

# ON THE IMPACT OF EMPTY BUCKETS ON THE FERRITE CAVITY CONTROL LOOP DYNAMICS IN HIGH INTENSITY HADRON SYNCHROTRONS \*

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## Abstract

Due to technical reasons two of ten buckets have to stay empty in the planned SIS100 synchrotron at the GSI Helmholtzzentrum für Schwerionenforschung. The planned low level RF control systems consist of linear P and PI type controllers. These are responsible to maintain a desired phase and amplitude of the gap voltage. In addition the cavity is controlled to follow a prescribed resonance frequency ramp. In SIS100 the acceleration will be performed by ferrite cavities with comparatively small quality factors. Therefore, effects resulting from transient beam loading have to be expected. Influences due to empty buckets are analysed in the frequency domain and particle tracking simulations are carried out to estimate the effect on the overall system with particular consideration of emittance growth and particle loss.

## INTRODUCTION

Beam-loading-effects may have a serious impact on the beam quality in high intensity synchrotrons. The new heavy ion synchrotron SIS100 as part of FAIR at GSI will have to deal with significantly higher beam currents than in the past. In order to guarantee a satisfying beam quality and to prevent emittance growth, detailed simulation studies are necessary to evaluate the effect on the cavity and its low level control loops. At SIS100 this problem will be even more important as two adjacent buckets, out of ten, have to stay empty due to technical reasons, so that transient effects have to be expected. Especially these induced parasitic frequencies are an open topic up to now and the planned control architecture has to be validated with respect to this issue. As the resonance frequency of a cavity has to be tunable over a wide range and classical resonator cavities would be too big, ferrite cavities are chosen to provide the accelerating voltage. The bandwidth of these cavities is not sufficiently narrow and therefore results from [1], dealing with similar problems, are not applicable.

## CAVITY MODEL AND CONTROL LOOPS

It has been shown that near operating point conditions the ferrite cavity dynamics may be well modeled as a lumped

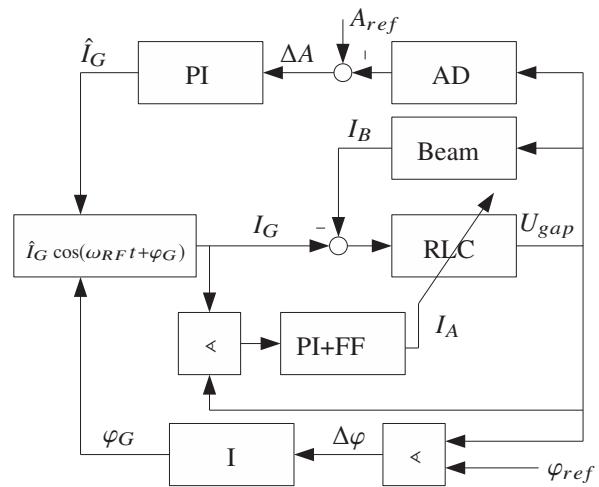


Figure 1: Block diagram of cavity and beam dynamics with control loops.

parallel RLC circuit with the differential equation [2]:

$$\ddot{U}_{gap} + \frac{1}{R_p C_p} \dot{U}_{gap} + \frac{1}{L_p C_p} U_{gap} = \frac{1}{C_p} \frac{d}{dt} (I_G - I_B),$$

with  $U_{gap}$  as the gap voltage,  $I_G$  being the generator current and  $I_B$  the beam current. While the capacitance is nearly constant over all operating conditions with  $C_p = 740 \text{ pF}$ , the inductance  $L_p$  and the resistance  $R_p$  show a time variant behaviour.  $L_p$  can be adjusted by applying a magnetizing bias current to the coils of the cavity. Experiments show a nearly quadratic dependence of the resonance frequency on the current in vicinity of the relevant operating conditions [3]. The corresponding regression curve can be stated as:

$$\left( \frac{\omega_0}{\text{Mrad s}^{-1}} \right)^2 \approx 2 \left( \frac{I_A}{\text{A}} \right) - 7.5.$$

and the corresponding inductivity follows to be:

$$L_p = \frac{1}{\omega_0^2 C_p} \approx \frac{1 \times 10^{-12} \text{ A s}^2}{(2I_A - 7.5 \text{ A}) C_p}$$

The resistance  $R_p$  shows distinct nonlinear dependencies on the frequency and the amplitude of the gap voltage  $U_{gap}$  and is modeled as in [4], but limited to the interval

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$[2k\Omega, 3k\Omega]$  to accord with [5]. The quality factor  $Q$  of the cavity can then be expressed by:

$$Q = R_p \sqrt{\frac{C_p}{L_p}} = R_p C_p \omega_0. \quad (1)$$

The planned low level RF control loops consist of an amplitude, phase and resonance control loop as depicted in Fig. 1, while the driving current of the generator is given by:

$$I_G = \hat{I}_G \cos(\omega_{RF} t + \varphi_G).$$

The amplitude control is a linear PI-type controller which acts on the generator current by adjusting  $\hat{I}_G$ . Its input is the output of the amplitude detector which is modeled by:

$$A(t) = \sqrt{2 \cdot \text{LPF}(U_{\text{meas}}^2(t))}.$$

A second order Bessel filter is used as a low pass filter (LPF). The phase detector is modeled as an ideal I/Q- detector and supplies phase measurements in the range of  $[-\pi, \pi)$  to the phase control and the resonance control loop. The phase control loop is responsible to track the desired phase of the reference voltage and is implemented as P-type frequency correction which therefore possesses inherent integral type behavior:

$$\varphi_G(s) = K_S \frac{1}{s} \Delta\varphi(s),$$

where  $\Delta\varphi$  is the phase discrepancy between the gap voltage and the reference voltage. Finally the resonance control loop has to adjust the resonance frequency of the cavity by applying the bias current. The input to the controller is the phase difference between the generator current  $I_G$  and the gap voltage  $U_{gap}$  as the cavity is operated in resonance under low beam current conditions when this phase shift vanishes. Again a PI controller is chosen, guaranteeing steady state accuracy. Additionally a feed-forward-path is added which non-linearly maps the desired resonance frequency to the desired bias current.

## ANALYSIS OF AN $^{238}\text{U}^{28+}$ CYCLE

The intended  $^{238}\text{U}^{28+}$  extremal cycle will only fill eight out of ten buckets and will possess significant beam current. Therefore this case is of particular interest for validating the planned control architecture. In order to analyse the frequency components of the beam current of a bunch train (BT) with empty buckets we can approximate a single bunch (SB) by a Gaussian distribution

$$I_B^{SB}(t) = \hat{I}_B \exp\left(\frac{-t^2}{2\sigma^2}\right),$$

with typical values of  $\sigma$  lying in the corresponding interval of 40 to 60 degrees. Its Fourier transform is again a normal distribution given by:

$$I_B^{SB}(\omega) = \hat{I}_B \sigma \sqrt{2\pi} \exp\left(\frac{-(\omega\sigma)^2}{2}\right).$$

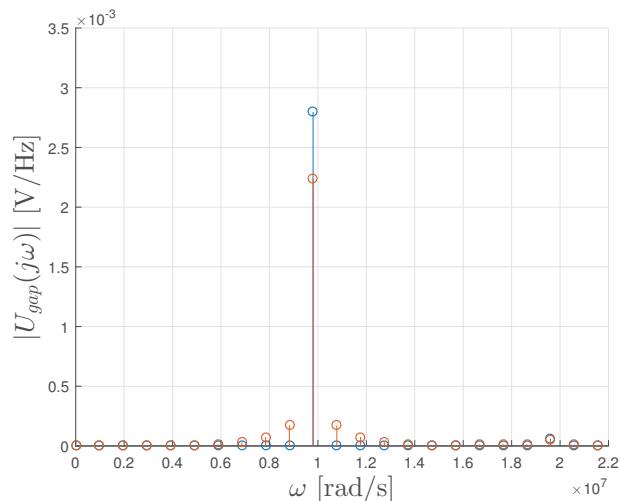


Figure 2: Magnitude spectrum response of the cavity to a beam current with unity amplitude: Ten filled buckets (blue), eight out of ten filled buckets (red).

The frequency distribution of a single bunch train can be obtained by delaying and adding the single bunch components:

$$I_B^{BT}(\omega) = I_B^{SB}(\omega) \sum_{k=1}^h \epsilon_k \exp(-j\omega k T_{RF}).$$

Herein  $h$  is the harmonic number,  $T_{RF}$  is the RF periodic time and  $\epsilon_k$  is either one or zero, depending on whether bucket number  $k$  is filled or empty. Finally the frequency components of the stationary orbiting bunch train are obtained by sampling of the above expression, as periodical continuation in the time domain is equivalent to sampling in the frequency domain with Dirac pulses:

$$I_B(\omega) = I_B^{BT}(\omega) \sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_R), \quad \omega_R = \frac{2\pi}{T_R},$$

with  $T_R$  being the revolution period time. We are particularly interested in the effect of the beam current on the gap voltage of the cavity. For the present  $^{238}\text{U}^{28+}$  case the smallest quality factor during injection follows to be  $Q = 14.5$ , from Eq. (1). Anticipating a cavity which is tuned to the resonance frequency, the beam current can be transformed by the transfer function

$$G_{cav}(s) = \frac{s/C_p}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2},$$

to obtain the induced gap voltage. Figure 2 shows in comparison the magnitude spectrum of the gap voltage which is induced by a beam current with unity peak ( $\hat{I}_B = 1$ ) at injection conditions, once in the case that all buckets are filled and in the case that only the first eight buckets are filled. Apparently higher harmonics are effectively damped in both cases. If no dipole oscillations are present, it can

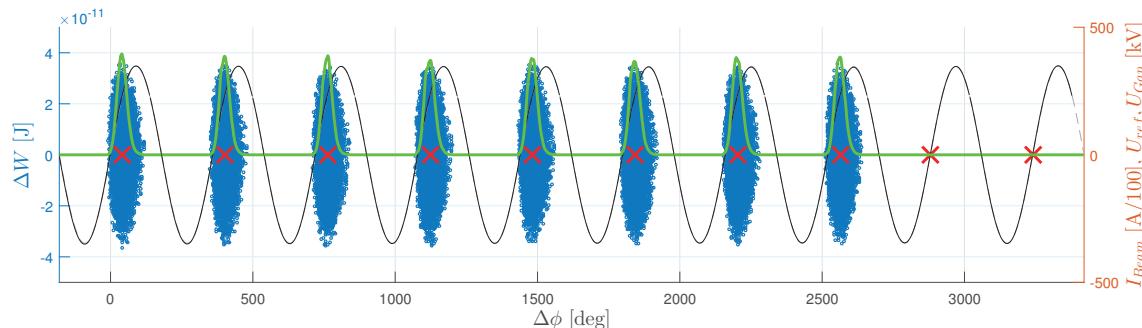


Figure 3: Complete bunch train during acceleration: particles (blue), beam current (green), gap voltage (black) and center of gravity (red).

be seen that no sidebands occur, if all buckets are filled. However the empty buckets induce sidebands which exist independently of beam dipole oscillations. Even though the sidebands are not symmetric we can estimate their effects by analyzing the resulting interference pattern. Thus the resulting additional disturbance is an amplitude modulated signal with the carrier frequency  $f_{RF} = 1/T_{RF}$  and the beat frequency  $f_R = 1/T_R$ , with  $f_{RF} = 10f_R$ :

$$U_d(t) = \hat{U}_d \cos(\omega_R t + \varphi_1) \cos(\omega_{RF} t + \varphi_2),$$

where the amplitude follows from the Fourier spectrum to be  $\hat{U}_d = 2/T_R |U_{gap}(j(\omega_0 + \omega_R))| \approx 54\text{V}$  for  $\hat{I}_B = 1\text{A}$ . Therefore one has to expect different influences of this disturbance on the single bunches of a bunch train. While the amplitude and phase control loops are designed to compensate the influence of the first harmonic over several RF periods, they are not able to eliminate the influence of this disturbance as it is beyond their bandwidth. However the amplitude is comparatively small and thus the influence should only be remarkable during low gap voltages, for example during injection.

Up to now no analytic stability criteria for the overall closed loop system is known. Therefore nonlinear macro-particle tracking simulations of the acceleration cycle have been carried out for the planned cycle parameters shown in Table 1. The buckets are injected in groups of two, letting the last two buckets empty. In contrast to the real system only one cavity is simulated and its gap voltage is multiplied by the total number of cavities, in order to obtain realistic relative beam loading ratios.

The shape of the complete bunch train during acceleration, near the highest gap voltage, is shown in Fig. 3. The peak current reaches about 4 A, while the RF voltage is still following its reference. The emittance growth depends on the bunch position. While the first bunch grows by 2.1% the sixth bunch shows an emittance growth of 8.4%. The objective of the simulation is to serve as a proof of principle for the closed loop control systems planned for SIS100. There is still the possibility to improve the beam quality, for example by temporarily de-activating individual cavities. Particle losses are hardly noticeable and will be dominated by

Table 1: Main Cycle Values, extract from [6]

Parameter	Value	Dimension
ion mass	238.05078	amu
number of ions	$5 \times 10^{11}$	
injection energy	197.57	MeV/u
extraction energy	2700	MeV/u
RF frequency	1.56 - 2.67	MHz
gap voltage (max)	372.53	kV
synchronous phase (max)	59.28	deg
ramping rate (max)	4	T/s
momentum compaction	0.005	

other effects. Therefore the proposed control architecture is suitable to deal with the expected beam loading.

## CONCLUSION

The influence of empty buckets has been analyzed in the frequency domain. It has been shown that the sidebands lead to beats which affect the single bunches differently. The results obtained in the overall simulation show that the planned low level RF systems are still able to deal with the beam loading effects during the  $^{238}\text{U}^{28+}$  scenario.

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