



# Learning attribute grammars for movement primitive sequencing

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## Abstract

*Movement primitives are a well studied and widely applied concept in modern robotics. However, composing primitives out of an existing library has shown to be a challenging problem. We propose the use of probabilistic context-free grammars to sequence a series of primitives to generate complex robot policies from a given library of primitives. The rule-based nature of formal grammars allows an intuitive encoding of hierarchically structured tasks. This hierarchical concept strongly connects with the way robot policies can be learned, organized, and re-used. However, the induction of context-free grammars has proven to be a complicated and yet unsolved challenge. We exploit the physical nature of robot movement primitives to restrict and efficiently search the grammar space. The grammar is learned by applying a Markov chain Monte Carlo optimization over the posteriors of the grammars given the observations. The proposal distribution is defined as a mixture over the probabilities of the operators connecting the search space. Moreover, we present an approach for the categorization of probabilistic movement primitives and discuss how the connectivity of two primitives can be determined. These characteristics in combination with restrictions to the operators guarantee continuous sequences while reducing the grammar space. In addition, a set of attributes and conditions is introduced that augments probabilistic context-free grammars in order to solve primitive sequencing tasks with the capability to adapt single primitives within the sequence. The method was validated on tasks that require the generation of complex sequences consisting of simple movement primitives using a seven-degree-of-freedom lightweight robotic arm.*

## Keywords

Movement primitives, movement primitive sequencing, probabilistic context-free grammar, attribute grammar, grammar induction, human-robot interaction

## 1. Introduction

Movement primitives (MPs) are a well-established concept in robotics. MPs are used to represent atomic, simple movements and are, therefore, appropriate for tasks consisting of a single stroke-based or rhythmic movement (Paraschos et al., 2018). They have been used in a large variety of applications, e.g., table tennis (Muelling et al., 2013), pancake flipping (Kormushev et al., 2010), and hockey (Paraschos et al., 2018). However, for more complex tasks, a single MP is often not sufficient. Such tasks require sequences of MPs for feasible solutions. Considering a set or library of MPs, such sequences can be generated in a variety of ways, including hidden Markov models (HMMs) (Kulic et al., 2012), mixture models (Lioutikov et al., 2017), and other hierarchical approaches (Stulp and Schaal, 2011). These approaches can be regarded as mechanisms that produce sequences of MPs, revealing a common, important downside: understanding these mechanisms

requires a significant amount of expert knowledge. However, a declared goal of robotics is the deployment of robots into scenarios where direct or indirect interactions with non-expert users are required. Therefore, more intuitive sequencing mechanisms for non-experts are necessary.

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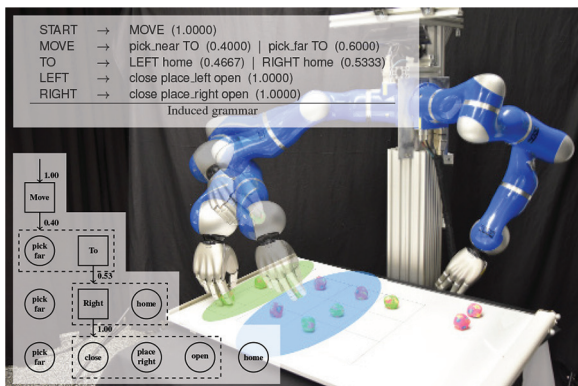
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This paper introduces the use of formal grammars for the sequencing of MPs. In particular, we focus on probabilistic context-free grammars (PCFGs) and propose a method to induce PCFGs from observed sequences of primitives. Formal grammars represent a formal description of symbols and rules, encoding the structure of a corresponding language. They have been studied extensively in both natural language processing and compiler construction, but have also been applied in a variety of fields such as molecular biology (Chiang et al., 2006), bioinformatics (Rivas and Eddy, 2000), computer vision (Kitani et al., 2005; Zhu and Mumford, 2007), and robotics (Dantam and Stilman, 2013; Lee et al., 2013; Sarabia et al., 2015). The choice of learning a probabilistic representation over a single deterministic plan is based on the insight that probabilistic representations of behavior are generally more robust to changes than deterministic representations, especially in dynamic environments. For instance, in a collaborative task, a fixed plan describing the behavior between a robot and a user would require the user to always behave according to the set plan. A distribution over plans allows for at least some flexibility as long as the plan is still within the distribution. Furthermore, PCFGs allow the implicit embedding of hierarchies within the rules of the grammar associating every produced sequence with at least one corresponding parse tree. Such a parse tree represents the derivation of the produced sequence in an intuitive way. Figure 1 shows a learned grammar for placing a stone in a game of tic-tac-toe, including the parse tree for a produced primitive sequence.

The understandability of the grammar itself is highly dependent on the size and structure of the grammar. The induction of concise but expressive grammars is considered non-trivial and, in the context of natural language, even an ill-posed problem. A common approach to grammar induction is to formulate the problem as a search problem where each possible grammar is a node in the search space and a set of operators generate the edges between those nodes. This search space can then be traversed through different search methods where a scoring function determines the



**Fig. 1.** The robot executes a turn in the tic-tac-toe game, represented as a sequence of MPs. The sequence was generated by a PCFG learned from previously labeled observations.

quality of each grammar. Stolcke (1994) suggested formulating the problem as a maximum *a posteriori* estimation where the scoring is defined as the posterior given the observations. In order to reduce the possibility of getting stuck in bad local optima, the search space was traversed via beam search. In this work we formulate the search as a Markov chain Monte Carlo (MCMC) optimization similarly to Talton et al. (2012), where the scores were defined as posteriors over the grammars given the observations.

To ease the learning of grammars, the proposed approach exploits the structure inherently present in the physical motions. We assume each segment of the observed sequences to be a sample from an underlying library of MPs (e.g., Lioutikov et al., 2017; Niekum et al., 2015).

Owing to the considerably smaller size of a primitive library compared with the corpus of a natural language, the observed sequences of even complex tasks show a simpler structure than a sentence of a natural language. Furthermore, the category of a MP, e.g., hand or arm movement, can be more easily deduced than the category of a word, e.g., verb or noun. We discuss how to determine the category of a MP later in this paper.

An important restriction that improves the induction of grammars for movements is that any produced sequence has to result in a continuous trajectory inside the state space. Therefore, any grammar that would produce a jump in the state space is invalid and has to be removed from consideration. In this work, we avoid such grammars directly by restricting the operators to exclusively produce valid grammars, by ensuring the connectivity between two consecutive MPs.

The contributions of this work are the induction of PCFGs for the sequencing of MPs. The posteriors are computed using a novel prior distribution that avoids many disadvantages of existing methods based on minimum description length and Dirichlet distributions. The search is formulated as a MCMC optimization where the proposed distributions are defined through restrictions placed upon the operators connecting the grammar search space. These restrictions include physical constraints presented in the domain of movements. This paper extends the work presented in Lioutikov et al. (2018) with details on the categorization and the assessment of connectivity of MPs. In addition, we enhance the induced grammars with attributes and an evaluation scheme for MP sequencing tasks. The presented method is evaluated on a tic-tac-toe task, where a grammar is induced that sequences primitives in order to pick up a stone and place it on a tic-tac-toe playing field. In addition, we evaluate the method on a collaborative chair assembly task, where the robot induced a grammar describing a sequence of required hand-over primitives.

## 2. Related work

MPs are usually used to solve tasks consisting of single, atomic stroke-based or periodic movements (Paraschos

et al., 2018). For more complex tasks, however, a sequence of primitives has to be applied. An example of such a task is the grasping, positioning, and cutting of a vegetable (Lioutikov et al., 2014) with dynamical movement primitives (DMPs) (Ijspeert et al., 2013). However, in the approach of Lioutikov et al. (2014) the sequences were not learned, but predefined. An approach combining the segmentation of observations and the learning of a sequencing mechanism was presented by Kulic et al. (2012). The primitives are encoded using HMMs and a graph structure is learned during the segmentation. This graph can be used subsequently to sequence the primitives. Another approach featuring a sequence graph was presented by Manschitz et al. (2014). The graph was learned from demonstrations through an agglomerative clustering scheme. In this work, we propose PCFGs as a means of sequencing MPs. Grammars bring the advantage of being a general method capable of representing hierarchies in a principled and intuitive manner.

Motion grammars (Dantam and Stilman, 2013) are extensions of context-free grammars (CFGs) modeling the discrete and continuous dynamics of hybrid systems (Dantam and Stilman, 2012). Motion grammars aim at fast task verification and have not yet been induced from observations. Dantam et al. (2011) and Sarabia et al. (2015) described the benefits of applying CFGs in human–robot interaction scenarios such as playing chess or making music collaboratively. Lee et al. (2012, 2013) used PCFGs to sequence discrete actions. Analogously to Stolcke (1994), the grammar was learned by applying a beam search for the maximal posterior inside the grammar space. The grammar space was traversed by applying the merge and chunk operators (Stolcke, 1994) to observed sequences. In contrast to the approach of Stolcke (1994), a  $n$ -gram-like frequency table was used to determine reoccurring patterns in the observations, hence, identifying candidate productions for the chunk operator. To avoid unintuitive, compact grammars, the prior definition, originally defined solely by the minimal description length, was extended by a log-Poisson term similar to the approach by Kitani et al. (2008). The meaning of the operators shall become clear later in this article.

Although we share the motivation of learning intuitive, PCFGs for primitive sequencing, our work differs from Lee et al. (2012) in several ways. We use a stochastic MP representation and actively take advantage of its properties to induce the grammar. We deviate from the common structure prior definition as an exponential distribution over the minimal description length and define the entire prior as a combination of several Poisson distributions. Furthermore, we use MCMC optimization to find the grammar maximizing the posterior, similarly to Talton et al. (2012), which is more robust to local optima than beam-search.

The grammar induction approach described in Talton et al. (2012) uses the Metropolis–Hastings algorithm (Andrieu et al., 2003) to learn grammars describing designs in various domains, such as websites and geometric

models. The prior is defined using the description length and the grammar learning is not used in any robotics context. In addition, the structure of the observed sequences differs significantly from our problem setting. In the approach of Talton et al. (2012), the observations and, hence, the starting points of the grammar induction are already hierarchical structures. Therefore, it is sufficient to traverse the grammar space using solely the merge and split operators. These operators allow the generalization and specialization of grammars, but are not able to introduce new hierarchies like the operator (Stolcke, 1994). In this work, we apply all three operators. To achieve the required irreducibility of the Markov chain, we additionally introduce the insert operator, negating the effects of the chunk operator.

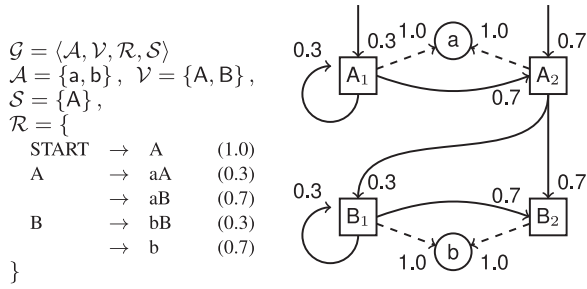
### 3. Background

Before presenting the induction of MP grammars, this section briefly introduces the general concepts of formal grammars and MPs.

#### 3.1. Formal grammars

A formal grammar is a description of a formal language in terms of symbols and production rules. The symbols,  $(\mathcal{A} \cup \mathcal{V})$ , are commonly separated into two disjoint sets called terminals  $\mathcal{A}$  and non-terminals  $\mathcal{V}$ , with the convention that terminals represent the atomic elements of the language, while non-terminals can be substituted with sequences of symbols. Each production rule  $r_\beta \in \mathcal{R}_\alpha$  substitutes the symbol sequence  $\alpha \in (\mathcal{A} \cup \mathcal{V})^+$  with  $\beta \in (\mathcal{A} \cup \mathcal{V})^+$ . A rule is commonly denoted as  $\alpha \rightarrow \beta$ , where  $\alpha$  and  $\beta$  are referred to as the left- and right-hand side, respectively. With these definitions, a grammar can be described as a four-tuple  $\mathcal{G} = \langle \mathcal{A}, \mathcal{V}, \mathcal{R}, \mathcal{S} \rangle$ , where  $\mathcal{R}$  denotes the set of all  $\mathcal{R}_\alpha$  and  $\mathcal{S}$  is the set of all starting symbols  $\mathcal{S} \subseteq (\mathcal{A} \cup \mathcal{V})^+$ . A grammar can contain multiple rules for the same left-hand side, i.e.,  $|\mathcal{R}_\alpha| > 1$ , resulting in a non-deterministic grammar. Weighting each  $\mathcal{R}_\alpha \in \mathcal{R}$  with a corresponding multinomial  $\boldsymbol{\rho}_\alpha \in \Delta^{|\mathcal{R}_\alpha|-1}$ , leads to a stochastic or probabilistic grammar. Furthermore, grammars can be recursive, i.e., a series of productions starting with a rule in  $\mathcal{R}_\alpha$  results in a sequence containing  $\alpha$ . The non-deterministic and recursive properties allow grammars to represent complex, hierarchical relations between symbols in relatively simply structured production rules.

Formal grammars are commonly classified via the Chomsky hierarchy. The most constrained and, therefore, least-expressive grammars in this hierarchy are the so-called regular grammars. Languages described by these grammars are known as regular expressions, e.g., the expression  $a^+b^+$  represents all sentences that consist of one or more  $a$  followed by one or more  $b$ . Probabilistic regular grammars are equivalent to HMMs. Similar to instantiated HMMs formal grammars explain observed sequences in so-called parse trees. Figure 2 shows a regular grammar

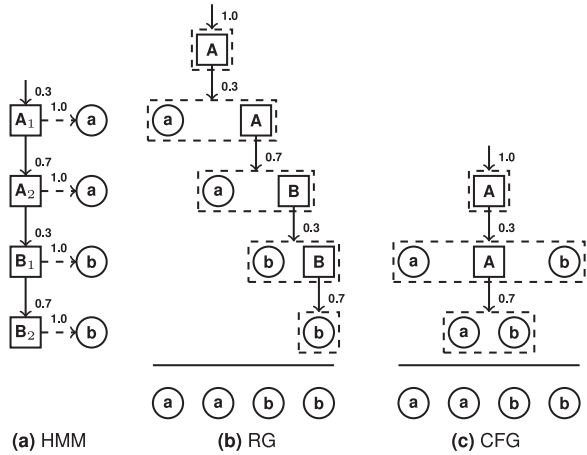


**Fig. 2. Left:** A regular grammar describing the language  $a^+b^+$ . The corresponding parse tree for the sequence  $aabb$  is shown in Figure 3b. **Right:** A HMM that is equivalent to the regular grammar. Squares describe hidden states and circles describe the emissions. Figure 3a shows an instantiated HMM for the sequence  $aabb$ .

**Table 1.** A CFG describing the language  $a^n b^n$ , i.e., the language of all sentences consisting of a number of a followed by the exact same number of b.

$\mathcal{G} = \langle \mathcal{A}, \mathcal{V}, \mathcal{R}, \mathcal{S} \rangle$	
$\mathcal{A} = \{a, b\}, \mathcal{V} = \{A\}, \mathcal{S} = \{A\}$	
$\mathcal{R} = \{$	
START	$\rightarrow A \quad (1.0)$
A	$\rightarrow ab \quad (0.7)$
	$\rightarrow aAb \quad (0.3)$
$\}$	

grammar. Furthermore, every infinite language satisfying the pumping lemma is also regular, as for instance the described language  $a^+b^+$ .



**Fig. 3.** Parse trees for the sequence  $aabb$  of (a) and (b) the HMM and the regular grammar shown in Figure 2. (c) Corresponding parse tree for the CFG shown in Table 1.

describing the expression  $a^+b^+$  as well as a corresponding HMM, while Figure 3 shows (a) an instantiated HMM and (b) a parse tree for the sequence  $aabb$  of the HMM and the regular grammar, respectively. The left-hand side of each rule is a single non-terminal and the right-hand side can be either a terminal or a terminal followed by a non-terminal. In addition, each production of a single rule is weighted by a probability, e.g., the non-terminal  $A$  produces the sequences  $aA$  and  $aB$  with a probability of 0.3 and 0.7, respectively. While the parse tree and the graph for the time series look fairly similar the description of the higher-level policy as a grammar is more intuitive to understand than the graphical model and is, therefore, better suited for non-expert users. However, given that HMMs are an extensively studied tool for the learning and analysis of time series, they are a much more common choice for the sequencing of discrete actions, such as MPs, than regular grammars. Despite their simplicity regular grammars can describe complex languages. In fact, every finite language is regular and can, therefore, be described by a regular

**3.1.1. PCFGs.** PCFGs are able to describe infinite languages that cannot be described by regular grammars or, therefore, by HMMs. An example for such a language is  $a^n b^n$ , which contains any sequence with  $n$  a that are always followed by the same number of b. Regular grammars do not contain any mechanism to keep track of the number of produced a. However, such languages can be described by CFGs such as that shown in Table 1. CFGs still have only a single non-terminal on the left-hand side, but can now contain an arbitrary sequence of terminals and non-terminals of the right-hand side. Despite the more complex language, the CFG is at least as intuitive as the previous regular grammar. Figure 3c shows the corresponding parse tree for the sequence  $aabb$ . The probabilistic extension of a CFG, as in the given example, is a so-called probabilistic or stochastic context-free grammar (PCFG). The non-terminal on the left hand side,  $A$ , can produce the sequences  $ab$  and  $aAb$  on the right-hand side with a probability of 0.7 and 0.3 respectively.

**3.1.2. Attribute grammars.** Attribute grammars are an enhancement of CFGs, where each terminal and non-terminal can be assigned multiple inherited or synthesized attributes. An inherited attribute belongs to a symbol on the right-hand side of a rule that obtains its value from attributes of the non-terminal of the left-hand side or other symbols on the right-hand side. A synthesized attribute is an attribute of the non-terminal on the left-hand side of a rule whose value is computed using attributes of the right-hand side symbols. The above example for instance can be transformed into an attribute grammar containing the attributes `depth` and `max_depth`.

The indices of  $A_1$  and  $A_2$  simply distinguish between the same non-terminal within a single rule. The synthesized attribute `depth` evaluates the number of recursions that occurred during the production of a sentence, while the inherited attribute `max_depth` defines how many recursions are at most supposed to occur. The latter is achieved by defining the condition that the second rule is only chosen if  $A_1.max\_depth > 0$ , resulting in sentences with at most

max\_depth number of a and b. Such conditions extend the expressiveness of attribute grammars beyond that of CFGs. For instance, the language  $a^n b^n c^n$  cannot be represented by CFGs. This language requires a context-sensitive grammar that encodes pre- and post-conditions within the context symbols. Alternatively, a simple counting attribute that keeps track of how many  $c$  have been produced can be added to a PCFG.

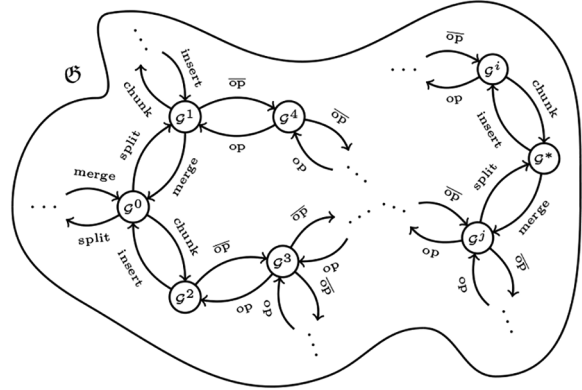
START	→	A	(1.00)
A <sub>1</sub>	→	ab	(0.70)
	→	A <sub>1</sub> .depth = 1	
	→	aA <sub>2</sub> b	(0.30)
		A <sub>1</sub> .depth = A <sub>2</sub> .depth + 1	
		A <sub>2</sub> .max_depth = A <sub>1</sub> .max_depth - 1	
		assert : A <sub>1</sub> .max_depth > 0	

**3.1.3. Grammar inductions.** Grammar inductions refers to the learning of formal grammars from sequences of terminals. Commonly the task is formulated as a search through a grammar space  $\mathfrak{G}$ , where the connections between grammars are represented as different operators, as illustrated in Figure 4. Such operators manipulate the set of production rules  $\mathcal{R}$  and the set of non-terminals  $\mathcal{V}$  accordingly. Starting from an initial grammar  $\mathcal{G}^0$  these operators are used to traverse the grammar space, searching for the optimal grammar  $\mathcal{G}^*$ . Various search strategies have been suggested, e.g., beam search (Lee et al., 2013; Stolcke, 1994) and MCMC optimization (Talton et al., 2012).

When searching for concise yet general grammars, a common problem is the induction of overly general grammars. For instance, the language  $a^*b^*$  is a superset of the language  $a^n b^n$ . Hence, a grammar representing  $a^*b^*$  can explain every sequence in an observed data set produced by the language  $a^n b^n$ . Therefore, the corresponding grammar is a valid entry in the grammar space. This general limitation is known as Gold’s law and states that a correct grammar cannot be learned from positive demonstrations alone (Gold, 1967). However, the induction method proposed in this work decreases the chances of inducing an overly general grammar by defining a posterior distribution that takes the observed data more strongly into account than other related methods.

### 3.2. MP representation

A MP encapsulates a movement or action as a discrete entity. Although simple point attractors are also sometimes referred to as MPs, MPs usually represent the shape of the movement in addition to a start and goal position. Furthermore, MPs are commonly parameterized, allowing for the modification of a MP originally learned from demonstrations. Two examples of parameterized MPs are



**Fig. 4.** The grammar space  $\mathfrak{G}$  contains all valid grammars  $\mathcal{G}^0 \dots \mathcal{G}^*$ . The space is traversed by applying operators  $op \in O = [\text{merge}, \text{split}, \text{chunk}, \text{insert}]$  on the current grammar. For every operator  $op$  generating  $\mathcal{G}'$  from  $\mathcal{G}$ , there exists an  $\overline{op}$  that generates  $\mathcal{G}$  from  $\mathcal{G}'$ , e.g.,  $\overline{\text{merge}} = \text{split}$ ,  $\overline{\text{chunk}} = \text{insert}$ .

the well-known DMPs (Ijspeert et al., 2013) and the more recent probabilistic movement primitives (ProMPs) (Paraschos et al., 2018).

In this paper, we choose ProMPs as primitive representation (Paraschos et al., 2018).<sup>1</sup> Each observed trajectory  $\tau$  is assumed to be sampled from the conditional distribution

$$p(\tau|\mathbf{w}) = \prod_t \mathcal{N}(\tau_t | \Phi_t \mathbf{w}, \Sigma_{\text{obs}}), \quad (1)$$

with  $\Sigma_{\text{obs}}$  being the observation noise. The feature matrix  $\Phi_t$  projects the trajectory  $\tau$  onto a lower-dimensional weight vector  $\mathbf{w}$  for every time step  $t$  as defined in Paraschos et al. (2018). The features  $\Phi_t$  are usually represented as radial basis functions for stroke-like movements and von Mises functions for rhythmic movements. The weight  $\mathbf{w}$  can be learned from the observed demonstration  $\tau$  by applying a maximum a posteriori optimization

$$\mathbf{w} = \arg \max_{\mathbf{w}'} p(\tau|\mathbf{w}') p(\mathbf{w}'), \quad (2)$$

with  $p(\mathbf{w}')$  denoting a prior over the weights. Depending on the prior choice the optimization yields different types of regressions. For instance, choosing a standard normal prior  $p(\mathbf{w}') = \mathcal{N}(\mathbf{w}'|0, \mathbf{I})$  yields a common ridge regression.

The projection of the demonstrated trajectories into lower-dimensional spaces is a common property of MPs, e.g., in DMPs (Ijspeert et al., 2013). However, ProMPs additionally define a Gaussian distribution over the projected trajectories

$$\mathbf{w} \sim \mathcal{N}(\mathbf{w}|\boldsymbol{\theta}), \boldsymbol{\theta} = (\boldsymbol{\mu}_w, \boldsymbol{\Sigma}_w), \quad (3)$$

with mean  $\boldsymbol{\mu}_w$  and covariance matrix  $\boldsymbol{\Sigma}_w$ . Every primitive is characterized through its parameters  $\boldsymbol{\theta}$  and the

corresponding state space distribution is obtained by integrating out the weights

$$p(\boldsymbol{\tau}|\boldsymbol{\theta}) = \int_{\boldsymbol{w}} p(\boldsymbol{\tau}|\boldsymbol{w})\mathcal{N}(\boldsymbol{w}|\boldsymbol{\theta})d\boldsymbol{w}, \quad (4)$$

$$= \prod_i \mathcal{N}(\boldsymbol{\tau}|\boldsymbol{\Phi}_i\boldsymbol{\mu}_w, \boldsymbol{\Phi}_i\boldsymbol{\Sigma}_w\boldsymbol{\Phi}_i^T + \boldsymbol{\Sigma}_{\text{obs}}) \quad (5)$$

For simplicity, in the remainder of this article we refer to  $\boldsymbol{\theta}$  as the primitive itself instead of its parameters.

#### 4. Problem statement

Given a set of demonstrations  $\mathcal{D} = \{\boldsymbol{d}_1, \boldsymbol{d}_2, \dots, \boldsymbol{d}_{|\mathcal{D}|}\}$  a set of primitives  $\Theta = \{\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \dots, \boldsymbol{\theta}_{|\Theta|}\}$ , each demonstration represents a labeled sequence of primitives  $\boldsymbol{d}_i \in \Theta^+$ . The goal of this work is to learn an attribute grammar  $\mathcal{G}_{\text{att}}^*$  that is concise and expressive yet has an easily comprehensible structure. For instance, given a set of demonstrations of turns in a game of tic-tac-toe, the following grammar is a concise representation of the possible sequences.

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 $\mathcal{D} = \{$ 
  ( pick_far, close, place_right, open, home ),
  ( pick_near, close, place_right, open, home ),
  ⋮
  ( pick_far, close, place_left, open, home ),
  ( pick_near, close, place_left, open, home )
 $\}$ 

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START → MOVE (1.00)
      MOVE.stone = START.stone
      MOVE.field = START.field
MOVE → pick_near TO (0.40) | pick_far TO (0.60)
      pick_near.stone = MOVE.stone pick_far.stone = MOVE.stone
      TO.field = MOVE.field TO.field = MOVE.field
TO → LEFT home (0.47) | RIGHT home (0.53)
    LEFT.field = TO.field RIGHT.field = TO.field
LEFT → close place_left open (1.00)
      place_left.field = LEFT.field
RIGHT → close place_right open (1.00)
       place_right.field = RIGHT.field

```

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The learned grammar represents a generalized structure over the observed demonstrations and allows the sampling of new sequences while the attributes allow for the adaptation of individual primitives. For instance, the attribute stones contains the position of the stone that is supposed to be played next and the primitives, e.g., pick\_near and pick\_far can now be conditioned on the passed down position. In addition, attributes are introduced that ensure a smooth and continuous trajectory across each sampled primitive sequence despite the adaptation of individual primitives. The desired grammar is learned by inducing a PCFG  $\mathcal{G}^*$  from the demonstrations  $\mathcal{D}$  and enhancing it afterwards with a general attribute scheme for sequencing MPs

$\mathcal{G}^* \Rightarrow_{\text{att}} \mathcal{G}_{\text{att}}^*$ . The set of terminals is defined as the set of primitives  $\mathcal{A} = \Theta$  and during the learning of the grammar the terminals, and hence the primitives are considered immutable, implying that the search space consists of grammars that only differ in  $\mathcal{S}$ ,  $\mathcal{V}$  or  $\mathcal{R}$ . Each grammar represents a node in the grammar space  $\mathcal{G}$ , where the directed edges between nodes are defined by operators. Operators manipulate the rule set  $\mathcal{R}$  of a grammar  $\mathcal{G}$  and consequently create a new grammar  $\mathcal{G}'$  while the grammar space itself is explored via a MCMC optimization. In order to optimize for grammars with concise and comprehensible structures a novel prior based on Poisson distributions is introduced. The grammar induction and the prior are presented in later in the paper.

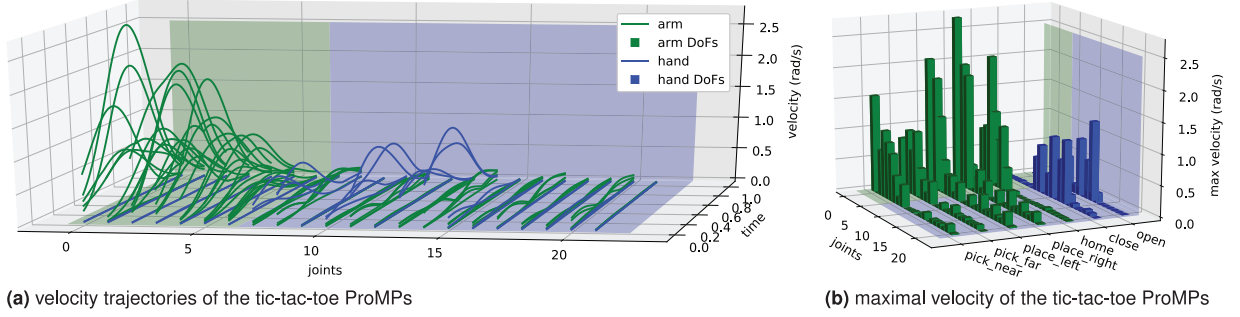
Given that the sequences produced by the grammar directly result in the movement of a robot, it is important that there are no state jumps at the transition between two consecutive primitives. Throughout this paper, this requirement is referred to as primitive connectivity. The connectivity of two primitives depends on the category of each primitive and the transition overlap between the primitives. The category of a primitive classifies which degrees of freedom are effectively controlled by the primitive. Primitives assigned to disjoint categories are not subject to the connectivity requirement. The transition overlap describes how much the end of one primitive and the beginning of the next primitive overlap. If the overlap is too small a smooth transition is unlikely and the connectivity requirement is violated. Next, we present a method for identifying the categories of primitive. Afterwards, we introduce an approach to compute the transition overlap between two subsequent ProMPs.

#### 5. Identifying the primitive category

Given a robotic platform with independent kinematic chains, e.g., an arm and a hand, each of these chains represents a category of movements. Such categories allow the relaxation of the connectivity requirement to hold only between primitives of the same category. Hence, the connectivity requirement of two subsequent primitives of the same category is independent of any primitive executed between them that does not belong to that category. Given that some primitives might contain significant movement across categories, we treat the identification of the primitive category as a simple multi-label classification problem. The classification is based on the degrees of freedom that are active during the movement and assigns each primitive  $\boldsymbol{\theta}$  to a category  $\mathcal{C}$ .

In this work, each primitive is represented as a single ProMP. The distribution over the trajectory  $\boldsymbol{\tau}$  given the parameters  $\boldsymbol{\theta}$  is defined in Equation (4). Assuming equidistant time steps, the distribution

$$p(\dot{\boldsymbol{\tau}}) = \prod_i \mathcal{N}(\dot{\boldsymbol{\tau}}|\dot{\boldsymbol{\Phi}}_i\boldsymbol{\mu}_w, \dot{\boldsymbol{\Phi}}_i\boldsymbol{\Sigma}_w\dot{\boldsymbol{\Phi}}_i^T + \boldsymbol{\Sigma}_{\text{obs}}), \quad (6)$$



**Fig. 5.** (a) Velocity trajectories of the tic-tac-toe ProMPs, showing the  $|\text{mean}| + 2\text{std}$  of the velocity distribution defined in Equation (6) for each ProMP. (b) Maximal velocity of the tic-tac-toe ProMPs, showing the maximal velocities as computed in Equation (7). The colors indicate the categories that were assigned to each ProMP and background colors highlight the category to which each joint belongs.

describes the velocity trajectories of the corresponding ProMP, with  $\dot{\Phi}_t$  being the first time derivative of the basis functions and  $\Sigma_{\text{obs}}$  denoting the observation noise with respect to the velocity trajectory. In order to identify the active degrees of freedom, we analyze the mean and standard deviation of the velocity distribution. In particular, we are interested in the maximal absolute velocity over the time steps and define the maximum velocity feature as

$$\psi^{\text{vel}}(\theta) = \max_t \left( |\dot{\Phi}_t \mu_w| + 2\sqrt{\text{diag}(\dot{\Phi}_t \Sigma_w \dot{\Phi}_t^T + \Sigma_{\text{obs}})} \right). \quad (7)$$

A primitive is considered active with respect to the category  $C_i$  if and only if any of the corresponding elements in  $\psi^{\text{vel}}(\theta)$  is above the threshold  $\epsilon_i$ , i.e.,

$$\theta \in C_i \Leftrightarrow \bigvee_{d \in \text{dof}(C_i)} \psi_d^{\text{vel}}(\theta) > \epsilon_i, \quad (8)$$

with  $\text{dof}(C_i)$  being the set of degrees of freedom associated with category  $C_i$  and  $\psi_d^{\text{vel}}(\theta)$  is the maximal velocity of the  $d$ th degree of freedom. The threshold  $\epsilon_i$  is chosen manually.

In the tic-tac-toe task, two different categories are distinguished, arm movements and hand movements. If two subsequent hand movements are connectible it does not matter how many arm movements are sequenced in between them. The connectibility requirement is still fulfilled. At the same time, hand movements can now be executed at arm configurations at which the hand movement has not been observed in the demonstrations. Figure 5 shows the velocity trajectories of the seven primitives used in the tic-tac-toe task where the colors of the trajectories indicate the identified category and the background color highlights the degrees of freedom associated with each category. Given the demonstrations, every primitive was assigned exactly one category, even though the described approach allows the association of multiple categories to a single primitive. While the given example could also have been solved by a simple annotation of the observed data, examples can be

thought of where the presented automated annotation is of great benefit. For instance, a significantly larger set of observed sequences or tasks that require multiple interacting kinematic chains, e.g., a bi-manual task.

## 6. Determining connectibility of primitives

In this section, we discuss how to automatically determine whether two ProMPs are connectible, that is, if two subsequent ProMPs would result in a jump in the state space or not. Given that ProMPs are a probabilistic trajectory representation it seems fitting to use probabilistic similarity measures, such as the Kullback–Leibler divergence or the Hellinger distance to test whether two ProMPs are connectible. However, considering that avoiding jumps in the state space is a spatial requirement probabilistic similarity measures can be misleading. We solve the connectibility problem in the spatial domain by treating each ProMP as a multidimensional tube. At every time step  $t$  a ProMP  $\theta_i$  can be approximated by a hyper ellipsoid

$$\mathcal{E}_t(\theta_i) = (c_t = \Phi_t \mu_w, S_t = \Phi_t \Sigma_w \Phi_t^T + \Sigma_{\text{obs}})$$

with center  $c_t$  and shape matrix  $S_t$ . Two ProMPs,  $\theta_i$  and  $\theta_j$ , are now considered connectible if the transition overlap between their corresponding tubes is larger than a predefined threshold

$$\theta_i \xrightarrow{\text{con}} \theta_j \Leftrightarrow \text{overlap}(\theta_i, \theta_j) \geq \epsilon_{\text{overlap}}.$$

The transition overlap from  $\theta_i$  to  $\theta_j$  is defined over the last ellipsoid of the preceding ProMP  $\mathcal{E}_T(\theta_i)$  and the first ellipsoid of the succeeding ProMP  $\mathcal{E}_1(\theta_{i+1})$

$$\text{overlap}(\theta_i, \theta_j) = \frac{\text{vol}(\mathcal{E}_T(\theta_i) \cap \mathcal{E}_1(\theta_j))}{\text{vol}(\mathcal{E}_T(\theta_i))},$$

with  $\text{vol}$  denoting the volume and  $\mathcal{E}_T(\theta_i)$  and  $\mathcal{E}_1(\theta_j)$  being the last ellipsoid of  $\theta_i$  and the first primitive of  $\theta_j$ , respectively. Hence, the transition overlap describes the

percentage of the end of  $\theta_i$  that is covered at the beginning of  $\theta_j$ .

Unfortunately computing the volume of the intersection between two hyper-ellipsoids is considered #P-complete (Bringmann and Friedrich 2010). We circumvent this problem by computing the overlap for each degree of freedom independently and choosing the minimal value as an approximation of the ellipsoidal overlap. As discussed in the previous section, the connectivity between two ProMPs is only considered if both ProMPs share at least one primitive category  $\mathcal{C}$

$$\text{overlap}(\theta_i, \theta_j) \approx \min_{d \in \text{dof}(\mathcal{C})} \frac{|\text{intersec}(\theta_i, \theta_j, d)|}{|c_{i,T,d} \pm n s_{i,T,d}|},$$

$$\text{intersec}(\theta_i, \theta_j, d) = c_{i,T,d} \pm n s_{i,T,d} \cap c_{j,1,d} \pm n s_{j,1,d},$$

where  $c_{i,T,d}$  and  $s_{i,T,d}$  are the  $d$ th element of the ellipsoid center and the  $d$ th standard deviation for primitive  $\theta_i$  at time step  $T$ . The constant  $n$  decides how many standard deviations wide the considered interval will be.

The threshold  $\epsilon_{\text{overlab}}$  can be defined either manually or derived from observations. Given that the learned grammar should at least be capable to reproduce the initially given observations all ProMPs that were pairwise connected in the observations have to be considered connectible. Therefore, the threshold

$$\epsilon_{\text{overlab}} = \alpha \min_{(\theta_i, \theta_j) \in \text{pairs}(\mathcal{D})} \text{overlap}(\theta_i, \theta_j),$$

is defined as a percentage  $\alpha$  of the minimal overlap value of all ProMPs that were connected in the observations. Here  $\text{pairs}(\mathcal{D})$  is a function that returns all consecutive primitive pairs in the set of demonstrations  $\mathcal{D}$ . We can now define two sets for each primitive. One set contains all primitives it is connectible to  $\overrightarrow{\text{Con}}(\theta)$  and the other set contains all primitives it is connectible from  $\overleftarrow{\text{Con}}(\theta)$

$$\overrightarrow{\text{Con}}(\theta_i) = \{\theta_j | \theta_j \in \Theta \wedge \theta_i \overrightarrow{\text{con}} \theta_j\},$$

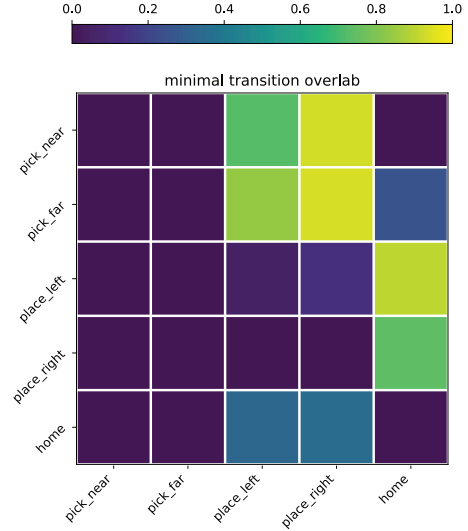
$$\overleftarrow{\text{Con}}(\theta_i) = \{\theta_j | \theta_j \in \Theta \wedge \theta_j \overrightarrow{\text{con}} \theta_i\},$$

Figure 6 illustrates the transition overlap between each pair of primitives of the tic-tac-toe task arranged in an adjacency-like matrix, where each row and column indicate which primitives belong into  $\overrightarrow{\text{Con}}(\theta)$  and  $\overleftarrow{\text{Con}}(\theta)$ , respectively.

Deciding the connectibility of two primitives using their overlap rather than purely basing it on the observations has the significant advantage of allowing to connect two primitives which might not have been observed connected during the demonstrations but are nevertheless safely connectible.

## 7. Inducing PCFGs for MPs

In this section, we introduce a grammar induction approach where the grammar search is defined as a maximum a posteriori problem and the grammar space is traversed using a



**Fig. 6.** The transition overlap for each arm primitives of the tic-tac-toe task. A value above the threshold  $\epsilon_{\text{overlab}} = 0.69$  signifies that the primitive at that row is connectible to the primitive of that column. The ellipsoids were  $n = 2$  standard deviations wide and the threshold was determined from the observations with  $\alpha = 0.95$ .

MCMC approach. We introduce a novel prior over the grammar structure based on three Poisson distributions allowing to define a desired grammar structure in more detail than common grammar priors. Furthermore, we discuss problems of common grammar priors and the advantages of the presented prior. We present four operators that allow the traversal of the grammar space and define distributions over each given a grammar. The proposal distribution of the MCMC approach is defined as a mixture over the operator distributions.

### 7.1. Learning grammars through posterior optimization

The posterior  $p(\mathcal{G}|\mathcal{D})$  describes how probable a given grammar  $\mathcal{G}$  is given the observed sequences  $\mathcal{D}$ . By applying Bayes theorem we can reformulate the posterior and, hence, the maximization as

$$\mathcal{G}^* = \arg \max_{\mathcal{G}} p(\mathcal{G}|\mathcal{D}) = \arg \max_{\mathcal{G}} p(\mathcal{D}|\mathcal{G})p(\mathcal{G}), \quad (9)$$

where  $p(\mathcal{D}|\mathcal{G})$  is the likelihood of the labeled demonstrations  $\mathcal{D}$  given the grammar  $\mathcal{G}$ . The likelihood is presented in the next section. Afterwards, we discuss common choices for the prior  $p(\mathcal{G})$  and, finally, we introduce a novel grammar prior based on Poisson distributions.

**7.1.1. Likelihood  $p(\mathcal{D}|\mathcal{G})$ .** The likelihood  $p(\mathcal{D}|\mathcal{G})$  is computed for each demonstration independently, yielding

$$p(\mathcal{D}|\mathcal{G}) = \prod_{d \in \mathcal{D}} p(d|\mathcal{G}). \quad (10)$$



Depending on the grammar  $\mathcal{G}$  the sequence  $\mathbf{d}$  could have been produced in multiple ways. Considering every possible derivation results in the sum-product formulation

$$p(\mathbf{d}|\mathcal{G}) = \sum_{\mathfrak{t} \in \mathbb{T}(\mathbf{d}, \mathcal{G})} \prod_{(A, r, \rho) \in \mathfrak{t}} \rho,$$

where  $\mathfrak{t}$  represents a single parse tree and  $\mathbb{T}(\mathbf{d}, \mathcal{G})$  denotes a function producing all feasible parse trees. The three-tuple  $(A, r, \rho)$  represents an edge in the parse tree  $\mathfrak{t}$  connecting the non-terminal  $A$  and its production  $r \in R_A$  with a probability of  $\rho \in \boldsymbol{\rho}_A$ . In this work, the function  $\mathbb{T}$  creating all possible parse trees for a given demonstration  $\mathbf{d}$ , is implemented by the Earley parser (Earley 1983). While the Earley parser suffers from a higher complexity compared with other parsers, it has the advantage that the parsed grammars do not have to be in any particular form.

*7.1.2. Grammar prior  $p(\mathcal{G})$ .* The grammar prior  $p(\mathcal{G})$  is commonly modeled as a joint distribution over the grammar probabilities  $\boldsymbol{\rho}_{\mathcal{G}} = \{\boldsymbol{\rho}_A | A \in \mathcal{V}\}$  and the grammar structure  $\mathcal{G}_{\mathcal{R}} = \{(A, R_A) | A \in \mathcal{V}\}$  (Stolcke 1994; Kitani, Sato, and Sugimoto 2008; Talton et al. 2012; Lee et al. 2013),

$$p(\mathcal{G}) = p(\boldsymbol{\rho}_{\mathcal{G}} | \mathcal{G}_{\mathcal{R}}) p(\mathcal{G}_{\mathcal{R}}). \quad (11)$$

The conditional  $p(\boldsymbol{\rho}_{\mathcal{G}} | \mathcal{G}_{\mathcal{R}})$  itself can be modeled as an independent joint distribution over the parameters of each non-terminal  $A \in \mathcal{V}$ ,

$$p(\boldsymbol{\rho}_{\mathcal{G}} | \mathcal{G}_{\mathcal{R}}) = \prod_{\boldsymbol{\rho}_A \in \boldsymbol{\rho}_{\mathcal{G}}} p(\boldsymbol{\rho}_A). \quad (12)$$

The dependency on the grammar structure is implicit, because the probabilities  $\boldsymbol{\rho}_A \in \boldsymbol{\rho}_{\mathcal{G}}$  depend on both the set of non-terminals  $\mathcal{V}$  and the productions for each non-terminal  $R_A$ . The parameters for each non-terminal  $\boldsymbol{\rho}_A \in \boldsymbol{\rho}_{\mathcal{G}}$  form a multinomial distribution, i.e.,  $\sum_{\rho \in \boldsymbol{\rho}_A} \rho = 1$ . Therefore, a Dirichlet distribution would be an obvious choice for the probability distribution over the parameters  $p(\boldsymbol{\rho}_A)$  for a single non-terminal  $A \in \mathcal{V}$ . A significant drawback of using a Dirichlet distribution is its factorial growth in the dimensionality of the multinomial. In fact, using an uninformative Dirichlet distribution, i.e., setting the concentration parameters to 1.0, will result in a probability density of  $p(\boldsymbol{\rho}_A) = (\dim(\boldsymbol{\rho}_A) - 1)!$  for any  $\boldsymbol{\rho}_A \in \boldsymbol{\rho}_{\mathcal{G}}$ .

To compensate for this growth, the structure prior  $p(\mathcal{G}_{\mathcal{R}})$  is usually modeled as an exponential distribution over the minimal description length (MDL) of the grammar structure  $\mathcal{G}_{\mathcal{R}}$ . Every symbol in the production rules, terminal and non-terminal, contributes to the MDL with  $\log_2 |\mathcal{A}| + |\mathcal{V}|$  bits, yielding the over all description length

$$\begin{aligned} \text{MDL}(\mathcal{G}_{\mathcal{R}}) &= \sum_{(A, R_A) \in \mathcal{G}_{\mathcal{R}}} \sum_{r \in R_A} \text{MDL}(r), \\ \text{MDL}(r) &= (1 + |r|) \log_2 |\mathcal{A}| + |\mathcal{V}|, \end{aligned}$$

A prior  $p(\mathcal{G}_{\mathcal{R}})$  defined as an exponential distribution over the MDL( $\mathcal{G}$ ) will prefer small and concise grammars.

However, such a prior can lead to grammars that are too compact to be intuitive for non-experts. In order to prefer grammars with a desired production length,  $\eta_r$ , the MDL has been extended with the log of a Poisson distribution with mean  $\eta_r$  (Kitani et al., 2008; Lee et al., 2013). Because of the factorial growth of the parameter prior  $p(\boldsymbol{\rho}_{\mathcal{G}} | \mathcal{G}_{\mathcal{R}})$  the structure prior is often additionally amplified with an exponential weighting term (Talton et al., 2012) to remain of significance for the overall grammar prior  $p(\mathcal{G})$  and, hence, the posterior  $p(\mathcal{G} | \mathcal{D})$ .

The likelihood  $p(\mathcal{D} | \mathcal{G})$  is defined as a product over the average of probabilities, which always results in  $p(\mathcal{D} | \mathcal{G}) \leq 1.0$ . However, the described grammar prior  $p(\mathcal{G})$  is the product of two probability densities, which will very quickly result in  $p(\mathcal{G}) \gg 1.0$  and, therefore, dominate the posterior.

*7.1.3. Novel prior.* The novel prior presented in this paper aims at inducing PCFGs that are easily understandable for non-experts. The key to achieving this goal is the grammar structure, rather than the grammar parameters. Therefore, we suggest a grammar prior, that does not explicitly model a Dirichlet distribution over the parameters, but instead implicitly considers the parameters in the overall grammar prior  $p(\mathcal{G})$ . We model the parameter prior and the structure prior jointly  $p(\mathcal{G}) = p(\boldsymbol{\rho}_{\mathcal{G}}, \mathcal{G}_{\mathcal{R}})$  as

$$p(\boldsymbol{\rho}_{\mathcal{G}}, \mathcal{G}_{\mathcal{R}}) = \frac{p(|\mathcal{R}| | \boldsymbol{\eta}_{\mathcal{R}})}{|\mathcal{R}|} \sum_{(A, R_A, \boldsymbol{\rho}_A) \in \mathcal{R}} \gamma(A, R_A, \boldsymbol{\rho}_A) \quad (13)$$

$$\gamma(A, R_A, \boldsymbol{\rho}_A) = p(|R_A| | \boldsymbol{\eta}_{R_A}) p(R_A | \boldsymbol{\rho}_A, \boldsymbol{\eta}_r), \quad (14)$$

where the probabilities over the number of rules  $p(|\mathcal{R}| | \boldsymbol{\eta}_{\mathcal{R}})$  and the size of each rule  $p(|R| | \boldsymbol{\eta}_{R_A})$  are modeled as Poisson distributions with means  $\boldsymbol{\eta}_{\mathcal{R}}$  and  $\boldsymbol{\eta}_r$ . The probability of each rule is modeled as a weighted average

$$p(R_A | \boldsymbol{\rho}_A, \boldsymbol{\eta}_r) = \sum_{r \in R_A, \rho \in \boldsymbol{\rho}_A} \rho p(|r| | \boldsymbol{\eta}_r), \quad (15)$$

over the probabilities of the corresponding productions. The weighting is given by the grammar parameters  $\rho \in \boldsymbol{\rho}_A$  and the probability of each production corresponds to the Poisson distribution over its length  $p(|r| | \boldsymbol{\eta}_r)$ , given a desired production length  $\eta_r$ . As all components are defined as discrete probabilities, the prior is always  $p(\mathcal{G}) \leq 1$ , eliminating the need for hard to tune weighting terms to cope with difficult scaling properties. Furthermore, the prior  $p(\mathcal{G})$  will now prefer grammars with  $\boldsymbol{\eta}_{R_A}$  productions per non-terminal with an average length of  $\boldsymbol{\eta}_r$  symbols per production. The hyperparameters  $\boldsymbol{\eta}_{\mathcal{R}}, \boldsymbol{\eta}_r$  can be set to achieve a desired simplicity of the grammar. By weighting each production  $r \in R_A$  with the corresponding grammar parameter  $\rho \in \boldsymbol{\rho}_A$  the prior gives more significance to production which are more likely to occur.

## 7.2. Traversing the grammar space $\mathfrak{G}$

To find the optimal grammar  $\mathcal{G}^*$ , it is necessary to define mechanisms that generate new grammars. A common choice is to define operators  $\text{op} \in \mathcal{O}$ , where  $\mathcal{O}$  denotes the set of all operators. Each operator  $\text{op}$  manipulates the rule set  $\mathcal{R}$  and consequentially the non-terminal set  $\mathcal{V}$  of a given grammar  $\mathcal{G}$ , therefore, creating a new grammar  $\mathcal{G}'$ . For each operator  $\text{op}$  we define a domain  $\Omega_{\text{op}}$  that  $\text{op}$  can act upon. The elements in  $\Omega_{\text{op}}$  depend on the operator itself and can be, for instance, non-terminals, pairs of non-terminals, or productions.

Each grammar represents a node in a grammar space  $\mathfrak{G}$ . The operators  $\text{op} \in \mathcal{O}$  represent directed edges in  $\mathfrak{G}$  between two grammars. The grammar space  $\mathfrak{G}$  is illustrated in Figure 4. After grammar  $\mathcal{G}'$  was created by applying an operator  $\text{op}$  on grammar  $\mathcal{G}$ , the grammar parameters usually have to be recomputed. In this work, the parameters are re-estimated for every new grammar  $\mathcal{G}'$  via the inside-outside algorithm (Baker 1979).

Not every possible grammar  $\mathcal{G}$  is suitable for sequencing MPs. Every sequence produced by  $\mathcal{G}$  has to guarantee a smooth, continuous trajectory within the state space of the MPs. In general, this means that a possible next primitive has to begin close to the end of the preceding primitive.

We restrict the grammar space  $\mathfrak{G}$  to only contain grammars that fulfill this connectivity requirement. The restriction is achieved by limiting the domain  $\Omega_{\text{op}}$  of each operator  $\text{op} \in \mathcal{O}$ , such that if grammar  $\mathcal{G}$  fulfills the connectivity requirement any grammar  $\mathcal{G}'$  resulting from an application of  $\text{op}$  on  $\mathcal{G}$  also fulfills the requirement. We incorporate the connectivity requirement into the definition of the two common operators merge and split.

**7.2.1. split.** The operator split divides the non-terminal  $A_i \in \Omega_{\text{split}}$  into two new non-terminals  $A_j, A_k$ . The productions  $\mathbf{R}_{A_i}$  are separated randomly into two corresponding, disjoint sets  $\mathbf{R}_{A_j}$  and  $\mathbf{R}_{A_k}$ , where neither of the resulting sets is empty. Each occurrence of  $A_i$  is randomly replaced by either  $A_j$  or  $A_k$ , where both  $A_j$  and  $A_k$  have to be selected at least once. The domain  $\Omega_{\text{split}}$  contains all non-terminals with at least two productions. Furthermore, every non-terminal in  $\Omega_{\text{split}}$  has to occur at least twice across all productions, including its own.

**7.2.2. merge.** The operator merge combines two non-terminals  $(A_j, A_k) \in \Omega_{\text{merge}}$  into a new non-terminal  $A_i$ . Correspondingly, the productions of are defined as the union  $\mathbf{R}_{A_i} = \mathbf{R}_{A_j} \cup \mathbf{R}_{A_k}$ . Every occurrence of  $A_j$  and  $A_k$  is replaced by  $A_i$ . If  $A_j$  and  $A_k$  contain productions that begin or end in very different MP state spaces a merging would endanger the connectivity requirement. We avoid this problem by restricting the domain  $\Omega_{\text{merge}}$  to only contain compatible non-terminal pairs. Assuming the sets  $\text{first}(A)$  and  $\text{last}(A)$  contain all possible primitives that could be at first or last position of any sequence produced starting from  $A$ .

Two non-terminals  $A_j$  and  $A_k$  are now considered compatible if

$$\bigcup_{\theta \in \text{first}(A_j)} \overleftarrow{\text{Con}}(\theta) = \bigcup_{\theta \in \text{first}(A_k)} \overleftarrow{\text{Con}}(\theta)$$

and

$$\bigcup_{\theta \in \text{last}(A_j)} \overrightarrow{\text{Con}}(\theta) = \bigcup_{\theta \in \text{last}(A_k)} \overrightarrow{\text{Con}}(\theta).$$

The split and merge operators negate each other and are capable of generalizing exiting hierarchies in grammars, however they lack the important ability to create new hierarchies. Therefore, we additionally utilize the chunk operator (Stolcke, 1994) and define the new insert operator that negates the effects of chunk.

**7.2.3. chunk.** The operator chunk creates a new non-terminal  $A$  with productions  $\mathbf{R}_A = \{r\}$ ,  $r \in (\mathcal{A} \cup \mathcal{V})^+ \wedge r \in \Omega_{\text{chunk}}$ . Every occurrence of the sequence  $r$  in a production in  $\mathcal{R}$  is replaced by  $A$ . The domain  $\Omega_{\text{chunk}}$  contains all possible subsequences of all productions in  $\mathcal{R}$ .

**7.2.4. insert.** The operator insert selects a non-terminal  $A \in \Omega_{\text{insert}}$  and replaces each occurrence of  $A$  with its production  $r \in \mathbf{R}_A$ . The domain  $\Omega_{\text{insert}}$  contains all non-terminals with exactly one production.

Given these four operators, we define the set of all possible operators as  $\mathcal{O} = \{\text{merge}, \text{split}, \text{chunk}, \text{insert}\}$ . Furthermore, the operators in  $\mathcal{O}$  are not exchangeable, i.e., if a grammar  $\mathcal{G}'$  was created by applying the operator  $\text{op}$  on grammar  $\mathcal{G}$ , there exists no operator in  $\mathcal{O} \setminus \{\text{op}\}$  that is able to produce  $\mathcal{G}'$  from  $\mathcal{G}$ .

## 7.3. Finding $\mathcal{G}^*$

Similarly to Talton et al. (2012), we search for the optimal grammar  $\mathcal{G}^* = \text{argmax}_{\mathcal{G} \in \mathcal{D}} \mathcal{G}(\mathcal{D})$  using MCMC optimization. A main advantage of MCMC over local search methods is that its stochastic exploration traverses the grammar space better than local search methods. Given the definition of the grammar score the corresponding landscape is highly multimodal. Often several operators that each lead to a lower scoring grammar are required to be executed sequentially in order to arrive at a new maximum. Even with a broad beam width, beam search often fails to surpass such valleys whereas MCMC owing to its stochasticity manages to reach at least better local optima and even offers theoretical guarantees to find the global optimum in the limit.

In Talton et al. (2012), the inputs are expected to already be hierarchical, restricting the grammar search to a reorganization of already existing productions by applying solely the merge and split operators. Given that our inputs are flat sequences, that is, pure sequences without hierarchy, of observed primitive samples, we additionally apply the chunk operator, that is capable of creating hierarchies

(Stolcke 1994). The insert operator ensures the irreducibility of the Markov chain. Analogously to Talton et al. (2012), we apply the Metropolis–Hastings algorithm. However, because Talton et al. (2012) solely uses the split and merge operator, in this article we directly define the proposal distributions  $q(\mathcal{G}'|\mathcal{G})$  as the probability of a split or a merge. In this work we define the proposal distribution as a mixture over the four operators  $\mathcal{O} = \{\text{merge, split, chunk, insert}\}$ ,

$$q(\mathcal{G}', \text{op}'|\mathcal{G}) = \sum_{\text{op} \in \mathcal{O}} p(\text{op}'|\mathcal{G}, \eta_{\text{op}'}) q_{\text{op}}(\mathcal{G}'|\mathcal{G}, \text{op}'),$$

with mixture components  $q_{\text{op}}(\mathcal{G}'|\mathcal{G}, \text{op}')$ . The mixture probability is defined as

$$p(\text{op}'|\mathcal{G}, \eta_{\text{op}'}) = \frac{\eta_{\text{op}'} (1 - \delta_{|\Omega_{\text{op}'|})}{\sum_{\text{op} \in \mathcal{O}} \eta_{\text{op}} (1 - \delta_{|\Omega_{\text{op}}|})} \quad (16)$$

where  $\eta_{\text{op}} \in \mathbb{R}$  is a weighting for the operator  $\text{op}$ ,  $\delta_{|\Omega_{\text{op}}|}$  denotes the Kronecker delta over the size of the domain  $\Omega_{\text{op}}$  for operator  $\text{op}$ . Given that the operators in  $\mathcal{O}$  are not exchangeable, a mixture component  $q_{\text{op}}(\mathcal{G}'|\mathcal{G}, \text{op}')$  should not contribute any probability mass if  $\text{op} \neq \text{op}'$ . This restriction is achieved by the Kronecker deltas  $\delta_{\text{op}', \text{op}}$  in the following mixture components.

**7.3.1.  $q_{\text{split}}(\mathcal{G}'|\mathcal{G}, \text{op}')$ .** Given that the split operator was applied to produce  $\mathcal{G}'$  from  $\mathcal{G}$ , there exist  $A_i \in \mathcal{V}$  and  $A_j, A_k \in \mathcal{V}'$ . The chance of randomly selecting  $A_i \in \Omega_{\text{split}}$  is  $1/|\Omega_{\text{split}}|$ . In addition, every production  $r \in \mathcal{R}_{A_i}$  was randomly assigned to either  $\mathcal{R}_{A_j}$  or  $\mathcal{R}_{A_k}$ , while each of those two sets had to be selected at least once. There are exactly  $2^{|\mathcal{R}_{A_i}|} - 2$  possibilities of assigning the productions to either  $\mathcal{R}_{A_j}$  or  $\mathcal{R}_{A_k}$ . Finally, the  $N_{A_i}$  occurrences of  $A_i$  across all productions in  $\mathcal{R}$  have been replaced by  $A_j$  or  $A_k$  in  $\mathcal{R}'$ . The chosen replacements have been one out of a total of  $2^{N_{A_i}} - 2$  possibilities, considering that  $A_j$  and  $A_k$  had to be chosen at least once. Combining the possibilities for assigning the productions and for assigning the occurrences results in redundancies, because there are always two combinations that will result in the same  $\mathcal{R}'$ . The overall probability of  $\mathcal{G}'$  being produced from  $\mathcal{G}$  by using a *split* operator is given as

$$q_{\text{split}}(\mathcal{G}'|\mathcal{G}, \text{op}') = \frac{\delta_{\text{op}', \text{split}}}{|\Omega_{\text{split}}|} \frac{2}{(2^{|\mathcal{R}_{A_i}|} - 2)(2^{N_{A_i}} - 2)}. \quad (17)$$

**7.3.2.  $q_{\text{merge}}(\mathcal{G}'|\mathcal{G}, \text{op}')$ .** The only stochastic part in the merge operator is the decision of which pair  $(A_i, A_j) \in \Omega_{\text{op}}$  is selected, therefore the probability for *merge* is given as

$$q_{\text{merge}}(\mathcal{G}'|\mathcal{G}, \text{op}') = \frac{\delta_{\text{op}', \text{merge}}}{|\Omega_{\text{merge}}|}. \quad (18)$$

**7.3.3.  $q_{\text{chunk}}(\mathcal{G}'|\mathcal{G}, \text{op}')$ .** Given that the domain  $\Omega_{\text{chunk}}$  already contains all possible subsequences of all

productions in  $\mathcal{R}$ , the probability for choosing one sequence at random is

$$q_{\text{chunk}}(\mathcal{G}'|\mathcal{G}, \text{op}') = \frac{\delta_{\text{op}', \text{chunk}}}{|\Omega_{\text{chunk}}|}. \quad (19)$$

**7.3.4.  $q_{\text{insert}}(\mathcal{G}'|\mathcal{G}, \text{op}')$ .** The domain  $\Omega_{\text{insert}}$  is already restricted to non-terminals with a single production, therefore the probability of insert is simply

$$q_{\text{insert}}(\mathcal{G}'|\mathcal{G}, \text{op}') = \frac{\delta_{\text{op}', \text{insert}}}{|\Omega_{\text{insert}}|}. \quad (20)$$

At every iteration of the Metropolis–Hastings algorithm a random new grammar is sampled from the proposal distribution  $\mathcal{G}', \text{op}' \sim q(\mathcal{G}', \text{op}'|\mathcal{G})$ . This new grammar is then accepted with a probability of

$$\text{acc}(\mathcal{G}', \text{op}'|\mathcal{G}) = \min\left(1, \frac{p(\mathcal{G}'|\mathcal{D})^{1/T} q(\mathcal{G}, \overline{\text{op}'|\mathcal{G}'})}{p(\mathcal{G}|\mathcal{D})^{1/T} q(\mathcal{G}', \text{op}'|\mathcal{G})}\right), \quad (21)$$

where  $T$  denotes a decaying temperature and  $\overline{\text{op}'}$  denotes the complementary operator to  $\text{op}'$ , i.e.,  $\overline{\text{split}} = \text{merge}$ ,  $\overline{\text{chunk}} = \text{insert}$ . If the new grammar was accepted it is set to the current grammar  $G \leftarrow G'$  and the next iteration begins. After a defined number of iterations, the grammar with the highest posterior is returned. For instance, Table 2 shows a grammar induced by the presented method given sequences of the previously described tic-tac-toe task. The semantically meaningful names of the non-terminals were chosen manually.

Given that the MCMC optimization finds high scoring grammar after only a few iterations, the hyper-parameter optimization is inexpensive. Furthermore, a good rule of thumb for the number of productions per non-terminal and the number of symbols per production are two and three, respectively, leaving the number of non-terminals the only free parameter of the presented prior.

**Table 2.** Grammar with the highest posterior after 400 iterations of the MCMC optimization. Grammar index 171 in Figure 8.

START	→	MOVE	(1.00)		
MOVE	→	pick_near TO	(0.40)		pick_far TO (0.60)
TO	→	LEFT home	(0.47)		RIGHT home (0.53)
LEFT	→	close place_left open			(1.00)
RIGHT	→	close place_right open			(1.00)

## 8. Enhancing PCFGs with attributes for MP sequencing

So far, the presented approach induces grammars that do not violate the connectivity requirement. However, connectivity as defined in this work only guarantees that the transition area of two consecutive primitives is large enough to produce a continuous state space trajectory. In order to ensure smooth trajectories the start of the subsequent

primitive has to be conditioned to the end of the current primitive. This can be achieved within the grammar formulation by introducing attributes. Furthermore, attributes can be used for defining points of interest that primitives need to reach for a successful execution. We introduce an evaluation scheme for MP sequencing tasks that enhance given PCFGs with attributes and conditions. The scheme generalizes to different MP sequencing tasks and, therefore, needs only little to no adaptation for specific tasks, with the exception of the initialization of the task-specific attribute values.

We define the following three attributes, common to primitive sequencing tasks.

- `transition` This attribute defines where the current primitive ends and the next primitive is supposed to start. It is solely defined for non-terminals, and ensures that the produced primitives result in a continuous state space trajectory.
- `endpoint` The endpoint of a MP. It is solely defined for terminals and after the terminal has been evaluated, the attribute of the left-hand side non-terminal is set to the `endpoint` of the corresponding primitive.
- `viapoints` An ordered list of points that are supposed to be traversed by the sequence of primitives. The points are given in the state space of the primitives. Once the first point is traversed by a primitive it is removed from the list and the next point is considered.

In addition to the attributes we define two conditions for necessary for the evaluation scheme. If preceded with an `assert` these conditions have to be satisfied for a successful evaluation.

- `reachable` Given a primitive and a point in the primitive state-space, this condition is satisfied if the point is reachable by the primitive. In this work, we use ProMPs over the joint configuration of the robot. A point given in the configuration of the robot is reachable by a particular primitive if it is within two times the standard deviation of the trajectory mean of the MP.
- `producible` Given a non-terminal this condition is satisfied if at least one of the corresponding right-hand sides is producible. A right-hand side is considered producible if all mandatory conditions are satisfied, given the current set of attributes.

The described attributes enhance CFGs for MP sequencing tasks, such that the sequenced primitives can be conditioned to state of the environment, e.g., the pose of an object. The conditions ensure a continuous state-space trajectory of the sequenced primitives, even in the case of primitive adaptations.

### 8.1. Evaluation scheme for the tic-tac-toe task

We explain the functionality of the attributes in detail using the example of the tic-tac-toe task. We start with the PCFG shown in Table 2. The grammar was induced from demonstrations as described in the previous section.

The production of the sequence always begins at the `START` non-terminal. We assign two points to the `viapoints` attribute. One for the position of a stone and one for the field the stone is supposed to be placed on. Furthermore, we set the `transition` attribute to the current position of the robot in the primitive state space. We use the literals `stone_pos`, `field_pos`, and `cur_pos` instead of the actual numerical values, where `assert` indicates that this condition must be satisfied otherwise the entire right-hand side is removed from consideration as a possible production of the corresponding non-terminal given the current attribute set. If the `START` non-terminal is not producible, the task is not solvable under the given attributes. Furthermore, if the `viapoints` list is not empty after evaluating `START` not all points were traversed and the task is not considered solved.

---

	<code>START.transition = cur_pos</code>	
	<code>START.viapoints = [stone_pos, field_pos]</code>	
	<code>assert: producible(START)</code>	
	<code>assert: empty(START.viapoints)</code>	
<code>START</code>	<code>→ MOVE</code>	(1.00)
	<code>MOVE.viapoints = START.viapoints</code>	
	<code>MOVE.transition = START.transition</code>	
	<code>assert: producible(MOVE)</code>	
	<code>START.transition = MOVE.transition</code>	
	<code>START.viapoints = MOVE.viapoints</code>	

---

An important convention in the attribute notation is that whenever a non-terminal appears as an argument of a condition or on the right-hand side of an assignment it has been evaluated before. For instance, the producibility of `START` and `MOVE` can only be asserted once the respective non-terminal has been fully evaluated.

The `MOVE` non-terminal contains multiple productions, each consisting of multiple symbols. The productions can be evaluated in parallel, i.e., the evaluation of each of the productions begins with the same set of attributes, independent of the changes that have occurred during the evaluation of the other productions. In contrast, the symbols of a single production are evaluated sequentially, i.e., every symbol begins with the attributes set after the evaluation of the previous symbol. As mentioned previously, terminals represent single MPs. It is important that a sequence of primitives does not contain any jumps in the state space, because a real robot platform will not be able to make significant changes in its configuration instantaneously. Therefore, we ensure that every selected primitive starts where the previous primitive ended. In the proposed

evaluation scheme, this is achieved by ensuring that the reachable condition holds for the primitive and the current transition point. If the primitive can start from the transition point, the `transition` attribute is set to the endpoint of the primitive afterwards. Furthermore, we define a function to traverse the `viapoint` list.

- `traverse` The function expects a terminal and a list of points. If the first point in the list is reachable by the terminal the corresponding primitive will traverse the point, the point will be removed from the list and the function evaluates to true.

Given that the possible adaptation of the primitive to the point could change the endpoint, `traverse` has to be evaluated before the `transition` point is adapted.

---

```

MOVE → pick_near TO (0.40)
      assert: reachable(pick_near, MOVE.transition)
      traverse(pick_near, MOVE.viapoints)
      MOVE.transition = pick_near.endpoint
      TO.viapoints = MOVE.viapoints
      TO.transition = MOVE.transition
      assert: producible(TO)
      MOVE.viapoints = TO.viapoints
      MOVE.transition = TO.transition

```

---

Only the evaluation for one of the two productions is shown. The evaluation of the other production is defined analogously, but with the terminal `pick_far` instead of `pick_near`. Despite that both of the productions next evaluate the `TO` non-terminal, the actual evaluations might differ due to two different sets of attribute values.

---

```

TO → LEFT home (0.47)
    LEFT.viapoints = TO.viapoints
    LEFT.transition = TO.transition
    assert: producible(LEFT)
    TO.viapoints = LEFT.viapoints
    TO.transition = LEFT.transition
    assert: reachable(pick_near, TO.transition)
    traverse(pick_near, TO.viapoints)
    TO.transition = pick_near.endpoint

```

---

Again two different possible productions are evaluated in parallel but only one is shown. The evaluation of the other production is defined equivalently, but with the non-terminal `RIGHT` instead of the `LEFT`. In contrast to the evaluation of `MOVE` the productions of `TO` require the evaluation of a non-terminal before the evaluation of a terminal.

## 8.2. A general evaluation scheme for sequencing tasks

A structure for both terminal and non-terminal evaluations is clearly evident. Every terminal `a` on the production of a

rule with non-terminal `A` on the left-hand side is evaluated using the statements and every non-terminal `B` on the right-hand side of a rule with non-terminal `A` on the left-hand side is evaluated using

---

```

assert: reachable(a, A.transition)
traverse(a, A.viapoints)
A.transition = a.endpoint
B.viapoints = A.viapoints
B.transition = A.transition
assert: producible(B)
A.viapoints = B.viapoints
A.transition = B.transition

```

---

The presented evaluation scheme is very general and can be applied to any MP sequencing task. Using not further specified via points has the advantage that the evaluation does not restrict which primitive traverses which point. For instance, in the case of an obstacle it might be sufficient that the obstacle is passed at some point, but it does not necessarily matter which primitive avoids it. However, the unspecified list of via points has a significant disadvantage. A primitive might require a certain via point, for instance `pick_near` and `pick_far` have to know where the stone is positioned in order to pick it up successfully. Nothing in the current scheme associates via points with a certain primitives. We solve this problem by introducing two additional attributes.

- `keywords` An unordered list of keywords. This attribute is assigned only to terminals before the evaluation and contains keywords identifying relevant points in the `targets` attribute.
- `targets` A dictionary that maps keywords to ordered lists of points. The points are defined in the primitive state space. A primitive containing a matching keyword in its `keywords` attribute extracts the first point in the corresponding list.

The evaluation scheme for terminals is now defined as

---

```

assert: reachable(a, A.transition)
for: key in a.keywords
  assert: key in A.targets
  assert: traverse(a, A.target[key])
traverse(a, A.viapoints)
A.transition = a.endpoint

```

---

We introduce the `for:` notation to indicate an iteration and the `in` notation to indicate the existence of an element in a list. The `targets` attribute can strongly influence the production of a sequence. The given target could be outside of the distribution of the primitive associated with the terminal. For instance, both terminals `pick_near` and `pick_far` have a `stone` keyword. If the `targets` attribute associates `stone` with a value outside of the `pick_near` primitive but within the `pick_far` primitive, the `assert` statement would only hold for `pick_far`, ensuring that every sequence

produced with this set of targets will contain a `pick_far` and never a `pick_near`. In this way, the target attributes directly influence the effective structure of the grammar.

The evaluation scheme for non-terminals only changes such that the `targets` attribute is additionally passed down and received afterwards, analogously to the `via_points` attribute.

### 8.3. Evaluating parallel attribute sets

We already established that the right-hand sides of a single non-terminal are evaluated in parallel. If more than one right-hand side does not violate any asserts, multiple parallel sets of attributes return from that non-terminal evaluation. Given that within one right-hand side the attributes are passed sequentially from symbol to symbol, the question arises which of the multiple attribute sets should be considered. A naive approach would be to select a random attribute set. However, one attribute set might result in an unproducible right-hand side while another might not. We address this problem by storing every attribute set corresponding to a producible right-hand side in an ordered list. The order is defined randomly, while being weighted with the probabilities of the right-hand sides. Only the first set of attributes is considered, unless the set results in an assert violation, then the attribute set is discarded and the evaluation continues with the next set in the list. If no sets are left, the right-hand side is considered unproducible. It is possible that a given set of `targets` results in an effective grammar structure that is not capable of producing any sequence of primitives. For instance, neither `place_left` nor `place_right` are able to place the stone outside of the playing field. Hence, if the corresponding target is set outside the playing field neither of the two productions of the `TO` terminal will be producible and the non-producibility will be propagated up until the start symbol. In this case, the grammar would return an empty sequence. This can easily be used to prompt the user that the current grammar cannot produce a sequence satisfying the given set of targets. Therefore, different targets or new demonstrations extending the grammar are required.

The presented attributes and evaluation scheme are independent of the actual task itself and generalize over MP sequencing tasks. The only attribute that has to be accessed and potentially adapted by the user are the `targets`. Hence, the remaining attributes and the evaluation scheme

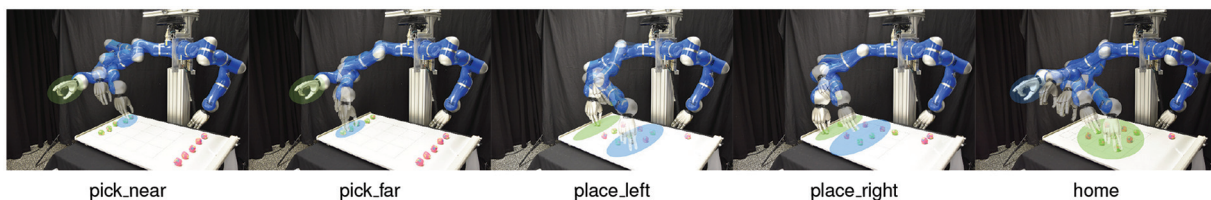
itself can be considered constants and can be hidden from the user, concealing necessary complexity that does not affect the representation of the behavior. We further simplify the presentation of the attribute grammar, by presenting the keywords of the `targets` attribute as grammar attributes themselves. By applying these simplifications we arrive at the attribute grammar as presented initially in the problem statement.

## 9. Experiments

We evaluated the proposed approach on several real robot tasks. First, we induced a grammar producing turns of the tic-tac-toe game. Second, we learned a grammar that assists a human with the assembly of a simple toolbox. In both tasks the necessary primitives were encoded as ProMPs (Paraschos et al., 2018). For each of the tasks, we compare the posterior resulting from our proposed prior, *Grammar Poisson*, with the one resulting from three common structure prior choices, *MDL*, *Poisson + MDL*, *Avg. Poisson*. The *MDL* prior is simply defined as an exponential distribution with the MDL as its energy (Talton et al., 2012). The *Poisson + MDL* prior weights the description language for every production with the Poisson probability over the length of the production (Kitani et al., 2008). Finally, the *Avg. Poisson* prior discards the MDL completely and is solely represented by a Poisson distribution over the average length of all productions (Lee et al., 2013). A major difference of the *Grammar Poisson* prior to the other discussed priors is that we do not model the distribution over the grammar parameters as a Dirichlet distribution but rather use them as a weighting for the average production length.

### 9.1. Learning a Grammar for Tic-Tac-Toe Turns

In this task we learned a grammar that allows the robot to play tic-tac-toe against a human. Each produced sequence corresponds to one turn of the game, i.e., picking a stone, closing the hand, placing the stone on the field, opening the hand and returning to the home position. The goal is not to learn the logic behind the game but rather the induction of an intuitive grammar producing valid turns. The segmentation of the demonstrations and, hence, the learning of the primitives was done beforehand via Probabilistic Segmentation (Lioutikov et al., 2017). The five resulting arm primitives are shown in Figure 7, where the green and



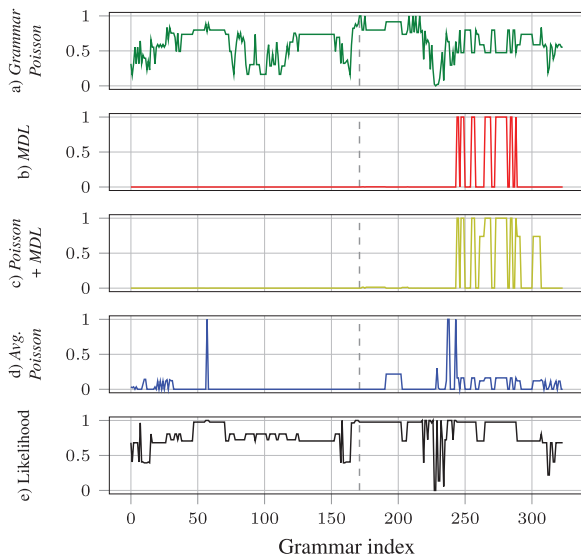
**Fig. 7.** The five arm primitives used in the sequences, representing turns in the tic-tac-toe game. While both `pick_near` and `pick_far` approach a stone from the home position, they differ in the stone positions they can reach. Similarly the primitives `place_left` and `place_right` position the stone in different areas of the playing field.

**Table 3.** Initial grammar. Grammar index 0 in Figure 8.

START	→	DEMO1 (0.33)		DEMO2 (0.20)
		DEMO3 (0.27)		DEMO4 (0.20)
DEMO1	→	pick_far close place_right open home (1.00)		
DEMO2	→	pick_near close place_right open home (1.00)		
DEMO3	→	pick_far close place_left open home (1.00)		
DEMO4	→	pick_near close place_left open home (1.00)		

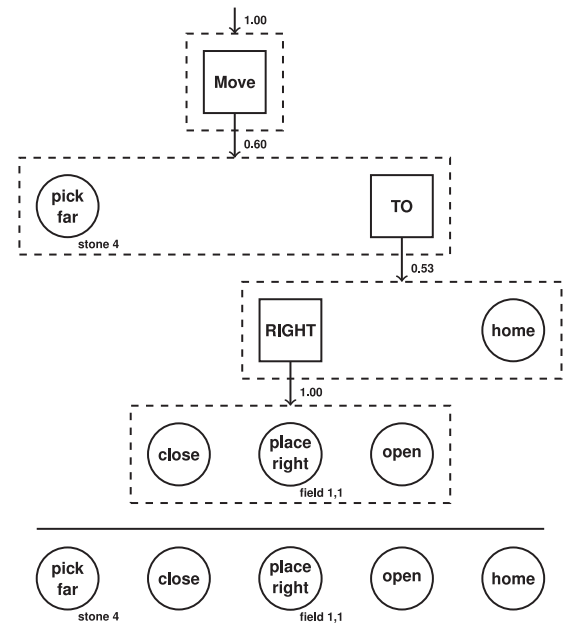
blue highlighted areas mark the start and end of the end-effector. While `pick_near` and `pick_far` are semantically similar, they actually differ quite substantially in the encoded joint trajectory of the robot and, hence, the segmentation algorithm separated those movements into two separate primitives. The same explanation holds for `place_left` and `place_right`.

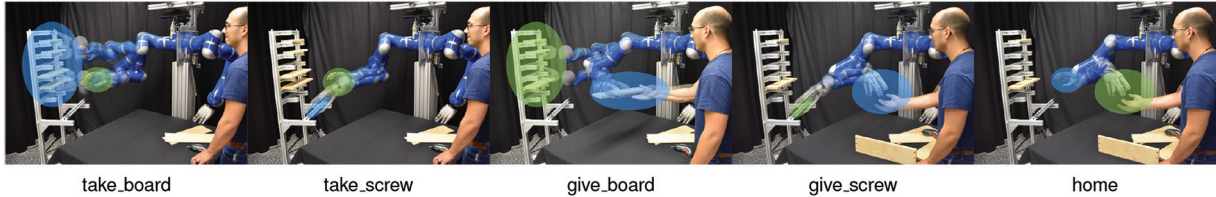
The grammar learning was initialized with 15 observations of 4 unique sequences, each consisting of 5 terminals. The initial grammar is given in Table 3. We initialized our approach with a desired number of rules  $\eta_{\mathcal{R}} = 5$ , the desired number of average productions per rule  $\eta_{\mathcal{R}} = 2$  and the desired average length of each production  $\eta_r = 3$ . The weights for each operator were set uniformly to  $\eta_{\text{op}} = 1$ ,  $\text{op} \in \mathcal{O}$ . The MCMC optimization was run for 400 steps and resulted in 324 accepted grammars. The corresponding normalized posteriors are shown in Figure 8 and the grammar with the highest posterior, grammar index 171 is given in Table 2. The induced grammar intuitively represents, that each produced sequence will move a near or a far stone to either the left or the right side of the playing field. Furthermore, after every closing of the hand there will be a later opening of the hand. A possible sequence produced by the grammar, including the corresponding

**Fig. 8.** The posteriors and the likelihood for the tic-tac-toe turn grammar. The vertical, dashed line indicates the index of the highest posterior (171), given the presented Poisson prior.

parse tree is seen in Figure 9. The parse tree includes keys and values assigned to the `keywords` and `targets` attributes. The production of the sequence was started with the attribute `targets = {stone : stone_pos, field : field_pos}` consisting of the position of the stone that should be played next, `stone_pos` and the field position `field_pos` on which the stone should be placed. For simplicity, the parse tree presented to the user replaces the actual position of `stone_pos` and `field_pos` but instead the numbering of the corresponding playing field cell.

The naming of the non-terminals was chosen manually after the grammar learning. An automated naming of the non-terminals corresponding to the semantics of the productions is outside of the scope of this paper and remains part of future work. Figure 8b–d shows the normalized posteriors corresponding to the three common priors. The  $x$ -axis corresponds to the different grammars traversed during the MCMC optimization, i.e., the grammar  $\mathcal{G}^i$ ,  $\text{op}^i \sim q(\mathcal{G}^i, \text{op}^i | \mathcal{G}^{i-1})$  was sampled from the proposal distribution around  $\mathcal{G}^{i-1}$  by applying  $\text{op}^i$ . The spiky behavior of the posteriors (b–d) is due to the uninformative Dirichlet prior for the grammar parameters and the exponential distribution over the *MDL*. Both of these factors can change significantly with a small change in the grammar, e.g., a merge creating a rule with many productions or a chunk reducing the length of a long production.

**Fig. 9.** A parse tree of a sequence produced by the learned grammar for tic-tac-toe turns. Non-terminals are presented as squares and terminals as circles. A dashed rectangle represents the production chosen by the parent terminal with the probability next to the connecting arrow. The solid line separates the final sequence from the producing parse tree. The grammar was enhanced with the presented attributes and evaluation scheme, where stone and field are two keys assigned to the keywords attributes of the `pick_far` and `pick_right` terminals, respectively.

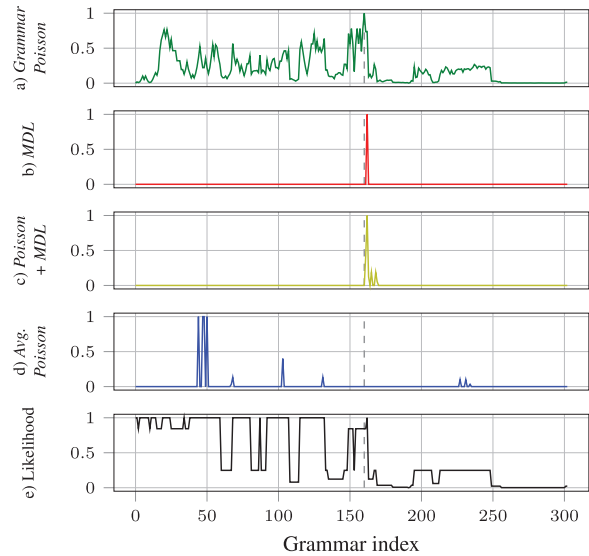


**Fig. 10.** The arm primitives of the box assembly task. The robot applies different primitives for grasping a board, take board, or picking a screw, take screw. Similarly the handover for boards and screws is encode in different primitives.

Furthermore, it is noticeable that the likelihood of the grammar  $p(\mathcal{G}^i|\mathcal{D})$  does not play significantly into the posteriors of (b–d), whereas our posterior (a) shows a much stronger dependency on the likelihood. This behaviour is explained by the fact that the likelihood as introduced in Equation is a probability mass function, but the three priors (*MDL*, *Poisson + MDL*, *Avg. Poisson*) are products of probability density functions. In contrast, our prior (*Grammar Poisson*) is defined as a probability mass function, averaging over multiple Poisson distributions. This definition prohibits the prior from completely dominating the likelihood. As a consequence, the proposed prior (*Grammar Poisson*) results in a posterior (a) that takes the given observations much more strongly into account than the posteriors in (b–d).

## 9.2. Learning a grammar for a simple toolbox assembly

This task shows the abstraction capabilities of our approach. The demonstrations were again segmented beforehand and resulted in the five arm primitives, shown in Figure 10, and four hand primitives, closing and opening the hand for both a board and a screw grasp. The set of demonstrations contained three different sequences, consisting of 40 terminals each. Every observation showed the grasping and handing over of four boards and four screws, either alternating between the board and the screw or starting with two boards and alternating subsequently. The approach was initialized with  $\eta_R = 9$ ,  $\eta_R = 2$ ,  $\eta_r = 2$ . The weights for the split and merge operators were set to one and the remaining two were set to two. The MCMC optimization ran for 400 iterations and 303 grammars were accepted. The posteriors for the accepted grammars are shown alongside the likelihood in Figure 11. The posteriors show similar behavior as in the previous task. Both the *MDL* and the *Poisson + MDL* have a maximum at 162, indicating that the corresponding grammar has the minimal description length of all accepted grammars. The *Avg. Poisson* prior has its maximum at 44 due to an average production length close to  $\eta_r$ . However, the corresponding grammar contains 14 rules with one production each. The grammar with the maximum posterior according to the *Grammar Poisson* prior is given at index 160 and presented in Table 4 and three produced sequences are shown in



**Fig. 11.** The posteriors and the likelihood for the box assembly task. The vertical, dashed line indicates the index of the highest posterior (160), given the presented Poisson prior.

Table 5. The grammar abstracts a full turn from taking a board or screw until going back to the home position. This subsequence was not marked in any way and was detected as a consequence of the grammar learning. The sequence occurred multiple times during each observation. Abstracting it into a non-terminal will therefore simplify the grammar significantly. Furthermore, the grammar encodes that a grasping of a board or a screw through the closing of the hand has to be eventually followed by the corresponding opening of the hand. The alternation between handing over a board and a screw is represented in the two rules for SBS and BSB and the rules for ASSEMBLE\_SB. The option of starting with two boards is encoded in ASSEMBLE\_BB.

## 10. Conclusion

In this work, we have introduced attribute grammars as a mechanism to sequence MPs. We have shown how to identify the categories of MPs and how to determine whether two ProMPs are connectible. The presented categorization approach is simple yet efficient, however, in future work we want to investigate more sophisticated approaches for the



**Table 4.** The grammar with the highest posterior for the box assembly task after 400 iterations of the MCMC optimization.

START	→	ASSEMBLE_SB	(0.5)
	→	ASSEMBLE_BB	(0.5)
BOARD	→	take_board GIVE_B home	(1.0)
SCREW	→	take_screw GIVE_S home	(1.0)
BSB	→	BOARD SCREW BOARD	(1.0)
SBS	→	SCREW BOARD SCREW	(1.0)
GIVE_S	→	close_screw give_screw open_screw	(1.0)
ASSEMBLE_BB	→	BOARD BOARD SBS ...	
		BOARD SCREW SCREW	(1.0)
GIVE_B	→	close_board give_board open_board	(1.0)
ASSEMBLE_SB	→	SBS BOARD SCREW BSB	(0.5)
	→	BOARD SBS BOARD SBS	(0.5)

**Table 5.** Three sample sequences produced by the induced assembly grammar. For the sake of brevity the non-terminals BOARD and SCREW have not been resolved further.

SCREW BOARD SCREW BOARD SCREW BOARD SCREW BOARD  
 BOARD BOARD SCREW BOARD SCREW BOARD SCREW SCREW  
 BOARD SCREW BOARD SCREW BOARD SCREW BOARD SCREW


clustering of parameterized time series such as the applied MPs. Furthermore, we have presented an approach that induces PCFGs from flat sequences of MP samples, i.e., no hierarchy in the observations, while taking advantage of a stochastic primitive representation. The novel grammar prior is defined over several coupled Poisson distributions, and eliminates the many complications that arise from both Dirichlet parameter priors and minimal description length-based structure priors. In our method, the hyper-parameters of the prior have a clear semantic interpretation, namely the number of productions for each non-terminal and the average length of each production. The posterior is learned using a MCMC optimization where the proposal distribution is formulated as a mixture model over four operators. We defined attributes and conditions of a general evaluation scheme for sequencing tasks. We enhanced an initially induced PCFG for making a move in a game of tic-tac-toe with the defined attributes and the evaluation scheme. While the MCMC optimization is less likely to get stuck in local optima than other suggested search strategies, such as beam search, it is not without fault. Depending on the complexity of the task with respect to the length of the observed sequences and the number of terminals, a significant number of samples are required to reach a promising area of the search space. Given that, the actual interest of grammar induction is not the exploration of the posterior, but rather the finding of the optimal grammar inside the search space, future work, will investigate the advantages of Monte Carlo tree search over MCMC for this particular challenge. Another future line of research is the goal to learn more general grammars while avoiding an over generalization, effectively defying Gold’s law. A possible approach is to take advantage of the grammar as a generative model and


introduce reinforcement learning techniques to improve the grammar after it has been induced from a given set of demonstrations.

## Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: The research leading to these results has received funding from the European Community’s Seventh Framework Programme (FP7-ICT-2013-10; grant agreement number 610878; 3rdHand). Furthermore, this project has received funding from the European Union’s Horizon 2020 research and innovation programme (grant agreement number 640554; SKILLS4ROBOTS). This article is also based on results obtained from a project commissioned by the New Energy and Industrial Technology Development Organization (NEDO).

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## Note

1. Note that the presented method is equally applicable when using other popular representations such as DMPs (Ijspeert et al., 2013), Gaussian mixture models (Calinon et al., 2007), and Gaussian processes (Schneider and Ertel, 2010).

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