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PAPER

Protecting coherence by environmental decoherence: a solvable paradigmatic model

Juan Mauricio Torres[1,2] **and Thomas H Seligman**[3,4]

[1] Instituto de Física, Benemérita Universidad Autónoma de Puebla, Apartado Postal J-48, Puebla 72570, Mexico
[2] Institut für Angewandte Physik, Technische Universität Darmstadt, D-64289, Germany
[3] Instituto de Ciencias Físicas, Universidad Nacional Autónoma de México, Cuernavaca, Morelos, Mexico
[4] Centro Internacional de Ciencias A. C., Cuernavaca, Morelos, Mexico

E-mail: jmtorres@ifuap.buap.mx

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Abstract

We consider a particularly simple exactly solvable model for a qubit coupled to sequentially nested environments. The purpose is to exemplify the coherence conserving effect of a central system, that has been reported as a result of increasing the coupling between near and far environment. The paradigmatic example is the Jaynes–Cummings Hamiltonian, which we introduce into a Kossakowski–Lindblad master equation using alternatively the lowering operator of the oscillator or its number operator as Lindblad operators. The harmonic oscillator is regarded as the near environment of the qubit, while effects of a far environment are accounted for by the two options for the dissipative part of the master equation. The exact solution allows us to cover the entire range of coupling strength from the perturbative regime to strong coupling analytically. The coherence conserving effect of the coupling to the far environment is confirmed throughout.

1. Introduction

Decoherence was and is not only a central theme of physics, but it is also a central problem for any practical implementation of quantum computation and quantum information schemes [1]. The source of decoherence is the surrounding environment to which the system under investigation couples invariably. Dynamical decoupling is a well established technique to isolate a physical system or to tailor a desired Hamiltonian evolution [2, 3]. Other dynamical control methods exploit the quantum Zeno effect to slow down decoherence processes [4–6]. In these techniques, one of the requirements is a periodic driving or measurement of the system. A natural question is whether intrinsic decay mechanisms of the environment can enhance the coherence of a central system. Recent studies have considered the coherence loss of a 'near' environment to stabilize the coherence of a central quantum system. This reaches from the limit of very fast decoherence of the near environment [7–11], which may actually lead in some limit to a protected subspace for the central system, all the way to perturbative treatments where all couplings are small [8, 9]. The great benefit in this approach is that one does not require to control the system dynamically. Numerical results for spin systems [10] and random matrix environments [8] seem to support such improvement throughout the range of coupling strength. In all cases there seem to exist options to improve the persistence of coherence of the central system considerably if it is already quite good to start with. Indeed it so seems, that weak coupling of the central system to the near environment is the only prerequisite for this method to be workable. The results are positive and interesting but a little counter intuitive.

Under such conditions an exactly solvable example is usually very enlightening and that is what we are going to present. We will use a fairly new technique to obtain exact solutions of the corresponding Kossakowski–Lindblad master equation [24]. We shall thus deepen the understanding of the role of nested environments for decoherence and simultaneously provide a non-trivial application of a new technique to solve open quantum problems. Our study focuses on two versions of a paradigmatic, simple, and exactly solvable model in quantum

optics: a Jaynes–Cummings (JC) model with dephasing and a damped JC model. The dephasing and the damping mechanisms are assumed to act solely on the cavity mode. The role of the central system is played by a two-level atom which is coupled to a single mode of an optical cavity acting as the near environment. Decoherence of the cavity is taken into account to mimic the effects of a far environment. We assume that the two types of decoherence mechanisms in the cavity which can be described in terms of a Markovian master equation in Kossakowski–Lindblad form [12, 13]. In a first approach we consider a dephasing model, with the number operator as Lindblad operator. This may not be very realistic, but it will turn out to be most illustrative due to its simple analytical treatment and the absence of competing effects: the Liouville operator can be expressed in terms of disconnected 4×4 matrices. In a second case, photon losses are considered by choosing the annihilation operator as Lindblad operator. This case has deep roots in the field and can be connected to the standard setting in the Haroche experiment [14] and according to Garraway's pseudo mode theory [15–17], it is equivalent to a two-level atom interacting with a continuum of modes.

The simple fact that we do get exact solutions in special but non-trivial situations for the decay of coherence of a rather complicated system is of great interest, as it will allow us to gain insight in possible mechanisms leading to protection of coherence in nested environments, that previous to this work were numerically detected and analytically derived for extreme situations.

We shall start by briefly defining the model and then proceed to discuss the simpler case, where the Lindblad operator is the number operator of the harmonic mode. Next, we will address the case of photon losses in the cavity in which the annihilation operator of the oscillator is considered as Lindblad operator. Finally, we shall discuss to what extent our considerations may shed light into known numerical and random matrix results [8–10], and discuss in what ways the basic result can be used to help control coherence.

2. The model

The JC model describes the interaction between a two-level atom with one mode of the electromagnetic field inside an optical cavity [18]. The Hamiltonian in the interaction picture with respect to the electromagnetic field energy is given by ($\hbar = 1$)

$$H = \delta\sigma_+\sigma_- + g\,(a\sigma_+ + a^\dagger\sigma_-), \tag{1}$$

where $\sigma_\pm$ are the raising and lowering operators of the two-level atom acting on the Hilbert space $\mathcal{H}_{\rm at} = \mathbb{C}^2$, while a and $a^\dagger$ are the cavity mode creation and annihilation operators that act on the Fock space $\mathcal{H}_{\rm cav} = F_+(\mathbb{C}) = \ell^2$ [23]. The complete Hilbert space is therefore the tensor product of the composite Hilbert spaces $\mathcal{H} = \mathcal{H}_{\rm at} \otimes \mathcal{H}_{\rm cav}$. The parameter g is the interaction strength between the two-level atom and the cavity, while δ is the detuning of the atomic transition frequency from the frequency of the mode. A general state of the system can be represented by the density matrix ρ that is an operator acting on $\mathcal{H}$. Its Hamiltonian dynamics is governed by the von Neumann equation $\dot\rho = -\mathrm{i}[H,\,\rho]$. We introduce decoherence effects in the system by adding the action on ρ of the generator $\mathcal{D}[A]$ defined in Lindblad form as

$$\mathcal{D}[A]\,\rho = A\rho A^\dagger - \frac{1}{2}(A^\dagger A\rho + \rho A^\dagger A). \tag{2}$$

The Lindblad operator A could in principle be chosen to act on the composite Hilbert space $\mathcal{H}$, as in the case of more realistic models that consider combined decay mechanisms in Lindblad form [21, 22]. However, in this work we restrict ourselves to operators A acting solely on the Hilbert space of the cavity $\mathcal{H}_{\rm cav}$. The aim is to treat the two-level atom as a central system, the cavity as a near environment and the effects of a far environment described by the Lindblad operators. The model an its connection to the nested environment description is shown in figure 1. In the next two sections we will consider first a dephasing and then a photon loss operator.

3. Dephasing of the cavity

Let us start our discussion by considering a situation that involves a dephasing mechanism in the cavity but without any loss of excitations. The dynamics of a model that includes this effect can be described by the following master equation

$$\dot\rho = \mathcal{L}_{\rm d}\rho = -\mathrm{i}[H,\,\rho] + \gamma\mathcal{D}[a^\dagger a]\,\rho \tag{3}$$

depending on the dissipator of equation (2) and with the Lindblad operator $A = a^\dagger a$. An important property of the Liouville operator $\mathcal{L}_{\rm d}$ is that it conserves the number of excitations of the operator $a^\dagger a + \sigma_+\sigma_-$. Therefore, if one considers initial states of the form $|{\rm e},\,n-1\rangle = |{\rm e}\rangle \otimes |n-1\rangle$, that is an excited atom and $n-1$ photons in the cavity, the time dependent density matrix of the system can be expressed as

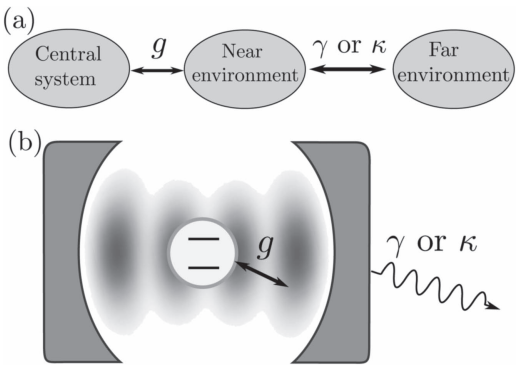

Figure 1. (a) Sketch of the nested environment setting: a central system couples to a near environment with strength g. The near environment interacts with the far environment with an effective coupling parameter either γ or κ. (b) Sketch of the Jaynes–Cummings model: a two-level atom (central system) interacts with an optical cavity (near environment) with interaction strength g. The cavity presents either dephasing at rate γ or photon losses at rate κ (to a far environment).

$$\rho(t) = v_1|\mathrm{g},\, n\rangle\langle \mathrm{g},\, n| + v_2|\mathrm{g},\, n\rangle\langle \mathrm{e},\, n-1| + v_3|\mathrm{e},\, n-1\rangle\langle \mathrm{g},\, n| + v_4|\mathrm{e},\, n-1\rangle\langle \mathrm{e},\, n-1|. \tag{4}$$

The time dependent coefficients are solution of the differential equation $\dot{\vec{v}} = L\vec{v}$, where $\vec{v} = (v_1,\, v_2,\, v_3,\, v_4)^\top$ is a column vector and L is a matrix that can be obtained from the Liouvillian $\mathcal{L}_\mathrm{d}$ and has the explicit form

$$L = \begin{pmatrix} 0 & \mathrm{i}g\sqrt{n} & -\mathrm{i}g\sqrt{n} & 0 \\ \mathrm{i}g\sqrt{n} & -\gamma + \mathrm{i}\delta & 0 & -\mathrm{i}g\sqrt{n} \\ -\mathrm{i}g\sqrt{n} & 0 & -\gamma - \mathrm{i}\delta & \mathrm{i}g \\ 0 & -\mathrm{i}g\sqrt{n} & \mathrm{i}g\sqrt{n} & 0 \end{pmatrix}. \tag{5}$$

Actually, the matrix L is one of many disconnected blocks that form the Liouvillan $\mathcal{L}_\mathrm{d}$ [24]. As it is a 4×4 matrix, the eigenvalues of L can always be calculated in closed form as shown in appendix A. However, we focus our attention to the resonant case as it captures the qualitative essence of the dynamics we want to describe and the resulting equations can be written in compact form. Deviations from this condition do not present a qualitative change in the long time behavior that we are interested. Therefore, in the resonant case, i.e., $\delta = 0$, the four eigenvalues of L are given by

$$l_0 = 0, \quad l_1 = -\gamma, \quad l_\pm^{(n)} = -\gamma\frac{1 \mp \sqrt{1-\eta_n}}{2}, \tag{6}$$

where we have introduced the dimensionless parameter

$$\eta_n = 16g^2 n/\gamma^2. \tag{7}$$

This parameter η_n can be seen as a rescaled interaction strength between atom and cavity, that tends to zero for increasing values of γ whenever the values of g and the photon number n are finite. As we are particularly interested in the coupled dynamics in this limiting case ($\gamma \gg g,\, n$), we analyze the eigenvalues by expanding them in terms of η_n which leads to the expression

$$l_\pm^{(n)} = -\frac{1 \mp 1}{2}\gamma - \frac{1}{4}\gamma\eta_n + \mathcal{O}[\eta_n^2]. \tag{8}$$

It follows from this expansion that the eigenvalues of L, which are also eigenvalues of the Liouville operator $\mathcal{L}_\mathrm{d}$, are insensitive to the coupling strength for sufficiently large values of the dephasing parameter γ. This is already an indication that the atomic system is 'protected' from the presence of the cavity by the dephasing mechanism.

Now we turn our attention to the dynamical properties of the atomic sub-system. By solving the differential equation and tracing over the photonic degree of freedom, one can evaluate the atomic density matrix which is diagonal in this case and is given by

$$\rho_\mathrm{at}(t) = \frac{1}{2}[1 - h_n(t)]|\mathrm{g}\rangle\langle \mathrm{g}| + \frac{1}{2}[1 + h_n(t)]|\mathrm{e}\rangle\langle \mathrm{e}|, \tag{9}$$

were we have considered the complex function

$$h_n(t) = \frac{\eta_n \mathrm{e}^{l_+^{(n)}t}}{2\eta_n + 4l_+^{(n)}/\gamma} + \frac{\eta_n \mathrm{e}^{l_-^{(n)}t}}{2\eta_n + 4l_-^{(n)}/\gamma}. \tag{10}$$

In the limit of $\eta_n \to 0$ ($\gamma \gg g,\, n$), the first term tends to 1 while the second tends to zero. This can be noted by taking into account the form of $l_\pm^{(n)}$ in equation (8) and of η_n in equation (7). Therefore, in this limiting case the probability of finding the atom in the excited state

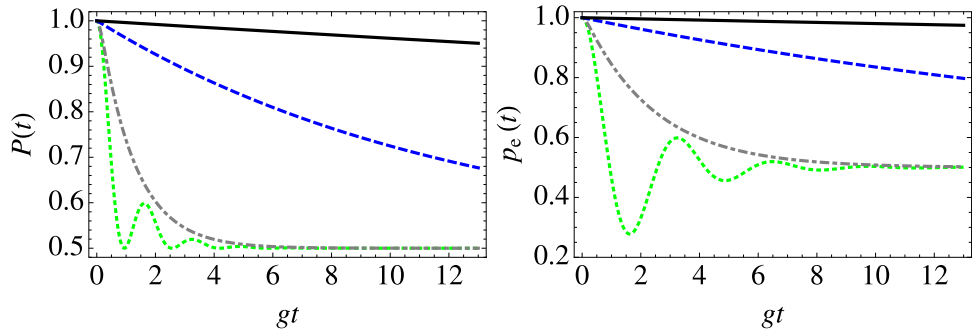

Figure 2. Left (right) plot: atomic purity $P(t)$ (excited state population $p_e(t)$) as a function of time in units of $1/g$ for different value of the cavity's dephasing parameter γ. The initial state is an excited atom and an empty cavity subject to the dynamics of equation (3). Dotted (green) line $\gamma/g = 1$, dotted–dashed (gray) line $\gamma/g = 10$, dashed (blue) line $\gamma/g = 100$, and full (black) line $\gamma/g = 1000$. Detuning is $\delta/g = 0$. Note: the purity $P(t)$ is plotted in the range of minimum to maximum purity ([0.5, 1]).

$$p_e(t) = \mathrm{Tr}\{\sigma_+\sigma_-\rho_{at}(t)\} = \frac{1 + h_n(t)}{2} \tag{11}$$

freezes at unit value. One can also consider purity of the atomic state. For this quantity one finds

$$P(t) = \mathrm{Tr}\{\rho_{at}^2(t)\} = \frac{1}{2} + \frac{\eta_n e^{-\gamma t} + f_n^+(t) + f_n^-(t)}{4(1 - \eta_n)},$$
$$f_n^{\pm}(t) = \frac{1}{2}(1 \pm \sqrt{1 - \eta_n})^2 e^{-\gamma(1 \mp \sqrt{1-\eta_n})t}. \tag{12}$$

In the limit of vanishing rescaled interaction strength η_n the purity tends to one, as it can be noted that $\lim_{\eta_n \to 0} f_n^{\pm}(t) = 1 \pm 1$.

In figure 2 we have plotted the purity $P(t)$ and atomic excitation probability $p_e(t)$ for different values of the dephasing parameter γ. The stationary state of the atomic sub-system is the totally mixed state. This explains the drop of purity as a function of time and the asymptotic value $1/2$ of the excitation probability. However, the basic effect is evident, for increasing values of γ the purity and excitation probability have a slower decay. Closer inspection shows that the two quantities are, in this case, closely related as $p_e(t)$ has to take the value $1/2$ for purity to reach the minimal value of $1/2$.

We close this section by pointing out that the expressions in equation (6) are valid for any value of the parameters as long as $\delta = 0$. However note that when $\eta_n > 1$, the eigenvalues are complex and therefore the imaginary part gives rise to oscillations in the dynamics, as corroborated by the green (dotted) curves in figure 2, where $\eta_1 = 16$. This behavior shows that for small values of γ or comparable with $g\sqrt{n}$, the cavity influences the evolution of the atom through the Hamiltonian interaction in equation (1).

4. Photon losses

A more common and realistic scenario is the case of photon losses from the cavity. This effect can be incorporated by describing the dynamics in terms of the Kossakowski–Lindblad master equation

$$\dot{\rho} = \mathcal{L}_1\rho = -i[H, \rho] + \kappa\mathcal{D}[a]\rho. \tag{13}$$

Here we have used again the dissipator defined in equation (2), but in this case we have considered the Lindblad operator $A = a$ which describes the damping mechanism due to photon losses in the cavity. The diagonalization of the Liouville operator $\mathcal{L}_1$ in equation (13) allows to evaluate the time evolution of any given initial state. It was noted in [24] that this procedure can be based on the diagonalization of the non-Hermitian Hamiltonian $K = H - i\kappa a^\dagger a/2$ which has the following eigenvalues

$$\varepsilon_j^{(n)} = \frac{2\delta + i\kappa - i2n\kappa}{4} + \frac{2\delta + i\kappa}{4}(-1)^j\sqrt{1 + \chi_n}, \tag{14}$$

with $j = 1, 2$ for $n > 0$, $j = 1$ for $n = 0$, and where we have introduced the rescaled interaction strength

$$\chi_n = 16g^2 n/(2\delta + i\kappa)^2. \tag{15}$$

In fact, the eigenvalues of $\mathcal{L}_1$ are given by the simple addition of two eigenvalues of K, i.e.,

$$\lambda_{j,k}^{(n,m)} = -i(\varepsilon_j^{(n)} - \varepsilon_k^{(m)*}), \tag{16}$$

with $k \in \{1, 2\}$ for $m \in \mathbb{N}_+$ and $k = 1$ for $m = 0$ (as for j and n) [24]. Therefore, analyzing the eigenvalues of K in equation (14) implies doing the same for the Liouvillian $\mathcal{L}_l$. The form of the solution is similar to the eigenvalues of equation (6) and an analogous expansion to the one in equation (8) can be performed. In this way one is able to analyze the eigenvalues in terms of an expansion in powers of χ_n that appears in the radical of equation (14). This procedure leads to the following expression

$$\varepsilon_j^{(n)} = -\mathrm{i}\frac{n\kappa}{2} + \frac{2\delta + \mathrm{i}\kappa}{2}\left(\delta_{j,2} + \frac{(-1)^j}{4}\chi_n + \mathcal{O}[\chi_n^2]\right), \tag{17}$$

where $\delta_{i,2}$ is the Kronecker delta, $j = 1, 2$ for $n > 0$, and $j = 1$ for $n = 0$. Note that the parameter χ_n tends to zero with increasing values of κ if g, n and δ remain finite. It follows from the expansion in equation (17) that the eigenvalues of the Liouville operator $\mathcal{L}_l$ are insensitive to the coupling strength for sufficiently large values of the photon decay rate κ. In this case we also get an indication that the atomic system is 'protected' from the cavity, this time due to photon losses. The result holds for arbitrary values of excitation number n and in this case for all the eigenvalues of the Liouville operator $\mathcal{L}_l$ as they are a sum of two eigenvalues of equation (14). In this example involving photon losses, the detuning can also be taken into account thanks to the simple diagonalization of $\mathcal{L}_l$ that breaks down to the diagonalization of 2×2 matrices.

Let us now study the dynamical features of the atomic sub-system. Using the approaches of [24, 25], it is possible to evaluate the eigensystem of $\mathcal{L}_l$ and in turn to calculate the time evolution of, in principle, any given initial condition. For the sake of simplicity, we consider the case where only one excitation is present in the system. Details of the calculations can be found in appendix B. We assume an arbitrary pure state of the atom and an empty cavity in the Fock state $|0\rangle$. The initial condition is then given by

$$|\Psi_0\rangle = (c_g|g\rangle + c_e|e\rangle) \otimes |0\rangle. \tag{18}$$

The corresponding reduced density matrix of the atomic sub-system for this particular initial state is found to be

$$\rho_{at}(t) = (1 - |c_e f(t)|^2)|g\rangle\langle g| + |c_e f(t)|^2|e\rangle\langle e| + c_e c_g^* f(t)|e\rangle\langle g| + c_e^* c_g f^*(t)|g\rangle\langle e| \tag{19}$$

with the complex function

$$f(t) = \frac{4g^2 e^{-\mathrm{i}\varepsilon_2^{(1)}t} + (\mathrm{i}\kappa + 2\varepsilon_1^{(1)})^2 e^{-\mathrm{i}\varepsilon_1^{(1)}t}}{4g^2 + (\mathrm{i}\kappa + 2\varepsilon_1^{(1)})^2}. \tag{20}$$

It can be seen that $\lim_{\chi_0 \to \infty} f(t) = e^{-\mathrm{i}\delta t}$, as from equation (17) it follows that in this limit the eigenvalues of the non-Hermitian Hamiltonian K tend to: $\varepsilon_1^{(1)} \to -\mathrm{i}\kappa/2$ and $\varepsilon_2^{(1)} \to \delta$. Therefore, for large values of the damping parameter κ with respect to g, the atom evolves freely under the influence of the free Hamiltonian $\delta\sigma_+\sigma_-$. This can be noted from equation (19) or by evaluating the population of the atomic excited state

$$p_e(t) = \mathrm{Tr}\{\sigma_+\sigma_-\rho_{at}(t)\} = |c_e f(t)|^2. \tag{21}$$

By tracing the square of the atomic density matrix one can find that the atomic purity as a function of time is given by the expression

$$P(t) = 1 + 2|c_e|^4|f(t)|^2(|f(t)|^2 - 1). \tag{22}$$

The form of $P(t)$ makes evident that the atom remains pure for longer times as κ increases with respect to g and remains completely pure in the limit $\chi_0 \to 0$ ($\kappa \gg g$).

Figure 3 shows the purity $P(t)$ and excited state population $p_e(t)$ of the atom as a function of time for different values of the decay parameter κ. The asymptotic behavior is explained by the knowledge of the steady state, which is the atom in the ground state and an empty cavity. The reason for this is the photon losses that drain all excitations in the system. The asymptotic steady state is actually a pure state which explains the re-emergence of the purity for large values of the interaction time. The excitation probability drops to zero to never revive also in accordance with the steady state involving the atom in the ground state. The important feature is, however, that all of this happens at larger time scales with increasing values of the photon losses κ. This means that very strong coupling of the cavity to its environment protects the atomic state. In the limit this state is frozen, i.e we again find a quantum Zeno effect.

We close this section by commenting on previous findings related this part of our work. Similar studies have been considered for a high finesse cavity coupled to a leaky cavity, were first numerical results where given by Imamoğlu [19] followed by analytical investigations of Nemes [20]. A two-level atom coupled to the continuum of modes was investigated by Kofman and Kurizki [4], who found analytical results for the decay of the excited state population including interruptions of the unitary dynamics in line with the quantum Zeno effect. In all these three cases [4, 19, 20], the authors find systematically that increasing the leakage of the cavity slows the decay of the central system. Note this is true with or without interruptions of the unitary evolutions, i.e., with or without repeated measurements. This implies that the obstruction of the decay in the atom by increasing leakiness of the cavity is not due to a quantum Zeno effect although it is enhanced by it (see figure 1 in [4]).

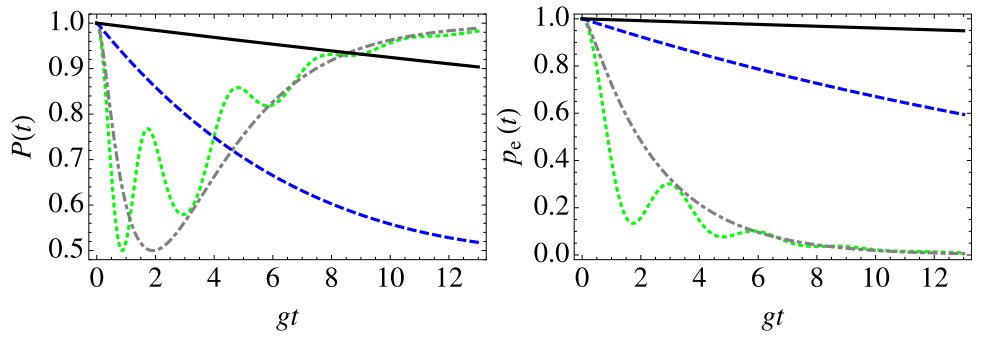

Figure 3. Left (right) plot: atomic purity $P(t)$ (excited state population $p_e(t)$) as a function of time in units of $1/g$ for different values of the cavity decay rate κ. The initial state is an excited atom and an empty cavity subject to the dynamics of equation (13). Dotted (green) line $\kappa/g = 1$, dotted–dashed (gray) line $\kappa/g = 10$, dashed (blue) line $\kappa/g = 100$, and full (black) line $\kappa/g = 1000$. Detuning is $\delta/g = 0.8$. Note: the purity $P(t)$ is plotted in the range of minimum to maximum purity ([0.5, 1]).

5. Conclusions

With the dephasing Lindblad operator we have a very simple example, where we can see the entire development of decoherence as a function of time for different parameters. The scaling behavior is readily established and we see, that the same equation describes the improvement of decoherence from the perturbative all the way to the strong coupling regime, as could be hoped from the rather general analytic results implicit in [7–9]. Yet in the perturbative regime for κ it seemed that for chaotic environments in the Fermi golden rule regime, i.e. with exponential coherence decay, the effect of the far environment was no longer noticeable. The latter effect is not seen in our system, where we always have a preserving effect of the coherence in the central system.

For the case of loss, the fact that we always return to a pure state is trivial, as the vacuum is the steady state, but the fact that the initial decoherence slows down as we increase the coupling to the far environment is non-trivial. We thus see, that while the same equation governs the system, the qualitative explanation using the quantum Zeno effect will only describe the strong coupling limit, as for weak coupling we have complete decoherence and later recoherence while there is a transition to another state. This becomes most clearly visible at the opposite end of the coupling range. Here we see oscillations both in the occupation number and in purity. The unitary Hamiltonian which causes an oscillation of the excitation between the spin and the oscillator is effective. For stronger coupling this dynamics loses importance until we reach a total freeze of dynamics including purity; this is in agreement with the findings of [8]. There the protection of coherence by decoherence of the environment was shown in a weak coupling regime by linear response considerations, which also preclude a quantum Zeno effect.

Thus our two simple models go a long way toward explaining what is going on in the matter of decoherence of a near environment protecting the central system. Yet, as is to be expected, some aspects of more realistic systems are not covered by the model behavior.

Acknowledgments

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Appendix A. Eigenvalues of L

In this appendix we present the exact eigenvalues of the 4×4 matrix L in equation (5). The eigenvalues are also roots of the fourth order characteristic polynomial of L:

$$Q(z) = z(4g^2 n\gamma + (4g^2 n + \gamma^2 + \delta^2)z + 2\gamma z^2 + z^3). \tag{A.1}$$

One of the solutions, namely $Q(0) = 0$, can be immediately identified by inspection of equation (A.1). The rest of the eigenvalues are roots of the third order polynomial $Q(z)/z$. Using the solution of the cubic equation in [26], the four zeros of $Q(z)$ and eigenvalues of L can be written as

$$z_0 = 0, \quad z_1 = -\frac{2}{3}\gamma + \left(s - \frac{q}{s}\right),$$
$$z_\pm = -\frac{2}{3}\gamma - \frac{1}{2}\left(s - \frac{q}{s}\right) \pm \mathrm{i}\frac{\sqrt{3}}{2}\left(s + \frac{q}{s}\right), \tag{A.2}$$

where we have introduced the following definitions

$$q = -\frac{\gamma^2 - 3\delta^2 - 12g^2 n}{9}, \quad r = \gamma\frac{\gamma^2 + 9\delta^2 - 18g^2 n}{27},$$
$$s = (r + \sqrt{q^3 + r^2})^{1/3}. \tag{A.3}$$

For the sake of simplicity we restrict our discussion to the case $\delta = 0$ in the main text.

Appendix B. Eigensystem of $\mathcal{L}_1$

Here we present details of the calculations using the eigensystem of the $\mathcal{L}_1$. The non-Hermitian Hamiltonian $K = H - \mathrm{i}\kappa a^\dagger a/2$ can be diagonalized in blocks in the basis $\{|n, j\rangle\}$ with the states

$$|n, 1\rangle = |n\rangle \otimes |\mathrm{g}\rangle, \quad n \geqslant 0$$
$$|n, 2\rangle = |n-1\rangle \otimes |\mathrm{e}\rangle, \quad n > 0. \tag{B.1}$$

The number state $|n\rangle$ describes a situation of n photons in the cavity, while $|\mathrm{e}\rangle$ and $|\mathrm{g}\rangle$ stands for the excited and ground state of the atom. The explicit form of the blocks of K is given by

$$K^{(n)} = \begin{pmatrix} -\mathrm{i}\frac{n\kappa}{2} & g\sqrt{n} \\ g\sqrt{n} & \frac{2\delta - \mathrm{i}(n-1)\kappa}{2} \end{pmatrix}, \tag{B.2}$$

for $n > 0$ and $K^{(0)} = 0$. The eigenvalues are given in equation (14). The diagonalization of the matrices $K^{(n)}$ can be accomplished with the transformation $R^{(n)\top} H^{(n)} R^{(n)}$, with

$$R^{(n)} = \begin{pmatrix} \cos\theta_n & -\sin\theta_n \\ \sin\theta_n & \cos\theta_n \end{pmatrix}, \tag{B.3}$$

with $\theta_n = \arctan[(2\varepsilon_1^{(n)} + \mathrm{i}n\kappa)/2g\sqrt{n}]$. The right and left eigenvectors of K are given by

$$|r_j^{(n)}\rangle = \sum_{j=1}^{2} R_{k,j}^{(n)} |n, k\rangle, \quad \langle q_j^{(n)}| = \sum_{j=1}^{2} R_{k,j}^{(n)} \langle n, k| \tag{B.4}$$

for $n > 0$ and the singlet $|0, 1\rangle$ for $n = 0$. It has been shown in [24] that the full eigensystem of the Liouville operator $\mathcal{L}_1$ in equation (13) can be constructed from the eigensystem of the non-Hermitian Hamiltonian K. With the knowledge of the full set of right (left) eigenvectors $\hat{\rho}_\lambda$ ($\check{\rho}_\lambda$) (labeled with the corresponding eigenvalue λ), one is able to evaluate the time evolution of any given initial condition ρ_0 as

$$\rho(t) = \mathrm{e}^{\mathcal{L}_1 t}\rho_0 = \sum_\lambda \mathrm{Tr}\{\check{\rho}_\lambda^\dagger \rho_0\}\mathrm{e}^{\lambda t}\hat{\rho}_\lambda. \tag{B.5}$$

For initial states $|\Psi_0\rangle\langle\Psi_0|$ of equation (18), the only contribution to (B.5) is given by the following set of 7 right eigenvectors of $\mathcal{L}_1$

$$\hat{\rho}_{1,1}^{(0,0)} = |0, 1\rangle\langle 0, 1|, \quad \hat{\rho}_{j,1}^{(1,0)} = |r_j^1\rangle\langle 0, 1|, \quad \hat{\rho}_{j,k}^{(1,1)} = |r_j^1\rangle\langle r_k^1| - \langle r_k^1|r_j^1\rangle|0, 1\rangle\langle 0, 1|. \tag{B.6}$$

The corresponding left eigenvectors are $\check{\rho}_{1,1}^{(0,0)} = \mathbb{I}$, $\check{\rho}_{j,1}^{(1,0)} = |q_j^1\rangle\langle 0, 1| + \ldots$ and $\check{\rho}_{j,k}^{(0,1)} = |q_j^1\rangle\langle q_k^1| + \ldots$, where the dots indicate a series of terms that we omit as they do not contribute to initial states describing one excitation in the system. The corresponding eigenvalues are $\lambda_{1,1}^{(0,0)} = 0$, $\lambda_{j,1}^{(1,0)} = -\mathrm{i}\varepsilon_j^{(1)}$ and $\lambda_{j,k}^{(1,1)} = -\mathrm{i}(\varepsilon_j^{(1)} - \varepsilon_k^{*(1)})$. With this subset of the eigensystem, it is possible to write the time evolution of the initial state in equation (18) as

$$\rho(t) = |0, 1\rangle\langle 0, 1| + \sum_{j,k=1}^{2} \langle q_j^1|\rho_0|q_k^1\rangle \mathrm{e}^{\lambda_{j,k}^{(1,1)} t}\hat{\rho}_{j,k}^{(1,1)} + \sum_{j=1}^{2}(\langle q_j^1|\rho_0|0, 1\rangle \mathrm{e}^{\lambda_{j,1}^{(1,0)} t}\hat{\rho}_{j,1}^{(1,0)} + \mathrm{H.c.}), \tag{B.7}$$

with $\rho_0 = |\Psi_0\rangle\langle\Psi_0|$. From equations (18) and (B.4) it follows that $\langle q_j^1|\Psi_0\rangle = c_\mathrm{e} R_{2,j}^{(1)}$ and $\langle\Psi_0|0, 1\rangle = c_\mathrm{g}^*$. By taking this into account and tracing over the photonic degrees of freedom on finds the reduced density matrix of the atomic system

$$\rho_\mathrm{at}(t) = |\mathrm{g}\rangle\langle\mathrm{g}| + \sum_{j=1}^{2}(c_\mathrm{e} c_\mathrm{g}^* (R_{2,j}^{(1)})^2 \mathrm{e}^{\lambda_{j,1}^{(1,0)} t}|\mathrm{e}\rangle\langle\mathrm{g}| + \mathrm{H.c.}) + |c_\mathrm{e}|^2 \sum_{j,k=1}^{2} (R_{2,j}^{(1)} R_{2,k}^{(1)*})^2 \mathrm{e}^{\lambda_{j,k}^{(1,1)} t}(|\mathrm{e}\rangle\langle\mathrm{e}| - |\mathrm{g}\rangle\langle\mathrm{g}|). \tag{B.8}$$

Finally, the expression in equation (19) is obtained by identifying that

$$f(t) = \sum_{j=1}^{2} R_{2,j}^{(1)} e^{-i\varepsilon_j^{(1)} t}. \tag{B.9}$$

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