E0 transition strengths from X(5) to the rigid rotor

A Krugmann[1], J Bonnet[1], N Pietralla[1] and R V Jolos[2]

[1] Institut für Kernphysik, Technische Universität Darmstadt, Darmstadt, Germany
[2] Joint Institute for Nuclear Research, Dubna, Russia

E-mail: krugmann@ikp.tu-darmstadt.de

Abstract. Relative and absolute $E0$ transition strengths $[\rho^2(E0)]$ on the transitional path between the X(5) solution and the rigid rotor limit have been evaluated [1] within the framework of the Confined β-Soft (CBS) rotor model. Relative $E0$ transition strengths between the β-vibrational band and the ground state band decrease with increasing angular momentum for a given potential stiffness. The Z-independent quantity $X \propto \rho^2(E0; 0_2^+ \to 0_1^+)/B(E2; 0_2^+ \to 2_1^+)$ has been traced between X(5) and the rigid rotor. It reaches the value $4\beta_M^2$ at the rigid rotor limit, as previously derived by Rasmussen. A new Inter-Band $E0-E2$ correlation observable $Y \propto \rho^2(E0; 0_2^+ \to 0_1^+)/B(E2; 0_2^+ \to 2_1^+)^2$ has be en proposed, which is independent on the absolute nuclear deformation and solely depends on the nuclear stiffness. Available data for X and Y are in satisfactory agreement with the CBS model.

1. Introduction

As a function of particle number, heavy nuclei can undergo rapid shape phase transitions with respect to their deformation. These shape phase transitions have been studied experimentally and theoretically for many years [2, 3]. This discussion intensified when Iachello proposed analytical solutions of the geometrical Bohr Hamiltonian near the critical points of various nuclear shape phase transitions [4, 5, 6]. The solutions labeled E(5) and X(5) have attracted a great deal of interest and initiated extensive research. Besides the characteristic excitation energy ratios $R_{4/2} = E(4_1^+)/E(2_1^+) = 2.20$ [for E(5)] and $R_{4/2} = 2.90$ [for X(5)] or the evolution of the $E2$ transition rates as a function of spin along the ground state band, the properties of the quadrupole-collective, excited 0^+ states including its $E2$ decays are considered as the identifying signatures of these models. Several experimental studies [7, 8, 9, 10] have dealt with these signatures.

Another prominent decay modes, in particular for 0^+ states, are $E0$ transitions. Brentano *et al.* [11] pointed out that in the interacting boson model the $E0$ transition rates to the ground state increase at the shape phase transitional point and continue to have larger values up to the SU(3) or the O(6) dynamical symmetries.

$E0$ transition strengths have not yet been reported for the X(5) solution. We have recently studied [1] $E0$ transition rates in the Confined $\beta-$Soft (CBS) rotor model [12]. This model interpolates between the X(5) solution and the rigid rotor limit as a function of one structural parameter. Using the CBS rotor model we have obtained relative $E0$ transition rates between bands with one, two or three nodes in the β wave function.

© 2010 IOP Publishing Ltd

License: CC BY-NC-SA 3.0 Unported - Creative Commons, Attribution, NonCommercial, ShareAlike
https://creativecommons.org/licenses/by-nc-sa/3.0/

2. CBS Rotor Model

Recently, we have studied [1] the evolution of $E0$ transition strengths for axially symmetric quadrupole deformation from Iachello's X(5) solution [5] towards the rigid rotor limit within the Confined β-Soft rotor model. The CBS rotor model represents an approximate analytical solution to the Bohr Hamiltonian [13]

$$H = -\frac{\hbar^2}{2B}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta} + \frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma} - \frac{1}{4\beta^2}\sum_k \frac{Q_k^2}{\sin^2(\gamma - \frac{2}{3}\pi k)}\right] + V(\beta,\gamma) \quad (1)$$

in the quadrupole shape parameters β and γ. Assuming a separable potential $V(\beta,\gamma) = u(\beta) + v(\gamma)$ the wave equation approximately separates into $\Psi(\phi,\theta,\psi,\beta,\gamma) = \xi_L(\beta)\,\eta_K(\gamma)\,D^L_{M,K}(\theta_i)$, where $D^L_{M,K}$ denotes the Wigner functions with θ_i being the Euler angles for the orientation of the intrinsic system. η_K denotes the appropriate wave function in γ. For sufficiently axially symmetric prolate nuclei one might consider a steep harmonic oscillator in γ [5]. $\xi_L(\beta)$ describes the part of the wave function depending on the deformation variable β. The CBS rotor model assumes for prolate axially symmetric nuclei an infinite square well potential $u(\beta)$, with boundaries at $\beta_M > \beta_m > 0$. For this potential the Schrödinger equation is analytically solvable. The ratio $r_\beta = \beta_m/\beta_M$ parameterizes the width of this potential, that is the stiffness of the nucleus in the β degree of freedom. For $r_\beta = 0$ the X(5) limit is obtained with large fluctuations in β. The rigid rotor limit without fluctuations in β corresponds to $r_\beta \to 1$. The full solution of the Bohr Hamiltonian can be analytically written and calculated in terms of Bessel functions of first and second kind.

The CBS rotor model well describes the evolution of low-energy 0^+ bands [12], ground bands of strongly deformed nuclei [14], and the dependence of relative moments of inertia as a function of spin in deformed transitional nuclei [15].

3. E0 Transition Rates

Electromagnetic transition rates can be calculated from the wave functions given above. For $E0$ transitions we obtain with Eqs. (3,4):

$$\rho^2_{\rm if}(E0) \;=\; \left(\frac{3Z}{4\pi}\right)^2 |\langle\psi_{\rm f}|\hat{\beta}^2|\psi_{\rm i}\rangle|^2 \quad (2)$$

between states that only differ in the β-dependent part of the wave function, *i.e.*, $\langle D_{\rm f}|D_{\rm i}\rangle = \langle\eta_{\rm f}|\eta_{\rm i}\rangle = 1$. As a function of the choice of the potential, *i.e.*, as a function of the nuclear deformation (β_M) and the nuclear stiffness against centrifugal stretching (r_β), we calculated the $E0$ transition rates [1].

4. E0 transitions in quadrupole collective models

The $E0$ transition strength is defined by

$$\rho^2_{\rm if}(E0) = \frac{|\langle\Psi_{\rm final}|\hat{T}(E0)|\Psi_{\rm initial}\rangle|^2}{(eR^2)^2} \quad (3)$$

with the $E0$ transition operator to lowest order in the deformation parameter

$$\hat{T}(E0)_{tr} = \frac{3}{4\pi}ZeR^2\,\hat{\beta}^2 \quad (4)$$

Data on $E0$ transitions have been reviewed by Wood *et al.* [16]. The $E0$ transition operator is similar in structure to the β-dependent part of the $E2$ transition operator

$$\hat{T}(E2)_{\Delta K=0} = \frac{3}{4\pi}ZeR^2\hat{\beta}\,, \quad (5)$$

again in the axially symmetric case with $\gamma = 0$.

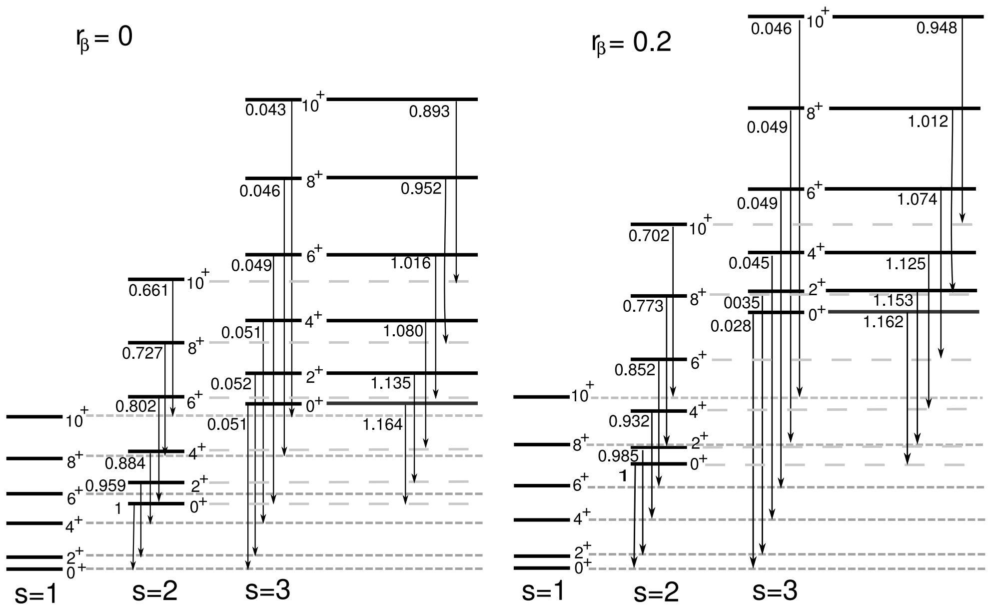

Figure 1. Partial level schemes for low-energy $K = 0$ bands obtained with the CBS rotor model with relative $E0$ transition strengths $\rho^2(E0,\, J_i \to J_f)/\rho^2(E0,\, 0_2^+ \to 0_1^+)$
left: for X(5) [r_β=0], $R_{4/2} = 2.90$
right: for r_β=0.2, $R_{4/2} = 3.10$

4.1. Stiffness-dependence of relative $E0$ *transition strengths*
First we derive the $E0$ transition strengths in the X(5) solution, *i.e.*, for the structural parameter $r_\beta = 0$. The width of the potential well, β_M, is considered as the sole free parameter and thus we normalize the $E0$ transition rates relative to the $0^+_{s=2} \to 0^+_{s=1}$ $E0$ transition. The left hand side of Fig. 1 shows these relative $E0$ transition rates for $r_\beta = 0$, which is X(5). Some of the values are also tabulated in Table 1. We note that the $\rho^2(E0)$ values are comparable in size for all those $E0$ transitions that have $\Delta s = 1$, while $\Delta s > 1$ $E0$ transitions are suppressed by an order of magnitude. The right hand side of Fig. 1 shows the relative $E0$ transition rates normalized again to the $0^+_{s=2} \to 0^+_{s=1}$ transition for the CBS model parameter $r_\beta = 0.2$, which corresponds to an intermediate situation with an excitation energy ratio $R_{4/2} = 3.10$. Again, we observe an approximate selection rule for $E0$ transitions between bands with $\Delta s > 1$.

4.2. Angular momentum dependence of relative $E0$ *transition strengths*
Next we compare relative $E0$ transition strengths for different width parameters r_β and different angular momenta J for transitions between states with $s = 2$ (β-band) and $s = 1$ (ground state band). The results are shown in Fig. 2. Here, the $E0$ transitions are again normalized to the $0^+_{s=2} \to 0^+_{s=1}$ transition for every width parameter r_β, separately. For increasing angular momentum, the model predicts a decreasing $E0$ transition rate. For X(5) the $10^+_{s=2} \to 10^+_{s=1}$ $E0$ transition rate is reduced by 38% relative to the corresponding $0^+_{s=2} \to 0^+_{s=1}$ transition. Notice also, that for rising potential stiffness, the relative $E0$ transition strengths rapidly approaches a constant value as a function of spin, which is typical for the rigid rotor limit.

4.3. Evolution of absolute $E0$ *rates*
Next we focus on absolute transition strengths. Fig. 3 shows $E0$ transition strengths for different angular momenta depending on the width parameter r_β. All transition strengths are normalized

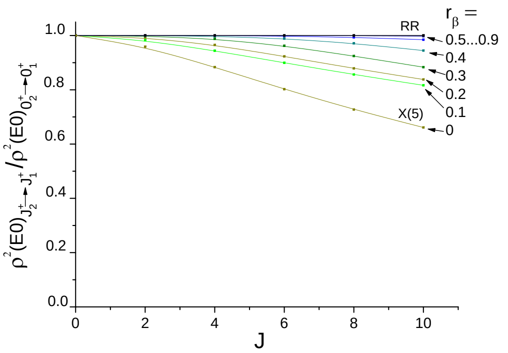

Figure 2. (Color online) Relative $E0$ transition strengths for various nuclear stiffnesses with r_β ranging from 0 to 0.9 in steps of 0.1.

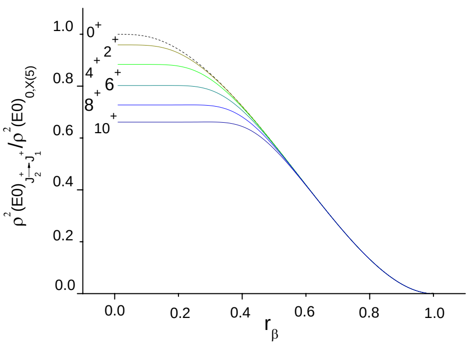

Figure 3. (Color online) $E0$ transition rates from s=2 to s=1 between states with angular momenta $J = 0 - 10$ as a function of r_β. The theoretical values are normalized to the $0_2^+ \to 0_1^+$ $E0$ transition strength in X(5), $\rho^2_{0,X(5)} \equiv \rho^2(E0, 0_2^+ \to 0_1^+; r_\beta = 0)$.

Table 1. $E0$ transition strengths in convenient unitsfor $r_\beta = 0$, 0.2 and 0.4.

transition	$10^3 \frac{\rho^2(E0)}{Z^2\beta_M^4}$								
	$s=2 \to s=1$			$s=3 \to s=1$			$s=3 \to s=2$		
$J_f \to J_i$	X(5)	$r_\beta = 0.2$	$r_\beta = 0.4$	X(5)	$r_\beta = 0.2$	$r_\beta = 0.4$	X(5)	$r_\beta = 0.2$	$r_\beta = 0.4$
0^+	1.81	1.70	1.31	0.092	0.048	0.013	2.10	1.98	1.52
2^+	1.73	1.68	1.30	0.093	0.059	0.015	2.05	1.96	1.52
4^+	1.60	1.59	1.30	0.092	0.076	0.019	1.95	1.92	1.52
6^+	1.45	1.45	1.28	0.088	0.084	0.027	1.84	1.83	1.51
8^+	1.31	1.31	1.23	0.083	0.083	0.037	1.72	1.72	1.49
10^+	1.19	1.20	1.16	0.078	0.078	0.047	1.61	1.61	1.46

to the transition $0_2^+ \to 0_1^+$ for X(5), *i.e.*, we plot the quantity $\frac{\rho^2(E0, J_f \to J_i; r_\beta)}{\rho^2(E0, 0_2^+ \to 0_1^+; r_\beta = 0)}$ for a fixed deformation β_M. For r_β approaching unity all $E0$ transition strengths drop to zero. The evolution of this decrease as a function of r_β depends on angular momentum as it was discussed above. $E0$ transition strengths from 0^+ states monotonically decrease with increasing r_β. $E0$ transition strengths between states with larger angular momenta are significantly effected only above a certain value of r_β, which depends on angular momentum. This behavior originates in the phenomenon of centrifugal stretching for states at higher spins, which causes for higher rotational frequencies a lower sensitivity to the potential at small values of β [12].

For higher stiffness ($r_\beta \gtrsim 0.5$) the decrease of $E0$ transition rates as a function of r_β is entirely dictated by the narrowing of the square-well potential. Independent on low enough spin, the $E0$ transition rates decrease almost linearly with increasing r_β. Table 1 provides an overview over the values of $E0$ transition strengths for the width parameters of $r_\beta = 0$, 0.2 and 0.4. In order to present some global values, the $E0$ transition strengths in Table 1 are divided by the appropriate powers of the nuclear charge Z and the scale of its deformation, β_M. A comparison of $E0$ transition strengths, predicted by the CBS rotor model, to experimental values is given

Table 2. Comparison of experimental values for $\rho^2(E0)$ for ^{152}Sm [17] with the CBS rotor model with stiffness parameter $r_\beta = 0.14$. The second line contains absolute predictions (see text). The values given in the 3rd line are normalized to the $E0$ strength of the $0_2^+ \rightarrow 0_1^+$ transition in ^{152}Sm.

transition	$0_2^+ \rightarrow 0_1^+$	$0_3^+ \rightarrow 0_1^+$	$0_3^+ \rightarrow 0_2^+$
$\rho^2(E0)$ experiment	0.058(6)	0.0007(4)	0.022(9)
$\rho^2(E0)$ CBS from $E2$	0.3041	0.0114	0.3534
$\rho^2(E0)$ CBS renormalized	0.058	0.0022	0.0674

in Table 2. Data on ^{152}Sm where the strengths of the $E0$ transition rates between the 0_1^+, 0_2^+ and the 0_3^+ states are known are used for example and were taken from Ref. [17]. The CBS values were calculated using a stiffness parameter $r_\beta = 0.14$ obtained in Ref. [12] for a good reproduction of the level scheme of ^{152}Sm.

Reproduction of the $E2$ excitation strength B($E2$, $0_1^+ \rightarrow 2_1^+$) $= 3.47 \pm 7\, e^2\text{b}^2$ results in a width parameter $\beta_M = 0.46$ and an average deformation of $\langle\beta\rangle = 0.29$. This parameter-free absolute prediction of the $0_2^+ \rightarrow 0_1^+$ $E0$ transition strength for ^{152}Sm coincides with the data within a factor of about five when no effective charges are considered.

Data and theory, furthermore, agree on the approximate selection rule $\Delta s = 1$, which suppresses the $0_3^+ \rightarrow 0_1^+$ $E0$ transition by one order of magnitude. Normalizing the theory to the $0_2^+ \rightarrow 0_1^+$ transition in the spirit of using an effective $E0$ charge yields a good description for the $E0$ decay rates of the 0_3^+ state. This is quite satisfactory.

5. Inter-band $E2$-$E0$-correlation observables

Finally we discuss observables that relate $E0$ and $E2$ transition strengths between the β-band and the ground state band. The Z-independent quantity X was introduced in the 1960s by Rasmussen [18]. He defined the ratio of the $E0$ transition strength $\rho^2(E0,\, 0_2^+ \rightarrow 0_1^+)$ and $B(E2,\, 0_2^+ \rightarrow 2_1^+)$ value between the β-band and the ground state band

$$X_{\text{Rasmussen}} = \frac{\rho^2(0_2^+ \rightarrow 0_1^+)\, e^2 R^4}{B(E2, 0_2^+ \rightarrow 2_1^+)} = 4\beta_{RR}^2 \tag{6}$$

within the rigid rotor model, where β_{RR} denotes the deformation of the nucleus in the rigid rotor limit. This result has lead to a wide-spread misconception about the evolution of $E0$ transition rates in deformed nuclei. Eq. (6) shows the increase of the $E0/E2$-ratio between ground state band and β-band as a function of the nuclear deformation. It does *not* express an increase of the $E0$ transition strength to the β-band for a sequence of nuclei approaching the Rigid Rotor Limit, *i.e.*, with increasing $R_{4/2}$ value ($\rightarrow 3.33$). In contrast, our previous results (see Fig. 3 and Table 1) show that the $E0$ strengths decrease with increasing stiffness of the nuclear quadrupole deformation. *I.e.*, for a typical deformation $\beta \lesssim 0.5$ we must expect the following consequence: the better the nucleus resembles the Rigid Rotor ($R_{4/2} \approx 3.33$), the larger is its stiffness, the smaller are the $E0$ and $E2$ transitions between the β-band and the ground state band.

In the CBS model we can trace the evolution of X as a function of nuclear stiffness. Therefore we consider the quantity

$$X_{\rm CBS} = \frac{\rho^2(0_2^+ \to 0_1^+)\, e^2 R^4}{B(E2, 0_2^+ \to 2_1^+)} \tag{7}$$

$$= \frac{\langle \Psi_{0_1^+} |\, \mathfrak{b}^2 \,| \Psi_{0_2^+} \rangle^2 \, \beta_M^2}{\langle \Psi_{2_1^+} |\, \mathfrak{b} \,| \Psi_{0_2^+} \rangle^2} \tag{8}$$

with $\mathfrak{b} = \frac{\beta}{\beta_M}$.

As plotted in Fig. 4, the ratio X converges to $4\beta_M^2$ in agreement with the limit obtained by Rasmussen in the rigid rotor model. Data on X exist for the nuclides ^{152}Sm and ^{156}Gd. They are given in Table 2 and included in Fig. 4. The predicted increase of X as a function of nuclear stiffness is obvious. However, the CBS overpredicts the data by a factor of about two. The ratio X/β_M^2 includes the scaling factor β_M^2 and therefore is not determined completely by data. It is costumary to extract β_M^2 from the $B(E2, 2_1^+ \to 0_1^+)$ in a consistent, albeit model-dependend way. The deviations between theory and data for X/β_M^2 seen in Fig. 4, may then be attributed to the usage of both, data on inter-band (X) and intra-band ($\beta_M^2 \propto B(E2, 2_1^+ \to 0_1^+)$) transitions.

In order to fully avoid this dependence on the scaling factor β_M and, thus, any model-dependence, we define the quantity

$$Y = \frac{\rho^2(0_2^+ \to 0_1^+)\, (e^2 R^4 Z)^2 \, (\frac{3}{4\pi})^2}{B(E2, 0_2^+ \to 2_1^+)^2}$$

$$= \frac{\langle \Psi_{0_1^+} |\, \mathfrak{b}^2 \,| \Psi_{0_2^+} \rangle^2}{\langle \Psi_{2_1^+} |\, \mathfrak{b} \,| \Psi_{0_2^+} \rangle^4} \tag{9}$$

as the ratio between two appropriate powers of inter-band transition strengths. This observable theoretically depends solely on the stiffness parameter r_β. Due to the monotonic relation between r_β and the $R_{4/2}$ ratio [12], the CBS model uniquely relates the observable Y to the $R_{4/2}$ ratio, which itself is a fundamental, easily obtainable characteristical observable. Figure 5 shows plots of Y as a function of r_β (left) and $R_{4/2}$ (right). Y increases monotonically with the stiffness parameter r_β and with the $R_{4/2}$ ratio, too. Y diverges in the rigid rotor limit. Experimental data points for ^{152}Sm and ^{156}Gd have been included in figures 4 and 5 for example. They have been obtained by evaluating the quantities X and Y as a function of the easily obtainable branching ratio Γ_{E0}/Γ_{E2} for the decays of the 0_2^+ state to the 0_1^+ and 2_1^+ members of the ground state band and the lifetime $\tau_{0_2^+}$ of the 0_2^+ state.

6. Summary

The analytical wave functions in the deformation coordinate β of the Confined β-Soft rotor model have been used to evaluate $E0$ transition strengths between low-energy $K = 0$ bands in axially symmetric nuclei [1]. As a function of the only structural parameter within the CBS rotor model, *i.e.*, r_β, which interpolates from X(5) to the rigid rotor, absolute $E0$-transitions strengths decrease with increasing r_β. The relative $E0$ strengths for a given value of the structural parameter r_β decrease with increasing angular momentum. The Z-independent quantity $X \propto \rho^2(E0; 0_2^+ \to 0_1^+)/B(E2; 0_2^+ \to 2_1^+)$ has been traced between X(5) and the rigid rotor. It reaches the value $4\beta_M^2$ at the rigid rotor Limit, as previously derived by Rasmussen. A new observable $Y \propto \rho^2(E0; 0_2^+ \to 0_1^+)/B(E2; 0_2^+ \to 2_1^+)^2$ has been proposed, which is independent on the absolute nuclear deformation and solely depends on the nuclear stiffness. Available data are in satisfactory agreement with the CBS model.

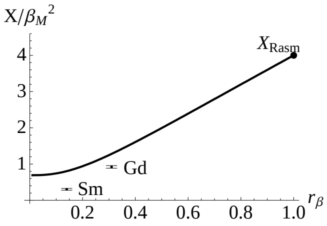

Figure 4. Ratio X_{CBS}/β_M^2 plotted as a function of the stiffness parameter r_β. A value of $X/\beta_M^2 = 4$ has been derived by Rasmussen [18] in the rigid rotor limit. The data points show the experimental values for ^{152}Sm [17] ($r_\beta = 0.14$) and ^{156}Gd [19, 20]($r_\beta = 0.31$).

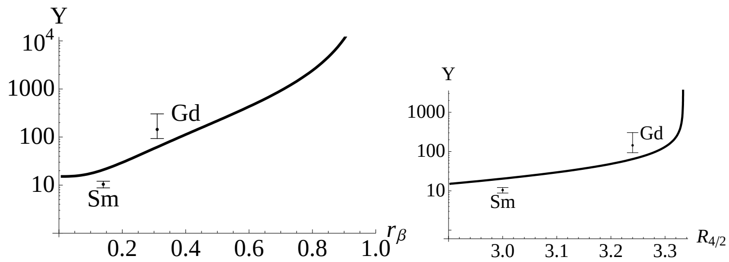

Figure 5. Ratio Y over r_β and $R_{4/2}$. The data points represent the experimental values of Y for ^{152}Sm($R_{4/2} \approx 3.00$) [17] and for ^{156}Gd ($R_{4/2} \approx 3.23$) [19, 20].

Acknowledgments

We thank F. Iachello, K. Heyde, P. von Brentano, G. Rainovski, O. Möller and V. Werner for stimulating discussions. Support by the DFG under Grant No. SFB 634, by the Helmholtz International Center for FAIR and by the DAAD within the German-Bulgarian exchange programme under Grant Nos. D/06/05918 and DAAD-09 is gratefully acknowledged.

References

[1] Bonnet J, Krugmann A, Beller J, Pietralla N and Jolos R V 2009 *Phys. Rev. C* **79** 034307
[2] Bohr A and Mottelson B 1975 *Nuclear Structure* vol 2 (Reading: W. A. Benjamin)
[3] Casten R F 2000 *Nuclear Structure from a Simple Perspective* second edition ed (Oxford: Oxford Univ. Pr.)
[4] Iachello F 2000 *Phys. Rev. Lett.* **85** 3580
[5] Iachello F 2001 *Phys. Rev. Lett.* **87** 052502
[6] Iachello F 2003 *Phys. Rev. Lett.* **91** 132502
[7] Clark R M *et al.* 2003 *Phys. Rev. C* **68** 037301
[8] Clark R M *et al.* 2004 *Phys. Rev. C* **69** 064322
[9] Casten R F and Zamfir N V 2001 *Phys. Rev. Lett.* **87** 052503
[10] McCutchan E A and Zamfir N V 2005 *Phys. Rev. C* **71** 034309
[11] von Brentano P, Werner V, Casten R F, Scholl C, McCutchan E A, Krücken R and Jolie J 2004 *Phys. Rev. Lett.* **93** 152502
[12] Pietralla N and Gorbachenko O M 2004 *Phys. Rev. C* **70** 011304
[13] Bohr A 1952 *Mat. Fys. Medd. K. Dan. Vid. Selsk.* **26**
[14] Dusling K and Pietralla N 2005 *Phys. Rev. C* **72** 011303
[15] Dusling K *et al.* 2006 *Phys. Rev. C* **73** 014317
[16] Wood J L, Zganjar E F, Coster C D and Heyde K 1999 *Nucl. Phys.* 323
[17] Artna-Cohen A 1996 *Nuclear Data Sheets* **79** 1
[18] Rasmussen J O 1960 *Nucl. Phys.* **19** 85
[19] Reich C W 2003 *Nuclear Data Sheets* **99** 753
[20] Klora J *et al.* 1993 *Nucl. Phys.* 1