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PAPER

Chimera states in three dimensions

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Abstract

The chimera state is a recently discovered dynamical phenomenon in arrays of nonlocally coupled oscillators, that displays a self-organized spatial pattern of coexisting coherence and incoherence. In this paper, the first evidence of three-dimensional chimera states is reported for the Kuramoto model of phase oscillators in 3D grid topology with periodic boundary conditions. Systematic analysis of the dependence of the spatiotemporal dynamics on the range and strength of coupling shows that there are two principal classes of the chimera patterns which exist in large domains of the parameter space: (I) oscillating and (II) spirally rotating. Characteristic examples from the first class include coherent as well as incoherent balls, tubes, crosses, and layers in incoherent or coherent surrounding; the second class includes scroll waves with incoherent, randomized rolls of different modality and dynamics. Numerical simulations started from various initial conditions indicate that the states are stable over the integration time. Videos of the dynamics of the chimera states are presented in supplementary material. It is concluded that three-dimensional chimera states, which are novel spatiotemporal patterns involving the coexistence of coherent and incoherent domains, can represent one of the inherent features of nature.

The chimera state represents one of the most fascinating discoveries in nonlinear science at the turn of the century. A new class of self-organizing patterns has been revealed as natural solutions of complex, homogeneous, and high dimensional dynamical systems of coupled oscillators, demonstrating the robust spatial co-existence of coherence and incoherence. It was discovered and identified more than a decade ago, in 2002, by Kuramoto and Battogtokh [1] for a model of identical but nonlocally coupled oscillators in the one-dimensional complex Ginzburg–Landau equation and its phase approximation, the Kuramoto model. The phenomenon was characterized as the spontaneous occurrence of a global symmetry breaking motion in the form of two incongruent clusters, one made of synchronized and phase locked oscillators and the other made of unsynchronized and drifting ones. A fully homogeneous medium of identical elements was unexpectedly found to be able to give rise to a specific symmetry breaking behavior, next to the previously often observed fully synchronized or fully desynchronized behavior.

Two years after, in 2004, this phenomenon was highlighted by Abrams and Strogatz in the famous paper [2] and rapidly triggered thereafter a broad interest in the nonlinear dynamics community, confirming and improving the understanding of such a striking solution for a diversity of homogeneous systems of coupled oscillators and their perturbations. For now, chimera-like behavior has been reported in numerous theoretical studies and observed in experiments from different fields, see review paper [3] and references therein. Such behavior has the potential to facilitate understanding of self-organized coherent–incoherent patterns which are widespread in nature. Of importance, e.g., are turbulent-laminar patterns in fluid dynamics of shear and pipe flows [4], bump states in neuroscience as spatially localized structures of persistent neuronal activity linked to

working memory [5–7], or isolated desynchronization in power grid networks [8]. In medicine, a prospective application is associated with viewing chimeras as models of spiral patterns formed on heart tissue during ventricular tachycardia and fibrillation, which is one of the primary causes of sudden cardiac death in humans [9–13]. See [3] for links to other possible applications of the chimera states.

Spatially extended processes in the real world are three-dimensional, and appropriate three-dimensional models are required to explore them. Nevertheless, most studies in the chimera field have been based on one-dimensional models and only a small part of them was for the 2D case. Can chimera states exist in three dimensions? If so, with what shapes and how robust are they? These principal questions have motivated recent research and results are reported in this paper.

An important prerequisite for obtaining chimera states is nonlocal coupling in the network. Indeed, models which have the striking spatiotemporal behavior of a chimera must not be locally coupled only. As PDEs imply local coupling at infinitesimal distances, they are not suitable for chimera modelling. To count the coupling non-locality, a model should contain an integral term with a kernel imposing how the coupling spreads through the space. Then, integro-differential equations can provide a suitable basis for theoretical and computational study of the chimera states. Such an approach was suggested, in [1] for the nonlocally coupled complex Ginzburg–Landau equation where the diffusion term was replaced by an integral one with an exponentially decaying kernel. The phase approximation of the problem is now known as the Kuramoto model. After discretization it has the form
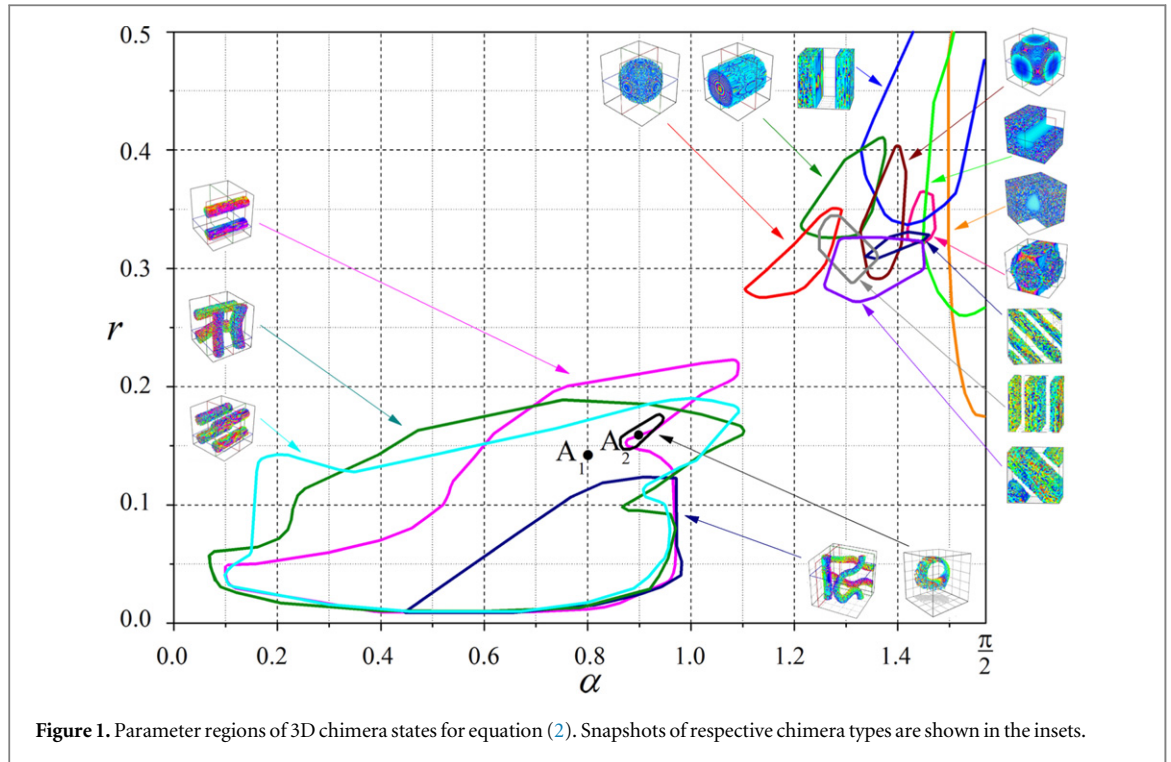
$$\dot{\varphi}_i = \omega + \frac{K}{N} \sum_{j=1}^{N} G_{ij} \sin\left(\varphi_j - \varphi_i - \alpha\right), \tag{1}$$

where φ_i is the phase of oscillator i, ω is the natural frequency, K is the coupling strength, and α is the coupling lag. In the original paper [1], the coupling function G was assumed of the form $G_{\exp}(x, k) = \frac{k}{2} \exp(-kx)$, $x = \frac{|j-i|}{N}$.

A year after, in 2003, this novel approach was extended to the two-dimensional case. Spiral waves with randomized core were identified for a class of three-component reaction-diffusion systems of biological relevance and for the two-dimensional Ginzburg–Landau equation with a nonlocal coupling term [14]. The respective phase approximation model, which is a two-dimensional counterpart of equation (1), was analyzed in [15]. It was argued that spiral patterns with an incoherent core can typically arise if some chemical components involved are diffusion-free, then the coupling is imposed by a modified Bessel function which is radially symmetric and decaying with the distance. This kind of space-temporal behavior, where the coherent region of a pattern is spirally rotating around an incoherent core, constitutes the second class of chimera states, different from those originally detected in [1]. In two dimensions, parameter regions for both chimera classes: (I) oscillating without rotation and (II) spirally rotating with an incoherent core, were obtained numerically and derived analytically [16, 17]. Moreover, it was found that they emerge in opposite corner parts of the parameter space and thus chimeras of different classes cannot co-exist.

A powerful tool for understanding the complex chimera dynamics is the thermodynamic limit $N \to \infty$, with the macroscopic coupling range $r = R/N$ tending to a constant. In this case, exact equations are obtained for chimera states which are stationary solutions with important macroscopic properties, such as the size and shape of the coherent/incoherent regions and the averaged frequencies of individual oscillators. This approach was proposed in the original papers [1, 2] (1D case) and [14, 15] (2D case) and developed later on by many authors, see [18–21] and [3] for more references. The thermodynamic limit approach can be apparently extended on the considered 3D case, but this paper is focused on the diversity of 3D chimera states in the finite-dimensional Kuramoto model.

The new term *chimera state* itself was suggested in [2] where equation (1) was analyzed with a cosine coupling function $G_{\cos}(x, A) = 1 + A \cos 2\pi x$. The case of a multimodal cosine coupling function was analyzed recently in [22, 23]. Topologically, the simplest form of nonlocal coupling causing chimeras is given by a step function $G_{\text{step}}(x, r) = 1/2r$, if $|x| \leqslant r$, and $= 0$ otherwise. It was introduced in the chimera study in [24], and is now widely used by many authors. Here r is a new control parameter—consistent with the radius of coupling—defined such that each oscillator φ_i is coupled with equal strength to all its nearest neighbors within radius r and not coupled beyond. Since the pioneering works [1, 2, 14, 15], chimera states have been reported and thoroughly studied with different models, network configurations, and coupling schemes. The main attention has been paid to a ring of oscillators, see [20–37] and to the case of two and three groups of oscillators [18, 38–40]. Robustness of the chimera states was examined from different viewpoints [41–43] and confirmed by recent experiments in various fields including chemistry [44–46], optical and microelectronic systems [47–50], and mechanics [51, 52]. Two-dimensional networks have been analyzed for an infinite plane [14, 15, 19], a periodic square topologically (equivalent to a T^2-torus) [16, 17, 45], and for the surface of a unit sphere S^2 [53]. 2D type (I) chimeras have been obtained in the form of stripes and spots, both coherent and incoherent [16, 17].

Figure 1. Parameter regions of 3D chimera states for equation (2). Snapshots of respective chimera types are shown in the insets.

Similar to the 1D case [32], they emerge in the right-upper part of the (r, α)-parameter region of interest, namely, $0 < r < 0.5$ and $0 < \alpha < \pi/2$. Spiral wave chimeras, i.e. of type (II), do not have 1D analogues. They fill up the opposite, left-down part of the parameter plane. As a result, (r, α)-parameter domains for the type (I) and type (II) chimeras are not intersecting, i.e., 2D chimeras of the two principal classes (I) and (II) cannot co-exist [16].

In this paper, a detailed numerical study of the chimera phenomenon, based on nonlocally coupled Kuramoto phase oscillators placed uniformly in a three-dim cube with periodic boundary conditions, is presented. A variety of striking spatiotemporal patterns of both types are detailed and the (r, α)-parameter regions for their existence are stated. The robustness of the three-dimensional chimera patterns is supported by the fact that they are obtained in numerical experiments with randomly chosen initial conditions. The corresponding videos (see supplemental data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/) provide visual confirmation. Numerical simulation was based on the Runge–Kutta solver DOPRI5 that has been integrated into software for large nonlinear dynamical networks [54], allowing for parallelized simulations with different sets of parameters and initial conditions. The simulations were performed on a computer cluster 'chimera' (http://nll.biomed.kiev.ua/cluster) and Ukrainian Grid Infrastructure providing distributed cluster resources and the parallel software [55]. The main bifurcation diagram in figure 1 below is a result of long-time parallel simulations (over a period of more than one year) of the model using random searches followed by a standard continuation procedure.

The dynamical system of concern is governed by a Kuramoto-like network of N^3 identical oscillators

$$\dot{\varphi}_{ijk} = \omega + \frac{K}{P^3} \sum_{(i',j',k') \in B_P(i,j,k)} \sin\left(\varphi_{i'j'k'} - \varphi_{ijk} - \alpha\right), \quad (2)$$

where φ_{ijk} are phase variables, indexes i, j, k are periodic mod N which induces a 3D torus structure on the array. The coupling is assumed to be long-ranged and isotropic: each oscillator φ_{ijk} is coupled with equal strength K to all its nearest neighbors $\varphi_{i'j'k'}$ within a range P, i.e. to those oscillators falling in the ball-like neighborhood

$$B_P(i, j, k) := \left\{(i', j', k'): (i' - i)^2 + (j' - j)^2 + (k' - k)^2 \leqslant P^2\right\},$$

where the distances $i' - i$, $j' - j$, and $k' - k$ are calculated regarding the periodic boundary conditions of the network.

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Without loss of generality, it is put in equation (2) $\omega = 0$ and $K = 1$. The phase lag parameter α is assumed to belong to the attractive coupling range from 0 to $\pi/2$ [5] (cr also [32]). The second control parameter in the model (2) is the coupling radius $r = P/N$. It varies from $r = 1/N$ (local coupling) to $r = 0.5$ (close to global coupling). Chimera states in the Kuramoto model arise at intermediate values of coupling radius, between $1/N$ and 0.5, referred to as nonlocal coupling [2, 24, 32]. There is one more parameter in the model (2), namely, the network size N^3. Numerical simulation of equation (2) have been performed for $N = 50$ and $N = 100$ and require the integration, respectively, of 125 000 and 10^6 nonlinear differential equations. No differences of importance in the system dynamics were observed for these two values of N. Naturally, such computation is beyond the capability of the modern personal computer and requires more powerful computer facilities and parallelized software.

Results of direct numerical simulation of model (2) in the two-parameter plane of the phase shift α and the coupling radius r are presented in figure 1. This figure reveals the appearance of regions of different kinds of 3D chimera states, outlined in shading (colors), at intermediate values of the radius of coupling and phase lag. Typical shapes of the chimera states obtained are illustrated in the insets. For better visual representation, they are centered with respect to the coordinate frame (as also done in the figures below and in the supplemental videos).

Parameter regions for type (I) chimeras, i.e. not-rotating, can be seen in the upper-right corner of the (α, r)-plane. Alternatively, type (II) chimeras, i.e. spirally rotating locate in the lower left part of the bifurcation diagram. It can be observed that the regions belonging to the two chimera types are separated from each other, whereas the regions for chimeras of the same type are essentially intersecting which causes multistability. It should be mentioned that the chimeras shown in figure 1 co-exist with the fully synchronized solution which is stable for the values of α and r considered. In addition, and consistent with 1D and 2D cases alike, stable rotating waves also arise at $r < 1/3$, so-called q-twisted states [56]. Usually, to obtain chimera states in numerical simulations it is necessary to use special initial conditions. However, as our numerical experiments show, most of the chimera states can be obtained by repeating calculations with randomly chosen initial conditions, which indicates large chimera basins. In our experiments with random initial conditions other sorts of space-temporal dynamics have been observed including apparently intermingled behavior when the trajectory switches between different chimera states which resembles heteroclinic cycling. The study of them is for the future and the following concentrates on the chimeras presented in figure 1.
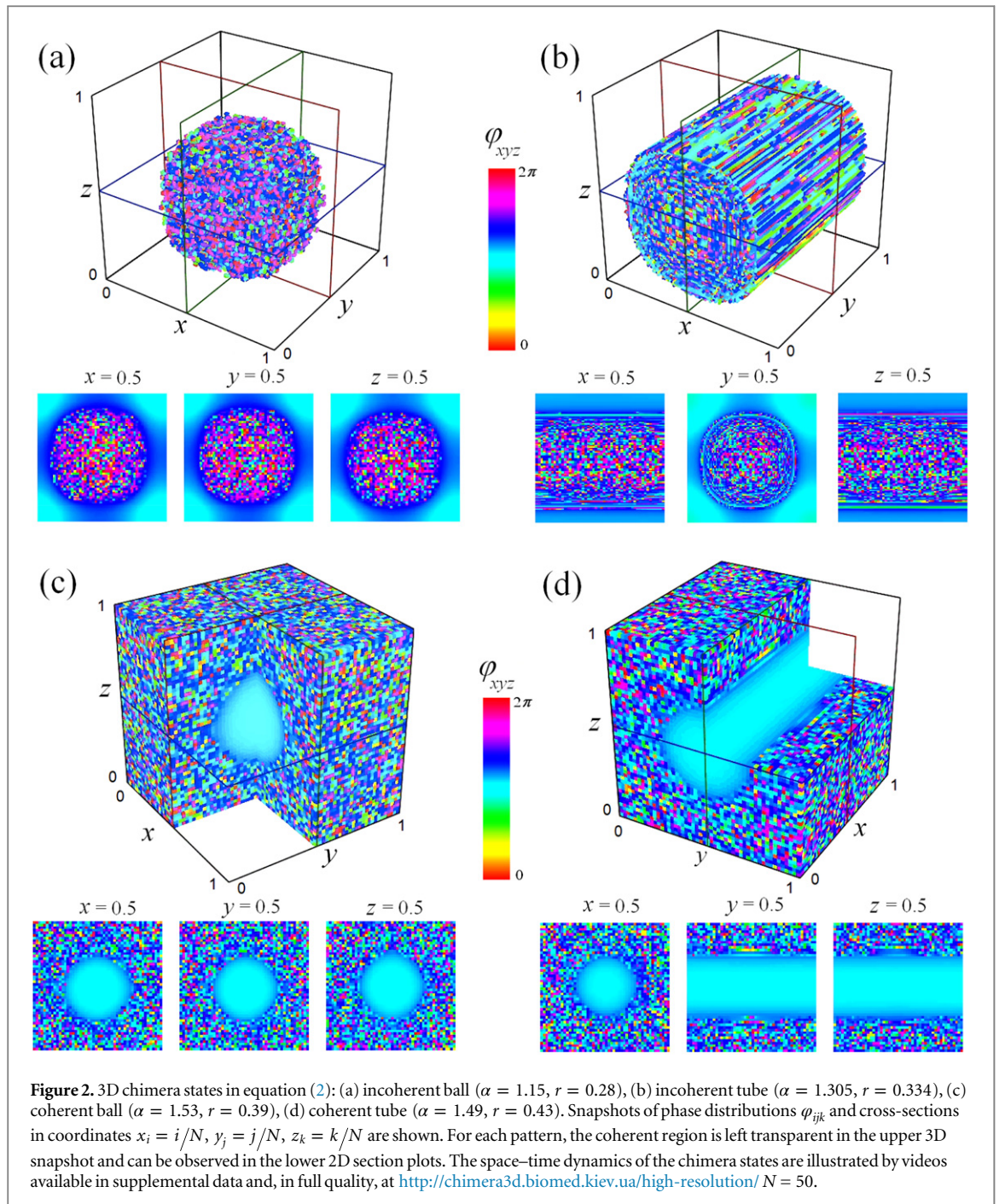
In figure 2, snapshots of four characteristic chimera states for model (2) are presented. These are an incoherent ball (a) and tube (b) in a coherent surrounding, and a coherent ball (c) and tube (d) in an incoherent surrounding. For clarity the coherent regions in the 3D snapshots are left transparent, but they can be seen in the 2D section plots below. These chimera types are the three-dimensional counterparts of strip and spot chimeras in two dimensions, see [16, 17]. As in two dimensions, they exist at large values of the parameters r and α (see figure 1 where parameter regions for their existence are specified). The mean frequencies of the individual oscillators have also the same bell-shape profile as in 1D and 2D cases, see figure 3. Due to the periodic boundary conditions in model (2), tube-like chimeras (b) and (d) can be represented as doughnut-shaped coherent/incoherent toroids in an incoherent/coherent surrounding, respectively.

Figure 4 illustrates two more exotic chimera states, also of type (I), which do not have 2D analogues: (a) a six-piece incoherent cross and (b) a four-piece coherent cross. The respective parameter regions can be seen in figure 1 and are in between incoherent and coherent ball and tube chimeras. The complementary coherent set of the six-piece chimera shown in figure 4(a) represents the same kind of cross but displayed in shifted coordinates $x \rightarrow x + 0.5$, $y \rightarrow y + 0.5$, $z \rightarrow z + 0.5$. Hence, both incoherent and coherent six-piece crosses exist simultaneously, but in different coordinates, and they, clearly, are perfectly attached to each other (see supplemental data for more details). The second chimera state of this kind is a coherent four-piece cross (figure 4(b)). In contrast to the six-piece cross (figure 4(a)), its complementary incoherent set in the shifted coordinates has only two pieces. As for an incoherent four-piece cross chimera, it has not been observed in our simulations when varying system parameters and initial conditions. Its existence in model (2) is an open problem.

As illustrated in the supplementary videos the six-cross chimera dynamics are rather stationary: the boundary surface between the coherent and incoherent regions is visually non-oscillating. This is not the case for the four-piece cross: it breathes in time such that the coherent–incoherent boundary performs, evidently, large scale chaotic oscillations.

Scroll waves with an incoherent core(s) constitute the second principal class of the three-dimensional chimera states and snapshots of the typical patterns of this kind are illustrated in figure 5. They emerge in the lower-left part of the (r, α)-parameter plane (see the bifurcation diagram in figure 1). In these chimera states,

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[5] The measure of coupling attractivity in the Kuramoto model is controlled by the phase lag parameter α: for $0 < \alpha < \pi/2$ the coupling is attractive, whereas for $\pi/2 < \alpha < 3\pi/2$ it is repulsive. In the first, attractive case the coherent state $\varphi_1 = \ldots = \varphi_{N^3}$ is stable coexisting with chimera states (if they do exist) or other possible states. As α crosses $\pi/2$, the coherent state losses its stability giving rise to so-called q-twisted states. See [32, 56, 57] for more details.

Figure 2. 3D chimera states in equation (2): (a) incoherent ball ($\alpha = 1.15$, $r = 0.28$), (b) incoherent tube ($\alpha = 1.305$, $r = 0.334$), (c) coherent ball ($\alpha = 1.53$, $r = 0.39$), (d) coherent tube ($\alpha = 1.49$, $r = 0.43$). Snapshots of phase distributions φ_{ijk} and cross-sections in coordinates $x_i = i/N$, $y_j = j/N$, $z_k = k/N$ are shown. For each pattern, the coherent region is left transparent in the upper 3D snapshot and can be observed in the lower 2D section plots. The space–time dynamics of the chimera states are illustrated by videos available in supplemental data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/ $N = 50$.
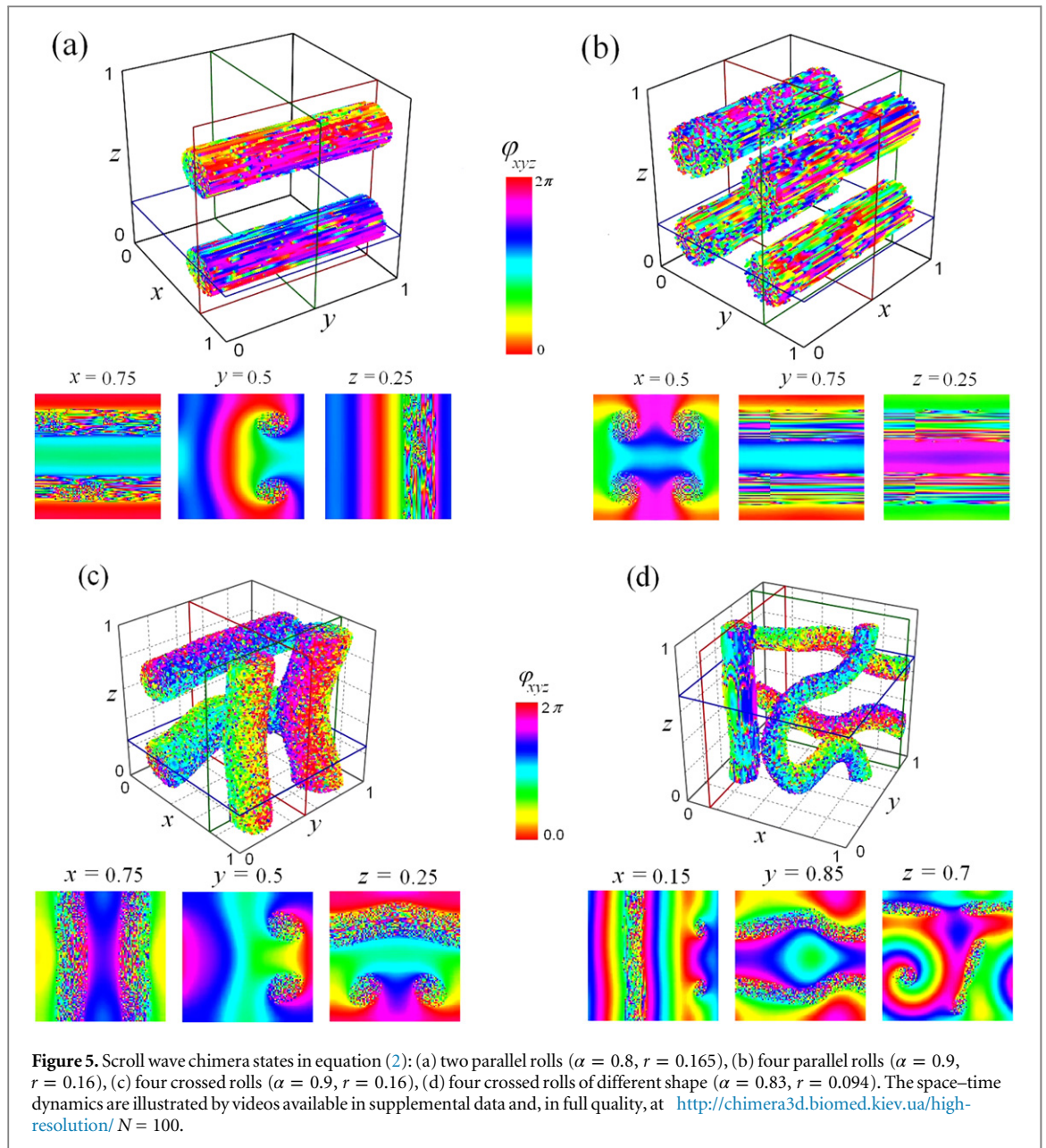
coherent regions are spirally rotating around incoherent rolls as can be visually seen in the supplementary videos. The rolls represent a three-dimensional 'swelling' of standard scroll wave filaments and their microscopic dynamics is apparently chaotic. Due to the periodic boundary conditions, the rolls can be considered to be closed in a ring (in circular coordinates) representing, thereby, a toroid—scroll wave ring with an incoherent core. The surrounding coherent dynamics may be twisted or not [58].

Scroll wave chimeras are characterized by the number of the incoherent rolls and their large-scale behavior. In figure 5, chimeras with two (a) and four (b)–(d) rolls are shown. The rolls here are stationary, i.e. not moving in a significant manner as it can be concluded from the supplementary videos. The four-roll chimeras in (b) and (c) differ in their position and their shape. Indeed, in (b) the rolls are parallel and symmetric and this is the 3D counterpart of the spiral chimera state obtained in [16]. In contrast, the rolls in (c) and (d) are pairwise located in perpendicular directions and, clearly, can not have any 2D analog. Compare [59] where the interactions of a pair of parallel scroll waves similar to shown in figure 5(a) was studied experimentally for three-dimensional excitable media in Belousov–Zhabotinsky reaction.

Figure 3. Spatial profiles of average frequencies for an incoherent ball (left panel) and an incoherent tube (right panel). Equation (2) was integrated up to $T = 10\,000$, then averaging was calculated at the time interval $\Delta T = 100$. Parameters are the same as in figures 2(a) and (b), respectively.

Figure 4. Cross-chimeras in equation (2): (a) incoherent six-pieces cross ($\alpha = 1.365$, $r = 0.342$) and (b) coherent four-pieces cross ($\alpha = 1.45$, $r = 0.35$). The space–time dynamics are illustrated by videos available in supplemental data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/ $N = 50$.
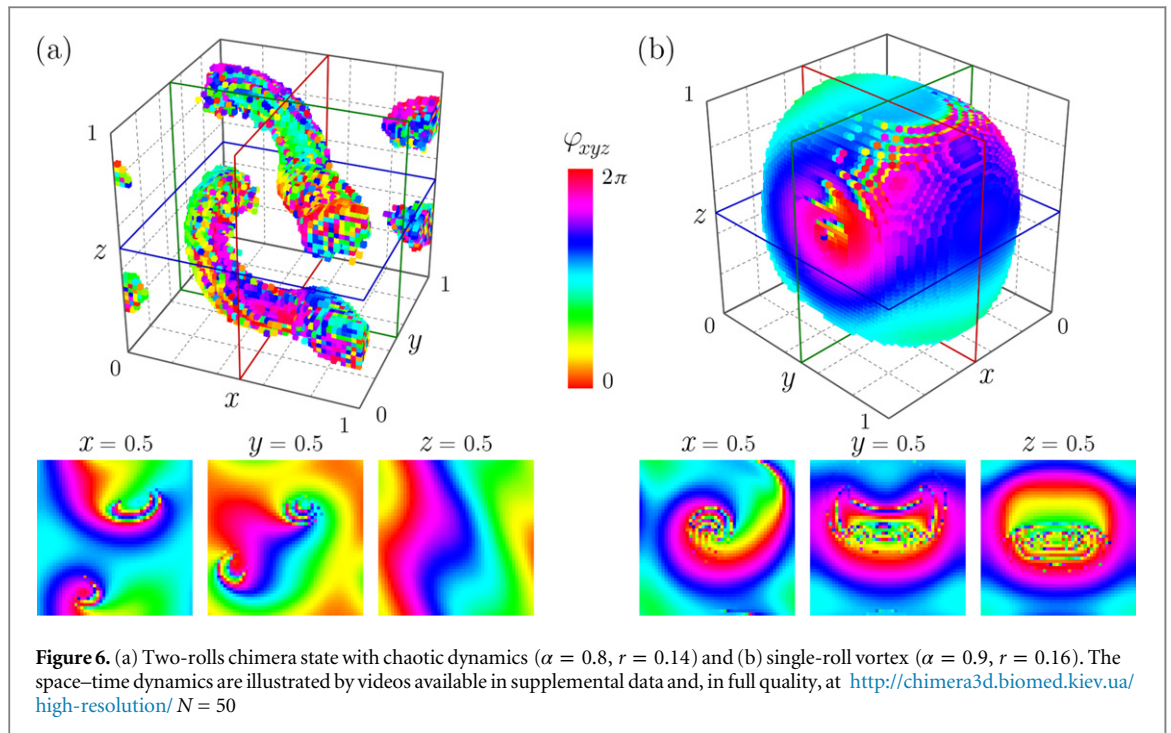
In numerical experiments, scroll wave chimeras have also been observed with more complicated behavior of the incoherent rolls, when the established large scale dynamics are not stationary, as in video 'figure 5(a)video. mkv', but periodic or even chaotic. Two examples of this kind can be found at the web page http://chimera3d. biomed.kiev.ua/high-resolution/. In the first such video 'two traveling rolls-1.mkv' ($\alpha = 0.9$, $r = 0.17$) the large

Figure 5. Scroll wave chimera states in equation (2): (a) two parallel rolls ($\alpha = 0.8$, $r = 0.165$), (b) four parallel rolls ($\alpha = 0.9$, $r = 0.16$), (c) four crossed rolls ($\alpha = 0.9$, $r = 0.16$), (d) four crossed rolls of different shape ($\alpha = 0.83$, $r = 0.094$). The space–time dynamics are illustrated by videos available in supplemental data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/ $N = 100$.

scale dynamics are rather simple: both rolls are symmetrically drifting with a constant velocity staying in parallel to one of the coordinate axes. In the second video 'two traveling rolls-2.mkv' ($\alpha = 0.9$, $r = 0.16$) the rolls remain parallel to the coordinate axis but behave in a more complicated, visually chaotic manner moreover, their shape is chaotically breathing in the process of movement[6].

An example of chaotic two-roll dynamics is presented in figure 6(a). This kind of behavior resembles scroll wave turbulence [61, 62] known also as Winfree turbulence [63, 64]. It is consistent with spatiotemporal chaos in three-dimensional excitable media and is thought to be relevant for understanding fibrillation of the heart [65]. The most surprising type (II) chimera state is that of a localized vortex with an incoherent core, as illustrated in figure 6 (b). This chimera state develops, rather fast, from random initial conditions, as can be seen in the supplementary video. The vortex contains a unique incoherent roll which is closed in a ring and rotating with a constant velocity. The transverse diameter of the roll is maximal in the center, it decreases and eventually vanishes at the periphery. The vortex exist in a small parameter region delineated in black in figure 1. Nonetheless, the origin of this fascinating pattern is not clear at present; we have also no ideas as for its possible analogues in other fields.

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[6] Traveling chimeras can also be observed in the one-dimensional case however, with additional conditions on the coupling function: multimodality [22] or asymmetry [60], or spatial inhomogeneity [23]. In the considered 3D case the chimera movement typically arises under fully symmetric conditions.

Figure 6. (a) Two-rolls chimera state with chaotic dynamics ($\alpha = 0.8$, $r = 0.14$) and (b) single-roll vortex ($\alpha = 0.9$, $r = 0.16$). The space–time dynamics are illustrated by videos available in supplemental data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/ $N = 50$

The scroll wave chimeras illustrated in figures 5 and 6, exist in wide regions of the parameter space which can be seen in the lower-left part of the main bifurcation diagram shown in figure 1. Moreover, these parameter regions are heavily intersecting. Indeed, numerical simulations confirm that in the conjoint domains several states of this type can be obtained starting from random initial conditions uniformly distributed on the circle (from $-\pi$ to $+\pi$). To illustrate this multistability phenomenon, model (2) has been integrated with $N^3 = 50^3$ oscillators for two cases: in the parameter points A_1 defined by $\alpha = 0.8$ and $r = 0.14$ and A_2 defined by $\alpha = 0.9$ and $r = 0.16$ (marked in figure 1). In each of the two cases, simulations started with 40 different random initial conditions and used up to $T = 5000$ time units. The following scroll wave chimeras and other space-temporal patterns were observed at the end of the integration:

Point A_1: three stationary two-roll chimera states (as in figure 5(a)), and one chaotic two-roll chimera state (as in figure 6(a)). The other 35 simulations resulted in different behavior as it was recorded at $t = 5000$, namely: 15 coherent fully synchronized states, two rotating wave solutions with one and two rotating coordinates respectively, 15 spatial chaos states with a coherent background, and four spatial chaos states with a rotating wave background.
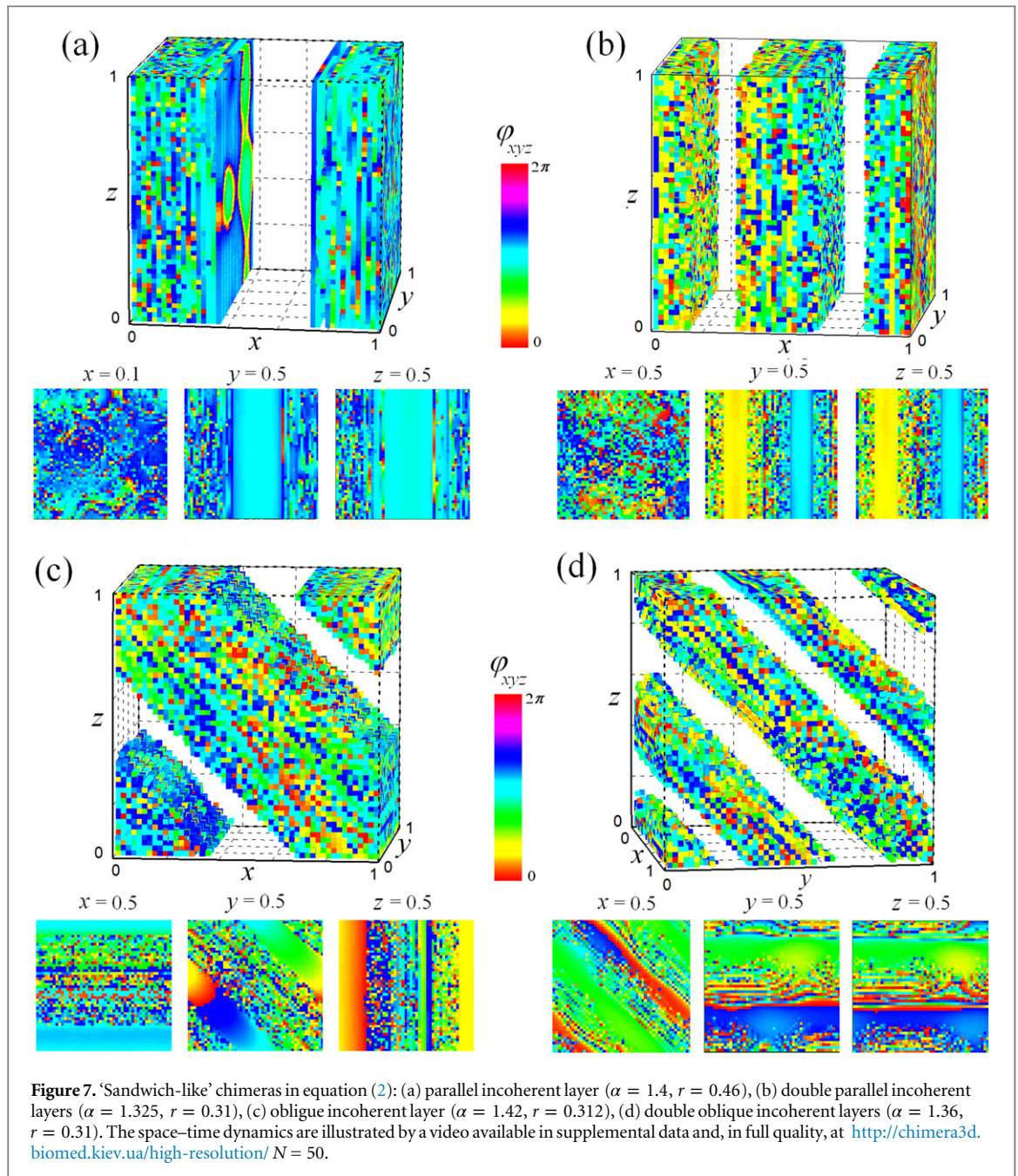
Point A_2: six moving two-roll chimera states (as in videos 'two traveling rolls-1.mkv' and 'two traveling rolls-2.mkv' at the webpage http://chimera3d.biomed.kiev.ua/high-resolution/), one four-roll parallel spiral chimera state (as in figure 5(b)), one four-roll crossed chimera state (as in figure 5(c)), one chaotic two-roll chimera state (as in figure 6(a)), and one localized single-roll vortex (as in figure 6(b)). The other 30 simulations resulted in the following different behaviors as observed at $t = 5000$: 17 coherent fully synchronized states, three rotating wave solutions, nine spatial chaos states with a coherent background, and one spatial chaos state with a rotating wave background (one coordinate rotates).

From these simulations scroll wave chimeras appear in 17.5% of experiments: 15% among them are regular, i.e. with stationary or drifting rolls only and 2.5%—chaotic scroll chimeras resembling 3D spiral chaos. The majority of the experiments—82.5%—result in fully or almost fully synchronized plane or rotating waves. It is worth noting that calculations of up to $t = 5000$ time units, and with 80 sets of initial conditions, provides only a brief insight into the high dimensional system dynamics. However, the limited simulations do demonstrate the rich and multistable behavior of nonlocally coupled phase oscillators for the three-dimensional case[7].

As a prospective challenging application, consider the co-existence of synchronous oscillations, with both regular and chaotic scroll waves, which resembles, phenomenologically, the three characteristic regimes in cardiovascular heart disease, namely, normal sinus rhythm, ventricular tachycardia, and ventricular fibrillation (see [3], chapter 7.2 and references therein). It is expected that further research in the direction of spatiotemporal control of the scroll wave behavior can contribute to understanding the mechanisms of this fatal

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[7] Rich mustistable structure of co-existing chimera dynamics in the 1D case is demonstrated by phase diagrams in [32]. Basins of attraction for chimera states are studied in [50, 66].

Figure 7. 'Sandwich-like' chimeras in equation (2): (a) parallel incoherent layer ($\alpha = 1.4$, $r = 0.46$), (b) double parallel incoherent layers ($\alpha = 1.325$, $r = 0.31$), (c) obligue incoherent layer ($\alpha = 1.42$, $r = 0.312$), (d) double oblique incoherent layers ($\alpha = 1.36$, $r = 0.31$). The space–time dynamics are illustrated by a video available in supplemental data and, in full quality, at http://chimera3d.biomed.kiev.ua/high-resolution/ $N = 50$.

disease which is the leading cause of death in the modern world. Such research could provide prerequisites for developing possible alternatives to massive defibrillator shocks, as well as designing new drag therapies, with the goal of a less traumatic transition from the pathologic scroll wave behavior observed in fibrillation to a healthy heart rhythm [9–13].

This report on three-dimensional chimera states in the Kuramoto model would not be complete without the mention of the states with alternating layers of coherence and incoherence, i.e. 'sandwich-like' chimeras which are illustrated in figure 7. Most probable are the patterns where the coherence/incoherence layers are parallel to one of the coordinate planes (figures 7(a) and (b)), which are the natural counterparts of the 2D strip chimeras [16]. The wide parameter region for this kind of behavior can be seen in the upper right corner of the bifurcation diagram in figure 1. Some other simulations show robust oblique layer structures, where the layers lie along one of the diagonal planes. Two examples of such states are presented in figures 7(c) and (d), with one and two incoherent layers, respectively. They resemble well known turbulent-laminar oblique structures which naturally arise as robust states in shear and Couette flows in fluid dynamics, see [4] and references therein. In numerical experiments of equation (2), however, oblique chimera states were obtained in insular parameter points at the intermediate values of the coupling radius *r*. Often, they arise after a long transient.

In conclusion, the very first observation of three-dimensional chimera states in the nonlocally coupled Kuramoto model, with a periodic topology equivalent to 3D torus T^3, has been reported. Two large families of the chimera states are obtained, which are (I) incoherent/coherent balls, tubes, crosses, and layers (figures 2, 4 and 7), and (II) incoherent rolls in spirally rotating coherent surrounding, which can behave in regular (figures 5 and 6(a)) and chaotic ways (figure 6(b)). Large numerical simulations indicate that these states are robust and that they constitute an essential part of the spatiotemporal network dynamics. By carefully inspecting the (r, α)-bifurcation diagram, the parameter regions for the states of interests have been obtained and it has been concluded that (I) stationary and (II) scroll wave chimeras inhabit large, but distinct, areas of the parameter space.

It is worth noting that the obtained three-dimensional coherent–incoherent patterns do not exhaust the emerging beauty of the fascinating chimera dynamics. A variety of multi-headed three-dimensional chimeras are certainly waiting to be discovered with further studies of the Kuramoto model and the other, more realistic systems. The authors believe that the chimera state is probably a common, and universal, phenomenon in networks of very different nature and that a variety of surprising applications await to be discovered.

Acknowledgments

References

[1] Kuramoto Y and Battogtokh D 2002 Coexistence of coherence and incoherence in nonlocally coupled phase oscillators *Nonlinear Phenom. Complex Syst.* **5** 380–5

[2] Abrams D M and Strogatz S H 2004 Chimera states for coupled oscillators *Phys. Rev. Lett.* **93** 174102

[3] Panaggio M and Abrams D 2015 Chimera states: coexistence of coherence and incoherence in networks of coupled oscillators *Nonlinerity* **28** 67–87

[4] Brethouwer G, Duguet Y and Schlatter P 2012 Turbulentlaminar coexistence in wall flows with Coriolis, buoyancy or Lorentz forces *J. Fluid Mech.* **704** 137172

[5] Compte A, Brunel N, Goldman-Rakic P S and Wang X-J 2000 Synaptic mechanisms and network dynamics underlying spatial working memory in a cortical network model *Cereb. Cortex* **10** 910

[6] Renart A, Song P and Wang X-J 2003 Robust spatial working memory through homeostatic synaptic scaling in heterogeneous cortical networks *Neuron* **38** 473–85

[7] Chow C and Coombes S 2006 Existence and wandering of bumps in a spiking neural network model *SIAM J. Appl. Dyn. Syst.* **5** 552–74

[8] Pecora L, Sorrentino F, Hagerstrom A, Murphy T and Roy R 2014 Cluster synchronization and isolated desynchronization in complex networks with symmetries *Nat. Commun.* **13** 4079

[9] Witkowski F, Leon L, Penkoske P, Giles W, Spanok M, Ditto W and Winfree A 1998 Spatiotemporal evolution of ventricular fibrillation *Nature* **392** 78–82

[10] Baylya P, KenKnight B, Rogers J, Johnson E, Ideker R and Smith W M 1998 Spatial organization, predictability, and determinism in ventricular fibrillation *Chaos* **8** 103–15

[11] Panfilov A 1998 Spiral breakup as a model of ventricular fibrillation *Chaos* **8** 57–64

[12] Fenton F and Karma A 1998 Vortex dynamics in three-dimensional continuous myocardium with fiber rotation: filament instability and fibrillation *Chaos* **8** 20–47

[13] Filippi S, Gizzi A, Cherubini C, Luther S and Fenton F H 2014 Mechanistic insights into hypothermic ventricular fibrillation: the role of temperature and tissue size *Europace* **16** 424–34

[14] Kuramoto Y and Shima S I 2003 Rotating spirals without phase singularity in reaction-diffusion systems *Prog. Theor. Phys. Suppl.* **150** 115–25

[15] Shima S I and Kuramoto Y 2004 Rotating spiral waves with phase-randomized core in nonlocally coupled oscillators *Phys. Rev.* E **69** 036213

[16] Omel'chenko O, Wolfrum M, Yanchuk S, Maistrenko Yu and Sudakov O 2012 Stationary patterns of coherence and incoherence in two-dimensional arrays of nonlocally-coupled phase oscillatorss *Phys. Rev.* E **85** 036210

[17] Panaggio M J and Abrams D M 2013 Chimera states on a flat torus *Phys. Rev. Lett.* **110** 094102

[18] Ott E and Antonsen T 2008 Low dimensional behavior of large systems of globally coupled oscillators *Chaos* **20** 037113

[19] Martens E, Laing C and Strogatz S 2010 Solvable model of a spiral wave chimeras *Phys. Rev. Lett.* **104** 044101

[20] Bordyugov G, Pikovsky A and Rosenblum M 2010 Self-emerging and turbulent chimeras in oscillator chains *Phys. Rev.* E **82** 035205

[21] Wolfrum M, Omel'chenko O, Yanchuk S and Maistrenko Yu 2011 Spectral properties of chimera states *Chaos* **21** 013112

[22] Xie J, Knobloch E and Kao H-C 2014 Multi-cluster and traveling chimera states in nonlocal phase-coupled oscillators *Phys. Rev.* E **90** 022919

[23] Xie J, Kao H-C and Knobloh E 2015 Chimera states in systems of nonlocal nonidentical phase-coupled oscillators *Phys. Rev.* E **91** 032918

[24] Omel'chenko O, Wolfrum M and Yu Maistrenko 2010 Chimera states as chaotic spatiotemporal patterns *Phys. Rev.* E **81** 065201

[25] Abrams D M and Strogatz S H 2006 Chimera states in a ring of nonlocally coupled oscillators *Int. J. Bifurcation Chaos* **16** 21–37

[26] Sakaguchi H 2006 Instability of synchronized motion in nonlocally coupled neural oscillators *Phys. Rev.* E **73** 031907

[27] Omel'chenko O, Maistrenko Yu and Tass P 2008 Chimera states: the natural link between coherence and incoherence *Phys. Rev. Lett.* **100** 044105

[28] Sethia G C, Sen A and Atay F M 2008 Clustered chimera states in delay-coupled oscillator systems *Phys. Rev. Lett.* **100** 144102

[29] Wolfrum M and Omel'chenko O 2011 Chimera states are chaotic transients *Phys. Rev. E* **84** 015201
[30] Omelchenko I, Maistrenko Yu, Hövel P and Schöll E 2011 Loss of coherence in dynamical networks: spatial chaos and chimera states *Phys. Rev. Lett.* **106** 234102
[31] Omelchenko I, Omel'chenko O, Hövel P and Schöll E 2013 When nonlocal coupling between oscillators becomes stronger: patched synchrony or multichimera states *Phys. Rev. Lett.* **110** 224101
[32] Maistrenko Yu, Vasylenko A, Sudakov O, Levchenko R and Maistrenko V 2014 Cascades of multi-headed chimera states for coupled phase oscillators *Int. J. Bifurcation Chaos* **24** 1440014
[33] Zakharova A, Kapeller M and Schöll E 2014 Chimera death: symmetry breaking in dynamical networks *Phys. Rev. Lett.* **112** 154101
[34] Sethia G C and Sen A C 2014 Chimera states: the existence criteria revisited *Phys. Rev. Lett.* **112** 144101
[35] Yeldesbay A, Pikovsky A and Rosenblum M 2014 Chimeralike states in an ensemble of globally coupled oscillators *Phys. Rev. Lett.* **112** 144103
[36] Jaros P, Maistrenko Yu and Kapitaniak T 2015 Chimera states on the route from coherence to rotating waves *Phys. Rev. E* **91** 022907
[37] Schmidt L and Krischer K 2015 Clustering as a prerequisite for chimera states in globally coupled systems *Phys. Rev. Lett.* **114** 034101
[38] Abrams D M, Mirollo R, Strogatz S H and Wiley D A 2008 Solvable model for chimera states of coupled oscillators *Phys. Rev. Lett.* **101** 084103
[39] Martens E 2010 Chimeras in a network of three oscillator populations with varying network topology *Chaos* **20** 043122
[40] Laing C 2012 Disorder-induced dynamics in a pair of coupled heterogeneous phase oscillator networks *Chaos* **22** 043104
[41] Laing C 2009 The dynamics of chimera states in heterogeneous Kuramoto networks *Physica D* **238** 1569–88
[42] Laing C, Rajendran K and Kevrekidis I 2012 Chimeras in random non-complete networks of phase oscillators *Chaos* **22** 013132
[43] Omelchenko I, Provata A, Hizanidis J, Eckehard Schöll E and Hövell P 2015 Robustness of chimera states for coupled FitzHugh–Nagumo oscillators *Phys. Rev. E* **91** 022917
[44] Taylor A F, Nkomo S and Showalter K 2012 Chimera and phase-cluster states in populations of coupled chemical oscillators *Nat. Phys.* **8** 662–5
[45] Nkomo S, Taylor A F and Showalter K 2013 Chimera States in populations of nonlocally coupled chemical oscillators *Phys. Rev. Lett.* **110** 244102
[46] Schmidt L, Schönleber K, Krischer K and Garca-Morales V 2014 Coexistence of synchrony and incoherence in oscillatory media under nonlinear global coupling *Chaos* **24** 013102
[47] Hagerstrom A M, Murphy T E, Roy R, Hövel P, Omelchenko I and Schöll E 2012 Experimental observation of chimeras in coupled-map lattices *Nat. Phys.* **8** 658–61
[48] Lager L, Penkovsky B and Maistrenko Yu 2013 Virtual chimera state for delayed-feedback systems *Phys. Rev. Lett.* **111** 054103
[49] Viktorov E, Habruseva T, Hegarty S, Huyet G and Kelleher B 2014 Coherence and incoherence in an optical comb *Phys. Rev. Lett.* **112** 224101
[50] Lager L, Penkovsky B and Maistrenko Yu 2015 Laser chimeras as a paradigm for multi-stable patterns in complex systems *Nat. Commun.* **6** 7752
[51] Martens E, Thutupalli S, Fourriére A and Hallatschek O 2013 Chimera states in mechanical oscillator networks *Proc. Natl Acad. Sci.* **110** 10563–7
[52] Kapitaniak T, Kuzma P, Wojewoda W, Czolczynski K and Maistrenko Yu 2014 Imperfect chimera states for coupled pendula *Sci. Rep.* **4** 7369
[53] Panaggio M J and Abrams D M 2014 Chimera states on the surface of a sphere *Phys. Rev. E* **91** 022909
[54] Salnikov A, Levchenko R and Sudakov O 2011 Integrated grid environment for massive distributed computing in neuroscience *Proc. 6th IEEE Workshop IDAACS'2011* pp 198–202
[55] Zynovyev M, Svistunov M, Sudakov O and Boyko Yu 2007 Ukrainian grid infrastructure practical experience *Proc. 4th IEEE Workshop IDAACS 2007 (Dortmund, Germany, 6–8 September 2007)* pp 165–9
[56] Wiley D, Strogatz S and Girvan M 2006 The size of the sync basin *Chaos* **16** 015103
[57] Girnyk T, Hasler M and Maistrenko Yu 2012 Multistability of twisted states in nonlocally coupled Kuramoto-type models *Chaos* **22** 013114
[58] Winfree A and Strogatz S 1983 Singular filaments organize chemical waves in three-dimensions: I. Geometrically simple waves *Physica D* **8** 35–49
      Winfree A and Strogatz S 1983 Singular filaments organize chemical waves in three-dimensions: II. Twisted waves *Physica D* **9** 65–80
      Winfree A and Strogatz S 1983 Singular filaments organize chemical waves in three-dimensions: III. Knotted waves *Physica D* **9** 333–45
[59] Kupitz D and Hauser M 2013 Interaction of a pair of parallel scroll waves *Phys. Chem. A* **117** 12711–8
[60] Bick C and Martens E 2015 Controlling chimeras *New J. Phys.* **17** 033030
[61] Alonso S and Panfilov A 2013 Negative tension of scroll wave filaments and turbulence in three-dimensional excitable media and application in cardiac dynamics *Bull. Math. Biol.* **75** 1351–76
[62] Alonso S, Sagues F and Mikhailov A S 2003 Taming Winfree turbulence of scroll waves in excitable media *Science* **299** 1722
[63] Winfree A 1994 Persistent tangled vortex rings in generic excitable media *Nature* **371** 233–6
[64] Winfree A 1994 Electrical turbulence in three-dimensional heart muscle *Science* **266** 1003–6
[65] Gray R A and Jalife J 1996 Spiral waves and the heart *Int. J. Bifurcation Chaos* **6** 415–35
[66] Martens E, Panaggio M and Abrams D 2015 Basins of attraction for chimera states arXiv:1507.01457