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OPTIMAL TUNING OF SHUNT PARAMETERS FOR LATERAL BEAM VIBRATION ATTENUATION WITH THREE COLLOCATED PIEZOELECTRIC STACK TRANSDUCERS

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ABSTRACT. Structural vibration may occur in mechanical systems leading to fatigue, reduced durability or undesirable noise. In this context, shunting piezoelectric transducers to RL-shunts can be an appropriate measure for attenuating lateral beam vibrations. The achieved vibration attenuation significantly depends on an adequate tuning of the shunt to the structural resonance mode. In this paper, an existing method for resonant shunt circuit tuning based on electrical impedances is extended for lateral vibration attenuation of the first mechanical mode of a beam with circular cross-section and three collocated resonantly shunted stack transducers. It is shown by numerical simulation that a presented electrical impedance model including only the first beam mode can be used for the shunt parameter optimization if higher beam modes are taken into account.

KEYWORDS: piezoelectric shunt damping, integrated piezoelectric transducers, electromechanical impedance, numerical optimization

1 INTRODUCTION

Structural vibration may occur in mechanical systems leading to fatigue, reduced durability or undesirable noise. In this context, shunting piezoelectric transducers to passive or active electrical circuits can be an appropriate measure. Generally, a piezoelectric transducer converts mechanical kinetic energy of a vibrating host structure into electrical energy according to the piezoelectric effect. Shunting the piezoelectric transducer with a resistance and an inductance, also named RL-shunt, an electrical resonant circuit with the inherent capacitance of the transducer is created. This electromechanical system acts comparable to a mechanical vibration absorber. As for a mechanical vibration absorber, the adequate tuning of the resonant shunt to the mechanical mode of interest is crucial for the reachable vibration attenuation. Different methods exist, e. g. [1, 2, 3, 4, 5, 6, 7], for the calculation of optimal shunt parameters. In [1] and [2], the optimal shunt parameters for resistance and inductance are calculated in a manner analogous to a mechanical vibration absorber when inertia, damping and stiffness is optimized. Furthermore [2] and [3] investigate the pole placement technique to optimally reduce the structural vibrations. [4] analytically shows the derivation of optimal shunt parameters by minimizing the H_∞ norm. All these approaches mainly focus on mechanical 1-dof systems with one single shunted transducer and are based on a mechanical impedance model of the structure. A method dealing with multiple collocated shunted transducers for vibration attenuation in the arbitrary lateral beam directions is not known. However, in dealing with piezoelectric shunt damping the use of electrical impedance models do have merits. All structural properties of the mode of interest affecting the transducers including the electromechanical coupling of structure and transducer can easily be obtained from one electrical impedance measurement of arbitrary structures. In [5], [6] and [7], the tuning of a resonantly shunted transducer has been performed based on an electrical impedance model with model parameters resistance, inductance and capacitance identified from an electrical impedance measurement without the need of complex dynamic tests or expensive finite elements calculations. Again, the electrical impedance model is only derived for an assumed mechanical 1-dof system only including one resonant shunt.

In this paper, the optimal tuning of three collocated and resonantly shunted transducers to one mechanical resonance mode is performed numerically based on an electrical impedance model. As an example, a beam with circular cross-section and piezo-elastic supports [8] under harmonic excitation is considered that allows vibration deflection in arbitrary lateral direction. In the piezo-elastic support, bending of the beam in arbitrary direction is transferred into an axial deformation of three collocated stack transducers which are used for lateral vibration attenuation with resonant shunts. The dynamic behavior of the beam with piezo-elastic supports represents a system with its first mechanical resonance mode that may not be adequately approximated by a single mechanical mode without taking into account the contribution of higher modes and, moreover, three collocated shunted transducers are taken into account.

After presenting the mathematical model of the full electromechanical vibrational state beam system with resonantly shunted transducers including 20 mechanical modes, a reduced electrical impedance model of the beam with resonantly shunted transducers is presented including only the first mode and a residual mode term. The equivalent network impedance from the first mode is derived and its singular absolute maximum value is minimized to obtain the optimal shunt parameters. Then, the optimal shunt parameters are used in the beam system to verify the proposed shunt tuning method. It is shown that the derived values for resistance R and inductance L optimally attenuate the vibration in the lateral beam y- and z-direction.

2 SYSTEM DESCRIPTION

The investigated system is a beam made of aluminum alloy EN AW-7075 with length l_b and circular solid cross-section of radius r_b, Fig. 1a. The beam properties bending stiffness EI_b and density ϱ_b are assumed constant across the entire beam length. The circular cross-section has no preferred direction of lateral deflection, so the beam may vibrate in any plane perpendicular to the x-axis. The beam is supported by two piezo-elastic supports A and B. Elastic spring elements made of spring steel 1.4310 in both supports A and B at location $x = 0$ and $x = l_\mathrm{b}$ bear lateral forces in y- and z-direction and allow rotation φ_y and φ_z in any plane perpendicular to the x-axis. In Fig. 1, the spring elements for both supports A and B are represented by lateral stiffness $k_{y,\mathrm{A}} = k_{z,\mathrm{A}} = k_{y,\mathrm{B}} = k_{z,\mathrm{B}} = k_\mathrm{l}$ in y- and z-direction and rotational stiffness $k_{\varphi_y,\mathrm{A}} = k_{\varphi_z,\mathrm{A}} = k_{\varphi_y,\mathrm{B}} = k_{\varphi_z,\mathrm{B}} = k_\mathrm{r}$ around the y- and z-axis. In each

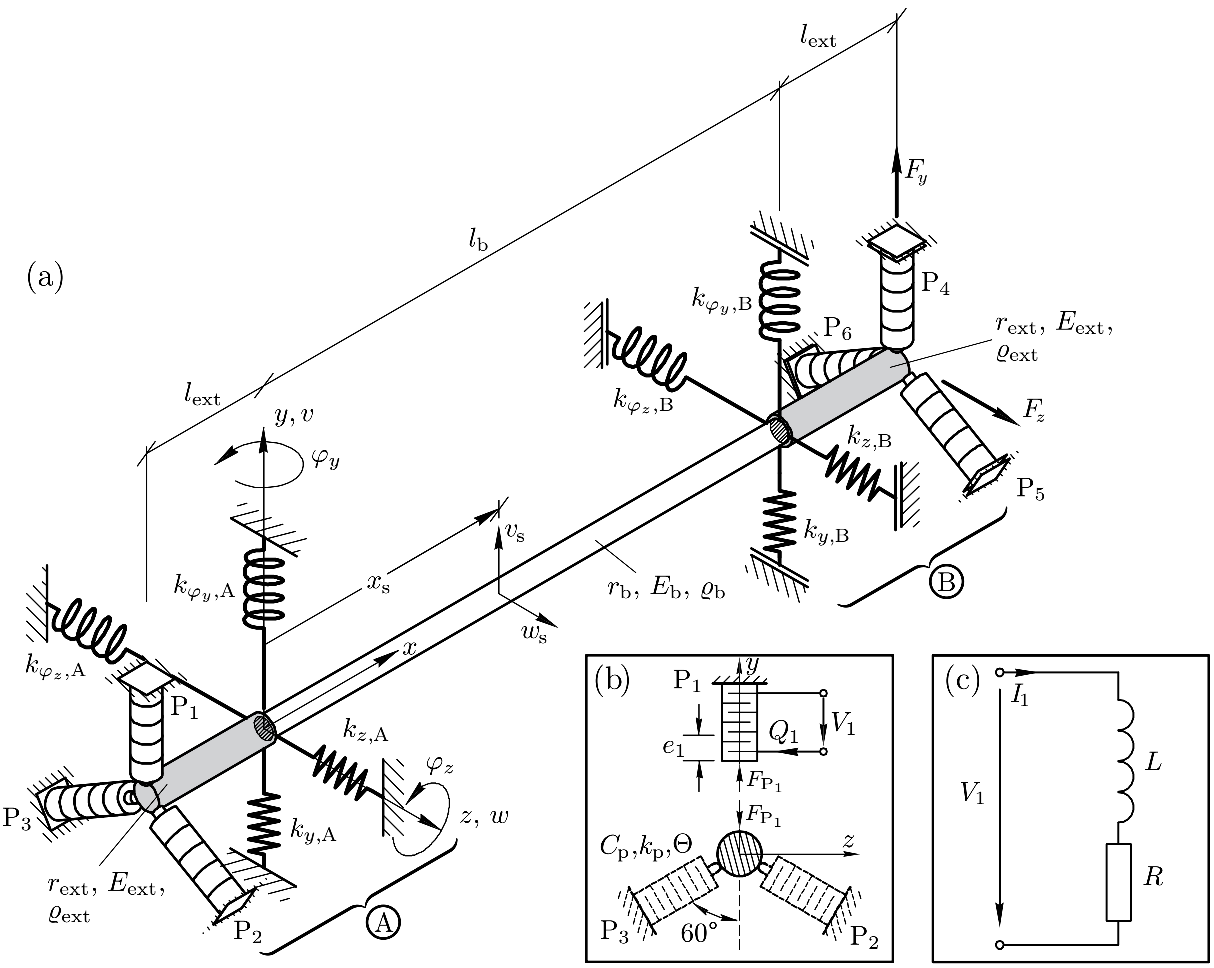


Figure 1: Beam system, (a) beam with piezo-elastic supports A and B, (b) arrangement of piezoelectric transducers, (c) shunt circuit connected to transducer P_1

piezo-elastic support A and B at $x = -l_{\mathrm{ext}}$ and $x = l_{\mathrm{b}} + l_{\mathrm{ext}}$, three piezoelectric stack transducers are collocated within the support at an angle of 120° to each other in y-z-plane orthogonal to the beam's x-axis. For lateral vibration attenuation of the beam in y-z-plane, piezoelectric transducers in support A are connected to a electrical shunts with inductance L and resistance R, Fig. 1c, with input current $I(t)$ shunt voltage $V(t)$. The forces $F_y(t)$ and $F_z(t)$ of the transducers P_4, P_5 and P_6 in support B excite the beam laterally in y-z-plane. The transducers are mechanically connected to the beam via a relatively stiff axial extension, shown in light gray, made of hardened steel 1.2312 with length l_{ext}, radius r_{ext}, constant bending stiffness EI_{ext} and density ρ_{ext}. With that, the beam deflection in y- and z-direction is transformed into the stack transducer's axial deformation and vice versa. Each piezoelectric transducer is represented by an uniaxial transducer model with relative axial elongation e_1, the electrical charge Q_1 at the transducer electrodes and the electrical potential difference V_1 between the electrodes, e. g. transducer P_1 in Fig. 1b. The resulting force F_{P_1} of piezoelectric transducer P_1 acts laterally on the axial extension. The transducers are PI P-885.51 stack transducers with the transducer force constant Θ, the capacitance C_p at constant mechanical stress and the mechanical stiffness k_p with short circuited electrodes. All geometric, mechanical and electrical properties of the beam with piezo-elastic supports and resonant shunts are given in Tab. 1. However, the piezoelectric transducer properties are independent from operating the transducers either shunted to RL-shunts in support A or as actuators for vibration excitation in support B. They are the same for all transducers used.

Table 1: Geometric, mechanical and electrical properties.

property	symbol	value	SI-units
beam length	l_b	$4.0 \cdot 10^{-1}$	m
beam radius	r_b	$5.0 \cdot 10^{-3}$	m
beam density	ϱ_b	$2.8 \cdot 10^{3}$	kg/m^3
beam Young's modulus	E_b	$7.4 \cdot 10^{10}$	N/m^2
axial extension length	l_{ext}	$7.5 \cdot 10^{-3}$	m
axial extension radius	r_{ext}	$6.0 \cdot 10^{-3}$	m
axial extension density	ϱ_{ext}	$7.8 \cdot 10^{-3}$	kg/m^3
axial extension Young's modulus	E_{ext}	$2.1 \cdot 10^{12}$	N/m^2
spring element lateral stiffness	k_l	$2.2 \cdot 10^{7}$	N/m
spring element rotational stiffness	k_r	$1.5 \cdot 10^{2}$	Nm/rad
modal damping ratios	$[\zeta_1,\ \zeta_2]$	$[3.1,\ 4.0] \cdot 10^{-3}$	–
position of sensor	x_s	$0.475\, l_b$	m
transducer stiffness	k_p	$5.0 \cdot 10^{7}$	N/m
transducer force constant	Θ	5.7	N/V
transducer capacitance	C_p	$1.6 \cdot 10^{-6}$	F
shunt inductance	L	0.24 to 0.72	H
shunt resistance	R	28 to 84	Ω

3 MATHEMATICAL MODEL OF THE BEAM WITH SHUNTED TRANSDUCERS

In this section, the mobility functions of the full beam system in y- and z-direction with resonantly shunted transducers are obtained from an electromechanical state space model including 20 mechanical modes. Furthermore, a reduced electrical impedance model is derived that describes the lateral beam vibration in y-direction with resonantly shunted transducers and that later is used to calculate the optimal values for shunt resistance and inductance for optimal vibration attenuation. Besides the first mechanical mode, the reduced impedance model also includes an residual term of 19 higher modes contributing to the first mode.

3.1 Mobility functions of the electromechanical model

The vibration attenuation capability of the harmonically excited beam with shunted transducers in piezo-elastic support A, according to Fig. 1, is investigated by studying the mobility of the beam in y- and z-direction obtained from a numerical state space model. First, an electromechanical state space model of the beam with open circuited transducer stiffness is derived in LAPLACE domain. The state space model includes the first 20 mechanical beam modes in y- and z-direction, as a result of a convergence verification that proves an adequate dynamic state representation. Second, the electrical admittances of the resonant shunts are connected electrically to the state space model of the beam. The $[4N \times 4N]$ global mass matrix $\boldsymbol{M}$ and global stiffness matrix $\boldsymbol{K}$ are obtained from a finite element discretization with N nodes and $N-1$ one-dimensional EULER-BERNOULLI beam elements. Each n node has four degrees of freedom, two translational displacements v_n and w_n in y- and z-direction and two rotational displacements φ_{y_n} and φ_{z_n} around the y- and z-axis. For further information about the model derivation process, the reader is kindly referred to [8].

State space model and electromechanical transfer function with open circuited electrodes

As state space input $\boldsymbol{u}(s)$, harmonic excitation forces $F_y(s)$ and $F_z(s)$ and electrical charges $\boldsymbol{Q}_\mathrm{A}(s) = [Q_1(s), Q_2(s), Q_3(s)]^T$ are summarized in the $[5 \times 1]$ state space input vector

$$\boldsymbol{u}(s) = \begin{bmatrix} F_y(s) \\ F_z(s) \\ \boldsymbol{Q}_\mathrm{A}(s) \end{bmatrix}, \qquad \text{and} \qquad \boldsymbol{y}(s) = \begin{bmatrix} v_\mathrm{s}(s) \\ w_\mathrm{s}(s) \\ \boldsymbol{V}_\mathrm{A}(s) \end{bmatrix} \tag{1}$$

is the $[5 \times 1]$ state space output vector including the lateral displacements $v_\mathrm{s}(s)$ and $w_\mathrm{s}(s)$ at the sensor location x_s and the transducer voltages $\boldsymbol{V}_\mathrm{A}(s) = [V_1(s), V_2(s), V_3(s)]^T$. The electromechanical state space model of the beam in Fig. 1 becomes

$$\begin{aligned} s\,\boldsymbol{x}(s) &= \underbrace{\begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{D}_\zeta \end{bmatrix}}_{[8N\times 8N]} \boldsymbol{x}(s) + \underbrace{\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \boldsymbol{M}^{-1}\boldsymbol{B}_\mathrm{F} & \boldsymbol{M}^{-1}\boldsymbol{B}_\mathrm{A}\frac{\Theta}{C_\mathrm{p}} \end{bmatrix}}_{[8N\times 5]} \boldsymbol{u}(s) \\ \boldsymbol{y}(s) &= \underbrace{\begin{bmatrix} \boldsymbol{b}_\mathrm{v} & 0\dots 0 \\ \boldsymbol{b}_\mathrm{w} & 0\dots 0 \\ -\boldsymbol{B}_\mathrm{A}^T\frac{\Theta}{C_\mathrm{p}} & 0\dots 0 \end{bmatrix}}_{[5\times 8N]} \boldsymbol{x}(s) + \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{C_\mathrm{p}} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{C_\mathrm{p}} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{C_\mathrm{p}} \end{bmatrix}}_{[5\times 5]} \boldsymbol{u}(s). \end{aligned} \tag{2}$$

In Eq. (2), $\boldsymbol{x}(s)$ is the $[8N \times 1]$ state vector. The zero and identity matrices $\mathbf{0}$ and $\mathbf{I}$ are of appropriate dimensions. $\boldsymbol{D}_\zeta$ is the global $[4N \times 4N]$ damping matrix and is determined by Rayleigh proportional damping $\boldsymbol{D}_\zeta = \alpha\, \boldsymbol{M} + \beta\, \boldsymbol{K}$, [9]. The proportional damping coefficients α and β are determined for modal damping rations $\zeta_{1/2}$ of the first two bending modes, identified in own experiments [10]. Furthermore, $\boldsymbol{B}_\mathrm{A}$ allocates the lateral forces of transducer P_1, P_2 and P_3 to node 1 and $\boldsymbol{B}_\mathrm{F}$ allocates the vibration excitation forces F_y and F_z of transducers P_4, P_5 and P_6 to node N. In Eq. (2), the displacements $v_\mathrm{s}(s)$ and $w_\mathrm{s}(s)$ are extracted from the state vector $\boldsymbol{x}(s)$ in Laplace domain by $[4N \times 1]$ vectors $\boldsymbol{b}_\mathrm{v}$ and $\boldsymbol{b}_\mathrm{w}$. As a result, the beam's eigenfrequencies f_oc with open circuited electrodes for neglected damping can be calculated by solving the characteristic equation

$$\det\left[\boldsymbol{K} - (2\pi f_\mathrm{oc})^2\, \boldsymbol{M}\right] = 0. \tag{3}$$

In short form, Eq. (2) can also be written as

$$\begin{aligned} s\,\boldsymbol{x}(s) &= \boldsymbol{A}\,\boldsymbol{x}(s) + \boldsymbol{B}\,\boldsymbol{u}(s) \\ \boldsymbol{y}(s) &= \boldsymbol{C}\,\boldsymbol{x}(s) + \boldsymbol{D}\,\boldsymbol{u}(s). \end{aligned} \tag{4}$$

Then, the electromechanical transfer function

$$\boldsymbol{G}(s) = \frac{\boldsymbol{y}(s)}{\boldsymbol{u}(s)} = \left[\boldsymbol{C}(s\,\mathbf{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D}\right] = \begin{bmatrix} \boldsymbol{G}_\mathrm{rF}(s) & \boldsymbol{G}_\mathrm{rQ}(s) \\ \boldsymbol{G}_\mathrm{VF}(s) & \boldsymbol{G}_\mathrm{VQ}(s) \end{bmatrix} \tag{5}$$

represents the relation of the inputs in $\boldsymbol{u}(s)$ to the outputs in $\boldsymbol{y}(s)$, [11]. In Eq. (5), $\boldsymbol{G}_\mathrm{rF}(s)$ connects the input forces $F_y(s)$ and $F_z(s)$ to the output displacements $v_\mathrm{s}(s)$ and $w_\mathrm{s}(s)$, $\boldsymbol{G}_\mathrm{rQ}(s)$ connects the input charges $\boldsymbol{Q}_\mathrm{A}(s)$ to the output displacements $v_\mathrm{s}(s)$ and $w_\mathrm{s}(s)$, $\boldsymbol{G}_\mathrm{rF}(s)$ connects the input forces $F_y(s)$ and $F_z(s)$ to the output voltages $\boldsymbol{V}_\mathrm{A}(s)$ and $\boldsymbol{G}_\mathrm{VQ}(s)$ connects the input charges $\boldsymbol{Q}_\mathrm{A}(s)$ to the output voltages $\boldsymbol{V}_\mathrm{A}(s)$.

Electromechanical model with with resonantly shunted transducers

The electrical admittances of the three resonant shunt circuits are connected to the three transducers in support A via electrical charges $\boldsymbol{Q}_\mathrm{A}(s)$ and voltages $\boldsymbol{V}_\mathrm{A}(s)$. For that, the circuit in Fig. 1c is considered in the transfer function Eq. (5). The circuit takes into account a damping resistance R and an ideal inductance L. As an example, the electrical shunt admittance of the RL-shunt connected to transducer P_1 in Laplace domain is

$$Y_\mathrm{RL,1}(s) = \frac{I_1(s)}{V_1(s)} = \frac{s\,Q_1(s)}{V_1(s)} = \frac{1}{R + Ls} \tag{6}$$

with current $I_1(s)$, voltage $V_1(s)$ and charge $Q_1(s)$ of transducer P_1. The electrical resonance frequency f_el of the shunted transducer is calculated by

$$f_\mathrm{el} = \left(2\pi\sqrt{LC_\mathrm{p}}\right)^{-1}. \tag{7}$$

The admittances $Y_{\mathrm{RL},2}(s)$ and $Y_{\mathrm{RL},3}(s)$ shunted to transducers P_2 and P_3 are written analogously to Eq. (6) where R and L are assumed have the same values each in all three shunts for a given choice according to Tab. 1. To connect the resonant shunts, the shunt (sh) transfer function $\boldsymbol{G}_{\mathrm{sh}}(s)$ relates the electrical charges $\boldsymbol{Q}_{\mathrm{A}}(s)$ to the voltages $\boldsymbol{V}_{\mathrm{A}}(s)$ via the shunt admittances $Y_{\mathrm{RL},1/2/3}(s)$ according to

$$\boldsymbol{Q}_{\mathrm{A}}(s) = \boldsymbol{G}_{\mathrm{sh}}(s)\,\boldsymbol{V}_{\mathrm{A}}(s), \tag{8}$$

with

$$\boldsymbol{G}_{\mathrm{sh}}(s) = \frac{1}{s}\begin{bmatrix} Y_{\mathrm{RL},1}(s) & 0 & 0 \\ 0 & Y_{\mathrm{RL},2}(s) & 0 \\ 0 & 0 & Y_{\mathrm{RL},3}(s) \end{bmatrix}. \tag{9}$$

From Eq. (5) and Eq. (8), the transfer function of the electromechanical beam system with shunted transducers is obtained to

$$\begin{bmatrix} v_{\mathrm{s}}(s) \\ w_{\mathrm{s}}(s) \end{bmatrix} = \left[\boldsymbol{G}_{\mathrm{rF}}(s) + \boldsymbol{G}_{\mathrm{rQ}}(s)\,\boldsymbol{G}_{\mathrm{sh}}(s)\left(\mathbf{I} - \boldsymbol{G}_{\mathrm{VQ}}(s)\right)^{-1}\boldsymbol{G}_{\mathrm{VF}}(s)\right]\begin{bmatrix} F_y(s) \\ F_z(s) \end{bmatrix}. \tag{10}$$

with lateral harmonic excitation input forces $F_y(s)$ and $F_z(s)$ and with output displacements $v_{\mathrm{s}}(s)$ and $w_{\mathrm{s}}(s)$ as described in [8]. For investigating the vibration attenuation capability in the following chapter, the mobility functions

$$G_y(s) = s\frac{v_{\mathrm{s}}(s)}{F_y(s)} \quad \text{and} \quad G_z(s) = s\frac{w_{\mathrm{s}}(s)}{F_z(s)}. \tag{11}$$

of the beam in y- and z-direction are considered. In Fig. 5, $G_y(s)$ and $G_z(s)$ are plotted as functions of frequency f in Hz using the conversion $s = j2\pi f$.

3.2 Impedance function of the reduced electrical model

In this section, a reduced electrical impedance model of the beam shown in Fig. 1 with resonantly shunted transducers including the first mechanical mode in y-direction is derived. The impedance model is extended by a residual mode term to also consider the contribution of the 19 higher mechanical modes. Finally, the equivalent network impedance seen from the first mechanical mode is derived and, in the next section, minimized by varying the shunt parameters L and R within the range given in Tab. 1 to obtain the optimal shunt parameters L_{opt} and R_{opt}.

The reduced impedance model is shown in Fig. 2, it only includes electrical components such as capacitances, resistances and inductances. The impedance model takes into account the shunted transducer P_1 with capacitance C_{p} and shunt parameters L and R, the modal approximation of the first mechanical mode in y-direction and an equivalent shunted transducer representing the shunted transducers P_2 and P_3 with capacitance C_{p} and shunt parameters L and R. The equivalent shunted transducer results from the transformation of the shunted transducers forces F_{P_2} and F_{P_3} in the y-direction. This simplification is justified since all three transducers and all three resonant shunts of the same values for C_{p}, L and R assuming the same choices of L and R in Tab. 1. The mechanical mode is considered using the second

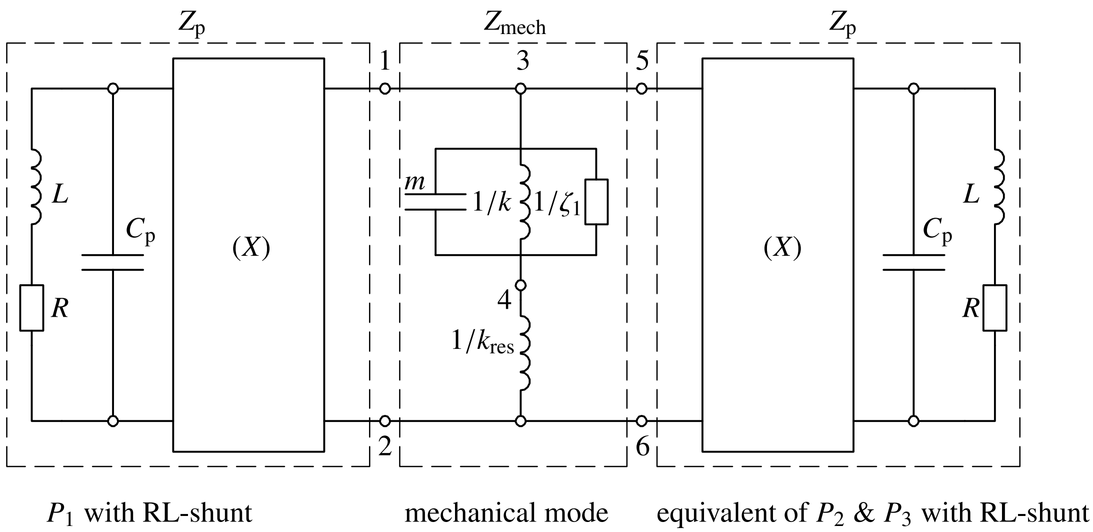


Figure 2: Reduced electrical impedance model of the beam with resonantly shunted transducers in y-direction

electromechanical analogy where the current I, voltage V, capacitance C, inductance L and resistance R are analog to the force F, velocity $\dot{v}$, modal mass m, modal compliance $1/k$, and modal damping admittance $1/\zeta_1$ as shown in [12]. Additionally the contribution of the mechanical modes 2 to 20 is considered by the residual compliance $1/k_{res}$. The modal parameters can be obtained from the full electromechanical system Eq. (2) according to [13]. For connecting the electrical impedances of the shunted transducers to the mechanical impedance of the mechanical mode, the dimensionless conversion factor X is considered. X represents the conversion between electrical and mechanical energy and may be considered as the electromechanical coupling of transducers and beam.

In order to find the optimal shunt parameters L_{opt} and R_{opt}, an equivalent impedance $Z_{eq}(s)$ of the network in Fig. 2 for the mechanical mode is derived in the LAPLACE domain, see Fig. 3, and its singular absolute maximum is minimized. The equivalent impedance

$$Z_{eq}(s) = \frac{V_{eq}(s)}{I_{eq}(s)} = \frac{Z_{mech}(s)\, Z_{34}(s)}{Z_{mech}(s) + Z_{34}(s)} \tag{12}$$

results from the parallel connection of the mechanical modal impedance

$$Z_{mech}(s) = \frac{1}{ms + \dfrac{k}{s} + \zeta_1} \tag{13}$$

of the first beam mode in y-direction between the terminals 3 and 4 and the network impedance Z_{34} seen from the terminals 3 and 4, see Fig. 3. The network impedance

$$Z_{34}(s) = \frac{Z_p(s)}{2} + Z_{res}(s) \tag{14}$$

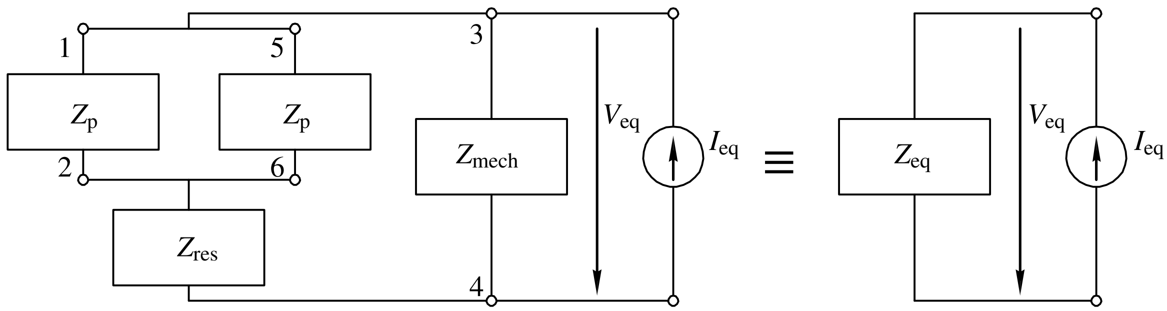


Figure 3: Equivalent network impedance of the impedance model in Fig. 2

results from the parallel connection of the resonantly shunted piezoelectric transducers

$$Z_p(s) = X^2\left(sC_p + \frac{1}{Ls + R}\right) \tag{15}$$

seen from the terminals 1 and 2 and 5 and 6, respectively, that in turn are in series connection with the impedance

$$Z_{res}(s) = \frac{s}{k_{res}} \tag{16}$$

of the residual mode term.

4 NUMERICAL TUNING OF THE RL-SHUNT PARAMETERS

To achieve a maximum vibration attenuation of the first bending mode of the beam in Fig. 1, the maximum singular absolute value of the equivalent impedance $Z_{eq}(s)$, according to Eq. (12), is minimized by varying the two free shunt parameters L and R in the range $0.24 \leq L \leq 0.72$ H and $28 \leq R \leq 84$ *Omega*. The considered frequency range is between $150 \leq f \leq 250$ Hz with the first resonance frequency $f_{oc} = 190.3$ Hz according to Eq. (3). The numerical minimization is carried out using a derivative-free algorithm, as implemented in the MATLAB Optimization Toolbox. For the applied electromechanical analogy according to [12], minimizing $Z_{eq}(s)$ will result in a optimal tuning for the mobility function $G_y(s)$. The ability of vibration attenuation with RL-shunts is compared to the undamped state with open circuited transducers electrodes and analyzed for the impedance function and the mobility functions.

4.1 Impedance function with optimal RL-shunt

Figure 4a shows the amplitude and phase response of the equivalent impedance $Z_{eq}(s)$ with open circuited transducers and with optimal parameters $L_{opt} = 0.48$ H and $R_{opt} = 56.2\,\Omega$ found by the algorithm. The electrical response function is attenuated and, as expected for a resonant shunt being comparable to an mechanical absorber [1, 2], shows the two characteristic bumps with equal amplitudes. As mentioned before, the residual term $Z_{res}(s)$ of higher modes 2 to 20 influences the vibration behavior with RL-shunts significantly. To see the influence, Fig. 4b clarifies that amplitude and phase response of the equivalent impedance $Z_{eq}(s)$ show different electrical resonance frequencies according to Eq. (7) in case $Z_{res}(s)$ is neglected and $Z_{res}(s)$ is considered in Eq. (12). For $R = 0$ and $L = L_{opt}$, the electrical resonance frequency becomes $f_{el} = 190.6\,\text{Hz}$ if $Z_{res}(s)$ is taken into account and $f_{el} = 210.5\,\text{Hz}$ if $Z_{res}(s)$ is not taken into account.

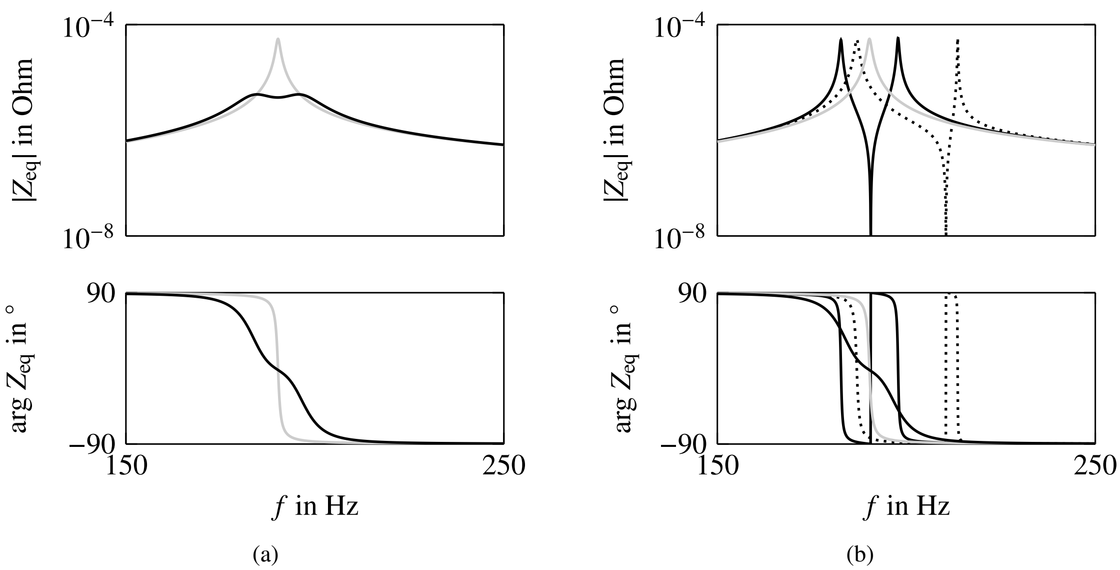
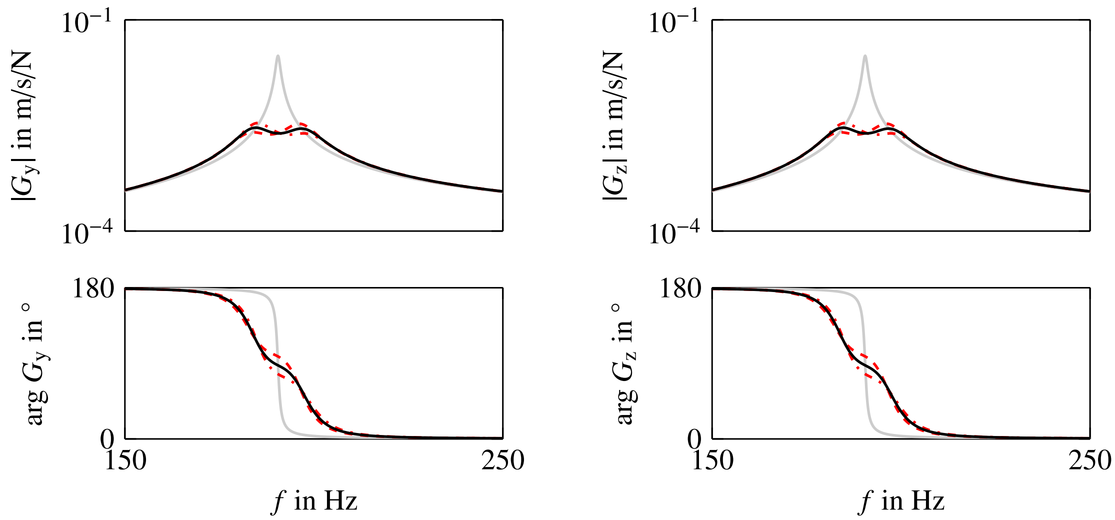


Figure 4: Electrical impedance function $Z_{eq}(s)$ with (a) open circuited transducers (——) and optimal parameters L_{opt} and R_{opt} (——) (b) influence of modes, open circuited transducers (——) and $R = 0$ and $L = L_{opt}$ residual term $Z_{res}(s)$ included (——) and residual term $Z_{res}(s)$ neglected (- - -)

4.2 Mobility functions with optimal RL-shunt

To verify the investigated tuning method, the shunt parameters L_{opt} and R_{opt} are used in the electromechanical model Eq. (10) of the beam in Fig. 1 to calculate mobility functions $G_y(s)$ and $G_z(s)$ in Eq. (11). Figure 5a and Fig. 5b show amplitude and phase response in y- and z-direction with open circuited electrodes and with resonantly shunted transducers P_1, P_2 and P_3. It can be seen, that with L_{opt} and R_{opt} the mobility functions $G_y(s)$ and $G_z(s)$ are equally attenuated. That is due to the assumed symmetric behavior of the full electromechanical model in y- and z-direction. Furthermore, Fig. 5 shows the mobility functions $G_y(s)$ and $G_z(s)$ for a deviating $L = L_{opt}\,(1 \pm 0.02)$, verifying that the vibration attenuation gets worse and L_{opt} and R_{opt} may be considered as an optimal shunt tuning.



(a) (b)

Figure 5: Mobility functions (a) $G_y(s)$ in y- and (b) $G_z(s)$ in z-direction with open circuit transducers ( ) and optimal tuned RL-shunts (——), and with deviations from the optimal vibration attenuation for L with $L = 0.98\, L_{\mathrm{opt}}$ (–·–) and $L = 1.02\, L_{\mathrm{opt}}$ (– –)

5 CONCLUSIONS

The method of optimally tuning a resonant shunt based on an electrical impedance model is successfully applied to a beam with circular cross-section subject to vibrate in arbitrary lateral directions in piezo-elastic supports. The method is extended in order to tune three resonant shunt circuits for three collocated piezoelectric transducers to attenuate the first lateral beam mode in y- and z-direction. An electromechanical model of the beam that includes 20 mechanical modes is presented and a reduced electrical impedance model of the beam with resonant shunts including the first mechanical mode only is derived to calculated the optimal shunt inductance and resistance. Furthermore, it is shown that that taking into account higher modes has a significant influence on the vibration behavior of the first mode. Therefore, a residual mode term is considered in the impedance model for optimal parameter tuning. In future investigations, the experimental identification of the impedance model parameters will be investigated more detailed as well as the application of the proposed method to the experimental beam with piezo-elastic supports.

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REFERENCES

[1] N. W. Hagood and A. H. von Flotow. Damping of structural vibrations with piezoelctric materials and passive electrical networks. *Journal of Sound and Vibration*, 146(2):243 – 268, 1991.

[2] M. Neubauer, R. Oleskiewicz, K. Popp, and T. Krzyzynski. Optimization of damping and absorbing performance of shunted piezo elements utilizing negative capacitance. *Journal of Sound and Vibration*, 298:84 – 107, 2006.

[3] B. de Marneffe. *Active and Passive Vibration Isolation and Damping via Shunted Transducers*. PhD thesis, Université Libre de Bruxelles, 2007.

[4] P. Soltani, G. Kerschen, G. Tondreau, and A. Deraemaeker. Piezoelectric vibration damping using resonant shunt circuits: an exact solution. *Smart Materials and Structures*, 23:1–11, 2014.

[5] M. A. Kozlowski, D. G. Cole, and R. L.. Clark. A comprehensive study of the RL series resonant shunted piezoelectric: A feedback controls perspective. *Journal of Vibration and Acoustics*, 133:011012–1 – 011012–10, 2011.

[6] J. Kim, Y.-H. Ryu, and S Choi. New shunting parameter tuning method for piezoelectric damping based on measured electrical impedance. *Smart Materials and Structures*, 9:868–877, 2000.

[7] R. Salloum, O. Heuss, B. Götz, and D. Mayer. Optimally tuned resonant negative capacitance for piezoelectric shunt damping based on measured electromechanical impedance. In *Proceedings of SPIE Vol. 9433 Industrial and Commercial Applications of Smart Structures Technologies*, San Diego, CA/USA, 2015.

[8] B. Götz, R. Platz, and T Melz. Lateral vibration attenuation of a beam with circular cross-section by a support with integrated piezoelectric transducers shunted to negative capacitances. *Smart Materials and Structures*, accepted and to be published.

[9] S. M. Khot and N. P. Yelve. Modeling and response analysis of dynamic systems by using ansys and matlab. *Journal of Vibration and Control*, 17:953 – 958, 2011.

[10] B. Götz, R. Platz, and T Melz. Lateral vibration attenuation of a beam with circular cross-section by supports with integrated resonantly shunted piezoelectric transducers. In *Proceedings of SMART2015 7th ECCOMAS Thematic Conference on Smart Structures and Materials*, Ponta Delgada, Azores, Jun. 3-6 2015.

[11] C. R. Fuller, S. J. Elliot, and P. A. Nelson. *Active Control of Vibration*. Number 0-12-269440-6. Academic Press, Inc., 1997.

[12] A. Lenk, R.G. Ballas, R. Werthschützky, and G. Pfeifer. *Electromechanical Systems in Microtechnology and Mechatronics*. Number 978-3-540-89320-2. Springer-Verlag Berlin Heidelberg, 2011.

[13] F. J. Fahy and P Gardonio. *Sound and Structural Vibration*. Number 9780080471105. Academic Press, 2007.