Supplemental Material for Chimera States in Three-Dimensions

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3D videos of the chimera states

Space-time dynamics of the chimera dynamics are illustrated by 3D videos available in Supplementary Data. In the full quality, the videos can be watched at http://chimera3d.biomed.kiev.ua/high-resolution/. In the videos, the phase of each oscillator $\varphi_{ijk}(t)$ is shown as a function of time t. For better visual control of the system dynamics the coherent part of the chimera in some videos is left transparent; in others, both coherent and incoherent regions are present but some part of them is cut. This is illustrated in Fig.1.

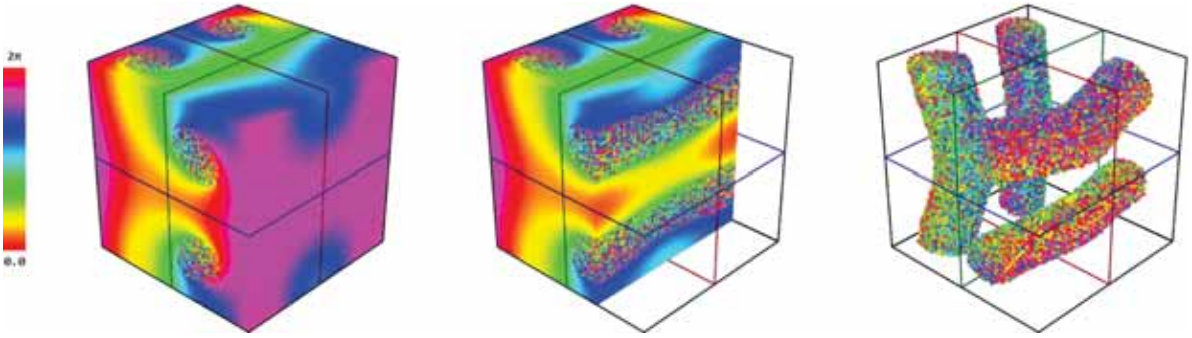

Figure 1. Different 3D representations of the four-rolls chimera state as in Fig.5c of the main text: (left) both coherent and incoherent chimera regions are shown; (middle) the same but 1/4th part of the chimera is cut; (right) coherent region is left transparent. Parameters and notions as in Fig.5c.

To distinguish the coherent and incoherent chimera's parts, phase difference between neighbouring oscillators φ_{ijk} and $\varphi_{i'j'k'}$ in the time moment t is estimated in a standard manner as

$$|\varphi_{i'j'k'}(\tau) - \varphi_{ijk}(\tau)| < \varepsilon$$

for all

$$\tau \in [t - \Delta T/2, t + \Delta T/2],$$

where the parameters ε and ΔT are chosen in an optimal way to get the best coherence-incoherence separation.

Figure2aVideo: Incoherent ball

Video shows the phase of each oscillator $\varphi_{ijk}(t)$ as a function of time in coordinates $x_i = i/N, y_j = j/N, z_k = k/N$, corresponding to Fig.2(a) in the main text. The coherent region is left transparent in the upper 3D image, its details can be followed in the lower 2D sections. 250 time units of simulation are shown with time step $\Delta t = 0.5$ and interpolation in-between. Parameters $\alpha = 1.15$, $r = 0.28$, and $N = 50$. The video is available in Supplemental Data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/.

Figure2bVideo: Incoherent tube

Video shows the phase of each oscillator $\varphi_{ijk}(t)$ as a function of time in coordinates $x_i = i/N, y_j = j/N, z_k = k/N$, corresponding to Fig.2(b) in the main text. The coherent region is left transparent in the upper 3D image, its details can be followed in the lower 2D sections. 250 time units of simulation are shown. Parameters $\alpha = 1.305$, $r = 0.334$, and $N = 100$. The video is available in Supplemental Data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/.

Figure2cVideo: Coherent ball

Video shows the phase of each oscillator $\varphi_{ijk}(t)$ as a function of time in coordinates $x_i = i/N, y_j = j/N, z_k = k/N$, corresponding to Fig.2(c) in the main text. 250 time units of simulation are shown with. Parameters $\alpha = 1.53$, $r = 0.39$, and $N = 50$. The video is available in Supplemental Data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/.

Figure2dVideo: Coherent tube

Video shows the phase of each oscillator $\varphi_{ijk}(t)$ as a function of time in coordinates $x_i = i/N, y_j = j/N, z_k = k/N$, corresponding to Fig.2(d) in the main text. 250 time units of simulation are shown. Parameters $\alpha = 1.49$, $r = 0.43$, and $N = 50$. The video is available in Supplemental Data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/.

Figure4aVideo: Incoherent 6-pieces cross

Video shows the phase of each oscillator $\varphi_{ijk}(t)$ as a function of time in coordinates $x_i = i/N, y_j = j/N, z_k = k/N$, corresponding to Fig.4(a) in the main text. 250 time units of simulation. Parameters $\alpha = 1.365$, $r = 0.342$, and $N = 50$. The video is available in Supplemental Data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/.

Coherent and incoherent part of the 6-pieces cross chimera have been printed at 3D printer, see Fig.2. Both coherent and incoherent 6-piece crosses exist simultaneously in different coordinates shifted by $(0.5, 0.5, 0.5)$ with respect to each other. They are slightly asymmetric due tothe chimera chaotic wandering property, see Ref. [12] in the main text, but perfectly attached to each other. Using many copies of these frames printed at 3D printer, it is possible to fill the whole R^3 space playing like a child with LEGO.

Figure4bVideo: Coherent 4-pieces cross

Video shows the phase of each oscillator $\varphi_{ijk}(t)$ as a function of time in coordinates $x_i = i/N, y_j = j/N, z_k = k/N$, corresponding to Fig.4(b) in the main text. 250 time units of simulation are shown. Parameters $\alpha = 1.45$, $r = 0.35$, and $N = 50$. The video is available in Supplemental Data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/.

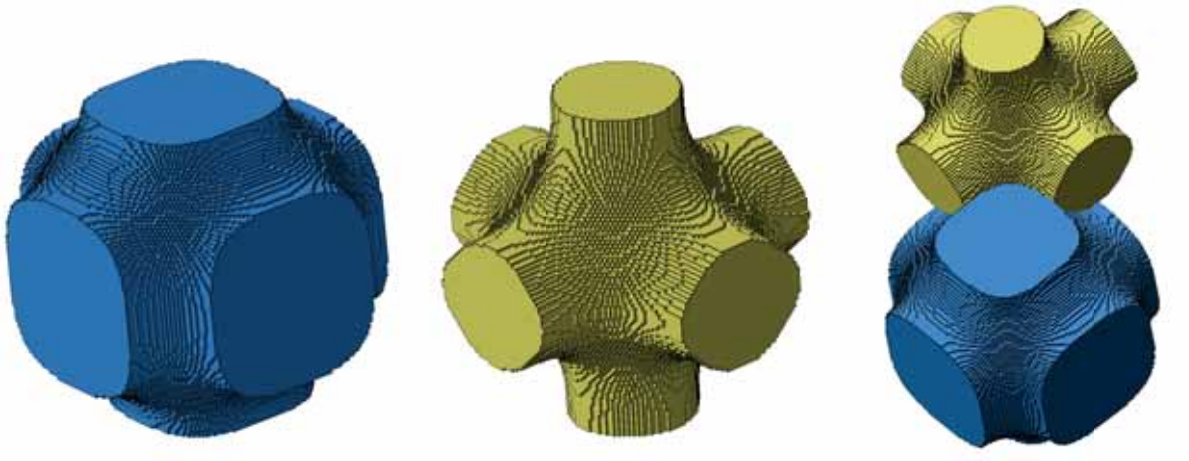

Figure 2. 6-pieces cross chimera in STL format prepared for printing at a 3D-printer: (left) incoherent part, (middle) coherent part, (right) coherent and incoherent parts perfectly attached to each other. Parameters as in Fig.4(a) in the main text.

Figure5aVideo: Scroll wave with two incoherent rolls

Video shows the phase of each oscillator $\varphi_{ijk}(t)$ as a function of time in coordinates $x_i = i/N, y_j = j/N, z_k = k/N$, corresponding to Fig.5(a) in the main text. Spirally rotating coherent region is left transparent in the upper 3D image, its details can be followed in the lower 2D sections. 300 time units of simulation are shown. Parameters $\alpha = 0.8$, $r = 0.165$, and $N = 100$. The video is available in Supplemental Data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/.

Figure5bVideo: Scroll wave with four incoherent rolls

Video shows the phase of each oscillator $\varphi_{ijk}(t)$ as a function of time in coordinates $x_i = i/N, y_j = j/N, z_k = k/N$, corresponding to Fig.5(b) in the main text. Spirally rotating coherent region is left transparent in the upper 3D image, its details can be followed in the lower 2D sections. 250 time units of simulation are shown. Parameters $\alpha = 0.9$, $r = 0.16$, and $N = 100$. The video is available in Supplemental Data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/.

Figure5cVideo: Scroll wave with four perpendicular incoherent rolls

Video shows the phase of each oscillator $\varphi_{ijk}(t)$ as a function of time in coordinates $x_i = i/N, y_j = j/N, z_k = k/N$, corresponding to Fig.5(c) in the main text. Spirally rotating coherent region is left transparent in the upper 3D image, its details can be followed in the lower 2D sections. 250 time units of simulation are shown. Parameters $\alpha = 0.9$, $r = 0.16$, and $N = 100$. The video is available in Supplemental Data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/.

Figure5dVideo: Scroll wave with four perpendicular incoherent rolls of dif-

ferent shape

Video shows the phase of each oscillator $\varphi_{ijk}(t)$ as a function of time in coordinates $x_i = i/N, y_j = j/N, z_k = k/N$, corresponding to Fig.5(d) in the main text. Spirally rotating coherent region is left transparent in the upper 3D image, its details can be followed in the lower 2D sections. 250 time units of simulation are shown. Parameters $\alpha = 0.83$, $r = 0.094$, and $N = 100$. The video is available in Supplemental Data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/.

Figure6aVideo: Chaotic scroll wave with two incoherent rolls

Video shows the phase of each oscillator $\varphi_{ijk}(t)$ as a function of time in coordinates $x_i = i/N, y_j = j/N, z_k = k/N$, corresponding to Fig.6(a) in the main text. Spirally rotating coherent region is left transparent in the upper 3D image, its details can be followed in the lower 2D sections. 600 time units of simulation are shown. Parameters $\alpha = 0.8$, $r = 0.14$, and $N = 50$. The video is available in Supplemental Data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/.

Figure6bVideo: Vortex with an incoherent roll

Video shows the phase of each oscillator $\varphi_{ijk}(t)$ as a function of time in coordinates $x_i = i/N, y_j = j/N, z_k = k/N$, corresponding to Fig.6(b) in the main text. The coherent region is left transparent in the upper 3D image, its details can be followed in the lower 2D sections. 1000 time units of simulation are shown. Parameters $\alpha = 0.9$, $r = 0.16$, and $N = 50$. The video is available in Supplemental Data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/.

Figure7aVideo: Incoherent layer

Video shows the phase of each oscillator $\varphi_{ijk}(t)$ as a function of time in coordinates $x_i = i/N, y_j = j/N, z_k = k/N$, corresponding to Fig.7(a) in the main text. The coherent region is left transparent in the upper 3D image, its details can be followed in the lower 2D sections. 1000 time units of simulation are shown. Parameters $\alpha = 1.4$, $r = 0.46$, and $N = 50$. The video is available in Supplemental Data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/.

Figure7bVideo: Double incoherent layer

Video shows the phase of each oscillator $\varphi_{ijk}(t)$ as a function of time in coordinates $x_i = i/N, y_j = j/N, z_k = k/N$, corresponding to Fig.7(b) in the main text. The coherent region is left transparent in the upper 3D image, its details can be followed in the lower 2D sections. 1000 time units of simulation are shown. Parameters $\alpha = 1.325, r = 0.31$, and $N = 50$. The video is available in Supplemental Data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/.

Figure7cVideo: Oblique incoherent layer

Video shows the phase of each oscillator $\varphi_{ijk}(t)$ as a function of time in coordinates $x_i = i/N, y_j = j/N, z_k = k/N$, corresponding to Fig.7(c) in the main text. The coherent region is left transparent in the upper 3D image, its details can be followed in the lower

2D sections. 1000 time units of simulation are shown. Parameters $\alpha = 1.42$, $r = 0.312$, and $N = 50$. The video is available in Supplemental Data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/.

Figure7dVideo: Double oblique incoherent layers

Video shows the phase of each oscillator $\varphi_{ijk}(t)$ as a function of time in coordinates $x_i = i/N, y_j = j/N, z_k = k/N$, corresponding to Fig.7(d) in the main text. The coherent region is left transparent in the upper 3D image, its details can be followed in the lower 2D sections. 1000 time units of simulation are shown. Parameters $\alpha = 1.36$, $r = 0.31$, and $N = 50$. The video is available in Supplemental Data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/.