# Shell model progress on neutrinoless double beta decay: nuclear matrix element uncertainties, neutrino exchange mechanism in seesaw models

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**Abstract.** We study the neutrinoless double beta decay in the context of the interacting shell model. Firstly, we estimate the uncertainties associated to the different approximations performed in the calculation of the nuclear matrix elements, which are necessary to obtain information about the neutrino masses. We then study the dependence of the nuclear matrix elements on the mass of the exchanged neutrinos and we discuss, within the type I seesaw model, which will be the contribution to the neutrinoless double beta decay rate of the different extra neutrinos, depending on their mass range. We also discuss how seesaw models could reconcile large rates of neutrinoless double beta decay with stringent cosmological bounds on neutrino masses.

## 1. Introduction

Neutrinoless double beta decay ($0\nu\beta\beta$ decay) is a second order weak process. In contrast to the well-known two-neutrino double beta decay ($2\nu\beta\beta$ decay), it requires neutrinos to be their own antiparticles in order to take place, this is, a positive measurement would proof the Majorana nature of neutrinos [1]. It would also be the first lepton number violating process detected to date. Moreover, the fact that these $\beta\beta$ processes happen in nuclei which are otherwise stable makes $0\nu\beta\beta$ decay the best candidate for establishing the Majorana nature of neutrinos. So far, there has been only one unconfirmed claim of detection $0\nu\beta\beta$ decay [2, 3], while several experiments will look for a $0\nu\beta\beta$ decay signal with unprecedented sensitivity in the next few years [4].

In addition, if one or several measurements are achieved, the nuclear matrix element (NME) of the transition can be used to find out which is the mechanism responsible for the decay and, in the case of the most simple light neutrino exchange, very valuable information about the neutrino masses will be obtained. In the latter case, the formula for the lifetime of the $0\nu\beta\beta$ decay is [5, 6]

$$\left(T_{1/2}^{0\nu\beta\beta}\left(0^+ \to 0^+\right)\right)^{-1} = G_{01}\left|M^{0\nu\beta\beta}(0)\right|^2 \left|\frac{m_{\beta\beta}}{m_e}\right|^2, \tag{1}$$

where $G_{01}$ is a kinematic factor, $M^{0\nu\beta\beta}(0)$ the NME, $m_e$ is the electron mass and $m_{\beta\beta} = \sum_k U_{ek}^2 m_k$ the so-called effective Majorana neutrino mass, with $m_i$ the neutrino masses and $U_{ek}$ the neutrino mixing matrix.

Consequently, it is of the utmost importance to obtain accurate nuclear matrix elements for the different $0\nu\beta\beta$ decay candidates.

## 2. NMEs and their uncertainties within the interacting shell model

Since the computation of the NMEs is rather challenging, a number of approximations are involved in any calculation. In consequence, the results will be subject to some uncertainties. Here we will make an estimate of the ones associated to the calculation of nuclear matrix elements in the context of the interacting shell model (ISM) [7]. This is one of the most popular methods that are employed to obtain these NMEs, even though other alternatives such as the quasiparticle random phase approximation (QRPA) [8, 9] can also provide valuable insight about which are the relevant nuclear structure elements in order to obtain reliable NMEs.

The ISM calculates the nuclear wavefunctions in relatively small valence spaces, typically one major harmonic oscillator shell or one spin-orbit shell. Adapted to each valence space there is an effective interaction which successfully describes the spectroscopy of the nuclei in the region. Consequently the first estimate we will have to do is that of the uncertainty introduced by the valence space and the effective interaction employed. The effect of having larger valence spaces was analyzed in Ref. [10], with the result that the NMEs increased by $\sim 15 - 20$ %. The same number was obtained in a QRPA calculation when it was quantified the effect of the orbits absent in a ISM valence space [11]. Since these results come from $0\nu\beta\beta$ decays in different regions, we take this $\sim 15 - 20$ % increase as a general estimate of the uncertainty due to the configuration space in ISM calculations.

The effect of the nuclear interaction was explored in Refs. [10] and [12], and a moderate dependence of $\sim 5 - 10$ % was found in both cases. Since the nuclei studied belong to different valence spaces, we will also take this figure as general.

ISM calculations of the $0\nu\beta\beta$ decay NMEs use the closure approximation, this is, summing over all the intermediate virtual states appearing in second order perturbation theory. This can be done since the transferred momentum in the process, $|\mathbf{p}| \sim 100$ MeV, is much larger than the energy difference between these virtual states, so that instead of the specific values of the energy of each state in the intermediate nucleus, an average value can be used. We estimate the error associated to this approximation to be around $5 - 10$ %, as has been suggested by QRPA calculations performed without this closure approximation [13]. This small dependence is also in agreement with the very soft variation that is seen in the NME as the average energy of the intermediate states is modified.

Until recently, another considerable source of uncertainty were the short range correlations (SRC). The reason to include them in the calculation is the fact that the nuclear operator needs to be regularized when acting in a truncated valence space, in the same manner as the nuclear interaction is. When this consistent regularization is not performed, new correlations, called SRC, have to be included in the calculation via a general prescription. However, such consistent calculations were performed recently, and it now seems that the SRC contribution is rather small and that proper UCOM [14] or Jastrow-type parametrizations can take these terms very well into account, with a precision of $\sim 5$ % [15, 16].

The above considerations can be classified as uncertainties related to the nuclear structure method used to compute de NMEs. On the other hand, also the weak nuclear currents that form the transition operation are subject to some approximations that translate into uncertainties in the NMEs.

Among these are the coupling constants and the form factors that appear in the nuclear currents. The greater uncertainty comes from the axial coupling. It is currently under discussion if it should be quenched from its bare value $g_A = 1.25$ to $g_A = 1.0$ or not [17, 12], as is required by the single $\beta$ and two-neutrino $\beta\beta$ decays where the operator is pure Gamow-Teller. However in the $0\nu\beta\beta$ decay the virtual neutrino makes the operator more involved, and as a consequence
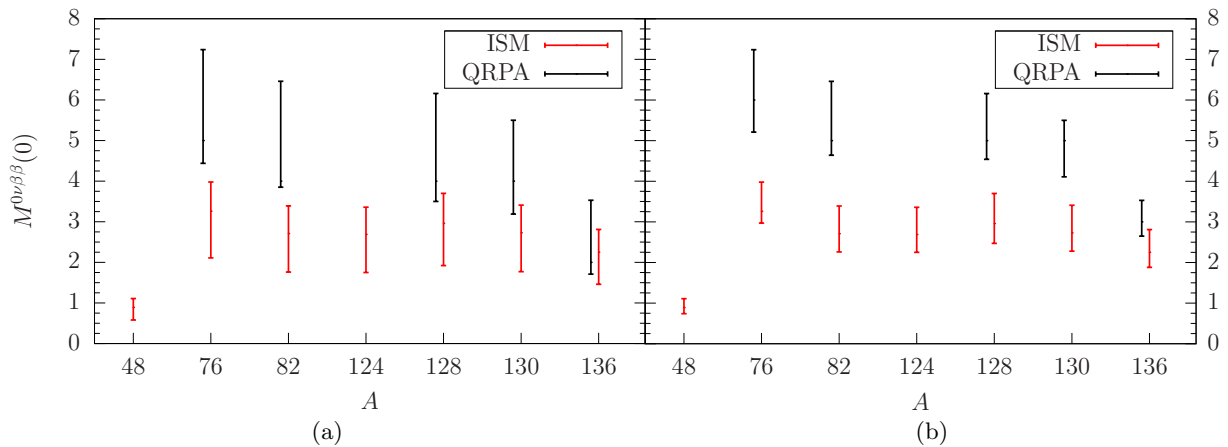
Figure 1: (1a) NMEs with their estimated uncertainties in ISM and QRPA calculations (taken from Ref. [16]), for different $0\nu\beta\beta$ decay emitters. (1b) Now the uncertainty in $g_A$ is not considered as the non-quenched value $g_A = 1.25$ is assumed for both ISM and QRPA calculations.

of this the particular Gamow-Teller $J^P = 1^+$ channel is not dominant anymore. In addition, depending on its relative sign, quenching it may result even in an enhancement of the NME. Taking this into account, a $\sim{}^{+5\,\%}_{-30\,\%}$ uncertainty can be estimated due to this effect. This is, together with the possible effect of larger valence spaces, the main source of uncertainty in ISM NME calculations and is also responsible for most of the spread found in QRPA results.

Another uncertainty related to these couplings comes from the form factors that drive their response at high momenta, commonly referred to as finite nuclear size (FNS) terms. Usually a dipole form is assumed, where a 'cutoff' parameter is required, taken from experiment. In this case, the variation in the NME due to different but reasonable values of the 'cutoffs' appearing in the FNS terms [18, 19] is less than 5 %.

Finally we can also estimate the possible effect of missing terms in the nuclear operator. The nuclear weak currents are obtained from a non-relativistic expansion in the impulse approximation, and usually only the lower order terms are kept. For instance, the effect of odd-parity terms in the currents with $p$-wave emitted electrons is neglected, as well as terms proportional to $\frac{\mathbf{p}^2}{m_N}$, where $m_N$ is the nucleon mass. In principle, these constitute corrections to the NME of the order of 1%, so we can conservatively estimate an additional uncertainty of less than 5 % in our results.

All in all the uncertainty in the valence space only moves the estimate up, the effect of axial quenching essentially moves it down, while the remaining contributions are expected to be Gaussian-distributed. Even though some of these errors may be correlated in a rather complicated way, as a first approximation we will take them as independent. Altogether, adding every contribution in quadrature we expect an overall uncertainty in the final NME of $\sim{}^{+25\,\%}_{-35\,\%}$.

We have used this estimate in Figure 1a, where we show for different $0\nu\beta\beta$ decay candidates the NMEs with their uncertainties calculated within the ISM, and compare them to the QRPA values of from Ref. [16]. The ISM valence spaces, interactions and the transition operator are described in detail in Ref. [7]. In both cases, the uncertainties are of the order of about 30%. When all sources of error are included, the results obtained by both methods become almost compatible in most cases, which is apparently in contradiction with the well-known fact that ISM NMEs are systematically larger than QRPA ones. The reason for this is that in Figure 1a NMEs with different $g_A$ are being compared, whereas a fair comparison between NMEs obtained by different methods only makes sense if they share the same $g_A$ value. Indeed, what makes

the error bars in Figure 1a so close is the fact that QRPA results with $g_A = 1.00$ resemble ISM non-quenched ($g_A = 1.25$) NMEs.

This is clarified in Figure 1b, where the same comparison between ISM and QRPA is made but ignoring the uncertainty associated to $g_A$ by assuming the non-quenched value $g_A = 1.25$. Here it is clear that ISM results are, except for the $A = 136$ case, systematically higher that the QRPA ones. The relatively small valence space of the ISM [8] or the absence of full correlations in QRPA calculations [20] have been proposed as explanations for these differences. To successfully understand them and hence to establish the actual value or the NMEs remains a major challenge for the theoretical $0\nu\beta\beta$ decay community.

### 2.1. Application to the $^{76}Ge$ decay

As an example, we will consider the case of the $^{76}$Ge decay. In Ref. [12] the NME was obtained with different effective interactions and SRC, resulting in the interval $2.81 < M^{0\nu\beta\beta}(0) < 3.52$. If we take into account the further uncertainties of the valence space, the FNS, the closure approximation, higher order terms in the transition operator and the $g_A(0)$ quenching, we end up with $2.11 < M^{0\nu\beta\beta}(0) < 3.98$.

We have used this result following the procedure described in Ref. [21] to derive bounds on the neutrino masses assuming the measurement claim of Ref. [3]. The result is $0.24$ eV $< m_{\beta\beta} < 0.89$ eV. This can be compared to constraints on neutrino masses from cosmology and neutrino oscillation data [22, 23]. This comparison has been performed in a recent work, Ref [24]. The conclusion is that some tension is found between cosmological results and the $0\nu\beta\beta$ decay claim if the mechanism for the latter is the exchange of the light neutrinos of the Standard Model (SM). In the following, we will study how a possible contribution of neutrinos other than the SM ones can contribute to eliminate this tension.

## 3. NMEs as a function of the neutrino mass

In order to study the role of additional neutrinos in $0\nu\beta\beta$ decay we first have to find which is the NME dependence on the neutrino mass, since in principle the exchange of any massive neutrinos may contribute to the $0\nu\beta\beta$ decay rate. This is done in Figure 2 for different decay candidates. As we notice, we distinguish only two different regions: up to $\sim 100$ MeV (the typical transferred momentum of the process), this is, while the momenta of the transferred neutrinos is larger than their mass, the NMEs are almost constant; in the second region, starting from $\sim 100$ MeV, the neutrino masses become larger than their momentum, and the NMEs decrease as $m_\nu^{-2}$. This behaviour, and the smooth transition between both regimes, can be understood in terms of the neutrino propagator $\sim \frac{1}{p^2+m_i^2}$, to whom the NME is proportional. No differences are found between the different decays studied.

The analysis of the previous section about the ISM NME uncertainties applies only to the case of light neutrino exchange. For very heavy neutrinos, the NMEs get very dependent on the SRC and FNS treatments. The reason is that when the mass becomes dominant compared to the transferred momentum there is a tendency in the operator to prefer the decaying nucleons to be unphysically close to each other, which has to be overcome by the FNS and SRC terms. In this case, if we study the estimated uncertainties in the same fashion as in the previous section we obtain a 15-20% for SRC uncertainties and 10% for FNS ones, which would lead to a final $\sim^{+35}_{-40}$ % uncertainty. However, one should take this number cautiously since the FNS approach for such heavy exchanged particles is not as reliable as for light ones [26], and the prescriptions for SRC have been obtained only in the context of light neutrino exchange.

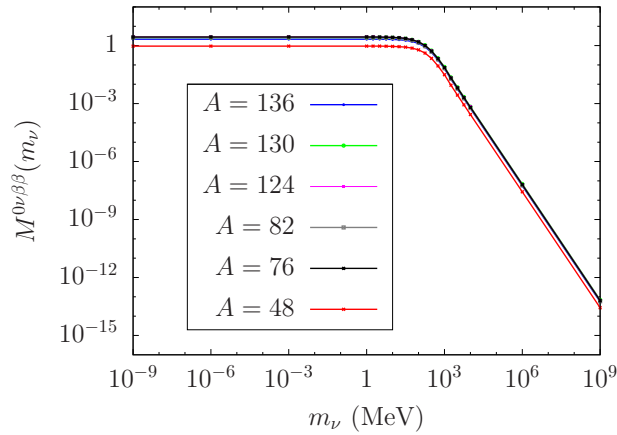Once we have the NMEs, in the case they are dependent on the neutrino mass Eq. (1) has

Figure 2: Nuclear matrix elements for the $0\nu\beta\beta$ decay as a function of the neutrino mass for different decay candidates. The data is available at Ref. [25].

to be replaced by

$$\left(T_{1/2}^{0\nu\beta\beta}\left(0^+ \to 0^+\right)\right)^{-1} = G_{01}\left|\sum_j U_{ej}^2 \frac{m_j}{m_e} M^{0\nu\beta\beta}(m_j)\right|^2. \tag{2}$$

Thus, in order to predict the consequences of these extra neutrinos in the $0\nu\beta\beta$ decay, we need some information about the mixing matrix elements $U_{ej}$. Since this implies physics beyond the SM, some particle physics model needs to be assumed. In the following we will concentrate in the most popular type I seesaw model [27, 28, 29, 30], where the extra neutrinos are the only particles apart from the SM ones. A complete discussion of $0\nu\beta\beta$ decay within seesaw models can be found in the recent work Ref. [24].

**4. Contribution of extra neutrinos to the $0\nu\beta\beta$ decay rate within the seesaw model**
Within this model, the following constraint, which origins in the vanishing of the $ee$ element of the Majorana mass matrix, must be fulfilled:

$$\sum_{i\in \text{SM}} m_i U_{ei}^2 + \sum_{I\in \text{extra}} m_I U_{eI}^2 = 0. \tag{3}$$

Then, three different scenarios may appear [24], depending on the masses of the extra neutrinos, which will be classified into light ($m_i < 100$ MeV) or heavy ($m_i > 100$ MeV):

- Only extra light neutrinos are introduced. Then the $0\nu\beta\beta$ decay is strongly suppressed, since

$$A \propto \left(\sum_{i\in \text{SM}} m_i U_{ei}^2 + \sum_{I\in \text{light}} m_I U_{eI}^2\right) M^{0\nu\beta\beta}(0) \approx 0, \tag{4}$$

  where $A$ stands for the amplitude of the process.

- Only extra heavy neutrinos are introduced. In this case the $0\nu\beta\beta$ decay is dominated by light SM neutrinos, since

$$A \propto \sum_{I\in \text{heavy}} m_I U_{eI}^2 \left(M^{0\nu\beta\beta}(m_I) - M^{0\nu\beta\beta}(0)\right) \approx \sum_{i\in \text{SM}} m_i U_{ei}^2 M^{0\nu\beta\beta}(0). \tag{5}$$

- Both extra light and heavy neutrinos are introduced. Now the $0\nu\beta\beta$ decay can be dominated by extra seesaw neutrinos, since

$$A \propto \left( \sum_{i \in \text{SM}} m_i U_{ei}^2 + \sum_{I \in \text{light}} m_I U_{eI}^2 \right) M^{0\nu\beta\beta}(0), \qquad (6)$$

and now both $m_i U_{ei}^2 \ll m_I U_{eI}^2$ and the constraint of Eq. (3) can be fulfilled at a time, given that the light and heavy extra states contributions cancel in this constraint. Of course, some degree of fine-tuning is required for this cancellation to take place. However, the present tension between the measurement claim of $0\nu\beta\beta$ decay and the present cosmological observations would be explained with a mild 50 % cancellation [24].

## Acknowledgments

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