

Pushing the space charge limit: electron lenses in high-intensity synchrotrons?

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Abstract. Several accelerator projects require an increase in the number of particles per bunch, which is constrained by the space charge limit. Above this limit the transverse space charge force in combination with the lattice structure causes beam quality degradation and beam loss. Proposed devices to mitigate this beam loss in ion beams are electron lenses. An electron lens imparts a nonlinear, localized focusing kick to counteract the (global) space-charge forces in the primary beam. This effort is met with many challenges, including a reduced dynamic aperture (DA), resonance crossing, and beam-beam alignment. This contribution provides a detailed study of idealized electron lens use in high-intensity particle accelerators, including a comparison between analytical calculations and pyORBIT particle-in-cell (PIC) simulations.

1. Introduction

Space charge represents a major intensity limitation in booster synchrotrons operating at low or medium energies. The limitation arises due to stopbands caused by envelope instabilities driven by the lattice structure [1, 2] or due to the space charge induced tune spread and its overlap with incoherent, nonlinear resonances [3]. Depending on the chosen working point and the intensity, coherent or incoherent effects can dominate or occur together. The space charge limit in booster synchrotrons is still an active field of research, both numerically as well as experimentally. After the success of electron lenses for beam-beam compensation in RHIC [4] the potential of such lenses to also compensate, at least partially, for space charge would allow to push the intensity in booster synchrotrons. However, one should keep in mind that space charge acts as a distributed defocusing error, whereas the beam-beam tune shift has a local source. Therefore localized lenses cannot be expected to be as efficient for space charge as they are for beam-beam compensation. One has to install several lenses around the ring instead of placing one lens close to the beam-beam interaction section. Furthermore, in order to be effective for space charge the electron current profile has to match the bunch profiles of the ions. For very short, relativistic bunches the optimum bunch overlap was studied in [5]. For booster synchrotrons with typically long bunches compared to the length of an electron lens, the matching of the electron current profile with the ion bunch profile might be easier to achieve experimentally. Different concepts for space charge compensation were studied in [6]. Electron lenses for space charge compensation were also discussed in [7]. The possibility of space charge compensation in booster synchrotrons was studied in [8] using analytical models for the envelope instability and the coherent dipole tune. The limitation of the compensation degree by the upward shift of the coherent dipole tune was pointed out. For the FNAL booster synchrotron a minimum number of lenses was estimated.



For the SIS synchrotron, used as a reference machine in this study, the incoherent resonances induced by the space charge field of the electron beam in the cooling section were analyzed in [9]. For electron cooling the tune shift induced by the electron beam is usually well below 0.1 and the ion beam currents are low. Therefore incoherent resonances dominate and were identified up to order 6 in the tracking studies together with a severe emittance growth. The present study relies on a simulation model for intense beams including the linear synchrotron lattice, the 2D self-consistent space charge force and simple kicks for the lenses. We compare the results of extended simulation scans to analytical models.

2. Space charge compensation

2.1. 2D Model

The compensation of space charge in bunches is a 3D problem. The transverse profile of the electron beams and its current profile ideally have to both match the transverse profile and the longitudinal bunch profile of the circulating ion bunches. In this study we assume ion bunches that are long compared to the interaction length with the electron beam. The current profile of the electron beam in the interaction section is assumed to follow exactly the ion bunch profile. If a certain space charge compensation degree is chosen for the bunch center, it will therefore hold also at the bunch ends. If we further assume that the relevant effects are faster than the synchrotron oscillation period, a 2D model for the bunch center slice is sufficient.

The space charge tune shift in the bunch center is (vertical plane):

$$\Delta Q_y \approx -\frac{NZ^2r_p}{2\pi\varepsilon_y\beta_0^2\gamma_0^3AB_f}, \quad (1)$$

where β_0 and γ_0 are the relativistic parameters for the ion beam, N is the total particle number in the ring, Z the charge state of the ions, A the mass number, B_f is the bunching factor, ε_y is the unnormalized emittance of the equivalent KV beam.

The beam-beam tune shift (co-propagating: $-$, counter-propagating: $+$) induced by one electron lens is

$$\Delta Q_y^e = \frac{1 \mp \beta_e\beta_0}{\beta_e} \frac{ZI_e L r_p}{2\pi A e c \varepsilon_y \beta_0^2 \gamma_0}. \quad (2)$$

Hereby we assume that the transverse profiles of both beams overlap ideally. I_e is the current of the electron beam, L is the length of the interaction section, β_e is the velocity of the electrons divided by the speed of light.

The degree of space charge compensation we define as

$$\alpha = \frac{\Delta Q^e}{\Delta Q}, \quad (3)$$

where ΔQ is the space charge tune shift and ΔQ^e the total beam-beam tune shift generated by N_e electron lenses. The GSI SIS heavy-ion synchrotron [11] is used as a reference case for the study. The SIS has a circumference of $C = 216$ m and $S = 12$ periodic sectors. The injection energy is 11.4 MeV/u. As a simple example, we assume each sector is an ideal FODO cell, and the electron lenses are distributed symmetrically around the ring and treated as thin lenses.

2.2. Stability considerations: Orbits and Envelopes

The transverse space charge force acts as a distributed (de)focusing error around the ring. In periodic focusing lattices, space charge alone only leads to incoherent structural or coherent parametric resonances. On the contrary, the electron lenses act as localized focusing errors. If they are spaced symmetrically around the synchrotron they define new periodic structure cells (for $N_e < S$). In addition the lenses result in additional nonlinear error resonances.

As a first-order effect, the limitation of the compensation degree due to the shift of the coherent tune has been pointed out in [8]. The focusing electron lenses cause a positive shift of the coherent tune (the frequency of the beam centroid oscillations)

$$Q_c = Q_0 + \alpha\Delta Q, \quad (4)$$

where Q_c is the coherent tune and Q_0 the bare tune. This has been experimentally shown at the SIS for both coasting and bunched ion beams in [12]. An important limitation for the total strength of the lenses is that Q_c should stay well below integers n in order for the closed orbit to remain stable. This limitation only depends on the total beam-beam tune shift and not on the number of lenses.

The number of lenses N_e is important for the perturbation of the optical functions. The maximum beating of the β -function caused by an electron lens j , treated here as a localized focusing error is (see e.g.[10])

$$\left(\frac{\Delta\beta}{\beta}\right)_{\max} \approx 2\pi\Delta Q^e/N_e, \quad (5)$$

where ΔQ^e is the total beam-beam tune shift. If we require that the maximum beta beating remains well below 1, only very small space charge tune shifts can be compensated with one lens (well below 0.1 in the SIS). For the typical space charge tune shifts in high current synchrotrons, of the order of 0.4 – 0.5, several (at least $N_e \gtrsim 3$ for the SIS) lenses are required to at least partially compensate the space charge tune shift, without causing a large beating amplitude of the optical functions.

It is important to note that for dc electron beams the beta beating could be reduced by corrector quadrupoles. However, for the desired pulsed electron beam operation a correction with fixed gradient quadrupoles is not possible.

The required number of lenses can also be estimated from the stability criteria for betatron oscillations in periodic focusing lattices:

$$\left| \cos(2\pi Q/N_e) - 2\pi \frac{\Delta Q^e}{N_e} \sin(2\pi Q/N_e) \right| < 1, \quad (6)$$

where the tune Q is the particle tune and ΔQ^e is the total beam-beam tune shift induced by the N_e electron lenses. The area in tune space with $|\dots| \geq 1$ are stopbands in which the amplitude of linear betatron oscillations grows exponentially. Including the space charge tune shift in the particle tune $Q = Q_0 + \Delta Q$ causes a shift of the stopbands, but leaves the stopband width unchanged. The stopband width is determined by the beam-beam tune shift and the density of bands by N_e . The distance between stopbands is $N_e/2$, which is also why we call those stopbands "180°". The stopband width increases as $\delta Q \approx 2\Delta Q^e$.

The 2D envelope equations (see for example [13]) can be solved numerically including the electron lenses, treated as thin, linear focusing elements. Thereby one can obtain space charge structural instabilities and their modification due to the lenses. The condition for a parametric resonance or envelope instability including a partial incoherent space charge compensation is

$$2(Q_0 - \alpha\Delta Q) - \Delta Q_2 = \frac{n}{2}S, \quad (7)$$

where Q_0 is the bare tune, ΔQ_2 is the coherent tune shift for the envelope modes, n is the harmonic number and $S = N_e$ is the number of structure cells, which in this case equals the number of electron lenses. It is important to note that the stopbands arising from (coherent) envelope instabilities, also called 90° stopbands, are different from the 180° stopbands due to localized gradient errors.

Both stopbands can overlap, as is the case in the SIS close to $Q_y = 3$ (see Figure 1). In this case the 180° stopband dominates, as it defines the linear stability for the single particle and envelope betatron oscillations.

3. Particle-In-Cell (PIC) tracking

The 2D envelope model, discussed in the previous section, allows for very fast tune scans and helps to separate coherent and incoherent effects, which occur together in Particle-In-Cell (PIC) tracking simulations. In our simplified model, incoherent resonances are excited by the (static) nonlinear beam-beam forces and by the $n = 0$ harmonic of the nonlinear space charge force. We use an initial waterbag distribution for the PIC ion macro-particles, which might be closer to the actual distribution of the injected beam in a booster synchrotron than a Gaussian. A 2D tune scan performed with the PIC simulation code pyORBIT [14] is shown in Figure 1. The elliptical transverse profile of the ion beam at the location of the lens is met with the round, Gaussian beam of the electron lens. The emittance growth observed after 500 cells is shown for $N_e = 3$ lenses. The reference high-intensity working point of the SIS is indicated as a white diamond. The overlapping 90° and (lens-induced) 180° stopbands as described in the previous section are shown clearly near $Q_y = 3$. Sixth order resonance lines appear near $Q_{x,y} = 3.75$, arising from localized, nonlinear focusing errors in the electron lenses. It should be noted that the order and location of these nonlinear resonance lines depend greatly on the number of lenses and beam profile matching. An example ion beam profile displaying sixth order resonance after 100 cells is shown in the bottom right-hand corner of the figure. Without the lenses, the simulation results only in the 90° stopbands, shifted upwards according to the tune shift in Equation (1). The widths of the 90° stopbands are unaffected by the lenses.

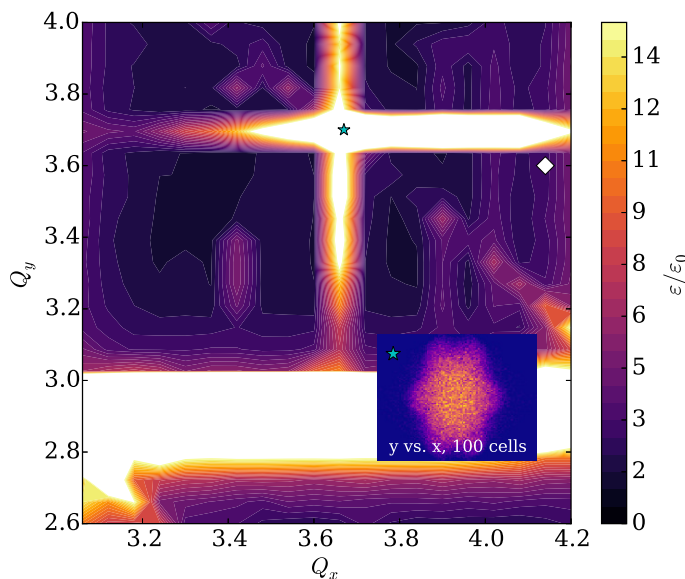


Figure 1. PIC simulation scan for $N_e = 3$, $\Delta Q_{x,y} = -0.3, 0.8$ and $\alpha = 1$. The SIS high-intensity working point is represented by a white diamond. 6^{th} -order nonlinear resonance is shown after 100 cells (star).

4. Conclusions and Outlook

Within a simplified 2D beam dynamics model we studied the (partial) compensation of large space charge tune shifts, typical for booster synchrotrons, by localized electron lenses. The model includes self-consistent 2D space charge and fixed (nonlinear) kicks from the electron lenses. We use the GSI SIS18 heavy-ion synchrotron without bends as the reference case, and only the

transverse space charge and lenses account for the 'error' sources in our simulation model. From the envelope model we obtain the beta beating amplitudes, the 180° stopbands (caused by the localized gradient errors) as well as the envelope instability or 90° stopbands (caused by space charge structural instabilities). The PIC simulation provides additional information on nonlinear resonances excited by the lenses, which cannot be resolved in the envelope model.

For any $N_e < S$ the unwanted effects caused by the lenses seem to dominate in our example study. One has to keep in mind that in our model many additional sources of errors are not included, for example systematic and random errors in the alignment of the lenses and in the 3D overlap between the ion and electron beams. The alignment errors of the lenses will add to the closed orbit instability at integer coherent tunes. The errors in the electron current for different lenses will lead to additional gradient errors stopbands. Also we did not include any lattice errors, which are usually the reason for the space charge limit without the lenses. Therefore we expect our model to be very optimistic.

Still there is room for further studies and possible improvements. For example, instead of trying to ideally overlap both beams one could use a transverse McMillan lens profile [15, 16]. This could reduce the effect of nonlinear resonances induced by the lenses, but would not affect the error stopbands or the beta beating. The compensation might work better for lower space charge tune shifts (0.1 or lower) and a space charge limit dominated by nonlinear lattice errors. This would require studies with a 3D simulation model including synchrotron motion over much longer time scales than in this study. We plan to conduct such long-term simulation studies for the FAIR SIS100 synchrotron, first within a frozen 3D space charge model.

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