Vibration Control for the Flexible Rotor with Piezoelectric Bearings Based on the Mixed Sensitivity Robust Controller

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Abstract. Active control of flexible rotors is a challenging issue in modern industries. This paper focuses on the synthesis of a mixed sensitivity robust controller for a linear parameter-varying (LPV) system. The objective is to control rotor vibration, especially when the rotor is passing the first two bending critical speeds. In the formulation of the problem for the controller, weighting functions are proposed based on the relationship between the desired shape of the open-loop transfer function and sensitivity functions of the closed-loop system. Recent research has highlighted the efficiency of mixed sensitivity robust controllers in stabilizing a wide range of magnetic bearing systems. Here, the method is extended to control the vibration of a piezoelectric bearing system. The experimental rotor features two unbalance-exited resonances within its operating range. Experimental results demonstrate good performance of the vibration reduction and the effectiveness of the design method.

Keywords. mixed-sensitivity robust controller; LPV; flexible rotor; piezoelectric bearing

1. Introduction

Rotor vibration is a limiting factor for high-speed rotating machinery, especially when the rotor has to pass bending critical frequencies. Typical applications are shafts in aircraft engines and in compressors of liquid helium refrigerators. Piezoelectric actuators are well suited for active vibration control systems, because they feature low weight, small dimensions and non-problematic behaviour in case of failure. Active piezoelectric bearings, offer high stiffness, low energy consumption and a broad frequency range of operation. However, the choice of the control strategy is crucial to the performance of the actively controlled rotor system with piezoelectric bearings.

Robust control strategies such as robust H∞ control methods have received extensive attention in the existing research. These methods are regarded as powerful tools to stabilize the rotor system or control vibration of the rotor. The desired performance, such as the interference rejection ability, could be obtained by minimizing the infinitive norms of selected closed-loop transfer functions. The multitude of H∞ controller design approaches can be split into two categories, the signal-based schemes and the mixed sensitivity design methods, which are also called loop-shaping based schemes [1, 2, 3]. Schittenhelm et al. propose a signal-based H∞ controller for piezoelectric bearings and give detailed description on the design procedure [4]. Based on this work, Becker et al. improve this control strategy by adopting a gain-scheduled H∞ controller based on the LPV system instead of a linear time-invariant (LTI) system [5].

However, this approach may cause complications because of the high number of considered transfer functions and the affiliated constraints which lead to a higher controller's order. The high controller order imposes higher requirements on the hardware and thus increases the overall costs of the active system. However, the mixed sensitivity H∞ controller only considers two main weights representing a complementary sensitivity function and a control sensitivity function of the closed-loop system. This simplifies the design procedure. Sahinkaya and Sawicki [6] summarize the difference between the mixed sensitivity H∞ control method and the signal-based H∞ control method in detail and demonstrate their superiority over PID controllers on a test rig with magnetic bearings. The publications [3, 7, 8] demonstrate that the mixed sensitivity H∞ controller can help the rotor to pass the bending critical speeds on the magnetic bearing systems.

The cited mixed sensitivity robust controllers are all based on LTI systems. However, systems with flexible rotors may depend highly on the time-varying rotational speed, making the LTI controller a challenging task to achieve good performance. For this reason, this paper follows the LPV model built by Becker et al. [9] and designs a scheduled gain controller, but the scheme of constructing the weighting functions proposed by this paper is different. The mixed sensitivity robust controller used by the author for the magnetic bearing system [10] is adjusted here to adapt the piezoelectric bearing system.

2. Test rig and modelling

The test rig used for validation control is shown in Figure1. The rotor comprises a thin shaft and two discs.

The rotor is supported by a passive bearing and an active piezoelectric bearing. The piezo-actuators are pre-loaded with springs and can be operated at the voltage between 0 and 1000 V. The maximum actuator displacement with an offset voltage of 500 V is ±30μm. Forces are measured by piezoelectric load washers, which are collocated with the actuators.

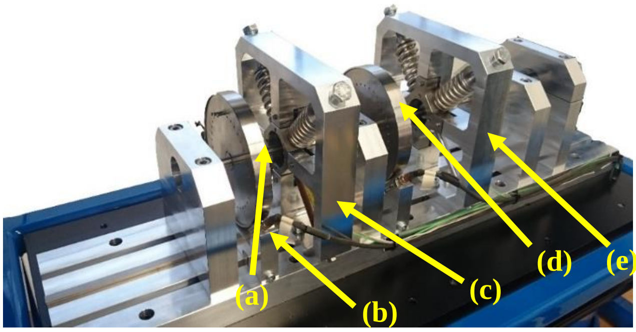


Figure 1. Test rig: (a) disk 1, (b) eddy-current sensor, (c) piezoelectric bearing, (d) disk 2, (e) passive bearing.

The maximum rotating speed of the system is 10,000 rpm, and the rotor has to pass the first two bending critical speeds (around 60 Hz and 130 Hz) within the operating range [11].

The mechanical system is approximated by a linear speed-dependent model. In order to obtain a model-based controller, the whole system model is constructed. Given the flexible rotor is operated at changing rotational speeds, a linear parameter-varying state space model with the form

$$\begin{cases} \dot{x} = A(\omega) + B_u u + B_d f \\ y = Cx + D_u u + D_d f \end{cases} \tag{1}$$

is proposed by [9], where x denotes the system states, u the control voltages applied to the active piezoelectric actuators, f the unbalance force of the rotor. It is worth noting that y is the active bearing force, instead of displacement signals, which is commonly used in magnetic bearing systems. That is a useful approach because the force from the actuator is measurable in piezoelectric bearing systems, opposed to magnetic bearing systems. It is also preferable because this avoids non-collocated sensor-actuator pairs.

The system matrix $\boldsymbol{A}(\Omega)$ depends linearly on the rotational speed Ω by

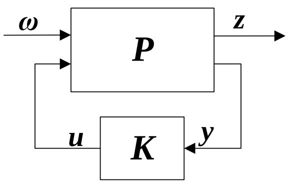
$$\boldsymbol{A}(\Omega) = \boldsymbol{A} + \Omega \boldsymbol{A}_{\Omega} \tag{2}$$

The state space equation is derived from motion equations of the rotor system, which are computed using finite element analysis. The finite element model is built on the basis of Timoshenko Beam theory using a MATLAB toolbox, which was developed by the Institute for Mechatronic Systems at TU Darmstadt. The dynamics of the electrical components, such as sensors, amplifiers, and anti-aliasing filters are also included in this model.

3. Robust controller design

3.1. Robust H∞ Controller

H∞ controller problems can be cast into the standard robust control configuration, shown in Figure 2, where $\boldsymbol{P}$ is the generalized plant, or the augmented plant, and $\boldsymbol{K}$ denotes the designed controller. The external inputs are denoted by the vector $\boldsymbol{\omega}$ while the performance outputs are represented by $\boldsymbol{z}$, which are assumed to be penalized via weighting functions. The weighting functions are used to shape the complementary sensitivity and control sensitivity of the closed-loop system.



$$\begin{bmatrix} \boldsymbol{z} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}_{11} & \boldsymbol{P}_{12} \\ \boldsymbol{P}_{21} & \boldsymbol{P}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \boldsymbol{u} \end{bmatrix}$$

Figure 2. H∞ control standard framework.

The augmented plant $\boldsymbol{P}$ is the system matrix from the inputs $\boldsymbol{\omega}$ and $\boldsymbol{u}$ to outputs $\boldsymbol{z}$ and $\boldsymbol{y}$. The corresponding closed-loop transfer function with same inputs and outputs is marked as $\boldsymbol{F}_l(\boldsymbol{P}, \boldsymbol{K})$. In fact, $\boldsymbol{F}_l$ is a lower linear fractional transformation (FTL) on mathematical issues [12].

$$\boldsymbol{F}_l(\boldsymbol{P}, \boldsymbol{K}) = \boldsymbol{P}_{11} + \boldsymbol{P}_{12}\boldsymbol{K}(I - \boldsymbol{P}_{22}\boldsymbol{K})^{-1}\boldsymbol{P}_{21} \tag{3}$$

Then the controller $\boldsymbol{K}$ is obtained by minimizing infinitive norm of the transfer function $\boldsymbol{F}_l$ in the form

$$\boldsymbol{K} = \arg\min_{\boldsymbol{K}} \|\boldsymbol{F}_l(\boldsymbol{P}, \boldsymbol{K})\|_{\infty} \tag{4}$$

In this paper, the MATLAB function ‘hinfgs’ is used to calculate the optimized H∞ controller. The key problems in designing a robust H∞ controller is the choice of weighting functions $\boldsymbol{W}$ and construction of the augmented plant model $\boldsymbol{P}$, which is shown in the following section.

3.2. Weighting Functions Design

The block diagram of mixed sensitivity H∞ control configuration is shown in Figure 3. Since the piezoelectric bearing system has two approximately symmetrical planes in the radial direction, its weights should be two-dimensional diagonal matrixes $\boldsymbol{w}$ with the subscripts ‘r’, ‘1’ and ‘2’ to express different weights. Every weight matrix has identical diagonal elements, denoted by the small letter $\boldsymbol{w}$ with corresponding subscripts. The weight transfer functions $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ are chosen by shaping the

Bode diagrams of the complementary sensitivity function '***T***' and the control sensitivity '***KS***' respectively, which is related to the open-loop transfer function of the system.

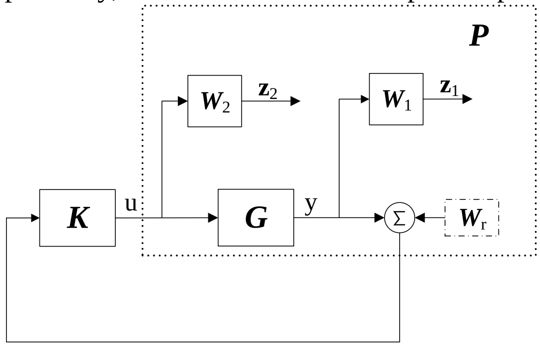


Figure 3. Configuration of the mixed sensitivity H∞ control.

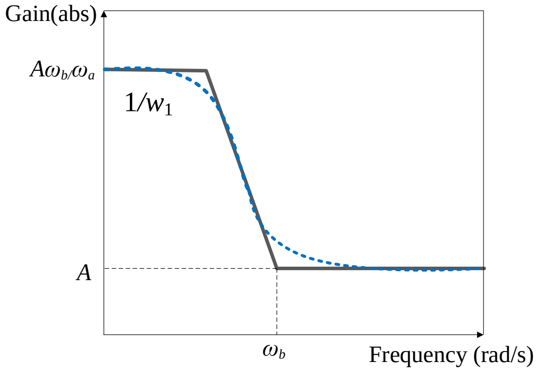


Figure 4. Typical shape of the inverse complementary weighting function $1/w_1$.

The complementary sensitivity function $\boldsymbol{T}$ represents the dynamic performance of closed-loop systems, which is the transfer function from reference input r to the performance output $\boldsymbol{y}$. It also can be seen from other perspectives as the ability of rejecting interference from output disturbance and high frequency sensor noise.

$$\boldsymbol{T}=\frac{\boldsymbol{GK}}{1+\boldsymbol{GK}}=\frac{\boldsymbol{L}}{1+\boldsymbol{L}} \tag{5}$$

From the above equation, it is concluded that the gain of the complementary sensitivity function is close to the open-loop transfer function $\boldsymbol{L}$ ($\boldsymbol{L}=\boldsymbol{GK}$) at high frequency [13]. The singular value of the open-loop function is expected to be decreased at a certain rate and be smaller at high frequencies to gain better rejection ability for disturbance and unmodeled dynamics [14]. Thus, the complementary sensitivity magnitude should be designed smaller at high frequencies. According to robust controller design principle

$$\left\|\boldsymbol{w}_1\boldsymbol{T}\right\|_\infty \le 1 \tag{6}$$

the classical shape of the upper bounds of the complementary sensitivity function $\boldsymbol{T}$, i.e. the approximate shape of the inverse complementary weighting function $1/\boldsymbol{w}_1$ is shown in Figure 4 [15]. The weighting function $\boldsymbol{w}_1$ may be illustrated by

$$\boldsymbol{w}_1=\frac{s+\omega_a}{A\left(s+\omega_a\right)} \tag{7}$$

where $1/\boldsymbol{w}_1$ is equal approximately to A at high frequencies. It represents the suppression magnitude of the disturbance and noise. Decreasing the value of A enhances the anti-interference ability, at the cost of increasing the control voltage. The crossover frequency $\boldsymbol{\omega}_b$ is approximately the bandwidth requirement of the controller, which should be the lowest limit for the frequency of the disturbance.

The control sensitivity ***KS*** represents the magnitude of the controller output from the reference signal r to the control signal u. The constraint for the maximum output voltage can be established by using a first order high pass filter [16]. This is required because the piezo actuators have a maximum driving voltage and would be damaged in case of excessive voltages. Therefore, control sensitivity ***KS*** is

The dashed pre-compensator $\boldsymbol{w}_r$ in Figure 4 is used to achieve deliberate command shaping [12]. It is used to distinguish the different penalization from the reference signal and the disturbance signal to performance outputs. As explained in Section 1, the flexible rotor has to pass its critical bending speeds. The disturbance paths caused by the mass unbalance must be considered and expressed in the piezoelectric actuator rotor system model. Here the weight $\boldsymbol{w}_r$ is selected around 20.

3.3. Augmented Plant

The state space realizations of above transfer functions can be represented as:

$$\boldsymbol{G}=\begin{bmatrix}\boldsymbol{A} & \boldsymbol{B}_d & \boldsymbol{B}_u \\ \boldsymbol{C} & \boldsymbol{D}_d & \boldsymbol{D}_u\end{bmatrix},\ \boldsymbol{W}_1=\begin{bmatrix}\boldsymbol{A}_1 & \boldsymbol{B}_1 \\ \boldsymbol{C}_1 & \boldsymbol{D}_1\end{bmatrix},\ \boldsymbol{W}_2=\begin{bmatrix}\boldsymbol{A}_2 & \boldsymbol{B}_2 \\ \boldsymbol{C}_2 & \boldsymbol{D}_2\end{bmatrix},\boldsymbol{W}_r=\boldsymbol{D}_r$$

It can be deduced that a possible state space realization for augmented plant P is

$$\boldsymbol{P}=\begin{bmatrix}\boldsymbol{A} & 0 & 0 & \boldsymbol{B}_d & 0 & \boldsymbol{B}_u \\ \boldsymbol{B}_1\boldsymbol{C} & \boldsymbol{A}_1 & 0 & \boldsymbol{B}_1\boldsymbol{D}_d & 0 & \boldsymbol{B}_3\boldsymbol{D}_u \\ 0 & 0 & \boldsymbol{A}_2 & 0 & 0 & \boldsymbol{B}_2 \\ \boldsymbol{D}_1\boldsymbol{C} & \boldsymbol{C}_1 & 0 & \boldsymbol{D}_1\boldsymbol{D}_d & 0 & \boldsymbol{D}_1\boldsymbol{D}_u \\ 0 & 0 & \boldsymbol{C}_2 & 0 & 0 & \boldsymbol{D}_2 \\ \boldsymbol{C} & 0 & 0 & \boldsymbol{D}_d & \boldsymbol{D}_r & \boldsymbol{D}_u\end{bmatrix}$$

Apart from the mentioned deriving method to get the system matrix from generalized inputs to generalized outputs, MATLAB functions ‘sconnect’ (in the LMI Toolbox), ‘sysic’ (Robust Control Toolbox), ‘iconnect’ (Mu Analysis and Synthesis Toolbox) can also be applied [17].

4. Experimental results

After a lot of trial and error, a group of weighting functions, which satisfy the mentioned conditions, are

$$\boldsymbol{W}_1=\boldsymbol{I}_2\omega_1=\boldsymbol{I}_2\frac{s+10}{14(s+68)},\ \boldsymbol{W}_2=\boldsymbol{I}_2\omega_2=\boldsymbol{I}_2\frac{s+400}{3s+1.2e5}$$

The final sensitivity functions in single radial direction, along with the inverse weighting functions obtained by the mixed sensitivity robust control method are displayed in Figure 5. Figure 6 presents the frequency response of the designed mixed-sensitivity controller, which is implemented in both radial axes.

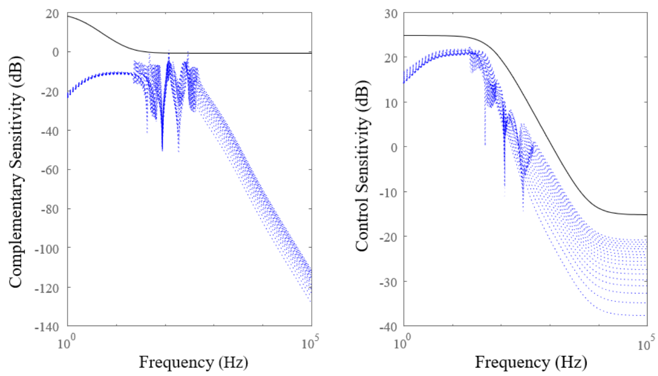


Figure 5. Comparison between inverse weights (-) and corresponding sensitivity transfer functions (--).

The designed continuous time controller is discretized using zero order hold on the inputs with a sample frequency of 3000 Hz and implemented using Simulink Real-Time, in order to verify the results on the test rig. The rotor of the test rig is running up slowly with an angular acceleration of 200 rpm/s and the signal amplitudes are recorded.

The control results shown in Figure 7 demonstrate that the vibration is highly reduced at both discs for the first two resonances. The maximum displacement of 0.81 mm occurred at the disc 1 and the first resonance is reduced by about 80% to 0.16 mm while the corresponding force is decreased 81%. At the same time, the maximum control voltage, around 300 V, is below the safety threshold of 500 V.

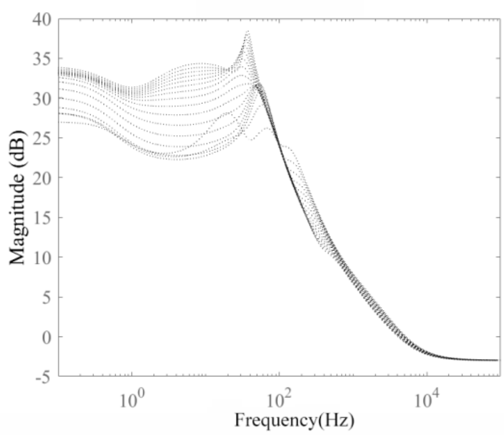


Figure 6. Mixed sensitivity H∞ controller for single radial direction.

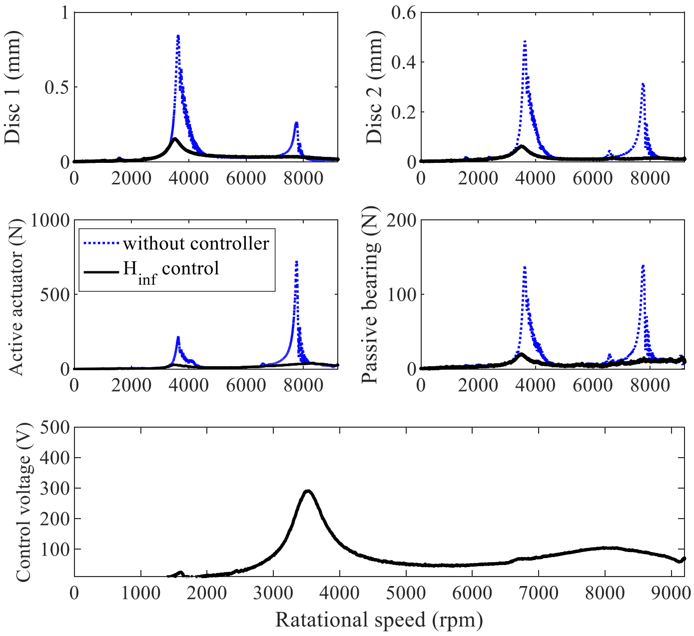


Figure 7. Experimental results of mixed sensitivity H∞ controller for the flexible rotor: no control (--); mixed sensitivity control (-).

5. Conclusions

In this paper, a mixed sensitivity robust control formulation for vibration isolation is proposed and has been verified on a test rig with a piezoelectric actuator rotor system. Since the rotor is thin and has two discs, the strong gyroscopic effects cause an approximately linear speed dependency of the test rig. Based on the LPV rotor system model, a gain-scheduled controller is designed. Experimental results show a good performance of the proposed control method. Not only the displacement at both discs has been reduced significantly, but also the vibration of bearing forces, especially near the first two resonances.

6. References

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