energies MDPI

**Supplementary Material S2**

**Governing equations for fluid and heat transport solved in FEFLOW® [46]**

The precise formulation of the set of equations is given in this supplementary material:

Under steady conductive conditions, the thermal energy equation simplifies to the common Poisson-type diffusion equation (1):

$$0=\nabla(\lambda\nabla T)+S \tag{1}$$

With ∇ the Nabla operator ($\nabla=(\partial/\partial x,\partial/\partial y,\partial/\partial z)$), T temperature, λ thermal conductivity of the rock [W/m*K] and S the internal radiogenic heat production [$\mu W/m^3$] as heat source.

Under similar assumptions of steady flow, the fluid mass balance simplifies to:

$$\nabla(\varrho^f q^f) = \varrho^f Q \tag{2}$$

With $\varrho^f$ fluid density and Q as Source/Sink term of mass [1/T] and $q^f$ the three dimensional Darcy velocity as:

$$\mathbf{q}^f = -\mathbf{K}(\nabla h) \tag{3}$$

With K being the hydraulic conductivity tensor [m/s] and h the hydraulic head [m].

For the coupled thermo-hydraulic simulation the time dependent mass conservation including varying viscosity ($\mu^f$) and density ($\varrho^f$) is formulated under Boussinesq approximation as (see also Diersch and Kolditz [69]):

$$\frac{dx}{dt}(n\rho) + \nabla(\mathbf{q}^f) = H \tag{4}$$

n is hereby the porosity. In order to account for the dependency of the main fluid parameters (viscosity and density) on the main state variables (T, h), Darcy velocity is now formulated as:

$$\mathbf{q}^f = -\mathbf{K}\left(\nabla h + \frac{\rho^f - \rho_0^f}{\rho_0^f}\mathbf{e}\right) \tag{5}$$

With e as the gravitational unit vector and $\left(\frac{\rho^f-\rho_0^f}{\rho_0^f}\right)$ as the buoyancy force induced by density variation between the reference fluid density $\rho_0^f$ and the fluid density $\varrho^f$. K is the hydraulic conductivity tensor, which depends on temperature via an equation of state for the fluid viscosity after Magri et al. [70]:

$$\mathbf{K} = \frac{\boldsymbol{\kappa}\rho_0^f g}{\mu^f(T)} \tag{6}$$

The equation of state for the fluid density is formulated in Magri et al. [71] and approximates the temperature and pressure dependency on the fluid density by a polynomial expression. Thereby the thermal expansion coefficient couples back variations of temperature on fluid density of the fluid.

The energy balance equation then reads as:

$$Q_T = (c\rho)_{fs}\frac{\partial T}{\partial t} + \nabla\left(c^f\rho^f T\mathbf{q^f}\right) - \nabla(\lambda\nabla T) \tag{7}$$

$Q_T$ is the heat source function and $(c\rho)_{fs}$ is the specific heat capacity of the fluid and solid.

Under steady state conditions the equation of state for energy balance for coupled fluid flow and heat transport in porous media is written as:

$$Q = \nabla\left(c^f\rho^f T\mathbf{q^f}\right) - \nabla(\lambda\nabla T) \tag{8}$$

And the heat transport is consisting of an advective flow part and a conductive part. Mixed convection is not supported as the buoyancy term is constant and therefore not able to trigger mixed convection.

## References

46. Diersch, H.-J., *FEFLOW - Finite Element Modeling of Flow, Mass and Heat Transport in Porous and Fractured Media*. 1 ed.; Springer-Verlag Berlin Heidelberg: Heidelberg, 2014.

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70. Magri, F.; Akar, T.; Gemici, U.; Pekdeger, A., Deep geothermal groundwater flow in the Seferihisar–Balçova area, Turkey: results from transient numerical simulations of coupled fluid flow and heat transport processes. **2010,** 10, (3), 388-405.

71. Magri, F.; Bayer, U.; Maiwald, U.; Otto, R.; Thomsen, C., Impact of transition zones, variable fluid viscosity and anthropogenic activities on coupled fluid-transport processes in a shallow salt-dome environment. **2009,** 9, (3), 182-194.