Efficiency Scaling: Influence of Reynolds and Mach Numbers on Fan Performance

The efficiency, pressure ratio, and shaft power of a fan depends on type, size, working medium, and operating condition. For acceptance tests, a dissimilarity in Reynolds number, Mach number, relative roughness, and relative blade tip clearance of the scaled model and prototype is unavoidable. Hence, the efficiency differs between model and prototype. This difference is quantified by scaling methods. This article presents a validated and physics-based, i.e., reliable scaling method for the efficiency, pressure ratio, and shaft power of axial and centrifugal fans operating at subsonic conditions. The method is validated using test results gained on standardized test rigs for different fan types, sizes, and operating conditions. For all scenarios, the presented scaling method provides a much reduced scaling uncertainty compared to the reference method described in ISO 13348. [DOI: 10.1115/1.4053172]

Keywords: fan, compressor, and turbine aerodynamic design, measurement techniques

1 Introduction

Performance scaling is an essential element in the preliminary design of new turbomachinery and in acceptance tests to verify specified performance data. Machine development is a lengthy, iterative, and time-consuming process. So a suitable and well-tried design is often scaled to the required specifications. For acceptance tests of large machinery, standardized test rigs are not economical due to the required dimensions and in situ tests are too inaccurate. The basis of scaling is similarity. Full similarity between model, marked by a prime (′), and prototype is preferable but in most cases not achievable. A scaling method quantifies the differences in performance between model and prototype.

This article gives an analytical description of how to scale efficiency, pressure coefficient, and power coefficient in the design point and at off-design capabilities. It is based on an axiomatic method with models of dominant loss mechanisms with a focus on Reynolds and Mach number dependence.

In scaling methods found in the relevant literature, usually only friction losses are considered. The presented method extends this toward inertia and induced losses for incompressible and slightly compressible flow. The method assumes geometric similarity, which is given when all geometric dimensions of the machine are scaled with the same scaling factor with the exception of roughness and gap width. Flow separation and the effects of three-dimensional secondary flows are not considered. Furthermore, no detailed analysis has been made to predict the surge region.

The method is used here for industrial fans. However, the analytical ansatz can also be applied to other turbomachinery. Hence, the loss mechanisms of the machine must be adapted.

A good scaling method must be (i) reliable, (ii) valuable, and (iii) universally applicable. (i) For them to be considered reliable, scaling methods must be physical rather than empirical. (ii) A scaling method is deemed valuable if the sum of model measurement uncertainty and scaling uncertainty (compared with Ref. [1]) is smaller than the uncertainty of an in situ measurement of the prototype.

Figure 1 shows schematically the measurement uncertainty of a model and a prototype for different Mach numbers Ma′ ≠ Ma and Reynolds numbers Re′ ≠ Re as well as the scaling uncertainty. (iii) To be universally applicable, an efficiency scaling method for fans must be valid in a wide range of specific speeds σ including axial and centrifugal fans.

In this article, a reliable, valuable, and universal scaling method is presented. The method was developed and validated in the last decade in close collaboration with industry, International Organization for Standardization (ISO) and academia.

1.1 Review on Scaling Methods. The first scaling method for the efficiency based on dimensional analysis [2] and Blasius’ friction law was published by Pfeiderer [3] in 1947:

\[ 1 - \eta = \left( \frac{\text{Re}}{\text{Re}} \right)^{m}, \quad m = 0.1 \]  

(1)

The shortcoming of Eq. (1) is the unrealistic asymptote \( \eta \to 1 \) for \( \text{Re} \to \infty \). The reason for this shortcoming is that inertia loss (often called kinematic loss) is ignored, whereas it is in fact always present. In 1948, Mühlemann [4] published Acke1er’s modified formula, which takes inertia loss into account by introducing a loss fraction of friction loss in total loss L:

\[ 1 - \eta = (1 - L) + L \left( \frac{\text{Re}}{\text{Re}} \right)^{m}, \quad m = 0.2 \]  

(2)
Ackeret assumes inertia and frictional losses to be independent of each other based on Froude’s hypothesis [5]. The assumed fraction of friction loss is \( L = 0.5 \). Pelz and Heß [6] investigated how the fraction \( L \) changes with flow coefficient \( \varphi \) and found that \( L \) is a function of the machine type and the operating conditions \( L = L(\text{specific speed} \, \sigma, \text{flow coefficient} \, \varphi) \). In addition, Wiesner [7] summarized optimized values for \( L \) and \( m \) for various turbomachinery types, e.g., pumps and compressors. The shortcoming of this fitting is its empirical nature, which limits the robustness of scaling.

Instead of separating inertia and friction losses by means of the fraction \( L \), Stoffel [8] and later Spurk [9] introduce an asymptotic efficiency \( \eta_0 = \lim_{Re\to\infty} \eta < 1 \). Therefore, Eq. (1) is meant to be

\[
\frac{\eta_{\infty} - \eta}{\eta_{\infty}} = \left( \frac{Re}{Re'} \right)^{-m}, \quad 0.1 < m < 0.2 \quad (3)
\]

But still, with \( L(\sigma, \varphi) \) and \( \eta_{\infty}(\sigma, \varphi) \), Eqs. (2) and (3) are identical with respect to the basic concept and idea of the scaling method.

Casey and Robertson [10] give a correction method for Reynolds number, size, and roughness effects on the performance of compressors. Empirical functions calibrated by a large number of measurements are used for their method. They distinguish between Reynolds number-dependent and number-independent losses, showing that the Reynolds number-dependent calibration coefficient varies with the machine type.

Pelz and Stonjek [11] published a method for scaling the efficiency and the pressure rise of fans derived from the total derivative in an axiomatic way. This method forms the basis for the work presented here. So far, Pelz and Stonjek’s method is limited to the influence of Reynolds number and roughness, which is extended here to include inertia and induced losses.

1.2 Independent Investigation of Reynolds and Mach Number Effect. For small Mach number, the major changes in performance, i.e., efficiency and pressure coefficient, are due to a change in the Reynolds number. The well-known Reynolds number effect is decisive for this and describes the nonlinear dependence of the surface friction on the Reynolds number. With increasing Mach number, the compressibility of the flow increases having several effects: First, wave phenomena may occur. At subsonic convection Mach number, the compressibility of the gas reduces the surface friction on the Reynolds number. With increasing Mach number, the compressibility of the flow increases having several effects: First, wave phenomena may occur. At subsonic convection Mach number, the compressibility of the gas reduces the surface friction on the Reynolds number. With increasing Mach number, the compressibility of the flow increases having several effects: First, wave phenomena may occur. 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The ratio of Mach and Reynolds number is expressed as follows:

\[
\frac{Ma}{Re} = \frac{v_1/a_1}{D} \quad (4)
\]

Here, \( v_1 \) denotes the kinematic viscosity, \( a_1 \) is the speed of sound, and \( D \) is the outer diameter of the fan. For axial turbomachinery, the outer diameter is that of the housing, and for radial ones, it is the outer impeller diameter. Subscript 1 denotes the condition at fan inlet. According to Jousen and Wutz [15] the mean free path of a molecule at inlet conditions is given for an ideal gas by

\[
\lambda_1 = \frac{\mu_1}{RT_1^2} \frac{p_1^{1.1}}{\sqrt{T_1}} \quad (5)
\]

with the dynamic viscosity \( \mu_1 \), individual gas constant \( R \), temperature \( T_1 \), pressure \( p_1 \), and isentropic exponent \( \gamma \).

Therefore, the ratio \( Ma/Re \) yields

\[
\frac{Ma}{Re} = \frac{\lambda_1}{D} = Kn \quad (6)
\]

which is known as the Knudsen number \( Kn \), i.e. the ratio of free path length \( \lambda_1 \) to a characteristic length being the diameter of the turbomachinery.

As a result, when testing a fan in ambient conditions given by constant pressure, constant temperature and gas unchanged, the Knudsen number \( Kn \) is constant and Reynolds and Mach number are proportional to each other as sketched in Fig. 1. In consequence, the effect of Reynolds number and Mach number on the machine performance is usually difficult to separate. It would be desirable to adjust the Knudsen number for overcoming this problem. To make it possible, a pressure chamber, outlined in Fig. 2, was built in which an entire fan test rig can be operated, see Ref. [16].

The variation of pressure \( p_1 \) changes the molecular mean free path \( \lambda_1 \) and allows a change of the Knudsen number \( Kn \). It is well known that for an ideal gas, the speed of sound and hence the Mach number are independent of pressure. In contrast, the Reynolds number increases with pressure. The circumferential speed \( u = \pi Dn \) can be used in turbomachinery as the characteristic speed for calculating Reynolds and Mach number. (The blade channel or blade related dimensionless numbers are derived by multiplying by the flow number). For constant rotational speed \( n \), the Mach number is constant and Reynolds number effects are investigated by changing the pressure \( p_1 \neq p_1' \). To investigate Mach number effects at constant Reynolds number, pressure and rotational speed have to fulfill the equation \( p_1'/p_1 = n/m' \). Thus, Reynolds and Mach number can be set independently of each other. To validate the scaling method presented here, the test results from measurements inside the pressure vessel are used, among other tests performed with scaled fans.

So far, none of the known scaling methods account for the compressibility. It is therefore necessary to develop a universal scaling method based on physics, which allows taking different effects into account. None of the existing methods is sufficient and flexible enough to serve as a starting point except the Spurk ansatz [9], which was extended by Pelz and Stonjek [11].

2 Methodology

The presented scaling method predicts the efficiency scaling \( \Delta \eta \), power coefficient scaling \( \Delta \lambda \), and the pressure coefficient scaling \( \Delta \phi \) independent of the flow coefficient \( \phi \). Moreover, the scaling method can be used in the best efficiency point and at off-design conditions.

2.1 Background of the Method. Efficiency and pressure coefficient depend on type, quality, size, and working medium and operating condition. The type of the fan is determined by the specific speed \( k_1 \) and the clearance between rotor and casing, which is also known as the relative gap width \( k_2 \). For axial fans, the gap is between the blade tip and the casing, for centrifugal fans between the impeller inlet and the inlet nozzle. Relative roughness and relative gap give an indication of the manufacturing quality and the manufacturing effort. The Reynolds number \( Re \) and the Mach number \( Ma \) characterize the size and speed, whereas the working medium is characterized by the isentropic exponent \( \gamma \). The flow coefficient \( \phi \)
describes the operating point. The independent dimensionless products can be found in Refs. [9] and [17]. They are summarized as follows:

- Flow coefficient \( \varphi : = 4V_1/(\pi D^2 u) \) (volume flowrate \( \dot{m}/\rho_1 \)
  \( \dot{m}/\rho_1 \), density \( \rho_1 \), circumferential velocity \( u = \pi D n \), rotational speed \( n \)),
- Specific speed \( \sigma : = \varphi^{1/2} w^{-3/4} \),
- Reynolds number \( Re := D u q_1/\mu_1 \) (dynamic viscosity \( \mu_1 \)),
- Mach number \( Ma := u/a_1 \) (speed of sound \( a_1 = \sqrt{RT_1} \), temperature \( T_1 \)),
- Relative roughness \( k_i := k/D \) (absolute roughness \( k \)), and
- Relative gap width or relative blade tip clearance \( s_i := s/D \) (absolute gap width or absolute blade tip clearance \( s \)).

The isentropic efficiency for an adiabatic operation of the machine is defined as follows:

\[
\eta : = \frac{\Delta h_{is}}{\Delta h_i} = \eta(\sigma, Re, Ma, \gamma, k_i, s_i, \varphi)
\]  
(7)

with the isentropic enthalpy change \( \Delta h_{is} \), the added total enthalpy \( \Delta h_i = P_S/m \), the mass flowrate \( \dot{m} \), and the shaft power \( P_S \).

With the fluid regarded as ideal gas, the isentropic enthalpy change becomes

\[
\Delta h_{is} = \frac{\gamma}{\gamma - 1} \left( \frac{p_2}{p_1} \right)^{(\gamma - 1)/\gamma} - 1 + \frac{c_i^2}{2} - \frac{c_f^2}{2}
\]  
(8)

The pressure coefficient is defined as follows:

\[
\psi : = \frac{2}{\eta} \frac{\Delta h_{is}}{u^2} = \psi(\sigma, Re, Ma, \gamma, k_i, s_i, \varphi)
\]  
(9)

From efficiency and pressure coefficient, a third dimensionless product, the power coefficient, can be derived that characterizes the work input:

\[
\lambda : = \frac{\psi}{\eta} = \frac{2 \Delta h_i}{u^2} = \lambda(\sigma, Re, Ma, \gamma, k_i, s_i, \varphi)
\]  
(10)

2.2 Efficiency Scaling. The inefficiency for an adiabatic machine (\( Q \equiv 0 \)) is defined as follows:

\[
\varepsilon : = 1 - \eta = \frac{\Delta h_i - \Delta h_{is}}{\Delta h_i} = \frac{h_i}{\Delta h_i}
\]  
(11)

Here, \( h_i \) is the enthalpy loss associated with the increase of entropy. Note that the first law of thermodynamics for an adiabatic machine reads \( \Delta h_i = \Delta h_{is} + h_i \), as illustrated in Fig. 3.

The enthalpy loss may be written in a dimensionless form, the loss coefficient:

\[
\zeta : = \frac{2h_i}{u^2}
\]  
(12)

Equation (11) is written in the equivalent form:

\[
\varepsilon = \frac{\zeta}{\lambda}
\]  
(13)

Differentiating Eq. (13) yields

\[
\frac{d\varepsilon}{\varepsilon} = \frac{d\zeta}{\zeta} - \frac{d\lambda}{\lambda}
\]  
(14)

With the differentials replaced by differences, the efficiency change reads

\[
\Delta \varepsilon = -\Delta \varepsilon = (1 - \eta) \left( \frac{\zeta' - \zeta}{\zeta'} - \frac{\lambda' - \lambda}{\lambda'} \right)
\]  
(15)

This approach is based on the method published by Pelz and Stonjek [11], which evolved from Spurk’s ansatz [9]. Since it is based on the differential of efficiency, it is exact and universally applicable. The method requires the modeling of loss coefficient \( \zeta \) and power coefficient \( \lambda \). Depending on the type of turbomachine (turbine, compressor, fan, pump, etc.), different loss sources and models are required.

In the following, the modeling of loss coefficients is focused on axial and centrifugal fans. The purpose is an universally applicable scaling method with a manageable amount of input parameters. Integral parameters (e.g., flow coefficient \( \varphi \), power coefficient \( \lambda \)) or easily measurable parameters (e.g., impeller diameter \( D \)) shall be used.

2.2.1 Loss Coefficient. Four different losses are present: friction loss \( \zeta_f \), inertia loss \( \zeta_i \), induced loss \( \zeta_{ind} \), and losses due to wave phenomena \( \zeta_w \). For fans operating under subsonic conditions, \( Ma < 1 \) wave phenomena can be neglected: \( \zeta_w \approx 0 \). According to Froude’s hypothesis [5], the losses are assumed to be independent of each other. Hence, in the approximation, the total loss is the sum of the individual losses. The loss coefficients may be added as long as the denominators of the individual loss coefficients are equal.

We choose the commonly used circumferential velocity \( u \), as shown in Eq. (12). All loss coefficients calculated with \( u \) are machine specific; no further index is used.

\[
\zeta = \zeta_f + \zeta_i + \zeta_{ind}
\]  
(16)

For loss modeling, the local velocity (relative velocity \( w \)) and length scale (chord length \( l \)) are needed. All loss coefficients and dimensionless parameters calculated with local scales will be marked by an asterisk (*)

That is, we have the global loss coefficient:

\[
\zeta : = \frac{h_i}{w^2/2} = \zeta(Re, Ma, k_i, s_i, \varphi)
\]  
(17)

The local loss coefficient:

\[
\zeta^* : = \frac{h_i}{w^2/2} = \zeta^*(Re^*, Ma^*, k_i^*, s_i, \varphi)
\]  
(18)

Hence, we have the transformation

\[
\zeta : = \zeta^*(w^2/2) = \zeta^*(\varphi, \text{shape})
\]  
(19)
For moderate Mach number, $Ma < 0.3$, the effect of friction (Reynolds number) and compressibility (Mach number) can be separated in a product ansatz:

$$\zeta^s(Ma) \approx F(Ma^s)\zeta^c(Mac^c = 0)$$  \hspace{1cm} (20)

This approach has proven to be valid and practical in modeling of kinetic losses like the Carnot shock loss. Thus, we have the transformation

$$\zeta = F(Ma^s)f(\varphi, \text{shape})\zeta^c(Re^c, Ma^c = 0, k^c)$$  \hspace{1cm} (21)

The correction function for compressibility is expressed as follows:

$$F(Ma^s) := \frac{\zeta^c(Re^c)}{\zeta^c(0)}$$  \hspace{1cm} (22)

where $\zeta^c(Re^c)$ is the loss coefficient for compressible and $\zeta^c(0)$ for incompressible flow.

For a centrifugal fan, the local volume flow in the impeller is increased by the gap volume flow, see Eq. (41). Hence, the local flow coefficient for the impeller is expressed as follows:

$$\varphi^f = \varphi(1 + \dot{V}_f/V)$$  \hspace{1cm} (23)

For the flow in the volute of a centrifugal fan as well as for axial fans, the local flow coefficient is $\varphi = \varphi^f$.

In the following, the loss coefficients $\zeta^s$, shape factors $f$, and the correction functions $F$ are presented and discussed.

### 2.2.2 Friction Loss Coefficient $\zeta_f$

Friction loss is present in all parts of the fan. Figures 4 and 5 show the simplified geometry of an axial and a centrifugal fan, which is divided into rotating and stationary parts. Prandtl’s boundary layer theory allows us mapping the flow along a blade surface to the flow along a flat plate.

When calculating the resistance of a flat plate, a distinction is made between laminar $Re^\prime < 10^5$ and turbulent flow $Re^\prime > 5 \cdot 10^5$. There is no reliable method for determining the transition Reynolds number. Hence, to limit uncertainties, we restrict the scaling to turbulent flow. In practice, this is acceptable, since most experiments are carried out with Reynolds numbers $Re^\prime > 5 \cdot 10^5$; de facto turbulent flow is present.

The loss coefficient for a turbulent flow along a flat plate is given by Gülich’s interpolation [18]$

$$\zeta^s_f = \frac{0.136}{\left[-\log_{10}(0.2k_f^s + 12.5/Re^c)^2\right]^{0.15}}$$  \hspace{1cm} (24)

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[Diagram of Axial fan]

Figure 4: Axial fan

[Diagram of Centrifugal fan]

Figure 5: Centrifugal fan

with the local length $l$, the local velocity $\bar{w}$, and the absolute roughness $k$. For $k$, the equivalent sand roughness shall be used [18,19].

The calculated local friction loss factor must now be transformed into the machine related consideration using a shape factor. According to Pelz and Stonjek [20], the shape factors are based on the different definitions of the loss coefficients $\zeta_f := 2h_f/\bar{w}^2$ and $\zeta_f^{\prime} := 2W/(A_w\bar{w}^3)$, with the drag $W$, the wetted surface $A_w$, and the relative velocity $\bar{w}$. The energy conservation is given by $h_f \dot{m} = W \bar{w}$, and with Eq. (19), the shape factor for friction yields

$$f_t = \frac{\zeta_f - 4A_w}{\pi D^2}\frac{1}{\bar{w}^3}\left(\frac{m}{u}\right)^3$$  \hspace{1cm} (27)

The formulas for the wetted surface $A_w$ and the relative velocity $\bar{w}$ for axial and centrifugal fans are summarized in Appendix A.

Compressibility correction functions are given by Krasnov [12] for turbulent flows, but the impact on the friction coefficient is low ($\approx 1%$ at $Ma = 0.3$). As a result, $f_t(Ma^s) = 1$.

### 2.2.3 Inertia Loss Coefficient $\zeta_i$

Inertia loss can be divided into incidence loss, Carnot shock loss and gain loss. Further losses, like trailing edge loss, are assumed to be negligible. The loss due to leakage flow through the clearance between blade tip and casing of axial fans is modeled as an induced loss, which is introduced in Sec. 2.2.4.

**Incidence loss.** In 1922, Thoma [21] published a model for the incidence loss for incompressible flows:

$$\zeta^s_{inc} = \frac{2h_i}{w^2} = \tan^2 \alpha$$  \hspace{1cm} (28)

with the incidence angle $\alpha$. Saul et al. [13] generalized Thoma’s ansatz to consider compressible flows. Even though this model is very simplistic, we will see in Sec. 7 that this ansatz shows reasonable results with low efforts. The derived correction function $F_{inc.}(Ma^s, i)$ is depicted in Appendix B in Fig. 18.

The incidence angle is $i = \beta_i - \beta$ for the rotor and $i = \alpha_i - \alpha$ for the stator, as shown in Figs. 6 and 7. Index “m” indicates variables in the mean section, “ax” indicates the axial velocity component, and blade angles are indicated by the index “b.”

The flow angles depend on the operating point, and the shape of the fan is given by the geometry. The flow angle at the impeller inlet of a centrifugal fan is expressed as follows:

$$\beta = \arctan\left(\frac{\dot{V}_i D}{4b_i \nu}\right)$$  \hspace{1cm} (29)

with the hub–tip ratio $\nu = D_i/D$ and the height at impeller inlet $b_i$. 

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For axial fans, the flow angles in the mean section are as follows:

$$\beta = \arctan \left( \frac{\varphi}{1 - \nu^2} \right)$$

$$\alpha = \arctan \left( \frac{\varphi (1 + \nu)}{\lambda (1 - \nu)} \right)$$

For more information, the derivation of the flow angles is added in Appendix C.

For axial fans, the flow angle at best efficiency point shall be used as the blade angle $\beta_b = \beta(\phi_{BEP})$ and $\alpha_b = \alpha(\phi_{BEP}, \lambda_{BEP})$. With this ansatz, we avoid further corrections of the incidence angle based on Lieblein’s experimental investigations [22], which are elaborate and much more input parameters, e.g., the real blade angle and twist, would be necessary.

The shape factor is calculated with Eq. (19):

$$f = \left( \frac{w}{u} \right)^2$$

with the local relative velocity $w$ at rotor or stator inlet, as shown in Table 3 in Appendix A. Therefore, the incidence shape factor for centrifugal fans yields

$$f_{inc} = \left( \frac{\varphi D}{4 \nu b} \right)^2 + \nu^2$$

and for axial fans

$$f_{inc} = \left( \frac{\varphi}{1 - \nu^2} \right)^2 + \left( 1 + \frac{\nu}{2} \right)^2$$

$$f_{inc,s} = \left( \frac{\varphi}{1 - \nu^2} \right)^2 + \left( \frac{\lambda}{2} \right)^2$$

Carnot loss. The Carnot loss is determined by

$$\zeta_C^* = \frac{2 h_l}{w^2} \left( 1 - \frac{A_1}{A_2} \right)^2$$

with the cross section ratio $A_1/A_2 < 1$ [23]. This equation is applicable to the Carnot loss at impeller outlet of centrifugal fans with a volute.

The cross section ratio is calculated with the cross section $A_1 = \pi Db$ and $A_2 = \pi DB$, with the impeller height at the impeller outlet $b$ and the volute height $B$, see Fig. 8. Hence, the cross section ratio yields $A_1/A_2 = b/B$.

Rist [14] generalized the Carnot loss for compressible flows, and the deduced correction function $F_C(M_{a*}, A_2/A_1)$ is shown in Appendix B in Fig. 19.

The shape factor for the Carnot loss at impeller outlet yields

$$f_C = \left( \frac{D \varphi^*}{4b} \right)^2$$

Gap loss. For centrifugal fans, the power loss $P_g$ due to gap leakage flow $\dot{m}_g$ is expressed as follows:

$$P_g = \dot{m}_g \Delta h$$

and is caused by the gap width $s$, which is shown in Fig. 8. The gap loss coefficient is defined with machine specific parameters as follows:

$$\zeta_g = \frac{2 h_l}{w^2} = \frac{2 P_g/\dot{m}}{u^2}$$

with the enthalpy loss through the gap $P_g/\dot{m}$. Equations (38) and (39) result in

$$\zeta_g = \frac{\dot{V}_g}{\dot{V}}$$

and the volume flow ratio $\dot{V}_g/\dot{V}$ is calculated with Eck’s formula [24]:

$$\dot{V}_g \dot{V} = s_x^4 \frac{2 \mu B}{\varphi} \sqrt{0.8 \psi - \frac{1}{4} \left( 1 - \nu^2 \right)} + \varphi^2 \nu^2$$

Stonjek’s investigations [25] with relative gap width difference of $(s_x - s_0)/(s_x = 0.8)$ show a negligible change in pressure coefficient.
\[(\psi' - \psi)/\psi \approx -0.045.\] 

Accordingly, the gap loss coefficient for centrifugal fans is given by

\[\Delta \zeta_g = 4\mu_g \frac{\psi'}{\psi} (s_i - s_i) \sqrt{0.8\psi - \frac{1}{4} (1 - \nu^2)} + \psi \nu^{-4} \] (42)

The cross section ratio is \(A_1/A_2 = 4\pi D_i s_i/(\pi D_s^2) = 4s_i/\nu.\)

2.2.4 Induced Loss. In an unshrouded axial fan, the tip clearance loss is treated as an induced drag derived and validated by Pelz and Karstadt [26]:

\[\zeta_{\text{ind}} = \frac{2h_1}{w} = C \cdot s_i \left(\frac{\psi'}{\psi}\right)^2 \] (43)

with the machine-dependent and dimensionless gap constant \(C\) (shape). Compressible effects are not observed by Karstadt and therefore neglected.

2.2.5 Summarized Loss Coefficients. For the scaling of axial fans with rotor (r) and stator (s) Eq. (16) yields

\[
\zeta = \zeta_{\text{rs}} + \zeta_{\text{inc,r}} + \zeta_{\text{inc,s}} + \zeta_{\text{ind}}
\] (44)

and for centrifugal fans with impeller (r) and volute (s), Eq. (16) results in

\[
\zeta = \zeta_{\text{rs}} + \zeta_{\text{inc}} + \zeta_{\text{r}} + \zeta_{\text{g}}
\] (45)

2.3 Power Coefficient. The power coefficient depends on the power transferred to the fluid \(\lambda_{\text{eq}}\), the additional power due to internal leakage flow \(\lambda_{\text{g}}\), and the friction between rotating impeller and casing \(k_c\). In this context, we use Froude’s hypothesis [5] again:

\[\lambda = \lambda_{\text{eq}} + \lambda_{\text{g}} + \lambda_{\text{f}} \] (46)

The power transferred to fluid is assumed to be constant, resulting in \(\Delta \lambda_{\text{eq}} = 0\).

2.3.1 Centrifugal Fans. The leakage flow of centrifugal fans was derived in Sec. 2.2.1. The additional power coefficient due to leakage flow is assumed to be the same than the gap loss \(\lambda_{\text{g}} = \zeta_{\text{g}}\). Hence, the scaling of power coefficient is equivalent to \(\Delta \lambda_{\text{g}} = \Delta \zeta_{\text{g}}\), see Eq. (42).

The friction between impeller disc and casing (outside the flow channel) is determined by Gülich’s ansatz [18]:

\[P_{\text{f,ax}} = \zeta_{\text{f,ax}} \frac{q \pi^3 D^2}{4} \] (47)

\[\zeta_{\text{f,ax}} = \frac{\pi}{2 \Re s_{\text{ax}}} + \frac{0.02}{(\Re/2)^{0.2}} \frac{1 + 2 s_{\text{ax}}/D \tilde{F}_{\text{f,ax}}}{1 + s_{\text{ax}}/D} \] (48)

\[\tilde{F}_{\text{f,ax}} = \left[\frac{\log(25/\Re)}{\log(0.4 s_{\text{ax}} + 25/\Re)}\right]^{2.15} \] (49)

\[k_{\text{f,ax}} \frac{2k}{D} \] (50)

with the friction coefficient \(\zeta_{\text{f,ax}}\), the axial gap width \(s_{\text{ax}}\) (see Fig. 8), and the correction function of the rough surface \(\tilde{F}_{\text{f,ax}}\).

The power loss coefficient is expressed as follows:

\[\lambda_{\text{f}} = \frac{2P_{\text{f,ax}}/h}{\pi^2} = \frac{2\zeta_{\text{f,ax}}}{\pi} \] (51)

and its scale yields

\[\Delta \lambda_{\text{f}} = \frac{2 \Delta \zeta_{\text{f,ax}}}{\pi} \] (52)

Finally, the scale of power coefficient for centrifugal fans is given by

\[\Delta \lambda = \Delta \lambda_{\text{g}} + \Delta \lambda_{\text{f}} \] (53)

2.3.2 Axial Fans. The leakage flow of axial fans is described by an induced drag, which does not have an effect on the power coefficient (\(\Delta \lambda_{\text{g}} = 0\)). This statement is supported by measurements on axial fans with different relative blade tip clearances \(s_f\) presented by Heß [27] and Karstadt [28].

The friction in the hub between rotor and stator is ignored \(\Delta \lambda_{\text{f}} = 0\) because the hub-tip ratio is mostly \(\nu < 0.6\) and causes low friction factors \(\lambda_{\text{f}} \propto \nu^5\) \((\nu^5 = 0.6^5 \approx 0.08)\). Hence, the power coefficient scale for axial fans is expressed as follows:

\[\Delta \lambda = 0 \] (54)

2.4 Pressure Coefficient Scaling. The scaling of efficiency, pressure coefficient, and power coefficient are given by

\[\Delta \eta = \eta - \eta', \quad \Delta \psi = \psi - \psi', \quad \Delta \lambda = \lambda - \lambda' \] (55)

and hence, the pressure coefficient of the prototype reads

\[\psi = \eta \lambda = (\eta' + \Delta \eta)(\lambda' + \Delta \lambda) \] (56)

Equation (56) is valid for all fans, but for axial fans, the power coefficient is constant \(\Delta \lambda = 0\), and in this case, the pressure coefficient scale results in

\[\psi = (\eta' + \Delta \eta)\lambda' = \psi(1 + \Delta \eta / \eta') \] (57)

In 2010, Heß [27] suggested this formula based on empirical observations.

3 Validation

To validate the previously deduced scaling method, experimental investigations are carried out on standardized test rigs for axial and centrifugal fans. An overview of all investigated operation points given by their Reynolds Re and Mach number Ma as well as their respective \(\sigma\) and \(D\) are presented in Fig. 9.

The test rig setup including the specific pressure vessel design is given in Appendix D. All test rigs were built according to ISO 5801:2010 [29]. The axial fan test rigs are described by Pelz and Heß [30] and Saul and Pelz [31,32]. In addition, the experimental

Fig. 9 Overview of all investigated fans and operation points at the Chair of Fluid Systems, TU Darmstadt
The setup of the measurements in the pressure vessel is published by Saul et al. [16].

For the validation of the introduced scaling method, two centrifugal (C) as well as two axial (A) fans of different scalings and roughness are chosen. The relevant data are summarized in Table 1. The designation of the fans consists of the type, i.e., C for centrifugal and A for axial fan, the fan number showing which fans are geometrically similar and the diameter in mm. In case of a change in roughness, the letter R is added.

In addition, experiments are conducted for three different working media: air, $\gamma = 1.4$; nitrogen $N_2$, $\gamma = 1.4$; and Argon Ar, $\gamma = 1.67$.

### Table 1 Seven fans used for the validation of the scaling method

<table>
<thead>
<tr>
<th>FAN</th>
<th>$\sigma$</th>
<th>$D$ (m)</th>
<th>$k_{+x} \times 10^6$</th>
<th>$k_{+y} \times 10^6$</th>
<th>$s_{+x} \times 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1_331</td>
<td>0.1</td>
<td>0.331</td>
<td>22.1</td>
<td>7.3</td>
<td>2.8</td>
</tr>
<tr>
<td>C1_1324</td>
<td>0.1</td>
<td>1.324</td>
<td>1.21</td>
<td>2.49</td>
<td>2.6</td>
</tr>
<tr>
<td>C2_224</td>
<td>0.3</td>
<td>0.224</td>
<td>17.9</td>
<td>26.8</td>
<td>1.6</td>
</tr>
<tr>
<td>A1_800</td>
<td>1.0</td>
<td>0.8</td>
<td>5.6</td>
<td>19.5</td>
<td>1.3</td>
</tr>
<tr>
<td>A2_250</td>
<td>1.5</td>
<td>0.25</td>
<td>37.0</td>
<td>83.7</td>
<td>1.0</td>
</tr>
<tr>
<td>A2_1000</td>
<td>1.5</td>
<td>1</td>
<td>12.0</td>
<td>7.2</td>
<td>1.0</td>
</tr>
<tr>
<td>A2_1000R</td>
<td>1.5</td>
<td>1</td>
<td>51.0</td>
<td>7.2</td>
<td>1.0</td>
</tr>
</tbody>
</table>

#### 3.1 Scaling of Fan Characteristics

Figure 10 shows the efficiency characteristic scaling of a high-pressure centrifugal fan ($\sigma = 0.1$). These investigations were conducted in the pressure vessel at constant Mach number $Ma = 0.1$. The efficiency $\eta$ is plotted against the flow coefficient $\phi$ and the reference Reynolds number is $Re = 1.26 \times 10^5$. Furthermore, the introduced scaling method is marked by “SCALING METHOD” (solid line), and the reference is the scaling method proposed by the ISO 13348:2007 (dashed line).

In the best efficiency point, the efficiency is underestimated by $\approx 0.5$ percentage points, which is also the largest deviation between scaling and measurement. Thereby, the introduced scaling method predicts 1.5 percentage points higher efficiencies than the scaling method defined by the ISO 13348:2007.

In addition, the presented scaling method offers the option of scaling the pressure coefficient. In Fig. 11, the pressure coefficient scaling of the previous example is shown.

The universal applicability is demonstrated in Fig. 12 by means of the efficiency scaling of an axial fan ($\sigma = 1.0$).

#### 3.2 Scaling of Friction

Now we focus on the best efficiency point to compare and validate the scaling results in a large Reynolds number range.

Therefore, Fig. 13 shows the validation for a medium-pressure centrifugal fan ($\sigma = 0.3$) at constant Mach number $Ma = 0.12$. The solid lines represent the efficiency scaling presented within this article and the dashed line gives the scaling as proposed by ISO 13348:2007. The reference is marked by a filled circle.

Again, the presented scaling method works much better than the method proposed by the ISO 13348:2007 and at the same time never overpredicts the efficiency.

A change of relative roughness $k_+$ and Reynolds number $Re$ is presented in Fig. 14 for an axial fan ($\sigma = 1.5$). The scaling starts at the model size fan (A2_250), and two different prototypes with
different relative roughnesses (A2_1000 and A2_1000R) are given. The effect of relative roughness is about 1.5%, and this scaling result is confirmed by the experimental investigations.

3.3 Scaling of Compressible Effects. To validate the scaling of compressible effects, we conducted experimental investigations in the pressure vessel for the high-pressure centrifugal fan ($\sigma = 0.1$). Figure 15 shows the efficiency at constant Reynolds number $Re = 1.4 \times 10^6$. For moderate Mach numbers $Ma < 0.37$, the presented scaling for compressible effects fits the measurements. A slight drop in efficiency can be detected for increasing Mach numbers.

3.4 Combined Effects of Compressibility and Friction. Figure 16 illustrates all considered effects of the presented scaling method in one contour plot. The application is done for a high-pressure centrifugal fan ($\sigma = 0.1$) with model (C1_331) and prototype (C1_1324) having different relative roughnesses. The Mach number $Ma$ is plotted against the Reynolds number $Re$ and the contour indicates the efficiency at the best efficiency point ($\eta = \eta_{BEP}$). The reference point is marked by the black and white split circle on the left side of the diagram. The circle size represents the measurement uncertainty ($\delta \eta$), and its surface color intensity illustrates the minimal deviation of measurement and scaling, $\min(\Delta \eta = \Delta \eta_{scale} \pm \delta \eta_{ref} \pm \delta \eta)$. The smaller and darker the circle, the lower is the measurement uncertainty and the deviation between measurement and scaling, respectively.

While the friction loss is scaled in horizontal direction, the inertia losses are scaled vertically. The discontinuity at Reynolds number $Re = 5.6 \times 10^6$ marks the change of model to prototype, resulting in an efficiency increase of 2.4 percentage points due to differing relative roughnesses. With the increase in rotational speed $n$, the Mach number $Ma$ and Reynolds number $Re$ increase, which is indicated by the black lines marked with $Ma = Kn Re$ and $Ma' = Kn' Re'$. At high Reynolds numbers, i.e., $Re > 10^7$, the measurements do not align with the predicted line. This results form an increased ambient temperature, which in turn lowers the Mach and Reynolds number. In Appendix E, a calculation example is given in Table 5.

3.5 Application to Other Machines. To demonstrate the universal applicability of the introduced scaling method, a centrifugal pump with $\sigma = 0.52$ is studied. The considered measurement data are given by Rotzoll [33], who investigated a centrifugal pump with a large Reynolds number variation of $Re_{max}/Re_{min} = 100$. As the roughness is unknown, the roughness of the impeller and the casing are estimated with $k_s = 50 \times 10^{-6}$ according to DIN 4766 considering the achievable roughness depths at that time. For the calculation of the friction losses between impeller and casing, the rear side of the impeller is simplified and assumed to be a disc. Furthermore, the Carnot loss at the impeller outlet is neglected as the
The presented scaling method is (i) reliable due to the physics base. Friction, inertia, and induced losses are calculated mathematically on a local level using loss models and transformed into machine related considerations. Furthermore, dimensionless independent parameters are used for the calculation: Reynolds number $Re$, Mach number $Ma$, relative roughness $k_r$, relative gap width $s_r$, operating point $\psi$, and the type of the fan $\sigma$. (ii) The good scaling results of our method demonstrates its value. In all experimental investigations, the scaling is much better than the reference method, which is the recommended method in the ISO 13348:2007. Moreover, the introduced method predicts the efficiency $\eta$ as well as pressure coefficient $\psi$ and power coefficient $\lambda$ in part load, best efficiency point, and over load conditions. Common scaling methods focus on the best efficiency point while neglecting off-design conditions. Yet, overload and partial load make up a significant operation time. (iii) The presented scaling method is applicable to fans in general, which is demonstrated by the conducted experimental investigations in the specific speed range up to $\sigma = 0.11.5$.

In comparison to computational flow dynamics simulations, the method is rather simple and the computing time is significantly lower (less than one second).

In addition, a new measurement method for fans to investigate compressible (Mach number) and frictional (Reynolds number) effects independent of each other is used to validate the scaling method. Implementing this introduced measurement method, the results are validated experimentally.

For moderate Mach numbers $Ma < 0.4$, the influence of the Mach number is low, which has been validated by experiments. In turn, the Mach number effect becomes interesting for centrifugal fans with low specific speeds $\sigma \leq 0.1$ and Mach numbers $Ma > 0.5$. Therefore, further investigations with different centrifugal fans at high Mach numbers $Ma > 0.5$ are necessary. Since these investigations represent a crossover to radial compressors, an extension of the method for these turbomachinery is also conceivable.

### Acknowledgment

The authors would like to thank the Arbeitsgemeinschaft industrieller Forschungsvereinigungen “Otto von Guericke” e.V. (AiF), the Bundesministerium für Wirtschaft und Technologie (BMWi), and the Forschungsvereinigung für Luft- und Trocknungstechnik (FLT) e.V. whose support made this work possible.

### Conflict of Interest

There are no conflicts of interest.

### Data Availability Statement

The authors attest that all data for this study are included in the paper.

### Nomenclature

- $a$ = speed of sound (m/s)
- $b$ = impeller width (m)
- $f$ = shape factor (–)
- $h$ = specific enthalpy (J/kg)
- $i$ = incidence angle (deg)
- $k$ = absolute roughness (m)
- $l$ = molecular mean free path length (m)
- $l =$ characteristic length scale (m)
- $m = exponent (–)$
- $n = rotational speed (1/s)$
- $p = pressure (Pa)$
- $s =$ blade tip clearance (m)
- $g = gap width (m)$
- $u =$ circumferential velocity (m/s)
- $w =$ relative velocity (m/s)
- $z =$ number of blades (–)
- $A =$ cross section ($m^2$)
- $B =$ volute depth (m)
- $C =$ gap constant for axial fans (–)
- $D =$ diameter (m)
- $F =$ correction function for compressible flow (–)
- $L =$ volute width (m)
- $L = independent load fraction (–)$
- $P =$ shaft power (W)
- $R =$ individual gas constant (J/(kg K))
- $T =$ absolute temperature (K)
- $m = mass flowrate (kg/s)$
- $V = volume flow rate (m^3/s)$
- $A_w =$ wetted surface ($m^2$)
- $F_1 =$ correction function for friction factor (–)
- $Kn =$ Knudsen number (–)
- $Ma =$ Mach number (–)
- $Re =$ Reynolds number (–)
- $\overline{u} =$ average of relative velocity (m/s)

### Greek Symbols

- $\alpha =$ flow angle of the absolute velocity (stator) (deg)
- $\beta =$ flow angle of the relative velocity (rotor) (deg)
- $\gamma =$ isentropic exponent (–)

---

**Fig. 17** Efficiency scaling in the best efficiency point $\eta_{BEP}$ of a centrifugal pump with $\sigma = 0.52$ versus Reynolds number $Re$ according to Rotzoll [33]
\[ \delta = \text{uncertainty operator} \quad (-) \]
\[ \Delta = \text{difference operator} \quad (-) \]
\[ \varepsilon = \text{inefficiency} \quad (-) \]
\[ \zeta = \text{loss coefficient} \quad (-) \]
\[ \eta = \text{efficiency} \quad (-) \]
\[ \kappa = \text{scaling factor} \quad (-) \]
\[ \lambda = \text{power coefficient} \quad (-) \]
\[ \mu = \text{dynamic viscosity} \quad (\text{kg}/\text{m s}) \]
\[ \mu_k = \text{gap constant for centrifugal fans} \quad (-) \]
\[ \nu = \text{hub-tip ratio} \quad (-) \]
\[ \psi = \text{kinematic viscosity} \quad (\text{m}^2/\text{s}) \]
\[ \Pi = \text{pressure ratio} \quad (-) \]
\[ \varrho = \text{density} \quad (\text{kg/m}^3) \]
\[ \phi = \text{flow coefficient} \quad (-) \]
\[ \psi = \text{pressure coefficient} \quad (-) \]

**Subscripts**

1 = inlet
2 = outlet
\( \infty = \text{infinity} \)
\( \text{+} = \text{dimensionless} \)
\( \text{ax} = \text{axial} \)
\( \text{b} = \text{blade} \)
BEP = best efficiency point
C = Carnot
f = friction
g = gap
i = inner
\( \text{in} = \text{index} \)
\( \text{inc} = \text{incidence} \)
\( \text{ind} = \text{induced} \)
1 = loss
\( \text{r} = \text{rotor} \)
\( \text{ref} = \text{reference} \)
s = stator
S = shaft
t = total
\( \text{tr} = \text{transition} \)
w = wetted surface
\( \text{w} = \text{wave drag} \)

**Superscripts**

\( ^{'} = \text{(first) model} \)
\( ^{''} = \text{second model} \)
\( ^{*} = \text{local scale} \)

Appendix A: Wetted Surfaces and Flow Velocities

Table 2 summarizes the wetted surfaces, and Table 3 presents all length scales and velocities in the machine and local scale.

Appendix B: Correction Functions for the Compressibility

The correction function for compressible flows for the incidence loss \( F_{\text{inc}} \) is given in Fig. 18 and for the Carnot loss in Fig. 19.

Appendix C: Derivation of the Flow Angles

For axial fans, the flow angle at rotor inlet is expressed as follows:

\[ \beta = \arctan \left( \frac{c_1}{u_m} \right) \]  

(C1)

The absolute velocity is \( c_1 = c_{\text{ax}} = V_1/A_1 \) for vortex free inlet condition. The volume flowrate is given by \( V_1 = \pi D^2 u_m/4 \), the cross section is \( A_1 = 4/\pi (D^2 - D_i^2) \) and the circumferential velocity at mean section is \( u_m = u(1 + \nu)/2 \). Hence, Eq. (C1) results in

\[ \beta = \arctan \left[ \frac{2 \nu}{(1 - \nu)(1 + \nu)} \right] \]  

(C2)
For constant inner and outer diameter \(D_i, D = \text{const.},\) the axial velocity is constant \(c_{ax} = \text{const.}\) as well as the circumferential velocity at mean section \(u_m = \text{const.}\). Therefore, the flow angle at stator inlet yields

\[
\alpha = \arctan \left( \frac{c_{ax}}{c_{u2m}} \right) \tag{C3}
\]

with the circumferential velocity component \(c_{u2m}\). Euler’s turbine equation reads

\[
P_S = \dot{m} u_m (c_{u2m} - c_{u1m}) \tag{C4}
\]

and with vortex free inlet conditions (\(c_{u1m} = 0\)) Eq. (C4) yields

\[
c_{u2m} = \frac{P_S}{\dot{m} u_m} \]

and with the definition of the power coefficient, we get

\[
\frac{c_{u2m}}{u_m} = \frac{\lambda}{1 + \nu} \tag{C5}
\]

The equivalent but dimensionless form of Eq. (C3) yields

\[
\alpha = \arctan \left( \frac{q(1 + \nu)}{\lambda(1 - \nu)} \right) \tag{C6}
\]

For centrifugal fans, we use Eq. (C1), but the cross section is given by \(A_1 = \pi b_i D_i\), with the blade width at impeller inlet \(b_i\) and instead of \(u_m\) the circumferential velocity is \(u_1 = v \cdot u\). Finally, the flow inlet angle of a centrifugal fan is

\[
\beta = \arctan \left( \frac{q D}{4 b_i v} \right) \tag{C7}
\]

**Appendix D: Investigated Fans and Experimental Setup**

The specifications of the investigated fan types are summarized in Table 4.

All test rigs run with air at ambient conditions except for the investigations in the pressurized vessel running with nitrogen (N\(_2\)) or argon (Ar). The setup of the centrifugal fan test rigs is shown in Fig. 20 for the medium scaled test rig. The flow goes from left to right and passes firstly the volume flowrate nozzle (1, A). The nozzle is a standardized part, but for higher measurement accuracy, the volume flow nozzle is calibrated with a total pressure comb (2, B) [34]. After the throttle (4), the flow straightener (5) is placed to lower the turbulence and inhomogeneities. The fan inlet conditions (total temperature and static pressure) are measured in the measuring chamber (6) at position (C). The test fan (8) is connected with a torque meter (9) to the engine (11). The torque meter is located between impeller and bearings allowing the measurement of the real torque without measuring the losses in the bearings. This type of test rig has a free outlet, and therefore (in measurement plane D), the total temperature has to be measured and the static pressure is equal to the ambient pressure. The small-scale test rigs have the same design, but all length are scaled down.

The axial fan test rig is shown in Fig. 21 with the flow direction from left to right. The basic design is very similar to the circumferential fan test rigs, but at position (5), an auxiliary fan is added to run the test fan at overload conditions, too. The measuring chamber (8) has flow straighteners and screens to homogenize the flow. They are placed upstream the measuring position (C). The electric drive for the test fan (11) is placed more than \(>2D\) away from the fan outlet, which guarantees a flow with a free outlet. The torque meter (10) is located inside the hub between bearings and the rotor. The small-scale test rig differs only in the position of the electric drive, which is placed inside the hub.

The pressure vessel setup is shown in Fig. 2 and includes one of the small scale test rigs. The ambient conditions are determined by measuring the temperature \(T_0\) and pressure \(p_0\). In addition, the gas is analyzed to determine the specific gas constant \(R\) and isentropic exponent \(\gamma\). A heat exchanger cools the vessel and guarantees a constant temperature during the measurements.

**Table 4 Specifications of all investigated fan types**

<table>
<thead>
<tr>
<th>(\sigma)</th>
<th>(D) in mm</th>
<th>(\Delta \beta) in °</th>
<th>(\nu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centrifugal</td>
<td>0.1</td>
<td>331, 1324</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>224, 896, 2240</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>177.5, 355</td>
<td>–</td>
</tr>
<tr>
<td>Axial</td>
<td>1.0</td>
<td>800</td>
<td>–18:6:12</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>800</td>
<td>–18:6:12</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>250, 1000</td>
<td>–18:6-6</td>
</tr>
</tbody>
</table>
Appendix E: Calculation Example

Table 5 Efficiency scaling at the best efficiency point $\eta = \eta_{BEP}$ of a high-pressure centrifugal fan ($\varepsilon = 0.1$)

<table>
<thead>
<tr>
<th>Var.</th>
<th>Model</th>
<th>Prototype</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>0.331 m</td>
<td>1.324 m</td>
</tr>
<tr>
<td>$D_i$</td>
<td>0.089 m</td>
<td>0.356 m</td>
</tr>
<tr>
<td>$l$</td>
<td>0.146 m</td>
<td>0.384 m</td>
</tr>
<tr>
<td>$b$</td>
<td>0.01 m</td>
<td>0.04 m</td>
</tr>
<tr>
<td>$b_i$</td>
<td>0.026 m</td>
<td>0.104 m</td>
</tr>
<tr>
<td>$B$</td>
<td>0.052 m</td>
<td>0.208 m</td>
</tr>
<tr>
<td>$L$</td>
<td>0.068 m</td>
<td>0.272 m</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>$\nu_b$</td>
<td>7.3 $\mu$m</td>
<td>1.6 $\mu$m</td>
</tr>
<tr>
<td>$\nu_s$</td>
<td>2.4 $\mu$m</td>
<td>3.3 $\mu$m</td>
</tr>
<tr>
<td>$s$</td>
<td>0.93 mm</td>
<td>3.44 mm</td>
</tr>
<tr>
<td>$s_{ax}$</td>
<td>7.8 mm</td>
<td>34 mm</td>
</tr>
<tr>
<td>$\mu_b$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\mu_s$</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$\Delta \psi$</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>Measurement</td>
<td>$\Delta \psi$</td>
<td>0.0905</td>
</tr>
<tr>
<td>$\varphi$</td>
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<td>0.021</td>
</tr>
<tr>
<td>$\psi$</td>
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<td>?</td>
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<tr>
<td>$\eta$</td>
<td>0.729</td>
<td>?</td>
</tr>
<tr>
<td>$\lambda$</td>
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<td>?</td>
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<tr>
<td>$\Delta \lambda$</td>
<td>0.068 m</td>
<td>0.272 m</td>
</tr>
<tr>
<td>$i$</td>
<td>0.026 m</td>
<td>0.104 m</td>
</tr>
<tr>
<td>$r$</td>
<td>0.04 m</td>
<td>0.172 m</td>
</tr>
<tr>
<td>$g$</td>
<td>0.104 m</td>
<td>0.356 m</td>
</tr>
<tr>
<td>$f$</td>
<td>36.286</td>
<td>36.448</td>
</tr>
<tr>
<td>$\Delta f$</td>
<td>0.0768</td>
<td>0.1</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.0041</td>
<td>0.0021</td>
</tr>
<tr>
<td>$\Delta \eta$</td>
<td>0.0768</td>
<td>0.021</td>
</tr>
<tr>
<td>$\Delta \psi$</td>
<td>1.28</td>
<td>0.172</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1.756</td>
<td>0.0768</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>1.756</td>
<td>0.172</td>
</tr>
</tbody>
</table>

Note: Fan C1_331 serves as a reference and fan C1_1324 as a geometrically similar prototype.

References