Extraction of the ANC from the ^{10}Be$(d,p)^{11}$Be transfer reaction using the ADWA method

J.Yang[1,2], **P.Capel**[1,3]

[1]Physique Nucléaire et Physique Quantique, Université Libre de Bruxelles, C.P.229, B-1050 Bruxelles, Belgium
[2]Afdeling Kern-en Stralingsfysica, Celestijnenlaan 200d-bus 2418, 3001 Leuven, Belgium
[3]Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany

E-mail: `jiecyang@ulb.ac.be`

Abstract. A halo nucleus is built from a core and at least one weakly bound neutron or proton. To understand this unique cluster structure, lots of efforts have been undertaken. During the past decades, the (d,p) reaction has been widely used in experiments and has become an important tool for extracting single-particle properties of nuclei. In this work, our goal is to obtain the Asymptotic Normalization Coefficient (ANC) of the halo nuclei ^{11}Be using the ADWA method. We perform the analysis for the ^{10}Be$(d,p)^{11}$Be stripping reaction at E_d=21.4, 18, 15, and 12MeV for the ground state and first excited state of the composite nucleus ^{11}Be. The experimental measurement was carried out at Oak Ridge National Laboratory by Schmitt *et al.*[1] The sensitivity of the calculations to the optical potential choice is also checked. Overall, the transfer process becomes more peripheral at lower energies and forward angles. Investigation in this area is the best way to extract a reliable ANC from the experimental data. For the ground state of ^{11}Be, the ANC obtained using our method (C=0.785$^{+0.029}_{-0.030}$ fm$^{-1/2}$) shows perfect agreement with the one obtained by Ab initio calculations (C_{Ab}=0.786 fm$^{-1/2}$) [2].

1. Introduction

Halo nuclei [3] constitute a unique class of exotic systems, which are mostly found in the neutron-rich region of the nuclear chart and exhibit a much larger matter radius than the stable nuclei do. The halo is a threshold effect arising for some weakly bound nuclei. Since the discovery of the first halo nucleus ^{6}He in 1986 [4], great interest has been put in the study of the halo phenomenon in nuclei both experimentally and theoretically. Up to now, the extraction of accurate structure information of halo nuclei remains as a big challenge due to their very short lifetime, which requires unusual experimental methods and accurate theoretical analyses.

Experimentally, the upgrade of rare isotope beam facilities worldwide provides us with new ways to explore these halo systems. As one of the most classical transfer reaction [5], (d,p) reaction plays an important role in extracting single-particle properties of nuclei with one neutron populating the unoccupied state of the target [6].

On the theoretical side, many reaction theories have been developed to study deuteron-induced reactions. The Distorted Wave Born Approximation (DWBA) [7] is one of the representative methods and received considerable success in describing transfer reactions and extracting spectroscopic information on nuclear structure. However, since this method does not present an adequate treatment of the exact three-body interaction, such as the effects caused

by the breakup of the deuteron, it is desirable to develop an alternative formulation, which does contain the dominant contributions from all the channels without losing the relative simplicity of the DWBA method. Johnson and Soper initially introduced an zero-range adiabatic prescription (ZR-ADWA) [8] to efficiently take into account the contributions from the breakup channels in transfer processes. Following this work, a finite-range version of the ADWA method (FR-ADWA) was proposed by Johnson and Tandy [9]. The accuracy and effective range of ADWA has been studied by direct comparison with the continuum-discretized coupled channels (CDCC) method and exact Faddeev-AGS calculations (FAGS) [10, 11]. For transfer cross sections at low energy ($E_d \approx 20$MeV), there is a remarkable agreement among the three models. Moreover, when the angular momentum transfer is small and the neutron-nucleus system is loosely bound, ADWA is a good candidate to predict the transfer cross sections.

In the present work, we are interested in studying the nucleus ^{11}Be. It is well known for its distinct $n+^{10}$Be halo structure [12]. With the aim to understand the halo nature of ^{11}Be, many different experimental methods (e.g. β decay [13], breakup [14], neutron knockout [15] and transfer reactions [1]) have already been carried out. Meanwhile, there has also been considerable effort put forward on the theoretical side to describe the structure and help explain the reactions involving ^{11}Be. Belyaeva *et al.* extracted several representative parameters [16] about the formation of neutron halo in the ground state as well as the excited state of this nuclei within the Coupled Reaction Channels (CRC) calculations. Recently, Ab initio calculations [2] got improved by including the three-nucleon forces and the ^{10}Be-n continuum effects when studying this nucleus. Thanks to these pioneering investigations, now we are able to make some attempts to check the structure information of ^{11}Be. The main purpose of this work is to obtain the ANC by comparing the experimental data with theoretical results, and to find out if our method could increase the reliability of such obtained values. We are going to look at the experimental data from Schmitt *et al.*'s work [1]. In their experiment, the ^{10}Be(d,p) reaction has been used in inverse kinematics at four beam energies to study the structure of ^{11}Be.

The structure of this paper is as follows: In Sec.2, we briefly present the ADWA theoretical framework. In Sec.3, we introduce the numerical inputs and different descriptions of the n-^{10}Be bound state. Finally we show the calculation results and have some discussions in Sec.4. The conclusion is given in Sec.5.

2. Theoretical Framework

Considering a stripping reaction of the form $A(d,p)B$, we adopt the three-body model to help build the theoretical framework. In a simple physical picture, this transfer reaction $A+d \rightarrow p+B$ can be viewed as a process in which the valence neutron n from the incident deuteron d populates an unoccupied state in the target nucleus A, producing a composite nucleus $B = n + A$. The proton, neutron and the core A form a three-body system. Within the three-body model, the interactions between them can be described by two-body potentials V_{np}, V_{nA} and U_{pA} during the collision process. U means the optical potential is complex in general. The transfer matrix elements of the (d,p) stripping reaction can be expressed in the post form as [5]

$$T_{post}(pB, dA) = \langle \chi^{(-)}_{pB} \varphi_{nA} | V_{pn} + U_{pA} - U_{pB} | \psi^{(+)}_{dA} \rangle, \qquad (1)$$

where $\chi^{(-)}_{pB}$ is the distorted wave function generated by the potential U_{pB} which is responsible for reproducing the elastic scattering on the p-B channel, φ_{nA} represents the wave function of the n-A bound state, and $\psi^{(+)}_{dA}$ corresponds to the exact wave function of this deuteron-target system. For the interaction terms, V_{pn}, the binding potential of deuteron, is the dominating term, and the rest part ($U_{pA} - U_{pB}$) is considered as the remnant term.

The primary concern here is the exact solution of the function $\psi^{(+)}_{dA}$ with respect to the three-body system. Since the DWBA method does not include the breakup component of the wave

function, we chose to use the adiabatic prescription that was initially introduced by Johnson and Soper [8] to take into account the contributions from the breakup channels in transfer processes. In the ADWA model, the three-body wave function is approximated by

$$\psi_{dA}^{(+)}(\boldsymbol{r'}, \boldsymbol{R}) \simeq \chi_{dA}^{(+)}(\boldsymbol{r'}, \boldsymbol{R})\varphi_{pn}(\boldsymbol{r'}), \qquad (2)$$

where $\chi_{dA}^{(+)}$ is the deuteron-target scattering wave function. It depends on the relative coordinates $\vec{r'}$ between the neutron and the proton and $\vec{R}$ between the n-p-centre of mass and the target A. φ_{pn} is the deuteron wave function. Once the transfer amplitude $T_{post}(pB, dA)$ is obtained, we can calculate the differential cross section

$$\left(\frac{d\sigma}{d\Omega}\right)^{th} = \frac{\mu_{dA}\mu_{pB}}{(2\pi\hbar^2)^2}\frac{k_p}{k_d}|T_{post}(pB, dA)|^2, \qquad (3)$$

in which μ_{iX} and k_i are the reduced mass and wave number for the relative motion between i and X in each of the two channels dA (elastic entrance channel) and pB (transfer channel).

Referring to the wave function for the relative motion between n and A that forms B in Eq. 1, let us consider a specific nucleus B that has a strong single-particle character. In this case, the valence neutron orbit has a well-defined asymptotic behavior at large distances where nuclear forces are vanishingly small [17],

$$\varphi_{nA}(r) \xrightarrow{r\to\infty} b_{n_r lj}\frac{W_{-\eta,\, l+\frac{1}{2}}(2\kappa r)}{r}, \qquad (4)$$

in which $b_{n_r lj}$ is the single-particle ANC (SPANC) defining the strength of the exponential tail of the n-A bound-state wave function, W is the Whittaker function, η is the Sommerfeld parameter for the bound state, and κ is the wave number. Note in order to well simulate what has been observed in experiments from the microscopic calculations, the bound-state wave function φ_{nA} in Eq. 1 should be replaced by the overlap function I_{nA}^{B}, which will take into account nuclear structure effects realistically. The overlap integrals are not longer normalized to unity, but to the spectroscopic factors (SF) $S_{n_r lj}$ while experiencing a similar asymptotic behavior [6]. In the single-particle approximation, the ANC and SFANC are related by $C_{lj}^2 = S_{n_r lj}b_{n_r lj}^2$. When the reaction is completely peripheral, the transfer differential cross sections should only be sensitive to the value of ANC without retaining information on the SF.

3. Transfer calculation

3.1. Numerical inputs

For the $^{10}Be(d,p)^{11}Be$ transfer reaction, the equivalent deuteron incident energy E_d is taken to be 21.4, 18, 15, and 12MeV refer to the experiment of Schmitt *et al.*[1] In our calculations, a finite-range version of the adiabatic potential developed by Johnson and Tandy is used for transfers from deuteron [18]. The nucleon-nucleus optical potentials are obtained from the global parametrization CH89 [19] without including the spin-orbit terms. Using ADWA approach, $U_{p-^{10}Be}$ and $U_{n-^{10}Be}$ are calculated at half the incident energy $E_d/2$, while the auxiliary potential $U_{p-^{11}Be}$ is obtained at the proton energy corresponding to the exit channel. For the p-n interaction, the Reid soft-core interaction [20] is chosen to get the appropriate wave function of the deuteron. All transfer calculations in this study were performed with FRESCO [21], and the adiabatic potentials are calculated using the front-end code of TWOFNR [22].

3.2. Description of ^{11}Be

As mentioned, ^{11}Be can be modeled as a neutron loosely bound to a ^{10}Be core. With the assumption that the ^{10}Be core is in its ground state (0^+), the $1/2^+$ ground state (g.s) of ^{11}Be

can be described by a $2s_{1/2} \otimes {}^{10}\text{Be}(0^+)$ configuration, and the $1/2^-$ excited state (ex.s) by a $1p_{1/2} \otimes {}^{10}\text{Be}(0^+)$ configuration. In this study, we adopt Gaussian potentials to describe the interaction between ${}^{10}\text{Be}$ and the valence neutron. The potential is defined as $V(r) = V_0 e^{-r^2/2r_0^2}$. We test nine sets of Gaussian potentials with different widths r_0 starting from 0.4fm to 2.0fm. The depth V_0 is adjusted based on the neutron binding energy (0.502MeV for the g.s; 0.182MeV for the ex.s). They are all listed in Tables 1 with the corresponding SPANCs $(b_{2s_{1/2}}, b_{1p_{1/2}})$ provided. Using such potentials, we can calculate the wave function of the n-${}^{10}\text{Be}$ bound state.

Table 1. Gaussian parameters of the n-${}^{10}\text{Be}$ potentials. The potential depths and SPANCs of the $2s1/2$ ground state $b_{2s1/2}$ and the $1p1/2$ first excited state $b_{1p1/2}$ are shown as well.

Potential	r_0 (fm)	$V_0(g.s)$ (MeV)	$b_{2s1/2}$ (fm$^{-1/2}$)	$V_0(ex.s)$ (MeV)	$b_{1p1/2}$ (fm$^{-1/2}$)
V_1	0.4	1314.6	0.601	869.4	0.068
V_2	0.6	592.3	0.632	387.3	0.085
V_3	0.8	337.8	0.664	218.4	0.100
V_4	1.0	219.2	0.697	140.2	0.114
V_5	1.2	154.4	0.732	97.7	0.127
V_6	1.4	115.1	0.769	72.1	0.140
V_7	1.6	89.3	0.807	55.4	0.152
V_8	1.8	71.6	0.846	44.0	0.165
V_9	2.0	58.8	0.888	35.8	0.177

The results of the g.s and the ex.s wave functions are shown in Fig. 1. According to the definition of SPANC in Eq.(4), all the wave functions shown here will exhibit a same tail after scaled by their SPANCs. This phenomenon can help us define the confidence interval to extract ANC.
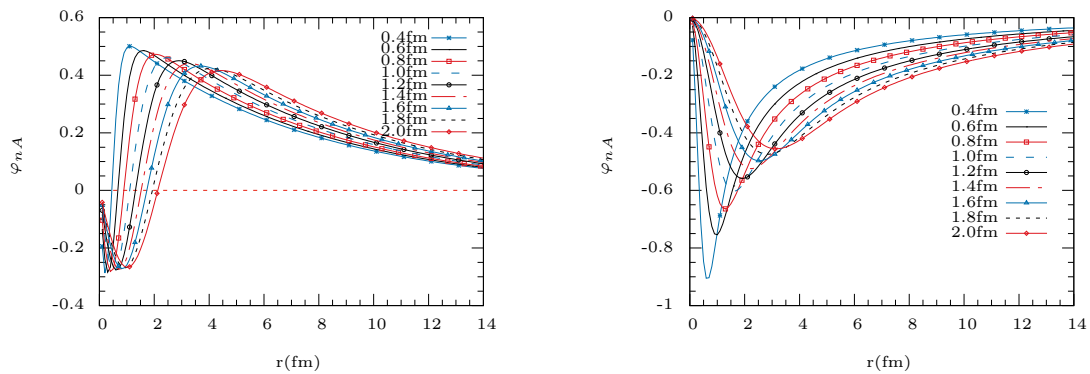

Figure 1. (Color online). Wave functions φ_{nA} obtained with nine different sets of Gaussian potentials listed in Tables 1 for the $1/2^+$ g.s of ${}^{11}\text{Be}$ (left) and for the $1/2^-$ excited state (right).

4. Results and Discussion

In this section, a series of calculation results is presented. Here we take two cases at different energies to clarify our analysis work and findings. On the left side of Fig.2, the top plot (a) shows the angular distribution of the cross sections calculated at 21.4MeV. With narrower

Gaussian potentials, the probability of occurrence of the reaction gradually decreases. At forward angles, there exist big discrepancies among the calculated cross sections with different potentials. Dividing the cross sections by the square of the SPANC, we obtain plot (b). From (a) to (b), these discrepancies are much reduced. Most curves begin to get on top of each other in the forward region except those of r_0=0.4fm, and 0.6fm. In order to gain quantitative insight into the variation between different distributions after scaling, we compare them taking the calculation at r_0=1.4fm as a reference:

$$\mathcal{R}_{r_0/1.4fm} = \frac{(\frac{d\sigma}{d\Omega})^{th}_{r_0}/(b_{n_r l j})^2_{r_0}}{(\frac{d\sigma}{d\Omega})^{th}_{1.4fm}/(b_{n_r l j})^2_{1.4fm}} - 1 \qquad (5)$$

in which the index r_0 corresponds to a set of Gaussian potential used for calculation. The results are presented in plot (c). Then we choose the empirical value $\pm 5\%$ set as the tolerance band to define the peripheral area of the reaction. One sees from this plot that only a small angular region meets our peripheral definition except the variations with respect to 0.4fm, and 0.6fm. For the ^{10}Be$(d,p)^{11}$Be(g.s) reaction at 21.4MeV, we have $\theta_{c.m.}$=0°–7° extracted as the angular range where the $b^2_{n_r l j}$ scaling works.

Coming to the right side of Fig.2, the reaction is carried out at E_d=12MeV. All the plots are organized following the same logical structure shown for the case of 21.4MeV. Through comparison between the results at two different energies, we can find that $b^2_{n_r l j}$ scaling works better at 12MeV, leading to a wider and more consistent angular region. The peripheral region in this case is $\theta_{c.m.}$=0°–17°. Similar analyses are also made at 18MeV and 15MeV, which are not presented here, to check the variation of the angular agreement range with the deuteron energy. The peripheral part is 0°–10° for 18MeV, and 0°–20° for 15MeV without considering the results of 0.4fm, and 0.6fm. After investigating all the results, we have two basic conclusions: a)the peripheral area of this transfer reaction is always found at forward angles; b)when the incident energy decreases, the reaction exhibits a more pronounced peripheral property.

4.1. ANC Extraction of the n-^{10}Be bound state

With the peripheral information obtained in the last section, we can extract the ANCs by performing the χ^2 analysis for the scaling of the theoretical results to the experimental ones. The related results are shown in Fig. 3. It can be seen that the ANC extraction is more reliable at lower energy for both cases since the reaction becomes more peripheral there. For 21.4MeV, we don't observe a plateau since at this energy the reaction is less peripheral and the interior part of the bound-state wave function plays a non-negligible role. Therefore this is not an ideal sample for ANC extraction. There is a problem with the results at 18MeV which are always smaller than the others. It has been seen in the analysis of Schmitt *et al.* [1] as well. However, the most reliable sets of data at 15MeV, and 12MeV lead to consistent results. The value of the ANC we extract for ^{11}Be is $0.785^{+0.029}_{-0.030}$ fm$^{-1/2}$ (C_{Ab}=0.786 fm$^{-1/2}$) for the g.s and $0.136^{+0.005}_{-0.005}$ fm$^{-1/2}$ (C_{Ab}=0.129 fm$^{-1/2}$) for the ex.s. In general, our results are in agreement with the ANC predicted from an Ab initio calculation [2] and also close to that found by Belyaeva *et al.* [16] with the CRC model. Moreover, the extracted ANC reproduces a good agreement at forward angles between theoretical and experimental results despite the 'always' overestimation of the calculations at 18MeV. Due to the fact that this analysis relies a lot on the accuracy of the experimental data, the conclusion of this analysis is that experimenters should focus on low energies and forward angles to extract precise ANCs.

4.2. The sensitivity to the optical potential choice

On the other hand, the angular distributions of cross sections calculated with ADWA model can depend strongly on the optical potential parameters. In this section, we would like to check the
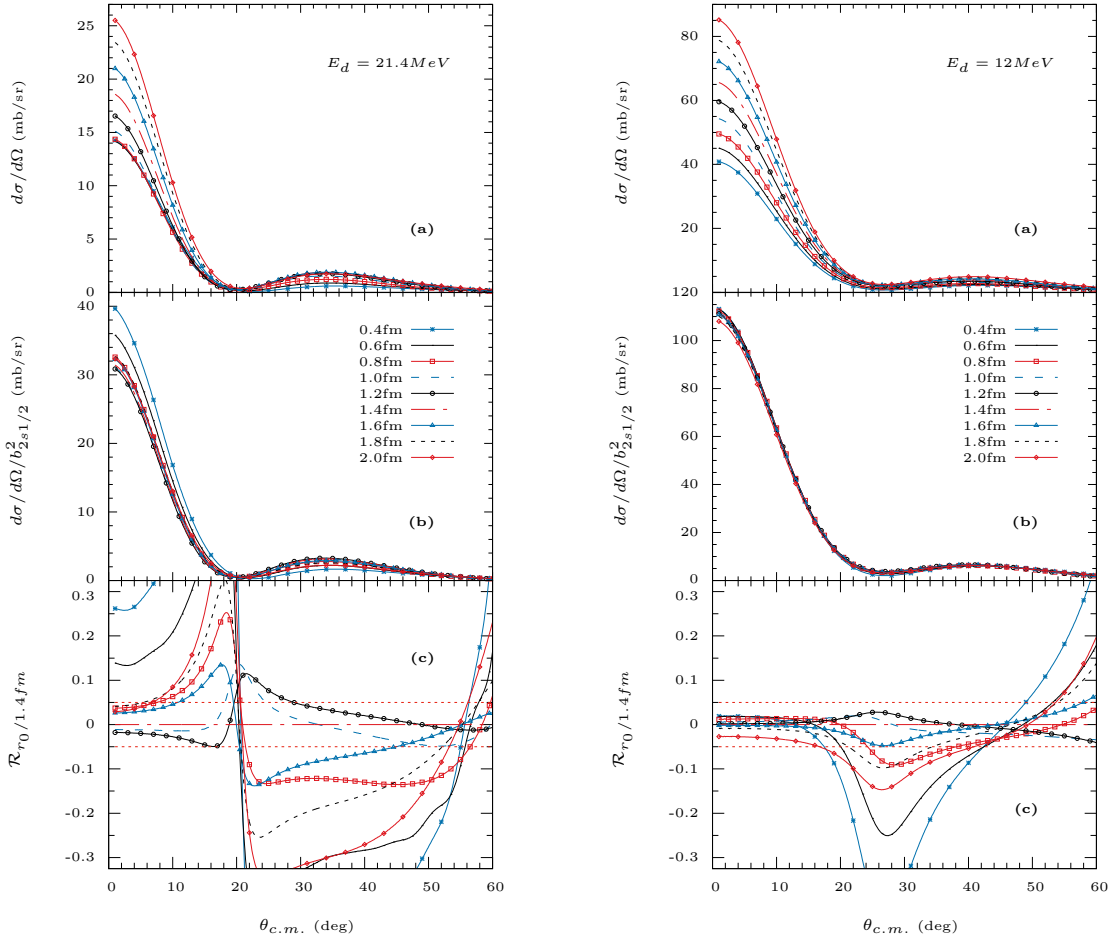
Figure 2. (Color online). Analysis of the cross sections of ^{10}Be$(d,p)^{11}$Be to the ground state of ^{11}Be with incoming deuteron energy of 21.4 MeV (left side) and 12MeV (right side).

influence of the nucleon-nucleus optical potentials on the calculations. In particular, we choose Koning & Delaroche potential (KD) [23] instead of CH89 to perform this series of analyses. Since the cross sections calculated with KD potentials lead systematically to larger cross sections than those with CH89, we can expect a smaller ANC obtained using the same method. The value of the extracted ANC is $0.755^{+0.028}_{-0.029}$ fm$^{-1/2}$ with a 3.9% deviation from the Ab initio one.

5. Conclusion

An analysis of the theoretical results of Schmitt *et al.*'s experiment has been performed with the aim to extract the ANC of the halo nucleus ^{11}Be using the ADWA method in this paper. Using Gaussian potentials of different widths to generate the wave functions of the $n+^{10}$Be bound states, we are able to define a peripheral area where the SPANC^2 scaling works well. By comparing between theoretical results at four energies, it is noticed that transfer reaction becomes more peripheral at lower energies and forward angles. Based on this knowledge, we proposed a systematic analysis to extract ANCs from experimental data in this region. The ANC extracted for the ground state of ^{11}Be (C_{lj}=$0.785^{+0.029}_{-0.030}$ fm$^{-1/2}$) is in excellent agreement with the value predicted by Ab initio calculations (C'_{lj}=0.786 fm$^{-1/2}$). The sensitivity of the calculations to the optical potential choice is also checked.
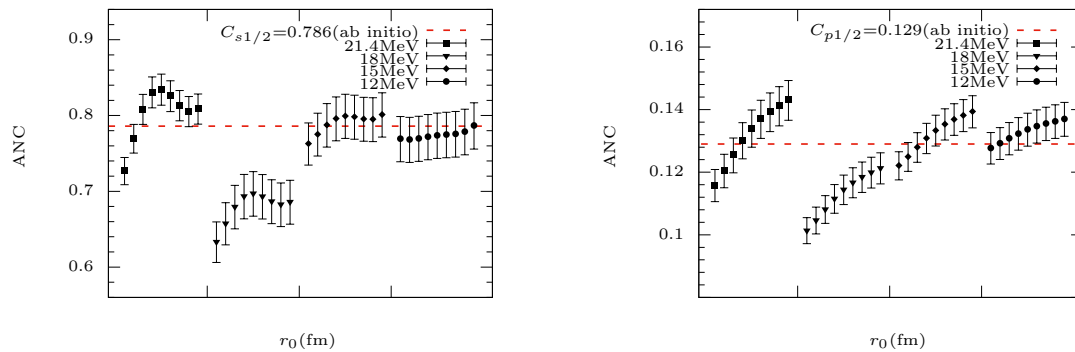
Figure 3. ANC extracted for the g.s (left side) and ex.s (right side) of ^{11}Be. The Ab initio result (C_{Ab}=0.786 fm$^{-1/2}$ for g.s or 0.129 fm$^{-1/2}$ for ex.s) [2] for comparison is presented by a red dashed line. From left to right in each grid of the horizontal axis, the data points start with the ANC extracted for r_0=0.4fm and end with that for r_0=2.0fm.

Overall, our study indicates that investigation at lower energies and forward angles for transfer reaction can ensure us the peripherality of the reaction and is the best way to obtain a reliable ANC from experimental data. Our method also suggests a good practice for this purpose.

References
[1] Schmitt K T, Jones K L, Bey A *et al.* 2012 *Phys. Rev. Lett.* **108**(19) 192701
[2] Calci A, Navrátil P, Roth R *et al.* 2016 *Phys. Rev. Lett.* **117**(24) 242501
[3] Al-Khalili J 2004 *An Introduction to Halo Nuclei* (Berlin, Heidelberg: Springer Berlin Heidelberg) pp 77–112
[4] Tanihata I, Hamagaki H, Hashimoto O *et al.* 1985 *Physics Letters B* **160** 380 – 384
[5] Camacho J G and Moro A M 2014 *The Euroschool on Exotic Beams* **Vol. IV** 39–66
[6] Thompson I J and Nunes F M 2009 *Nuclear Reactions for Astrophysics, by Ian J. Thompson, Filomena M. Nunes, Cambridge, UK: Cambridge University Press, 2009*
[7] Satchler G 1964 *Nuclear Physics* **55** 1 – 33
[8] Johnson R C and Soper P J R 1970 *Phys. Rev. C* **1**(3) 976–990
[9] Johnson R and Tandy P 1974 *Nuclear Physics A* **235** 56 – 74
[10] Nunes F M and Deltuva A 2011 *Phys. Rev. C* **84**(3) 034607
[11] Upadhyay N J, Deltuva A and Nunes F M 2012 *Phys. Rev. C* **85**(5) 054621
[12] Tanihata I, Kobayashi T, Yamakawa O *et al.* 1988 *Physics Letters B* **206** 592 – 596
[13] Madurga M, Borge M, Alcorta M *et al.* 2009 *Physics Letters B* **677** 255 – 259
[14] Fukuda N, Nakamura T, Aoi N *et al.* 2004 *Phys. Rev. C* **70**(5) 054606
[15] Aumann T, Navin A, Balamuth D P *et al.* 2000 *Phys. Rev. Lett.* **84**(1) 35–38
[16] Belyaeva T L, Perez-Torres R, Ogloblin A A *et al.* 2014 *Phys. Rev. C* **90**(6) 064610
[17] Pang D Y, Nunes F M and Mukhamedzhanov A M 2007 *Phys. Rev. C* **75**(2) 024601
[18] Wales G and Johnson R 1976 *Nuclear Physics A* **274** 168 – 176
[19] Varner R, Thompson W, McAbee T, Ludwig E and Clegg T 1991 *Physics Reports* **201** 57 – 119
[20] Reid R V 1968 *Annals of Physics* **50** 411 – 448
[21] Thompson I J 1988 *Computer Physics Reports* **7** 167 – 212
[22] Igarashi M and Toyama M 2008 *computer program TWOFNR, University of Surrey version*
[23] Koning A and Delaroche J 2003 *Nuclear Physics A* **713** 231 – 310