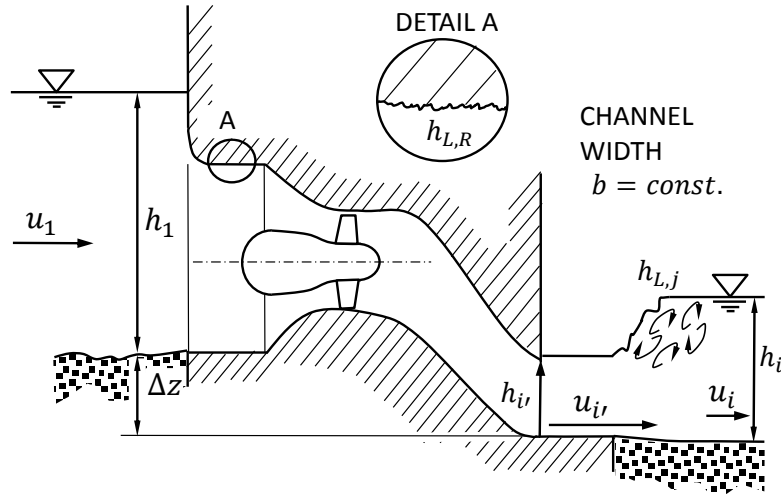


Figure 4: Draft tube without diffuser (Case I)

Case II: Draft Tube with Diffuser (see Figure 1)

In case II the flow is converted from state i' to state a inside the diffuser. As a consequence of this conversion, the total head of the tail water decreases. Also there are no hydraulic jump losses. Hence the convertible fraction of the effective head H_{eff} increases

$$\frac{H_{T,II}}{H_{eff}} = 1 - \frac{h_a}{H_{eff}} - \frac{Q^2}{2g b_2^2 h_a^2 H_{eff}} - \frac{h_{L,II}}{H_{eff}}. \quad (14)$$

The increase of turbine head then is

$$\Delta H_{T,I,II} = H_{T,II} - H_{T,I}. \quad (15)$$

The pressure drop at the rotor outlet, which is mentioned by various authors, can be explained by equation (15). The diffuser enhances the coefficient of performance C_p (see figure 2) but the optimal coefficient of performance is not attained for case II. A simple design strategy for a diffuser operating at $C_p/\eta = 0.5$ is proposed in the section 7.

Case III: Submerged Diffusers and Carnot Shock Losses

An important loss mechanism for draft tube outlets is the Carnot shock loss. It occurs when the draft tube outlet is submerged by the tail water column ($h_D < h_2$, see Figure 5). Submerging can occur due to backwater from downstream dams or as a

consequence of the operating point of machines, i.e. volume flow rate of watercourses.

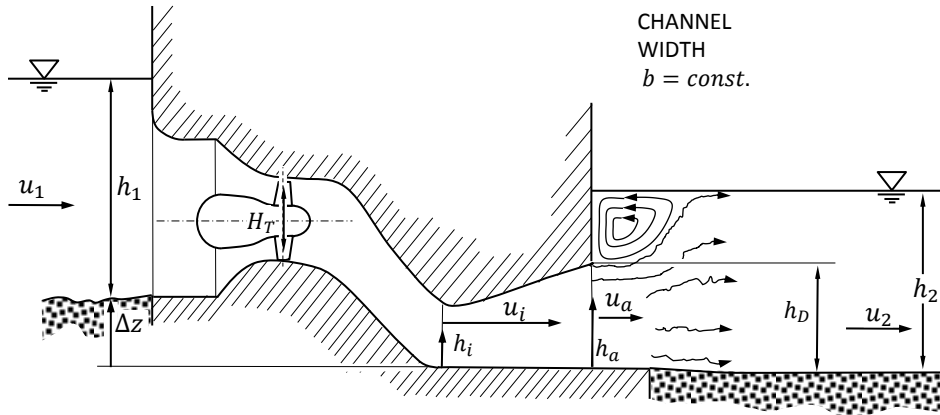


Figure 5: Submerged diffuser outlet

The Carnot shock loss can be established by strictly axiomatic equations. It is a consequence of the conservation of momentum and Bernoulli's equation which itself can be established under use of the conservation of momentum [3]. The result of this derivation is the Carnot shock loss for an open channel-flow (see Figure 5) of constant width b , volume flow rate Q

$$h_{L,C} = \frac{Q^2}{2g b^2 h_D^2} \left(1 - \frac{h_D}{h_2}\right)^2. \quad (16)$$

With the dimensionless operating parameters q_+ and h_+ , defined in Figure 2, and the dimensionless diffuser outlet height $h_D^+ := h_D/H_{eff}$ the dimensionless Carnot shock loss reads

$$h_{L,C}^+ = \frac{h_{L,C}}{H_{eff}} = \frac{1}{2} \frac{q_+^2}{h_+^2} \left(1 - \frac{h_D^+}{h_+}\right)^2 \quad (17)$$

As a consequence of the first law of thermodynamics the hydraulic efficiency due to Carnot shock losses is

$$\eta_C = \frac{1}{1 + \frac{h_{L,C}}{H_T}}, \quad (18)$$

with $H_T = H_1 - H_2 - h_{L,C} - h_L$. Substituting equation (18) into equation (4) the coefficient of performance with Carnot shock loss included is

$$\frac{C_{P,Carnot}}{\eta} = \frac{\eta_C}{2} \left(\frac{5}{2}\right)^{5/2} q_+ \left(1 - h_+ - \frac{1}{2} \frac{q_+^2}{h_+^2}\right). \quad (19)$$

Figure 6 shows a grey contour plot for the coefficient of performance without dissipative losses C_P/η (see equation 4). The black lined contour plot in Figure 6, with the same values for isoclines, shows an exemplary coefficient of performance with Carnot shock loss included (see equation (19)). For this example the dimensionless diffuser outlet height $h_D^+ = h_D/H_{eff}$ equals the optimal tail water height $h_{+,opt}$. For values $h_+ < h_D^+$ there is no Carnot shock loss, hence both contour plots are identical ($C_{P,Carnot}/\eta = C_P/\eta$). For values $h_+ > h_D^+$ nonzero Carnot shock losses occur, hence $C_{P,Carnot}/\eta < C_P/\eta$ for all values of dimensionless tail water height h_+ and dimensionless tail water flow rate per width unit q_+ .

7 Diffuser Design for Optimal Operation

As it is mentioned in the first section the optimal operating point is established by [1]. A diffuser should be designed to work in this operating point. For a rectangular cross section at the outlet (see Figure 7), the optimal height of the diffuser is the optimal tail water height $h_{+,opt} = 2/5$ (see Figure 2), hence $h_{D,opt} = h_{2,opt} = \frac{2}{5}H_{eff}$. The optimal volume flow rate per width unit $q_{2,opt} = g^{1/2}(2/5 H_{eff})^{3/2}$ can be adjusted by variation of the width of the diffuser. For optimal operation the width calculates

$$b_{D,opt} = b_{2,opt} = \frac{Q}{q_{2,opt}} . \quad (20)$$

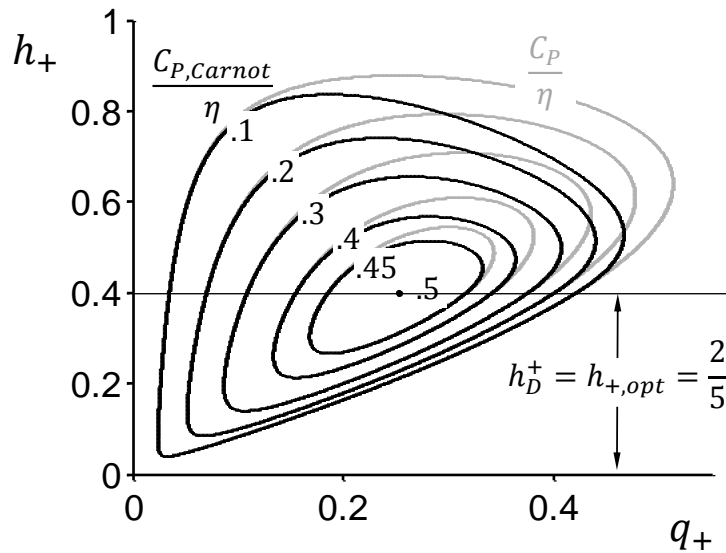


Figure 6: Coefficient of Performance with Carnot shock losses included

8 Conclusion

For an adequate discussion of efficiency in hydropower, the coefficient of performance is the measure to be discussed. Limitation on the discussion of hydraulic efficiency can be misleading.

For an adequate explanation of the function of a draft tubes diffuser, free surface effects have to be considered. These effects cannot be described by Bernoulli's equation but by the first energy equation and mass conservation.

The geometry of the diffuser can contribute to achieve the optimal coefficient of performance $C_p/\eta = 0.5$.

Losses due to dissipation can be modelled analytically and be included in the coefficient of performance as it is shown in section 8. Then the gained energy can be calculated by consideration of operation point and dissipative losses.

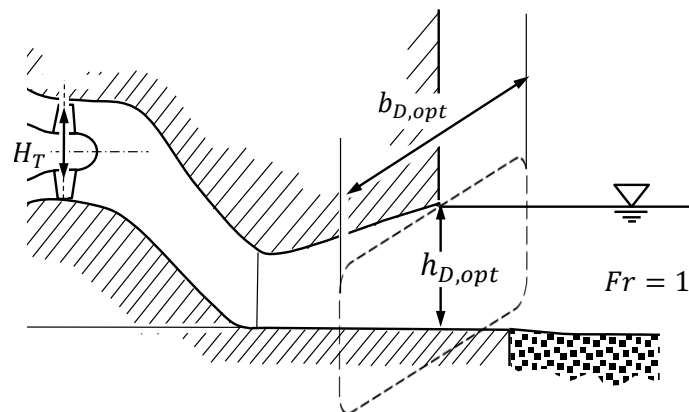


Figure 7: Rectangular diffuser for optimal operation

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