EMPLOYING HYDRAULIC TRANSMISSION FOR LIGHT WEIGHT DYNAMIC ABSORBER

Tobias Corneli* and Peter Pelz* **

Technische Universität Darmstadt, Chair of Fluid Systems, Magdalenenstr. 4,
D-64289 Darmstadt, Germany*
Industrial Science GmbH powered by IAV, Alexanderstr. 25,
D-64283 Darmstadt, Germany**
E-Mail: peter.pelz@fst.tu-darmstadt.de

A new dynamic absorber concept, called Fluid Dynamic Absorber (FDA), is presented. The absorber employs hydrostatic transmission to reduce weight and material need. At the same time the functionality compared to classical dynamic absorber is improved.

The absorber is built out of a double-sided piston of cross section A connected by elastic elements (spring, beam, …) to the vibrating structure. Both piston sides communicate due to a closed loop pipe of cross section $a = \alpha^{-2}A$ and length L. Due to the piston movement the fluid mass $m_F = \rho La$ is accelerated. The piston movement and the fluid movement is geared by the factor α^2. With this transmission factor the effective absorber mass is given $m_v = \alpha^2 m_F \gg m_F$.

The concept of hydraulic absorber is known already to reduce the dynamic force transmission by hydraulic mounts. Up to now hydraulic absorbers are not used to reduce structural vibrations.

Within the paper the design, prototype, physical model and experimental validation of the Fluid Dynamic Absorber is shown. A critical discussion and comparison to classical dynamical damper is given. Doing so, the advantages in performance (damping behavior), application and effort (reduced weight, reduced package) become clear.

Keywords: Absorber, oscillations, weight reduction
Target audience: Mobile Hydraulics, structural mechanics, systems

1 Introduction

In consequence of rising costs in energy and basic materials there is a demand on light weight constructions especially for on road mobile machineries e.g. cars, trucks and mobile working machines. A mass reduction accompanies commonly with a reduction in stiffness of the construction. Hence, the eigenfrequencies of the system shift down to lower values. If they accord with the operating frequencies undesired oscillation problems occur. To overcome them tuned mass dampers (tmd's) are usually installed. This approach looks like fighting fire with fire due to the fact that at piece of the reduced weight is added again to remove the vibrations. Here our invention called Fluid Dynamic Absorber (FDA) /2/ ties in. We use a hydraulic transmission as well as an effective inertial system to reduce the weight of tuned mass dampers in a meaningful manner. In this paper the results of a FDA applied at a two mass oscillator as well as the results of an equivalent classical tuned mass damper applied at the same oscillator are presented. The FDA attains a weight reduction of approximately 30% compared to the classical one. The second advantage of our invention is the retrenchment of the dissipative component (damper). Due to the application of an oscillating liquid column pressure losses are integrated into the inertia of the FDA. Hence, the developed system combines the inertia and the dissipative element in one. This is beneficial since most of the research and development activity at tuned mass dampers is related to the dissipation. Several damping systems were published and patented in the last decades: e.g. a solid friction based damper and a fluid damper in /7/, a tuned mass damper using shock absorbers in /6/ and damping with granulate material was published in /4/ and /7/.

The idea to use a hydraulic transmission for tuned mass dampers is based on the hydraulic engine mount. The theory of this kind of machine element and its function are presented in /8/ and /9/. The optimization of stiffness and damping constant of the FDA are based on the theory of /4/. The whole theory of the FDA is presented in /1/.

For the outline of the paper: At first the basic idea of the FDA is pointed out in detail before the describing equations are derived. Based on the mechanical vibrations a two mass oscillator is constructed to compare the efficiency of the FDA compared to a classical tuned mass damper. In the section Results the disposed theories are validated with a test rig. For the test rig a two mass oscillator in the design of a multi storey frame is used.

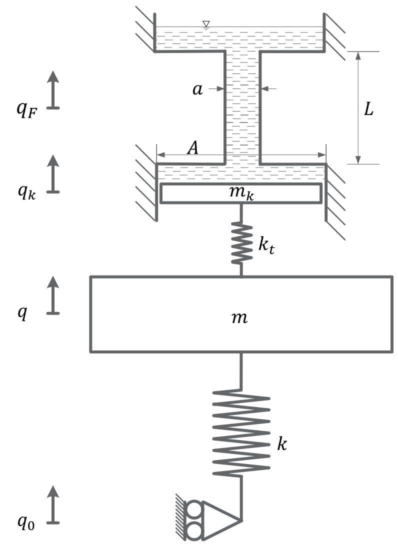

Figure 1: Sketch of the Fluid Dynamic Absorber.

2 Theory of the FDA

Figure 1 shows a schematic sketch of an oscillator with an added FDA. The oscillator consists out of mass m and a spring with stiffness k. The system has one degree of freedom denoted by the coordinate q. The excitation effects throughout the displacement q_0. To calm down the vibrations at the eigenfrequency $\omega = \sqrt{k/m}$ a FDA is tied in via a spring of stiffness k_t.

In principle the absorber consists out of a hydraulic piston of mass m_k and piston skirt A, a cylinder and the fluid of density ϱ. The cylinder has two different surface cross sections A and a and is connected at an inertial system. Through the surface change a transmission is generated and the bearing reaction is stabilised at the inertial system. The two coordinates of the hydraulic piston q_k and the fluid in the liquid column q_F are related by the following kinematic relationship

$$q_F = \frac{A}{a} q_k \tag{1}$$

Hence, the FDA has only one independent degree of freedom. Compared to a common tuned mass damper the inertia m_t of the absorber is composed by the mass of the hydraulic piston m_k and the transmitted inertia of the fluid. The area ratio A/a will be denoted in the following as transmission factor α.

The describing equations of motion for the system presented in *Figure 1* are the equation of motion for the oscillator

$$m\ddot{q} + k_t(q - q_k) + kq = kq_0 \tag{2}$$

and the equation of motion for the hydraulic piston

$$m_k\ddot{q}_k + k_t(q_k - q) + pA = 0. \tag{3}$$

To obtain an expression for the pressure in Equation (3) Bernoulli's equation is integrated among the stream tube of length L. If the influence of the gravity potential as well as the influence of the velocity $u^2/2$ are neglected the pressure depends on the inertia and the sum of pressure losses

$$p = \varrho L\ddot{q}_F + \sum\Delta p_v. \tag{4}$$

As yet the pressure losses in Bernoulli's equation above avoid to express Equation (2) and Equation (3) in terms of the coordinates q and q_k. So that the FDA can be compared with a common tuned mass damper and the weight reduction can be quantified. The losses in *Figure 1* are composed additively by the entrance and outlet losses and the friction losses in the column of length L. Commonly in liquid filament theory the pressure losses are expressed by a toll ζ. Hence the pressure losses are given by the expression

$$\sum\Delta p_v = \frac{\varrho}{2}\dot{q}_F^2\sum\zeta\ . \tag{5}$$

In Equation (5) the pressure losses are proportional to the square of the velocity. Hence, a linearization is used to keep the system of ordinary differential equations linear. For linearization it is assumed that the dissipated work per load cycle of a linear and a squared damping element are equal. After some analysis the following expression for the linearized damping coefficient can be obtained

$$\bar{b}_F = \frac{4}{3\pi} A\varrho\Omega\alpha^2\hat{q}_k\sum\zeta \tag{6}$$

With above damping constant the pressure losses can be expressed in terms of a linear damping constant $\bar{b}_F$

$$\sum\Delta p_v = \frac{\bar{b}_F}{A}\dot{q}_k. \tag{7}$$

Inserting Equation (1), Equation (4), Equation (7) and the expression for the fluid mass in the liquid $m_F = \varrho La$ into Equation (2) and Equation (3) leads to the following system of linear ordinary differential equations for the oscillator and the FDA

$$\begin{bmatrix} m & 0 \\ 0 & m_k + \alpha^2 m_F \end{bmatrix}\begin{bmatrix} \ddot{q} \\ \ddot{q}_k \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \bar{b}_F \end{bmatrix}\begin{bmatrix} \dot{q} \\ \dot{q}_k \end{bmatrix} + \begin{bmatrix} k + k_t & -k_t \\ -k_t & k_t \end{bmatrix}\begin{bmatrix} q \\ q_k \end{bmatrix} = \begin{bmatrix} kq_0 \\ 0 \end{bmatrix}. \tag{8}$$

Compared to a common tuned mass damper as presented in /3/ two differences occur. The first difference applies to the mass matrix in Equation (8). The fluid column of mass m_F is transmitted by the factor α^2. Hence, the mass of the FDA is given by the sum of fluid mass m_F and the mass of the hydraulic piston m_k whereas the inertia of the absorber is given by the virtual mass

$$m_v = m_k + \alpha^2 m_F. \tag{9}$$

From above equation it can be figured out quite easily why the FDA affords to save weight in a meaningful manner compared to a common tuned mass damper: The main inertia is generated due to the transmission of the fluid mass m_F. In a theoretical limit case the transmission factor α^2 tends to infinity and the weight of the absorber can be neglected compared to the main mass. Indeed life is not as easy as it seems to be in a limit value consideration. In a practical application the second difference compared to a tuned mass damper has to be taken into account. Commonly a tuned mass damper comprises out of three components: the spring, the solid body as inertia and a damper. Owing the principles involved the FDA has an integrated damper throughout the pressure losses. From component and cost effort this is an advantage. But from an adjustment point of view the coupling of inertia and dissipation is a challenge that avoids the approach described in the limit value consideration. From Equation (6) it can be figured out that the damping constant depends on the transmission ratio α, the toll ζ and the amplitude of the hydraulic piston $\hat{q}_k$. The toll of the intake and outlet losses itself depends again on the transmission ratio. The friction factor of the pipe depends on the Reynolds number Re and hence on the flow velocity. The flow velocity is linked throughout the kinematic to the hydraulic transmission. As pointed out the damping of the FDA is highly coupled with the transmission ratio. Due to the fact that a FDA as well as a tuned mass damper has an optimum damping factor the optimisation has to be conducted iteratively. And the parameters fluid mass, transmission ratio and damping constant have to be taken into account. In an optimum case the damping of the system is a bit lower than the optimum one and the final adjustment is fitted via a choke. Therefore, the maximum transmission ratio of the FDA and hence the saved weight is limited by the dissipation of the system.

3 Application of the FDA

In the previous section the principle idea of the FDA has been presented and the describing equations of motion have been derived. The exhibited sketch in *Figure 1* has according to some considerations a theoretical character: From the first point of view only oscillations in vertical direction to the inertial system can be reduced. And from the second point of view it cannot be assumed that an inertial system is achievable in such an easy manner as presented in the sketch. Therefore, in the following section a more general system will be pointed out. Furthermore a test rig and a prototype of the FDA will be presented.

3.1 General Application of the FDA

To overcome the described challenges related to the achievability of an inertial system and the direction of the oscillations the system presented in *Figure 2* is considered in the following section. *Figure 2* consists out of a simple mass oscillator and a FDA. The oscillator has a generic structure of a storey frame as it is commonly used in vibration test rigs. At first a 2-dimensional multi storey frame with one mass m and two coach springs of stiffness $k/2$ is considered. The degree of freedom is denoted by the coordinate q. This oscillator can be easily extended in a modular manner. The FDA is fixed at the grounding of the multi storey frame. It consists out of the double acting piston with piston surface area A and mass m_k. Its degree of freedom is indicated by the coordinate q_k. The piston is bounded in a cylinder with the same cross section area. Piston and cylinder are fitted by a running fit $H7g6$. This fit enables to disclaim a dynamic sealing due to the fact that the losses are negligible. For the test rig presented below the leakage flow is approximately 1% of the whole fluid flow. The losses are estimated while using a Couette flow. At both ends of the cylinder the oscillating liquid column of mass $m_F = \varrho La$ is connected via a pipe. Hence, the hydraulic circuit is closed and cavitation can be avoided. The key issue was is to connect the oscillating mass with the hydraulic piston. For the generic test rig a pendulum construction of length l was used. One end of the pendulum was gibed on the oscillating mass and on the other end the pendulum was connected with the piston through a slot hole. The slotted hole allows the piston to conduct a horizontal movement due to the fact that a pendulum describes an orbit denoted by the coordinate φ. The restorable element is realized through two springs of stiffness $k_k/2$. The springs are mounted with an inclination angle ψ at a distance l_k away from the fulcrum of the pendulum. Changes of the inclination angle allow modifications in the absorber frequency if the eigenfrequency of the oscillator was defined incorrectly or the inertia of the absorber is changed. The length $L_k/2$ is the length of the fluid reservoir at both sides of the piston. It has to be larger than the maximum amplitudes $\hat{q}_k$ of the absorber. Otherwise the cylinder is a mechanical end stop and the absorber can't work as predicted.

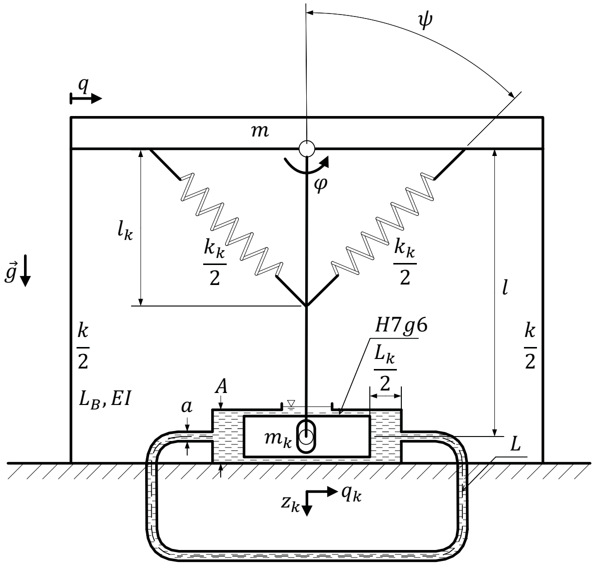

Figure 2: Application of the tuned mass damper with hydraulic transmission.

3.2 Application of the FDA at a multi degree of freedom system

For the in *Figure 2* presented system with a single degree of freedom the application of a pendulum seems to be reasonable to decouple the reacting point and the supporting point of the FDA. But for a system with more degrees of freedom that vibrates in the first mode the recommended solution does not appear realisable. This can be explained if skyscrapers are used as examples: These buildings have a height of several hundred meters, so that the use of a rigid beam would acquire many resources and introduce lots of weight into the system. E.g. a beam made of steel with a cross section of 0.1×0.1 m would have a mass of approximately eight tons per hundred meters. Therefore we thought about the coupling point of the absorber. Common tuned mass dampers are mounted at the point of the largest displacement where the inertia of the absorber has its greatest influence. For a multi storey frame that vibrates in the first mode a tuned mass damper is mounted at the top as presented for a two storey frame in *Figure 3*. Our invention has throughout the quadratic influence of the transmission factor a great potential for weight reduction and lightweight construction of tuned mass dampers. Therefore we considered a change in the coupling point of the Fluid Absorber. As presented in *Figure 4* our absorber is mounted close to the grounding. For the two storey frame we choose the first floor as coupling point. Thus a gear reduction in the excitation of the absorber is approved. To achieve a weight reduction with the FDA the gear reduction of the excitation has to be a smaller influence than the transmission of the inertia. To evaluate the influence of the coupling point the concept of kinetic equivalent masses is used. The kinetic equivalent mass for discrete masses is calculated based on the equation

$$m_{eff} = m_i X^2(x_i) \tag{10}$$

given in /5/. In the presented equation m_i denotes the mass of the i-th node of the multi degree of freedom system. And the variable $X(x_i)$ represents the maximum displacement of the vibration normalized with the maximum displacement of the node whereas the absorber is mounted at the investigated eigenfrequency.

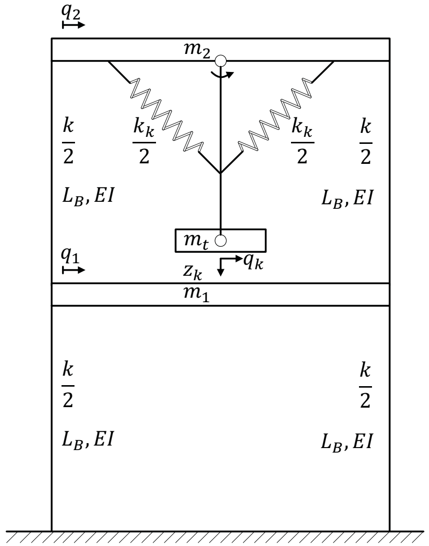

Figure 3: Conventional tuned mass damper.

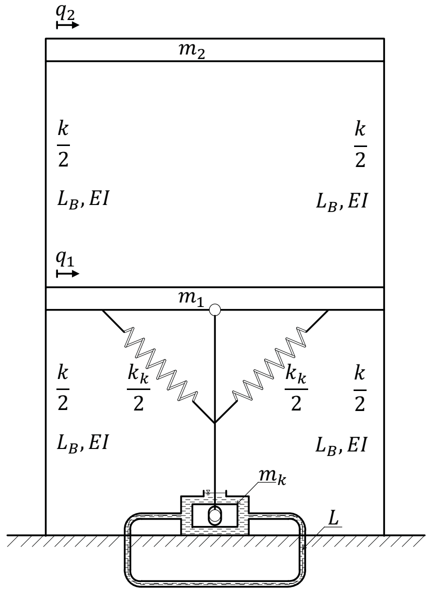

Figure 4: Tuned mass damper with hydraulic transmission.

3.3 Test Rig

In this section the conceived theories so far are validated using the two storey frame described above. The test rig consists out of two nodes of mass $m_1 = m_2 = 11.75$ kg. The nodes can be interpreted as floors and have a dimension of 180×180 mm. The storeys were connected via four steel trusses. Coach springs are used as steel trusses due to the fact that large displacements are expected when no absorber is installed. Commonly vibrations of skyscrapers are excited throughout wind loadings. In this case frequencies of approximately 1 Hz have to be cancelled out. Due to the fact that the system is excited through an imbalanced mass mounted on the top of the system the excitation force is proportional to the square of the excitation frequency. Hence, for low frequencies the excitation would be too small and might not be visible. We chose a frequency of approximately 3 Hz as first eigenmode. The chosen frequency is an compromise between realistic conditions and expected displacements. Since the test rig is also used as demonstrator without measurement equipment and the displacements should be visible with the naked eye. For the steel trusses we choose a cross section area of 20×4 mm. From the deflexion curve the length of them is specified at 440 mm. With the known stiffness and masses the response characteristics of the two storey frame without any absorber as well as the kinetic equivalent masses of the stated system can be designated. The effective kinetic mass of the system at the first eigenfrequency can be figured out when the characteristic response from *Figure 7* is taken into account. At a frequency ratio of $\eta = \Omega/\omega_1 = 1$ the kinetic equivalent mass of an absorber positioned as presented in *Figure 3* yields to $m_{eff}^{q_2} = 16.25$ kg and for the system as presented in *Figure 4* to $m_{eff}^{q_1} = 42.25$ kg. The superscript in the kinetic equivalent masses denotes the coordinate that is used for the nominalization of the displacements $X(x_i)$ and hence the mounting position of the absorber. With the quotient of the kinetic equivalent masses the influence of the mounting position of the absorber can be quantified by the gear reduction factor

$$\chi = \frac{m_{eff}^{q_1}}{m_{eff}^{q_2}} \tag{11}$$

The above presented factor χ is the quotient of the kinetic equivalent mass of a common tuned mass damper coupled at an arbitrary node divided by the kinetic equivalent mass of a common tuned mass damper coupled at the node of maximum displacement. It denotes about which factor the inertia of the non-optimum positioned absorber has to be greater than the inertia of the optimum positioned one in order to obtain the same characteristic response. For our test rig we choose an inertia of $m_t = 2.25$ kg for the common tuned mass damper depicted in *Figure 3*. To obtain an equivalent characteristic response for the mounting position at coordinate q_1 we need an inertia of $\chi m_t = 5.85$ kg for our FDA.

In the following the construction as well as the dimensioning of the FDA is presented. In *Figure 5* a half section of the absorber is displayed. The sketch shows the pendulum with the slotted hole, the piston of cross section area A, the cylinder which covers the piston and the pipe of cross section area a and Length L that contains the liquid column. In *Figure 6* an image of the FDA is presented. It contains piston, cylinder and the sealing cap of the cylinder with the pipe connections. For the dimensioning of the FDA we had three aims:

- Our whole construction containing the equipment presented in *Figure 5* and *Figure 6* as well as the fluid should have a lower weight than the used common tuned mass damper.
- The characteristic response of the system with FDA should be better than that one achieved with the common tuned mass damper
- The pipe for the liquid column should be a standard semi finished product.

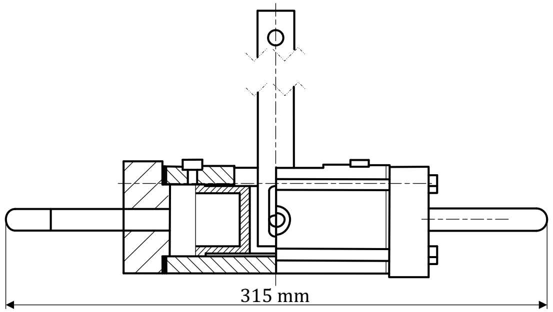

Figure 5:Half section of the Fluid Dynamic Absorber

Figure 6:Parts of the FDA

To fulfil our key claim of a light weight construction the inertia of the FDA $m_v = m_k + \alpha^2 m_F$ has to be greater than 5.85 kg and the mass of the whole system has to be smaller than 2.25 kg. From an iterative improvement out of piston displacements, inertia and manufacturing effort we figured out that the minimum length of the cylinder has to be 130 mm. With the sealing cap we end up of a total length of 170 mm for the construction presented in *Figure 6*. For the estimated cylinder length we need a minimum pipe length of 500 mm to connect both chambers of the double-sided piston. The diameter of the piston was also improved until the weight of the whole construction consisting out of the parts presented in *Figure 6* plus the pipe was smaller than 1.5 kg. We ended up with an inner diameter of the cylinder of $D_I = 40$ mm and an equipment weight of approximately 1.48 kg. With the known inner diameter and the weight of the piston $m_k = 0.17$ kg as well as the length of the liquid column we determined an inner pipe diameter of $d_I = 10$ mm. With the fluid mass of $m_F = 0.033$ kg we achieved an inertia of 8.54 kg. The total inertia of the FDA implies additionally the inertia of the piston (0.17 kg) as well as the inertia of the fluid in the balancing chambers in front of the piston (0.08 kg). The total weight of the fluid yields approximately 0.113 kg and almost 70% are not geared and located in the balancing chambers. Compared to the total weight of the FDA $m_A \approx 1.58$ kg the geared fluid mass $m_F \approx 0.033$ kg can be neglected. The total inertia of the FDA sums up to $m_v = (0.17 + 0.08) + 16^2(0.033) \approx 8.7$ kg.

With above result of the dimensioning process all considerations are fulfilled. With the total inertia of about $m_v \approx 8.7$ kg it can be expected that a better retirement of the oscillations is achieved compared to a common tuned mass damper. And the total weight of the system FDA is lower than the weight of the common tuned mass damper.

The optimum damping was determined iteratively in the experimental investigations through commensurate high coulomb friction in bearings. For the optimum damping we used oil with a kinematic viscosity ν of 10 mm^2/s at 20°C.

To prove the stated assumptions of the previous sections the characteristic response curves were measured with the test rig. Therefore an acceleration sensor was used. The sensor was mounted at all storeys as well as at the tuned mass damper and the FDA. Throug a double integration of the measured acceleration the displacement was evaluated. In the diagrams presented in *Figure 7*, *Figure 8* and *Figure 9* the nondimensional response curve is presented depending on the excitation frequency. The excitation frequency is normalized by the first eigenfrequency of the two storey frame. The displacement is normalized using the product of the mass ratio of the imbalance mass and the mass of a storey multiplied by the eccentricity e of the imbalance mass $[(m_U/m)e]$.

4 Results

In this section the experimental results on the test rig are presented. The section results is divided into three parts. At first the displacements of the reference system without an absorber are presented. They are used to determine the kinetic equivalent masses. Afterwards the improvements that can be achieved while using a common tuned mass damper are presented. Finally the results using the FDA are published. In all three cases the black solid line denotes the calculations and the grey triangles illustrate the measurements.

4.1 Reference System

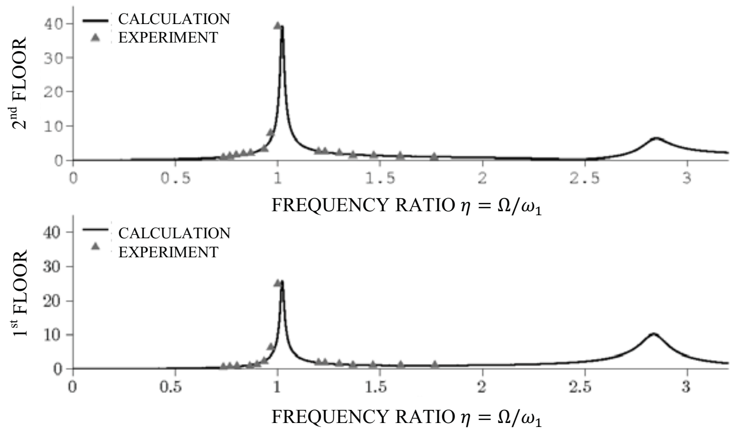

Figure 7: Characteristic Response of the two storey frame without a tuned mass damper.

In *Figure 7* the characteristic response of the reference system is shown. The first eigenfrequency is measured at 2.9 Hz and used for the normalisation. As it can be seen the expected and the measured eingenfrequency as well as the rest of the transmission behaviour fit quite well. The maximum displacements at the 2[nd] Floor are greater than 40 mm and hence visible with the unaided eye. Therefore, the system is useable for our application. We have large oscillations at a defined frequency that can be cancelled out while using a tuned mass damper and a FDA.

4.2 Common Tuned Mass Damper

The results presented in *Figure 8* are achieved with a common tuned mass damper of inertia $m_t = 2.25$ kg at optimum adjusted conditions for the stiffness. The optimisation was conducted in analogy to Den Hartog /3/. With a common tuned mass damper it was possible to reduce the maximum oscillations by a factor of 5 compared to the reference system.

4.3 FDA

The results archived with a FDA are presented in *Figure 9*. In comparison to the results from a common tuned mass damper shown in *Figure 8* the maximum oscillations is reduced by a factor of 4 again. And in comparison to the reference system the maximum oscillations is reduced by a factor of 20.

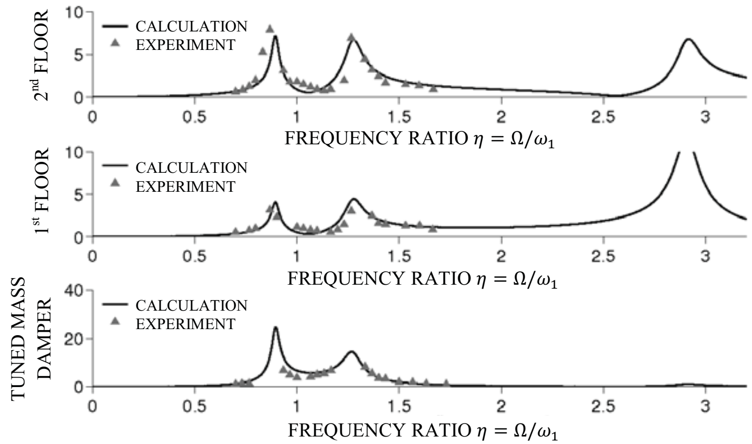

Figure 8: Characteristic Response of the two storey frame with a tuned mass damper.

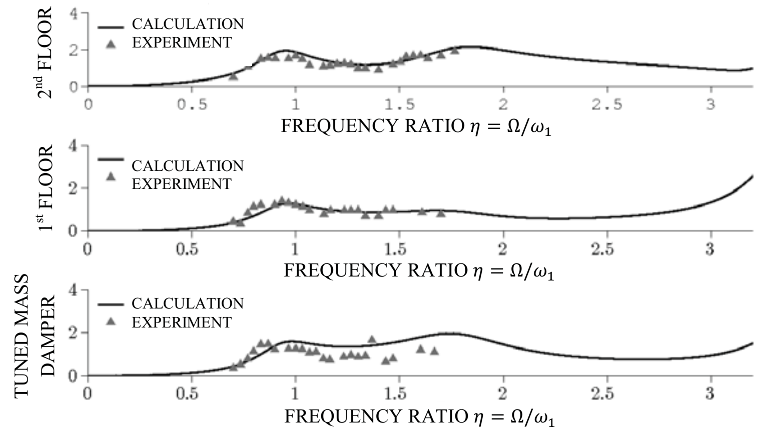

Figure 9: Characteristic Response of the two storey frame with a FDA.

5 Summary and Conclusion

An invention for passive vibration reduction called FDA has been presented. A key claim for the invention is to save weight compared to a classic tuned mass damper while using a hydraulic transmission. This claim can be fulfilled if an inertial system is used to capture the bearing forces of the transmission. We presented the response characteristic of a FDA as well as the response characteristic of a tuned mass damper for a two storey. The total equipment weight of the presented FDA is 30% lower than that one of the classic tuned mass damper. At the same time the maximum amplitudes of the storey frame can be reduced using the FDA by a factor of 4 compared to that one of a tuned mass damper. For the improvements in weight and response characteristic two reasons can be identified: First with the hydraulic transmission we can introduce a higher inertia into the system with less weight. Secondly through the pressure losses of the oscillating liquid column we are able to adjust the damping while using different viscosities of the Fluid. For common tuned mass damper it is difficult to find an adjusted damping system.

As a disadvantage of our system the higher manufacturing cost compared to a tuned mass damper needs to be mentioned. This is mainly due to the fact that for the inertia of a FDA a hydraulic piston as well as a hydraulic cylinder are needed while a tuned mass damper gets along with a solid body. However, the higher cost effort for the inertia of a FDA can be compensated through the pressure losses of the liquid column and the accompanied retrenchment of the dissipative element.

Nomenclature

Variable	*Description*	*Unit*
a	Cross section of the liquid column	[m²]
A	Cross section of the hydraulic piston	[m²]
$\overline{b}_F$	Linearized damping constant	[kg/s]
d_I	Inner pipe diameter	[m]
D_I	Inner piston diameter	[m]
e	Eccentricity of the imbalance mass	[m]
E	Elastic modulus	[kg/ms²]
g	Gravity of earth	[m/s²]
I	Area moment of inertia	[km²]
k	Spring stiffness of the oscillating system	[kg/s²]
k_k	Spring stiffness of the pendulum absorber	[kg/s²]
k_t	Spring stiffness of the FDA	[kg/s²]
l	Pendulum length	[m]
l_k	Distance between fulcrum and spring connection	[m]
L	Length of the liquid column	[m]
L_B	Length of the coach spring	[m]
L_K	Length of the balancing chamber	[m]
m	Oscillating mass	[kg]
m_{eff}	Kinetic equivalent mass	[kg]
m_F	Fluid mass	[kg]
m_k	Mass of the hydraulic piston	[kg]
m_t	Inertia/mass of the tuned mass damper	[kg]
m_U	Imbalance mass	[kg]
m_v	Virtual mass – inertia of the FDA	[kg]
p	Pressure	[kg/ms²]
Δp_v	Pressure losses	[kg/ms²]
q	Coordinate of the oscillating mass	[m]
q_0	Coordinate of excitation	[m]
q_F	Coordinate of the liquid column	[m]
q_k	Coordinate of the hydraulic piston	[m]
u	Velocity	[m/s]
X	Maximum displacement of the vibration normalised with the maximum displacement of the node whereas the absorber is mounted	[-]
α	Transmission Ratio	[-]
ζ	Toll	[-]
η	Frequency ratio	[-]
ν	Kinematic Viscosity	[m²/s]
ϱ	Density	[kg/m³]
φ	Coordinate of the pendulum	[°]
χ	Gear reduction factor	[-]
ψ	Inclination angle of the spring	[°]
ω	Eigenfrequency	[1/s]
Ω	Excitation frequency	[1/s]

References

/1/ Corneli, T., *Theorie Konstruktion und Machbarkeit des Hydrotilgers,* Interner Forschungsbericht TU Darmstadt, 2013.

/2/ Corneli, T., Pelz, P, Thurner, J., *Tilgereinrichtung für ein schwingungsfähiges Gebilde,* Industrial Science GmbH, Patent, Darmstadt, 2013.

/3/ Den Hartog, J. P., *Mechanical Vibrations*, Dover Books, 1984.

/4/ Langer, W., *Querschwingungen hoher, schlanker Bauwerke mit kreisförmigem Querschnitt*, Mitteilung aus dem Institut für Leichtbau und ökonomische Verwendung von Werkstoffen, Dresden, pp 184–197, 1969.

/5/ Markert, R., *Umdruck zur Veranstaltung Numerische Verfahren in der Strukturdynamik*, Technische Universität Darmstadt, Fachbereich Maschinenbau, Fachgebiet Strukturdynamik, 2007.

/6/ Pacht, H; *Schwingungsuntersuchungen an stählernen Turmbauwerken wie Maste und Schornsteine aus Sicht der Praxis*, VDI - Berichte Nr. 221, Schwingungstagung, pp 127–133, 1974.

/7/ Petersen, C., *Schwingungsdämpfer im Ingenieursbau*, Maurer Söhne, 2001.

/8/ Spurk, J., *Hydraulisch Gedämpfte Motorenauflager*, Ernst Becker Gedächtniskolloquium, 1985.

/9/ Spurk, J. and Andrä R., *Theorie des Hydrolager*. Automobil - Industrie, Jun 1985, 112.