

## Novel nuclear structure aspects of the $0\nu\beta\beta$ -decay

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**Abstract.** We explore the influence of the deformation on the nuclear matrix elements of the neutrinoless double beta decay (NME), concluding that the difference in deformation -or more generally in the amount of quadrupole correlations- between parent and grand daughter nuclei quenches strongly the decay. We correlate these differences with the seniority structure of the nuclear wave functions. In this context, we examine the present discrepancies between the NME's obtained in the framework of the Interacting Shell Model and the Quasiparticle RPA. In our view, part of the discrepancy can be due to the limitations of the spherical QRPA in treating nuclei which have strong quadrupole correlations. We surmise that the NME's in a basis of generalized seniority are approximately model independent, *i. e.* they are "universal".

### 1. Introduction

The double beta decay is a rare weak process which takes place between two even-even isobars when the single beta decay is energetically forbidden or hindered by large spin difference. The two neutrino beta decay is a second order weak process—the reason of its low rate—, and has been measured in a few nuclei. The  $0\nu\beta\beta$  decay is analog but requires neutrinos to be Majorana fermions. With the exception of one unconfirmed claim [1], it has never been observed, and currently there is a number of experiments either taking place or expected for the near future—see e.g. ref. [2]—devoted to detect this process and to set up firmly the nature of neutrinos. Furthermore, the  $0\nu\beta\beta$  decay is also sensitive to the absolute scale of the neutrino mass, and hence to the mass hierarchy. Since the half-life of the decay is determined, together with the masses, by the nuclear matrix element for the process, its knowledge is essential to predict the most favorable decays and, once detection is achieved, to settle the neutrino mass scale and hierarchy.

Two different methods were traditionally used to calculate the NME's for  $0\nu\beta\beta$  decays, the quasiparticle random-phase approximation and the shell model in large valence spaces (ISM). The QRPA has produced results for most of the possible emitters since long [3, 4, 5]. The ISM, that was limited to a few cases till recently [6], can nowadays describe (or will do it shortly) all the experimentally relevant decays but one, the decay of  $^{150}\text{Nd}$ . Other approaches, that share a common prescription for the transition operator (including higher order corrections), and for the treatment of the short range correlations (SRC) and the finite size effects, are the Interacting Boson Model [7], and the Projected Hartree Fock Bogolyubov method [8].

The expression for the half-life of the  $0\nu\beta\beta$  decay can be written as [9]:

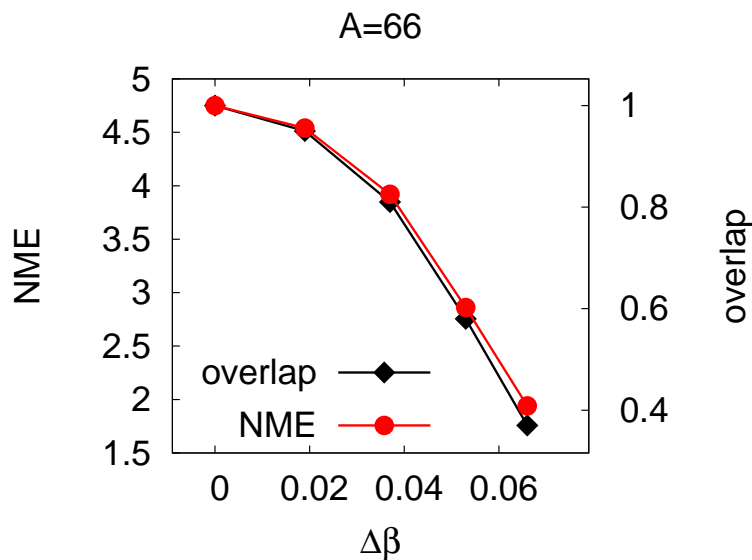
$$\left(T_{1/2}^{0\nu\beta\beta}(0^+ \rightarrow 0^+)\right)^{-1} = G_{01} \left|M^{0\nu\beta\beta}\right|^2 \left(\frac{\langle m_{\beta\beta} \rangle}{m_e}\right)^2, \quad (1)$$

where  $\langle m_{\beta\beta} \rangle = \left|\sum_k U_{ek}^2 m_k\right|$  is the effective neutrino mass, a combination of the neutrino mass eigenvalues  $m_k$ .  $U$  is the neutrino mixing matrix and  $G_{01}$  is a kinematic factor dependent on the charge, mass and available energy of the process.  $M^{0\nu\beta\beta}$  is the nuclear matrix element of the neutrinoless double beta decay operator, which has Fermi, Gamow-Teller and Tensor components. The kinematic factor  $G_{01}$  depends on the value of the coupling constant  $g_A$ . In addition, some calculations use different values of  $r_0$  in the formula  $R=r_0 A^{1/3}$ . It is therefore convenient to define:

$$M'^{0\nu\beta\beta} = \left(\frac{g_A}{1.25}\right)^2 \left(\frac{1.2}{r_0}\right) M^{0\nu\beta\beta} \quad (2)$$

In this way the theoretical  $M'^{0\nu\beta\beta}$ 's are directly comparable among them irrespective of the values of  $g_A$  and  $r_0$  employed in their calculation, since they share a common  $G_{01}$  factor—the one computed with  $g_A = 1.25$  and  $r_0=1.2$  fm. Thus, the translation of the  $M'^{0\nu\beta\beta}$ 's into half-lives is transparent.

## 2. Pairing and Quadrupole; The Influence of Deformation



**Figure 1.**  $^{66}\text{Ge} \rightarrow ^{66}\text{Se}$  NME,  $M'^{0\nu}$ , as a function of the difference in deformation induced by the extra quadrupole interaction added to  $^{66}\text{Se}$ .

An important issue regarding the  $0\nu\beta\beta$  decay is the role of the correlations; pairing that drives the nucleus toward a superfluid state and quadrupole that favors deformed intrinsic shapes. It has been shown recently that the  $2\nu\beta\beta$  is hindered by the difference in deformation between the initial and final nuclei [10, 11]. For the neutrinoless mode, the calculations [6] indicate that the pairing interaction favors the decay and that, consequently, the truncations in seniority, which

quench the pair breaking action of the quadrupole correlations, produce an overestimation of the values of the NME's. On the other hand, the NME's are also reduced when the parent and grand-daughter nuclei have different deformations [12, 13].

We have chosen to study the (unphysical) transition between the mirror nuclei  $^{66}\text{Ge}$  and  $^{66}\text{Se}$  in order to have a clearer view of the effect of the deformation in the NME's. This transition has the peculiarity that the wave functions of the initial and final nuclei are identical (provided Coulomb effects are neglected) and consequently it is easier to disentangle the contributions of the  $0\nu\beta\beta$  operator and the nuclear wave functions to the NME. The calculations are carried out in the valence space r3g with the effective interaction gcn28:50. The SRC are modeled by a Jastrow factor with the Spencer and Miller parametrization [14], although it has been shown recently that, once the finite size of the nucleon has been taken into account by a dipole form factor, softer options are more realistic [15, 16]

To increase the deformation of a given nucleus we add to the effective interaction a term  $\lambda Q \cdot Q$ . Fig. 1 shows the results when the final nucleus has been artificially deformed by adding an extra quadrupole-quadrupole term. Notice in the first place that for  $\lambda=0$  both nuclei are deformed with  $\beta \sim 0.2$ . In spite of that, the NME is a factor of two larger than the values obtained for the  $A=76$  and  $A=82$  decays in the same valence space and with the same interaction. Hence, even if the two  $A=66$  partners are deformed, the fact that their wave functions are identical enhances the decay. Nevertheless, the NME is still far from its expected value in the superfluid limit ( $\text{NME} \sim 8$ ). The figure shows that the reduction of the NME as the difference in deformation increases is very pronounced. For the values of  $\lambda$  between 0.0 and 0.2, the difference in deformation parameter between parent and grand daughter grows from zero to about 0.1. In addition, the NME follows closely the overlap between the wave function of one nucleus obtained with  $\lambda=0$  and the wave function of the same nucleus obtained with  $\lambda \neq 0$ . This means that, if we write the final wave function as:  $|\Psi\rangle = a|\Psi_0\rangle + b|\Psi_{qq}\rangle$ , the  $0\nu\beta\beta$  operator does not connect  $\Psi_0$  and  $\Psi_{qq}$ . This behavior of the NME's with respect to the difference of deformation between parent and grand daughter is common to all the transitions between mirror nuclei that we have studied ( $A=50$ ,  $A=110$ ) and to more realistic cases like the  $A=82$  decay that we have examined in detail in [17]. Therefore we can submit that this is a robust result.

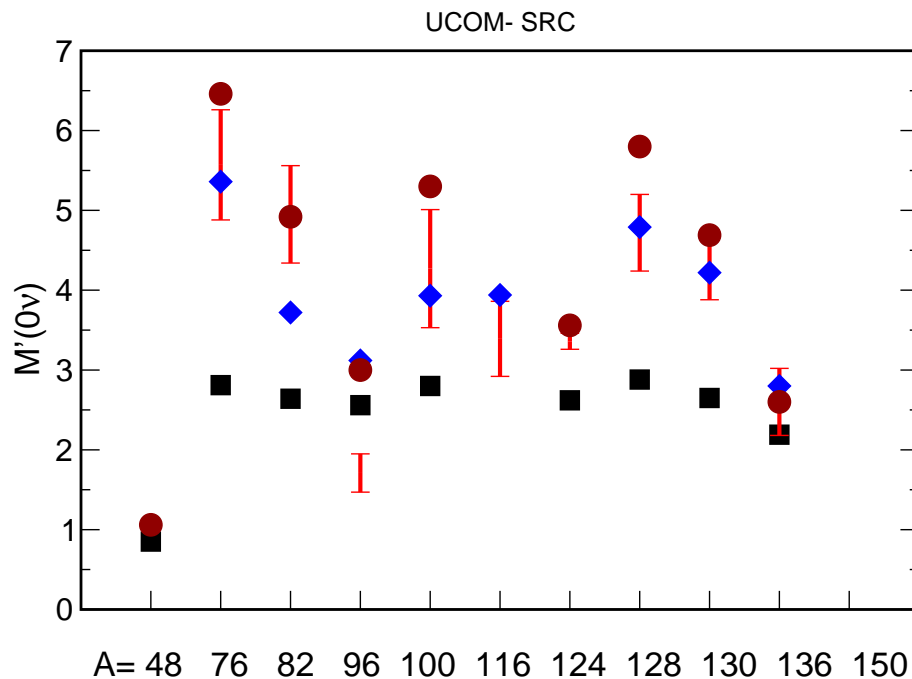
### 3. The NME's and the seniority structure of the nuclear wave functions

We can also analyze the results of the preceding section in terms of the seniority structure of the wave functions of parent and grand daughter nuclei. Indeed when  $\Delta\beta=0$  both  $^{66}\text{Ge}$  and  $^{66}\text{Se}$  have identical wave functions. The probabilities of the components of different seniority are given in table 1. It is seen that changing  $\beta$  from 0.22 (mildly deformed) to 0.30 (strongly deformed) increases drastically the amount of high seniority components in the wave function, provoking a seniority mismatch between the decaying and the final nuclei. This leads to very large cancelations of the nuclear matrix elements of the decay, as shown also in table 1.

**Table 1.** The seniority structure of the wave functions in the  $A=66$  mirror decay

	$s = 0$	$s = 4$	$s = 6$	$s = 8$	$s = 10$
$\Delta\beta=0$	39	43	7	10	1
$\Delta\beta=0.08$	6	32	21	31	10
	$M_F^{0\nu}$	$M_{GT}^{0\nu}$	$M_T^{0\nu}$	$M^{0\nu}$	
$\Delta\beta=0$	-2.02	3.95	0.08	5.16	
$\Delta\beta=0.08$	-0.76	1.65	0.02	2.12	

ISM: full(squares) ,  $s_m=4$ (circles) ; QRPA: Tu(bars) , Jy(diamonds)



**Figure 2.** The neutrinoless double beta decay nuclear matrix elements  $M'^{0\nu\beta\beta}$  for ISM and QRPA calculations treating the SRC with the UCOM approach. Tu, QRPA results from ref. [18] and Jy, QRPA results from refs. [3, 4]. The ISM results for A=96 and A=100 are preliminary

Coming back to the physically relevant decays, we compare in figure 2 the ISM and QRPA NME's. In both approaches, the SRC are taken into account in the UCOM framework [19] and  $g_A=1.25$  is adopted. We have discussed elsewhere that the discrepancies between both approaches show the following trends: when the nuclei that participate in the decay have a low level of quadrupole correlations, as in the decays of  $^{96}\text{Zr}$ ,  $^{124}\text{Sn}$  and  $^{136}\text{Xe}$ , the calculations tend to agree. On the contrary, when the correlations are large, the QRPA in a spherical basis seems not to be able to capture them fully. As the effect of the correlations is to reduce the NME's, the QRPA produces NME's that are too large in  $^{76}\text{Ge}$ ,  $^{82}\text{Se}$ ,  $^{100}\text{Mo}$ ,  $^{128}\text{Te}$ , and  $^{130}\text{Te}$ . Indeed, when the ISM calculations are truncated to maximum seniority  $s_m=4$ , which is the leading order of the ground state correlations in the QRPA (corresponding to the two quasi-particle contribution), they follow closely the QRPA results, as can be seen also in figure 2. Notice that only when the ISM calculations are converged at this level of truncation the two approaches do produce similar NME's.

We compare in table 2 the seniority structure of the wave functions of the ISM and QRPA, in some of the cases for which the latter are available [20]. It is seen that the differences are important and share a common trend: in the QRPA, the seniority structure of parents and grand daughters is much more similar than in the ISM. According to what we have seen in the A=66 case, this is bound to produce larger NME's in the QRPA than in the ISM, as it is actually the case. To make this statement quantitative, we have developed the ISM matrix elements in a basis of generalized seniority

**Table 2.** The seniority structure of the wave functions in the ISM and QRPA

	$s = 0$	$s = 4$	$s = 6$	$s = 8$	$s = 10$	$s = 12$	$s = 14$	$s = 16$
ISM								
$^{48}\text{Ca}$	97	3	-	-	-	-	-	-
$^{48}\text{Ti}$	59	36	4	1	-	-	-	-
$^{76}\text{Ge}$	43	41	7	8	1	-	-	-
$^{76}\text{Se}$	26	41	11	16	4	1	-	-
$^{82}\text{Se}$	50	39	10	1	-	-	-	-
$^{82}\text{Kr}$	44	41	6	8	1	-	-	-
$^{128}\text{Te}$	70	26	3	1	-	-	-	-
$^{128}\text{Xe}$	37	41	9	10	2	-	-	-
QRPA								
$^{76}\text{Ge}$	55	33	-	10	-	2	-	-
$^{76}\text{Se}$	59	31	-	8	-	2	-	-
$^{82}\text{Se}$	56	32	-	9	-	2	-	-
$^{82}\text{Kr}$	54	34	-	11	-	2	-	-
$^{128}\text{Te}$	52	34	-	11	-	3	-	-
$^{128}\text{Xe}$	40	37	-	17	-	5	-	1

$$M_{F,GT,T} = \sum_{\alpha,\beta} A_{\nu_i(\alpha)} B_{\nu_f(\beta)} \langle \nu_f(\beta) | O_{F,GT,T} | \nu_i(\alpha) \rangle$$

where the A's and B's are the amplitudes of the different seniority components of the wave functions of the initial and final nuclei. Obviously, when we plug the ISM amplitudes in this formula, we recover the ISM NME's. But, what shall we obtain if we put the QRPA amplitudes instead? Indeed, we get approximately the QRPA NME's! (5.73 for A=76 and 4.15 for A=82). Therefore as we had anticipated, the seniority mismatch of the initial and final wave functions, which is severely underestimated in the QRPA calculations, explains most of the discrepancy between the two descriptions. In addition, this result strongly suggests that there is some kind of universal behavior in the NME's of the neutrinoless double beta decay when they are computed in a basis of generalized seniority. If this is so, the only relevant difference between the different theoretical approaches would reside in the seniority structure of the wave functions that they produce.

**Table 3.** The GT NME's of the A=48 decay in the generalized seniority basis

$^{48}\text{Ti}$	$s = 0$	$s = 4$	$s = 6$	$s = 8$
$^{48}\text{Ca } s = 0$	3.95	-3.68	-	-
$^{48}\text{Ca } s = 4$	0.00	-0.26	0.08	-0.02

A very spectacular example of the cancellation of the NME by the seniority mismatch is provided by the  $^{48}\text{Ca}$  decay. In Table 2 we have included also the seniority structures of the two nuclei, and we see that they are very different. If we now examine the values of the matrix elements  $\langle \nu_f(\beta) | O_{GT} | \nu_i(\alpha) \rangle$  we find the values listed in Table 3. There are two large matrix elements one diagonal and another off-diagonal of the same size and opposite sign. If the two nuclei were dominated by the seniority zero components one should obtain  $M_{GT} \sim 4$ . If  $^{48}\text{Ti}$  were a bit more deformed,  $M_{GT}$  will be essentially zero. The value produced by the KB3 interaction is

0.75 that is more than a factor five reduction with respect to the seniority zero limit. Earlier work on double beta decays in a basis of generalized seniority (limited to  $s=0$  and  $s=4$  components) showing also this kind of cancellations can be found in ref. [21]

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