Radial profile of moments of instantaneous axial velocity of a fully pulsed axisymmetric jet

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Abstract. Radial distribution of the first and higher moments of instantaneous axial velocity of a fully pulsed round jet were investigated along 80 diameters downstream from the jet exit. Response of the unsteady flow to the varying Reynolds and Strouhal numbers thus allowing a series of distinct pulses were sampled using a single wire hot-wire anemometer. It was found that the first moments, as compared to steady jet phenomena, do not maintain self-similarities over the flow domain but their shapes remain unchanged. At a lower Reynolds and Strouhal number, widening radial profiles of the first moment are obtained. Transverse profiles of the third and fourth turbulence moments explain the nature of random process in the mechanically excited jet. Their values were found to remarkably depart from the Gaussian values and to be significantly higher than those of the steady jet across the observed jet widths hence yielding more intense-highly irregular motions and more spreading jet. For all the tested cases, it was demonstrated that the direct effects of the controlled parameters on the flow field become stronger in the outer region than in the inner conterpart of the jet width.

Keywords: hot-wire anemometer, mass flow rate, pulsing frequency, unsteady jet

1. Introduction

Research interest into turbulence behavior in *free* shear flows has been a long tradition as attempts to understand their unique characteristics in various technical devices such as: mixing processes, sprays, and thrust augmentation. In principle, the turbulent free shear flows have simple flow domains which have no interaction with any solid surfaces and are triggered by mean velocity differences. Various techniques to generate the flows include jets, wakes, and mixing layers. In this paper, however, we restrict our attention to *round* jet which remains classical flows of relevance in the mixing processes, sprays, combustor systems, and other technical devices. This is because the velocity statistics of the

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jet, described by the mean velocity and higher order turbulence moments are valuable in comprehending the nature of random process in the jet.

Much study on the turbulent round jet has been performed in the past from a relatively low to very high Reynolds number. As a central point in this present study, we signify a subtantial contribution of Dahm and Dimotakis [5] in which the unique characteristics of Reynolds number defined-flow structures were revealed; displaying qualitatively turbulence scales and jet width. *Interestingly*, from the planar images of their jet evolution [5] it was found that there is *no* dependence of the jet width on the varied Reynolds numbers but the finer turbulence scales caused by scale separation were immediately seen at a higher Reynolds number. A few years later, experimental studies by Panchapakesan and Lumley [7] and Hussein, Capp et. al [8] confirmed the self-determination of the jet width [5] demonstrating a linear spreading rate of the jet propagation and relatively similar empirical constants of the jet growth under a large range of the Reynolds number. From this point of view, it therefore is encouraging to optimize the functionality of the jet indicated by a more spreading jet by introducing an additional non-dimensional parameter to control the jet development. It is desired that the additional parameter would create stronger turbulence in which eventually would affect the jet width.

In this paper, velocity and higher order turbulence statistics of a *mechanically* exited jet namely fully pulsed jet was experimentally studied. The jet was defined by a large range of the Reynolds and Strouhal numbers based on its nozzle diameter. Physically, this unsteady jet produced a series of distinct pulses as a result of the modulation imposed. We assessed the first, third, and fourth moments of instantaneous axial velocity across the jet field along further downstreams with the use of a hot-wire anemometer. Comparisons of the higher order turbulence moments with those of a steady round jet [13] are also illustrated. Henceforth, this article is devoted to furthering an understanding on the nature of random process in the pulsed jet.

2. Research Method

The pulsed jet was generated from a circular nozzle of 12.77 mm diameter and pulsing valve, located in the Jet Laboratory, the Division of Mechanical Engineering, the University of Queensland. The apparatus is illustrated in Figure 1. This is the same facility which was used by Bremhorst and Harch [3], Bremhorst and Hollis [4] with the exception that the jet exit diameter was halved giving a 12.77 mm diameter [6, 12]. The nozzle consisted of a 50 mm extension with a honeycomb flow straightener, and mesh cone. A 200 mm disc was mounted on the nozzle to avoid directly entrainment of the fluid behind the nozzle, hence allowing the flow near the jet exit to be axisymmetric.

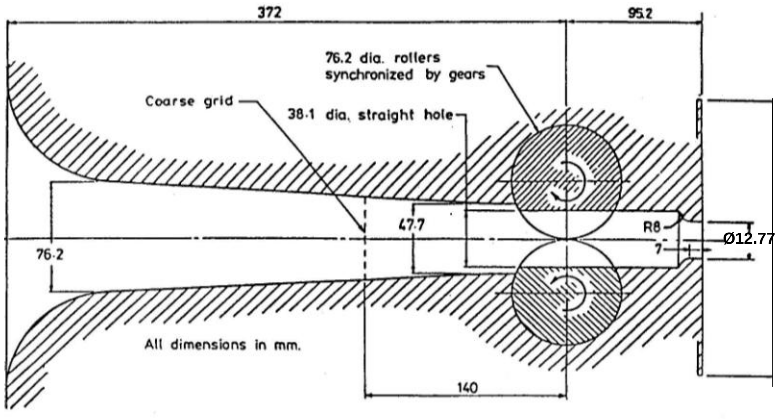

Figure 1. Pulsed jet apparatus
(reproduced from Winter [12])

Flow field of the jet was sampled by a hot-wire anemometer operated in a constant temperature mode. The hot-wire probe was a single normal wire which was made of Sigmond Cohn alloy 851 (79%Pt, 15%Rh, and 6%Ru) mounted on the prongs with a spot-welding technique. An overheat ratio of 1.3 was used to maintain the wire velocity sensitivity. The Kolmogorov length scale was estimated from the work of Gehrke [6] giving a value of 0.124 mm. A 10.16 μm diameter wire was used with the active length of 2 mm in order to give good spatial resolution. Hot-wire calibration gave a ±0.03% accuracy for the extended power-law equation improved by the look-up table method and 1.22% rms relative error of the fluctuating velocity component. Statistical accuracy evaluation found the sampling frequency, number of sample, and sampling time of 1000 Hz, 50,000 and 50 seconds; respectively to be chosen for accurate and repeatable measurements. It allowed a 0.26%, 0.45%, 1.10%, and 4.38% uncertainty of the estimated mean value, second, third, and fourth order turbulence moments; respectively.

Radial profile and propagation tests were initially to be performed to evaluate the symmetrical velocity distribution and propagation of the jet. For the preliminary tests [9], the jet was generated at the bulk exit velocity and pulsing frequency of 27.6 m/s and 10 Hz, respectively. It was shown that the jet is axisymmetric and follows a straight line trajectory [9]. Afterwards, the primary measurements were implemented at the bulk exit velocities of 13.7 and 34.4 m/s to obtain the radial profiles of mean axial velocity along 80 diameters downstream from the jet exit. The pulsing frequencies of 10 and 25 Hz were set to produce the pulsed jet.

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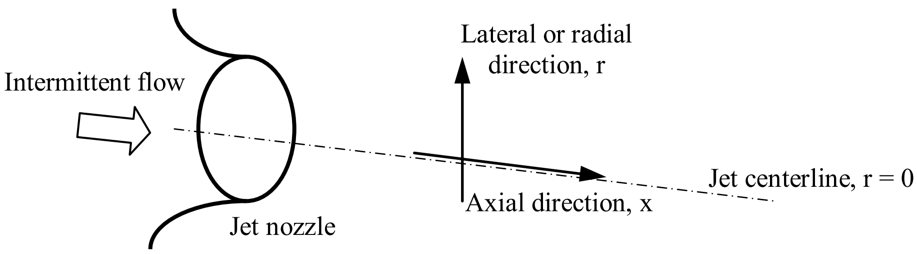

Figure 2. A schematic of the jet field

Flow field of the jet is illustrated in Figure 2. At a fixed point in the jet field, the instantaneous axial velocity of the jet, U_i (x, r) consists of the mean axial velocity, U (x, r) and the aggregate turbulence or fluctuating component, u (x, r). This was to be the case as the statistics does not depend on the time and circumferential coordinate, i.e the shapes of velocity distribution across the field in the horizontal and vertical planes were similar [10]. The Reynolds decomposition as in Equation 1 then holds for the evaluation of the mean axial velocity and fluctuating component. Since the flow was statistically stationary [10], estimation of the desired mean axial velocity can be obtained from a discrete equation over a finite time period as expressed in Equation 2 where N is number of samples.

$$U_i(x,r) = U(x,r) + u(x,r) \tag{1}$$

$$\hat{U}(x,r) = \frac{\sum_{i=1}^{n} U_i(x,r)}{N} \tag{2}$$

A Gaussian [4, 7] and exponential [3] equations, depending on the turbulent flow-sensors used (i.e laser Doppler and hot-wire anemometers), can be applied to represent the radial profile of the first moment of instantaneous axial velocity of the pulsed axisymmetric jet as in Equation 3 and 4; respectively. From their observation [4, 7], it was found that α was equal to 0.693 offering a good Gaussian curve fit for the radial profiles as in Equation 1.

$$\frac{U(x,r)}{U_0(x,r=0)} = e^{-\alpha\left(\frac{r}{r_{1/2.U}}\right)^2} \tag{3}$$

The exponential equation with α = 0.44 [3] to provide a good fit of the radial profiles of the mean centerline axial velocity, is expressed as in

$$\frac{U(x,r)}{U_0(x,r=0)} = \left[1+\alpha\left(\frac{r}{r_{1/2.U}}\right)^2\right]^{-2}. \tag{4}$$

The third and fourth moment of turbulence can be determined with the aid of the Reynolds decomposition and the discrete equation. The third, $\overline{u^3}$ and fourth, $\overline{u^4}$ moments can be formulated as in

$$\overline{u(x,r)^3} = \overline{(U_i(x,r)-U(x,r))^3} \tag{5a}$$

or rewritten as

$$\overline{u(x,r)^3} = \frac{1}{(N-1)}\left[\sum_{i=1}^{N} U_i(x,r)^3 - NU(x,r)^3\right], \tag{5b}$$

and

$$\overline{u(x,r)^4} = \overline{(U_i(x,r)-U(x,r))^4} \tag{6a}$$

or simplified as

$$\overline{u(x,r)^4} = \frac{1}{(N-1)}\left[\sum_{i=1}^{N} U_i(x,r)^4 - NU(x,r)^4\right], \tag{6b}$$

respectively. Similar expressions to define the radial, V and azimuthal velocity components, W can also be used. Nevertheless, in this present study we only focus on the measurements of axial velocity component, U.

Other higher order turbulence moments which are dimensionless quantities, including: skewness and kurtosis or flatness factors can be determined with the help of equation 5a – 6b. The skewness, S_u and flatness, F_u of the fluctuating velocity, u are formulated, respectively, as in

$$S_u = \frac{\overline{u(x,r)^3}}{(\overline{u(x,r)^2})^{\frac{3}{2}}} \tag{7}$$

and

$$F_u = \frac{\overline{u(x,r)^4}}{(\overline{u(x,r)^2})^2} \tag{8}$$

These quantities are valuable in the case of turbulent flows to judge how far from Gaussian distribution a particular random variable of the flow might be. In the case of *strict* Gaussian distribution, the value of S_u and F_u are 0 and 3, respectively. Therefore, the closer a random variable with those two values, the more stationary or steady the random process is.

3. Research Results and Discussion

Some results of radial measurements are presented in Figures 3 – 4 displaying the normalized velocity signals at x/d = 20. The instantaneous axial velocity, U_i and time, t are normalized by the mean exit velocity, U_e and time for one complete cycle, t_p; respectively. For the pulsing frequency, f_p of 25 Hz, the value of t_p was 0.035 seconds. As can be seen in Figures 3 and 4, under a constant mean exit velocity the levels of U_i/U_e are relatively the same as the pulsing frequency is increased. However, from the measurements, an increase in the pulsing frequency resulted in the weakening axial aggregate turbulence components.

Appreciations for the radial profiles of the first moment under varying jet exit velocity and pulsing frequency over 80 diameters downstream are illustrated in Figures 5 – 6. As depicted in Figure 5, narrower radial profiles are produced as the Strouhal number is increased at the lower mass flow rate. The similar pattern was also demonstrated by Bremhorst and Harch [3] but with a higher mass flow rate. However, the narrower profiles as the consequence of increasing Strouhal number are relatively less evident at the higher jet exit velocity. Moreover, the narrower profile are also seen as the Reynolds number was increased at the lower Strouhal number (corresponding to the pulsing frequency of 10 Hz), as illustrated in Figure 6. Conversely, at the pulsing frequency of 25 Hz the narrower profiles as a result of the increasing mass flow rate are relatively less obvious. The present study was the *pioneering* work to establish the narrow radial profiles in response to the increasing Reynolds number as compared to the former experimental studies by Bremhorst and Harch [3], Bremhorst and Hollis [4], Gehrke [6], and Winter [12].

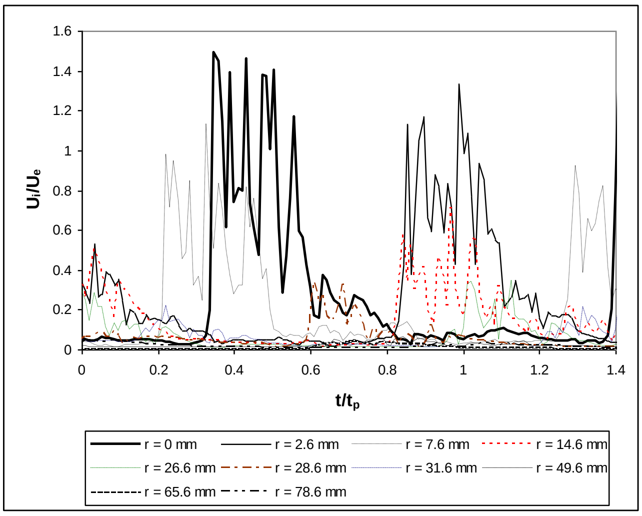

Figure 3. Velocity signals of radial measurement in horizontal direction at U_e = 34.4 m/s, f_p = 10 Hz, x/d = 20.

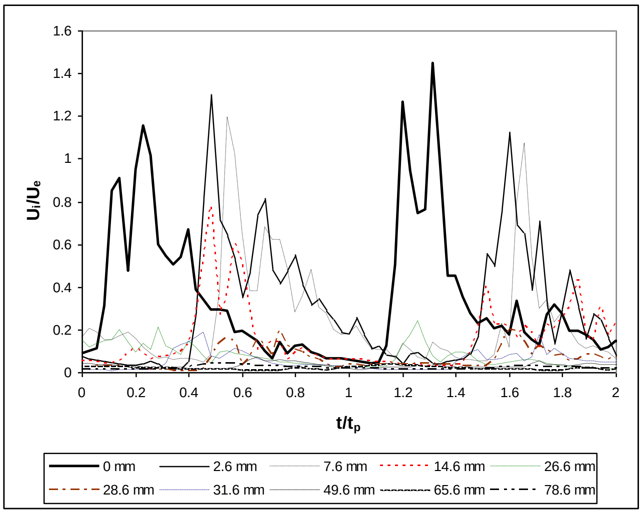

Figure 4. Velocity signals of radial measurement in horizontal direction at U_e = 34.4 m/s, f_p = 25 Hz, x/d = 20.

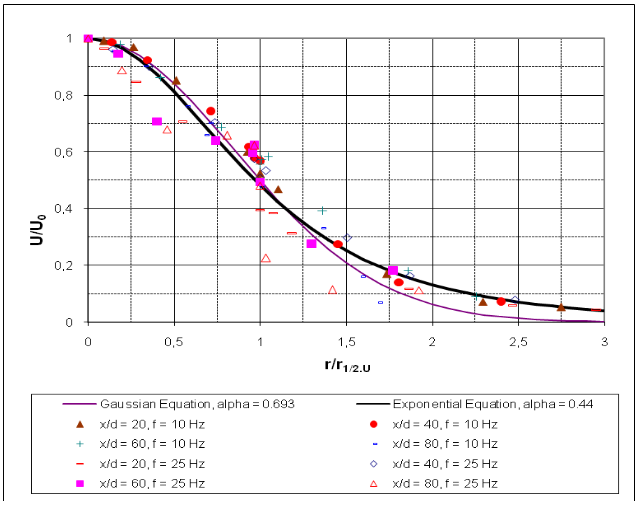

Figure 5. Radial profiles of the first moment of instantaneous axial velocity at U_e = 13.7 m/s.

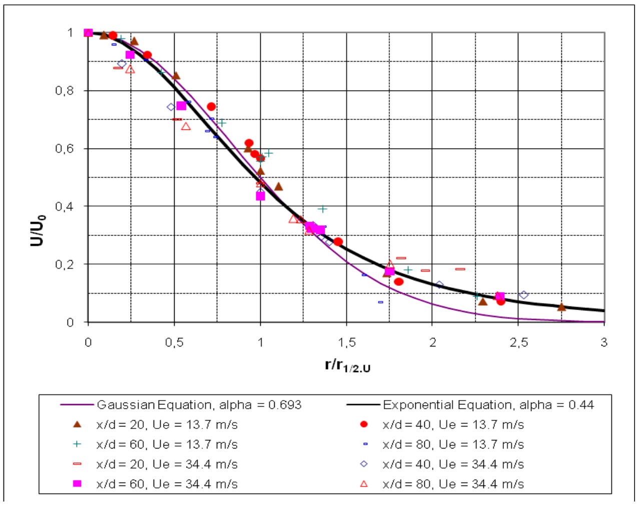

Figure 6. Radial profile of the first moment of instantaneous axial velocity at f_p = 10 Hz.

Comparison of the radial profile of the first moments between steady [2, 5, 7, 8] and pulsed jets are given in Figure 7. It is clearly seen that the significant changes in the radial profile are not the case for the steady jets investigated by Panchapakesan and Lumley [8], Hussein et. al [7], Bogey and

Bailly [2] under a very large range of the Reynolds numbers. From the red continuous line, all the laser Doppler anemometer (LDA) experimental [7, 8] and simulated [2] results of the jets over 140 diameters downstream ranging from a Reynolds number of 5000 up to 95,500 collapsed into a single curve, i.e a Gaussian curve (equation 3). This self similarity is an indication that there is no dependence of the flow structure of the steady jets on varying mass flow rate thus the steady jet width is not affected by controlling the Reynolds number [5]. The simulated results by Bogey and Bailly [2] were obtained by using Large Eddy Simulation (LES). Concerning the widening radial profiles of the pulsed jet at the lower Strouhal number, under different Reynolds numbers Tanaka [11] and Atassi, Charnay et. al [1] also demonstrated the similar evidences for their unsteady axisymmetric jets. In addition, it is seen that the present results depart from the Gaussian curve fit (Equation 3) thus agreeing with the exponential curve fit (Equation 4) but with different constants, for example as seen in Figure 8. Using a hot-wire sensor, similar trends on the radial profiles were also shown by Bremhorst and Harch [3] and Gehrke [6].

Transverse profiles of the skewness and flatness factors of the fluctuating components at 80 nozzle widths downstream which are in the pulsed dominated- and high turbulence steady jet regions [9] are shown in Figures 9 – 12. The results are plotted on the normalized positions in the radial direction associated with the jet width. As can be seen from the figures, both factors vary from the strict Gaussian values ($S_u = 0$ and $F_u = 3$) over the whole region. For all tested cases, the values of both factors have lower values in the locations close to the jet centerline and reach much larger values in the outer regions from the middle of jet widths ($\frac{r}{x} \approx 0.14$). Local maximum values ($S_{u,max}$ and $F_{u,max}$) are registered at the edges of the jet for all cases. That the nearer Gaussian values are found in the inner regions of the jet widths indicates that the influences of the Reynolds and Strouhal numbers on the turbulent statistics in the inner regions are weaker relative to those in the outer regions ($\frac{r}{x} \geq 0.14$). The physical phenomena in the outer regions are best explained by visual observations of the steady jet of Dahm and Dimotakis [5] at an instant of time as there are more intense-highly irregular interactions between the turbulent flow and the ambient fluid in the outer region. Moreover, in all cases, the further downstream where the pulsed dominated-region gradually collapsed [9] the more decreasing the values of both factors over the jet widths thus reaching relatively closer to the Gaussian values.

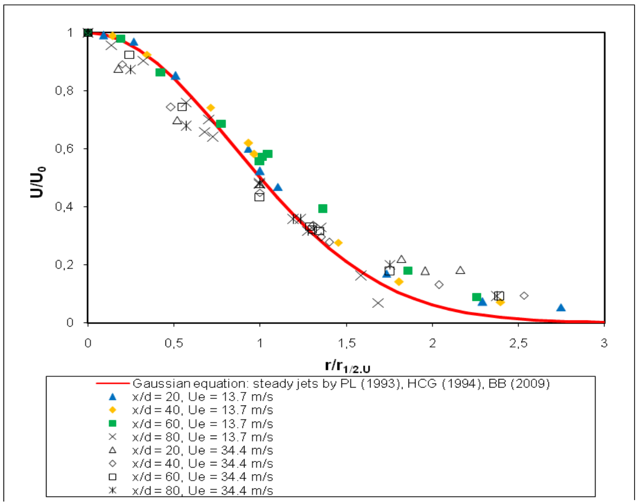

Figure 7. Comparison of the radial profiles between steady and pulsed jets.

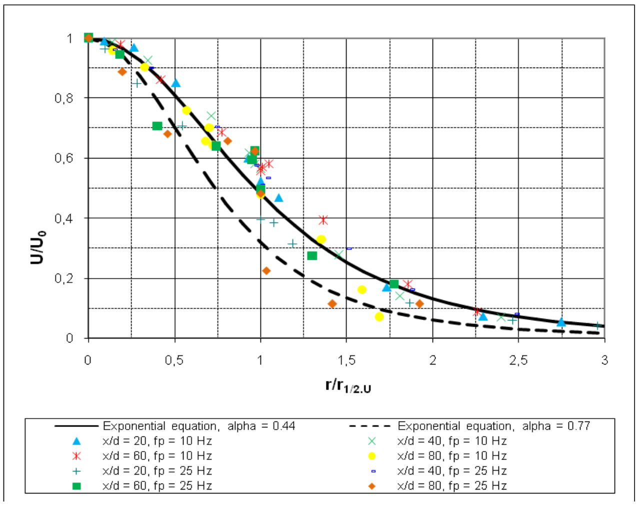

Figure 8. Exponential curve fits (Equation 4) of the first moment of pulsed jet.

Comparison of the skewness and flatness factors between the steady and pulsed jets are given in Figures 13 – 14. Both factors of the steady jet were experimentally studied by Wygnanski and Fiedler [13] at a position of 75 diameters from the jet nozzle at a high Reynolds number of 10^5 using a hot-wire sensor. Based on this benchmark, both factors of the pulsed jet at the position of x/d = 80 are plotted together with those earlier results [13] but we only focus on the highest Reynolds number used in which produced the narrower radial profiles relative to ones at the lower Reynolds number. It is important to note that the location of 80 diameters in the pulsed jet is supposedly the high turbulence steady jet region [9]. With this scheme, the comparison is sufficiently reasonable to judge the turbulence nature in the original steady and 'modulated steady' jet i.e. the unsteady one when already switched to a high turbulence steady jet.

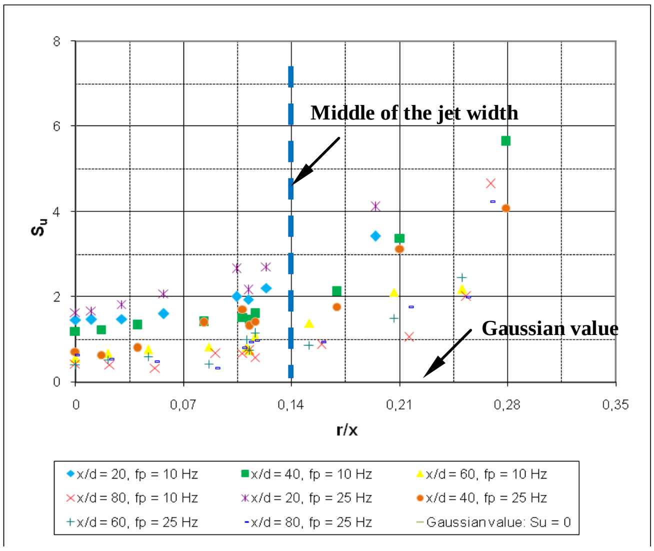

Figure 9. Skewness at U_e = 13.7 m/s.

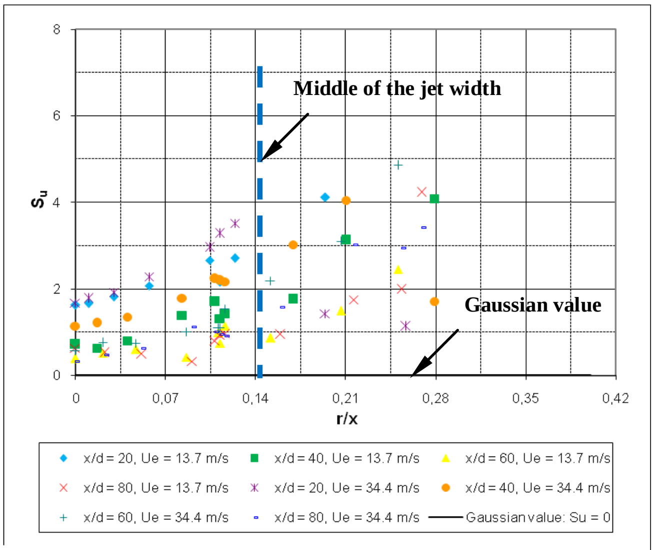

Figure 10. Skewness at f_p = 25 Hz.

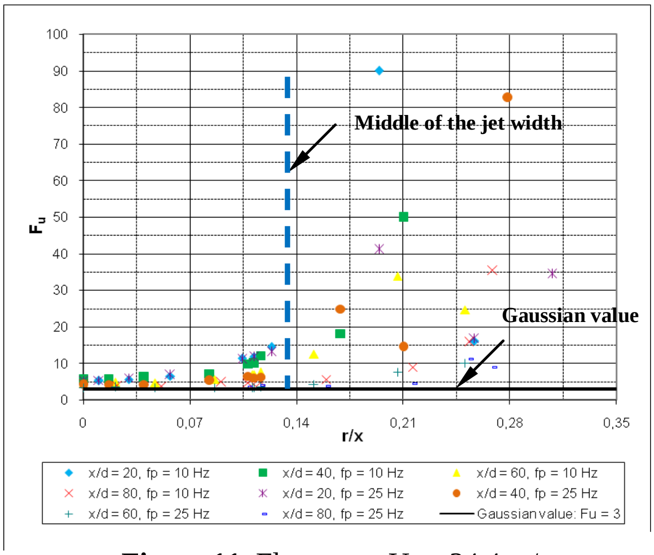

Figure 11. Flatness at U_e = 34.4 m/s.

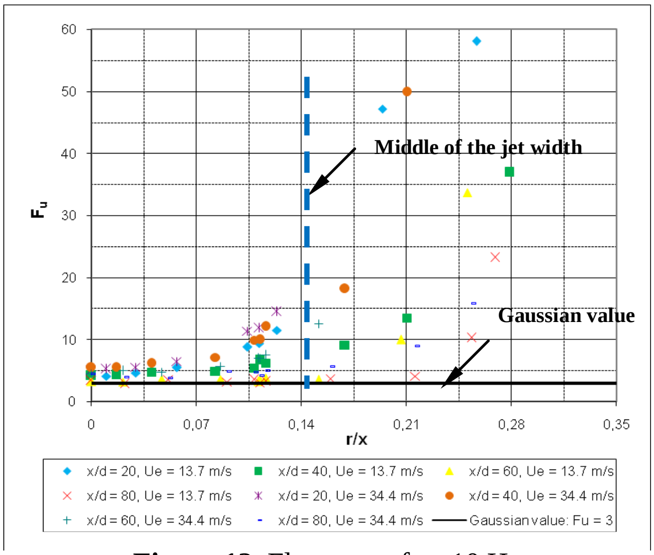

Figure 12. Flatness at f_p = 10 Hz.

From the figures, as the turbulent flow sensors were moved away from the jet centerlines, the value of both factors for the jets elucidate the aforesaid phenomena thus the departures from the strict

Gaussian values over the jet widths result. Concerning the turbulent statistics in the steady jet, the skewness in the inner region is relatively close with the Gaussian one (S_u = 0) for the locations very close to the jet centerline but that is not the case for the flatness factors as the closeness to the Gaussian value (F_u = 3) remains even up to the border of the inner region. Furthermore, comparing to the pulsed jet this is not the case in the inner region where it has the higher values than those of the steady couterpart. However, when comparing the values with those in the outer region of the pulsed jet, the influences of the Reynolds and Strouhal numbers on the turbulent statistics become significantly stronger in the outer region than in the inner region as previously explained. Such phenomena also occur for the steady jet but are only relevant for the influence of the Reynolds number as the jet is only defined by the parameter. As a *focal* point, it was found for the two distinct jets that the more intense-highly irregular motions occur in the outer region associated with the higher values of both factors than those of strict Gaussian values. The skewness and flatness factors of the pulsed jet were found to be substantially higher than those of the steady jet as a results of the modulations imposed. From Figure 13 and 14, it is also seen that the pulsed jet has a qualitatively wider flow field reaching the jet edge up to around $\frac{r}{x} \approx 0.28$ than that of the steady jet which reaches the jet edge of around $\frac{r}{x} \approx 0.22$.

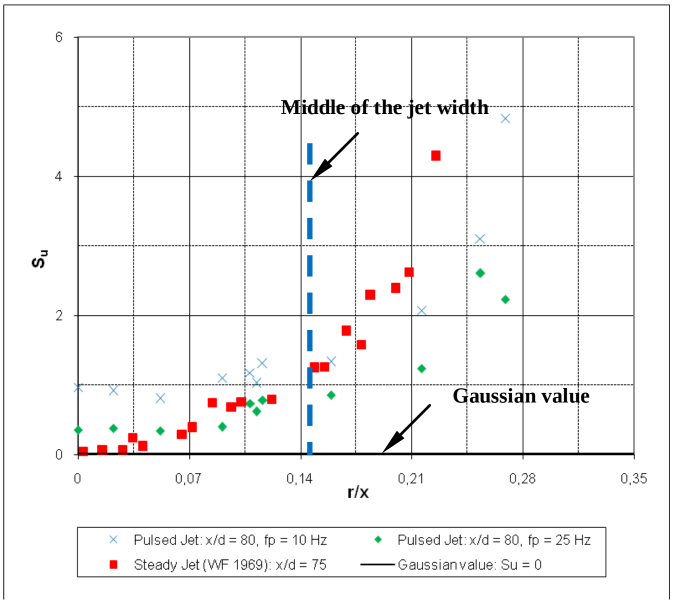

Figure 13. Comparison of the skewness between the steady and pulsed jets

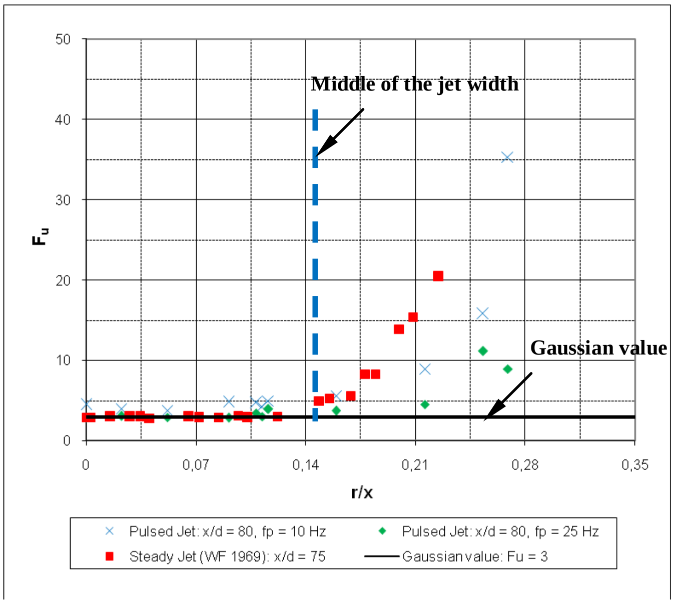

Figure 14. Comparison of the flatness between the steady and pulsed jets

4. Conclusion

Present results feature the responses of unsteady flow in a fully pulsed round jet to a range of the Reynolds and Strouhal numbers. The ensemble average technique allows evaluations of the first and higher moments of turbulence within a high degree of accuracy and proves a statistically stationarity of the intermittent flow [10]. Instead of the Reynolds number, imposing the additional parameter i.e the Strouhal number to define the pulsed jet certainly results in a significant change in the flow structure of the jet thus the self similarity in the first moment as is the case for a steady jet is no longer seen. Over the steady jet, key features of the pulsed jet include a wider jet width and more intense-highly irregular motions as a result of the mechanically excitations. However, a less fluctuating-pulsed jet is established at an increasing pulsing frequency. Therefore, the current findings offer the benefits for mixing enhancement in various industrial processes such as: sprays, combustor systems by using the fully pulsed axisymmetric jet as is the case in the earlier works [3, 4, 6, 12].

For the practical purposes, the present scheme also highlights the established knowledge on controlling the flow field of the fully pulsed jet. A less fluctuating-pulsed jet and narrower radial profile of the first moment are obtained at a constant mass flow rate with an amplified Strouhal number. Furthermore, at a constant Strouhal number as the mass flow rate is increased the augmentation of the fluctuating components at the jet centerline, u_0' and the narrower radial profiles of the first moment yield. For practical reasons, as an attempt to enhance the mixing process it therefore is desired to have a more fluctuating-pulsed jet and widening radial profile by setting a low mass flow rate and Strouhal number.

Transverse profiles of the third and fourth moments of the fluctuating components for the fully pulsed jet illustrate the nature of random process and were found to remarkably depart from the Gaussian values under the controlled parameters. The skewness and flatness factors of the mechanically excited jet were also found to be significantly higher than those of the steady jet over the jet widths thus leading to more intense-highly irregular interactions between the turbulent flow and the ambient fluid as well as more qualitatively spreading jet. Furthermore, stronger effects of the Reynolds and Strouhal numbers on the flow field take into account in the outer region than in the inner part of the jet width.

Concerning the complicated results of the radial profile of the first moment at the higher Reynolds number and Strouhal number, the present study can therefore be the *stimulus* for a further research in order to advance the knowledge on the turbulence nature in the jet. The crucial point here is to determine the *limit* whether the pulsed jet is still acceptable for the mixing enhancement compared to the steady jet. To have an efficient mixing, typically, one would avoid the narrower radial profiles produced at the higher jet exit velocity and pulsing frequency. As the radial profiles gradually become narrow under the increasing controlled parameters, the further research would therefore be useful to expand the upper limits of the current used parameters.

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