

Length scale analysis in wall-bounded turbulent flow by means of Dissipation Elements

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Abstract. The Dissipation Element (*DE*) method is used to analyse the geometric structure of turbulent pattern for several scalar fields in a turbulent plane channel flow obtained by Direct Numerical Simulations (*DNS*) of the Navier-Stokes equations. We show that both the probability density function (*pdf*) and the number of *DE* exhibit a clear scaling behavior as a function of the wall. Further, a remarkable insensitivity of the *pdf* is observed with respect to the Reynolds number and the choice of the scalar.

1. Introduction

The turbulent scalar field obtained by Direct Numerical Simulations (*DNS*) is divided into various finite size regions by identifying local pairs of minimal and maximal points in the scalar field $\phi(x, y, z, t)$ where $\nabla\phi = 0$. Any scalar quantity like the components of the velocity, the vorticity vector, the kinetic energy and its dissipation could be chosen as such a scalar field ϕ . Gradient trajectories of finite length starting from every point in the scalar field in the directions of ascending and descending scalar gradients will necessarily reach a minimum and a maximum point. The set of all points and trajectories belonging to the same pair of extremal points defines a dissipation element. Accordingly, the decomposition of the domain into dissipation elements follows from the structures of the flow itself and is not arbitrary. Different from classical length scale concepts in turbulence theory *DE* is domain filling which means that any turbulent scalar field can be completely decomposed into such elements. The shape of a *DE* is highly irregular. Nevertheless, the Euklidian distance ℓ between its extremal points and the absolute value of the scalar difference $\Delta\phi$ at these two points are appropriate parameters to uniquely parameterize the geometry and the field variable structure of a *DE*.

The authors in Wang & Peters (2006) report for the case of homogeneous shear turbulence that the mean *DE* length is in the order of the Taylor scale defined as $\lambda = (10\nu k/\epsilon)^{1/2}$. Our present results confirm this for the turbulent channel flow as well although it is a statistically inhomogeneous flow in the wall-normal direction y .

The turbulent channel flow investigated presently exhibits characteristic wall-normal layers namely, viscous sublayer, buffer layer, logarithmic region and the core/defect region. Our main focus will be to explore the influence of solid walls on the *DE* distribution, i.e. the wall-normal dependency of statistical parameters such as the number of *DE*, the *DE* length ℓ and its relation

to classical length scales. In addition, the influence of the choice of the friction Reynolds number Re_τ as well as of the scalar variable ϕ is studied. The probability density function *pdf* assumed to be a function with the dependencies $P(\ell, \Delta\phi, y, Re_\tau)$ is examined closely.

DNS of turbulent channel flow

Corresponding to the respective wall-normal layer different turbulence phenomena are dominant leading to inhomogeneity in all wall-normal statistics. This typically applies to the turbulent length scales as well. While sufficiently far from the wall large scale length scales ($\ell \gg \eta$) are widely independent of the influence of viscous forces the latter become very influential approaching the wall. The anisotropic largest turbulent scales correspond to the integral length scale which are of the order of channel height.

A spectral numerical method using Fourier series in the horizontal streamwise (x) and spanwise (z) directions and Chebyshev polynomial expansion in the wall-normal (y) direction is applied to solve the three-dimensional time-dependent incompressible Navier-Stokes equations in the dimensionless form (for details see Lundbladh *et al.* (1999))

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{Re_\tau} \nabla^2 \mathbf{u}, \quad (1)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (2)$$

where \mathbf{x} , t , \mathbf{u} and p are respectively the position vector, time, the velocity vector and pressure. Time integration is performed using a third order Runge-Kutta scheme for the advective and forcing terms and second order Crank-Nicolson for the viscous terms. While periodic boundary conditions are applied in the homogeneous streamwise (x) and spanwise (z) directions no-slip boundary condition is adopted at the channel walls where $\mathbf{u}(x, y = \pm 1, z) = 0$. $\mathbf{u} = (u, v, w)$ denote the streamwise, wall-normal and spanwise velocity components.

Presently, the channel flow at two Reynolds numbers $Re_\tau = u_\tau h / \nu$ are analyzed, where $u_\tau = \sqrt{\tau / \rho}$ is the friction velocity, τ is the mean wall-shear stress at the wall and ρ is the density (see table 1). The numerical resolution $N_x \times N_y \times N_z$ reads $512 \times 257 \times 256$.

Table 1. Simulation parameters. h denotes the channel half-height.

-	Re_τ	L_x/h	L_z/h
Case 1	180	2π	π
Case 2	360	2π	π

Dissipation Element statistics

The probability density function (*pdf*) of the Euclidian length ℓ , the mean scalar difference conditioned on ℓ , i.e. $\langle \Delta\phi | \ell \rangle$, and in particular its dependence on the wall distance y , describing each element will be analyzed in detail in the present work. The *pdf* marks the distribution of the element lengths and the latter shows the scaling behavior of the scalar difference with respect to the length. Hence, rather generally, the present *pdf* is a function with the dependencies $P(\ell, \Delta\phi, y, Re_\tau)$ satisfying the normalization condition

$$\int_0^\infty P \, d\ell = 1. \quad (3)$$

The mean or expectation value for the DE length is defined as

$$\ell_m = \int_0^\infty \ell P d\ell . \quad (4)$$

For the analysis of DE we investigate three scalar field variables, the fluctuation of the streamwise velocity component u , the turbulent kinetic energy k and its dissipation rate ϵ , which is highly intermittent, at two Reynolds numbers being a factor of two apart. The latter two are defined as

$$k = \frac{1}{2} (u^2 + v^2 + w^2) , \quad (5)$$

$$\epsilon = \frac{1}{2} \nu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 , \quad (6)$$

where ν is the kinematic viscosity.

The linear length ℓ_m of *DE* averaged over the horizontal directions x, z is plotted in figure 1(a) against the wall-normal direction $y/h \in [0, 1]$. In the vicinity of the wall regions including viscous and buffer layers one can notice a very steep rise with a local peak. For the logarithmic layer and the core region there is a characteristic linear behavior up to about $y/h \approx 0.8$ as evidence for a linear grow of the *DE* size in this region such as

$$\ell_m \sim y + c . \quad (7)$$

Around the center of the channel with the weakest shear the size of the *DE* remains largely constant. The curves indicate that small elements are mostly located near the wall whereas elements are larger in average in the center of the channel with the highest velocity. Since more extremal points are produced for the scalar of the turbulent kinetic energy k - and even more for its dissipation rate ϵ - whose distances define the lengths of the *DE* its elements are smaller compared to the fluctuation of the streamwise velocity u . In accordance with Wang & Peters (2006) we find that *DE* length scales excellent with the known classical Taylor length scale λ .

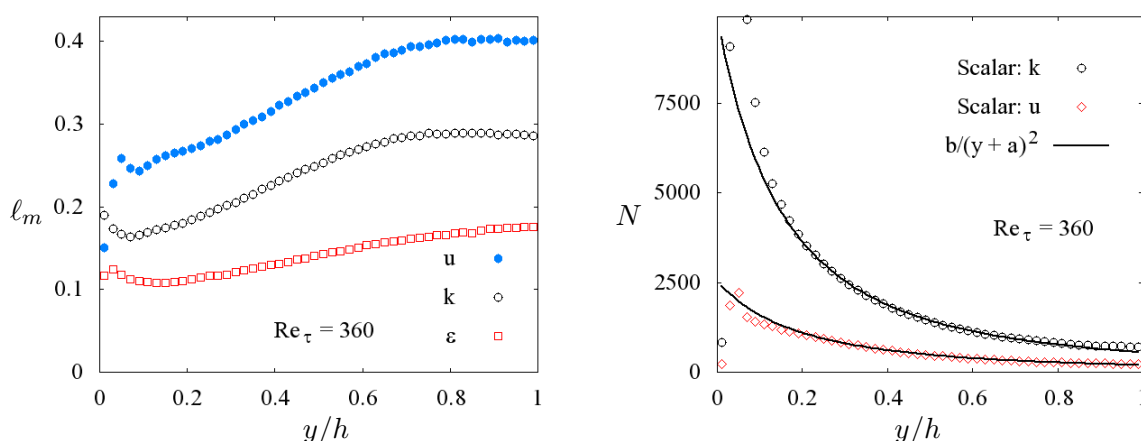


Figure 1. Mean *DE* length ℓ_m (a) and the number of *DE* (b) as a function of the wall-normal direction.

Based on the geometric properties i.e. in particular space-filling character of *DE* and their linear increase along the wall-normal direction as shown above we derive a scaling relation for the number of *DE* as a function of y : $N(y) = b/(y+a)^2$, where a and b are constants (figure

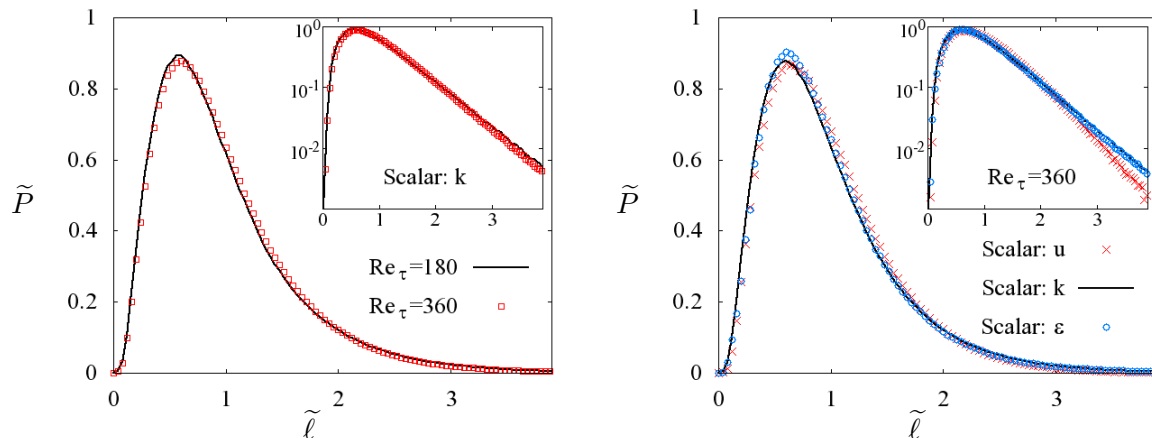


Figure 2. Comparison of the overall *pdf* of the entire channel for different (a) Reynolds numbers and (b) scalar variables.

1(b)) to be taken from the data. The new scaling law shows that only weak discrepancies in the very center of the channel and in the near-wall regions highlighting the strong influence of solid walls.

The figures in 2 illustrate the dependence of the marginal *pdf* of the Euclidian DE length on (a) Reynolds number and (b) different scalar variables. Here, $\tilde{P} = \ell_m P$ and $\tilde{\ell} = \frac{\ell}{\ell_m}$ are invariants where ℓ_m is the mean DE length defined in equation (4). All *pdf* feature a clearly non-Gaussian distribution. For small DE the curves show a very steep rise before reaching a maximum at around $\ell \approx 0.6\ell_m$ marking the highest probability for DE length scales. With further increasing DE length an exponential decay becomes evident which can be seen even better in the small inset figures. Figure 2 (a) reveals a very interesting insensitivity with respect to Reynolds number for the scalar of k as an example. Despite the relatively big gap between both Re their *pdf* coincide almost perfectly. The same conclusion applies to the dependency regarding to the choice of the scalar variable as can be observed in figure 2 (b). All three curves exhibit a very good agreement with each other almost throughout the entire spectrum of DE length scales. Only around the maximum and in the far-tail of the *pdf* one can see small deviations.

At this point *pdf* for k and ϵ seem to be closer than the velocity component u which decays faster as displayed in the semi-logarithmic illustration. Nevertheless, the obvious insensitivity with respect to the choice of the scalar variable implies a pronounced DE isotropy for all DE spectrum.

In figures 3 *pdf* of DE length have been investigated for three characteristic wall-normal layers of a turbulent channel flow exemplarily for the case of k to explore the influence of the distance from the wall. The very thin viscous sublayer has been excluded to avoid ambiguity of the DE definition at the wall. The layers defined in terms of the wall distance are as follows.

- buffer region: $5 < y^+ < 30$
- log region: $y^+ \geq 30, y/h < 0.3$
- core region: $0.3 \geq y/h < 1$

where $y^+ = yu_\tau/\nu$.

The *pdf* in figure 3 (a) are normalized only according to equation (3) while in figure 3 (b) the length scale has also been normalized by the corresponding local mean length ℓ_m of the channel layer. As can be seen, without re-scaling of the *pdf* figure 3 (a) features huge deviations indicating a strong impact of the wall distance on the *pdf*. The *pdf* closer to the

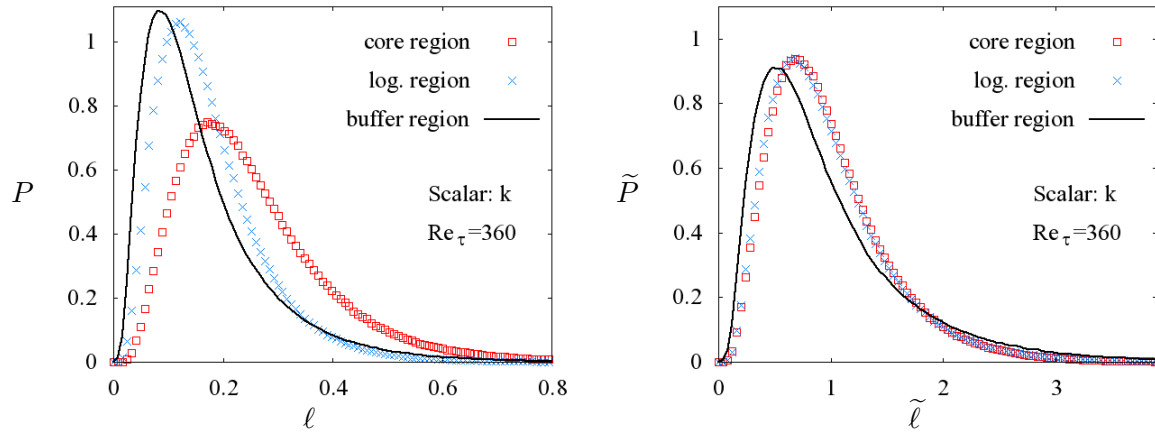


Figure 3. pdf for characteristic wall-normal layers where in (a) ℓ is not normalized while in (b) ℓ is normalized with ℓ_m .

wall exhibit a narrower shape while at the same time the maximum peak moves towards smaller element lengths. Thus, one can conclude that smaller elements can be encountered near the wall whereas larger elements are mostly located in the far-wall regions. This is in agreement with the observations on the basis of figure 1 where the DE length distribution illustrates the influence of the wall distance. Furthermore, with its broader shape *pdf* for the core region reveals that this layer consists of a wider spectrum of length scales than the logarithmic and the buffer regions.

On the other hand, the *pdf* curves exhibit an evident similarity if ℓ is normalized with the according mean length, i.e. $\tilde{\ell} = \ell/\ell_m$. This behavior is a strong indication toward a Lie scaling group and its corresponding similarity (Oberlack (2001)). Thus, the rescaled similarity variable $\tilde{P} = \ell_m P$ is plotted in figure 3 (b). An almost complete collapse of the *pdf* is displayed for the logarithmic and the core regions which together amount to more than 90% of the entire channel. In contrast, *pdf* for the near-wall buffer region is substantially different from others as a result of predominant wall and viscous effects. The lack of linear scaling of the mean DE length in the buffer layer which was observed in the far-wall regions is certainly the reason for this deviation.

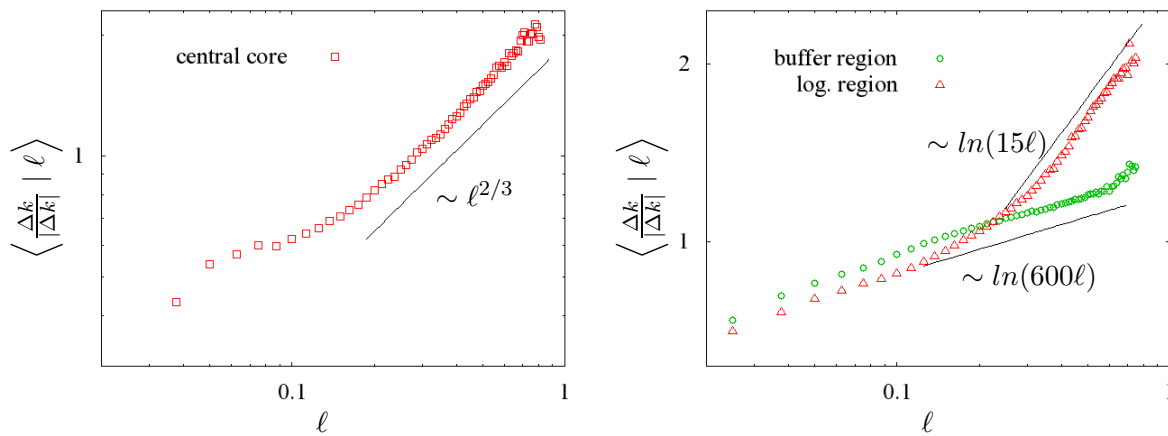


Figure 4. Conditional mean scalar differences for different wall-normal channel layers. (a) logarithmic, (b) semi-logarithmic.

Finally, we examine the conditional mean scalar differences between the values at the extremal points conditioned on the length of the corresponding dissipation element for the instantaneous turbulent kinetic energy k . Therefore, the first order conditional moment is investigated to measure its scaling along wall-normal distance. The first moment based on gradient trajectories is non-zero since the value of the turbulent kinetic energy increases per definition monotonically along a trajectory from the minimum to the maximum point.

Hence, in figure 4 conditional mean scalar difference is plotted for different wall-normal layers. In (a) we focus on a restricted region in the very center region (central core) corresponding to $0.8 \leq y/h \leq 1$ featuring the highest degree of isotropy and the weakest shear in present flow regime. For sufficiently large DE the logarithmic plot reveals an algebraic scaling which is remarkable close to the Kolmogorov exponent of $2/3$. Approaching the wall, of course, this scaling is likely to break under the influence of shear induced anisotropy as can be confirmed by the semi-logarithmic plots. In the near-wall buffer layer and the logarithmic layer where turbulent kinetic energy scales as u_τ^2 we observe a $\ln(\ell)$ law equivalent to the classical K^{-1} law in this very region (Perry *et al.* (1986)). In the buffer layer this behavior is extended to an even wider spectrum of elements.

Conclusion

Dissipation Element (*DE*) method has been applied to the wall-bounded turbulent channel flow to analyze the geometrical structure of turbulent length scales. Different statistics such as the probability density function *pdf* for different turbulent scalar variables, the conditional mean scalar difference of the turbulent kinetic energy and the DE length have been studied. The dependency of this statistics on the wall-normal distance in terms of characteristic wall-normal regions (buffer, logarithmic and core regions) has been investigated. Generally, strong influence of the wall could be observed in all statistics except for rescaled *pdf* for different channel layers yielding invariant forms of the *pdf*. Mean *DE* size was found to have clear linear scaling with respect to the distance from the wall as well as the conditional mean scalar differences between the extremal points of *DE*. We showed that in the very center of the channel Kolmogorov's $2/3$ scaling holds whereas layers closer to wall feature a logarithmic law rather than a power law.

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