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No-Core MCSM calculation for ^{10}Be and ^{12}Be low-lying spectra

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Abstract. The low-lying excited states of ^{10}Be and ^{12}Be are investigated within a no-core Monte Carlo Shell Model (MCSM) framework employing a realistic potential obtained via the Unitary Correlation Operator Method. The excitation energies of the 2_1^+ and 2_2^+ states and the B(E2; $2_{1,2}^+ \rightarrow 0_{g.s.}^+$) for ^{10}Be in the MCSM with a standard treatment of spurious center-of-mass motion show good agreement with experimental data. Some properties of low-lying states of ^{10}Be are studied in terms of quadrupole moments and E2 transitions. The E2 transition probability of ^{10}C, the mirror nucleus of ^{10}Be, is also presented with a good agreement to experiment. The triaxial deformation of ^{10}Be and ^{10}C is discussed in terms of the B(E2) values.

1. Introduction

In the last decades, many progresses have been made in obtaining an accurate representation of realistic nucleon-nucleon (NN) potentials. One can construct a two-body potential phenomenologically by fitting experimental data on nucleon-nucleon (NN) scattering, as it is done in the Argonne V18 potential [1], the CD-Bonn potential [2] and the Nijmegen potentials [3]. Alternatively, the chiral N^{3}LO potential can be constructed in the framework of chiral effective field theory [4–6]. By using these realistic nuclear interactions, *ab initio* nuclear many-body calculations have been performed. In Green's Function Monte Carlo (GFMC) calculations the exact ground-state wave function is calculated by treating the many-body Green's functions in a Monte Carlo approach [7–9]. The GFMC calculations of light nuclei up to ^{12}C with the Argonne interaction reproduce the experimental nuclear binding energies and radii as well as the spectra. Another *ab initio* approach for nuclei up to A=14 is the No-Core Shell Model (NCSM) [10–12].

The straightforward application of those realistic interactions in nuclear many-body calculations is difficult due to the strong short-range repulsion and tensor correlation. The Unitary Correlation Operator Method (UCOM) is one of the methods to tackle this problem by introducing a unitary transformation [13–15]. In the UCOM approach two unitary transformation operators are defined, a central correlation operator and a tensor correlation operator, which correspond to two most important correlations: the central correlations induced by the strong short-range repulsion and the tensor correlations, respectively. The so-called UCOM potential can be used in various kinds of many-body calculations, such as no-core shell model calculations [16–18].

In the shell model calculations, the direct diagonalization of the Hamiltonian matrix in the full valence-nucleon Hilbert space is difficult, as the dimension of such a space becomes larger and larger when one moves from light nuclei to heavier nuclei. One way to overcome this difficulty, the Quantum Monte Carlo Diagonalization (QMCD) method has been proposed for solving quantum many-body systems with a two-body interaction [19–22]. The QMCD can describe not only the ground state but also excited states, including their energies, wave functions and hence transition matrix elements. Thus, on the basis of the QMCD method, the Monte Carlo Shell Model (MCSM) has been introduced [23] for nuclear shell model calculations [24–27]. An extrapolation method in the Monte Carlo Shell Model has been proposed very recently [28–30]. We shall present, in this paper, for the first time, how such *ab initio* no-core MCSM calculations work. The results to be shown in this paper will play a key role in judging as to whether one should move ahead to more systematic calculations with the revised method or not.

2. Low lying spectra of ^{10}Be and ^{12}Be

The theoretical framework and numerical detail are not presented in this paper. The details of these parts can be found in Ref. [31].

The convergence of low-lying excitation energies as a function of the MCSM dimension has been examined firstly. In the MCSM calculation, the diagonalization is performed in a subspace comprised of 25 to 50 optimally generated basis states. The size (dimension) of this subspace is quite small compared to that of the entire Hilbert space taken in the direct diagonalization in the conventional shell model. This advantage will be even more obvious for heavier nuclei by the fact that the full diagonalization in $e_{\max} = 3$ is hardly feasible with other calculational techniques available presently.

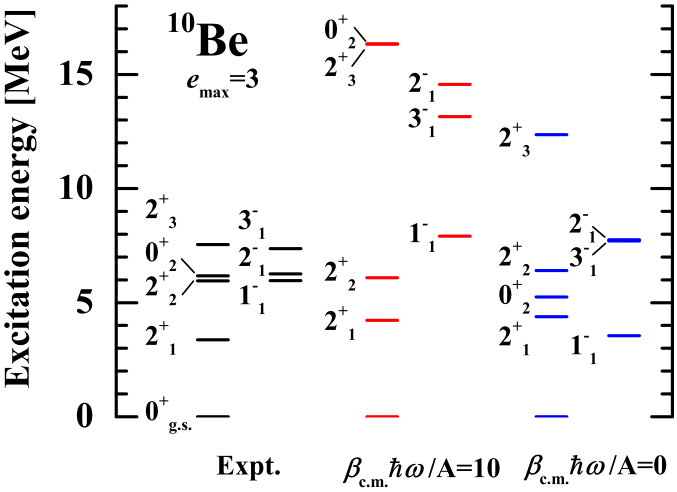

Figure 1. (Color online) Some low-lying spectra of ^{10}Be in the $e_{\max} = 3$ model space. Black bars indicate experimental levels. Red levels are theoretical results obtained with the suppression of spurious center-of-mass motion ($\beta_{c.m.} \cdot \hbar\omega/A = 10$ MeV). Blue levels are obtained without removing the spurious center-of-mass motion.

We now discuss properties of the 0^+_1 and $2^+_{1,2}$ states of ^{10}Be. Figure 1 shows energy levels of these states. Some other low-lying states of ^{10}Be are shown also. While the MCSM results are about 1 MeV and 0.6 MeV higher than the experimental values for 2^+_1 and 2^+_2, respectively, the basic patterns and scale are reproduced well by the MCSM calculation. In particular, the low-lying 2_2^+ level is a characteristic indicator of triaxial deformation, as discussed later.

We now investigate these excited states in terms of the quadrupole moments and E2 transitions. The quadrupole moments of protons and neutrons for the 2_1^+ and 2_2^+ states of ^{10}Be are calculated. One finds that beyond MCSM dimension of 30, those quadrupole moments reach stable values. The nucleus ^{10}Be has a negative quadrupole moment for the 2_1^+ state. In contrast, the 2_2^+ state shows a positive quadrupole moment. These features are also predicted in Ref. [1]. We note that the protons have stronger deformation than neutrons in both states of ^{10}Be, because there are two valence protons and four valence neutrons in the p-shell in major configurations, and the former produce stronger deformation than the latter.

The B(E2) values from the $2_{1,2}^+$ states to the ground state of ^{10}Be are evaluated. Some B(E2) values are calculated also for the mirror nucleus, ^{10}C, in the isospin formalism, as shown in Table 1. This table indicates that MCSM value of $\mathrm{B(E2}; 2_1^+ \to 0_{g.s.}^+)$ appears to be in rather good agreement with the corresponding experimental data [32, 33] for both ^{10}Be and ^{10}C. This is of certain importance because from the viewpoint of the liquid-drop model, B(E2) value is proportional to Z^2, and thereby the value of ^{10}C is expected to be larger than the corresponding one of ^{10}Be, by a factor of $6^2/4^2$ in a naive expectation. While we take only bare charge ($e_p = e$ and $e_n = 0$ with e being the unit charge), we can still produce almost the same values of $\mathrm{B(E2}; 2_1^+ \to 0_{g.s.}^+)$ of ^{10}Be and ^{10}C. This is because although there are two more protons in ^{10}C than in ^{10}Be, they do not necessarily increase quadrupole deformation, partly due to the $0p_{3/2}$ closed-shell formation.

The nuclei ^{10}C and ^{10}Be belong to the same isospin multiplet of T=1. In the notation of Timmer [34], which makes direct use of the isospin formalism, the B(E2) value of ^{10}C should be smaller than that in ^{10}Be, as $T_z = -1$ for ^{10}C and $T_z = 1$ for ^{10}Be.

Assuming that the 0_1^+ and 2_1^+ states of ^{10}Be belong to the same $K = 0$ rotational band, the intrinsic quadrupole moment Q_0 can be evaluated from the $\mathrm{B(E2}; 0_1^+ \to 2_1^+)$ value and the spectroscopic quadrupole moment. Q_0 evaluated by the spectroscopic quadrupole moment is 20.5 e fm^2, which is consistent to the one (21.6 e fm^2) extracted from the $\mathrm{B(E2}; 2_1^+ \to 0_1^+)$ value. This similarity seems to suggest an axially symmetric deformation in the yrast band. On the other hand, the $\mathrm{B(E2}; 2_2^+ \to 2_1^+)$ is sizable, which hints at a notable triaxial deformation of ^{10}Be. $\mathrm{B(E2}; 2_2^+ \to 0_1^+)$= 0.32 e^2 fm^4 leads us to a triaxial deformation with $\gamma = 11.4°$ in the Davidov-Fillipov model [35]. Thus, the present results are of interest in view of nuclear shapes, although it may be an open question as to whether the classical picture of shapes can make sense in such light nuclei. However, for ^{12}Be, We definitely need a larger model space, and it is not tractable presently.

In summary, for the first time, we have applied the no-core MCSM with realistic UCOM-transformed interactions to the investigation of structure of ^{10}Be and ^{12}Be. We calculate some low-lying states of ^{10}Be and ^{12}Be in an $e_{\max}$=3 model space. The results for the 2_1^+ and 2_2^+ states of ^{10}Be show a reasonable agreement with experimental data. Some properties of low-lying states of ^{10}Be are studied in terms of quadrupole moments and E2 transitions.

Table 1. (a) $\mathrm{B(E2}; 2_1^+ \to 0_{g.s.}^+)$, (b) $\mathrm{B(E2}; 2_2^+ \to 0_{g.s.}^+)$ and (c) $\mathrm{B(E2}; 2_2^+ \to 2_1^+)$ values (e^2 fm^4) of ^{10}Be and those values of the mirror nucleus ^{10}C obtained by the MCSM and the experimental data [32, 33].

	^{10}Be			^{10}C		
	(a)	(b)	(c)	(a)	(b)	(c)
Exp.	9.2(3)	0.11(2)		8.8(3)		
MCSM	9.29	0.32	3.28	9.30	2.15	12.81

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