Quasi-Analytical Description of a Double Slit Planar Dielectric Waveguide as Broadband Dispersion Compensating Element

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***Abstract*—We demonstrate dispersion compensation of dielectric waveguides by a double slit dielectric structure by a quasi-analytical solution. The concept is then confirmed with a full-wave numerical simulation of realistic rectangular waveguides.**

I. Analytical Dispersion Relation of a Double Slit Waveguide

Low-loss propagation of terahertz signals through waveguides is a key requirement for applications where free space propagation is not possible. While metallic structures perform well at microwave frequencies, issues with surface roughness, the finite conductivity and the skin depth severely increase their losses at THz frequencies [1]. Dielectric waveguides offer a solution to these issues. For propagation of THz pulses, however, dispersion severely broadens the pulse. The higher the frequency, the more field is concentrated within the dielectric with higher refractive index than the surrounding, giving rise to positive dispersion.

In this work, a waveguide topology that mitigates the dispersion caused by dielectric slab waveguides (SWG) is proposed and analyticaly characterized. The proposed dispersion compensating waveguide (DCW) is fully dielectric with two lower refractive index slits symmetrically arranged, as shown in Fig. 1. For a planar structure, infinite in the y axis, an analytical expression can be obtained that allows to determine the propagation constant, β. Following the analysis from [2] and [3] for multilayer slab waveguides and simplyfying the problem by considering that the permitivity in the outer areas and in the slits is ϵ_o, with the waveguide being made out of a single, lossless, dispersionless material with $\epsilon_1 > \epsilon_0$, the following solutions are derived from source-less Maxwell equations at a dielectric medium with transversal magnetic field (TM mode) in rectangular coordinates, for which the electric field is perpendicular to the slit interface.

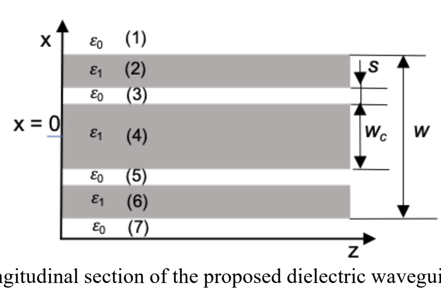

Fig. 1. Longitudinal section of the proposed dielectric waveguide. The wave propagates along the z direction.

Evanescent fields in region (1) and (7) of Fig. 1, respectively, are given by

$$E_z^{(1)} = A_1 e^{-q_0 x} e^{-j\beta z}$$

$$E_z^{(7)} = A_7 e^{q_0 x} e^{-j\beta z} \tag{1}$$

Field on dielectric areas (2), (4) and (6):

$$E_z = (A_m \sin px + B_m \cos px) e^{-j\beta z} \tag{2}$$

Field within slits (3) and (5):

$$E_z = (A_m \sinh q_0 x + B_m \cosh q_0 x) e^{-j\beta z}, \tag{3}$$

where $p^2 = \omega^2 \epsilon_1 \mu - \beta^2$, $q_0^2 = \beta^2 - \omega^2 \epsilon_0 \mu$ and m is the region according to Fig. 1. In the following, we omit the term $e^{-j\beta z}$ which is just a multiplicative factor to all equations.

As the longitudinal E-field (E_z) and the transversal H-field (H_y) must be continous through the interfaces between the different layers, equating the fields at each side of each boundary yields a set of equations from which it is possible to determine the dispersion relation and the fields, as shown in [3]. The fields can be expressed as:

$$\begin{bmatrix} E_z^{(1)} \\ jH_y^{(1)} \end{bmatrix} = \begin{bmatrix} e^{-q_0 x} & 0 \\ 0 & -\dfrac{\omega\epsilon_0}{q_0} e^{-q_0 x} \end{bmatrix} \begin{bmatrix} A_1 \\ A_1 \end{bmatrix} = M_1(x) \begin{bmatrix} A_1 \\ A_1 \end{bmatrix}, \tag{4}$$

$$\begin{bmatrix} E_z^{(7)} \\ jH_y^{(7)} \end{bmatrix} = \begin{bmatrix} e^{q_0 x} & 0 \\ 0 & -\dfrac{\omega\epsilon_0}{q_0} e^{q_0 x} \end{bmatrix} \begin{bmatrix} A_7 \\ A_7 \end{bmatrix} = M_7(x) \begin{bmatrix} A_7 \\ A_7 \end{bmatrix}, \tag{5}$$

$$\begin{bmatrix} E_z^{(m)} \\ jH_y^{(m)} \end{bmatrix} = \begin{bmatrix} \sin px & -\dfrac{\omega\epsilon_1}{p}\cos px \\ \dfrac{\omega\epsilon_1}{p}\sin px & \cos px \end{bmatrix} \begin{bmatrix} A_m \\ B_m \end{bmatrix} = M_m(x) \begin{bmatrix} A_m \\ B_m \end{bmatrix} \tag{6}$$

at $m = (2), (4), (6)$ and

$$\begin{bmatrix} E_z^{(m)} \\ jH_y^{(m)} \end{bmatrix} = \begin{bmatrix} \sinh q_0 x & \dfrac{\omega\epsilon_0}{q_0}\cosh q_0 x \\ \dfrac{\omega\epsilon_0}{q_0}\sinh q_0 x & \cosh q_0 x \end{bmatrix} \begin{bmatrix} A_m \\ B_m \end{bmatrix} = M_m(x) \begin{bmatrix} A_m \\ B_m \end{bmatrix} \tag{7}$$

at $m = (3), (5)$.

Subsequently, expressing the constants A_m and B_m in terms of A_{m-1} and B_{m-1} at the boundaries of each region, we obtain the expression

$$\begin{bmatrix} e^{-q_0 x_1} & -(N_{11} + N_{12}) \\ -\omega\epsilon_0/q_0 e^{-q_0 x_1} & -(N_{21} + N_{22}) \end{bmatrix} \begin{bmatrix} A_1 \\ A_7 \end{bmatrix} = 0. \tag{8}$$

Where

$$N = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} = M_2(x_1) M_2^{-1}(x_2) M_3(x_2) \ldots M_6^{-1}(x_6) M_7(x_6) \tag{9}$$

and x_m is the coordinate of the interface between the regions (m) and $(m+1)$.

The dispersion relation can be obtianed from the zeroes of the determinant of eq. (8). This is the only part where a numerical solution is required within this analysis.

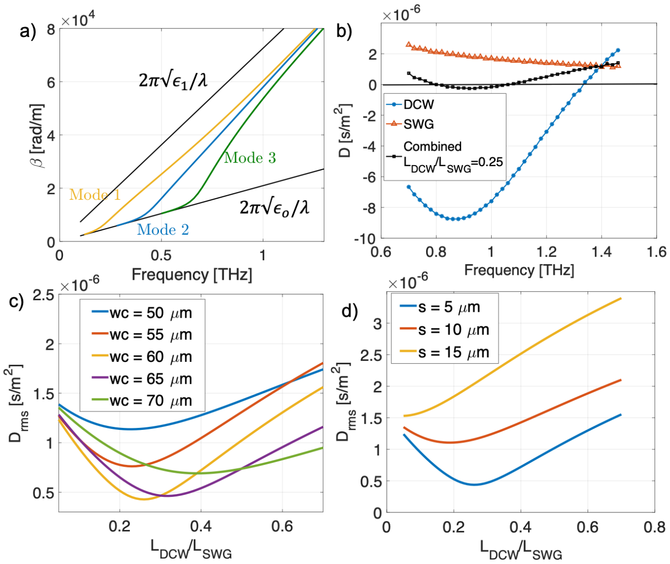

Fig. 2. a) Dispersion relation obtained analytically for the first three modes of the DCW with $w_c = 60\ \mu m$, $s = 5\ \mu m$ and $w = 200\ \mu m$. b) Dispersion (D) for the DCW, SWG and the combination of both given $L_{DCW}/L_{SWG} = 0.25$. The dimensions of the DCW waveguides are $w_c = 60\ \mu m$, $w = 200\ \mu m$. The SWG has a width of $w = 200\ \mu m$ and $\epsilon_1 = 12\epsilon_0$. c) Analytical rms dispersion for different sizes of the central slab (w_c), with $s = 5\ \mu m$. d) Analytical rms dispersion for different slit widths (s), with $w_c = 60\ \mu m$.

The resulting dispersion diagram for the first three modes is shown in Fig. 2(a) with $w_c = 60\ \mu m$, $s = 5\ \mu m$ $w = 200\ \mu m$ and $\epsilon_1 = 12\epsilon_0$.

II. Broadband Dispersion Compensation

Using the same approach as in [2], the dispersion relation for the fundamental TM mode of a SWG with width $w = 200\ \mu m$ and $\epsilon_1 = 12\epsilon_0$ is calculated. As the material is considered dispersionless, dispersion of the SWG and DCW can be identified by $D_{DCW} = -\frac{2\pi C_0}{\lambda^2}\frac{\partial^2 \beta_{DCW}}{\partial \omega^2}$ and $D_{SWG} = -\frac{2\pi C_0}{\lambda^2}\frac{\partial^2 \beta_{SWG}}{\partial \omega^2}$. Fig. 2(b) shows that the dispersion of both waveguide topologies have opposite sign, with the dispersion of the DCW being much higher than the SWG, thus allowing for dispersion compensation using a DCW with much shorter length than the dispersive slab.

The root-mean-square dispersion over a target frequency range is a good measure for the accumulated dispersion of the structure formed by the two concatenated waveguides. Assuming that the effect of the transition is negligible, the rms dispersion is then

$$D_{rms} = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}[D_{DCW}(f_i)L_{DCW} + D_{SWG}(f_i)L_{SWG}]^2}}{L_{DCW} + L_{SWG}} \qquad (10)$$

where L_{SWG} and L_{DCW} are the lengths of each segment and the frequencies f_i are summed over the (discretized) target frequency range, here 900 GHz-1300 GHz. Fig. 2(c) shows the optimization of the central slab width for a constant slit size of $s = 5\ \mu m$ that reaches its minimum at $w_c = 60\ \mu m$ for a length of the DCW of approximately ¼ of the SWG. Fig.2(d) shows the reduction of rms dispersion with the slit width, s, for fixed $w_c = 60\ \mu m$ and $w = 200\ \mu m$. In all cases, the dimension of

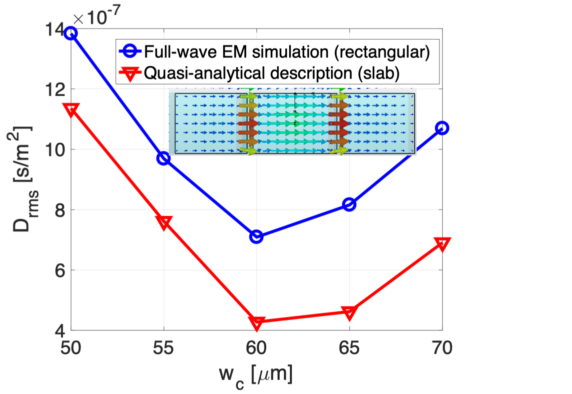

Fig. 3. Comparison between the full-wave EM simulation of the dispersion compensation of a double-slit waveguide shape with $s = 5\ \mu m$, $w = 200\ \mu m$ and a height of 50 μm with the analytical analysis of a planar DCW. The plotted graphs depict the lowest dispersion achieved for a given slab width (w_c), which may correspond to different lengths of the DCW. Inset: section of the EM simulation with the E-field of the fundamental mode at a frequency of 1.1 THz.

the slab matches with the outer dimension of the DCW (200 μm) and the permittivity of the dielectric is set to $\epsilon_1 = 12\epsilon_0$.

III. Comparison to Full-Wave EM Simulation

A similar result is obtined when β is calculated by means of an EM simulation, but now for finite dimension in the y-direction, i.e. a height of 50 μm. Fig. 3 shows that the waveguide dispersion is optimized for the same size of the central slab, $w_c = 60\mu m$. This proves that the simple analytical solution can provide the correct optimization result for dispersion compensation, despite neglecting the third dimension. The higher absolute value of the dispersion at the EM simulation is given by the finite height of this waveguide.

IV. Summary

We have demonstrated dispersion compensation of a dielectric waveguide using a compensating dielectric waveguide with two symmetrical slits through an analytical 2D approach, neglecting the vertical extension of the waveguide. We have confirmed that the approach is able to predict the correct compensation geometry even for a 3D structure by a numerical EM simulation.

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