



**THE INFLUENCE OF REYNOLDS NUMBER AND  
ROUGHNESS ON THE EFFICIENCY OF AXIAL AND  
CENTRIFUGAL FANS –  
A PHYSICALLY BASED SCALING METHOD**

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**SUMMARY**

To scale model test data there is a technical and economical need for a correction method, which fulfills five tasks: It should be (i) physically based, (ii) understandable and easy to apply, (iii) universal, i.e. applicable to centrifugal as well as to axial machines of different specific speed. More over the method should (iv) account for the aerodynamic quality of the machine and should (v) be reliable not only at the point of peak efficiency but also at off peak condition. Up to now, no method meets all five tasks. To fill that gap, a method developed at TU Darmstadt together with FLT (Forschungsvereinigung für Luft- und Trocknungstechnik e. V.) is introduced, validated, and critically discussed in comparison with other methods.

**INTRODUCTION**

More and more, the value and acceptance of a fan is determined not only by pressure characteristics but also by acoustic characteristics, erosion resistance and most important the efficiency of the fan.

Pressure rise and efficiency of a turbomachine changes if the following physical quantities are varied:

- machine size, given by the impeller diameter  $D$ ,
- rotational speed  $\pi n$ ,
- kinematic viscosity  $\nu$ ,
- density  $\rho$ ,
- compressibility measured by the speed of sound  $a$ ,
- typical roughness height  $K$ ,
- gap width (centrifugal fans: gap between shroud and inlet) or tip clearance  $S$  (axial fans).

Scaling methods serve to predict the change in efficiency  $\dot{V}$  and pressure rise  $\dot{V}$  with the change of one or more of the listed

physical parameters for a volume flow  $\dot{V}$ . The shape of the machine is described by a finite number of dimensionless parameters, such as the ratio of chord length to the impeller diameter  $\kappa_l = l/D$ . By means of dimension analysis the number of independent parameters can be reduced by 3. This yields to  $\eta = \eta(\varphi, Re, Ma, k, s, shape)$ ,  $\psi = \psi(\varphi, Re, Ma, k, s, shape)$ . The dimensionless products are

- flow coefficient  $\varphi = 4\dot{V}/uD^2\pi$  with the circumferential speed given by  $u = \Omega D/2$ ,
- Reynolds number  $Re = uD/\nu$ ,
- Mach number  $Ma = u/a$ ,
- relative roughness  $k = K/L$  with the characteristic length  $L$ ,
- relative gap width or tip clearance  $s = S/L$ ,
- efficiency  $\eta = 1 - P_1/P$  wherein  $P$  is the applied power and  $P_1$  is the sum of dissipative losses,
- and pressure coefficient  $\psi = 2Y/u^2$ .

### Machine type, size and aerodynamic quality

Comparing two machines of same shape but different Reynolds number and Mach number, there will be a difference in efficiency. Interpreting the Reynolds number as a ratio of inertial forces to viscous forces, the relative losses will be reduced with increasing Reynolds number. This effect is known as *size effect*. Because  $Re \sim size^2 \sim speed$ , Reynolds number changes not only with size but also with rotational speed. Thus the term *Reynolds effect* instead of *size effect* is more common. The machine shape together with the surface roughness, and – in the case of axial machines – the tip clearance, determine the *aerodynamic quality* of the machine. The quality of the machine may alter from one manufacture to another. The *machine type*, axial or centrifugal, is determined by the specific speed  $\sigma$ . A dimensional analysis is not unique. Hence it is possible to transfer the pair of values  $(\varphi, \psi)$  to the pair of values  $(\sigma, \delta)$ , i.e. specific speed, specific diameter. Employing the transformation  $\varphi = 1/(\delta^3\sigma)$ ,  $\psi = 1/(\delta^2\sigma^2)$  and assuming low Mach number flow yields

$$\psi, \eta = \psi, \eta(\varphi, Re, Ma \ll 1, k, s, shape) \Rightarrow \eta = \eta(\sigma, Re, k, s, shape) \text{ or} \quad (1)$$

$$\eta = \eta(type, size, quality) .$$

Figure 1 shows Eq. 1 for turbomachines determined by TU Darmstadt [1].

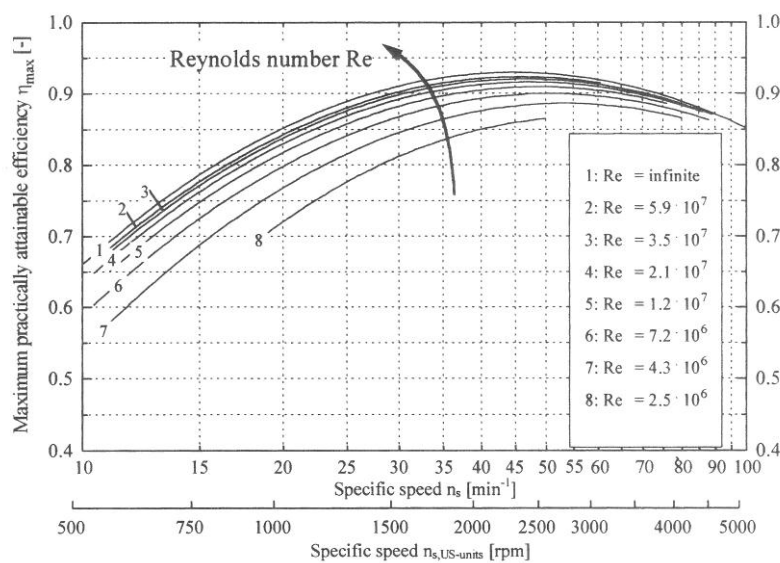


Figure 1: Efficiency vs. specific speed  $\sigma = n_s/(157.8 \text{ min}^{-1})$  for different Reynolds numbers [1].

In 2012, a new European standard for pumps will be available, where the minimal efficiency curve is defined on the basis of Eq. 1 (see [2]). In future only those pumps are allowed to be sold in the European Community, which fulfill the standard

$$\begin{aligned} \eta(\text{type}, \text{size}, \text{quality}) &> \eta_{\text{req.}}(\text{type}, \text{size}) \\ \text{or } \eta(\sigma, Re, \text{quality}) &> \eta_{\text{req.}}(\sigma, Re) \end{aligned} \quad (2)$$

### The need for a scaling method

For several reasons, reliable, easy to apply and general valid scaling laws are needed for design but also application engineers. The scaling laws are needed for the purposes of

- calculating the behavior of a full-scale (smaller or greater) machine from model test data obtained from a scaled machine,
- knowing the characteristic of one machine family containing machines of different scale but measuring only one machine,
- predicting the change in efficiency with changed rotational speed for the same machine,
- predicting the loss of efficiency with increased surface roughness

and other reasons. The scaling methods needed should

- be physically based and (hence) reliable,
- understandable and easy to apply,
- universal, i.e. applicable to centrifugal as well as to axial machines of different specific speed.
- account for the aerodynamic quality of the machine and should
- be reliable not only at the best point but also at off peak condition.

### Short review on scaling methods

The first physically based scaling method can be traced back to Pfleiderer in the year 1946 [3]. He was guided by the thought, that the inefficiency  $1 - \eta$  is proportional to the friction factor  $c_f$ . For hydraulic smooth surface  $c_f \sim Re^\alpha$  the ratios of inefficiencies from full scale to model (subscript “m”) yields

$$\frac{1 - \eta}{1 - \eta_m} = \left( \frac{Re}{Re_m} \right)^\alpha \quad (3)$$

According to pipe flow analogy with turbulent flow and hydraulically smooth wall  $\alpha$  was set to  $-0.25 \dots -0.1$ . Ackeret in 1948 [4] improved the method of Pfleiderer, by taking inertia losses into account

$$\frac{1 - \eta}{1 - \eta_m} = V \left[ 1 + \left( \frac{Re}{Re_m} \right)^\alpha \right], \quad (4)$$

where the loss factor  $V$  was arbitrarily set to 1/2. Heß and Pelz [5] considered the loss factor to be dependent on the flow coefficient  $V(\varphi)$  to account for an increase of inertia losses at off design operation.

Casey and Robinson [6] published an empirical scaling method where the difference in efficiency is given by

$$\Delta\eta = \eta - \eta_m = -B_{\text{ref}}(\sigma) \frac{\Delta c_f}{c_{f,m}} \quad (5)$$

$B_{\text{ref}}$  is an empirically determined function of specific speed. The disadvantage of both Eq. 4 and Eq. 5 is the following: Both methods need empirical functions which are machine dependent. Hence

there is always an uncertainty in applying both methods. It is the task of this work, to omit as far as possible any empirical relation to gain a truly universal, physically based scaling method.

By doing so, we get more physical insight in the dynamics of turbomachines in general and especially fans which in turn may be used to improve the quality of machines.

Up to now, no method, including all standards for turbomachine acceptance and performance tests, meet all listed tasks. In contrast, the model published here for the first time, does fulfill all tasks. It has three aspects, first the concept of the efficiency master curve, second the scaling of efficiency and third the scaling of the flow coefficient. The three parts will be introduced step by step in the next section.

## THE THREE STEPS OF EFFICIENCY SCALING

### 1. THE CONCEPT OF MASTERCURVE

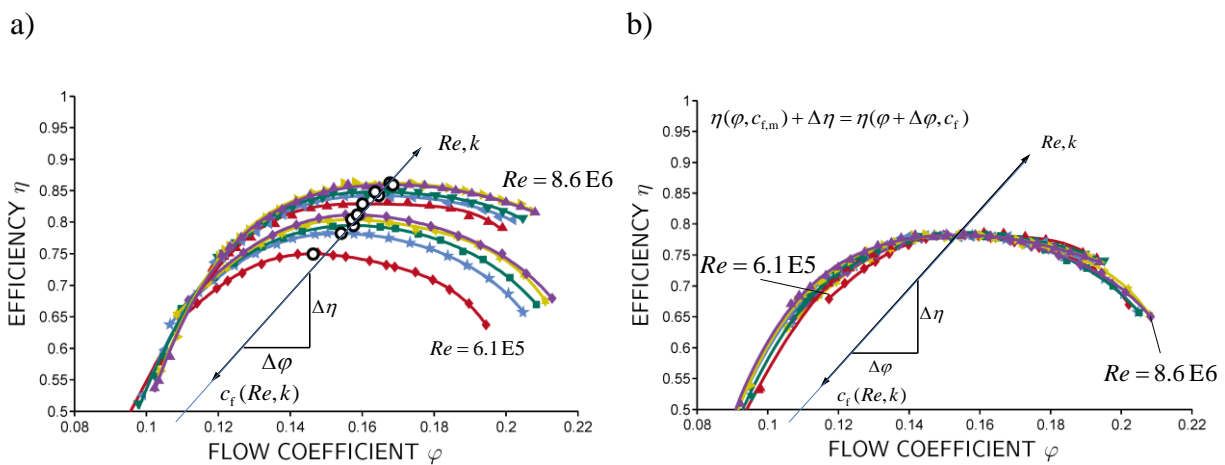


Figure 2: Performance characteristics of axial fans of specific speed  $\sigma = 1.46$  tested at the Chair of Fluid Systems Technology, TU Darmstadt, with the stagger angle  $\Delta\beta_0 = -6^\circ$ . The Reynolds number differs from  $6.1E5$  to  $8.6E6$ . The markers in a) designate the peak efficiency points. For the fan description see Heß [7].

Fig. 2a shows the dependency of efficiency versus flow coefficient for a typical axial fan at different Reynolds numbers [7]. Obviously the peak efficiency points are all aligned along one straight line. This is an observation which is seen at all measurement data, no matter if the fan is centrifugal or axial. In fact in the following we will show that this effect is due to the difference in boundary layer from suction side to pressure side of the blades. At the moment we would like to draw the attention to another point. If we shift the measured efficiency curves along that straight line we end up with one single curve which we call *master efficiency curve of the turbomachine* (see Fig. 2b).

The term “master curve” is adopted from the field of rheology. All of the molecular dynamic information, which is important for the relaxation behavior of a linear and thermo rheological simple material, is contained in the master curve. In analog we state: All the loss distribution going from part load to the best point to over load is given by one efficiency curve of only one rotational speed end hence one Reynolds number. This curve is pinned in the  $\eta, \varphi$ -plane by the position of its peak efficiency point.

The concept is shown in Fig. 2b, where the coincidence of the efficiency curve is shown after the shift. If a machine does have a master curve, the condition

$$\eta(\varphi, c_{f,m}) + \Delta\eta = \eta(\varphi + \Delta\varphi, c_f), \text{ with } \Delta\eta \sim \Delta\varphi \quad (6)$$

is fulfilled. The subscript “m” stands for model or reference friction factor. Even though we give the Eq. 6 here for completeness, it is important to mention, that our visible impression by comparing

Fig. 2a and Fig. 2b is much more convincing than arguing with an equation like Eq. 6 (in the appendix it is shown, that Eq. 6 is fulfilled also for a high Mach number flow in a turbocharger compressor).

As announced, the proportionality in Eq. 6 will be discussed later in more detail. Eq. 6 holds for all efficiency curves measured at our own laboratory for different specific speeds i.e. for pumps or fans, axial or centrifugal. The Reynolds number together with the relative roughness  $k$  do in combination determine the boundary layer thicknesses  $\delta_+ = \delta/L$  and hence the fluid displacement and momentum losses. Hence, in the context of the current considerations, it is sufficient and more over very convenient to use the relation

$$\delta_+ \sim c_f = c_f(Re, k) . \quad (7)$$

At that point one could discuss different friction loss models. Since the choice of the special friction loss model is only a minor detail of the principle physical concept of our approach we will postpone this to the appendix of this paper. More important is the relation of the friction factor and the dimensionless boundary layer thickness. We will use this relation in the following.

## 2. SCALING THE EFFICIENCY $\eta$

In contrast to most other methods, we start our analysis by the definition of the efficiency. We omit the adjective isentropic even though

$$1 - \eta := \frac{P_1}{P} \quad (8)$$

is the isentropic efficiency which becomes clear analyzing the conservation of energy carefully.  $P_1$  denotes power losses due to dissipation which results in an irreversible increase of entropy. The total differential of Eq. 8 gives (see Spurk [8])

$$-d\eta = (1 - \eta) \frac{dP_1}{P_1} - (1 - \eta)^2 \frac{dP}{P_1} \approx (1 - \eta) \frac{dP_1}{P_1} = (1 - \eta) \frac{dc_f}{c_f} . \quad (9)$$

Suppose the efficiency is 0.8, then the inefficiency, defined as  $\varepsilon := 1 - \eta$ , is 0.2 and the square of the inefficiency is 0.04. Thus the second term in Eq. 9 is much smaller compared the other terms. Hence we neglect this second order term. On the other side, the logarithmic change of the power loss  $dP_1/P_1$  is equal to the logarithmic change of the friction factor  $dc_f/c_f$ . With the definition of the inefficiency Eq. 9 reads in the easy to remember form

$$\frac{d\varepsilon}{\varepsilon} = \frac{dc_f}{c_f} , \quad (10)$$

i. e. *the logarithmic change of inefficiency and friction factor are equal*. From Eq. 9 the truly physical and strait forward scale up formula

$$-\Delta\eta = (1 - \eta_m) \frac{\Delta c_f}{c_{f,m}} \quad (11)$$

is given here for the first time. From the introduction (Eq. 1) it is clear, that the inefficiency  $(1 - \eta_m)$  accounts for three aspects, first the specific speed of the machine and hence the *type of machine* (from centrifugal to axial), *the size of the machine* and the specific *quality of the machine* with respect to efficiency.  $\eta_m = \eta_m(\sigma, Re, \text{quality}) = \eta_m(\text{type}, \text{size}, \text{quality})$ . Casey's equation is similar to Eq. 11 but of far less generality. Comparing Eq. 5 with Eq. 11 the nature of the empirical function  $B_{\text{ref}}(\sigma)$  becomes clear. It is equal to the inefficiency  $(1 - \eta_m)$  of the machine. Hence in Eq. 11 not only the specific speed is taken into account but also the quality of machine.

### 3. SCALING THE FLOW COEFFICIENT $\varphi$

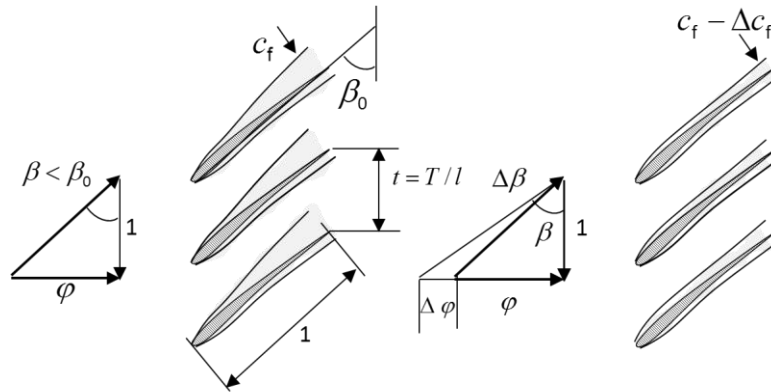


Figure 3: Uneven boundary layers on pressure and suction side. The difference between the left and right cascade is the Reynolds number and hence the boundary layer thickness. To achieve the optimal flow angle in the right cascade the flow coefficient  $\varphi$  has to be changed to  $\varphi + \Delta\varphi$ .

In this section we give the physical reason for the proportional relation  $\Delta\eta \sim \Delta\varphi$  and in fact give an analytic equation for it. We start the discussion with a schematic sketch of the flow through a blade row. Figure 3 left shows the situation at a small Reynolds number and hence high friction factor. Figure 3 right shows schematically the geometric similar machine at a higher Reynolds number flow. Since the profile length  $l$  is taken to be the natural length scale, the dimensionless boundary layer thickness  $\delta/l$  is given by the friction factor  $c_f$  as shown in Fig. 3. As usual all velocities are measured in multiples of the circumferential speed  $u$ . Hence the length of the adjacent side of the velocity triangle is one and the opposite side is given by the flow coefficient  $\varphi = c_m/u$  (here  $c_m$  is as usual the absolute meridian velocity component). Increasing the rotational speed or machine size will result in a thinning of the boundary layer. Here the different response from suction to pressure side to a change in Reynolds number is important. The relative change on the suction side is much greater in comparison to the pressure side. This point is made clear in the classical work of Schlichting and Scholz published in the year 1950 [9]. The change of the peak efficiency point follows for small changes is given by

$$\Delta\varphi = -\frac{1}{C(t, \beta_0)} \Delta c_f . \quad (12)$$

The constant  $C(t, \beta_0)$  is a function of stagger angle and dimensionless blade spacing  $t = T/l$ . Replacing the change in friction factor in Eq. 12 by the efficiency Eq. 11, we end up with

$$\frac{\Delta\eta}{\Delta\varphi} = C(t, \beta_0) \frac{1 - \eta_m}{c_{f,m}} . \quad (13)$$

which is already the desired result. Some further work has to be done to determine the correct value of the constant. Up to know the important Eq. 12 was introduced more or less on the basis of working hypotheses. In the truly analytic work of Schlichting and Scholz [9] the mentioned lack of flow angle

$$\beta < \beta_0 \quad (14)$$

is explained due to the boundary layers in blade cascade. Hence the boundary layers function like a change in stagger angle there is a change in optimal flow angle, where the inertia losses are minimal. This yields to a shift of the peak efficiency point to higher flow coefficients with higher Reynolds numbers.

As a determination of the change in  $\beta$  by the change of friction factor the relations from Schlichting were evaluated. A Taylor expansion of the result for a cascade

$$\beta = \arctan \left[ \tan \beta_0 \left( 1 - \frac{1}{t \sin \beta_0} c_f \right)^2 \right] \quad (15)$$

with  $\Delta\beta = \beta_0 - \beta$  determines the constant  $C(t, \beta_0)$  for a cascade to be given by

$$C(t, \beta_0) = \frac{t}{2} (\sin \beta_0 + \cos \beta_0) \quad (16)$$

If the hub-tip ratio is small and the effective diameter similar to the outer diameter of the rotor it counts

$$\Delta\varphi \approx \Delta\beta \quad (17)$$

Introducing Eq. 16 results in the already given Eq. 12. Even though Eq. 16 is only valid for axial fans we use a constant value of 0.4 for all fans in this work no matter if the fans are centrifugal or axial.

## VALIDATION OF THE METHOD

The validation of the scale-up formula is performed with test data from two axial fans with a diameter of 1000 mm, 250 mm and two centrifugal fans with 2240 mm, 896 mm diameter. Except Reynolds number, Mach number and relative roughness the two axial fans and the two centrifugal fans are similar to each other.

Table 1 shows an overview of the fans tested by the Chair of Fluid Systems Technology (FST). The small centrifugal model fan (sm) is currently being built and no measurement data is available yet. The performance characteristic of the full scale axial fan consists of few points only and is not taken into consideration within this work.

*Table 1: Characteristic data of the fans tested at the Chair of Fluid Systems Technology. Fans that are not considered in this work are greyed out.*

	centrifugal fan			axial fan		
designation	sm	lm	fs	sm	lm	fs
scale factor	1/10	1/2.5	1	1/10	1/2.5	1
diameter in m	0.22	0.89	2.24	0.25	1	2.5
rel. roughness	-	1E-5	2.2E-5	3.6E-5	1.2E-5	2.1E-5
Reynolds number / 1E6	0.6...1.6	1.6...6.5	6.5...16	0.6...1.5	4.3...8.6	11, 22

Two of the axial fans and one of the centrifugal fans listed in Tab. 1 are installed in laboratory of the Chair of Fluid Systems Technology (Fig. 4). The full scale centrifugal fan (fs) was analyzed by a scientific assistant from FST on manufacturer's test stand [10].

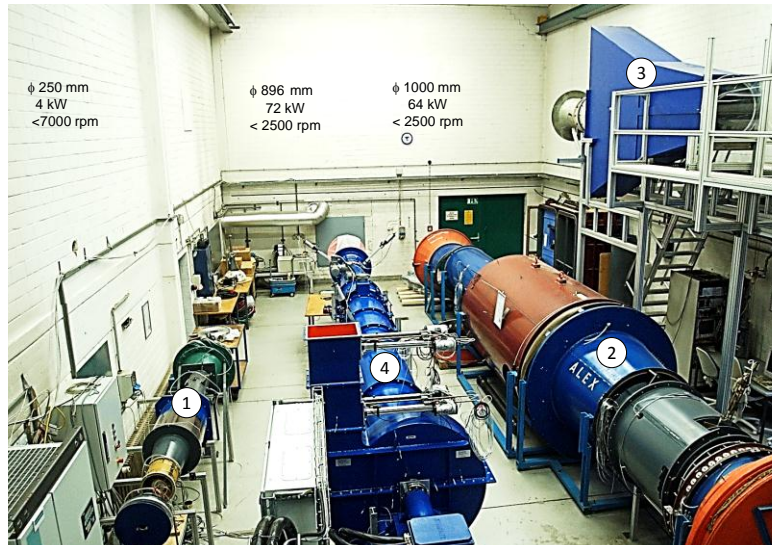


Figure 4: Fan test stands at the Chair of Fluid Systems Technology. 1 Axial fan (sm), 2 axial fan (lm), 3 axial fan (aero acoustics), 4 centrifugal fan (lm).

The test stands are built following DIN 24163 [11] and the measurement of the performance characteristics is evaluated complying with the VDI 2044 [12] guidelines. The shaft power of the fans is measured by a flying mount torque transducer, hence we need not take account in mechanical losses in bearings and gaskets. The achieved test data has a variation below 2.3 % points of total efficiency. The variation of the Reynolds number within one machine is achieved by varying the rotational speed.

### Validation of the method with data of centrifugal fan

Figure 5 shows the measured characteristics from the centrifugal fan and the predicted characteristic using different scaling methods mentioned in introduction. The scaling is done from the large model machine (lm) to the full scale machine (fs). The predicted performance characteristic using the new method (Pelz and Stonjek) shows the best agreement to measured data. The prediction obtained by Casey's and Ackeret's method underestimates the performance significantly.

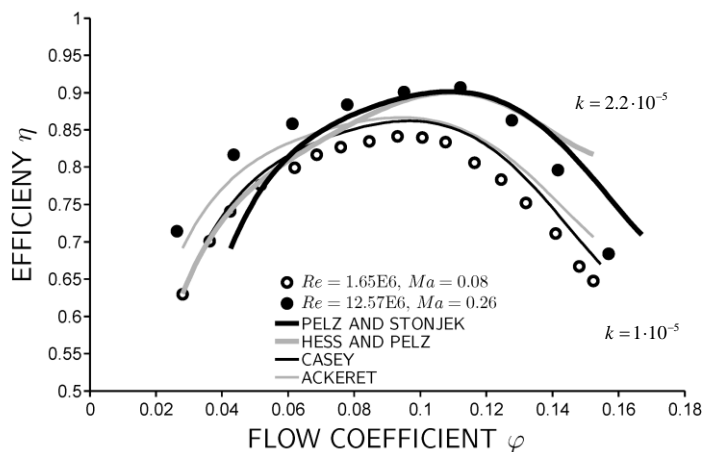


Figure 5: Characteristic of the centrifugal fan with  $\sigma = 0.29$  scaled up with different methods. The method of Heß is based on the Ackeret method.

It has to be pointed out, that this fan has a high flow coefficient compared to common centrifugal fans and might therefore not be a typical example.



### Validation of the method with data of axial fan with different stagger angles

The described scaling method has been validated with different fan geometries. To obtain different geometries on the same test rig, the stagger angle of the axial fan has been varied. If  $\beta_0$  is the stagger angle of the design point, the angle was varied from  $\Delta\beta_0 = +6^\circ$  to  $\Delta\beta_0 = -12^\circ$ . Figures 6 and 7 show the measured data and the predicted performance characteristics. The scaling is done from the lowest Reynolds number of the small model machine (sm) to the highest Reynolds number of the large model machine (lm). Since the introduced method is valid only for the hydraulic efficiency, the prediction of the characteristic is good for small variations from design point stagger angle. The low stagger angle of  $\Delta\beta_0 = -12^\circ$  results in significant deviations between measurement and predictions regardless of used scaling method.

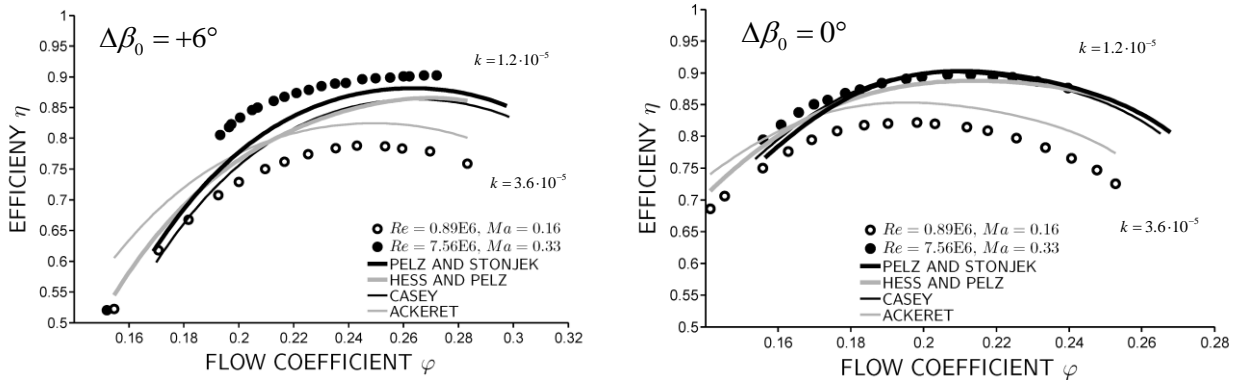


Figure 6: Axial fan with stagger angle  $\Delta\beta_0 = +6^\circ$  ( $\sigma = 1.64$ ) and  $\Delta\beta_0 = 0^\circ$  ( $\sigma = 1.49$ ).

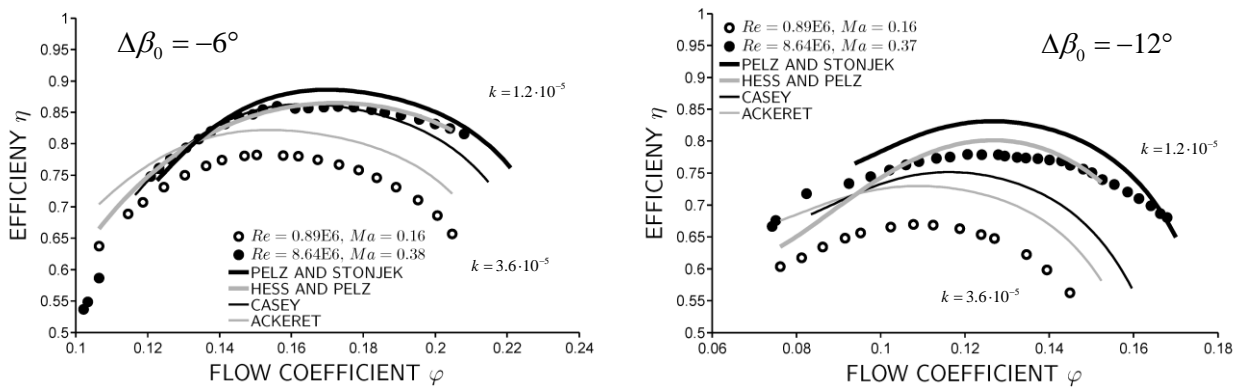


Figure 7: Axial fan with stagger angle  $\Delta\beta_0 = -6^\circ$  ( $\sigma = 1.46$ ) and  $\Delta\beta_0 = -12^\circ$  ( $\sigma = 1.53$ ).

### CONCLUSION

A new method for scaling up the efficiency and the pressure rise of fans has been introduced in this work. The method has essential advantages compared to previous introduced scale up methods

- simple application
- physical motivation for the scaling effect and the shift in flow rate, only one free parameter
- good results

The facts of discussed scaling methods are summarized in Tab. 2. More work has to be done regarding to the determination of the constant for the shift in flow rate. Furthermore, especially for centrifugal fans scaling the clearance losses and the disc friction losses separately could be

necessary. Nevertheless, the method shows good agreement to the test data within the scope of the fans analyzed at the Chair of Fluid Systems Technology.

Table 2: Overview of discussed scaling methods.

	DESCRIPTION	RATING	EFFICIENCY	FLOW COEFFICIENT	PRESSURE COEFFICIENT
PELZ, STONJEK (2011)	+ Physically based + Simple	+ Good validation	$\Delta\eta = -(1-\eta_m) \frac{\Delta c_f}{c_{f,m}}$	$\frac{\Delta\eta}{\Delta\varphi} = C \frac{1-\eta}{c_f}$	$\frac{\psi}{\psi_m} = \frac{\eta}{\eta_m}$
HESS, PELZ (2010)	- Empiric - Complicated	+ Good validation	$\frac{1-\eta}{1-\eta_m} = 1 + V(\varphi) \left[ \frac{c_f}{c_{f,m}} - 1 \right]$		$\frac{\psi}{\psi_m} = \frac{\eta}{\eta_m}$
CASEY, ROBINSON (2011)	- Empiric - Complicated	- Failing with centrifugal fans	$\Delta\eta = -B_{\text{ref}} \frac{\Delta c_f}{c_{f,m}}$	$\frac{\Delta\varphi}{\varphi} = C_{\text{ref}} \frac{\Delta\eta}{\eta}$	$\frac{\Delta\psi}{\psi} = D_{\text{ref}} \frac{\Delta\eta}{\eta}$
ACKERET (1948)	- Empiric + Simple	- Bad validation	$\frac{1-\eta}{1-\eta_m} = V \left[ 1 + \left( \frac{Re}{Re_m} \right)^{-0.2} \right]$		-

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## APPENDIX

### Determination of the friction factor

In general the friction factor  $c_f$  is a function of Reynolds number and relative roughness  $k$

$$c_f = c_f(k, Re) \quad . \quad (17)$$

Although relative changes are used for the friction factor, it is necessary to determine it from the model machine as a starting value. The friction factor can be determined in different ways depending on the modeling. The usage of the pipe friction analogy or the plate friction analogy are common approaches.

To use the  $c_f$  values from experimental investigations on plates or pipes it is necessary to transform the Reynolds number commonly used for turbomachines to the Reynolds number of the plate respectively the Reynolds number of the pipe in respect to geometry.

For axial fans, it can be derived as

$$Re_{P/R} = \frac{L}{D} \frac{1}{1 - \nu^2} \frac{1}{\sin(\beta_0)} \varphi Re \quad (18)$$

and for centrifugal fans

$$Re_{P/R} = \frac{1}{4} \frac{L}{b_2} \frac{1}{\sin(\beta_0)} \varphi Re \quad , \quad (19)$$

whereas  $\nu$  is the ratio of hub diameter to outer diameter for axial fans and  $b_2$  is the blade height at rotor outlet for centrifugal fans. Depending on the applied analogy (plate or pipe) the characteristic length  $L$  has to be replaced with either the blade length  $l$  or the hydraulic diameter  $D_h$  of the blade channel.

In case of the pipe friction analogy, there exists a well-known interpolation function from Colebrook [13], which is valid for the entire turbulence region from hydraulically smooth to hydraulically rough:

$$\frac{1}{\sqrt{c_f}} = -2 \lg \left( \frac{2.51}{Re\sqrt{c_f}} + 0.27k \right) . \quad (20)$$

This function is plotted in Fig. 8. We see the change in friction factor from the small model to the full scale machine. In this work, the pipe friction analogy is used. Therefore the friction factor  $c_f$  is determined with Eq. 20.

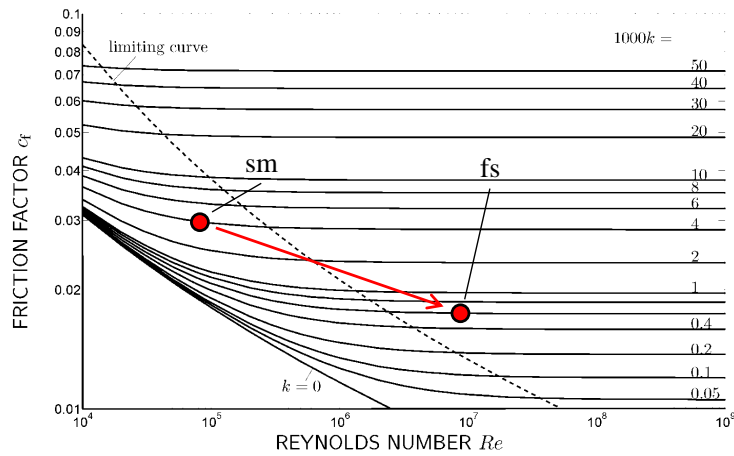


Figure 8: Colebrook's interpolation (Eq. 20) and scaling from small model (sm) to full scale machine (fs).

### Scaling of the pressure coefficient

Heß and Pelz [7] have shown, that the change in power coefficient with the Reynolds number is small. Based on the definition of hydraulic efficiency

$$\eta = \frac{Y}{Y + Y_1} = \frac{\psi}{\psi_{th}} , \quad (21)$$

in which  $\psi_{th}$  is the theoretical characteristic related to Euler's law of turbomachinery, we form from the measured values for  $\psi$  and  $\eta$

$$\psi_{ideal} := \frac{\psi}{\eta} . \quad (22)$$

$\psi_{ideal}$  for the axial fan with different stagger angles is shown in Fig. 9.

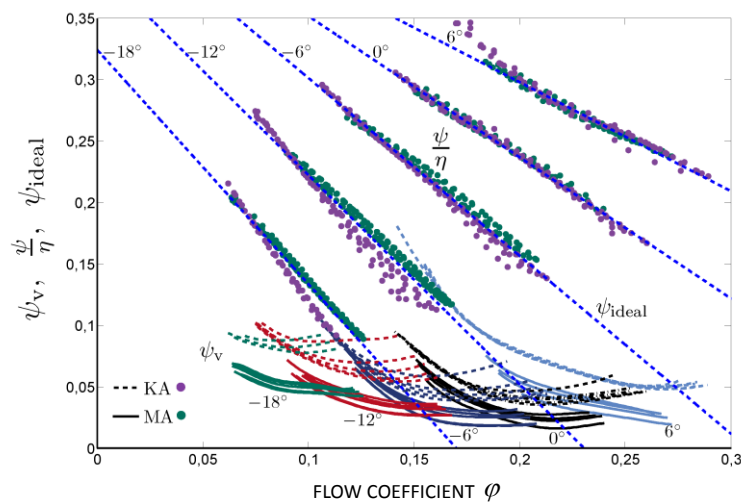


Figure 9: Ideal pressure coefficient and pressure loss coefficient [7].

Obviously,  $\psi_{ideal}$  just as  $\psi_{th}$  is a linear function of  $\varphi$ . The deviation from the theoretical characteristic is caused by the slip factor and the varying work distribution over blade height.

Hence, the scaling of the pressure coefficient can be accomplished following Heß and Pelz [5]:

$$\frac{\psi_m}{\psi} = \frac{\eta_m}{\eta} \quad (23)$$

It should be pointed out, that scaling the pressure coefficient by Eq. 23 must not be applied, if the disc friction losses become dominant or if the relative tip clearance from model to full scale machine changes. But measurement data from centrifugal fan of FST allow the assumption, that disc friction losses may be neglected.

### Scaling of tip clearance losses

Hess and Pelz [5] have shown, that tip clearance losses in axial fans are independent of Reynolds number. Hence, scaling of tip clearance can be accomplished using the loss model proposed in Karstadt and Pelz [14].

### Scaling of Mach number effects

Nakhjiri and Pelz [15] showed for high Mach and high Reynolds number flow the validity of the Eq. 6.

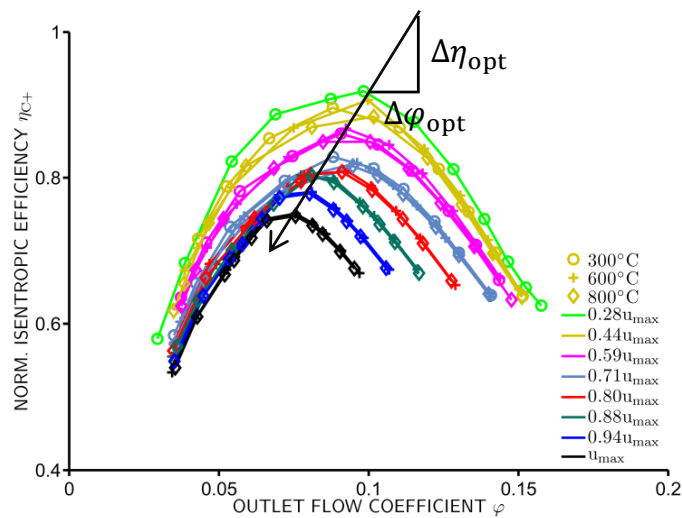


Figure 10: Change in efficiency with increase of Mach number for a compressor of a turbocharger. Measurements done by Nakhjiri and Pelz [15].