General relativistic effects on accretion disk neutrinos

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Abstract. The strong gravitational field of a black hole changes the neutrino fluxes emitted from matter accreting around it. We present effects of general relativity on the neutrino fluxes emerging from black hole accretion disks. The changes on the fluxes have implications on the nucleosynthesis resulting from the interaction of the emitted neutrinos and matter outflowing such disks.

1. Introduction

Black hole-neutron star, neutron-neutron star mergers and core collapse supernovae can result in a disk rapidly rotating around a black hole [1, 2]. These disks are interesting for several reasons. As core collapse supernovae they emit a large amount of neutrinos that can be detected at several facilities [3, 4]. The energy released during the merger has been proposed as a source for the generation of gamma ray bursts [5, 6, 7]. Also they have been suggested as the possible site for the production of heavy elements via rapid-neutron captures (r-process) [8, 9, 10, 11, 12, 13].

The presence of a black hole changes the properties of space-time around it affecting the spectra of radiation emerging from the matter located in its vicinity. Since black hole accretion disks are an important source of neutrinos, it is relevant to consider the effects of general relativity on their energies and trajectories. Several studies have been carried out for photons emerging from accretions disks [14, 15, 16]. These studies considered that the radiation is observed at a distant point form the source and that the disk has a smooth flat surface. They showed a strong impact of the general relativistic effects on the emerging photon spectra. To date, studies of the effects of general relativity on neutrinos emitted from accretion disks have focused on neutrino pair annihilation rates and the production of gamma ray bursts [17, 18, 19]. In this work we discuss general relativistic effects such as energy shifts and bending of trajectories when the observer (or absorption point) of the neutrino is in the vicinity of the disk. Besides studying a flat surface we also consider the neutrinos being emitted from the last point of scattering which corresponds to a puffy uneven surface. The results have strong implication in the synthesis of heavy nuclei as discussed in [20].

2. Formalism

In order to study the effects of general relativity on neutrino fluxes it is necessary to calculate energy shifts and bending of trajectories of neutrinos that leave the neutrino surface and arrive at some observation point. The last one could be a point on the trajectory of hot matter that has been ejected from the disk and that can interact with the neutrinos producing new elements.

2.1. Neutrino Spectra

The effective neutrino flux observed at some distance r_{ob} from the black hole is

$$\phi^{eff} = \frac{1}{4\pi} \int d\Omega_{ob} \times \phi_{ob}(E_{ob}), \tag{1}$$

where $d\Omega_{ob}$ is the solid angle that the source subtends as seen by the observer and

$$\phi_{ob}(E_{ob}) = \frac{g_\nu c}{2\pi^2 (\hbar c)^3} \frac{E_{ob}^2}{\exp(E_{ob}/T_{ob}) + 1}, \tag{2}$$

is the observed flux with $g_\nu = 1$. Equation 1 can be re-writen in terms of the emitted neutrino temperature T_{em}, which corresponds to the temperature at the surface of last scattering (neutrino surface) as

$$\phi^{eff} \propto \frac{1}{4\pi} \int d\Omega_{ob} \times \frac{E_{ob}^2}{\exp(E_{ob}(1+z)/T_{em}) + 1}, \tag{3}$$

where we have used the fact that the energy transforms as $E_{ob} = (1+z)E_{em}$, with z the redshift. According to Equation 1, to obtain the observed fluxes we need to determine the redshifts and the solid angle that the source describes on the observer's sky. Bellow we discuss the steps we follow to accomplish these tasks.

2.2. Neutrino trajectories

The solid angle $d\Omega_{ob}$ that the disk describes on the observer's sky can be written as

$$d\Omega_{ob} = \sin\xi d\xi \times d\alpha, \tag{4}$$

where ξ is the angle that a neutrino forms with the line that joins the center of the black hole and the observation point r_{ob} (radial direction), and the angle α describes the point where the neutrino hits the plane perpendicular to the radial direction (on the observer's sky). These angles are equivalent to θ and φ (in spherical coordinates) of a frame centered at the observer. However, to avoid confusion with the spherical coordinates used to described the disk we have used different greek letters. The details of how to find the angles ξ and α for a non-rotating black hole are given in [20]. Here we summarize the general steps.

Following the neutrino trajectories allows one to find ξ and α (see Figure 1). Because of the black hole presence neutrino trajectories are given by null geodesics. In the Schwarzchild metric null geodesics are described by the equation of motion (in a frame centered at the black hole) [21],

$$\int_{\varphi_{em}}^{\varphi_{ob}} d\varphi = \pm \int_{r_{em}}^{r_{ob}} \frac{dr}{r\sqrt{\frac{r^2}{b^2} - \left(1 - \frac{r_s}{r}\right)}}, \tag{5}$$

where the position of the observer(emitter) is $(r_{ob}, \varphi_{ob})((r_{em}, \varphi_{em}))$. r and φ are the spherical coordinates of these positions. The sign of equation 5 depends on whether the neutrino is approaching or leaving the center of the black hole. In the above equation b, the impact parameter at infinity, is the ratio between the neutrino energy E and its angular momentum L, $b = L/E$. The impact parameter b is a constant along the neutrino trajectory. Finally $r_s = 2M$ with M the black hole mass.

It is possible to solve Equation 5 for b when the initial and final points (r_{em}, φ_{em}) and (r_{ob}, φ_{ob}) are known. Once b is determined the angle ξ between the neutrino trajectory and the radial direction at any point r can be obtained from

$$b = \frac{r\sin\xi}{\sqrt{1 - r_s/r}}. \tag{6}$$
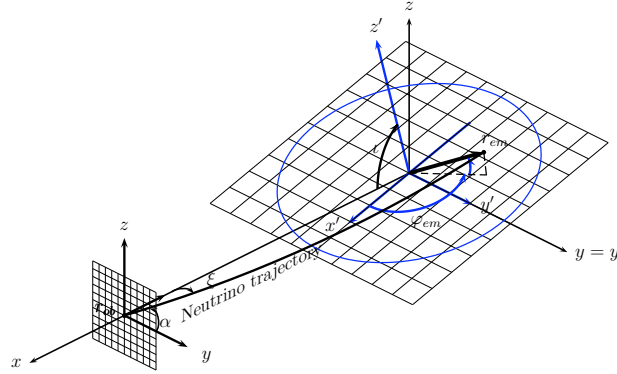
Figure 1. A neutrino is emitted from $(r_{em}, \varphi_{em}$ and observed at (r_{ob}, φ_{ob}). The z-axes of the observer and emitter coordinate frames form an angle ι. On the observer's sky the neutrino forms the angles α and ξ.

The angle α in Equation 4 can be found from relationships between spherical triangles formed by the equatorial plane of the disk, the xy plane in the observer's sky, and the plane described by the neutrino trajectory and the line joining the center of the black hole and the observer.

2.3. Energy shifts
The red-shift is given by [22],

$$1 + z = \frac{\left(p_t u^t + p_r u^r + p_\theta u^\theta + p_\varphi u^\varphi\right)_{em}}{\left(p_t u^t + p_r u^r + p_\theta u^\theta + p_\varphi u^\varphi\right)_{ob}}, \tag{7}$$

where the subindexes *em* and *ob* indicate that the quantities above should be computed with the observer or emitter coordinates accordingly. p is the projection of the neutrino 4-momentum on the 4-velocity u of the emitter/observer, $p_\beta u^\beta$. We consider both the observer and the emitter rotating around the black hole in stationary orbits with angular velocities Ω_{em} and Ω_{ob} respectively. If the black hole has a spin a then the redshifts are given in the Kerr metric by

$$1 + z = \frac{\left[-g_{tt} - 2g_{t\varphi}\Omega_{ob} - g_{\varphi\varphi}\Omega_{ob}^2\right]_{ob}^{1/2}}{\left[-g_{tt} - 2g_{t\varphi}\Omega_{em} - g_{\varphi\varphi}\Omega_{em}^2\right]_{em}^{1/2}} \times \frac{(1 + \Omega_{em}b\cos\eta)}{(1 + \Omega_{ob}b\cos\eta)}, \tag{8}$$

with $g_{\mu\nu}$ the components of the metric tensor [22]. η is the angle that the neutrino trajectory makes with the plane of the disk, which can be found using the law of cosines for spherical triangles, and is given by $\cos\eta = \cos\alpha\sin\iota$, where ι is the angle that the line joining the observer and the center of the black hole forms with the equatorial plane. Finally the angular velocity (calculated for the observer or emitter accordingly) is

$$\Omega = \frac{M^{1/2}}{r^{3/2} + aM^{1/2}}, \tag{9}$$

where a is the black hole spin, which is a input parameter in the disk models.

3. Results

We applied the above formalism to a disk formed during a black hole-neutron star merger. The disk model corresponds to a 3D hydrodynamical simulation [24, 23]. We use as an input for our calculations a snapshot of such simulation. We post-process the data to find neutrino surfaces and therefore neutrino temperatures as described in Ref [4]. The neutrino surfaces found are not smooth. They present sharp variations in temperature, density and height. We adopt the points on these surfaces to be the emission points of the neutrinos r_{em} and for this work we locate the observer at coordinates $r_{ob} = (x, z) = (40, 48)$ km above the neutrino surfaces. The r_{ob} coordinates correspond to the initial point (above the disk surface) of the trajectory of matter ejected from the disk used in the nucleosynthesis studies of Ref. [20].

Figure 2 compares results for effective neutrino fluxes when general relativistic effects are included and when they are ignored (Newtonian fluxes). The red lines are the Newtonian fluxes for electron neutrinos and antineutrinos. The black lines take into account redshifts and bending of trajectories. It can be seen that relativistic effects strongly impact the fluxes specially at high energies.

Since general relativistic effects depend on the position of the emitter and observer it is important to consider the geometry of the neutrino surfaces. To study this effect we consider two scenarios. The first one is the "flat disk" approximation. In this approximation, while the neutrino temperatures are determined by the surface of last scattering, the neutrino trajectories are started from the midplane (z=0) of the disk. The second one is the "puffy disk" where the neutrino trajectories begin at the neutrino surface.

Figure 3 shows the difference between the effective fluxes for electron neutrinos and electron antineutrinos when the flat disk approximation is used versus considering the three dimensional neutrino surfaces. In both cases general relativistic effects have been included. In the case of a puffy surface the neutrinos are emitted farther from the black hole and thus redshift effects are smaller. However, the hottest neutrinos in the puffy disk are emitted from a surface that tilts towards the black hole, making the geometrical factors more important and resulting in smaller fluxes when compared to the flat disk fluxes.

4. Conclusions

We have studied the effects that bending of trajectories and energy shifts have on the neutrino fluxes emitted from an accreting disk around a black hole. We found that these general relativistic effects have a strong impact on the effective neutrino fluxes in particular to the hottest neutrinos which are emitted closer to the black hole. We also studied the consequences that the neutrino surface geometry has on the resulting fluxes. We found that the geometry has an important impact and that a flat disk approximation results in larger neutrino fluxes compared with a puffy disk scenario. When neutrinos are emitted they can interact with outflowing matter from the disk. Both general relativistic effects and geometry considerations influence the ratio of protons to neutrons having important consequences for the synthesis of heavy elements [20]. General relativistic effects should be taking into account in nucleosynthesis studies of black hole accretion disks.

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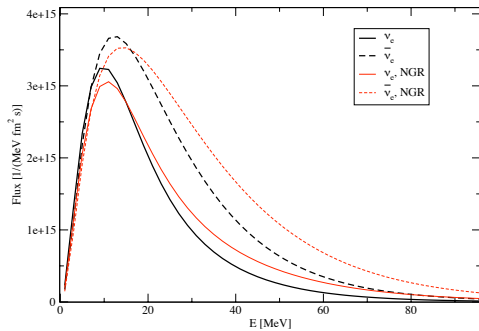
Figure 2. Electron neutrino and antineutrino effective fluxes as seen at a point above the neutrino surfaces located at $(x, y) = (40, 48)$ km. The black thick lines show the fluxes when bending of trajectories and energy shifts have been included. The red thin lines (NGR) correspond to the fluxes when general relativistic effects are ignored.
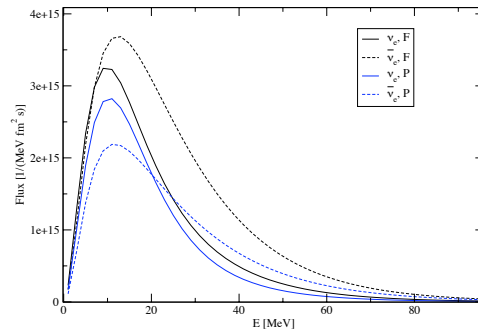
Figure 3. Effective neutrino fluxes obtained at a point above the neutrino surfaces located at $(x, y) = (40, 48)$ km. General relativistic effects are included. The black lines (F) correspond to a flat thin disk. The blue lines (P) show the fluxes when the uneven geometry of the neutrino surface is used.

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