Chapter 8

Conclusions

In this work, mesh morphing techniques were analyzed, extended, and used to build linear spaces of three-dimensional shapes. The shape space has proven to be a useful concept to describe a shape in terms of other shapes. This notion is, indeed, intuitive for modeling (as shown for facial animations) and representing abstract data. The approach is particular strong for animated sequences of shapes. Such sequences were automatically converted into a representation in a linear space, allowing for efficient progressive streaming and storing of geometric animations.

However, we believe that representing shapes as compounds of other shapes is a universal concept with many more applications than we can even think of at this point.

8.1 Summary of contributions

To review, the primary contributions of this work as described in this dissertation are:

Mesh morphing Several contributions to the field of mesh morphing have been made, which are describe in the following.

- **Feature alignment** A mesh morphing technique for topological spheres is introduced, which allows for particular easy feature specification. Features are specified as vertex-vertex correspondence. In contrast to other works, no minimum user interaction is necessary (such as in dissection-type methods) while allowing to specify as many features as wanted.
- **Mesh merging** A new algorithm for generating one mesh from two input meshes (mesh overlay) is presented, generalizing the overlay problem to arbitrarily shaped objects. In contrast to other approaches, the algorithm is asymptotically optimal.
- **Local control** In image morphing one can easily specify which region transforms when in the morph, e.g., first the nose, then the eyes and then

the rest of a face. We present similar techniques for mesh morphing. In particular Laplacian coordinates were introduce, representing mesh geometry in a differential way. This description is also useful for free-form deformation.

Vertex path A method to generate intuitive and well behaving vertex paths in morphing is introduced. The paradigm employed is that the object is transformed rigidly as much as possible to avoid unnecessary deformations. This is achieved by dissecting the shape into a simplicial complex and defining optimal simplex morph. With this approach also the interior of a shape is treated and not only the boundary.

Visualization of multiparameter data Morph spaces are shown to be a useful way to generate glyphs and icons in the visualization of multiparamter data. Additionally, the paradigm allows for a flexible and intuitive generation of the visualization.

Geometric animations Spaces of meshes are shown to be a particularly elegant and effective way to generate, store, and communicate geometric animations. The resulting compression is progressive and the achieved compression ratios progress over previous work. A preprocessing step first decouples rigid motion from the animation specification, thus, making the inherently linear process more effective.

8.2 Future research directions

This work has many ways to be extended or completed, partly because the idea of shape space has no limits in its application. The following subjects seem particular valuable or interesting:

Topology in mesh morphing Mesh morphing still suffers from differences in topology of the input. While some algorithms handle explicit topology changes, none can automatically deal with this problem. This is of paramount importance because of the nature of real world meshes (e.g., originating from range scanners): They contain some holes and tunnels, which may or may not be part of the model being represented. From a geometry point of view these small errors have negligible impact, however, mesh morphing algorithms will surely fail on this type of input.

It seems that this problem originates from the erroneous mesh. This is only partly true: At least two applications suffer from the topology problem in general.

1. Imagine a torus to be morphed into a large statue holding a small ring so that the statue has the same topology as the torus. The topologically correct mapping connects the tunnels in both models independent of their different geometric importance.

2. Imagine the statue to be morphed into another statue without the ring (i.e. a topological sphere). Why should the user be concerned with a very small feature?

Ideally, the user should be able to freely specify correspondence without any topological restrictions.

However, it might be that meshes are not suited for this application in general. We are currently analyzing how to use point set representations [Pfister et al. 2000; Malgouyres & Lenoir 2000] of shapes for morphing. The point set accurately describes the boundary of the sphere, however might be deformed into a point set representing a shape with a different topology.

Affinely independent representation For interpolation problems of all sorts intrinsic geometry representations seem useful. Ideally, the representation of geometry should be independent to affine transformations. We have presented Laplacian Coordinates, which are independent of translation and linear in the original coordinates. Linearity is important to ease numerical computations.

The basic idea of affinely independent coordinates is to represent each vertex as an affine sum of its neighbors. This representation is linear in the original coordinates and, by definition, insensitive to affine transforms. However, every subset (neighbors or a larger neighborhood) of vertices might be affinely dependent in $\mathbb{R}^3$ so that vertices cannot be represented as an affine combination in this subset.

It is interesting to characterize shapes that allow such an affine independent representation, i.e. where each sufficiently large subset of vertices is a base of $\mathbb{R}^3$. This representation has numerous applications in shape recognition, free-form deformation, or morphing.

Handling of attributes Even if coordinates are morphed in a pleasing way, it is not clear what to do with attributes such as normals. For example, if normals describe a crease in one model but the corresponding edge in the other model is smooth, when has the crease to disappear?

It seems worthwhile to give a precise answer to all these detail questions. Morphing, in the end, is a very esthetic subject and failure in the details leads to displeasing results.

Robust implementation Morphable meshes resulting from the merging process contain a lot of sliver triangles, which make them particularly prone to fail in further processing. Also small topological failures in the input cause most morphing algorithms to fail.

What is needed is a freely available, robust implementation of mesh morphing techniques. It seems that to date none of the publicized mesh morphing techniques are implemented in public domain code. This would allow to experiment and improve the code using state-of-the-art geometric computation techniques such as arbitrary precision predicates.

Affine independence of shapes The approach presented here constructs spaces of meshes with one consistent topology, where the base meshes are described by vertex vector. Compound shapes are just linear combinations of the vertex vectors. For many applications it is necessary to have an affinely independent basis. This can be computed on the basis of the vertex vectors in our approach. However, the affine independence of vertex vectors does not guarantee the affine independence of shapes. Imagine a square with one interior vertex connected to the boundary vertices. Wherever the interior vertex is located the shape is the same. Yet, two different vertex configurations are affinely independent.

It would be interesting to code geometry in such a way that different affine independent geometry descriptors imply affine independent shapes. It seems the problem of absolute coordinates lies in the possible redundancy: If redundant information is stored a degree of freedom is added in the representation, which could lead to independent description of dependent shapes. This implies that topology should be exploited when defining geometry. However, the question how to do this is open.

Shape registration Eigenfaces are an early example of using a linear space for recognition of objects. We would like to see spaces of meshes to be used for recognition. Together with the above mentioned affinely independent description of geometry they represent a good candidate space which is easy to search.

Feature-oriented SVD The SVD used to compute the basis for a shape space so far takes into account only geometry. Sometimes, however, small geometrical features are of special importance and should get an extra weight. An example could be a human figure standing on the floor: The contact of human and floor is of special importance because the human protruding the floor or floating are recognized easily by a human observer.

An idea could be to deform the geometric space so that regions of particular interest get enlarged. Then a regular SVD could be used. In the example above, the space around the wall would be enlarged, so that differences in this area get larger.

References

ALEXA, M. 1999. Merging polyhedral shapes with scattered features. In *Proceedings of the International Conference on Shape Modeling and Applications (SMI-99)*, B. Werner, Ed. IEEE Computer Society, Los Alamitos, CA, 202–210.

ALEXA, M. 2000. Merging polyhedral shapes with scattered features. *The Visual Computer 16*, 1, 26–37. ISSN 0178-2789.

ALEXA, M. 2001a. Local control for mesh morphing. In *Proceedings of the International Conference on Shape Modeling and Applications (SMI-01)*, B. Werner, Ed. IEEE Computer Society, Los Alamitos, CA, 209–215.

ALEXA, M. 2001b. Mesh morphing star. *Eurographics 2001 State of The Art Reports*, 1–20. ISSN 1017-4656.

ALEXA, M. 2002a. Differential coordinates for mesh morphing and deformation. *The Visual Computer 18*. to be published.

ALEXA, M. 2002b. Recent advances in mesh morphing. *Computer Graphics Forum 21*, 2. to be published.

ALEXA, M., BEHR, J., AND MÜLLER, W. 2000. The morph node. *Web3D - VRML 2000 Proceedings*, 29–34. ISBN 1-58113-211-5.

ALEXA, M., BERNER, U., HELLENSCHMIDT, M., AND RIEGER, T. 2001. An animation system for user interface agents. In *WSCG 2001 Conference Proceedings*, V. Skala, Ed.

ALEXA, M., COHEN-OR, D., AND LEVIN, D. 2000. As-rigid-as-possible shape interpolation. *Proceedings of SIGGRAPH 2000*, 157–164. ISBN 1-58113-208-5.

ALEXA, M. AND MÜLLER, W. 1998a. The morphing space. GRIS-98-2, Technische Universität Darmstadt.

ALEXA, M. AND MÜLLER, W. 1998b. Visualization by metamorphosis. *Visualization '98 Late Breaking Hot Topics*.

ALEXA, M. AND MÜLLER, W. 1999a. The morphing space. *Seventh International Conference in Central Europe on Computer Graphics and Visualization (Winter School on Computer Graphics)*. ISBN 80-7082-490-5. Held in University of West Bohemia, Plzen, Czech Republic, 10-14 February 1999.

ALEXA, M. AND MÜLLER, W. 1999b. Visualization by examples: Mapping data to visual representations using few correspondences. *Joint EUROGRAPHICS - IEEE TCVG Symposium on Visualization*.

ALEXA, M. AND MÜLLER, W. 2000. Representing animations by principal components. *Computer Graphics Forum 19,* 3 (August), 411–418. ISSN 1067-7055.

ARAD, N. AND REISFELD, D. 1994. Image warping using few anchor points and radial functions. *Computer Graphics Forum 14,* 1 (January), 35–46.

ARONOV, B., SEIDEL, R., AND SOUVAINE, D. 1993. On compatible triangulations of simple polygons. *Computational Geometry: Theory and Applications 3*, 27–35.

BAO, H. AND PENG, Q. 1998. Interactive 3d morphing. *Computer Graphics Forum 17,* 3, 23–30. ISSN 1067-7055.

BEDDOW, J. 1990. Shape coding of multidimensional data on a microcomputer display. In *Proceedings of Visualization 90*. 238–246.

BEHR, J. 1999. AVALON - A VR/AR system using VRML as application description language. http://www.zgdv.de/
avalon.

BERTIN, J. 1983. *Semiology of Graphics*. The University of Wisconsin Press.

BLANZ, V. AND VETTER, T. 1999. A morphable model for the synthesis of 3d faces. *Proceedings of SIGGRAPH 99*, 187–194. ISBN 0-20148-560-5. Held in Los Angeles, California.

BOYER, R. AND SAVAGEAU, D. 1985. *Places Rated Almanac*. Rand McNally.

BRENT, R. P. 1973. *Algorithms for Minimization without Derivatives*. Prentice-Hall, Englewood Cliffs, N.J.

BRODLIE, K. W. 1993. A classification scheme for scientific visualisation. In *Animation and Scientific Visualisation: tools and applications*, R. A. Earnshaw and D. Watson, Eds. Academic Press, 125–140.

BUJA, A., MCDONALD, J. A., MICHALAK, J., AND STUETZLE, W. 1991. Interactive data visualization using focusing and linking. *Visualization '91*, 156–163.

CHAZELLE, B. 1990. Triangulating a simple polygon in linear time. *Proceedings of the 31st Annual Symposium on Foundations of Computer Science*, 220–230.

CHAZELLE, B. AND PALIOS, L. 1989. Triangulating a non-convex polytope. In *Proceedings of the 5th Annual Symposium on Computational Geometry (SCG '89)*, K. Mehlhorn, Ed. ACM Press, Saarbrücken, FRG, 393.

CHENG, H., EDELSBRUNNER, H., AND FU, P. 1998. Shape space from deformation. *Pacific Graphics '98*. Held in Singapore.

CHERNOFF, H. 1973. The use of faces to represent points in k-dimensional space graphically. *Journal of the American Statistical Association 68*, 361–368.

CLEVELAND, W. S. 1985. *The Elements of Graphing Data*. Wadsworth. ISBN 0-534-03730-5.

COHEN, S., ELBER, G., AND BAR-YEHUDA, R. 1997. Matching of freeform curves. *Computer-aided Design 29,* 5, 369–378.

COHEN-OR, D. AND CARMEL, E. 1998. Warp-guided object-space morphing. *The Visual Computer 13,* 9-10, 465–478. ISSN 0178-2789.

COHEN-OR, D., SOLOMOVICI, A., AND LEVIN, D. 1998. Three-dimensional distance field metamorphosis. *ACM Transactions on Graphics 17,* 2 (April), 116–141. ISSN 0730-0301.

DE BERG, M., VAN KREVELD, M., OVERMARS, M., AND SCHWARZKOPF, O. 1997. *Computational Geometry – Algorithms and Applications*. Springer-Verlag, Berlin Heidelberg.

DECARLO, D. AND GALLIER, J. 1996. Topological evolution of surfaces. *Graphics Interface '96*, 194–203. ISBN 0-9695338-5-3.

DEERING, M. AND SOWIZRAL, H. 1997. *Java3D Specification, Version 1.0*. Sun Microsystems, 2550 Garcia Avenue, Mountain View, CA 94043, USA.

DÖRNER, R., LUCKAS, V., AND SPIERLING, U. 1997. Ubiquitous animation - an element-based concept to make 3d animations commonplace. In *Viusal Procedings SIGGRAPH '97*. ACM Press.

DYN, N. 1989. Interpolation and approximation by radial and related functions. In *Approximation Theory VI*. Vol. 1. Academic Press, 211–234.

ECK, M., DEROSE, T., DUCHAMP, T., HOPPE, H., LOUNSBERY, M., AND STUETZLE, W. 1995. Multiresolution analysis of arbitrary meshes. *Proceedings of SIGGRAPH 95*, 173–182. ISBN 0-201-84776-0. Held in Los Angeles, California.

ECKSTEIN, I., SURAZHSKY, V., AND GOTSMAN, C. 2001. Texture mapping with hard constraints. *Computer Graphics Forum 20*, 3, 95–104. ISSN 1067-7055.

EDELSBRUNNER, H. 1999. Deformable smooth surface design. *Discrete and Computational Geometry 21,* 1 (Jan.), 87–115.

EKMAN, P. AND FRIESEN, W. V. 1978. *Facial Action Coding System (Investigator's Guide)*. Consulting Psychologists Press, Inc., Palo Alto, California, USA.

FINKE, U. AND HINRICHS, A. 1995. Overlaying simply connected planar subdivisions in linear time. In *Proceedings of the 11th Annual Symposium on Computational Geometry*. ACM Press, New York, NY, USA, 119–126.

FLOATER, M. S. 1997. Parametrization and smooth approximation of surface triangulations. *Computer Aided Geometric Design 14,* 3, 231–250. ISSN 0167-8396.

FLOATER, M. S. 2001. Convex combination maps. *Algorithms for Approximation IV*.

FLOATER, M. S. 2002. One-to-one piecewise linear mappings over triangulations. *Math. Comp. to appear*.

FLOATER, M. S. AND GOTSMAN, C. 1999. How to morph tilings injectively. *Journal of Computational and Applied Mathematics 101*, 117–129.

FREITAG, L., JONES, M., AND PLASSMANN, P. 1999. A parallel algorithm for mesh smoothing. *SIAM Journal on Scientific Computing 20,* 6 (Nov.), 2023–2040.

FUJIMURA, K. AND MAKAROV, M. 1998. Folder-free image warping. *Graphical Models and Image Processing 60,* 2 (March), 100–111.

GARLAND, M. AND HECKBERT, P. S. 1997. Surface simplification using quadric error metrics. *Proceedings of SIGGRAPH 97,* 209–216. ISBN 0-89791-896-7. Held in Los Angeles, California.

GOLUB, G. H. AND VAN LOAN, C. F. 1989. *Matrix Computations,* Second ed. Johns Hopkins Series in the Mathematical Sciences, vol. 3. The Johns Hopkins University Press, Baltimore, MD, USA. Second edition.

GOTSMAN, C. AND SURAZHSKY, V. 2001. Guaranteed intersection-free polygon morphing. *Computers & Graphics 25,* 1 (February), 67–75. ISSN 0097-8493.

GREGORY, A., STATE, A., LIN, M., MANOCHA, D., AND LIVINGSTON, M. 1998. Feature-based surface decomposition for correspondence and morphing between polyhedra. *Computer Animation '98*. Held in Philadelphia, Pennsylvania, USA.

GREGORY, A., STATE, A., LIN, M. C., MANOCHA, D., AND LIVINGSTON, M. A. 1999. Interactive surface decomposition for polyhedral morphing. *The Visual Computer 15,* 9, 453–470. ISSN 0178-2789.

GUMHOLD, S. 2000. Personal communication on embedding meshes.

HOPPE, H. 1996. Progressive meshes. *Proceedings of SIGGRAPH 96,* 99–108. ISBN 0-201-94800-1. Held in New Orleans, Louisiana.

HORMANN, K. AND GREINER, G. 2000. Mips: an efficient global parametrization method. In *Curve and Surface Design: Saint-Malo 1999*, P. S. P.-J. Laurent and L. L. Schumaker, Eds. Vanderbilt University Press, 153–162.

HORMANN, K., GREINER, G., AND CAMPAGNA, S. 1999. Hierarchical parametrization of triangulated surfaces. In *Vision, Modeling and Visualization '99,* B. Girod, H. Niemann, and H.-P. Seidel, Eds. infix, 219–226.

HUBELI, A. AND GROSS, M. 2001. Multiresolution feature extraction for unstructured meshes. In *IEEE Visualization 2001*. 287–294. ISBN 0-7803-7200-x.

ISO JTC124. 1997. VRML-97. ISO/IEC 14772-1.

ISO JTC/WG11. 1997. MPEG 4. ISO/IEC 14496-1, ISO/IEC 14496-2.

KANAI, T. AND SUZUKI, H. 2000. Approximate shortest path on a polyhedral surface based on selective refinement of the discrete graph and its applications. *Proc. Geometric Modeling and Processing 2000*, 241–250.

KANAI, T., SUZUKI, H., AND KIMURA, F. 1997. 3d geometric metamorphosis based on harmonic map. *Pacific Graphics '97*. Held in Seoul, Korea.

KANAI, T., SUZUKI, H., AND KIMURA, F. 1998. Three-dimensional geometric metamorphosis based on harmonic maps. *The Visual Computer 14,* 4, 166–176. ISSN 0178-2789.

KANAI, T., SUZUKI, H., AND KIMURA, F. 2000. Metamorphosis of arbitrary triangular meshes. *IEEE Computer Graphics & Applications 20,* 2 (March/April), 62–75. ISSN 0272-1716.

KARNI, Z. AND GOTSMAN, C. 2000. Spectral compression of mesh geometry. *Proceedings of SIGGRAPH 2000*, 279–286. ISBN 1-58113-208-5.

KARYPIS, G. AND KUMAR, V. 1998. Multilevel k-way hypergraph partitioning. Tech. Rep. 98–036, Department of Computer Science and Engineering, University of Minnesota, Minneapolis, MN 55455.

KENT, J., PARENT, R., AND CARLSON, W. E. 1991. Establishing correspondences by topological merging: A new approach to 3-d shape transformation. *Graphics Interface '91,* 271–278.

KENT, J. R., CARLSON, W. E., AND PARENT, R. E. 1992. Shape transformation for polyhedral objects. *Computer Graphics (Proceedings of SIGGRAPH 92) 26,* 2 (July), 47–54. ISBN 0-201-51585-7. Held in Chicago, Illinois.

KIRBY, M. AND SIROVICH, L. 1990. Application of the karjunen-loeve procedure for the characterization of human faces. *IEEE Transactions on Pattern Analysis and Machine Intelligence 12,* 1, 103–108.

KIRKPATRICK, S., GELLATT, C. D., AND VECCHI, M. P. 1983. Simulated annealing. *Science 220,* 671.

KLEIN, R. 1998. Multiresolution representations for surfaces meshes based on the vertex decimation method. *Computers & Graphics 22,* 1 (February), 13–26. ISSN 0097-8493.

KOBBELT, L. 2000. sqrt(3) subdivision. *Proceedings of SIGGRAPH 2000*, 103–112. ISBN 1-58113-208-5.

KOBBELT, L., CAMPAGNA, S., AND SEIDEL, H.-P. 1998. A general framework for mesh decimation. *Graphics Interface '98,* 43–50. ISBN 0-9695338-6-1.

LAZARUS, F. AND VERROUST, A. 1997. Metamorphosis of cylinder-like objects. *The Journal of Visualization and Computer Animation 8,* 3, 131–146. ISSN 1049-8907.

LAZARUS, F. AND VERROUST, A. 1998. Three-dimensional metamorphosis: a survey. *The Visual Computer 14,* 8-9, 373–389. ISSN 0178-2789.

LEE, A., DOBKIN, D., SWELDENS, W., AND SCHRÖDER, P. 1999. Multiresolution mesh morphing. *Proceedings of SIGGRAPH 99,* 343–350. ISBN 0-20148-560-5. Held in Los Angeles, California.

LEE, A. W. F., SWELDENS, W., SCHRÖDER, P., COWSAR, L., AND DOBKIN, D. 1998. Maps: Multiresolution adaptive parameterization of surfaces. *Proceedings of SIGGRAPH 98*, 95–104. ISBN 0-89791-999-8. Held in Orlando, Florida.

LEE, S.-Y., CHWA, K.-Y., SHIN, S. Y., AND WOLBERG, G. 1995. Image metamorphosis using snakes and free-form deformations. *Proceedings of SIGGRAPH 95*, 439–448. ISBN 0-201-84776-0. Held in Los Angeles, California.

LENGYEL, J. E. 1999. Compression of time-dependent geometry. *1999 ACM Symposium on Interactive 3D Graphics*, 89–96. ISBN 1-58113-082-1.

LÉVY, B. 2001. Constrained texture mapping for polygonal meshes. In *Proceedings of ACM SIGGRAPH 2001*. Computer Graphics Proceedings, Annual Conference Series. ACM Press / ACM SIGGRAPH, 417–424. ISBN 1-58113-292-1.

LÉVY, B. AND MALLET, J.-L. 1998. Non-distorted texture mapping for sheared triangulated meshes. *Proceedings of SIGGRAPH 98,* 343–352. ISBN 0-89791-999-8. Held in Orlando, Florida.

LINDSTROM, P. AND TURK, G. 1998. Fast and memory efficient polygonal simplification. *IEEE Visualization '98,* 279–286. ISBN 0-8186-9176-X.

LOOP, C. AND DEROSE, T. 1990. Generalized b-spline surfaces of arbitrary topology. *Computer Graphics (Proceedings of SIGGRAPH 90) 24,* 4 (August), 347–356. ISBN 0-201-50933-4. Held in Dallas, Texas.

MAGNENAT-THALMANN, N., PRIMEAU, E., AND THALMANN, D. 1988. Abstract muscle action procedures for human face animation. *The Visual Computer 3,* 5, 290–297.

MALGOUYRES, R. AND LENOIR, A. 2000. Topology preservation within digital surfaces. *Graphical Models 62,* 2 (March), 71–84. ISSN 1524-0703.

MICHIKAWA, T., KANAI, T., FUJITA, M., AND CHIYOKURA, H. 2001. Multiresolution interpolation meshes. In *9th Pacific Conference on Computer Graphics and Applications*. IEEE, 60–69. ISBN 0-7695-1227-5.

MULLER, D. E. AND PREPARATA, F. P. 1978. Finding the intersection of two convex polyhedra. *Theoretical Computer Science 7,* 2, 217–236.

MÜLLER, W. AND ALEXA, M. 1998. Using morphing for information visualization. *Workshop on New Paradigms in Information Visualization and Manipulation (NPIV '98)*, 76–79.

MÜLLER, W., ALEXA, M., RIEGER, T., AND BRAUN, N. 2000. Ein flexibles Präsentationssystem für User-Interface-Agenten. *Workshop Digital Storytelling (DISTEL)*, 163–175. ISBN 3-8167-5566-6.

NGO, T., CUTRELL, D., DANA, J., DONALD, B., LOEB, L., AND ZHU, S. 2000. Accessible animation and customizable graphics via simplicial configuration modeling. *Proceedings of SIGGRAPH 2000*, 403–410. ISBN 1-58113-208-5.

OSTERMANN, J. 1998. Animation of synthetic faces in mpeg-4. *Computer Animation '98*. Held in Philadelphia, Pennsylvania, USA.

PARKE, F. I. 1979. Computer graphic models for the human face. *Proc. COMPSAC, The IEEE Computer Society's Third International Computer Software and Applications Conference*.

PARKE, F. I. 1982. Parameterized models for facial animation. *IEEE Computer Graphics & Applications 2*, 61–68.

PFISTER, H., ZWICKER, M., VAN BAAR, J., AND GROSS, M. 2000. Surfels: Surface elements as rendering primitives. *Proceedings of SIGGRAPH 2000*, 335–342. ISBN 1-58113-208-5.

PICKETT, R. M. AND GRINSTEIN, G. G. 1988. Iconographics display for visualizing multidimensional data. *Proceedings of IEEE Conference on Systems, Man, and Cybernetics*, 514–519.

PINKALL, U. AND POLTHIER, K. 1993. Computing discrete minimal surfaces and their conjugates. *Experimanetal Mathematics 2*, 1, 15–36.

POLTHIER, K. 2000. Conjugate harmonic maps and minimal surfaces. Tech. Rep. Preprint No. 446, TU Berlin, SFB 288.

PRAUN, E., SWELDENS, W., AND SCHRÖDER, P. 2001. Consistent mesh parameterizations. *Proceedings of SIGGRAPH 2001*, 179–184. ISBN 1-58113-292-1.

PREPARATA, F. P. AND SHAMOS, M. I. 1985. *Computational Geometry: An Introduction*. Texts and Monographs in Computer Science. Springer-Verlag, Berlin, Germany.

PRESS, W. H., TEUKOLSKY, S. A., VETTERLING, W. T., AND FLANNERY, B. P. 1992. *Numerical Recipes in C: The Art of Scientific Computing (2nd ed.)*. Cambridge University Press, Cambridge. ISBN 0-521-43108-5.

RHEINGANS, P. 1992. Color, change, and control for quantitative data display. In *Visualization '92*. 252–259.

RUPRECHT, D. AND MULLER, H. 1995. Image warping with scattered data interpolation. *IEEE Computer Graphics & Applications 15,* 2 (March), 37–43.

SCHROEDER, W. J. 1997. A topology modifying progressive decimation algorithm. *IEEE Visualization '97*, 205–212. ISBN 0-58113-011-2.

SCHROEDER, W. J., ZARGE, J. A., AND LORENSEN, W. E. 1992. Decimation of triangle meshes. *Computer Graphics (Proceedings of SIGGRAPH 92) 26,* 2 (July), 65–70. ISBN 0-201-51585-7. Held in Chicago, Illinois.

SEDERBERG, T. W., GAO, P., WANG, G., AND MU, H. 1993. 2d shape blending: An intrinsic solution to the vertex path problem. *Proceedings of SIGGRAPH 93*, 15–18. ISBN 0-201-58889-7. Held in Anaheim, California.

SEDERBERG, T. W. AND GREENWOOD, E. 1992. A physically based approach to 2d shape blending. *Computer Graphics (Proceedings of SIGGRAPH 92) 26,* 2 (July), 25–34. ISBN 0-201-51585-7. Held in Chicago, Illinois.

SHAPIRA, M. AND RAPPOPORT, A. 1995. Shape blending using the star-skeleton representation. *IEEE Computer Graphics & Applications 15,* 2 (March), 44–50.

SHAPIRO, A. AND TAL, A. 1998. Polyhedron realization for shape transformation. *The Visual Computer 14,* 8-9, 429–444. ISSN 0178-2789.

SHOEMAKE, K. AND DUFF, T. 1992. Matrix animation and polar decomposition. *Graphics Interface '92*, 258–264.

SPANIER, E. H. 1966. *Algebraic Topology*. McGraw-Hill, New York.

SUN, Y. M., WANG, W., AND CHIN, F. Y. L. 1997. Interpolating polyhedral models using intrinsic shape parameters. *The Journal of Visualization and Computer Animation 8,* 2 (April-June), 81–96. ISSN 1049-8907.

SURAZHSKY, T. AND ELBER, G. 2001. Matching free form surfaces. *Computers & Graphics 26,* 1, ??–?? ISSN 0097-8493.

TAL, A. AND ELBER, G. 1999. Image morphing with feature preserving texture. *Computer Graphics Forum 18,* 3 (September), 339–348. ISSN 1067-7055.

TAUBIN, G. 1995. A signal processing approach to fair surface design. *Proceedings of SIGGRAPH 95*, 351–358. ISBN 0-201-84776-0. Held in Los Angeles, California.

TUFTE, E. R. 1983. *The Visual Display of Quantitative Information*. Graphics Press, Cheshire, Connecticut.

TURK, M. AND PENTLAND, A. 1991. Eigenfaces for recognition. *Journal of Cognitive Neuro Science 3,* 1, 71–86.

TUTTE, W. T. 1963. How to draw a graph. *Proc. London Mathematical Society 13*, 743–768.

WATERS, K. 1987. A muscle model for animating three-dimensional facial expression. *Computer Graphics (Proceedings of SIGGRAPH 87) 21,* 4 (July), 17–24. Held in Anaheim, California.

WEB3D CONSORTIUM. 1999a. H-Anim. *http://ece.uwaterloo.ca:80/˜h-anim*.

WEB3D CONSORTIUM. 1999b. X3D. http://www.web3d.org/x3d.

WOLBERG, G. 1998. Image morphing: a survey. *The Visual Computer 14,* 8-9, 360–372. ISSN 0178-2789.

ZHANG, Y. 1996. A fuzzy approach to digital image warping. *IEEE Computer Graphics & Applications 16,* 4 (July), 34–41. ISSN 0272-1716.

ZIGELMANN, G., KIMMEL, R., AND KIRYATI, N. 2002. Texture mapping using surface flattening vie multi-dimensional scaling. *IEEE Transactions on Visualization and Computer Graphics to appear*.

ZÖCKLER, M., STALLING, D., AND HEGE, H.-C. 2000. Fast and intuitive generation of geometric shape transitions. *The Visual Computer 16,* 5, 241–253. ISSN 0178-2789.

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