

COMPARISON OF LLRF CONTROL APPROACHES FOR HIGH INTENSITY HADRON SYNCHROTRONS: DESIGN AND PERFORMANCE*

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Abstract

A usual and effective way to realize closed-loop controllers is to use cascaded SISO feedback and to rely on some kind of linear PID (proportional, integral, derivative) structure often with parameters adjusted manually. Such a control may not reach optimal performance if the system is coupled or non-linear. Regarding intense beams, longitudinal beam loading can be compensated by detuning. But the coupling between phase and amplitude (or I and Q component) highly depends on the tuning, that is on the resonant frequency of the cavity. It is derived that cavity and beam dynamics show bi-linear nature, i.e. belong to a well investigated class of non-linear systems with appropriate control strategies available [1, 4].

Different controller designs are compared. The performance evaluation is based on macro-particle tracking simulations.

Notations

Small deviations from the steady-state sine and cosine components of signals are denoted by s and c respectively, b , v , i refer to the beam current, gap voltage and current respectively. All other used symbols and parameters are summarized in Table 1.

MODEL

Bi-linear Systems

Dynamical systems are often described by systems of n differential equations of first order called **state space** representation. A bi-linear system can be described by

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \sum_{j=1}^m u_j(t)\mathbf{N}_j\mathbf{x}(t). \quad (1)$$

Beam Dynamics

Longitudinal beam dynamics are often detached from the cavity servo control presented here, by the architecture of the (overall feedback) system that is split into a local cavity feedback and a global longitudinal beam feedback loop due to multiple cavities and their spatial separation. Only the stationary case is considered as this paragraph is just included to show the impact of bi-linear systems on longitudinal beam dynamics. The dynamics of a single particle exposed to a sinusoidal voltage (3) are given by $\dot{\varphi} = aE$, $\dot{E} = bv(t)$ with the linear synchrotron frequency $\omega_s = \sqrt{-ab\hat{V}}$. The dynamics of

a bunched beam are approximately described using the I/Q-components of the fundamental of the beam current $i_{b,1}(t) = \hat{I}_{b,1}((1 + c_b(t))\cos(\omega t) + s_b(t)\sin(\omega t))$. For infinitesimal small bunches it follows

$$\ddot{s}_b + \omega_s^2 s_b = -\omega_s^2 (c_v(1 + c_b) + s_v s_b) \text{ and} \\ \ddot{c}_b + (2\omega_s)^2 c_b = \omega_s^2 (c_v s_b - 2s_v c_b)$$

Linearization around zeros of this bi-linear system yields the well-known linear dipole (and quadrupole) dynamics $\ddot{s}_b + \omega_s^2 s_b = -\omega_s^2 c_v$ (and $\ddot{c}_b + (2\omega_s)^2 c_b = -2\omega_s^2 s_v \cdot 0$).

Cavity Dynamics

With $\sigma^{-1} = 2RC$ and $\omega_R^{-2}(t) = L(t)C$ the differential equation of the resonator shown in Fig. 1 is given by

$$\omega_R^{-2}(t)\dot{v}(t) + \omega_R^{-2}(t)2\sigma v(t) + \int v(t)dt = \\ = \omega_R^{-2}(t)2\sigma Ri(t) \quad (2)$$

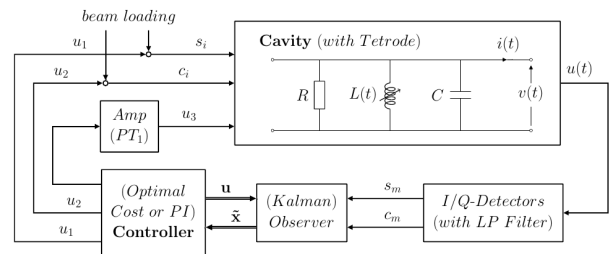


Figure 1: System layout with equivalent lumped element RLC circuit for the cavity.

Inserting the **I/Q modulated** signals

$$i(t) = \hat{I} \left((1 + s_i(t)) \sin(\omega t) + \left(c_i(t) - \frac{\omega_R^2 - \omega^2}{2\sigma\omega} \right) \cos(\omega t) \right) \\ v(t) = \hat{V} \left((1 + s_v(t)) \sin(\omega t) + c_v(t) \cos(\omega t) \right) \quad (3)$$

and the corresponding derivatives into the differential Eq. (2) yields Eqns. (4). These two differential equations are of 2nd order in the voltage, resulting in a minimal realization of 4th order. With the states x_i and inputs u_i as denoted in Eqns. (4) the system can be written in the bi-linear form (1).

It can be seen that even when the cavity is in resonance, i.e. beam loading is not significant or not compensated by detuning, amplitude and phase loop are slightly coupled during transient processes dependent on the quality factor $Q = \omega_R/2/\sigma$ of the cavity.

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$$\begin{aligned}
 \ddot{s}_v(t) &= -2\sigma\dot{s}_v(t) - (\omega_R^2(t) - \omega^2)s_v(t) + 2\omega\dot{c}_v(t) + 2\sigma\omega c_v(t) - \underbrace{(\omega_R^2(t) - \omega^2)}_{u_3} + 2\frac{\dot{\omega}_R(t)}{\omega_R(t)} \underbrace{(\dot{s}_v(t) + 2\sigma s_v(t) - \omega c_v(t) - 2\sigma s_i(t))}_{\omega_R^2(t)x_1} \\
 &\quad + 2\sigma \underbrace{(\dot{s}_i(t) - \omega c_i(t))}_{u_2} \\
 \ddot{c}_v(t) &= -2\sigma\dot{c}_v(t) - (\omega_R^2(t) - \omega^2)c_v(t) - 2\omega\dot{s}_v(t) - 2\sigma\omega s_v(t) + 2\frac{\dot{\omega}_R(t)}{\omega_R(t)} \underbrace{(\dot{c}_v(t) + 2\sigma c_v(t) + \omega s_v(t) - 2\sigma c_i(t) - \frac{\omega_R^2(t) - \omega^2}{\omega})}_{\omega_R^2(t)x_2} \\
 &\quad + 2\frac{\dot{\omega}_R(t)}{\omega} \dot{\omega}_R(t) + 2\sigma \underbrace{(\dot{c}_i(t) + \omega s_i(t))}_{u_1}
 \end{aligned} \tag{4}$$

SYSTEM LAYOUT

I/Q-Detectors

The measured values $s_m(t)$ and $c_m(t)$ for the states $s_v(t)$ and $c_v(t)$ are obtained by multiplying the gap voltage $v(t)$ with reference signals, $2\sin(\omega t)$ and $2\cos(\omega t)$ respectively, followed by Bessel-low-pass-filters with the edge frequency ω_d .

Actuators

The dynamics of the amplifier (tetrode) for the generator current $i(t)$ driving the gap voltage $v(t)$ can be included in the RLC circuit. The amplifier (transistor) for the bias current is modeled as a PT1 element, $\dot{i}_v(t) + f_a i_v(t) = f_a u_a(t)$. Mainly based on [2] the relationship between the squared eigenfrequency and the bias current $i_v(t)$ used for the tuning of the cavity was found to be linear or rather affine in good approximation, e.g. $\omega_R^2(t) = 0.5 \cdot 10^{11} \frac{1}{\text{As}^2} i_v(t) + \mathcal{C}$ as in [2]. This omission of nonlinear characteristics facilitates analytical controller design. With $u_3 := \omega_R^2(t) - \omega^2$ this results in $\dot{u}_3 + f_a u_3 = C_a u_a$.

Overall System

Because of the dynamics of the amplifier for the bias current the new input u_a replaces u_3 which becomes an internal state and the overall system dynamics are no more bi-linear but still input affine. Combining all the subsystems, the dynamics can be written in the form $\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}\mathbf{u}$, used for the setup of the MIMO state controller and observer. The model can also be used to analyze the stability margins of existing servo loops.

CLOSED-LOOP FEEDBACK

The design of the MIMO controller is based on Eqns. (4), the design of the SISO PI-controllers on the simplified decoupled cavity dynamics $\dot{s}_v(t) + \sigma s_v(t) = \sigma s_i(t)$, $\dot{c}_v(t) + \sigma c_v(t) = \sigma c_i(t) - u_3/2/\omega$, commonly used when considering stationary beam loading (e.g. in [3]).

MIMO Optimal Control

Let \mathbf{Q} be a symmetric, positive semi-definite matrix and \mathbf{R} be positive definite and symmetric.

A necessary condition for the (quadratic) cost function $J = \frac{1}{2} \int_0^\infty \mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{R} \mathbf{u} dt$ to be minimal is $\partial H(\mathbf{x}, \mathbf{u}, \lambda) / \partial \mathbf{u}|_{\mathbf{u}=\mathbf{u}_0} = 0$ with the Hamiltonian

$$H(\mathbf{x}, \mathbf{u}, \lambda) = \frac{1}{2}(\mathbf{x}^T \mathbf{Q} \mathbf{x} + \mathbf{u}^T \mathbf{R} \mathbf{u}) + \lambda^T (\mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}\mathbf{u})v$$

and $\lambda \in \mathbb{R}^n$. This yields $\mathbf{u}_0 = -\mathbf{R}^{-1} \mathbf{B}^T \lambda$.

The Hamilton-Jacobi-Bellman equation gives the sufficient condition $\partial \mathbf{V} / \partial t + H(\mathbf{x}, \mathbf{u}_0, \partial \mathbf{V} / \partial \mathbf{x}) = 0$.

Due to the infinite upper bound of the cost integral, the solution \mathbf{V} does not depend on time, i.e. $\partial \mathbf{V} / \partial t = 0$.

Under the condition that $\partial \mathbf{P}(\mathbf{x})\mathbf{x} / \partial \mathbf{x}$ is symmetric given in [1] the form $\partial \mathbf{V} / \partial \mathbf{x} = \mathbf{P}(\mathbf{x})\mathbf{x}$ is permitted. This results in $\frac{1}{2}(\mathbf{x}^T \mathbf{Q} \mathbf{x} + (-\mathbf{R}^{-1} \mathbf{B}^T \mathbf{P}(\mathbf{x})\mathbf{x})^T \mathbf{R}(-\mathbf{R}^{-1} \mathbf{B}^T \mathbf{P}(\mathbf{x})\mathbf{x})) + (\mathbf{P}(\mathbf{x})\mathbf{x})^T (\mathbf{A}(\mathbf{x})\mathbf{x} + \mathbf{B}(-\mathbf{R}^{-1} \mathbf{B}^T \mathbf{P}(\mathbf{x})\mathbf{x})) = 0$ or $\mathbf{P}^T(\mathbf{x})\mathbf{B}\mathbf{R}^{-1} \mathbf{B}^T \mathbf{P}(\mathbf{x}) - 2\mathbf{P}(\mathbf{x})^T \mathbf{A}(\mathbf{x}) = \mathbf{Q}$, a Riccati-like equation often used in control theory. For bi-linear systems the optimal control $\mathbf{u}_0(\mathbf{x}) = -\mathbf{K}(\mathbf{x})\mathbf{x}$ with $\mathbf{K}(\mathbf{x}) = \mathbf{R}^{-1} \mathbf{B}^T \mathbf{P}(\mathbf{x})$ follows from the Riccati equation with a symmetric matrix $\mathbf{P}(\mathbf{x})$. The feedback is calculated on the assumption that the loss of bi-linearity does not affect the optimality of the Riccati-controller too much.

Extended Kalman Filter

An observer is used to determine the not measurable inner states \mathbf{x} of a system by the simultaneous use of a model of the system driven by the same input. The extended Kalman filter is the most frequently used observer for non-linear systems. The matrix \mathbf{L} , used to feed back the difference between the estimated output $\tilde{\mathbf{y}}$ and the measured output \mathbf{y} , i.e. $\dot{\tilde{\mathbf{x}}} = \tilde{\mathbf{A}}\tilde{\mathbf{x}} + \tilde{\mathbf{B}}\mathbf{u} + \tilde{\mathbf{L}}(\tilde{\mathbf{y}} - \mathbf{y})$, is determined in the same way as the matrix \mathbf{K} of the optimal state feedback.

SISO PI-Controllers

With observer the eigenvalues of the *amplitude* loop can be placed freely on $\lambda_{1/2} = \tilde{\lambda}$ and the corresponding gains are $K_{P,s} = 2\tilde{\lambda}/\sigma + 1$ and $K_{I,s} = -\tilde{\lambda}^2/\sigma$. If the *phase* loop is designed equally due to the same dynamics, a good value for the proportional gain of the *bias* loop is $K_{P,\omega} = \sigma(2\tilde{\lambda}\sqrt{\tilde{\lambda}(2f_a + \tilde{\lambda})} - 2\tilde{\lambda}^2 - 2\tilde{\lambda}f_a - f_a^2)/(8C_a\tilde{\lambda})$, whereas for the integral gain $K_{I,s}$ a simple analytical expression could not be found.

Without observer the dynamics of the I/Q detectors have to be considered in the control setup. Together with the simplified cavity dynamics and the PI-feedback $s_i(t) = K_{P,s}s_v(t) + K_{I,s} \int s_v(t) dt$ the characteristic polynomial of the closed *amplitude* loop is $(\lambda^2 + \sqrt{3}\omega_d\lambda + \omega_d^2)(\lambda + \sigma)\lambda - \sigma\omega_d^2(K_{P,s}\lambda + K_{I,s})$

Where by choosing $K_{P,s} = -\omega_d/\sqrt{27}/\sigma$ and $K_{I,s} = -\omega_d/\sqrt{27}$ the zeros or eigenvalues of the system are $\lambda_{1/2/3} = -\omega_d/\sqrt{3}$ and $\lambda_4 = -\sigma$.

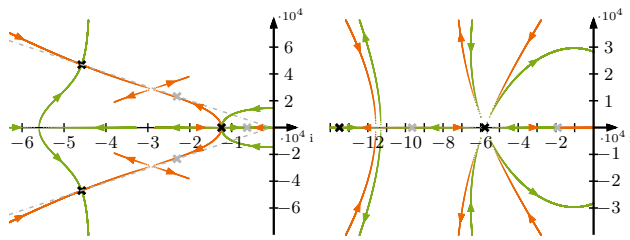


Figure 2: Root-locus plots of the bias loop for $\tilde{\lambda} = -\omega_d/\sqrt{3}$ with observer (left) and of the phase/amplitude loop w/o observer (right): variable $K_p, K_i, \tilde{\lambda}$

SIMULATIONS AND CONCLUSIONS

It is shown, that separate PI-controllers cannot yield optimal performance if the cavity is strongly detuned. Therefore macro particle tracking is performed inspired by a scenario where four batches from the SIS18 each consisting of two bunches a $5 \cdot 10^{11}$ particles are injected in sequence into the SIS100 of the FAIR-Project at GSI. The simulations are based on the SIS100 parameters of FAIR (www.gsi.de) with similar cavities as in the SIS18.

After the injections at $t = 5$ s, when the detuning is maximal, the amplitude of the voltage is abruptly reduced to show the coupling of the amplitude loop.

Table 1: Summary of Parameters

Name	Symbol	Value	Unit
Synchrotron			
Harmonic Number	h	10	
RF-Frequency	ω	$2\pi \cdot 2.4$	MH
Transition Energy	γ_t	16.8	
Bending Radius		50	m
Circumference		1083.6	m
Duty Cycle SIS18		1	s
Cavity			
Resistance	R	5	k Ω
Capacitance	C	740	pF
(Half) Bandwidth	σ	135	kHz
Quality Factor	Q	56	
Voltage Amplitude	\hat{V}	1.88	kV
Actuator (Bias Current)			
Response Time	f_a^{-1}	1	ms
Gain	C_a	$5 \cdot 10^{13}$	
Detectors (Bessel Filter)			
Cutoff Frequency	ω_d	100	kHz
Beam			
Uranium 28+			
Bunch Length	σ_t	0.27	μ s
	σ_φ	119	$^\circ$
Energy per Nucleon	W_{kin}	1	GeV/u
Synchrotron Frequency	f_{syn}	56.5	Hz

MIMO Optimal Control

The best results were obtained with the MIMO Optimal control shown in Figure 3 (left).

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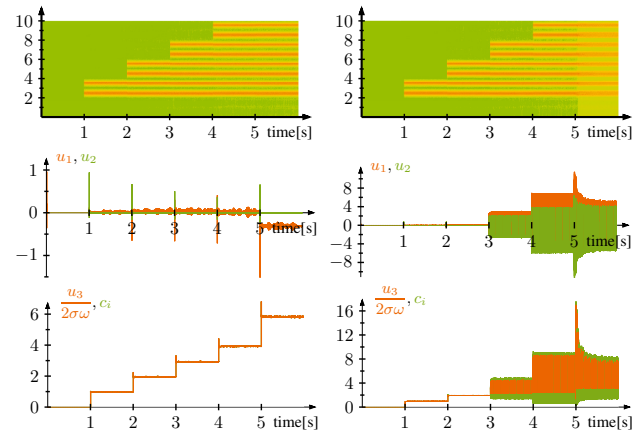


Figure 3: Waterfall plot of the beam, controller inputs u_1 and u_2 and detuning $u_3/2/\sigma/\omega$ with estimated c_i for the MIMO control (left) and the observer-based SISO PI-controllers (right).

Extended Kalman Filter

The design of a Kalman observer may need some successive optimization, but its realization is quite safe, because it could be (extensively) tested in parallel to a running closed-loop system. Its use may also provide additional insight into the dynamical processes.

SISO PI-Controllers

With observer at strong detuning the bias loop starts to oscillate at a high frequency, but with appropriate mean value. Nevertheless the performance of the amplitude is quite bad, when the cavity is detuned.

Without observer the optimal gains of the bias control could not be determined analytically. Therefore different pole configurations were tested. The best results were obtained for the poles marked in Figure 2. The beam could be stabilized up to a total of less than $1 \cdot 10^{12}$ particles. Then the bias loop became exponentially instable. Lower controller gains postpone this problem at the expense of the beam quality.

OUTLOOK

A dead time of about 1μ s is included in the simulations but may also be included in the design procedure. An analytical roof of the optimality and an explicit expression for the controller gain \mathbf{K} for any or almost any cost function would be nice.

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