## Gauge invariance and the $\kappa$ - $g_{\ell}$ relation

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**Abstract.** The connection between the enhancement factor of the photonuclear E1 sum rule and the orbital angular momentum g-factor of a bound nucleon is discussed in the framework of the Landau-Migdal theory for isospin asymmetric nuclear matter, and compared to empirical informations.

The enhancement factor  $(1 + \kappa)$  of the photonuclear E1 sum rule and the orbital angular momentum g-factor  $(g_{\ell})$  of a bound nucleon have been the subject of research for a long time[1, 2]. The values of  $\kappa$  for heavy nuclei, obtained by integrating over the GDR Lorentzian curves, are[4]  $\kappa \simeq 0.2 - 0.3$ , while the empirical angular momentum g-factors in the lead region are[5]  $g_{\ell}(p) \simeq 1.14, g_{\ell}(n) \simeq -0.07$  for protons and neutrons. The relation between these quantities was first discussed by Migdal et al, Fujita and Hirata, and Arima et al [1], and received much attention recently [2, 3].

In a nuclear matter picture, the effective orbital g-factors can be determined completely general from gauge invariance. In the nonrelativistic limit one obtains [3]

$$g_{\ell}(p) = 1 - \frac{1}{3} \frac{M}{M^*} F_1(pn) \frac{N}{A} \beta, \qquad g_{\ell}(n) = \frac{1}{3} \frac{M}{M^*} F_1(pn) \frac{Z}{A} \beta.$$
(1)

Here M and  $M^*$  are the free and effective nucleon masses <sup>1</sup>,  $F_1(pn)$  is the  $\ell = 1$  Landau-Migdal parameter for the proton- neutron interaction, and  $\beta = \left[1 - ((N-Z)/A)^2\right]^{-\frac{1}{3}}$ .

On the other hand, the total photoabsorption cross section in the long wave length limit can be expressed in terms of the current-current correlation function as  $\sigma(\omega) = (4\pi/\omega) \operatorname{Im} \Pi(\omega)$ , and divided into two pieces: One piece (A) comes from particle-hole cuts, including the effects of higher excited states (2p-2h) via their real parts (renormalized RPA), and the other (B) comes from cuts at higher excitation energies. In the nuclear matter picture, it can be shown that in the energy non-weighted sum rule  $S = (\operatorname{TRK}) (1 + \kappa_A + \kappa_B)$ , it is the A-part which is related to the orbital g-factors by the simple relation [3]

$$1 + \kappa_A = g_\ell(p) - g_\ell(n) \,. \tag{2}$$

By this identification, it is reasonable to assume that  $\kappa_A$  is dominated by the GDR ( $\kappa_A \equiv \kappa_{\text{GDR}}$ ), while the main contributions to  $\kappa_B$  come from the strength function beyond the GDR.

<sup>1</sup> The definition of  $M^*$  in terms of the Fermi velocity is  $v_F = p_F/M^*$ . We also note that  $F_1(pn) = F_1 - F'_1 < 0$ .

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The above relations imply that  $g_{\ell}(p) = 1 + (N/A) \kappa_{\text{GDR}}$  and  $g_{\ell}(n) = -(Z/A) \kappa_{\text{GDR}}$ . To get an idea how these relations compare to data, we integrate <sup>2</sup> the Lorentzian curves fitted to GDR data to get  $\kappa_{\text{GDR}}$ , and from this we obtain  $g_{\ell}(p)$  and  $g_{\ell}(n)$ . The results are shown in Figs. 1 - 4. It is clear from these figures that one has to consider surface and shell effects, but for most cases the results for the effective orbital g-factors agree qualitatively with empirical and theoretical analyses of nuclear magnetic moments [5].

The orbital g-factors are also related to the M1 sum rule for the scissors mode [6]. It would be very interesting to see whether the analyses of the scissors mode and the GDR lead to consistent results for the g-factors.

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Figure 1. Integrated photoabsorption cross sections (crosses).



**Figure 3.** Values of  $g_{\ell}(p)$  obtained from  $\kappa_{\text{GDR}}$  of Fig.2.

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 $^{2}$  We integrate the Lorentzian curves for even-even nuclei given in Ref.[4] from 7 MeV to 40 MeV. The extracted orbital g-factors then correspond to the neighboring odd A nuclei.



Figure 2. Values of  $\kappa_{\text{GDR}}$  obtained from Fig.1.



**Figure 4.** Values of  $g_{\ell}(n)$  obtained from  $\kappa_{\text{GDR}}$  of Fig.2.