Novel superconductor/magnet resonant configurations: Exact analytic representations of the Meissner state and the critical state

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Abstract. We derive exact analytic representations of the Meissner state and the critical state of a current-carrying superconductor strip located inside a cylindrical magnetic cavity. Our results show that, when the distance between the superconductor and the magnet is small, the penetration of magnetic flux fronts is strongly reduced as compared to the respective situation in an isolated strip. Even for total currents almost matching the critical current of the strip, the major part of the superconductor remains flux free, so that AC losses in it are greatly depressed; a fact which facilitates the use of such a configuration as a coaxial-type transmission line. From our generic representation it is possible to derive current profiles in cavities of various other closed geometries too by means of conformal mapping of the basic configuration addressed.

1. Introduction

Relatively high AC losses in superconductor cables and strips present a substantial problem for the implementation of superconductors in high-frequency [1] and low-frequency [2] applications. Recently, a suggestion for reducing AC losses in composite superconductor cables based on the idea of shielding out an applied magnetic field as well as the magnetic interaction between circular-cylindrical cable filaments by coating them with soft-magnet sheaths was put forth [3]. Theoretical modelling suggests that AC losses in a cable made up of N superconductor filaments may be reduced by a factor of $N-1$ due to magnetic decoupling of the filaments [3]. Experimental studies on composite superconductor/magnet cables and tapes confirm the reduction of AC losses due to the referred magnetic shielding effects [4,5]. On the other hand, AC losses caused by the current self-induced magnetic field in individual filaments with elliptical cross sections cannot be reduced by coaxial magnetic sheaths; they even grow up to a factor of 2.5 [4]. In contrast to this, AC losses caused by the current self-induced magnetic field in individual superconductor strips are expected to greatly decrease when the strips are exposed to suitably designed magnetic environments [6-8].

Exact analytic representations of the sheet current distributions in superconductor strips located between two high-permeability magnets occupying infinite half-spaces were derived previously [6,7]; these configurations allowed to find the respective current distributions for various other topologically *open* shielding geometries by application of the method of conformal mapping. Utilization of the latter tool for analyzing sheet current distributions and AC losses in the presence of topologically *closed*

magnetic environments of practical interest requires corresponding reference results. An establishment of such results is the focus of the present communication.

2. Theoretical model

We consider an infinitely extended type-II superconductor strip of width $2w$ located inside a cylindrical cavity of radius a in an infinitely extended magnet of relative magnetic permeability $\mu > 1$, the symmetry axis of this configuration coinciding with the z-axis of a Cartesian coordinate system x, y, z, as depicted in figure 1. Assuming the thickness of the strip to be small compared to its width, variations of the current over the thickness of the strip may be ignored and, for mathematical convenience, the state of the strip characterized by the sheet current J alone.

2.1. Meissner state of the superconductor strip

In the flux-free Meissner state, the distribution of the sheet current over the width of the superconductor strip, $-w < x < w$, is governed by the integral equation [7]

$$\int_{-w}^{w} dx' J(x') \left(\frac{1}{x - x'} + \frac{q}{x - a^2/x'} \right) = 0 \tag{1}$$

together with the normalization requirement

$$\int_{-w}^{w} dx' J(x') = I. \tag{2}$$

Here, $q = (\mu - 1)/(\mu + 1)$ is the strength of the image current induced by the magnetic cavity and I is the total transport current carried by the strip. In the limit $\mu \to \infty$, or equivalently $q \to 1$, equation (1) has the exact analytic solution

$$J(x) = \frac{2I}{\pi^2} \frac{\left(a^2 + w^2\right)\left(a^4 - x^4\right)}{\left(a^2 + x^2\right)^2 \sqrt{\left(a^2 + x^2\right)^2 w^2 - \left(a^2 + w^2\right)^2 x^2}}$$
$$\times \left\{ K\left(\frac{2aw}{a^2 + w^2}\right) + \frac{\left(a^2 + x^2\right)^2 w^2 - \left(a^2 + w^2\right)^2 x^2}{\left(a^2 + w^2\right)^2 x^2} \Pi\left(\frac{\left(a^2 + x^2\right)^2 w^2}{\left(a^2 + w^2\right)^2 x^2}, \frac{2aw}{a^2 + w^2}\right) \right\}, \tag{3}$$

where K and Π denote complete elliptic integrals of the first and, respectively, third kind. Sheet current profiles calculated on the basis of equation (3) for a range of the geometrical parameters involved are shown in figure 2. As a remarkable feature, they all reveal sharp peaks at the edges of the strip.

We comment that the distribution of the sheet current given by equation (3) is also representative for finite values of the relative permeability, $\mu \geq 200$, with an inaccuracy of less than 1% [7]; it is valid too if the infinitely extended hollow magnet is replaced by a magnetic sheath of finite thickness d, provided that the inequality $\mu d/w \gg 1$ is satisfied [9].
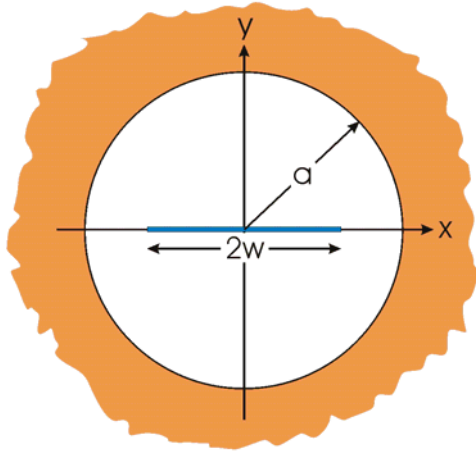
Figure 1. Cross-sectional view of the superconductor strip (bold line) located inside the cylindrical cavity in an infinitely extended magnet (shaded region). Cartesian coordinates x, y, z adapted to the strip are marked.
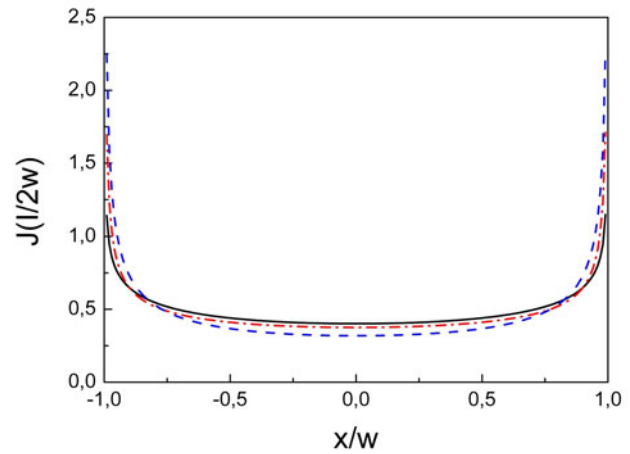

Figure 2. Distribution of the sheet current over the flux-free strip for the configuration of figure 1, with $a/w = 1.01$ (solid line), 1.1 (dashed-dotted line) and infinity (dashed line).

2.2. Critical state of the superconductor strip

When magnetic flux penetrates the superconductor strip in its critical state, the distribution of the sheet current is controlled by the pinning of magnetic vortices. In conformity with Bean's hypothesis for superconductors not exposed to a magnetic environment [10], the sheet current adopts its critical value J_c throughout the flux-penetrated regions of the strip, whereas the magnetic field component normal to the strip vanishes in the flux-free regions of the strip. Using the distribution of the sheet current given by equation (3), such a state can be constructed for the magnetically shielded strip of figure 1 as well. Proceeding in the spirit of previous work [11], a distribution of the sheet current prevails with magnetic flux penetrated from the edges of the strip, but with the central zone of half-width $b < w$ left flux free:

$$J(x) = \begin{cases} J_c \dfrac{\left(a^2 + b^2\right)\left(a^2 - x^2\right)}{\left(a^2 + x^2\right)^2} \phi_b(s(x)) & \text{for } 0 \leq |x| < b, \\ J_c & \text{for } b \leq |x| < w, \end{cases} \tag{4}$$

where

$$\phi_b(s) = \frac{a^2 + b^2}{\pi s^2} \left\{ \frac{c}{\sqrt{c^2 - s^2}} \arctan \sqrt{\frac{\left(h^2 - b^2\right)\left(c^2 - s^2\right)}{\left(b^2 - s^2\right)\left(c^2 - h^2\right)}} - \arctan \sqrt{\frac{h^2 - b^2}{b^2 - s^2}} \right.$$
$$\left. + \frac{\sqrt{b^2 - s^2}}{b} \arctan \sqrt{\frac{b^2 w^2 - b^4}{a^4 - b^2 w^2}} \left[1 - \Pi\left(\frac{b^2}{s^2}, \frac{b}{c}\right) \middle/ \mathrm{K}\left(\frac{b}{c}\right) \right] \right\} \tag{5}$$

with $s(x) = x\left(a^2 + b^2\right)\big/\left(a^2 + x^2\right)$, $h = w\left(a^2 + b^2\right)\big/\left(a^2 + w^2\right)$ and $c = \left(a^2 + b^2\right)\big/2a$. The width of the flux-free zone is controlled by the total transport current in the strip and by the geometry of the magnetic environment.

Sheet current profiles obtained from equation (4) for a range of the geometrical parameters involved, with a fixed value of b, are shown in figure 3. This exhibits a flattening of the current profile together with an increase in the magnitude of the total current up to saturation, when the radius of the magnetic cavity is reduced, precisely as in the case of topologically open magnetic cavities [6,7]. On the other hand, there is also a remarkable difference to the latter case, viz. the absence of a maximum of the sheet current profiles in the centre of the strip, resulting in a lower enhancement of the total current due to the shielding effect of the (concave) cylindrical magnetic cavity as opposed to the shielding effect of (convex) open magnetic cavities. Magnetic shielding also entails a reduction of the depth of penetration of magnetic flux into the strip, $\Delta(I) = w - b$, as compared to the depth in the situation without a magnetic environment, $\Delta_0(I) = w\left(1 - \sqrt{1 - (I/2wJ_c)^2}\right)$ [12]. As appears from figure 4, in a wide range of values of the total current, $\Delta(I) << \Delta_0(I)$, which means that AC losses here will be strongly reduced, too. Nevertheless, the general question of the efficacy of magnetic
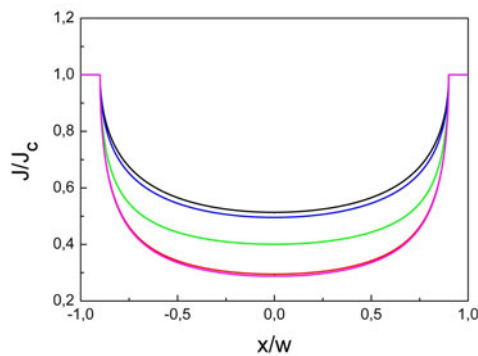

Figure 3. Distribution of the sheet current over the flux-free zone of the partly flux-filled strip delineated by $b/w = 0.9$ for the configuration of figure 1, with $a/w = 1.001, 1.01, 1.1, 2$ and infinity (from the upper curve down).
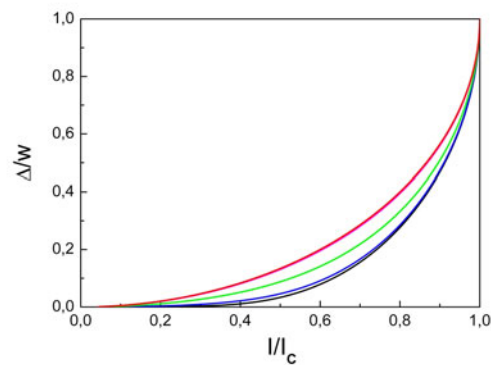
Figure 4. Dependence of the depth of penetration of magnetic flux into the strip on the normalized total current for the configuration of figure 1, with $a/w = 1.001, 1.01, 1.1, 2$ and infinity (from the bottom curve up).

shielding of a superconductor strip by a magnetic environment of closed topology should be studied further using the conformal mapping technique, for which the distributions of the sheet current given by equations (3) and (4) provide ideal starting points.

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